# Ministerial Weights and Government Formation: Estimation Using a Bargaining Model 

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#### Abstract

This article proposes a method to estimate relative ministerial weights in parliamentary democracies. Specifically, we present a bargaining model of government formation and estimate structural parameters of the model using data for (i) who the formateurs are, (ii) what each party's voting weight is, and (iii) what ministerial seats each party obtains. We also measure the effects of voting weights and formateur advantage on bargaining results. We apply our proposed method to the case of Japan. Our estimation results show that political players value pork-related posts (such as the Minister of Construction) much more than prestigious ones (such as the Minister of Foreign Affairs). We also find that there is a significant formateur advantage, whereas voting weights do not have a significant scale effect, which is consistent with the findings for European democracies.


## 1. Introduction

In parliamentary democracies, parties bargain over ministerial seats when a new government is formed. Although allocations of seats are publicly observable, the number of posts each party obtains tells us little about how much the party actually gains because ministerial seats may be of different importance. This naturally leads us to ask the following questions: How do ministerial posts differ in their importance, and how can we measure these differences in ministerial weights?

Not only do they reveal the actual gains of parties in government formations, ministerial weights are also indicative of two intrinsic factors in a ministry:

[^0]policy importance and pork-barrel spending. A ministry whose policy area is important to political parties may carry a high weight. Likewise a ministerial post that distributes or strongly influences pork-barrel spending may also be highly weighted. Thus, estimated ministerial weights would tell us which ministries are important to parties. Despite the importance of this question, however, the existing literature provides no statistical method for estimating ministerial weights per se. As Ansolabehere et al. (2005:556) put it, it would seem at first "difficult . . . to measure the relative valuation of the ministries beyond the obvious difference between Prime Minister and all others." In the present article, we propose a way to measure it.

In addressing this primary question, we also consider (i) how voting weights translate into gains from allocated seats (i.e., the scale effect of party size) and (ii) how large the advantage of being a formateur (i.e., a party chosen to form a government) is. More specifically, we ask whether bigger parties gain more than their voting weights out of the government formation process and whether becoming a formateur provides a party with an extra share in the cabinet formation process. A seminal study by Gamson (1961) presented a hypothesis, often called "Gamson's Law," that the share of cabinet posts for a party is proportional to its relative size in the coalition (so that there is no formateur advantage). Testing this hypothesis has been one of the main issues in the empirical literature of government formation since Gamson (1961). In the present study, we reconsider these questions by allowing cabinet posts to have different levels of importance.

The method we propose in the present article combines a bargaining model of government formation ${ }^{1}$ with a maximum likelihood (ML) estimation: We first present a bargaining model of government formation and then estimate the model structurally. This procedure allows researchers to interpret estimated parameters as model primitives of the formal model. ${ }^{2}$ We use variation of the data and the structure of the bargaining model to recover ministerial weights and other parameters. The data required for estimation are (i) who the formateurs are, (ii) what each party's voting weight is, and (iii) what ministerial seats each party obtains; all these are publicly observable data resulting from politicians' behavior. We emphasize that in applying the proposed method, researchers should employ a bargaining game that properly captures characterizing features of government formation.

The case for which we demonstrate our method is Japan. We estimate the model by employing a bargaining game based on the historical stylized facts of the Japanese government formation. Our estimation results show that political

[^1]players value pork-related posts (such as the Minister of Construction and the Minister of Transport) much more than prestigious ones (such as the Minister of Foreign Affairs and the Minister of Justice). We also find that the scale effect does not exist, although formateur advantage is significant. This implies that how much each party obtains in a government formation is almost proportional to its size for nonformateur parties even in an ex post sense (i.e., even after a first proposer is selected), whereas a formateur party gains more than its voting weights. This empirical result confirms Gamson's (1961) earlier predictions only for non-formateur parties.

The rest of the article is organized as follows. After reviewing the related literature in the next subsection, we present a model of government formation in Section 2. Data are described in Section 3, and the econometric specification and the estimation method are presented in Section 4. Section 5 presents the results and offers a counterfactual experiment. In Section 6, we discuss how our model and method can be applicable to any country or even for a comparative analysis of several countries. We conclude the article with some remarks in Section 7.

### 1.1 Related Literature

There are a number of important contributions to the empirical study of government formation. Browne and Franklin (1973) and Browne and Frendreis (1980) are among the first to do empirical investigations on allocation of ministerial posts. They maintained a strong assumption that all cabinet posts have the same weights. ${ }^{3}$ Warwick and Druckman (2001) employed the ranking of the importance of ministers reported by Laver and Hunt (1992) ${ }^{4}$ to reconsider the empirical relationship between cabinet post allocation and seat shares. Recently, Ansolabehere et al. (2005) conducted a more comprehensive empirical analysis on the issue. They investigate the above relationship under two alternate assumptions: (i) the assumption that all cabinet posts are equally valuable and (ii) the assumption that the relative value of the Prime Minister is three times higher than that of other ministers. In this article, we improve on the existing analysis by removing the ad hoc weighting assumptions on ministerial weights. Rather, we target the ministerial weights of estimation. Although we employ data from Japan, our results with respect to the scale effect and formateur advantage are similar to most of the existing studies on postwar European democracies: Parties' gains are proportional to voting weights for nonformateur parties, and there is a significant formateur advantage.

[^2]This article is also related to studies that structurally estimate models of government formation. Merlo (1997) uses a stochastic multilateral bargaining model to study both the duration of government formation and government stability and the effect of deadlines for such variables. Diermeier et al. (2003) extend Merlo's (1997) approach and identify the effects of constitutional features on the stability of governments by using data from nine European countries. Our contribution to this literature is that we deal with the allocation issue in the bargaining model, whereas Merlo (1997) and Diermeier et al. (2003) focused on other aspects, namely when and how a coalition is formed.

## 2. Bargaining Model

In this section, we describe a bargaining model that captures specific features of government formation in Japan during the period from 1958 to 1993, when the Liberal Democratic Party (LDP) maintained a majority in the House of Representatives. First, we consider a multilateral sequential bargaining model of government formation based on the observations of the stylized facts. Then, in the following sections, we explain the data we use and estimate the model.

Before presenting the formal model, some comments on the features of Japanese government formation and model assumptions are in order (see Appendix A for a more detailed discussion on our modeling assumptions and the features of Japanese government formation). First, we model the game of government formation as a bargaining model among the factions of the LDP because government formation was a process among the LDP factions during the period from 1958 to 1993. We take a faction in the LDP as a player in the bargaining game that we consider below. Second, we employ the unanimity rule for agreement following the observations that the LDP maintained a majority through the period, that an LDP faction of a significant size always obtained cabinet posts, that no faction has ever left the LDP, and that no vote of no-confidence was voted for by any LDP factions. We consider this game to be an alternating-offer, random-proposer model. We assume this because factions could have rejected the offer and voted for a no-confidence resolution to choose the proposer again and restart the process. ${ }^{5}$ Our model is an extension of that of Baron and Ferejohn (1989). We generalize the recognition probability as a function of the voting weights of the factions and employ unanimity, instead of a majority, as the agreement rule instead of majority. Third, we assume that factions evaluate posts in an identical way. As we argue in Appendix A, factions are understood to have little difference in policy interests. There is also no reason to assume that some factions value money differently, since they use it in the elections in a very similar way. Hence, we assume that the values of cabinet seats are identical in the eyes of all the factions.

