The economics of predation: What drives pricing when there is learning-by-doing?
—Online Appendix—

David Besanko* Ulrich Doraszelski† Yaroslav Kryukov‡

November 13, 2012

*Kellogg School of Management, Northwestern University, Evanston, IL 60208, d-besanko@kellogg.northwestern.edu.

†Wharton School, University of Pennsylvania, Philadelphia, PA 19104, doraszelski@wharton.upenn.edu.

‡Tepper School of Business, Carnegie Mellon University, Pittsburgh, PA 15213, kryukov@cmu.edu.

A1 Omitted expressions

Below we provide several expressions that are omitted from the main text as well as further details on the concept of inclusive price.

A1.1 Firms' decisions: Conditional expectation of scrap value and setup cost

Exit decision of incumbent firm. Given the assumed distribution for scrap values, the probability of incumbent firm 1 exiting the industry in state $\mathbf{e}'$ is

$$\begin{aligned}
\phi_1(\mathbf{e}') &= E_X\left[\phi_1(\mathbf{e}', X_1)\right] \\
&= \int \phi_1(\mathbf{e}', X_1) dF_X(X_1) = 1 - F_X(\widehat{X}_1(\mathbf{e}')) \\
&= \begin{cases} 1 & \text{if} \quad \widehat{X}_1(\mathbf{e}') < \overline{X} - \Delta_X, \\ \frac{1}{2} - \frac{\left[\widehat{X}_1(\mathbf{e}') - \overline{X}\right]}{2\Delta_X} & \text{if} \quad \widehat{X}_1(\mathbf{e}') \in [\overline{X} - \Delta_X, \overline{X} + \Delta_X], \\ 0 & \text{if} \quad \widehat{X}_1(\mathbf{e}') > \overline{X} + \Delta_X \end{cases}
\end{aligned}$$

and the expectation of the scrap value conditional on exiting the industry is

$$\begin{aligned}
E_X\left[X_1 | X_1 \geq \widehat{X}_1(\mathbf{e}')\right] &= \frac{\int_{F_X^{-1}(1-\phi_1(\mathbf{e}'))}^{\overline{X}+\Delta_X} X_1 dF_X(X_1)}{\phi_1(\mathbf{e}')} \\
&= \frac{1}{\phi_1(\mathbf{e}')}\left[Z_X(0) - Z_X\left(1 - \phi_1(\mathbf{e}')\right)\right],
\end{aligned}$$

where

$$Z_X(1-\phi) = \frac{1}{\Delta_X^2}\begin{cases} -\frac{1}{6}\left(\overline{X} - \Delta_X\right)^3 & \text{if} \quad 1-\phi \leq 0, \\ \frac{1}{2}\left(\Delta_X - \overline{X}\right)\left(F_X^{-1}(1-\phi)\right)^2 + \frac{1}{3}\left(F_X^{-1}(1-\phi)\right)^3 & \text{if} \quad 1-\phi \in \left[0, \frac{1}{2}\right], \\ \frac{1}{2}\left(\Delta_X + \overline{X}\right)\left(F_X^{-1}(1-\phi)\right)^2 - \frac{1}{3}\left(F_X^{-1}(1-\phi)\right)^3 - \frac{1}{3}\overline{X}^3 & \text{if} \quad 1-\phi \in \left[\frac{1}{2}, 1\right], \\ \frac{1}{6}\left(\overline{X} + \Delta_X\right)^3 - \frac{1}{3}\overline{X}^3 & \text{if} \quad 1-\phi \geq 1 \end{cases}$$

and

$$F_X^{-1}(1-\phi) = \overline{X} + \Delta_X \begin{cases} -1 & \text{if} \quad 1-\phi \leq 0, \\ -1 + \sqrt{2(1-\phi)} & \text{if} \quad 1-\phi \in \left[0, \frac{1}{2}\right], \\ 1 - \sqrt{2\phi} & \text{if} \quad 1-\phi \in \left[\frac{1}{2}, 1\right], \\ 1 & \text{if} \quad 1-\phi \geq 1. \end{cases}$$

Entry decision of potential entrant. Given the assumed distribution for setup costs, the probability of potential entrant 1 not entering the industry in state $\mathbf{e}'$ is

$$
\begin{aligned}
\phi_1(\mathbf{e}') &= E_S\left[\phi_1(\mathbf{e}', S_1)\right] \\
&= \int \phi_1(\mathbf{e}', S_1) dF_S(S_1) = 1 - F_S(\widehat{S}_1(\mathbf{e}')) \\
&= \begin{cases} 1 & \text{if} \quad \widehat{S}_1(\mathbf{e}') < \overline{S} - \Delta_S, \\ \frac{1}{2} - \frac{\left[\widehat{S}_1(\mathbf{e}') - \overline{S}\right]}{2\Delta_S} & \text{if} \quad \widehat{S}_1(\mathbf{e}') \in [\overline{S} - \Delta_S, \overline{S} + \Delta_S], \\ 0 & \text{if} \quad \widehat{S}_1(\mathbf{e}') > \overline{S} + \Delta_S \end{cases}
\end{aligned}
$$

