The Nonequivalence of the Dividend and Earnings Approaches to Equity Valuation

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Abstract

Accounting theory treats a wide class of equity valuation approaches as equivalent. For example, under clean surplus accounting, the earnings approach is viewed as identical to the discounted dividends approach. Empirical research, however, typically finds that the two valuation approaches do not predict market prices equally well.

This paper offers a theoretical explanation for this apparent anomaly, by modeling agents who imperfectly perceive a firm’s prospects and comparing the earnings-based valuation with the dividends-based valuation. Clean surplus always holds, but because of the perceptual limits, the models have too many degrees of freedom to remain equivalent.

The differences in valuation do not always show up in market prices. This provides an explanation for two additional empirical puzzles. First, market data may mask the information content of dividends. Second, the market may appear to react more strongly to dividend announcements than to initiations.

Key Terms: Equity Valuation, Residual Income, Dividends, Perceptions.

JEL Classifications: M41, D82, C65

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1 Introduction

Accounting is the language of business. As with other languages, it contains messages that are not about the objective state of the world, but about the world as individuals subjectively perceive it. In a setting with subjective information, the best a shared language can achieve is communication of approximate information—what one agent perceives as he or she believes another might understand it. This idea of a need for approximate reporting is captured in practice with aggregation of information and with notions of materiality, and is also seen in the theoretical accounting literature (e.g., Dikolli and Vaysman (2003) or Kanodia, Singh, and Spero (2004)).

This paper investigates one such setting, namely the earnings and dividends approaches to equity valuation.\(^1\) I assume clean surplus accounting, but allow for cases where agents only have imperfect perceptions of accounting income and of the premium of fundamental value over book value. The firm’s current owners refine their perceptions of these quantities during an accounting period, and then report to the market in an approximate way what they have learned, either by issuing an income statement or by making a dividend announcement.

When both income and the premium over book value are known and can be stated exactly, the two approaches to valuation have long been known to be equivalent. Once the reports reflect approximate information, however, this equivalence no longer holds. The reason is that, with approximate reports, valuations are restricted to intervals, rather than emerging as exact points. The intervals obtained from the dividends and earnings approach overlap, but in general have different upper and lower bounds. Roughly speaking, the approximate nature of the reports introduces additional degrees of freedom, which clean surplus accounting no longer suffices to resolve.

For related reasons, equilibrium prices also emerge as intervals. That is, any realization in an equilibrium interval is a possible equilibrium price.\(^2\) However, the set of possible equilibrium prices may be the same under the dividends and earnings approach, even when the two approaches justify

\(^1\)Dickhaut and Eggleton (1975) investigate the converse question, namely, how accounting language affects perception. The general question of how language shapes perception is addressed in Korzybski (1994). Ahn (2000) discusses the context-sensitive nature of perception; see also Kamp (1981) and, for philosophical foundations, Husserl (1913).

\(^2\)This roughly corresponds to the notion of rationalizability in Pearce (1984). I am indebted to Ed Green for this observation.
different valuation intervals. The reason is that the current owner’s valuation determines the lower bound on equilibrium prices, while the market’s valuation determines the upper bound. If the models differ only on the market’s lower bound, and both these values are weakly below the current owner’s private lower bound, then no difference will show up in the set of possible prices.

I find in particular that a dividend omission can refine the market’s upper bound on valuation, and thus leads to observable effects on the set of possible prices. On the other hand, the introduction of a dividend or the payment of a planned dividend in general will not show up in market data. Thus two puzzles about dividends are resolved: first, the asymmetric market response to dividend omissions versus initiations (Healy and Palepu (1988) and Michaely, Thaler, and Womack (1995)) is a consequence of the omissions’ lowering the market’s upper bound on valuation. Secondly, the apparent low information content of dividends (Watts (1973) and Watts (1976)) follows from the fact that dividend introductions affect the market’s lower bounds.

The organization of the rest of this paper is as follows: the next section reviews the literature on the earnings and dividend approaches. Section three discusses the argument for the equivalence of the two approaches under the assumption of clean surplus, and provides reasons that the argument can break down. Section four gives a model with one trading period in which earnings and the premium over book value are only approximately known. This section compares valuation based on an income statement with valuation based on a dividend announcement. Section five gives some results on the economic consequences of the results of the model. Section six concludes.

2 Background and Literature Review

The earnings approach to equity valuation goes back at least to the 1930s. Preinreich (1936) presents a valuation model based on a company’s original book value, its residual income, the market return on equity capital, and a final liquidating dividend. Elsewhere, Preinreich (1938) uses an analogous formula in the context of valuation of a productive asset, arguing that the asset’s value is its book value plus discounted residual income, which he calls “excess profits.”

Preinreich also discusses the theoretical equivalence between the earnings approach to valuation and
the dividends approach, which values an equity at the present value of future dividends. The latter is introduced in Fisher (1907), who expresses skepticism about the use of accounting information in valuation. In Preinreich (1936), the argument is summed up as follows:

Prof. Fisher devotes considerable space in his works to the proof that capital value can be obtained only by discounting “services” (which he calls “income”) and not by discounting “earnings.” That is true, although accountants have long been using an alternative method of computation, which is equally correct. Goodwill is commonly obtained by discounting “excess earnings.” If the original investment . . . is added to the goodwill, the same capital value results as from the discounting of “services.”

The above quotation is more appropriately applied to an infinite-horizon model, which appears in Preinreich (1932). In the finite-horizon case, the final value used in the earnings approach is the ending premium over book value (Preinreich’s notion of goodwill in the above quotation, related to the unrecorded goodwill in Feltham and Ohlson (1996)). By contrast, the dividends approach uses a terminal price or liquidating dividend as the final value. Ignoring this difference leads to spurious discrepancies between the two valuation formulae, as noted in Lundholm and O’Keefe (2001b).

More recent work (Feltham and Ohlson (1995), Ohlson (1995)) restates the theoretical equivalence of the dividends and earnings approaches to valuation, and also of an approach based on free cash flow. Yet despite the apparent consensus that each of these approaches ought to yield identical valuations, empirical researchers consistently obtain different results when using each approach, and argue that one approach outperforms another in its match with observed prices.

Empirical comparisons with market prices generally acknowledge the assumed theoretical equivalence of the dividends and earnings approaches. Abarbanell and Bernard (2000) argue that the earnings approach is more convenient to work with, and justify the approach by appealing to its equivalence to the discounted dividend model. Francis, Olsson, and Oswald (2000), on the other hand, argue against the equivalence of the approaches from a practitioner’s viewpoint:

In theory, the models yield identical estimates of intrinsic values; in practice, they will differ if the forecasted attributes, growth rates, or discount rates are inconsistent.
This statement corresponds to the view that a researcher may find one set of assumptions natural when working with the dividends approach, and may find entirely different assumptions natural under the earnings approach. For example, Frankel and Lee (1998) and Bradshaw (2004) assume that dividends follow a fixed policy (namely, matching recent dividend payouts); this adds information to a dividend model beyond what is in the actual dividend payouts, and may not lead to a reasonable assumption on terminal values for an analyst working with the residual income model.

Penman and Sougiannis (1998) take a similar position, arguing that the dividends and earnings approaches are equivalent in the infinite horizon model, but that, beyond a finite horizon, neither model is estimable. If a finite horizon is all that one has available, then the natural assumption for the user of either model is to set what cannot be estimated to zero. In the discounted dividends approach, this means assuming that the last estimated dividend is a liquidation; in the earnings approach, this means assuming that ending book value equals ending intrinsic value. Thus, the natural assumptions differ across approaches. Since terminal values deal with data that are not estimable, it is unsurprising that users of the different approaches will get different results.

Lundholm and O’Keefe (2001b) take issue with this viewpoint, arguing that whatever assumptions are made with one approach need to be made with any other in order to produce a fair horse race. The difficulty, however, is that the earnings approach only requires an estimate of how far book value can separate from intrinsic value, whereas the dividends approach requires an estimate of a terminal price. It is entirely possible that neither the terminal price nor the terminal book value can be estimated, yet that there is a stable relationship between price and book value. Nonetheless, the standard theoretical assumptions are that all the data in each model are well-defined and measurable. Therefore, Lundholm and O’Keefe argue that the cause of empirical differences across approaches must be violation of the theoretical assumptions (they emphasize dirty surplus) or inconsistencies in methodology.

