Introduction to the Course:

- **What is Finance 440 and who should take it?**
- **Who am I? – Who are you?**
- **First Stop: The course web page:**
  www.kellogg.nwu.edu/faculty/eberly/htm/d40
  - Announcements?
  - Lecture outlines
  - Problem sets
  - Supplementary materials

**Course Structure**

- **Required readings:**
  Brealey-Myers, 7th edition
  Course notes and articles in case pack

- **Class meetings: lecture & discussion**
  Be prepared
  Questions go both ways

- **Absorbing the material**
  Weekly Problems & Review Sessions
  Recommended problems
  Problem Sets (required)
  Study groups
  Office hours & Appointments

- **Evaluating your progress**
  6 Cases/Homeworks: each 5% of the grade
  Participation (class & groups): 5% of the grade
  Midterm: Take home, 25% of the grade
  The Midterm is a free option
  Final Exam: Cumulative, 38% of the grade
  Final grade distribution
Course Philosophy and Expectations

- Theory and Empirics
- Using models
- What is expected of you?
  - Preparation for class: we will move quickly
  - Class participation
  - Making up missed work
  - Classroom etiquette
  - Strict application of the Honor Code
- What can you expect of me?
  - Lecture outlines (on the Web page)
  - Address questions (in or out of class)
    Email: eberly@nwu.edu
    Phone: 847-467-1840; Fax: 847-491-5719
  - A Quantitative course

Introduction to Finance

I. Valuation and Investment Decisions

*Big Question: What projects should the firm adopt?*

Tool Building:
- present value techniques
- portfolio theory
- capital asset pricing model (CAPM)
- option pricing techniques
- the yield curve

II. Financing Decisions

*Big Questions: How should a firm's projects be financed? How should cash be distributed?*

Framework for decision-making:
- efficient market theory
- capital structure theory and practice
- payout policy theory and practice
- WACC and adjusted present value techniques
Goals for Topic 1

Understand present and future value concepts and their application to a variety of valuation problems.

Outline of Topics

• The NPV Rule
• Present Value Concepts
  o Future value, present value
  o Annuity values, perpetuity values
• Applications
• Bonds and Bond Pricing
  o Types of bonds
  o How interest rates are quoted
  o Application: mortgages
I. The NPV Rule

How do we go about answering the first fundamental question, “What projects should the firm take on?”

\[ \text{The firm should invest in a project if taking the project returns a higher value than the costs of undertaking the project.} \]

That is, a manager should choose projects with a positive net present value:

\[ \text{Net Present Value (NPV) = Project Value - Project Cost} \]

A. Benefits of The NPV Rule:

1. Projects are accepted if they increase shareholder value (i.e., they increase the price in the WSJ)
2. Allows separation of ownership and control.
3. Simplicity

B. Estimating net present value requires valuing cash flows:

1. arriving at different points in the future
2. with different degrees of uncertainty or risk

Accounting for these two problems provides the framework for determining value.

Thus, finance can be said to be the study of the effect of time and uncertainty on value.
II. Present Value Concepts

A. Fundamental Intuition: The time value of money

A dollar today is worth more than a dollar next year because it can earn interest.

**Example 1-1.** Invest $3.00 for one period at 10% interest.

You earn $0.30 in interest, producing $3.30 total.

\[(1+0.10)\times3.00 = 3.30\]

Invest $3.00 for two periods at 10% per period. At the end of the first period, you have $3.30, which then earns 10% interest.

So you have \((1+0.10)\times3.30 = (1+0.10)^2\times3.00 = 3.63\)

Invest $3.00 for three periods at 10% interest per period, you get

\[(1.10)^3\times3.00 = 3.99\]

**Notation:** \((1+r)^n = F/P(r,n)\)

"Future Value Factor"

B. Present Value

Rather than asking how much we get from investing, we can ask how much we need to invest to get a certain amount. This is the idea of present value.

**Example 1-2:** How much do we need to invest now at 10% to have $3.30 at the end of one period?

We already know the answer is $3.00; but if we didn't, we'd solve for it this way:

\[(1.10)x = 3.30\]

or

\[x = 3.30/(1.10) = 3.00\]

Thus, $3.00 is the present value of $3.30 at “time 1” when the interest rate is 10%.