and the expectation of the setup cost conditional on entering the industry is

$$
\begin{aligned}
E_S\left[S_1 | S_1 \leq \widehat{S}_1(\mathbf{e}')\right] &= \frac{\int_{\overline{S}-\Delta_S}^{F_S^{-1}(1-\phi_1(\mathbf{e}'))} S_1 dF_S(S_1)}{(1 - \phi_1(\mathbf{e}'))} \\
&= \frac{1}{\phi_1(\mathbf{e}')} \left[Z_S\left(1 - \phi_1(\mathbf{e}')\right) - Z_S\left(1\right)\right],
\end{aligned}
$$

where

$$
Z_S\left(1-\phi\right) = \frac{1}{\Delta_S^2} \begin{cases} -\frac{1}{6}\left(\overline{S} - \Delta_S\right)^3 & \text{if} \quad 1 - \phi \leq 0, \\ \frac{1}{2}\left(\Delta_S - \overline{S}\right)\left(F_S^{-1}(1-\phi)\right)^2 + \frac{1}{3}\left(F_S^{-1}(1-\phi)\right)^3 & \text{if} \quad 1 - \phi \in \left[0, \frac{1}{2}\right], \\ \frac{1}{2}\left(\Delta_S + \overline{S}\right)\left(F_S^{-1}(1-\phi)\right)^2 - \frac{1}{3}\left(F_S^{-1}(1-\phi)\right)^3 - \frac{1}{3}\overline{S}^3 & \text{if} \quad 1 - \phi \in \left[\frac{1}{2}, 1\right], \\ \frac{1}{6}\left(\overline{S} + \Delta_S\right)^3 - \frac{1}{3}\overline{S}^3 & \text{if} \quad 1 - \phi \geq 1 \end{cases}
$$

and

$$
F_S^{-1}(1-\phi) = \overline{S} + \Delta_S \begin{cases} -1 & \text{if} \quad 1 - \phi \leq 0, \\ -1 + \sqrt{2\left(1-\phi\right)} & \text{if} \quad 1 - \phi \in \left[0, \frac{1}{2}\right], \\ 1 - \sqrt{2\phi} & \text{if} \quad 1 - \phi \in \left[\frac{1}{2}, 1\right], \\ 1 & \text{if} \quad 1 - \phi \geq 1. \end{cases}
$$

A1.2 Learning-by-doing: Marginal revenue and inclusive price

$mr_1(p_1, p_2(\mathbf{e})) = p_1 - \frac{\sigma}{1 - D_1(p_1, p_2(\mathbf{e}))}$ is the marginal revenue of incumbent firm 1 with respect to quantity and therefore analogous to the traditional textbook concept. To see this, let $q_1 = D_1(p_1, p_2(\mathbf{e}))$ be demand and $p_1 = P_1(q_1, p_2(\mathbf{e}))$ inverse demand as implicitly defined by $q_1 = D_1(P_1(q_1, p_2(\mathbf{e})), p_2(\mathbf{e}))$. The marginal revenue of incumbent firm 1 is

$$
MR_1(q_1, p_2(\mathbf{e})) = \frac{\partial\left[q_1 P_1(q_1, p_2(\mathbf{e}))\right]}{\partial q_1} = q_1 \frac{\partial P_1(q_1, p_2(\mathbf{e}))}{\partial q_1} + P_1(q_1, p_2(\mathbf{e})). \qquad \text{(A1)}
$$

Define $mr_1(p_1, p_2(\mathbf{e})) = MR_1(D_1(p_1, p_2(\mathbf{e})), p_2(\mathbf{e}))$ to be the marginal revenue of incumbent firm 1 evaluated at the quantity $q_1 = D_1(p_1, p_2(\mathbf{e}))$ corresponding to prices p_1 and $p_2(\mathbf{e})$. Then we have

$$\frac{\partial P_1(D_1(p_1, p_2(\mathbf{e})), p_2(\mathbf{e}))}{\partial q_1} = \left[\frac{\partial D_1(p_1, p_2(\mathbf{e}))}{\partial p_1}\right]^{-1} = -\frac{\sigma}{\left[1 - D_1(p_1, p_2(\mathbf{e}))\right] D_1(p_1, p_2(\mathbf{e}))}. \tag{A2}$$

Substituting equation (A2) into equation (A1), it follows that $mr_1(p_1, p_2(\mathbf{e})) = p_1 - \frac{\sigma}{1 - D_1(p_1, p_2(\mathbf{e}))}$.

A1.3 Industry structure, conduct, and performance: Consumer and producer surplus

Consumer surplus in state $\mathbf{e}$ is

$$CS(\mathbf{e}) = \sigma \log \left\{ \exp\left(\frac{-p_0}{\sigma}\right) + \sum\nolimits_{n=1}^{2} \exp\left(\frac{-p_n(\mathbf{e})}{\sigma}\right) \right\}.$$