Despite these debates, the theoretical equivalence of the infinite-horizon and of the properly adjusted finite-horizon approaches is not in dispute in the above works.\(^3\) I argue here instead that there are good theoretical reasons for the approaches to yield different results. To make this argument, I

\(^3\)However, the managerial literature views performance metrics based on realized cash flows as different from those based on realized residual income. See Anctil (1996), Arya, Baldenius, and Glover (1999), or Reichelstein (2000).
drop the assumption that all of the data necessary for both approaches are estimable; this matches
Preinreich’s original justification for introducing a finite horizon model. A firm may be able to
perceive its accounting earnings fairly accurately, yet may have only a vague sense of what its
premium over book value is. Equivalently, the firm may have no reliable estimate of earnings in the
remote future. In this case, an income statement provides the market with information that is close
to what the firm knows. Conversely, if the firm has a good sense of future changes in equity that
are not captured by accounting income, it can signal this information by setting target dividends.4

The theoretical argument here would therefore suggest that dividends contain information beyond
what is found in earnings statements. Watts (1973) argues that any such incremental information
in dividends over earnings is negligible. He uses estimated unexpected dividends to predict market
prices and obtains low coefficients. Note, however, that his methodology assumes firms have a target
dividend rate (as in Lintner (1956)); in the present setting, the specifying of a target rate is already
incremental information beyond earnings. Laub (1976) and Pettit (1976) both challenge Watts’
methodology, moreover, and argue that the incremental information in dividends is substantial.
This matches the findings in the Pettit (1972), who partitions the set of firms to identify settings
where dividends convey more information.

Other empirical work shows incremental information in dividends, particularly in the omissions of
expected dividends or decreases in dividends; examples are Grullon and Michaely (2002), Aharony
here gives an explanation for this result, as the incremental information in dividends is generally
unobservable with the initiation of a dividend but is observable when an expected dividend is
missed. That is, the information role of dividends is symmetric, but the ability to observe it is not.

Since I argue in this paper that the dividend and earnings approaches are theoretically different, I
now turn to a more precise review of the argument for their equivalence, and a discussion of how
the argument can collapse.

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4This signaling role of dividends is similar to the ideas in Bhattacharya (1979), Miller and Rock (1985), and John
and Williams (1985). For discussion, see Allen and Michaely (2003).
3 The Residual Income and Dividends Models

3.1 Derivation of the Models and the Equivalence Argument

The argument that the earnings and dividends valuation approaches are equivalent begins with an assumption of clean surplus accounting. This says that the ending book value of shareholders’ equity equals its starting value plus income less dividends:

$$S_{t+1} = S_t + I_{t+1} - D_{t+1}.$$  

Here $S$ represents shareholders’ equity, $D$ represents dividends, and $I$ represents income. Letting $r$ represent the rate of return on equity capital, one can decompose income into normal income $rS_t$ and residual income $I_{t+1} - rS_t \equiv \rho_{t+1}S_t$. The clean surplus equation can then be written as

$$S_{t+1} = (1 + r + \rho_{t+1})S_t - D_{t+1}. \quad (1)$$

The discounted dividends model says that, in period $t$, the value of the equity is the present value of the expected future dividend stream:

$$P_t = \sum_{\tau=1}^{\infty} \frac{D_{t+\tau}}{(1+r)^\tau} = \frac{D_{t+1}}{1+r} + \frac{P_{t+1}}{1+r}. \quad (2)$$

The earnings approach, or residual income model, says that the period $t$ value of the equity is the book value of shareholders’ equity plus the present value of the expected future residual income stream:

$$P_t = S_t + \sum_{\tau=1}^{\infty} \frac{\rho_{t+\tau}S_{t+\tau-1}}{(1+r)^\tau} = S_t + \frac{\rho_{t+1}S_t}{1+r} + \frac{P_{t+1} - S_{t+1}}{1+r}. \quad (3)$$

Subtracting the book value of shareholders’ equity $S_t$ from both sides of Equation 3 gives an interpretation of this model: the current premium of intrinsic value over book value equals the discounted value of next period’s residual income plus the discounted value of next period’s premium.

The first part of the equivalence argument, due to Preinreich (1932), is then as follows:

**Proposition 3.1.** Given the clean surplus equation, the residual income model is derivable from the dividend model.
Proof. Equation 2 can be rewritten as

\[ P_t = \frac{D_{t+1}}{1+r} + \frac{S_{t+1} + P_{t+1} - S_{t+1}}{1+r}. \]

Plugging in the clean surplus equation (1) then gives

\[ P_t = \frac{D_{t+1}}{1+r} + \frac{(1+r)S_t}{1+r} + \frac{\rho_{t+1}S_t}{1+r} - \frac{D_{t+1}}{1+r} + \frac{P_{t+1} - S_{t+1}}{1+r}, \]

so that

\[ P_t = S_t + \frac{\rho_{t+1}S_t}{1+r} + \frac{P_{t+1} - S_{t+1}}{1+r}, \]

which is just the residual income model as stated in Equation 3.

Notice that the dividend at date \( t + 1 \) drops out of the present value calculation when written in the form of Equation 3. This is a form of dividend-irrelevance; see Miller and Modigliani (1961).

Conversely, it is straightforward to show the rest of the equivalence argument:

**Proposition 3.2.** Given the clean surplus equation, the dividend model is derivable from the residual income model.

*Proof.* Analogous to the proof of Proposition 3.1

### 3.2 Where the Equivalence of the Models Breaks Down

Both the residual income and discounted dividends models depend on information being estimable infinitely far into the future. For the residual income model, this requires meaningful definitions of purchases by customers who have not yet been born and product offerings by competitors that do not yet exist. For the dividends model, this requires meaningful beliefs about decisions of Boards of Directors that have not yet been appointed.\(^5\)

Relaxing these assumptions can mean that the data used in one approach are more informative than the data used in the other. To see this, note that the clean surplus equation (1) says that

\(^5\)This is related to the idea that the state space is known to all agents. See Stecher (2004) and Karni (2004).
ending book value of shareholders’ equity equals starting book value plus income less distributions. Solving this for income gives
\[ I_{t+1} = S_{t+1} - S_t + D_{t+1}, \]
which says that income equals the change in book value plus distributions. If income is known, then clean surplus provides one equation in two unknowns (change in book value and distributions). As long as the Board of Directors has not chosen next period’s dividend, knowledge of income does not provide any information on dividends or on the change in book value of equity. Thus, without violating clean surplus, there can be information available for use in the earnings approach without there being information for use in the dividends approach.

Conversely, the Board of Directors may set a dividend policy of paying a given amount to shareholders next period, irrespective of what income may be—for example, the Board may choose to set dividends to 0. Again, the clean surplus equation provides only one equation in the two unknowns: the dividend policy means that all income is retained, but does not provide any information on the change in book value or on earnings.

Thus, the firm’s income stream and its dividend stream need not be estimable over the same horizons. Recognizing this difficulty, Preinreich (1932) makes the horizon an explicit part of the model. Beyond the horizon, all terms in the given sequence are viewed as undefined. The approach of Penman and Sougiannis is based on this idea: their interest is in valuation rules that do not depend on what happens after the finite horizon. Whether valuation rules should have this property of being insensitive to truncation underlies the debate between Penman (2001) and Lundholm and O’Keefe (2001a, 2001b). The next subsection discusses the mathematics of this issue in detail.

### 3.3 Choice Sequences and Brouwer’s Continuity Principle

The purpose of this subsection is to discuss the mathematical consequences of Preinreich’s notion of a finite horizon—that is, of considering infinite sequences whose terms are only defined finitely far into the future.

There is an extensive mathematical literature on sequences with incompletely defined terms, dating
back at least to Borel (1912), who considers sequences with terms chosen

\[ \ldots \text{either entirely arbitrarily, or [by] imposing some restrictions which leave some arbitrary-ness.}^{6} \]

Such sequences, called *choice sequences* (because their terms have free choice), play an important role in the intuitionist approach to constructive mathematics.\(^7\) While choice sequences are not discussed explicitly in the economics and accounting literature to the best of my knowledge, they correspond to what Preinreich and what Penman and Sougiannis describe in their models with finite horizons.