Similarly, $3.00 is the present value of $3.99 at “time 3” when the interest rate is 10%, which we could compute as:

\[PV = 3.99/(1.10)^3 = 3.00\]

**Notation:** \(1/(1+r)^n = P/F(r,n)\)

"Present Value Factor"
C. Value Additivity

The present value of a stream of future cash flows is simply the sum of the present values of the individual cash flows.

Consider the stream of future cash flows:

$3.30 at time 1, 3.3, 3.63, 3.99$
$3.63 at time 2, 0, 1, 2, 3$
$3.99 at time 3, $

The present value of this stream (at, say 10%) is the amount you need to invest today to produce the stream.

We already know that an investment of $3.00 produces each of the components of the stream.

So to produce the entire stream would require an investment of $9.00, which is the present value of the stream of cashflows.

In general,

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_T}{(1+r)^T}$$

D. Perpetuity and Annuity Formulas

These are very useful short-cut formulas to use when cashflows are all the same or grow at a constant rate.

A perpetuity is an infinite stream of identical cash flows.

Using the formula for the sum of a geometric series, it turns out that:

$$PV(\text{perpetuity}) = \frac{C}{r}$$

(It is important to note that this formula is correct only if the first payment arrives at time 1.)

A growing perpetuity is an infinite stream of cash flows that grow at a constant rate.

The cash flow stream is: $C, (1+g)C, (1+g)^2C, (1+g)^3C, \ldots$ with the first cash flow arriving at time 1.

$$PV(\text{growing perpetuity}) = \frac{C}{r-g}$$

An annuity is a finite stream of constant payments.
If the first cash flow, $C$, arrives at time 1 and the last arrives at time $T$, the present value of an annuity is

$$PV(\text{annuity}) = \frac{C}{r} \left(1 - \frac{1}{(1+r)^T} \right)$$

Note that the cash flow stream to an annuity can be constructed by taking the stream to a perpetuity that starts at time 1, and subtracting off the stream to a perpetuity that starts at time $T+1$.

$$\frac{C}{r} - \frac{C/r}{(1+r)^T} = \frac{C}{r} \left(1 - \frac{1}{(1+r)^T} \right) = C \times \frac{1}{P/C(r,T)}$$

**Notation:**

$$\frac{1}{r} \left(1 - \frac{1}{(1+r)^T} \right) = P/A(r,n)$$

“Annuity Present Value Factor”

The formula for a **growing annuity** is in your notes.

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**Example 1-3:** What is the present value of $5,000 per year, starting in one year, for 7 years? Assume $r = 11.75\%$.

Solve for: $5,000 \times P/A(11.75, 7) =

$$\frac{5000}{\frac{1}{.1175}} \left(1 - \frac{1}{\left(\frac{1}{.1175}\right)^7} \right) = $23,000.82$$

**Example 1-4:** What is the future value in year 7?

$5,000 \times F/A(11.75, 7) =

$$\frac{5000}{\frac{1}{.1175}} \left(\frac{1}{\left(\frac{1}{.1175}\right)^7}\right) = $50,058.30$$

= $5,000 \times P/A(11.75\%, 7) \times F/P(11.75\%, 7)$
E. Methods of Calculation

1. A Universal Financial Calculator

5 buttons:

- **n**: number of payments
- **i**: interest rate per payment period
- **PV**: (-) present value or principal
- **PMT**: constant periodic payment amount
- **FV**: future value or face value

The calculator solves the equation:

\[ 0 = PV + \frac{PMT}{(1+i)} + \frac{PMT}{(1+i)^2} + \ldots + \frac{PMT}{(1+i)^N} + \frac{FV}{(1+i)^N} \]

**Warning:** To avoid inadvertent errors, set your calculator to 1 payment per year. If there are multiple periods in a year, adjust the interest rate and payment to the amount per period, and enter the number of periods for **N**.

2. Spreadsheets

The financial functions in all standard spreadsheets (e.g., Lotus, Excel, Quattro) work on the same principle:

- `@PV(n, i, pmt, fv)` gives the present value
- `@FV(n, i, pmt, pv)` gives the future value, etc.