The producer surplus of firm 1 in state $\mathbf{e}$ is

$$\begin{aligned} PS_1(\mathbf{e}) = 1\left[e_1 > 0\right] \Bigg\{ & D_0(p_1(\mathbf{e}), p_2(\mathbf{e})) \phi_1(\mathbf{e}) E_X \left[X_1 | X_1 \geq \widehat{X}_1(\mathbf{e})\right] \\ & + D_1(p_1(\mathbf{e}), p_2(\mathbf{e})) \left\{ p_1(\mathbf{e}) - c(e_1) + \phi_1(e_1+1, e_2) E_X \left[X_1 | X_1 \geq \widehat{X}_1(e_1+1, e_2)\right] \right\} \\ & + D_2(p_1(\mathbf{e}), p_2(\mathbf{e})) \phi_1(e_1, e_2+1) E_X \left[X_1 | X_1 \geq \widehat{X}_1(e_1, e_2+1)\right] \Bigg\} \\ - 1\left[e_1 = 0\right] \Bigg\{ & D_0(p_1(\mathbf{e}), p_2(\mathbf{e})) \left(1 - \phi_1(\mathbf{e})\right) E_S \left[S_1 | S_1 \leq \widehat{S}_1(\mathbf{e})\right] \\ & + D_1(p_1(\mathbf{e}), p_2(\mathbf{e})) \left(1 - \phi_1(e_1+1, e_2)\right) E_S \left[S_1 | S_1 \leq \widehat{S}_1(e_1+1, e_2)\right] \\ & + D_2(p_1(\mathbf{e}), p_2(\mathbf{e})) \left(1 - \phi_1(e_1, e_2+1)\right) E_S \left[S_1 | S_1 \leq \widehat{S}_1(e_1, e_2+1)\right] \Bigg\}. \end{aligned}$$

The first set of terms represents the contingency that firm 1 is an incumbent that participates in the product market and receives a scrap value upon exit; the second set the contingency that firm 1 is an entrant that incurs a setup cost upon entry.

A2 Additional figures and tables

Below we provide additional figures and tables to supplement those in the main text.

A2.1 Equilibrium correspondence: Metrics of industry conduct and performance

Figures A1–A5 complement Figure 3 in the main text. They illustrate the equilibrium correspondence by plotting $\overline{p}^{\infty}$, CS^{∞}, TS^{∞}, CS^{NPV}, and TS^{NPV} against ρ, σ, and $\overline{X}$, respectively.

A2.2 Equilibrium correspondence: Aggressive equilibria with little learning-by-doing

Figure A6 supplements footnote 21 in the main text. It illustrates that aggressive equilibria can arise for $\rho = 0.99$ and $\sigma = 0.10$ and $\rho = 0.98$ and $\sigma = 0.30$ where there is practically no learning-by-doing.

A2.3 Equilibrium correspondence: Multiple equilibria

Figure A7 shows the number of equilibria that we have identified for combinations of ρ and σ, ρ and $\overline{X}$, and σ and $\overline{X}$, respectively. Darker shades indicate more equilibria.

We have found 81 equilibria for $\rho = 0.45$ and $\sigma = 0.9$. In Figures A8–A10 we present some of them to further illustrate the differences between equilibria alluded to in footnote 25 in the main text. Figure A8 presents equilibria #35 and #36 that are fairly similar to each other;[1] the differences between values and policies in any state are less than 9%.

Figure A9 presents equilibria #35 and #5 that are much less similar to each other; the differences between values and policies in some state are 114%. These equilibria differ in the location of the trench: The pricing decision has a single trench along the e_1-axis at $e_2 = 5$ in equilibrium #35 and at $e_2 = 2$ in equilibrium #5. Due to the delayed onset of predation-like behavior, the industry is much more likely to evolve into a mature duopoly in equilibrium #35 than in equilibrium #5.

Figure A10 presents equilibria #14 and #15. In these equilibria the pricing decision has double trenches along the e_1-axis at $e_2 = 2$ and $e_2 = 5$. While the location of the trenches is the same, these equilibria differ in the depth of the trenches.

A2.4 Isolating predatory incentives: Accommodative equilibrium

Table A1 complements Table 6 in the main text. The left and middle panels illustrate the decomposition (10) for the accommodative equilibrium at the beginning of Section 4. The pricing decision is driven by the advantage-building/baseline and advantage-denying/baseline

[1] Equilibria are numbered arbitrarily.