Whether a function of a choice sequence ought to depend entirely on an initial segment depends on whether there is more information about the sequence than its numerical values. Such information can come in the form of restrictions, stating what legal values for the terms in the sequence might be without necessarily specifying the terms exactly. For example, a company might have a rule written into its charter specifying a dividend policy (e.g., paying $1 per share any time a dividend is declared). This does not state the value of any future dividend, but it does force the dividend stream to be a binary sequence. Given an interest rate \( r \), this information is insufficient for deriving an exact present value by either the discounted dividends or earnings approach. Nevertheless, the dividends approach cannot assign a value to the share below 0 or above \( 1/r \). By contrast, the residual income formula cannot provide any bounds on the share’s possible values from the above data: the firm needs more than just a charter for the residual income formula to assign a valuation.

The above example illustrates that, despite the equivalence argument holding when all dividends and income terms are well-defined, the bounds on valuation can differ under each approach when some terms are unspecified. Moreover, it is clear that knowledge of a dividend policy can refine the possible valuations under the dividends approach beyond what can be said based on well-defined terms in the dividend stream. These points appear formally in the model of the following section.

Equivalent restrictions need not bind with equal force. Consider the company in the last example,

\(^6\)The translation of Borel’s quotation is from Troelstra and van Dalen (1988).

\(^7\)See Brouwer (1921) for an early example, Heyting (1956) and Dummett (2000) for an overview, and Brouwer (1907) for the original development of intuitionistic mathematics. For modern applications, see Spitters (2003).
with one minor change: rather than having the dividend policy written into the company’s charter, imagine that the Board of Directors has announced a policy of paying $1 per share every time a dividend is declared. The sequences of payments for the company will be identical in each case, as long as the Board of Directors does not change its policy. However, the policy is only a provisional restriction, which a future Board of Directors might change.

If at each date, the composition of the Board of Directors can only be known finitely far into the future, then a weaker form of the argument in Penman and Sougiannis and in Preinreich, called *Brouwer’s continuity principle*, seems appropriate. In the present context, Brouwer’s continuity principle says this: if a valuation can be calculated for every sequence, and if a sequence might be ill-specified beyond some horizon (say, beyond the $m$th term), then any other sequence that has the same terms in a finite horizon and the same restrictions on finitely many more terms (say $n$ more) should return the same valuation. In other words, one can peek beyond the horizon to see what restrictions can hold, but requiring restrictions at all terms is tantamount to mandating that all restrictions be definitive. In the extreme case of lawless sequences, where no policies are in place and only finitely many terms are well-defined, Brouwer’s continuity principle requires that only the finite initial segment of defined terms be used in a valuation formula.

In the context of valuation approaches, it is natural to view infinite dividend and income streams as choice sequences: this allows for terms to be perceptible finitely far into the future, and perhaps restricted but not fully perceptible beyond some horizon. Plugging sequences whose terms cannot be fully known into a valuation formula yields an open interval, rather than an exact valuation. The observed terms and the possibilities on the unknown terms can be used to provide a lower bound and upper bound on defensible valuations. Intuitively, sequence with a given initial segment can be thought of as an open neighborhood in an appropriate topology (see Calude (2003) or Vickers (1988) for a rigorous treatment). The valuation of such sequences captures Brouwer’s notion of continuity, in the sense that the formulae become mappings that carry open sets to open sets; see Gebellato and Sambin (2001) for discussion.

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8Originally in handwritten lecture notes from 1917, and subsequently published in (Brouwer 1918, Brouwer 1927).

Note that if valuations are thought of as reservation prices, then the setting where dividend and income streams are choice sequences only provides a reservation price interval. At prices below the lower bound on this interval, a seller will never sell the equity, while a buyer will always buy it. Symmetrically, at prices above the upper bound on this interval, a buyer is unwilling to sell the equity, while a seller is willing to sell it. At any point inside these bounds, a potential trader may rationally choose either to trade or not to trade. In Section 5, I discuss the subtleties that this leads to for analyzing observed market data.

4 An Example with One Trading Period

4.1 The Model

The preceding discussion argues that valuation models applied to imperfectly perceived sequences yield neighborhoods of justifiable values. The model developed in this section is used throughout the rest of the paper to show how the firm can communicate a refinement in its perceptions of its prospects. I contrast communication through an income statement with communication through a dividend announcement or a statement of changes in financial position.

There is a set \( I = \{A, B\} \) of two agents. Agent \( A \) is thought of as the initial owner of a firm, while agent \( B \) is thought of as the market.

There are two dates, \( T = \{0, 1\} \). Each agent has an initial endowment of one unit of the unique good at date 0, and nothing at date 1, when consumption takes place. Both agents have access to a public storage technology. An investment of \( S_0 \) at date 0 in the public technology gives the investor \( S_0(1+r) \) at date 1. The value of \( r \) is common knowledge.

Agent \( A \) also has a private technology, on which the return is imperfectly perceived. If \( A \) invests \( S_0 \) initially in the private technology, then at date 1, the investment yields \( S_0(1+r+\rho) + \gamma \). Depending on the reporting environment, if \( A \) reports accounting income, then \( r + \rho S_0 \) can be called income but \( \gamma \) cannot. Thus, \( \rho S_0 \) is the residual income from the private technology, which is proportional to the size of the investment, while \( \gamma \) is thought of as the difference between the ending book value
of equity and the liquidation value. There is no requirement that $\gamma$ must vary proportionately to $S_0$. It is assumed that $S_0$ is common knowledge.

At date 0, both agents have identical perceptions about the private technology. Specifically, they can restrict $\rho$ and $\gamma$ to open intervals:

$$\rho \in (\underline{\rho}, \overline{\rho}) \quad \gamma \in (\underline{\gamma}, \overline{\gamma}).$$

The interpretation of these restrictions is that neither agent has enough experience with the private technology to distinguish investments for any values of $\rho$ or $\gamma$ within these intervals. That is, the intervals can be thought of as capturing residual income and the premium over book value within some common just noticeable difference.

During period 0, $A$ gains experience with the private technology and can therefore perceive $\rho$ or $\gamma$ (or both) more precisely. $A$ may learn the exact values of either of these parameters, or may refine the intervals to which they belong. When $A$ refines the interval on $\rho$, I write the refined interval as $(\rho', \rho'')$, where it is to be understood that $\underline{\rho} < \rho' < \rho'' < \overline{\rho}$. Refinements of the bounds on $\gamma$ to $(\gamma', \gamma'')$ are analogous.

After learning about the parameters, $A$ issues a report to $B$. The report has one of three forms:

**Income Statement** $A$ announces the accounting earnings. Since $r$ and $S_0$ are common knowledge, I interpret the income statement as $A$ stating a value for $\rho$.

**Dividend Declaration** $A$ announces that, at date 1, a payment of $D_1$ is assured. The dividend declaration can have one of the following possible structures:

**Targeted** $A$ has precommitted to paying a specified dividend value $\hat{D}$ if possible, and 0 otherwise.

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10 Adding such a requirement does not change the analysis, although it makes the form of the model similar to that in Lucas and Prescott (1971) and in Hayashi (1982). However, keeping $\gamma$ as a lump-sum adjustment gives the model the same general structure as the residual income model as presented in Equation 3.

11 Agent $A$ may also learn nothing about either parameter. This turns out to be a special case of when $A$ refines the interval to which a parameter belong, and all the results carry over to this special case with only mechanical changes being made.
Proportionate. $A$ has precommitted to paying a divided as some fixed proportion $\lambda \in (0, 1]$ of the largest possible dividend that can be promised, and 0 if it is impossible to commit to a positive dividend.

Unspecified. $A$ has not made any prior commitments, and simply announces a dividend.

Statement of Changes in Financial Position. $A$ announces changes in working capital—i.e., changes in the value of the investment that can be paid out at date 1. Income from $\rho$ and changes in valuation from $\gamma$ are not separated.