Last, it might be worth making explicit that we do not consider possible strategic relations between one government formation and another. We assume this because players' equilibrium payoffs are uniquely determined in the class of strategies we consider (stationary strategies), as will be explained in Section 2.2

[^3](Proposition 1). There is no analytical gain in considering a "repeated game" of government formation because the players' equilibrium payoffs of this extended game are just the discounted sum of the equilibrium payoffs at each stage game under stationary strategies. Note that if we do not confine the class of strategies to stationary ones, then, as Proposition B. 1 in Appendix B shows, any reasonable allocation can constitute an equilibrium even in each stage game of bargaining (and even under subgame perfection). Therefore, we cannot connect the model prediction to the data unless we connect the infinite possibilities of the real world to one observation, which is impossible.

### 2.1 Bargaining in Japanese Government Formation

We consider government formation as a bargaining game $\Gamma(\theta)$, where $\theta$ denotes a vector of model primitives. Throughout the article, we consider a complete information environment. Thus, each element in $\theta$ is observable to all the players in game $\Gamma(\theta)$. This game is a multilateral sequential infinite-horizon bargaining game with random proposers and with a unanimity rule. Let the set of all players (factions) be denoted by $I^{\text {gov }}=\{1, \ldots, n\}$. One of the primitives in $\theta$ whose effect is of particular interest in the literature is the voting weights of the parties (see, e.g., Gamson [1961], Browne and Frendreis [1980], and Ansolabehere et al. [2005]). We use the relative voting weights (or proportion of seats) of the parties in the government and denote them by $w=\left\{w_{i}\right\}_{i=1}^{n}$, where $w_{i} \in$ $[0,1]$ denotes player $i$ 's relative voting weights and $\sum_{i=1}^{n} w_{i}=1$ holds. ${ }^{6}$

Players bargain over the surplus from government formation, which is normalized to one. An allocation to players is denoted by $y \in Y$, where $Y$ is defined by

$$
Y=\left\{\left\{y_{i}\right\}_{i=1}^{n} \mid y_{i} \in[0,1], \quad \forall i \in I, \quad \sum_{i=1}^{n} y_{i} \leq 1\right\}
$$

so that $y_{i}$ is the amount that player $i$ obtains from government formation. Notice here that the size of the surplus is fixed and normalized to 1 . The players' preferences are expressed by the utility function $u_{i}(y)=y_{i}$, that is, we assume transferable utility. These features of the bargaining game $\Gamma(\theta)$ are common to standard bargaining models of government formation as in the works of Baron and Ferejohn (1989) and Diermeier et al. (2003).

Another issue of interest in the literature is the formateur advantage. In many models of government formation, a formateur is selected randomly if no party holds a majority of the seats in the legislature. Letting $r=\left\{r_{j}\right\}_{j=1}^{n}$, we write $r_{i}=1$ to mean player $i$ is a formateur and $r_{i}=0$ to mean he is not. We assume that there is only one formateur.

Whether player $i$ is a formateur may be the result of some random event. The way we formulate the procedure of the game is as follows. In the first period, a faction, $p \in I^{\text {gov }}$, is randomly recognized as a proposer with the probability of

[^4]$\frac{w_{p} \exp \left(\alpha w_{p}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}$,
where $\alpha$ is a one-dimensional parameter. The proposer offers an allocation of cabinet seats and side-payments, which we represent in terms of payoff vector $y=\left(y_{1}, \ldots, y_{n}\right)$. After observing the offer, factions sequentially respond whether they accept or reject the offer. We assume that the unanimity agreement is necessary for government formation; if all the factions have accepted the offer, the offer is implemented as proposed, and a government is formed. If not, the game goes to the second stage, and faction $i$ (which can coincide with the same $p$ in the previous period) is randomly recognized with the probability of
$\frac{w_{i} \exp \left(\alpha w_{i}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}$.
This formulation enables us to test the scale effect of factions. If an estimated $\alpha$ is large, a larger faction is increasingly more likely to be recognized as a proposer. If $\alpha$ is equal to zero, it implies that the recognition probability is $w_{i}$, that is, the probability is exactly proportional to the faction's voting weight.

The following procedure is exactly the same as in the first period, and the game continues until all the factions accept an allocation offer. We assume that factions discount the future with a common discount factor $\delta \in(0,1)$. If an allocation, $y$, is agreed to in stage $\tau$, faction $i$ will obtain $\delta^{\tau-1} y_{i}$. Otherwise, all factions will have a payoff of 0 .

Up to this point, our model has $\alpha$ and $\delta$ as primitive parameters. In the next subsection, we argue why $\alpha$ is interpreted as the parameter regarding the scale effect and $\delta$ as the parameter regarding the formateur effect.

To solve the model for an equilibrium, we define the following concepts. A history is a specification of a finite sequence of the actions taken on each date in the sequence up to a certain point. A strategy for faction $i$ is a sequence of actions that specifies what to do at every history where it must act, and a strategy profile is an $n$-tuple of strategies, one for each faction. A strategy profile is subgame perfect if and only if no faction can make itself strictly better off by deviating from its strategy on any single date.

Let an equilibrium allocation be denoted by $y^{*}(\theta)=\left\{y_{i}^{*}(\theta)\right\}_{i=1}^{n}$. In order to connect the model prediction to data observation, we need the following condition: the equilibrium allocation $y^{*}(\theta)$ of the bargaining game $\Gamma(\theta)$ is unique. This condition requires that the equilibrium concept generates a unique equilibrium allocation. Note that uniqueness of equilibrium allocation does not necessarily mean uniqueness of equilibrium. In Section 6, we return to this condition to discuss the applicability of the method to other government formations.
2.2 Uniqueness of Equilibrium Payoffs

The model has multiple subgame-perfect equilibria (SPEs). In a similar class of multilateral bargaining models (see Sutton [1986] and Baron and Ferejohn [1989]), any individually rational payoff is shown to constitute an SPE
outcome for $\delta$ close to 1 . Proposition B. 1 in Appendix B shows that a stronger version of the result applies to our model with the unanimity rule as well. That is, with subgame perfection alone, the above stated condition requiring uniqueness of the equilibrium allocation is not satisfied.

Facing this multiplicity of equilibrium allocations, the literature turned its focus to stationary subgame-perfect equilibrium (SSPE), that is, SPE in the class of stationary strategies. A strategy profile is called stationary if it does not depend on the current date and past history. Eraslan (2002) recently showed that we can find a unique pair of equilibrium payoffs (though there can be multiple SSPE) by focusing on stationary strategies in a more general model with a $q$-quota majority agreement. The model here requires unanimity for agreement. In the following proposition, we obtain a closed-form solution for the unique equilibrium payoffs of the model.

Proposition 1 (Eraslan 2002). In SSPE, factions agree in the first period. The SSPE payoff is unique. The ex-ante (before a proposer is chosen) payoff for a faction $i$ is,
$E\left(y_{i}^{*}(\alpha, \delta, w)\right)=\frac{w_{i} \exp \left(\alpha w_{i}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}$,
where $E$ denotes an expectation operator. The ex post payoff for the proposer $i$ (such that $r_{i}=1$ ) in the first period and the one for nonproposer $j$ (such that $r_{j}=$ 0 ) are, respectively,
$y_{i}^{*}(\alpha, \delta, r, w)=1-\sum_{j \neq i} \delta \frac{w_{j} \exp \left(\alpha w_{j}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}$
and
$y_{j}^{*}(\alpha, \delta, r, w)=\delta \frac{w_{j} \exp \left(\alpha w_{j}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}$.

Proof. Replace the $q$-majority in Eraslan's (2002) proofs by unanimity and make discount factors equal for all players.

This characterization has a natural and intuitive meaning; a faction that is chosen as a proposer in the first period is making an offer so that any other faction does not make itself better off by rejecting that offer and going to the next stage. As a result, all the factions agree with that offer in the first period. In other words, the proposer in the first stage can make an offer that is favorable enough for that proposer and for the other parties to ensure that the other parties will not regret the proposal because if they reject it the next proposal may be less favorable.