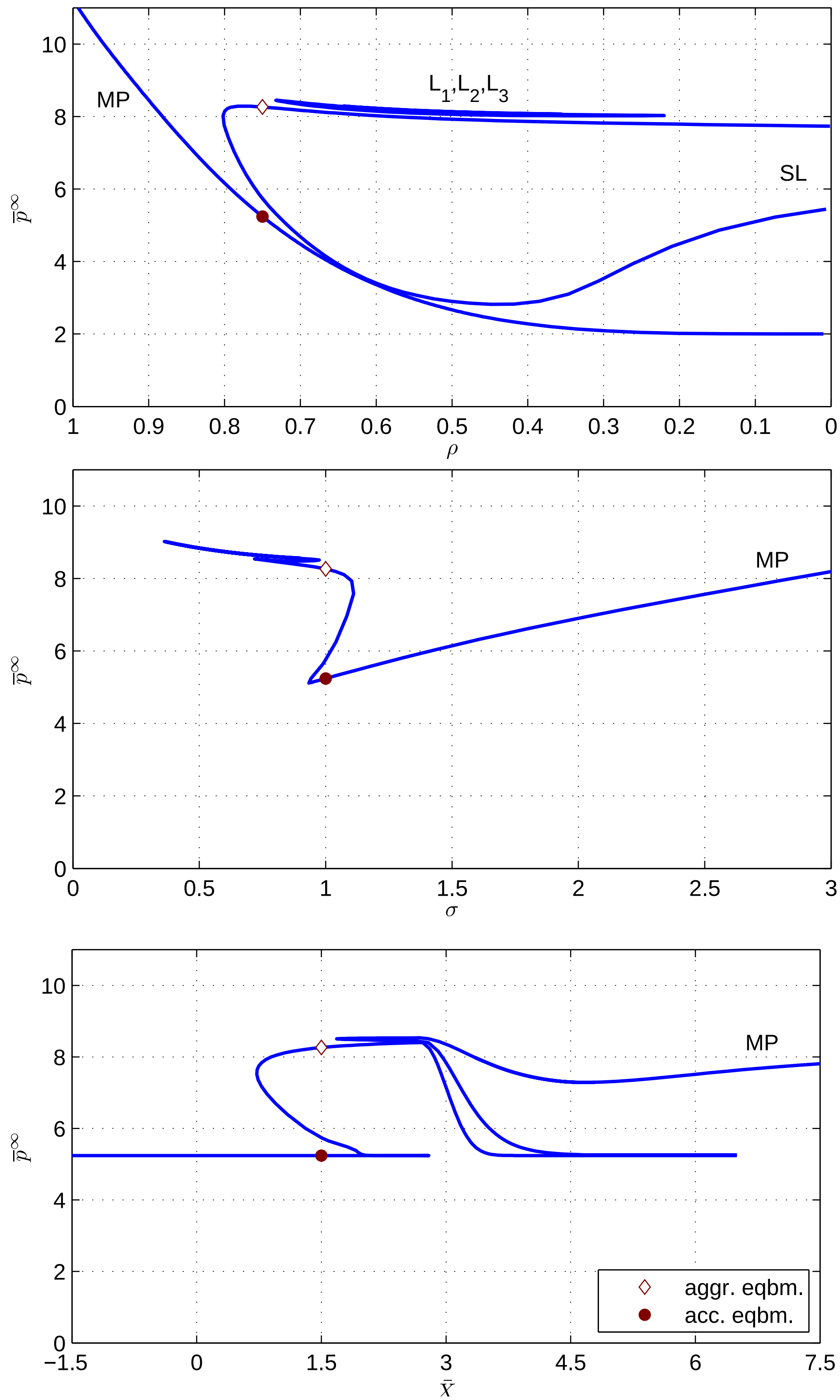

Figure A1: Expected long-run average price. Equilibrium correspondence: slice along $\rho \in [0,1]$ (upper panel), $\sigma \in [0,3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