Remark. The targeted dividend policy matches the assumption in much of the dividends literature (Lintner 1956, Watts 1973, Watts 1976, Laub 1976). The proportionate dividend policy is also frequently studied (Dutta and Reichelstein 2004). Unspecified dividends are studied in generalizations of the residual income model, for example in Ohlson and Juettner-Nauroth (2003). The Statement of Changes in Financial Position is an announcement of the funds known to be available for distribution at date 1, and thus represents a maximally informative dividend announcement. Until 1987, this statement was one of the standard financial reports in the United States, after which it was replaced by the Statement of Cash Flows (APB 1971, FASB 1987). Stephens and Govindarajan (1990) describe these statements and related measures of funds available for distribution.

Once $A$ issues the report, the market opens (still during period 0), and any trade that may occur takes place. After markets close, nothing happens until date 1, when $\rho$ and $\gamma$ are realized and the owner of the private technology receives the liquidating value.

All reporting rules obey the following convention: Agent $A$ only reports amounts that are guaranteed. For example, if $A$ were to learn nothing, the largest dividend that could be declared would be $\max\{S_0(1+r+\rho) + \gamma, 0\}$, since anything higher is possibly more than $A$ will be able to pay out. Similarly, the highest residual income rate that $A$ could report in this case is $\rho$.

4.2 Reporting Under Full Information

This subsection analyzes the case where the firm $A$ refines its perceptions perfectly of both residual income and the premium over book value, prior to issuing its report. The purpose of evaluating this
case is to restrict attention to the effects of the imprecision in the reporting language. Subsequently, I consider the interaction between perceptual limits and reporting.

Initially, both agents perceive the same bounds on $\rho$ and $\gamma$. The present values that can be assigned to the private technology at the start of date 0 are therefore:

$$P_0 = \left( S_0 + \frac{\rho S_0 + \gamma}{1 + r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1 + r} \right).$$

Since $\rho$ and $\gamma$ belong to open intervals $(\rho, \overline{\rho})$ and $(\gamma, \overline{\gamma})$, neither the upper bound nor lower bound of $P_0$ is attainable. The limits on this interval correspond to the calculation of the value under the residual income approach, using the bounds as worst and best case scenarios.

To use the discounted dividends approach, there would need to be some known yield at date 1. The most that $A$ could initially promise is $S_0(1 + r + \rho + \gamma)$, leaving an ex dividend date 1 value in $(0, S_0(1 + r + \rho + \gamma))$. It is clear that discounting these values leads to the same valuation interval under the dividends approach as is obtained above under the earnings approach. The reason is that there is no new information on either earnings or dividends.

Suppose that, prior to issuing any report, $A$ observes $\rho$ and $\gamma$. Then $A$ knows that the present value of the investment of $S_0$ is

$$P_0^A(\rho, \gamma) = \{ S_0 + \frac{\rho S_0 + \gamma}{1 + r} \}.$$

If $A$ releases an income statement, $B$ learns $\rho$ and refines the interval of present values to

$$P_0^B(I) = \left( S_0 + \frac{\rho S_0 + \gamma}{1 + r}, S_0 + \frac{\rho S_0 + \gamma}{1 + r} \right).$$

The set of present values based on $A$’s knowledge is a singleton, while those present values $B$ can justify is an open interval. The reason is that the income statement does not provide any information about $\gamma$. In a sense, the information in the income statement is too specific: it perfectly the ambiguity in $\rho$, but $B$ would be better informed by seeing the aggregate resolution of ambiguity.\(^\text{12}\)

To analyze the dividends approach, imagine first that $A$ releases a statement of changes in financial position. This tells $B$ what $A$ can commit to pay at date 1, which is $(1 + r + \rho)S_0 + \gamma$. $B$ now can

\(^{12}\)For other settings where aggregation increases information content, see Arya, Glover, and Mittendorf (2004).
assign a unique valuation:

\[ P^B_0(C) = \{ S_0 + \rho S_0 + \gamma \}. \]

Other forms of dividend declarations are less informative. If \( A \) did not initially specify any target dividend and announces a commitment to pay dividend \( D_1 > 0 \), then the announcement can be viewed as a dividend initiation. In this case, \( B \) can infer

\[(1 + r + \rho)S_0 + \gamma \geq D_1.\]

Because the dividend announcement does not identify earnings or the premium over book value, \( B \) cannot use the dividend to restrict the upper bound on possible valuations:

\[ P^B_0(D_1 > 0|\text{no policy}) = [\frac{D_1}{1 + r}, S_0 + \frac{\rho S_0 + \gamma}{1 + r}). \]

The interval is closed on the left because \( B \) knows that \( A \) has full information. Note that this valuation says that the present value is the discounted value of the dividend plus the discounted value of the remaining liquidating value.

A dividend omission is only meaningful if \( A \) had precommitted to paying some target dividend, and cannot guarantee that amount. When no dividend policy is in place, the declaration of a zero dividend is simply uninformative:

\[ P^B_0(D_1 = 0|\text{no policy}) = (S_0 + \frac{\rho S_0 + \gamma}{1 + r}, S_0 + \frac{\rho S_0 + \gamma}{1 + r}). \]

As discussed above, a prespecified dividend policy provides additional information, which can be used in the dividends approach to valuation. If \( A \) has a policy of targeting a specific dividend, say \( \hat{D} > 0 \), then the dividend announcement states whether earnings and the premium over book value are at least \( \hat{D} \). Thus, if the dividend of \( D_1 = \hat{D} \) is declared, then

\[ P^B_0(D_1 = \hat{D}|\text{targeted policy}) = [\frac{\hat{D}}{1 + r}, S_0 + \frac{\rho S_0 + \gamma}{1 + r}). \]

On the other hand, if the dividend is omitted, then \( A \) cannot guarantee \( \hat{D} \). This restricts the upper bound on \( B \)'s possible valuations:

\[(1 + r + \rho)S_0 + \gamma < \hat{D},\]

\[\Rightarrow P^B_0(D_1 = 0|\text{targeted policy}) = (S_0 + \frac{\rho S_0 + \gamma}{1 + r}, \frac{\hat{D}}{1 + r}). \]
As long as the dividend target is ex ante feasible \((0 < \hat{D} < (1 + r + \rho)S_0 + \gamma)\), this interval is smaller than the defensible valuations \(B\) had prior to the dividend announcement.

The targeted dividend policy hence partitions the range of possible payouts at the amount of the stated dividend. This is because of \(A\)'s full information. It will be shown below that, once \(A\) receives only partial information on either income or the premium over book value, the valuations justified under an omission overlap with those justified when the dividend is paid.

A proportionate dividend policy is likewise informative in both the case when \(A\) can commit to a dividend and when \(A\) cannot. In the first case, for fixed \(\lambda \in (0, 1]\), a positive dividend announcement tells \(B\) that

\[
D_1 = \lambda((1 + r + \rho)S_0 + \gamma)
\]

\[
\Rightarrow \quad P^B_0(D_1 > 0|\text{proportionate policy}) = \{S_0 + \frac{\rho S_0 + \gamma}{1 + r}\} = \{\frac{D_1}{\lambda(1 + r)}\}.
\]

Dividing the dividend by the payout rate \(\lambda\) gives \(B\) the exact future value, so the report is equivalent to the statement of changes in financial position and is fully revealing.

If \(A\) does not declare a dividend when the proportionate policy is in place, then \(B\) knows \((1 + r + \rho)S_0 + \gamma) \leq 0\), so that

\[
P^B_0(D_1 = 0|\text{proportionate policy}) = (S_0 + \frac{\rho S_0 + \gamma}{1 + r}, 0].
\]

Unless \(A\)'s private technology is highly destructive (i.e., unless \((1 + r + \rho)S_0 + \gamma \leq 0)\), the lack of a dividend is informative under the proportionate policy.

**Theorem 4.1.** Assume that \(A\) learns income and the premium over book value perfectly. Then the income statement, the dividend announcement, and the statement of changes in financial position all yield different sets of justified present values. Moreover, the dividend announcement contains information in addition to that in the income statement under the following parameter restrictions:

1. For an unspecified dividend policy, if \(D_1 > (1 + r + \rho)S_0 + \gamma\), then a dividend initiation provides a greater lower bound on valuation than the income statement. A zero dividend with an unspecified policy is never informative.
2. For a targeted dividend policy, if \( \hat{D} > (1 + r + \rho)S_0 + \gamma \), then a dividend declaration provides a greater lower bound on valuation than the income statement. If \( \hat{D} < (1 + r + \rho)S_0 + \gamma \), then the dividend omission provides a smaller upper bound than the income statement.