We use this equilibrium characterization for the estimation of the model in the following sections. The implication of this characterization is as follows. The
positive (or negative) value of $\alpha$ implies increasing (decreasing) returns to the scale of the size of a faction, whereas $\alpha=0 \mathrm{implies}$ constant returns to scale. The value of $\delta$ is low if the formateur has an advantage in obtaining seats, whereas a higher $\delta$, close to 1, implies little formateuradvantage. Before turning to the empirical part of this article, we make further arguments for employing this model.

### 2.3 Discussion of the Model

The SSPE of the model gives us further justification for employing this model. The first feature is about the timing of the agreement. The model predicts that the agreement is immediate. As described in Appendix A, the Japanese governments were formed in a very short time during the period from 1958 to 1993. It took at most only 3 days after the Prime Minister-designate was selected by the Diet, whereas the mean time to government formation in Italy is 4.98 weeks. ${ }^{7}$ Compared with other democracies, Japan's bargaining period is exceptionally short. We can call this an "immediate" formation when considering the time necessary for the Prime Minister-designate to make an offer. Hence, we can say that the model prediction exactly matches the historical observation that the cabinet formation was immediate.

Another feature of the SSPE is that all the players have a positive payoff. This results from the assumption of unanimity. For example, if we employ the majority rule for agreement, there should be a significant number of factions that cannot obtain a cabinet post. This equilibrium characterization made us to choose unanimity as a decision rule because the historical facts presented in Appendix A show that any faction of significant, nonnegligible voting weights obtained cabinet seats in most of the governments.

## 3. Data

We use Japanese data for the period from 1958 to $1993 .{ }^{8}$ We collected data on the number of LDP seats in the lower house, the size of factions at the time of cabinet formation, and the allocation of cabinet seats to factions, including the identity of the Prime Minister's faction. We collected this information from Sato and Matsuzaki (1986) and Kitaoka (1995). ${ }^{9}$ Table 1 presents the descriptive statistics of the data.

Forty-four cabinets were formed during this period, and the LDP maintained a majority in the lower house throughout the period. Factions in the LDP

[^5]Table 1. Descriptive Statistics

|  | Mean | SD | Min | Max |
| :--- | :--- | :--- | :--- | :---: |
| Number of faction | 8.43 | 2.02 | 5 | 12 |
| Faction size (total of LDP seats $=1$ ) | 0.114 | 0.067 | 0.007 | 0.281 |
| Number of cabinet posts by a faction | 2.57 | 2.11 | 0 | 9 |
| Size of Prime Minister's faction | 0.178 | 0.039 | 0.090 | 0.281 |

changed over time, and the number of factions ranged between 5 and 12 , with an average of 8.4 factions. ${ }^{10}$ Because we use separate data for each faction in each government, the total number of equations is 415 , though the number of observations of formed governments is 44 .

Figures 1-3 present the main features of our data. Figure 1 depicts the histogram of the relative size of the factions. Factions bigger than 0.2 are very few, whereas those smaller than 0.2 are more prevalent, at about the same proportion to one another. Figure 2 shows the histogram of the size of the Prime Minister's faction. A readily apparent pattern of the data in these figures is that the bigger the size of a faction, the more likely it is that it will be chosen as a proposer. This validates our model setup, which assumes that the probability to be recognized as a proposer is a function of the faction size.

Figure 3 depicts the relationship between the proportion of cabinet posts (i.e., the number of cabinet posts one faction obtained out of all the cabinet posts) and the faction size. There seems to be a positive correlation as overall. ${ }^{11}$ This, however, looks different if we look into it more carefully. The relationship is not clear for the faction with Prime Minister posts. ${ }^{12}$ This also confirms our assumption that cabinet posts have different levels of importance.

No significant bureaucratic reform was implemented during the data period. ${ }^{13}$ The numbers of cabinet posts are fairly constant, with 21 cabinet posts being constant throughout this period. Minor changes occurred in the early

[^6]

Figure 1. Histogram of Faction Size.

1970s when three new agencies were created (three posts were added accordingly) and in 1984 when one agency was closed. ${ }^{14}$ There was no significant change in the electoral system during the data period.

## 4. Econometric Specification and Estimation Method

### 4.1 Econometric Specification

Allocations that researchers can observe in the data are those of ministerial seats, whereas what player $i$ cares about is its own payoff, $y_{i}$. Thanks to Proposition 1, we have already established that the equilibrium allocation $\left\{y_{i}^{*}\right\}_{i=1}^{n}$ satisfies the uniqueness condition as in Section 2.1. To proceed further toward empirical investigation, we explain how to connect the model predictions to observations. To do so, we first introduce some notations.

Let $K \equiv\{1, \ldots, k\}$ denote the set of ministerial posts, and let $x_{i}=\left[x_{i 1}, \ldots\right.$, $\left.x_{i k}\right] \in\{0,1\}^{k}$ denote the allocation of cabinet seats to player $i$, where $x_{i j}=1$ means that player $i$ obtained the post of Minister $j$, and $x_{i j}=0$ otherwise. The

[^7]

Figure 2. Histogram of Factions of Prime Ministers.
relative weights of the ministers to be estimated are $\beta=\left[\beta_{1}, \ldots, \beta_{k}\right]^{\prime} \in[0,1]^{k}$. We assume that $\beta$ does not depend on player identity, that is, all players evaluate ministerial posts identically. To keep our argument consistent with the normalization in $\Gamma(\theta)$, we normalize the sum of the weights to be 1 , that is, $\sum_{j=1}^{k} \beta_{j}=1$. Now, $x_{i} \beta$ represents the value of ministerial posts player $i$ obtains.

Now, we decompose player $i$ 's ex post payoff, $y_{i}$, into the part related to an observable (to researchers) part and an unobservable part. Specifically, we employ the following specification:
$y_{i}=x_{i} \beta+\varepsilon_{i}$.
We interpret the second part, $\varepsilon_{i}$, as the net amount of side-payments received by player $i$ (players are assumed to be able to make and receive monetary sidepayments). In this specification, the gain in the bargaining game for any party is the sum of the rent from ministerial posts and side-payments. Although the first part, $x_{i} \beta$, takes only discrete values given $\beta, y_{i}$ can take any value in $[0,1]$ if we further assume that side-payments take continuous values. This specification allows for any division of the surplus over which the players bargain, which we want to consider as the "pie."

As it seems that there is no factor that systematically affects $\varepsilon_{i}$ across players and that side-payments are not observable, we treat $\varepsilon_{i}$ as a random variable


Figure 3. Proportion of Cabinet Posts and Faction Size. Lines are OLS fitted lines for (i) Prime Minister's faction (line with steeper slope) and (ii) factions without Prime Minister's posts.
drawn from an identical distribution. ${ }^{15}$ Side-payments across parties, however, are not independently distributed. The reason is that they add up to a constant value, giving $\left\{\varepsilon_{i}\right\}_{i=1}^{n}$ a correlation of -1 . In order to avoid technical difficulties, we assume that the side-payments will add up to 0 , that is, there is no outside fund. Then, side-payments have to satisfy the budget balance, $\sum_{l=1}^{n} \varepsilon_{l}=0$, or $\sum_{j \neq i} \varepsilon_{j}=-\varepsilon_{i}$, for any $i \in I^{\mathrm{gov}}$.

Hence, we have only $n-1$ degrees of freedom for $n$ draws of $\varepsilon_{i}$. A player receives some side-payment, which has a perfect negative correlation with the net side-payment of all the other players in the government, whereas $\varepsilon_{i}$ are independent among those players.