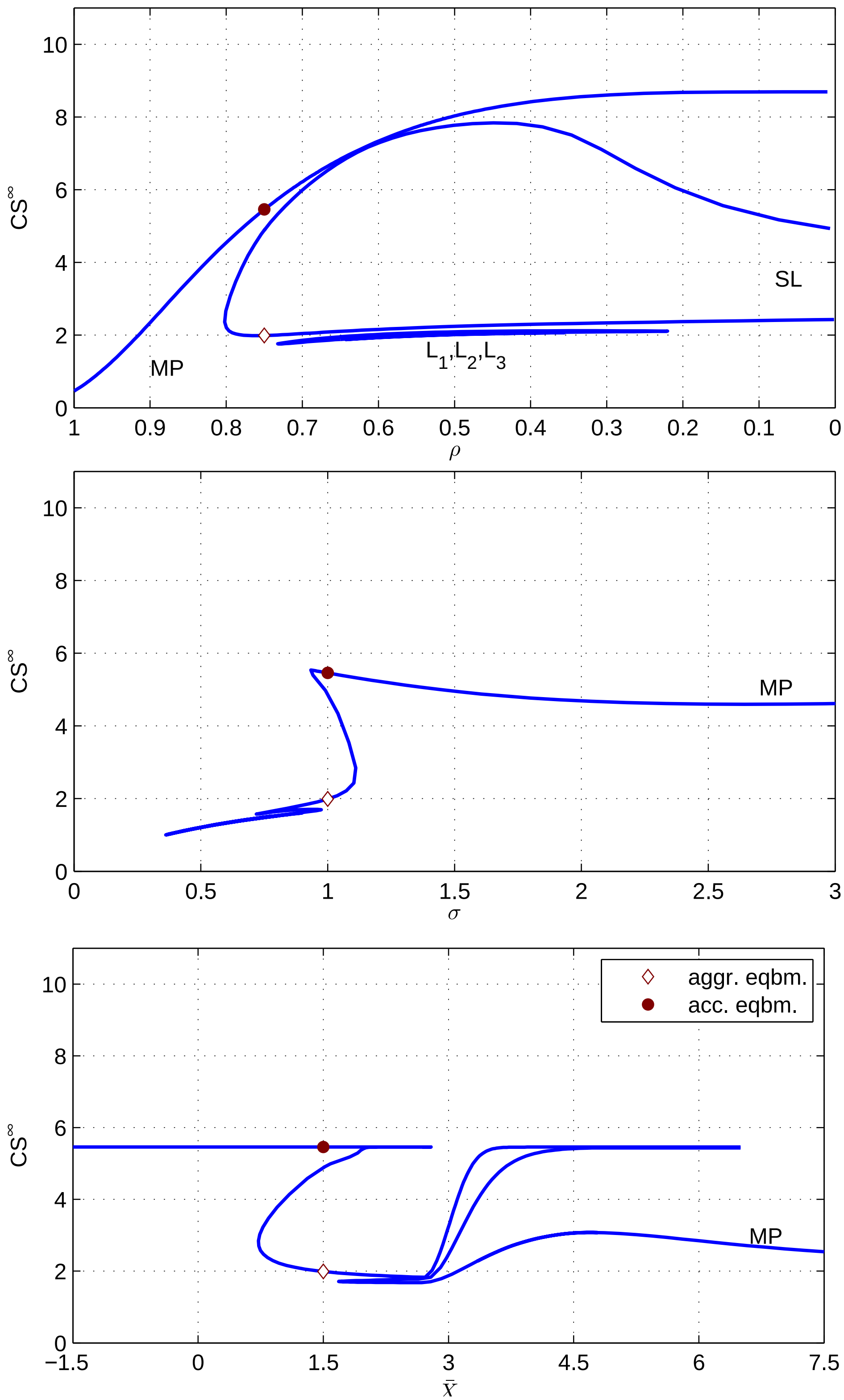

Figure A2: Expected long-run consumer surplus. Equilibrium correspondence: slice along $\rho \in [0, 1]$ (upper panel), $\sigma \in [0, 3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

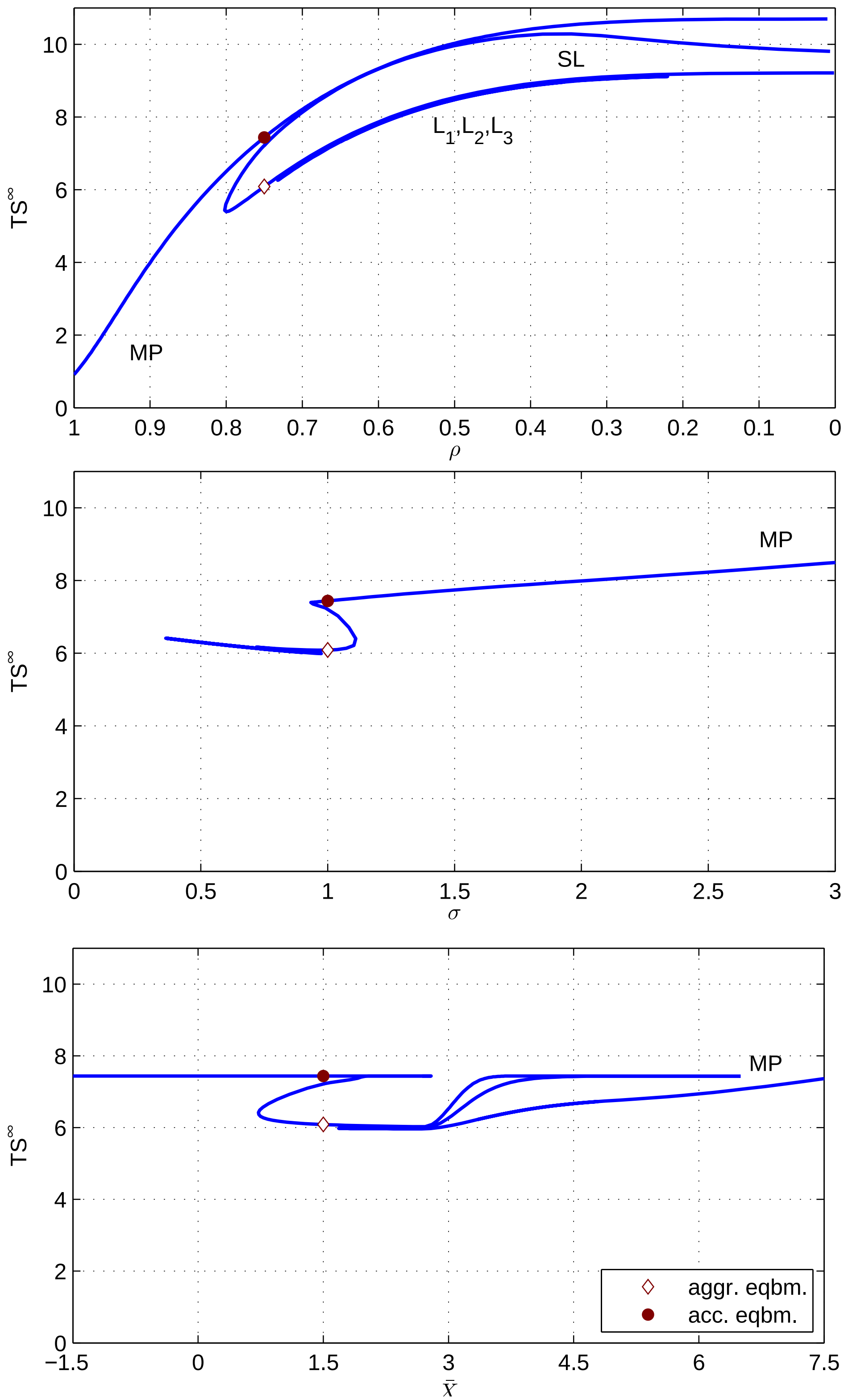

Figure A3: Expected long-run total surplus. Equilibrium correspondence: slice along $\rho \in [0, 1]$ (upper panel), $\sigma \in [0, 3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