3. For a proportionate dividend policy, a dividend declaration is fully revealing, while the income statement is not. If \( (1 + r + \rho)S_0 + \gamma > 0 \), then the dividend omission provides a smaller upper bound than the income statement.

The income statement can, conversely, contain information not in the dividend announcement. However, neither the income statement nor the dividend announcement contains information in addition to that contained in the statement of changes in financial position.

Consequently, both the earnings and the dividends approaches can yield more informative valuations, unless the firm issues a statement of changes in financial position. In that case, the dividends approach is always more informative.

The proofs of this and other main results are in the appendix.

4.3 Partial Information

In the case considered so far, agent \( A \) obtains perfect information about both earnings and the premium over book value, prior to having to issue a report. I now consider the case where \( A \) receives only partial information on at least one of these quantities.

4.3.1 Case 1: One quantity learned perfectly, the other refined

The situation where \( A \) learns \( \rho \) exactly and refines \( \gamma \) is analogous to an infinite horizon model, where the earnings at each date form a choice sequence. In such a setting, current period income may be known perfectly before the report is issued, while subsequent periods’ earnings may not be fully determined yet may be restricted in some ways.

Conversely, if agent \( A \) perceives the premium over book value exactly but only can perceive income
approximately, then the model is also analogous to an infinite horizon model where the earnings at each date form a choice sequence. In this interpretation, even current period earnings are indistinguishable from alternative calculations that would have been possible (e.g., use of estimates, numbers that did not differ materially from those calculated, etc.). However, there may be restrictions on the discounted infinite sum of earnings, as for example in Ohlson (2003).

The private valuations to agent $A$ are symmetric in these cases. Upon observing $\rho$ and refining $\gamma$ to $(\gamma', \gamma'')$, $A$’s valuation interval is:

$$P_0^A(\rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \gamma'}{1 + r}, S_0 + \frac{\rho S_0 + \gamma''}{1 + r}).$$

Conversely, upon observing $\gamma$ and refines $\rho$ to $(\rho', \rho''$, $A$’s valuation interval is:

$$P_0^A((\rho', \rho''), \gamma) = (S_0 + \frac{\rho' S_0 + \gamma}{1 + r}, S_0 + \frac{\rho'' S_0 + \gamma}{1 + r}).$$

If agent $A$ issues an income statement, then $B$ can learn about $\rho$ but not about $\gamma$. Consequently, in the case where $A$ learns $\rho$ perfectly,

$$P_0^B(I | A \text{ learns } \rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \gamma'}{1 + r}, S_0 + \frac{\rho S_0 + \gamma''}{1 + r}).$$

Thus, the income statement refines both the upper and the lower bound on $B$’s valuations, but does not fully disclose either of $A$’s bounds on the valuation.

On the other hand, if $A$ observes $\gamma$ and refines $\rho$ to $(\rho', \rho'')$, then the income statement is only informative about the lower bound on income. The information in the premium over book value is not part of the income statement, and the lower bound is the most income that $A$ can guarantee. Therefore, in this case,

$$P_0^B(I | A \text{ learns } (\rho', \rho''), \gamma) = (S_0 + \frac{\rho' S_0 + \gamma}{1 + r}, S_0 + \frac{\rho'' S_0 + \gamma}{1 + r}).$$

In each case, $B$’s lower bound is strictly below $A$’s lower bound and $B$’s upper bound is strictly above $A$’s upper bound.

The statement of changes in financial position, by combining the effects of income and the premium over book value, does give the lower bound exactly. Moreover, $B$ can use the statement of changes in financial position to infer a refinement to the upper bound. In the case where $A$ learns $\rho$ exactly,
B can use the fact that $\gamma' > \gamma$ to deduce

$$\rho S_0 + \gamma' > \rho S_0 + \gamma \Rightarrow \rho < \min\{\frac{(\rho S_0 + \gamma') - \gamma}{S_0}, \bar{\rho}\}.$$  

Similarly, if A learns $\gamma$ exactly, then B can use the fact that $\rho' S_0 > \rho S_0$ to deduce

$$\rho' S_0 + \gamma > \rho S_0 + \gamma \Rightarrow \gamma < \min\{(\rho' S_0 + \gamma) - \rho S_0, \bar{\rho}\}.$$  

Based on these refinements, B’s valuation intervals are

$$P^B_0(C|A \text{ learns } \rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \gamma'}{1 + r}, S_0 + \min\{\frac{\rho S_0 + \gamma'}{1 + r} + \frac{\gamma - \gamma}{1 + r}, \bar{\rho} S_0 + \bar{\gamma}\}),$$

and

$$P^B_0(C|A \text{ learns } (\rho', \rho''), \gamma) = (S_0 + \frac{\rho' S_0 + \gamma}{1 + r}, S_0 + \min\{\frac{\rho' S_0 + \gamma}{1 + r} + \frac{(\rho - \rho) S_0}{1 + r}, \bar{\rho} S_0 + \bar{\gamma}\}).$$

In sum, the following holds:

**Proposition 4.1.** When agent A observes one parameter perfectly and the other imperfectly, the statement of changes in financial position fully reveals A’s lower bound on valuation, while the income statement gives B a lower bound strictly below A’s. Both statements give upper bounds to B above that of A, but B’s upper bound from the income statement is strictly lower than B’s upper bound from the statement of changes in financial position.

**Proof.** Immediate from comparing $P^A_0$ above with $P^B_0(I)$ and $P^B_0(C)$. □

**Remark:** Proposition 4.1 says that the income statement limits the degree to which B can pay too much for the equity. Conversely, the statement of changes in financial position prevents B from missing opportunities, by keeping B from underestimating the lower bound on the equity’s value. These effects are symmetric, but their observability in market prices is not: the information in the income statement restricts observable prices, while that in the statement of changes in financial position prevents non-trades. I discuss this in greater detail below in Section 5.

If instead A initiates a dividend $D_1$ and had not previously specified a dividend policy, then B’s valuation interval becomes

$$P^B_0(D_1 > 0|\text{no policy}) = (\frac{D_1}{1 + r}, S_0 + \frac{\bar{\rho} S_0 + \bar{\gamma}}{1 + r}).$$
This is almost identical to the full information case, except now the left endpoint of this interval is excluded as a possible valuation. The reason is that $A$’s valuation is an open interval, so $A$ can only guarantee amounts strictly below the lower bound on $A$’s private valuations. On the other hand, if there is no policy in place and $A$ does not declare a dividend, then $B$ receives no useful information. Thus, in the absence of a dividend policy, the information in dividends is nearly identical to the full information case, differing only in that the left endpoint of $B$’s valuation interval is now excluded.

If there is a target dividend of $\hat{D}$ and a dividend is declared, then $B$ again can bound the valuation interval below at $\hat{D}/(1+r)$. Conversely, a dividend omission bounds $B$’s valuation interval above.