We observe a similar feature in other variables as well. The number of seats is fixed in each government formation, and each post is assigned to only one player. This implies that the information on the allocation of posts contained in

[^8]the data for $n-1$ players is equal to the information contained in the data for $n$ players. That is to say that the data $\left\{x_{l}\right\}_{l=1}^{n}$ has the following property: $\sum_{l=1}^{n} x_{l}=[1, \ldots, 1]$, or $\sum_{j \neq i} x_{j}=\left[1-x_{i 1}, \ldots, 1-x_{i k}\right]$. In other words, information on the post allocation of one player is always redundant. Similarly, the sum of the voting weights is fixed at 1 , that is, $\sum_{l=1}^{n} w_{l}=1$, or $\sum_{j \neq i}^{n} w_{j}=$ $1-w_{i}$ for any $i \in I^{\text {gov }}$.

We can find the voting weights of one player if we know the size of all the other players. Thus, we can also ignore the information on the voting weights of one government party because it is redundant.

Following this reasoning, we ignore the data for one player and use only $n-1$ equations for each government formation. This enables us to ignore the correlation among $\varepsilon_{i}$. If we use only $n-1$ players for estimation, $\varepsilon$ are no longer correlated since they have $n-1$ degrees of freedom. If we denote the party to be ignored by $p$ (proposer), we have the following $n-1$ equations for a government formation: $y_{i}^{*}(\alpha, \delta, r, w)-x_{i} \beta=\varepsilon_{i}$ for any $i \in I^{\text {gov }} \backslash\{p\}$.

From Proposition 1, which provides the expression for $y_{i}^{*}(\alpha, \delta, r, w)$, we have
$x_{i} \beta+\varepsilon_{i}=1-\sum_{j \neq i} \delta \frac{w_{j} \exp \left(\alpha w_{j}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}$
for proposer's faction $i$, and
$x_{j} \beta+\varepsilon_{j}=\delta \frac{w_{j} \exp \left(\alpha w_{j}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}$
for nonproposer $j \in I^{\text {gov }} \backslash\{i\}$. Therefore, each government formation provides us with a system of $n$ equations as shown above. As discussed previously, we ignore the data for one faction and use only $n-1$ equations for each government formation for consistency. This enables us to ignore the correlation among $\varepsilon_{i}$. For convenience, we choose the faction of the proposer to be removed from our estimation. ${ }^{16}$ Thus, we have the following $n-1$ equations for a government formation:
$\delta \frac{w_{j} \exp \left(\alpha w_{j}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}-x_{j} \beta=\varepsilon_{j}$
for any nonproposer factions $j \in I^{\text {gov }} \backslash\{i\}$.
On this system of $n-1$ equations for each government, we have a trivial solution of $\delta=0, \beta_{1}=1$, and $\beta_{k}=0$ for $k \in K \backslash\{1\}$ if we ignore $\boldsymbol{\varepsilon}=\left(\varepsilon_{1}, \ldots, \varepsilon_{n}\right)$. That is, when we estimate using the above specification, estimations may give values very close to $\delta=0, \beta_{1}=1$, and $\beta_{k}=0 .{ }^{17}$ This,

[^9]however, should not be a reasonable solution as the weights of ministers other than the Prime Minister are 0 . We avoid this trivial solution by dividing both sides of the equation by $\delta$ to prevent $\delta$ from being 0 , that is,
$\frac{w_{j} \exp \left(\alpha w_{j}\right)}{\sum_{l=1}^{n} w_{l} \exp \left(\alpha w_{l}\right)}-\frac{x_{j} \beta}{\delta}=\frac{\varepsilon_{j}}{\delta}$.

### 4.2 Estimation Method

We estimate the primitive parameters of the model, that is, $\beta$ (ministerial weights), $\alpha$ (scale effect), and $\delta$ (formateur effect) using the ML method. We index each government in the data with additional subscript $t \in$ $\{1, \ldots, T\}$, where $T$ is the total number of governments in the data. Note that, in general, the number of players in government $n$ depends on $t$, so we express this dependence by $n(t)$. Some of the other primitives also depend on $t$; the data in government formation $t$ consist of (i) who the formateurs are ( $r_{t}=$ $\left.\left[r_{1, t}, \ldots, r_{n(t), t}\right]\right)$, (ii) how the voting weights are distributed $\left(w_{t}=\left[w_{1, t}, \ldots\right.\right.$, $\left.w_{n(t), t}\right]$, and (iii) how the ministerial posts are allocated to the parties ( $x_{t}=$ $\left.\left[x_{1, t}, \ldots, x_{n(t), t}\right]\right)$. The unobservable part, $\varepsilon_{i, t}$, also depends on $t$.

As we have argued above, there is no reason to assume that the distributions of $\varepsilon_{i, t}$ are nonidentical; thus, for the purpose of this article, we will assume them to be identically distributed. We also have no reason to assume that side-payments are systematically correlated among parties or across time. ${ }^{18}$ Also, $\varepsilon_{i, t}$ cannot take any value out of $[-1,1]$. Hence, we assume that $\varepsilon_{i, t}$ follows an independently and identically distributed (i.i.d). Beta distribution of the first kind with a mean of 0 (whose density is $f(\cdot)$ ) and estimate the model using the ML method. Let $\mathbf{r}=\left(r_{1}, \ldots, r_{T}\right), \mathbf{w}=\left(w_{1}, \ldots, w_{T}\right)$, and $\mathbf{x}=\left(x_{1}, \ldots\right.$, $\left.x_{T}\right)$. Then, the likelihood function can be written as

$$
\begin{aligned}
& L(\beta, \alpha, \delta, \sigma \mid \mathbf{r}, \mathbf{w}, \mathbf{x})=\prod_{t=1}^{T} \prod_{i=1}^{n(t)-1} f\left(\varepsilon_{i, t}^{\prime} \mid r_{i, t}, w_{i, t,}, x_{i, t} ; \quad \beta, \alpha, \delta, \sigma\right) \\
& =\prod_{t=1}^{T} \prod_{i=1}^{n(t)-1} \frac{1}{(2 / \delta)^{2 \sigma-1} \operatorname{Beta}(\sigma, \sigma)}\left(\frac{1}{\delta}+\frac{w_{i, t} \exp \left(\alpha w_{i, t}\right)}{\sum_{l=1}^{n} w_{l, t} \exp \left(\alpha w_{l, t}\right)}-\frac{x_{i, t} \beta}{\delta}\right)^{\sigma-1} \\
& \quad \times\left(\frac{1}{\delta}-\frac{w_{i, t} \exp \left(\alpha w_{i, t}\right)}{\sum_{l=1}^{n} w_{l, t} \exp \left(\alpha w_{l, t}\right)}+\frac{x_{i, t} \beta}{\delta}\right)^{\sigma-1} .
\end{aligned}
$$

where $\operatorname{Beta}(\sigma, \sigma)$ is the beta function with the symmetric imposition (expressed by parameter $\sigma$ ). ${ }^{19}$

[^10]As we have $\mathbf{r}, \mathbf{w}$, and $\mathbf{x}$ as observable data, we can obtain an ML estimate by maximizing the log-likelihood function with respect to $\beta, \alpha, \delta$, and $\sigma$, given the data. ${ }^{20}$ Standard errors (SEs) can be obtained by the nonparametric bootstrap method. ${ }^{21}$ More specifically, we randomly drew observations from the original data, without replacing the original data, the same number of times as the original number of observations. We repeated this process 500 times and used the average as the point estimates and the standard deviations as the SEs for the parameters.

## 5. Results

### 5.1 Estimates

The results are presented in Table 2. The reported SE is obtained using the bootstrap method.