3. Annuity Examples Revisited: Using A Financial Calculator

**Example 1-5:** PV of $5,000 per year for 7 years @ r=11.75%?

- **FV** = 0
- **PMT** = annuity amount = 5000
- **N** = 7
- **i** = 11.75%
- **PV** = ?

\((-23,000.82)\)

**Example 1-6:** What is P/A(11.75%, 7)?

- **FV** = 0
- **PMT** = annuity amount = 1
- **N** = 7
- **i** = 11.75%
- **PV** = ?

\((-4.6)\)

Notice that 4.6($5,000) = $23,000
III. Applications

A. Planning for Retirement

Suppose it is your 25th birthday.

You will be able to save for the next 25 years, until age 50 (payments on birthdays 26 through 50, 25 in all).

Between age 50 to 60, your income will just cover your expenses.

Finally, you expect to retire at age 60 and live until age 80 (20 years of retirement).

Suppose that you want to guarantee yourself a supplemental income of $30,000 per year after retirement (withdrawals on birthdays 61 to 80, 20 in all).

Assume that the interest rate is \( r = 12\% \).

How much should you put away every year, for the next 25 years, starting at the end of this year?

Work backwards:

1. How much will you need at age 60?

2. How much will you need at age 50?

3. How much must you put away, every year for 25 years?
Changing Interest Rates

Suppose that we know that:

- the interest rate one year rate for the next year will be $r_1$,
- the one year interest rate for the following year will be $r_2$.

Then the present value of a future sum two years out is:

$$PV = \frac{F}{(1 + r_1)(1 + r_2)}$$

In general, for a cash flow $n$ periods out

$$PV = \frac{F}{(1 + r_1)\cdots(1 + r_n)}.$$ 

We will return to this issue when we discuss the yield curve. For now we will assume that interest rates are constant.

The Effect of Personal Taxes

Taxes are an expense, and so must be subtracted from revenues to find cashflows.

The effect of taxes must also be taken into account in the discount rate:

This is because taxes affect the opportunity cost of an investment, since interest income on most bonds is taxable at the tax rate on ordinary income.

For instance, if your marginal personal tax rate is 35% and you receive an interest payment of $100, you will have to pay $35 in taxes, leaving only $65.

The after tax interest rate = $r \cdot (1 - T)$, where $r$ is the return before taxes and $T$ is your tax rate.

*The general rule is: discount after-personal-tax cashflows at an after-personal-tax discount rate.*

This rule is important for personal finance applications. The valuation of corporate cashflows, however, usually abstracts from personal taxes.
B. Taxable Lottery Winnings.

The before-tax interest rate is 10% per year.

You win the lottery which either:

a) pays you income of $120,000 at the end of each year for 20 years or

b) pays you a lump sum of $1,000,000 now.

Either way, your winnings are taxable and your tax rate is 30%.
Do you choose (a) or (b)?

Say you expect to retire after 10 years, and that you expect your marginal tax rate to fall to 15%. How does this change your calculation?

Option (a)

Option (b)
What happens if your tax rate changes when you retire?

Nominal vs. Real Interest Rates: The Effect of Inflation

The nominal interest rate is the rate that you see in the market.
The real interest rate is the rate that would be found if there were no inflation.

What is the relationship between nominal and real interest rates?
Suppose there is no inflation and the interest rate is 10%.

I am willing to give up $100 today in return for $110 next year.
Now, suppose that the inflation rates jumps to 5%.

_My $110 is going to be worth 5% less in terms of purchasing power._

Therefore I require 10% to compensate me for the _time value of money_.

and also an additional 5% to compensate me for the _loss in purchasing power_.

In this example, the _real_ interest rate is 10%, while the nominal interest rate, with 5% inflation is

\[
(1 + .1)(1 + .05) - 1 = \\
.1 + .05 + .05(.1) = .155 = 15.5\%
\]

In general, if

- \( r_r \) = real interest rate
- \( r_n \) = nominal interest rate
- \( E(i) \) = expected inflation rate

Then

\[
r_n = (1+r_r)(1+E(i)) - 1
\]

The _market or nominal_ interest rate takes into account inflationary _expectations_ and therefore the loss in value of the dollar in the future.
Example: Inflation and Future Value

You want to put away enough money today to guarantee yourself $100,000 in 30 years. You want this $100,000 to be in terms of today's purchasing power.