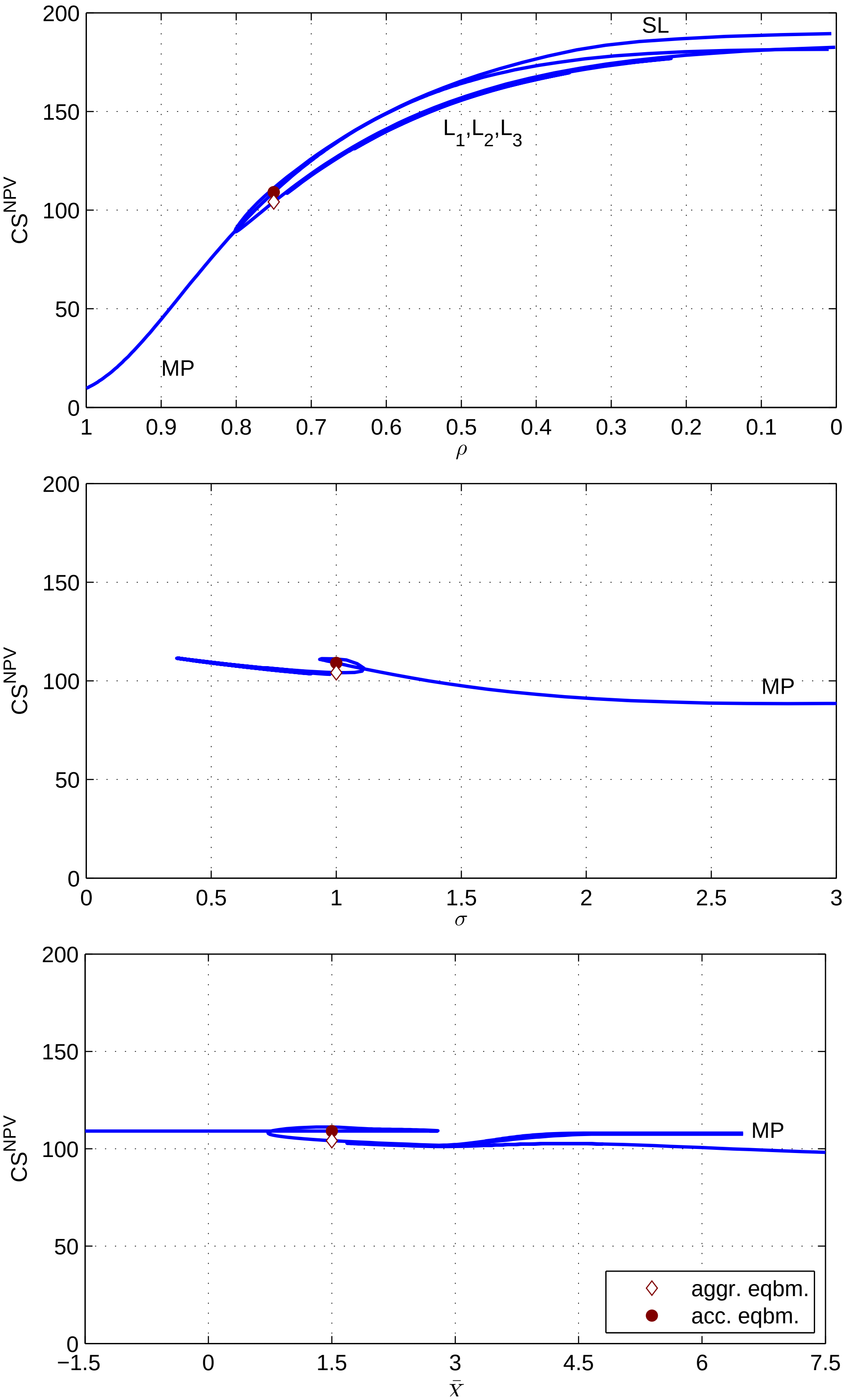

Figure A4: Expected discounted consumer surplus. Equilibrium correspondence: slice along $\rho \in [0, 1]$ (upper panel), $\sigma \in [0, 3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