The last column in Table 2 reports the results with a restriction that $\alpha=0$. The results are almost the same as the ones without the restriction. The likelihood decreases only by 0.22 . The likelihood ratio test cannot reject the hypothesis $\alpha=0$ even at the $1 \%$ level. The inability to reject the hypothesis is also clear from the robust SE of $\alpha$ reported in Table 2. Hence, we cannot reject the hypothesis when there is no scale effect of the voting weights.

### 5.2 Discussion

The above results tell us a number of important things. First, we find that the post of the Prime Minister has by far the highest value. This should not be surprising. The power of the Prime Minister results from many factors: (i) constitutionally, he is the head of the cabinet, and all the cabinet decisions need his signature; (ii) the budget has to be signed by the Prime Minister before it is submitted to the legislature; and (iii) he can also control the legislative process by having the power to dissolve the legislature. Although we do not know how much each of these factors contributes, we are not surprised that the Prime Minister has the highest value.

The second observation is that pork-related posts such as the Ministers of Construction and Transport have the next highest values to the Prime Minister. These are alleged to be "dirty" posts. Ramseyer and Rosenbluth (1993) mention the Ministry of Construction as having been characterized as "a politically

[^11]Table 2. Parameter Estimates

|  | $(1)$ | $(2)$ |
| :--- | ---: | ---: |
| Log-likelihood | -623.1998 | -622.9787 |
| $\sigma$ | $377.7728(27.8948)$ | 377.6669 |
| $\alpha$ | $-0.3573(0.5592)$ | 0.0000 |
| $\delta$ | $0.8058(0.0491)$ | 0.8054 |
| Prime Minister | $0.2632(0.0531)$ | 0.2595 |
| Transport | $0.0597(0.0081)$ | 0.0599 |
| Construction | $0.0583(0.0102)$ | 0.0587 |
| Economic planning | $0.0532(0.0094)$ | 0.0533 |
| Agriculture | $0.0492(0.0061)$ | 0.0498 |
| Defence | $0.0460(0.0071)$ | 0.0460 |
| Finance | $0.0455(0.0086)$ | 0.0461 |
| Labor | $0.0433(0.0072)$ | 0.0428 |
| International trade and industry | $0.0409(0.0076)$ | 0.0410 |
| Cabinet secretary | $0.0405(0.0210)$ | 0.0430 |
| Health and welfare | $0.0396(0.0070)$ | 0.0399 |
| Science and technology | $0.0378(0.0083)$ | 0.0379 |
| Management and coordination | $0.0370(0.0073)$ | 0.0369 |
| Home affairs | $0.0340(0.016)$ | 0.0344 |
| Education | $0.0340(0.0080)$ | 0.0333 |
| Posts and telecomunications | $0.0309(0.0067)$ | 0.0311 |
| Foreign affairs | $0.0281(0.0067)$ | 0.0274 |
| Justice | $0.0234(0.0066)$ | 0.0233 |
| Hokkaido development | $0.0210(0.0076)$ | 0.0210 |
| National public safety | $0.0144(0.0115)$ | 0.0147 |

SEs are in parentheses. Ministers are in the order of weight. Ministers with weights higher than the average weight (excluding the Prime Minister) are in bold fonts. Column (2) corresponds to the case with the restriction $\alpha=0$.
driven pork wagon" (124). There are many other academic as well as journalistic accounts for the claim that these two posts have strong influence on porkbarrel projects (see, e.g., Woodall 1996:110-119). The Ministry of Construction is in charge of the construction of dams, bridges, and roads. The Ministry of Transport controls the procurement process of ports, airports, railways, and highways. Among the alleged pork-related posts, the estimated value of the Ministry of Posts and Telecommunications is not high. Although it is well known that postmasters were among the strongest supporters of the LDP in the national elections (see, e.g., Maclachlan 2004), our results suggest that their activities were less relevant to the actual gains of the LDP. ${ }^{22}$

A third point, which is most clearly exemplified by the Minister of Foreign Affairs, is that ministerial weights are not necessarily correlated with the seniority of the appointed politicians. It is often said that the Ministers of Foreign

[^12]Affairs and of Justice are very prestigious positions for senior politicians (see, e.g., Sato and Matsuzaki [1986] and Kato and Laver [1998]). However, the values of these posts to the factions are very low in our results. For instance, the value of the Minister of Foreign Affairs (0.0281) is half of that of the Minister of Construction (0.0583). Remember that our model focuses on the bargaining among factions and the value of the ministers from the factions' perspective. The factions have politicians of diverse seniority, and our result demonstrates that seniority was a minor concern in cabinet formation from a faction's perspective.

Fourth, $\delta=0.806$ implies that there is a significant formateur advantage. If $\delta=1$, there is no formateur advantage, and the parties' gains are only dependent on their size. However, $\delta<1$ implies that a proposer gains more than a nonproposer even if their size is the same. For example, if a faction has the size of 0.2 , then in the absence of a scale effect, $\delta=0.8$ implies that being a proposer will give the faction 0.36 , whereas the gain is 0.16 if the faction is not a proposer.

Finally, $\alpha=0$ cannot be statistically rejected at a conventional level. This implies that we cannot reject the hypothesis that there is no significant scale effect of faction sizes when bargaining over cabinet seats. Specifically, cabinet post allocations are proportional to the size of the factions. This proportionality, however, applies only to the parties that are not the Prime Minister's. With the estimated $\alpha=0$ and $\delta=0.8$, the Gamson hypothesis applies to nonproposer factions, whereas the proposers gain more than their size. This result is consistent with that of Ansolabehere et al. (2005), though their estimation results are based on European data with exogenously determined ministerial weights.

### 5.3 A Counterfactual Experiment

Here we show a simple counterfactural experiment to see how a change in our bargaining environment affects the equilibrium allocation. Specifically, we consider the effect of a merger of the factions on the allocation of a pie. An advantage of a structural estimation is that we can measure the effect of an exogenous change in an equilibrium framework. In the present article, however, experiments would not provide very interesting results because the model cannot predict the actual allocation of the posts, though the model can predict the share of the pie that each faction will obtain.

Now suppose that there are five factions and that they all have the same share of seats, that is, $w_{i}=0.2, \forall i=1, \ldots, 5$. With the estimated parameter, the Prime Minister's faction has a size of 0.3553 , and each of the other four factions has 0.1611 . Now, assume that two of the factions that are not the Prime Minister's merge. Then we can compute the change in the size of the pie. Table 3 shows the portion of the pie each party obtains. Since the scale effect is small, the overall effect is not very large. Also, because of the negative scale effect, the merged faction gains less (it has only 0.3086 after the merger compared to the sum of the premerger shares of 0.3222 ).

Table 3. Sizes of the pie before and after the merger

|  | Before merger | After merger |
| :--- | :---: | :---: |
| Prime Minister's factions | 0.3553 | 0.3599 |
| Merged faction | 0.1611 | 0.3086 |
| Other factions | 0.1611 | 0.1657 |

## 6. Applicability to Other Government Formations

In this section, we discuss how our bargaining model and estimation procedure can be applied to other government formations. First, we describe a class of estimable bargaining models. The main requirement is that the bargaining model has a unique equilibrium outcome, which is the same as the uniqueness condition described in Section 2.1. Second, we discuss the econometric specification and the estimation method.