The nominal (market) interest rate is 8%.
The inflation rate is known to be 5%.

1. Find out how much $100,000 in today's purchasing power will be in actual (nominal) dollars in 30 years:

2. Find out how much we need to put away today in order to achieve this FV in 30 years:

An alternative way to look at this problem is to consider what would happen if there were no inflation.

(Verify that this is true: redo the previous problem using the exact real interest rate and make sure that you get the same result in the last step)

This case is an example of an …

Important General Rule:
Always discount nominal cashflows at a nominal interest rate, and real cashflows at a real interest rate.
IV. Bonds and Bond Pricing

Asset prices are present values.

We will use this insight to look at valuation models for bonds and stocks, beginning with bonds.

- Bonds promise a set of fixed future payments.
- The value of a bond, and its market price, equal the present value of these promised payments.
- Bond holders generally have no control rights except in bankruptcy.
- Bonds are a type of debt (make a loan = buy a bond, get a loan = sell a bond).

A. Types of Bonds

1. Coupon Bonds

- constant fixed coupon (interest) payment, quoted as % of face value
- lump-sum return of face value at maturity

\[
\begin{array}{ccccccc}
-P & cF & cF & cF & cF & cF & cF+F \\
0 & 1 & 2 & 3 & 4 & 5 & \\
\end{array}
\]

- \(c\) = coupon rate
- \(F\) = face value
- \(P\) = price

- price = PV of payments
- issue is often at par = face value
- common frequency for coupons is annual or semiannual
The Return on a Coupon Bond

\[ \text{total return} = \frac{P_t - P_0 + cF}{P_0} \]

this has two components:

\[ \% \text{ capital gain} = \frac{P_t - P_0}{P_0} \]

\[ \text{current yield} = \frac{cF}{P_0} \]

Investors care about total return.

Bond prices adjust so that expected total returns are similar across bonds with similar risk, maturity, and liquidity.

2. Floating Rate Bonds

- multi-period bonds
- rate is reset each period based on rate index (e.g., LIBOR, Treasury)
- growing popularity

Example: 3 year floating rate bond at 1 year, LIBOR+1%

<table>
<thead>
<tr>
<th>-P</th>
<th>P(LIBOR_0+1%)</th>
<th>P(LIBOR_1+1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

3. Fixed Payment Bonds

- constant payments, gradual amortization
- e.g., mortgage loans, auto loans
B. How Interest Rates are Quoted

1. Simple Interest

With simple interest, one simply applies the interest rate to the initial balance. Thus, no interest is earned on the interest paid (no compounding).

Example 1-7: Simple Interest
Suppose that you invest $100 for two years, with simple interest of 10% being paid.

Interest in Year 1 = $100 x .10 = $10
Interest in Year 2 = $100 x .10 = $10

Initial Balance = $100
Total at the end of two years = $120

2. Compound Interest

Often an annual rate will be compounded, but with a compounding interval of less than a year.

Example 1-8: Compound Interest
If I earn an 8% rate that is compounded semi-annually, then my dollar will grow so that at the end of the year it will be

$1(1 + r/2)^2 = 1(1.04)^2 = 1.0816$

If I compound the 8% every month, then my dollar will grow so that at the end of the year it will be

$1(1 + r/12)^{12} = 1(1.00667)^{12} = 1.0830$

If I compound the 8% every day, then my dollar will grow so that at the end of the year it will be

$1(1 + r/365)^{365} = 1(1.00002192)^{365} = 1.0833$

In general, the formula for total interest earned in a year is

$1(1 + r/k)^k$

k = number of compounding intervals per year

For t years, this becomes $1(1 + r/k)^{tk}$
Example 1-9: Using the General Formula
Suppose that you wish to invest $200 for 20 years at 12%, with monthly compounding. Then, at the end of 20 years you would have:

$200 \left(F/P, r/k, tk\right) = $200 \left(F/P, 1\%, 240\right)

= $200(1.01)^{240} = $2178.51

Continuous Compounding

Notice that in every case, the amount that earned goes up with the number of compounding intervals. There is a limit to how much the dollar can grow for a given interest rate. It is given by

\[ FV = 1e^{rt} \]

\[ t = \text{the number of years} \]
\[ e = 2.718281828. \]

Example 1-10: Continuous Compounding
Suppose that a bank offers you an 8% interest rate compounded continuously. If you invest $1 for one year, at the end of the year you will have:

\[ 1e^{0.08} = 1.0833 \]

If you invest the $1 for 10 years, you would have

\[ 1e^{0.08(10)} = 1e^{0.8} = 1 x 2.226 = 2.226 \]

Similarly the present value for continuous compounding is given by

\[ PV = 1e^{-rt} = 1/e^t \]
3. Effective Annual Rates

The quoted interest rate is known as the annual percentage rate (APR) or because it is given on an annual basis.