In the case where $A$ learns earnings exactly and refines the premium over book value, not paying the target dividend $\hat{D}$ tells $B$ that

$$(1 + r + \rho)S_0 + \gamma' < \hat{D}.$$ 

Since $\gamma' > \gamma$, $B$ can infer that

$$(1 + r + \rho)S_0 + \gamma < \hat{D} \Rightarrow (1 + r + \rho)S_0 + \bar{\gamma} < \hat{D} + \bar{\gamma} - \gamma.$$ 

Accordingly, the valuation $B$ can assign from the dividend omission is:

$$P_B^0(D_1 = 0|\text{targeted policy}; \rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \gamma}{1+r}, \min\{\frac{\hat{D} + \bar{\gamma} - \gamma}{1+r}, S_0 + \frac{\bar{\rho} S_0 + \bar{\gamma}}{1+r}\}),$$ 

and the valuation from paying the targeted dividend is

$$P_B^0(D_1 = \hat{D}|\text{targeted policy}; \rho, (\gamma', \gamma'')) = (\frac{D_1}{1+r}, S_0 + \frac{\bar{\rho} S_0 + \bar{\gamma}}{1+r}).$$

When $A$ instead learns the premium over book value exactly and refines earnings, the valuation interval for $B$ is identical if the targeted dividend is paid, and changes in the symmetric way when the dividend is omitted:

$$P_B^0(D_1 = 0|\text{targeted policy}; (\rho', \rho''), \gamma) = (S_0 + \frac{\rho' S_0 + \gamma}{1+r}, \min\{\frac{\hat{D} + (\bar{\rho} - \rho)S_0}{1+r}, S_0 + \frac{\bar{\rho} S_0 + \bar{\gamma}}{1+r}\}).$$

Under the proportionate dividend policy, the information is the same as the information in the statement of changes in financial position whenever a dividend is declared. If $A$ has a proportionate dividend policy in place and omits the dividend, then $(1 + r + \rho)S_0 + \gamma' \leq 0$ in the case where $A$
learns income exactly, and \( (1 + r + \rho)S_0 + \gamma \leq 0 \) in the case where \( A \) learns the premium over book value exactly. This is identical to the case of an omission under the targeted policy, except that the target value \( \hat{D} \) is replaced by 0, since a proportionate dividend can be arbitrarily close to zero.

By the above argument in the case of the targeted dividend policy, \( B \)'s valuations in the cases of an omission are

\[
P_0^B (D_1 = 0 | \text{proportionate policy}; \rho, (\gamma', \gamma'')) = (S_0 + \frac{\rho S_0 + \gamma}{1 + r}, \frac{\gamma - \gamma}{1 + r}),
\]

and

\[
P_0^B (D_1 = 0 | \text{proportionate policy}; (\rho', \rho''), \gamma) = (S_0 + \frac{\rho S_0 + \gamma}{1 + r}, \frac{\gamma - \gamma}{1 + r}).
\]

### 4.3.2 Case 2: Both learned partially

The last case to consider is where \( A \) refines both \( \rho \) and \( \gamma \) but continues to be incapable of distinguishing their true values from nearby values. To interpret this in the infinite horizon setting, imagine that earnings in every period are restricted, but that there are limits in how well earnings in any period can be measured.

The valuations \( A \) can justify at the time of issuing a report are now

\[
P_0^A ((\rho', \rho''); (\gamma', \gamma'')) = (S_0 + \frac{\rho' S_0 + \gamma'}{1 + r}, S_0 + \frac{\rho'' S_0 + \gamma''}{1 + r}).
\]

The income statement can convey \( \rho' \) to \( B \), but provides no information on \( \rho'' \) or on \( \gamma \). Consequently,

\[
P_0^B (I) = (S_0 + \frac{\rho' S_0 + \gamma'}{1 + r}, S_0 + \frac{\gamma - \gamma}{1 + r}).
\]

Thus, the income statement does not refine the upper bound on \( B \)'s valuations, and does not fully reveal \( A \)'s lower bound.

The statement of changes in financial position conveys the lower bound exactly, since \( \rho' S_0 + \gamma' \) are combined. As this report says nothing about \( \rho'' \) or \( \gamma'' \), it provides no information for refining the upper bound; i.e.,

\[
P_0^B (C) = (S_0 + \frac{\rho' S_0 + \gamma'}{1 + r}, S_0 + \frac{\gamma - \gamma}{1 + r}).
\]

This leads immediately to the following:
Proposition 4.2. If A receives partial information on both income and the premium over book value, then the statement of changes in financial position is strictly more informative than the income statement.

Proof. Compare $P^B_0(C)$ and $P^B_0(I)$ in this case: the statement of changes gives a higher lower bound on $B$’s valuations, and neither refines the upper bound.

Any dividend policy must be based on $\rho'$ and $\gamma'$. Thus, the following holds:

Proposition 4.3. If A receives partial information on both income and the premium over book value, a dividend omission is uninformative.

Proof. Since the dividend is zero, there is no refinement of the lower bound of possible valuations for $B$. On the other hand, the upper bound depends entirely on $\rho''$ and $\gamma''$, neither of which affects the dividend announcement.

This means the following result holds:

Corollary 4.1. If A receives partial information on both income and the premium over book value, the targeted and unspecified dividend policies convey the same information. Also, under this assumption, the proportionate policy is either identical to the statement of changes in financial position in its information content (i.e., when a dividend is declared) or uninformative.

4.4 Valuation with Truncated Calculations

Suppose now that $B$ were to calculate the valuation models based on either the dividends alone or on the book value plus discounted residual income. That is, consider the truncated calculations

Dividends $V(D) \equiv D_1/(1 + r)$, provided $D_1 > 0$, and undefined otherwise; and

Earnings $V(I) \equiv S_0 + \rho S_0/(1 + r)$, where $\rho$ is inferred from an income statement.
These two values correspond to the case where everything beyond the finite horizon is ignored.

The results from these calculations are as follow:

**Proposition 4.4.**
1. The value from the dividend calculation is the lower bound on $B$’s valuation interval if the dividend policy is unspecified or targeted, or if there is a proportionate dividend policy and $\lambda = 1$.

2. The value from the dividend calculation is strictly below the lower bound on $B$’s valuation interval if the dividend policy is proportionate and $\lambda < 1$.

3. The value from the earnings calculation is an interior point of $B$’s valuation interval if and only if $\gamma < 0$ and either $\gamma > 0$ or $\rho$ is sufficiently large.

In other words, the valuation from the truncated residual income model is a justified valuation provided the difference between book value and market value can be positive or negative. By contrast, the valuation from the truncated dividend model is not justified whenever the dividend-based valuation is a left-open interval (i.e., in every case except when $A$ has full information).

### 5 Economic Consequences

In all but the full information case, the valuations to both the firm’s current owner $A$ and the market $B$ are open intervals. It is unclear why trade would occur in the full information case: if the current owner has a unique reservation price, then the market can never be made better off by trade and has the possibility of being made worse off. Accordingly, this section focuses in trade when the firm does not have full information.

Assume both $A$ and $B$ have preferences that are strictly increasing in their final outcomes—i.e., that a higher payment at date 1 is, other things being equal, more desirable for each agent than a lower payment. The problem is that the equity’s valuation is restricted to an interval but otherwise indeterminate. Accordingly, I make the following assumption:

**Assumption 5.1.** Let $(x, y)$ and $(x', y')$ be two ordered pairs in $\mathbb{Q} \times \mathbb{Q}$ or in $\mathbb{R} \times \mathbb{R}$. Assume $x < y$
and \( x' < y' \). If \( y \leq x' \), then both agents prefer \((x', y')\) to \((x, y)\), written
\[
(x, y) \prec (x', y')
\]
For fixed \( z \in \mathbb{R} \) and \((x, y) \in \mathbb{Q} \times \mathbb{Q} \) (or in \( \mathbb{R} \times \mathbb{R} \)), assume \((x, y) \prec \{z\} \equiv y \leq z\), and \( \{z\} \prec (x, y) \equiv z \leq x \).

**Remark:** From a formal point of view, there is no need to restrict attention to open intervals with rational endpoints. On the other hand, if one takes seriously that reporting is a language, it is worth noting that rational numbers can be communicated with finite amounts of information. Moreover, there is no loss of generality in limiting the intervals to ones with rational endpoints, as they form the base of a topology on the real line; see Negri and Soravia (1999) for a discussion of convenient mathematical properties. One can also map preferences on rational intervals to interval-valued utilities, following a procedure similar to that in Bridges and Mehta (1995).\(^{13}\)

The relation \( \prec \) is irreflexive and transitive. However, it is not negatively transitive:

**Lemma 5.1.** Let \((x, y), (x', y'), (x'', y'')\) be ordered pairs in \( \mathbb{Q} \times \mathbb{Q} \) or \( \mathbb{R} \times \mathbb{R} \). Then

1. \( \neg((x, y) \prec (x, y)) \).

2. \((x, y) \prec (x', y')\) and \((x', y') \prec (x'', y'') \rightarrow (x, y) \prec (x'', y'')\).

3. \(\neg((x, y) \prec (x', y')) \) and \(\neg((x', y') \prec (x'', y'')) \) does not imply \(\neg((x, y) \prec (x'', y''))\).