### 6.1 Class of Estimable Bargaining Models

One distinct feature of Japanese government formation considered above is that all the players (LDP factions) join a government formation. In general (in European countries and recent Japan), however, we rarely observe all the political players joining a government formation. In order to analyze such cases, one needs to make a distinction between the set of players in the government, $I^{\text {gov }}=\{1, \ldots, n\}$, and the set of the remaining players, who are out of the government, $I^{\text {out }}=\{n+1, \ldots, N\}$, where $n$ is an integer that satisfies $2 \leq$ $n \leq N$ and $N$ is the total number of political players. If we model coalition formation, $n$ will be an endogenous variable. Players bargain over the surplus from government formation, which is normalized to one. An allocation to the players is denoted by $y \in Y$, where $Y$ is defined by

$$
Y=\left\{\left\{y_{i}\right\}_{i=1}^{N} \mid y_{i} \in[0,1] \quad \forall i \in I, \quad \sum_{i=1}^{N} y_{i} \leq 1\right\}
$$

so that $y_{i}$ is the amount that player $i$ obtains from government formation. For parties that are out of the government, that is, $i \in I^{\text {out }}$, we write $y_{i}=0$. Note that the size of the surplus is fixed and normalized to 1 . Players' preferences are expressed by the utility function $u_{i}(y)=y_{i}$, that is, transferable utility is assumed. These features of the bargaining game $\Gamma(\theta)$ are common to the standard bargaining models of government formation as in the works of Baron and Ferejohn (1989) and Diermeier et al. (2003). As described in Section 2.1, we need the following requirement for the model in order to connect the model prediction to the data observation:

Condition 1. The equilibrium allocation $y^{*}(\theta)$ of the bargaining game $\Gamma(\theta)$ is unique.

The method we present here does not depend on the equilibrium concept applied to the game $\Gamma(\theta)$. The above condition requires that the equilibrium concept generates a unique equilibrium allocation. Note that uniqueness of equilibrium allocation does not necessarily mean uniqueness of equilibrium. ${ }^{23}$

One of the primitives in $\theta$ whose effect is of particular interest in the literature is the voting weights of the parties. We use the relative voting weights of the parties in the legislature and denote them by $w=\left\{w_{i}\right\}_{i=1}^{N}$, where $w_{i} \in[0$, 1] denotes player $i$ 's relative voting weights and $\sum_{i=1}^{N} w_{i}=1$ holds. The effect of the voting weights on the equilibrium allocation is measured by function $h\left(w ; \theta_{w}\right)$ with parameter $\theta_{w}$ (possibly multidimensional).

Another issue of interest in the literature is the formateur advantage. In many models of government formation, a formateur is selected randomly if no party holds a majority of the seats in the legislature. We can also incorporate this factor into our specification by making $y_{i}^{*}(\theta)$ dependent on the realization of uncertainty. We write $r_{i}=1$ to mean player $i$ is a formateur and $r_{i}=0$ to mean he is not. We assume that there is only one formateur. Whether player $i$ is a formateur may be a result of some random event, depending on the model specifications. Letting $r=\left\{r_{i}\right\}_{i=1}^{n}$, we measure the effect of a formateur's identity on the equilibrium allocation by function $g\left(r ; \theta_{r}\right)$ with parameter $\theta_{r}$ (possibly multidimensional).

We include $\boldsymbol{r}$ in the model primitives, $\theta$, so that our bargaining game $\Gamma(\theta)$ starts with the formateur's identity determined. It will be clear that there is no loss in this formation because our empirical analysis does not use information on how the formateur is chosen. We express player $i$ 's ex post payoff by $y_{i}^{*}(\theta)=y_{i}^{*}\left(h\left(w ; \theta_{w}\right), g\left(r ; \theta_{r}\right), \theta^{\prime}\right)$ where $\theta=\left(w, r, \theta_{w}, \theta_{r}, \theta^{\prime}\right)$ and $\theta^{\prime}(\mathrm{if}$ any) denotes the model primitives other than $w, r, \theta_{w}$, and $\theta_{r}$.

### 6.2 Econometric Specification and Estimation Method

The difference between our model and the Japanese case is that there are parties that are not in the government. We assume $y_{i}=0$ for the parties out of the government, $I^{\text {out }}=\{n+1, \ldots, N\}$. For the parties in the government, that is, $I^{\text {gov }}=\{1, \ldots, n\}$, the specification is the same as in the Japanese case described in Section 4. Thus, we keep the linearity assumption of player $i$ 's ex post payoff: $y_{i}=x_{i} \beta+\boldsymbol{\varepsilon}_{\boldsymbol{i}}$, where $\boldsymbol{\varepsilon}_{\boldsymbol{i}}$ corresponds to the unobservable heterogeneity in player $i$ 's gain in government formation as discussed above. We then have $y_{i}^{*}\left(h\left(w ; \theta_{w}\right), g\left(r ; \theta_{r}\right)\right)-x_{i} \beta=\boldsymbol{\varepsilon}_{i}$ and can construct a likelihood function similar to the one in Section 4 written as

[^13]\[

$$
\begin{aligned}
& L\left(\beta, \theta_{w}, \theta_{r}, \sigma \mid \mathbf{r}, \mathbf{w}, \mathbf{x}\right)=\prod_{t=1}^{T} \prod_{i=1}^{n(t)-1} f\left(\varepsilon_{i, t} \mid r_{t}, w_{t}, x_{i, t} ; \beta, \theta_{w}, \theta_{r}, \sigma\right) \\
& =\prod_{t=1}^{T} \prod_{i=1}^{n(t)-1} \frac{1}{2^{2 \sigma-1} \operatorname{Beta}(\sigma, \sigma)}\left[1+\left(y_{i}^{*}\left(h\left(w_{t} ; \theta_{w}\right), g\left(r_{t} ; \theta_{r}\right)\right)-x_{i, t} \beta\right)\right]^{\sigma-1} \\
& \quad \times\left[1-\left(y_{i}^{*}\left(h\left(w_{t} ; \theta_{w}\right), g\left(r_{t} ; \theta_{r}\right)\right)-x_{i, t} \beta\right)\right]^{\sigma-1},
\end{aligned}
$$
\]

where $\mathbf{r}=\left(r_{1}, \ldots, r_{T}\right), \mathbf{w}=\left(w_{1}, \ldots, w_{T}\right), \mathbf{x}=\left(x_{1}, \ldots, x_{T}\right)$, and $\operatorname{Beta}(\sigma, \sigma)$ is the beta function with the symmetric imposition (expressed by parameter $\sigma$ ).

One can then estimate the primitive parameters of the model, that is, $\beta$ (ministerial weights), $\theta_{w}$ (scale effect), and $\theta_{r}$ (formateur effect) using the ML method. SEs can be obtained by the bootstrap method.

## 7. Concluding Remarks

This article has structurally estimated the relative ministerial weights in parliamentary democracies. Specifically, we have combined a bargaining model of government formation with ML estimation. The data required for the estimation are (i) who the formateurs are, (ii) what each party's voting weight is, and (iii) what ministerial seats each party obtains. By using the Japanese data from the period between 1958 and 1993, we find that the Ministers of Construction and Transport, who allegedly have a strong influence on pork-barrel spending, have high estimated values, whereas the estimates for the Ministers of Foreign Affairs and Justice have low weights, although they are regarded as the more prestigious ministers. We also find that voting weights do not have a significant effect on the returns to scale, that is, the factions' bargaining power is almost identical to their size. Finally, we find evidence of formateur advantage.

Our method can be applied to other parliamentary democracies such as European parliamentary countries by employing an appropriate bargaining game. One of the benefits of studying European parliamentary democracies is that one can conduct a comparative analysis of ministerial weights. It would be interesting to know how the difference in these weights depends on certain institutional and noninstitutional features. In such cases, some modifications would be necessary to our maintained assumption that each party values the ministries identically. Although this assumption is reasonable for Japan during the period in the present study, it may not be the case in some European countries where parties are ideologically divergent and have different preferences over policies. This and other interesting issues on government formation are left for future research.

## Appendix A

Features of Government Formation in Japan, 1958-1993
In this appendix, we describe distinct characteristics of government formation in Japan during the period from 1958 to 1993, which our formal model is based upon.