The effective annual rate (EAR) takes into account the compounding and asks what rate is earned over the entire year. The rate, \( r' \), is given by

\[
1 + r' = (1 + r/k)^k
\]

Example 1-11: Computing Effective Annual Rates

We computed the amount earned on a dollar invested with various rates of compounding for an A.P.R. of 8%:

<table>
<thead>
<tr>
<th>Compound Intervals</th>
<th>Final Sum</th>
<th>EAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.0800</td>
<td>8.00%</td>
</tr>
<tr>
<td>2</td>
<td>$1.0816</td>
<td>8.16%</td>
</tr>
<tr>
<td>12</td>
<td>$1.0830</td>
<td>8.30%</td>
</tr>
<tr>
<td>365</td>
<td>$1.0833</td>
<td>8.33%</td>
</tr>
<tr>
<td>( \infty )</td>
<td>$1.0833</td>
<td>8.33%</td>
</tr>
</tbody>
</table>

So, the effective annual yield goes up, given the Annual Percentage Rate, as the number of compounding periods increase.

C. Application: Mortgage Calculations

You borrow $150,000 to buy a house. Your mortgage rate is 9% per year (.75% per month) and the term is 30 years. Ignore taxes.

(i) What is your monthly mortgage payment?

(ii) What portion of the first payment is interest?
What portion is principal?

(iii) After 5 years of payments, mortgage rates drop to 6% per year.

Suppose you plan on living in the house for 4 more years at which time you will sell the house and pay off the mortgage.

How much in fees would you be willing to pay to refinance the remaining balance on your mortgage at the new lower rate of 6% per year?

Assume that the term of your new mortgage would be 30 years.
(i) A standard mortgage loan is an annuity with a present value equal to the amount borrowed. The monthly payment is:

(ii) The interest paid in the first month

The principal paid is:

Thus the principal balance going into the second month is:

(iii) This is the time line for your mortgage decision:

\[
\begin{array}{ccccccc}
0 & 5 & 9 & \ldots & 30 & 35 \\
\text{old mortgage} & \text{new mortgage} & \text{(sell house)}
\end{array}
\]

Deciding whether or not to refinance requires several steps:

**Step 1:** Find the remaining principal today (which is the payoff value of the old mortgage):

This is due to the rule:

*The payoff value of a loan is the present value of the remaining payments, discounted at the stated interest rate on the loan.*

*(Assuming no prepayments were made.)*

**Step 2:** Find the monthly payment under the new mortgage:

**Step 3:** Calculate the present value of future cashflows over the next four years in both cases and compare.
Refinance:

Don't Refinance:

Difference:

V. Summary
Key Concepts:

THE TIME VALUE OF MONEY: A DOLLAR TODAY IS WORTH MORE THAN A DOLLAR NEXT YEAR BECAUSE IT CAN EARN INTEREST.

ASSET PRICES ARE PRESENT VALUES.

Definitions:

NET PRESENT VALUE (NPV) = PROJECT VALUE - PROJECT COST

A PERPETUITY IS AN INFINITE STREAM OF IDENTICAL CASH FLOWS,

A GROWING PERPETUITY IS AN INFINITE STREAM OF CASH FLOWS THAT GROW AT A CONSTANT RATE.

AN ANNUITY IS A FINITE STREAM OF CONSTANT PAYMENTS

Notation:

\[(1+r)^n = F/P(r,n), \quad \text{"Future Value Factor"}\]

\[1/(1+r)^n = P/F(r,n), \quad \text{"Present Value Factor"}\]

\[1 \left(\frac{1}{(1+r)^n}\right) = P/A(r,n), \quad \text{"Annuity Present Value Factor"}\]