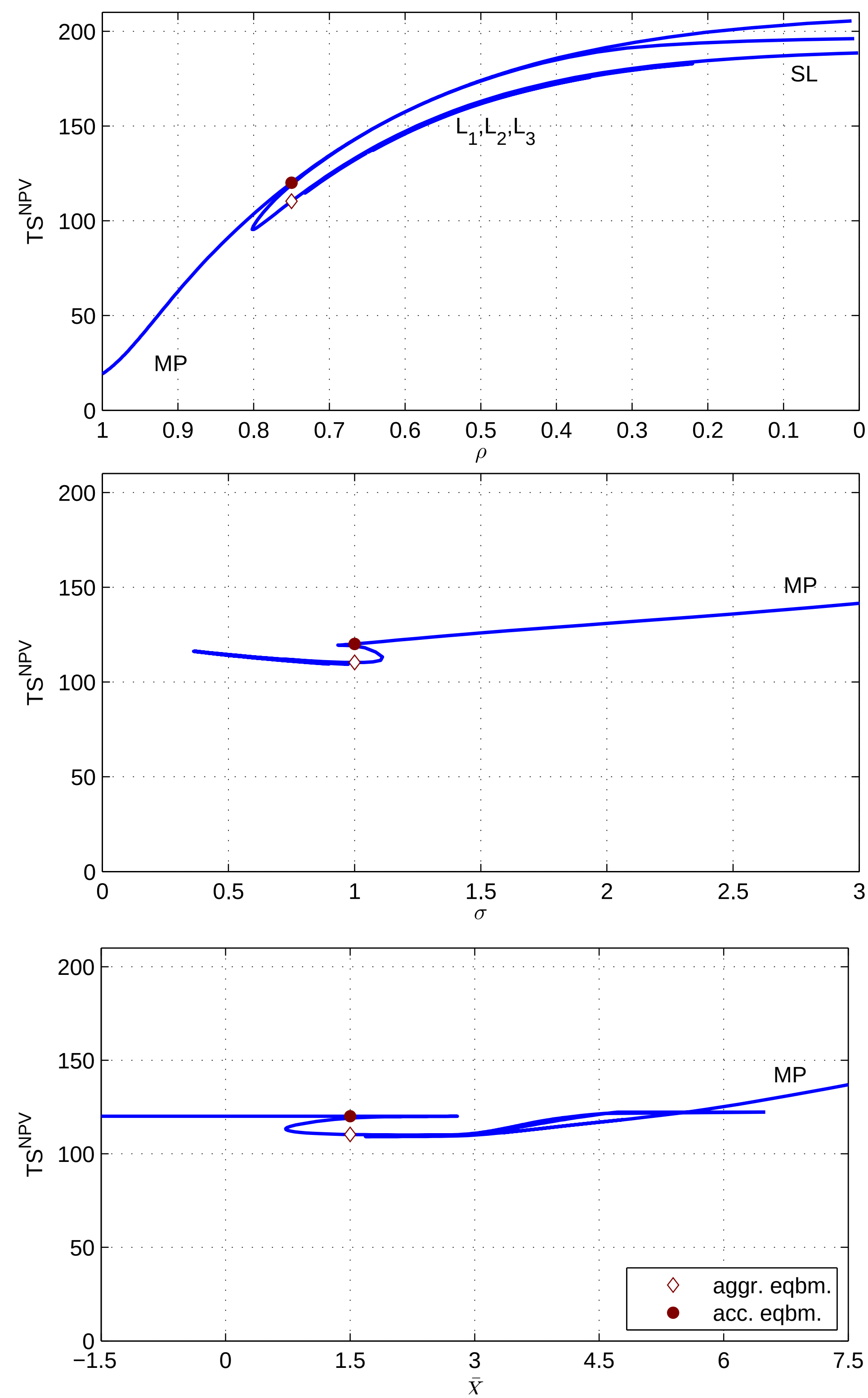

Figure A5: Expected discounted total surplus. Equilibrium correspondence: slice along $\rho \in [0, 1]$ (upper panel), $\sigma \in [0, 3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