Japan employs a parliamentary regime with a bicameral legislature. The legislature is called the Diet, which consists of the House of Representatives (the lower house) and the House of Councilors (the upper house). In the House of Representatives, a single nontransferable voting (SNTV) system, which (usually) has three to five seats per district, was used until 1993, whereas the members of the House of Councilors were chosen by proportional representation. ${ }^{24}$ The Prime Minister is the head of the cabinet and is designated by the legislature. The House of Representatives can make the final decision if the two Houses disagree on the designation for more than 10 days. Once designated, the Prime Minister-designate selects the cabinet members; the majority of those selected must be legislators. After all the cabinet members are selected by the Prime Minister-designate, the Emperor appoints him as the Prime Minister.

A member of the cabinet can be appointed to multiple posts, but each ministerial post can accommodate only one person. The Cabinet Law (Law No. 5 of 1947) specifies the maximum number of cabinet members. As the Prime Minister has the sole authority to form the cabinet, he can also dismiss and reform it. The reformation process is the same as the formation of a new cabinet. The members of the cabinet must resign en masse once the House of Representatives passes a no-confidence resolution. In such a case, a new Prime Minister-designate is chosen by the Diet in the same way as described above.

The following facts from the period between 1958 and 1993 provide us with some details about the actual process of government formation in Japan.

1. The LDP maintained a majority in the House of Representatives, ${ }^{25}$ and the Prime Minister-designates were always LDP presidents.
2. No LDP faction voted for a no-confidence resolution after government formation. Only one no-confidence resolution was passed in the Diet. ${ }^{26}$
3. LDP factions played the main role in government formation.
4. No LDP faction left the LDP, and LDP factions of significant size obtained cabinet posts in most of the cabinets.
5. All the cabinets were formed within 3 days following the selection of a Prime Minister-designate. ${ }^{27}$
[^14]First, the LDP maintained a majority in the House of Representatives during the period between 1955 and 1993. This implies that the actual process of choosing a Prime Minister was an internal process by the LDP. The LDP candidates for Prime Minister, who were also the presidents of the LDP, ${ }^{28}$ always won the vote of designation. The fact that the Prime Minister-designates were from the LDP and the fact that no LDP factions voted for a no-confidence resolution imply that the government formations (and not only the choice of the Prime Minister) were an internal process in the LDP. Hence, we think that both the choice of the Prime Minister and the cabinet formation were internal processes in the LDP.

Second, LDP factions have been the primary internal organizational unit of the LDP and have played a central role in cabinet formation (see, e.g., Leiserson [1968], Sartori [1976], Kohno [1992], Ramseyer and Rosenbluth [1993], and Woodall [1996]). The LDP factions are said to have few differences with respect to their preferences on policy issues. For example, Ramseyer and Rosenbluth (1993:211) cite an interview with one leader of an LDP faction, who says that factions are not distinguishable based on policy areas.

In the actual process, the Prime Minister offers a proposal of cabinet posts to all the factions. The factions respond to the offer by agreeing or by requesting more and/or different seats, and then the offer is revised. If the factions do not agree with the revised offers, they can ask for a change in the LDP leadership or leave the LDP and/or call for a vote of no-confidence. Historically, none of these disagreements has happened. The President of the LDP has never been replaced right after the selection of a Prime Minister-designate. During the period between 1958 and 1993, no faction left the LDP, and no vote of noconfidence was ever agreed upon by any LDP factions. We interpret these facts as follows: the cabinet formation process was under unanimous agreement among all the factions. ${ }^{29}$ Another fact is that any faction of significant size obtained cabinet seats in most of the cabinets. This also supports our interpretation that the agreements were unanimous because nonunanimous agreements should (at least theoretically) always result in no cabinet seats for some factions.

Historically, no cabinet formation required more than 3 days. We interpret this as an immediate agreement for two reasons. First, the Prime Ministerdesignate needs time to put together his offer to the factions carefully. Yet, the offer was still agreed upon, and the cabinet is formed, within a short period

[^15]of time. Second, a comparison with another country suggests that the agreement was immediate. Merlo (1997) provides data for government formation in postwar Italy. Compared with his data, in which the mean is 4.98 weeks, and the maximum, 18 weeks, we interpret that an agreement within 3 days is immediate.

## Appendix B

Any Individually Rational Payoff Constitutes an SPE Allocation
In Section 2.2, we mention the multiplicity of SPEs. The following proposition shows that as long as players do not perfectly discount the future payoff (i.e., as long as $\delta$ is not zero), we can construct any pair of individually rational payoffs as an SPE allocation.

Proposition B.1. Any individually rational payoff can be supported as an SPE payoff for any $\delta \in(0,1)$.

Proof. We prove the proposition by construction. Let $n$ be the number of players and fix an arbitrary feasible allocation, $s \in S \equiv\left\{s \in R_{+}^{n}: \sum_{i=1}^{n} s_{i} \leq\right.$ 1\}. Denote an allocation for punishing player $j$ as $r^{j}=\left[r_{1}^{j}, \ldots, r_{n}^{j}\right]$, where $r_{j}^{j}=0$ and $r_{i}^{j}=s_{i}+\frac{s_{j}}{n-1}$ for $i \neq j$. Similarly, denote an allocation for punishing players $j$ and $k$ as $r^{j k} \stackrel{n-1}{=}\left[r_{1}^{j k}, \ldots, r_{n}^{j k}\right]$, where $r_{j}^{j k}=r_{k}^{j k}=0$ and $r_{i}^{j k}=s_{i}+\frac{s_{j}+s_{k}}{n-2}$, for $i \neq j, k$. In the same way, define allocations for punishing more than two players by $r^{j k l}$ and $r^{j k l m}$.

Consider the following strategy profile. (1) In the first stage, a recognized proposer offers $s$, and all the players accept it. (2) If there is a deviation from (1) by player $j$, the proposer chosen in the next stage offers $r^{j}$, and all the other players (including $j$ ) accept it. (2') If there is a deviation from (2) by player $j$, repeat (2). ( $2^{\prime \prime}$ ) If there is a deviation from (2) by another player $k$, proposer offers $r^{j k}$, and all the other players (including $j$ and $k$ ) accept it. (3) If there is a deviation from $\left(2^{\prime \prime}\right)$ by player $j$ or $k$, repeat $\left(2^{\prime \prime}\right) .\left(3^{\prime}\right)$ If there is a deviation from (3) by yet another player, $l$, the proposer offers $r^{j k l}$, and all the other players (including $j, k$, and $l$ ) accept it. (4) For deviations by additional players, construct the rest of the strategy in the same way.

Any deviation from this profile results in the continuation payoff of zero for the deviating player, whereas no deviation always produces a nonnegative payoff to the player. This does not depend on what value $\delta$ takes. Thus, no faction can make itself strictly better off by deviation, and $s$ is sustained as a SPEs.