In other words, an increasing preference ordering on exact outcomes extends to a strict partial ordering on imperfectly perceived outcomes. The failure of negative transitivity makes the following immediate:

**Corollary 5.1.** Let \((x, y) \approx (x', y') \equiv \neg((x, y) \prec (x', y')) \) and \(\neg((x', y') \prec (x, y))\). Then \( \approx \) is not an equivalence relation; in particular, it is intransitive.

---

\(^{13}\)An alternative approach to utility under incomplete preferences is to work with vector-valued utilities, as outlined in Dubra, Maccheroni, and Ok (2001). Viewing utilities as open intervals is more in keeping with the spirit of perceptual limits that motivate the present approach.
Because of Corollary 5.1, it seems sensible not to view $\approx$ as indifference, and to treat preferences as incomplete when the universe includes imperfectly perceived objects.\(^{14}\)

The following assumption is made in order to discuss equilibrium under incomplete preferences:

**Assumption 5.2.** Let $i \in \{A, B\}$. Let $p$ be a given price. Then:

1. If $P_i^o < \{p\}$, then $i$ is willing to sell and unwilling to buy the equity.
2. If $\{p\} < P_i^o$, then $i$ is willing to buy and unwilling to sell the equity.
3. If $\{p\} \approx P_i^o$, then $i$ may or may not be willing to trade the equity.

The assumption means that equilibrium prices behave as follows:

**Lemma 5.2.** Suppose $P_o^A$ is an open interval. Then equilibrium prices are given by

$$(\inf P_o^A, \sup P_o^B).$$

If $P_o^A$ is a singleton or a left-closed interval, then the above is replaced with its closure on the left.

If both $P_o^A$ and $P_o^B$ are singletons, then the equilibrium price is their intersection.

The lemmata justify the following result:

**Theorem 5.1.** Suppose $A$ does not learn income or the premium over book value perfectly, but refines at least one of these. Then the equilibrium prices are the same, whether $A$ reports an income statement, a statement of changes in financial position, or a dividend announcement of any of the types considered.

When $A$ learns either quantity perfectly, Theorem 5.1 no longer holds. For example, the income statement and the statement of changes in financial position may yield different equilibrium prices.

\(^{14}\)The relation $\approx$ is a tolerance relation, i.e., it is reflexive and symmetric. The viewpoint here is that indifference relations ought to be transitive. If this requirement is relaxed, one could follow the approach in Schreider (1974) or Fishburn (1987).
Theorem 5.2. Suppose $A$ learns income perfectly and the premium over book value imperfectly. Then the equilibrium prices that result from $A$ reporting an income statement are a proper subset of those resulting from $A$ reporting a statement of changes in financial position.

Conversely, suppose $A$ learns the premium over book value perfectly and income imperfectly. Then the equilibrium prices that result from $A$ reporting an income statement either coincide with or are a superset of those resulting from $A$ reporting a statement of changes in financial position. In particular, if the initial perceptual limits on income are sufficiently small relative to those on the premium over book value, then the statement of changes in financial position induces a smaller set of possible equilibrium prices.

The next result gives the relationship between dividends and equilibrium prices.

Theorem 5.3. 1. An unspecified dividend policy never restricts the set of equilibrium prices.

2. A targeted dividend policy or a proportionate dividend policy can contain restrict the set of equilibrium prices beyond what the income statement or statement of changes in financial position can do.

The above results in particular offer an explanation for why dividends may seem uninformative. A dividend initiation or a payment in keeping with a dividend policy raises $B$’s lower bound on valuation, but this is always weakly lower than $A$’s lower bound. Since the market prices are bounded below by $A$’s lower bound on valuation, the information in a dividend payment or initiation is likely to be unobservable. The dividend prevents $B$ from rejecting some trades, but has no effect on possible prices.

Similarly, Theorem 5.3 explains why there would appear to be a strong market reaction to dividend omissions. When a dividend is omitted, agent $B$ refines the upper bound on possible valuations, and it is $B$’s upper bound that restricts the supremum of possible market prices. Thus the apparently stronger market reaction to dividend omissions is a consequence of the asymmetric ability to observe trades versus non-trades.
6 Concluding Remarks

It is widely accepted that the dividends and earnings approaches to valuation are theoretically equivalent. This viewpoint rests on the notion that each model (under the assumption of clean surplus) can be derived from the other.

The derivability of a dividends or residual income model from the other depends, however, on more than clean surplus. In particular, each term in the both models must be estimable arbitrarily far into the future. The clean surplus equation only specifies that earnings must be divided between changes in book value and dividends. This rule can be known to be in place, and earnings can be estimable, even when the actual division into book value and distributions has not yet been determined. Conversely, a dividend policy may be fixed without income being known at all. (Consider for example a policy of not paying any dividends for the next ten years: several terms in the dividend sequence are known, even before anything is learned about earnings.)

With these observations as motivation, this paper models imperfectly perceived accounting earnings and an imperfectly perceived premium over book value. Depending on what the firm learns before reporting to the market, and on what the market can perceive about the firm’s sources of information, there are cases where earnings were more informative than dividends, and there are cases where the converse holds. Various types of dividend announcements are investigated: dividends come in the form of a declaration with no policy in place, a pre-stated policy (of either a target dividend or a proportion of funds planned to be paid out) or a statement of the funds available for distribution. Each of these has different information content to the market.

The perceptual limits generally make both the firm’s and the market’s valuation of an equity open intervals rather than single-valued. Equilibrium market prices are also intervals, which may differ from the firm’s or market’s valuations. Thus, the information in reports and its effects on the firm or the market are not always apparent from observed prices.

The argument that dividends contain little or no information beyond what is in market prices seems to fail here, yet a reason emerges for why one would draw this inference. When dividends are positive, they do not restrict the set of equilibrium prices generated by the income statement and by the
statement of changes in financial position (i.e., the statement of funds available for distributions). However, if a dividend is omitted when a dividend policy is in place, the dividend announcement does restrict equilibrium prices, by refining the upper bound on the market’s valuation. Thus, the results here explain the market’s stronger reaction to dividend omissions than to initiations or payments.

Future work might explore the role of voluntary disclosure in providing upper bounds on valuation, and thus may add to the theoretical understanding of voluntary disclosure (Dye 1986, Gigler 1994, Verrecchia 1983). Recent empirical work has found evidence that firms whose earnings are relatively less correlated with abnormal returns are more likely than other firms to provide supplemental balance sheet information (Chen, DeFond, and Park 2002) or pro forma earnings statements (Lougee and Marquardt 2004). The former might be viewed as the firm providing the market with information on the premium over book value (more specifically, recording information that had previously not been recorded), while the latter might be viewed as the firm releasing an upper bound as well as a lower bound on income.
A Proofs

Proof of Theorem 4.1:

Proof. The differences in the information content of the dividends announcements, the income statement, and the statement of changes in financial position are immediate by comparing $P_0^B(\cdot)$ under the various reporting schemes.

The statement of changes in financial position is fully revealing, so none of the other statements can improve on this statement.

When the dividend policy is unspecified and $D_1 = 0$, there is no information in the announcement.

When the dividend is positive, either for the unspecified or the targeted dividend policy, the announcement provides a lower bound on $P_0^B$ at the present value of the dividend. Because the dividend can be based on either $\rho S_0$ or $\gamma$ or their combination, these lower bounds can be higher than those under the income statement (which does not refine $\gamma$).

When there is a dividend policy in place, the announcement that $D_1 = 0$ informs $B$ that $(1 + r + \rho)S_0 + \gamma$ is insufficient to pay a dividend under the policy. Thus, $D_1 = 0$ provides an upper bound on $P_0^B$ that depends on both $\rho$ and $\gamma$ (and, in the case of the targeted dividend policy, on $\hat{D}$), whereas the upper bound generated by the income statement contains no information on $\gamma$. \qed

Proof of Proposition 4.4:

Proof. 1. If the dividend policy is unspecified, then the lower bound on $P_0^B(D_1|D_1 > 0) = D_1/(1+r)$, irrespective of what $B$ knows about $A$’s information structure. Similarly, for the targeted dividend policy, the lower bound on $P_0^B(D_1 > 0|\text{targeted policy})$ is always $\hat{D}/(1+r)$. The proportionate dividend policy has a lower bound on $P_0^B(D_1 = 0|\text{proportionate policy}) = D_1/(\lambda(1+r)) = D_1/(1+r)$ when $\lambda = 1$. In each of these cases, the lower bound is $V(D)$.