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[^1]:    1. The existing literature has two distinctive approaches to government formation. The first approach considers that the surplus from government formation depends on the cabinet post allocation and coalition (e.g., Austen-Smith and Banks [1988, 1990] and Laver and Shepsle [1990, 1998]). The second approach assumes the surplus from government formation to be unchanged by the cabinet post allocation and parties in the coalition (e.g., Baron [1991, 1993, 1998] and Baron and Ferejohn [1989]). Following the second strand of the literature, the model in the present article assumes that the surplus does not change.
    2. Studies employing this method (often called structural estimation) in political economics include Coate and Conlin (2004), Diermeier et al. (2003, 2005), Gowrisankaran et al. (2005), Merlo (1997), and Shachar and Nalebuff (1999).
[^2]:    3. For an overview of the empirical literature, see, for example, Browne and Dreijmanis (1982), Laver and Schofield (1990), Strom (1990), Warwick (1994), and Laver and Shepsle (1996).
    4. Laver and Hunt (1992) measure the ministerial values by surveying major politicians in European democracies. Similarly, Kato and Laver (1998) survey Japanese political scientists about the ministerial ranking (excluding the Prime Minister) at the time of the 1996 general election; the Minister of Finance is judged the most important, followed by the Minister of Foreign Affairs. The results reported in this article do not necessarily contradict the results of Kato and Laver (1998). The reason we do not contradict each other is that they estimate the "importance" subjectively using evaluations by scholars, who are not players in politics, whereas we estimate the values of cabinet posts by using only the publicly observable behavioral data.
[^3]:    5. This did not happen in Japan during our data period. In our model, this is off the equilibriumpath behavior, that is, this should not happen in an equilibrium.
[^4]:    6. We can use the voting weights in the legislature. The reason for using the relative weights among government parties is for notational simplicity, and doing so does not affect our results.
[^5]:    7. See Merlo (1997) for details. Diermeier et al. (2003) also report that the mean number of attempts to form a government is 1.73 for eight parliamentary democracies in Western Europe.
    8. The LDP was formed in 1955 and maintained a majority until 1993 in the House of Representatives (see Section 3). The factions, however, were not clearly defined in the first couple of years of LDP's existence. We were not able to collect reliable data on how large each faction was and to which faction each member was affiliated with during the period from 1955 to 1957. Kohno (1992:371) also reports: "During the LDP presidential election in 1957, these leaders began to form alliances . . ., and by the end of 1957 eight factions had emerged as distinct organizational features of the LDP."
    9. We also consulted Asahi Shimbun (various issues), one of the leading daily newspapers in Japan, when information in Sato and Matsuzaki (1986) and Kitaoka (1995) was not consistent or was lacking.
[^6]:    10. For the estimation, we treated legislators who did not belong to any faction as an independent faction. The average number of seats that independent legislators obtained in each government is 0.75 . We define their size by $1 /($ number of LDP seats).
    11. Browne and Frendreis (1980) study this relationship in 132 European coalitional governments. We ran the same specification to obtain the following (numbers in parentheses are SEs):
    $\underset{R^{2}=0.663}{\text { of Seat }}=\underset{(0.006)}{-0.158} \times$ Const $+\underset{(0.043)}{1.147} \times$ Relative Seat Share.
    12. Dividing the data into the Prime Minister's faction and other factions, the results of the regressions are very different. For the Prime Minister's faction, the relationship is not very clear as the following high SEs and low $R^{2}$ suggest (numbers in parentheses are SEs):
    $\underset{R^{2}=0.007}{\%}$ of Seat $=\underset{(0.053)}{-0.151} \times$ Const $+\underset{(0.292)}{0.537} \times$ Relative Seat Share.
    On the other hand, the relationship for non-Prime Minister factions is more robust:
    $\underset{R^{2}=0.6951}{\%}$ of Seat $=\underset{(0.005)}{-0.012} \times$ Const $+\underset{(0.038)}{1.038} \times$ Relative Seat Share.
    13. The cabinet posts in this article are different from the current ones due to a significant bureaucratic reform in 2001.
[^7]:    14. The Director General of the Environment Agency (1971), the Director General of the Okinawa Development Agency (1972), and the Director General of the National Land Agency (1974) are the three posts that were added during this period. The Director General of the Administration Management Agency is the post that was removed.
[^8]:    15. One might argue that in government formation political parties give and receive side-payments strategically. We believe, however, that the allocation of ministerial seats is of more importance than side-payments in government formation and that "[g]overnment ministries are the most tangible manifestations of policy payoffs to governing parties" (Browne and Franklin 1973:454). Also, our estimation results provide a very low variance for the distribution of $\varepsilon_{i}$.
[^9]:    16. Our results do not depend on the choice of a faction we do not use. This is because the data on $n-1$ factions have exactly the same amount of information as the data on $n$ factions for the reasons discussed earlier.
    17. This is a trivial answer to the system of equations if we ignore $\varepsilon$. The answer is true for any value of $r, w$, and $\{x\}_{l=1}^{n}$.
[^10]:    18. Remember that the correlation arising from the budget balance is eliminated by using only $n(t)-1$ parties rather than $n(t)$ parties, as described above.
    19. Now, our Beta distribution of the first kind has support $[-1 / \delta, 1 / \delta]$ and a mean of 0 . See McDonald (1984) for the Beta distribution of the first kind. The notation $\varepsilon_{i, t}^{\prime}$ is used for the (onedimensional) random variable to distinguish it from the original definition of $\varepsilon_{i, t}$.
[^11]:    20. First, the parameter $\delta$ is identified by the fact that the Prime Minister party's equilibrium payoff has a different functional form than that of the non-PM parties. This is clear when one considers the situation where two parties have exactly the same seat allocation, but one party has the PM seat and the other does not. All $\beta_{k} s$ are also separately identified unless two or more posts are always occupied by one faction, that is, $\operatorname{rank}(\mathbf{x})=k$. We find that this is not the case with our data. In particular, the Prime Minister and the Cabinet Secretary usually belong to the same faction, but we still find four exceptions (out of 44 government formations).
    21. Bootstrapping is a method for estimating the distribution of an estimator by resampling the data and then treating them as if they were the population. Note that there is no need to introduce a new parameter when conducting the bootstrap calculation. For a more detailed explanation, see, for example, Horowitz (2001).
[^12]:    22. Maclachlan (2004:282) points out that "[i]n contrast to other electoral supporters of the LDP such as the farmers, big and small businesses, and the construction industry, the postmasters have neither the numbers nor the geographic concentration to function as voting blocs in their own right."
[^13]:    23. For example, although the sequential bargaining game by Baron and Ferejohn (1989) can have a continuum of equilibria in the class of stationary (i.e., time-, and history-independent) strategies, the equilibrium allocation is unique as shown by Eraslan (2002).
[^14]:    24. More precisely, in the House of Councilors, until 1980, a low-magnitude SNTV system was used in the prefectural districts along with the national SNTV system. Before the 1983 election, the prefectural districts were unchanged, but the national SNTV election was replaced by a national closed-list Proportional Representation (PR).
    25. The LDP was not in a majority position from 1983 to 1986. During that time, it formed a coalition with the New Liberal Club (NLC), which was a part of the LDP in substance and only merged with the LDP in 1986. The NLC was formed by a number of young LDP legislators who left the LDP after a corruption scandal in 1976. It stayed in opposition for 6 years (the LDP maintained a majority during these 6 years) before forming a coalition with the LDP in 1983.
    26. In 1980, the Diet passed the no-confidence resolution with the absence of 69 LDP legislators. However, this was not related to government formation.
    27. During the period between 1958 and 1993, cabinets were formed within 2 days after the selection of the Prime Minister-designates. One exception is that it took 3 days when the second Ohira cabinet was formed (November 6-9, 1979).
[^15]:    28. The only exception took place in 1979 , when the LDP had two candidates. Once designated, Prime Minister-designate Masayoshi Ohira allocated seats to all the factions with significant size, including the ones who voted against him when he was approved by the Diet.
    29. Factions may get frustrated with the cabinet formation. We, however, think that they still agreed even though they may have been frustrated. For example, the Fukuda faction thought they were underrepresented in the formation of the Tanaka Cabinet in 1972. They never really tried to form a hostile coalition nor called for a no-confidence resolution, though they could have succeeded in it and restarted the cabinet formation process. (The Fukuda faction had 56 seats, and the opposition had 207 seats. Hence, a vote for a no-confidence resolution could have won by 263-228.)