2. If the dividend policy is proportionate and $\lambda < 1$, then the lower bound on $P_0^B(D_1 > 0|\text{proportionate policy}) = D_1/(\lambda(1+r)) > D_1/(1+r) = V(D)$. 
3. Since \( \rho \) comes from the income statement, it is the lower bound on what \( A \) can report for income, while no information is provided on \( \gamma \). Consequently, the lower bound on \( P_{0}^{B}(I) \) is 
\[
S_{0} + \rho S_{0}/(1 + r) + \gamma/(1 + r),
\]
which is strictly less than \( V(I) \) iff \( \gamma/(1 + r) < 0 \Rightarrow \gamma < 0 \).

If \( B \) can determine that the value of \( \rho \) from the income statement is exact, then the upper bound on \( P_{0}^{B}(I) \) is 
\[
S_{0} + \rho S_{0}/(1 + r) + \gamma/(1 + r),
\]
which exceeds \( V(I) \) iff \( \gamma > 0 \). If \( B \) cannot bound \( \rho \) above, then the upper bound on \( P_{0}^{B}(I) \) is 
\[
S_{0} + \rho S_{0}/(1 + r) + \gamma/(1 + r),
\]
which exceeds \( V(I) \) by \((\overline{\rho} - \rho)S_{0} + \gamma)/(1 + r)\). This is positive if either \( \gamma > 0 \) or, given \( S_{0} \), if \( \rho \) is sufficiently larger than \( \rho \).

\( \square \)

Proof of Lemma 5.1:

Proof. 1. If \( (x, y) \prec (x', y') \), then \( x < y \); i.e., the interval must be nondegenerate, by the definition of \( \prec \). On the other hand, \( (x, y) \prec (x, y) \) requires \( y \leq x \). So \( \prec \) is irreflexive.

2. If \( (x, y) \prec (x', y') \), then \( y \leq x' < y' \). If \( (x', y') \prec (x'', y'') \), then \( y' \leq x'' \), and therefore \( y < x'' \).

So \( \prec \) is transitive.

3. Observe \( \neg((0, 2) \prec (1, 3)) \) and \( \neg((1, 3) \prec (2, 4)) \), but \( (0, 2) \prec (2, 4) \).

\( \square \)

Proof of Lemma 5.2:

Proof. \( A \) will not sell the equity for any price below \( \inf P_{0}^{A} \), while \( B \) will not pay more than \( \sup P_{0}^{B} \). For any price in this interval, at least one party may or may not be willing to trade, so there is not necessarily incentive for \( A \) to raise the asking price or \( B \) to lower the bid at any given interior point.

\( \square \)

Proof of Theorem 5.1:
Proof. Consider the case where $A$ learns $\rho \in (\rho', \rho'')$ and $\gamma \in (\gamma', \gamma'')$; other cases are similar. Here

$$P_A^A((\rho', \rho''); (\gamma', \gamma'')) = (S_0 + \frac{\rho'S_0 + \gamma'}{1 + r}, S_0 + \frac{\rho''S_0 + \gamma''}{1 + r})$$

so the lower bound on the equilibrium prices is $S_0 + (\rho'S_0 + \gamma')/(1 + r)$.

1. $P_B^B(I) = (S_0 + (\rho'S_0 + \gamma')/(1 + r), S_0 + (\overline{\rho}S_0 + \overline{\gamma})/(1 + r))$, because the income statement does not tell $B$ what the upper bounds on $\rho$ or $\gamma$ are. Thus, the upper bound on the equilibrium prices is $S_0 + (\overline{\rho}S_0 + \overline{\gamma})/(1 + r)$, giving prices of

$$\left( S_0 + \frac{\rho'S_0 + \gamma'}{1 + r}, S_0 + \frac{\overline{\rho}S_0 + \overline{\gamma}}{1 + r} \right).$$

2. $P_B^B(C) = (S_0 + (\rho'S_0 + \gamma')/(1 + r), S_0 + (\overline{\rho}S_0 + \overline{\gamma})/(1 + r))$, by a similar argument. Thus, the upper bound on $P_B^B(C)$ is the same as under $P_B^B(I)$.

3. Suppose $D_1 > 0$, under any of the dividend policies. Then the dividend refines the lower bound on $P_B^B$ but does not refine the upper bound.

4. Suppose $D_1 = 0$, and the dividend policy is unspecified or proportionate. Then the announcement does not refine the upper bound on $P_B^B$.

5. Suppose $D_1 = 0$, and there is a targeted dividend policy. Then, as discussed above, the dividend restricts the values for $\rho'$ and $\gamma'$, but does not provide any information on $\rho''$ or on $\gamma''$. Consequently, the upper bound on $P_B^B$ is not refined.

\[\square\]

Proof of Theorem 5.2:

Proof. Recall $P_A^A(\rho, (\gamma', \gamma'')) = (S_0 + (\rhoS_0 + \gamma')/(1 + r), S_0 + (\rhoS_0 + \gamma'')/(1 + r))$. So the left endpoint on the equilibrium price interval is $S_0 + (\rhoS_0 + \gamma')/(1 + r)$. From the income statement, $B$ infers $\rho$ but does not learn about $\gamma$, so the upper bound on $\gamma$ remains $\overline{\gamma}$. Therefore, the equilibrium price intervals when $A$ reports an income statement is

$$\left( S_0 + \frac{\rhoS_0 + \gamma'}{1 + r}, S_0 + \frac{\rhoS_0 + \overline{\gamma}}{1 + r} \right).$$
If \( A \) instead reports a statement of changes in financial position, then the upper bound of \( P_0^B(C) \) was seen above to be \( S_0 + (\rho S_0 + \gamma' + (\overline{\gamma} - \gamma))/(1 + r) \). Since \( \gamma' - \gamma > 0 \), it follows that

\[
\overline{\gamma} + \gamma' - \gamma > \overline{\gamma} \Rightarrow \sup P_0^B(C) > \sup P_0^B(I).
\]

Hence, the statement of changes in financial position induces a higher upper bound on the prices \( B \) might be willing to accept as a buyer.

When \( A \) learns \( \gamma \) exactly and \( \rho \) imperfectly, the income statement does not refine the upper bound on \( P_0^B \). Thus, the equilibrium prices in this case are

\[
(S_0 + \frac{\rho' S_0 + \gamma}{1 + r}, S_0 + \frac{\overline{\rho} S_0 + \overline{\gamma}}{1 + r}).
\]

The upper bound on \( P_0^B(C) \) was seen above to be \( S_0 + ((\overline{\rho} - \rho) S_0 + \rho' S_0 + \gamma)/(1 + r) \). Thus, the statement of changes in financial position yields a lower upper bound on equilibrium prices iff

\[
\overline{\rho} S_0 + \overline{\gamma} > (\overline{\rho} - \rho) S_0 + \gamma \Rightarrow \overline{\gamma} > (\rho' - \rho) S_0.
\]

\[\square\]

**Proof of Theorem 5.3:**

**Proof.** 1. When \( D_1 > 0 \), the dividend bounds \( P_0^B \) below. Since the equilibrium prices are bounded below by \( \inf P_0^A \), the information in the dividend has no effect on the observed equilibrium prices. On the other hand, if \( D_1 = 0 \) and there is no specified policy, the announcement does not restrict \( P_0^B \) at all.

2. Suppose \( A \) learns \( \rho \) precisely and \( \gamma \) imperfectly or not at all. Then

\[
\sup P_0^B(D_1 = 0|\text{proportionate policy}) = (\hat{D} + \overline{\gamma} - \gamma)/(1 + r),
\]

which for sufficiently low \( \hat{D} \) will be below the upper bounds on \( P_0^B(I) \) or \( P_0^B(C) \). Analogous results hold when \( A \) learns \( \gamma \) precisely and \( \rho \) imperfectly or not at all.

3. Analogous to the previous item, with \( \hat{D} \) replaced by 0.

\[\square\]
References


REFERENCES


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