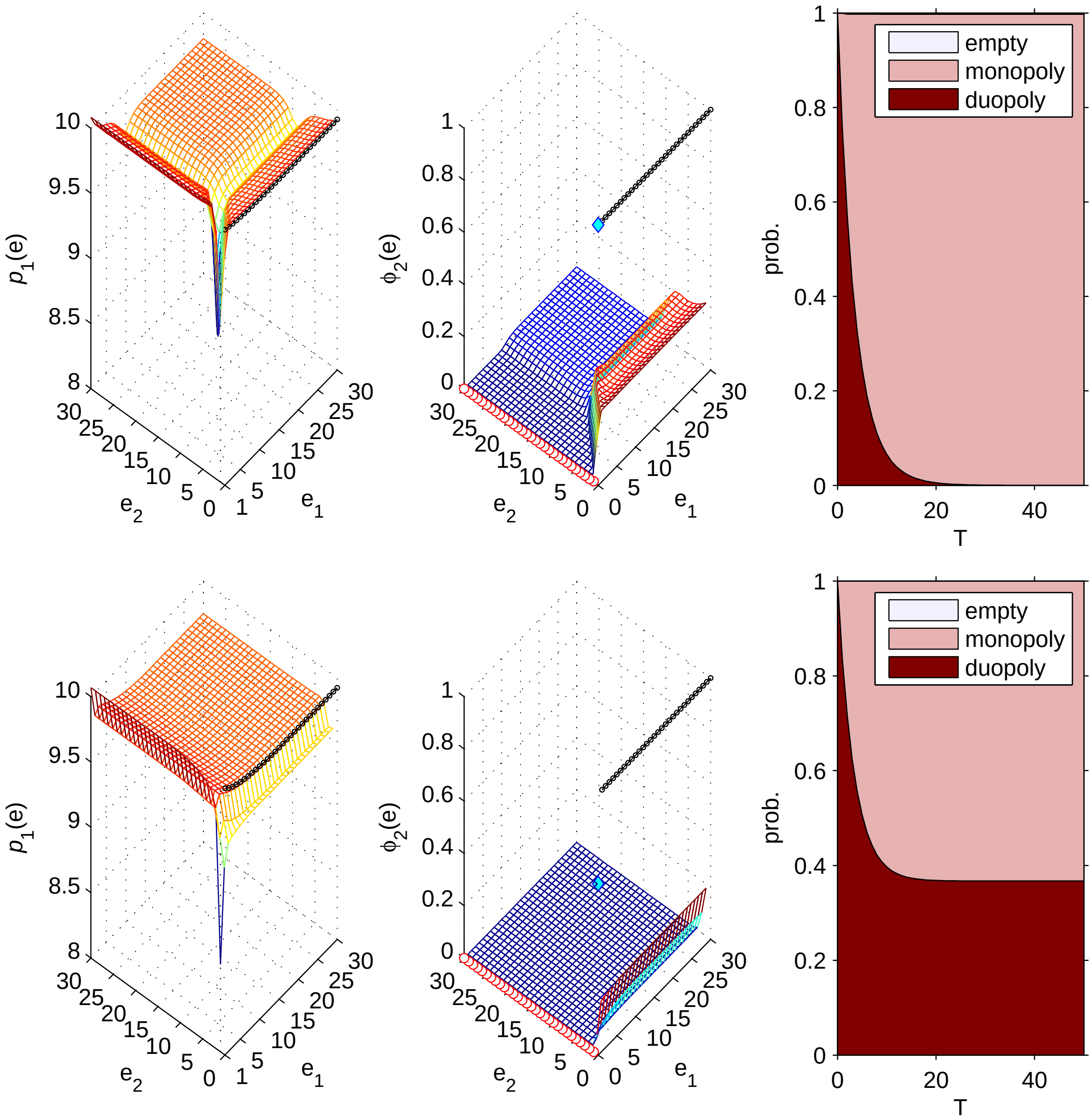

Figure A6: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $\mathbf{e} = (1, 1)$ at $T = 0$ (right panels). Aggressive equilibrium for $\rho = 0.99$ and $\sigma = 0.10$ (upper panels) and $\rho = 0.98$ and $\sigma = 0.30$ (lower panels).

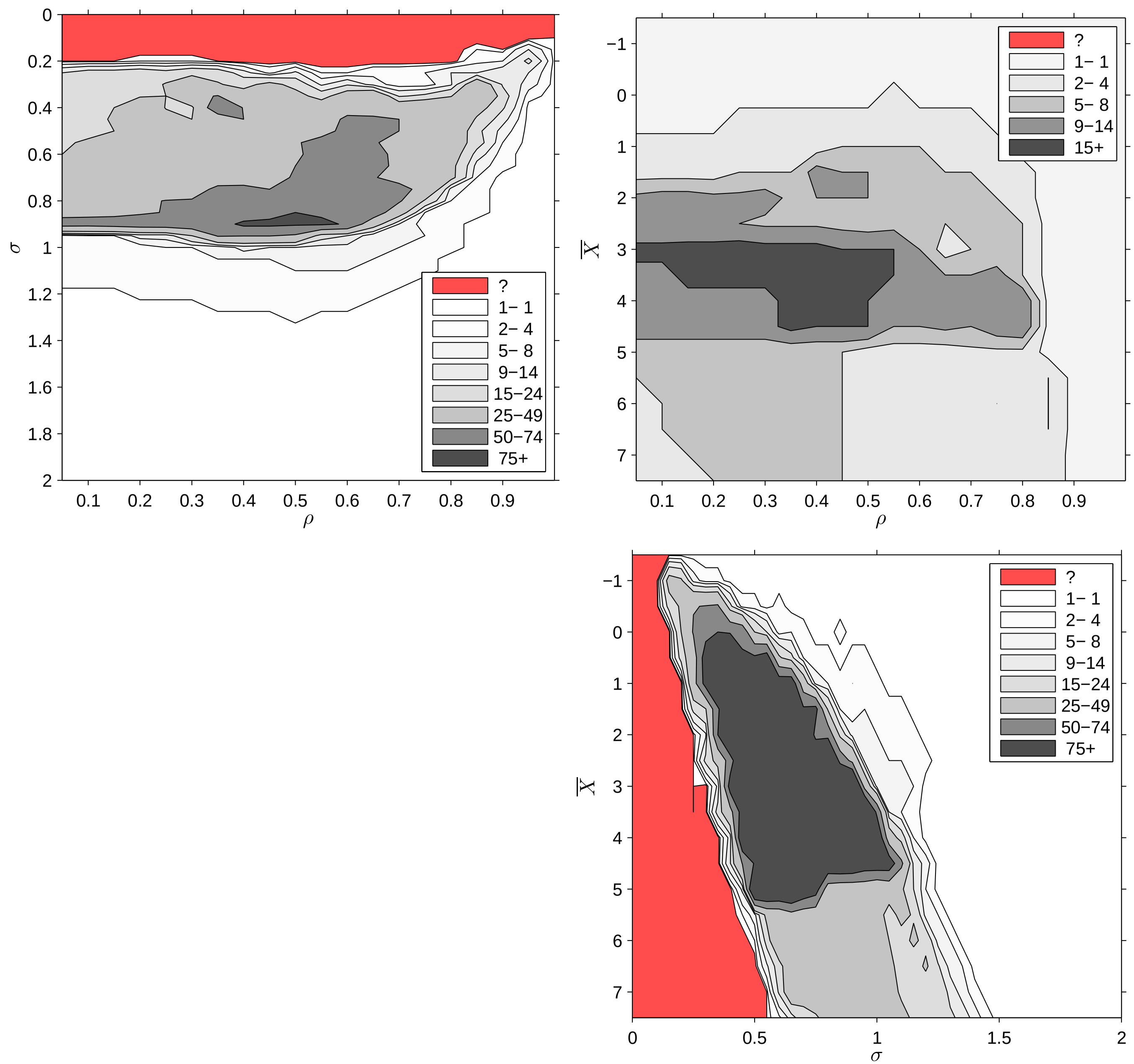

Figure A7: Number of equilibria. Equilibrium correspondence: slice along $(\rho, \sigma) \in [0, 1] \times [0, 3]$ (upper left panel), $(\rho, \overline{X}) \in [0, 1] \times [-1.5, 7.5]$ (upper right panel), and $(\sigma, \overline{X}) \in [0, 3] \times [-1.5, 7.5]$ (lower right panel). ? indicates that the homotopy algorithm crashed.

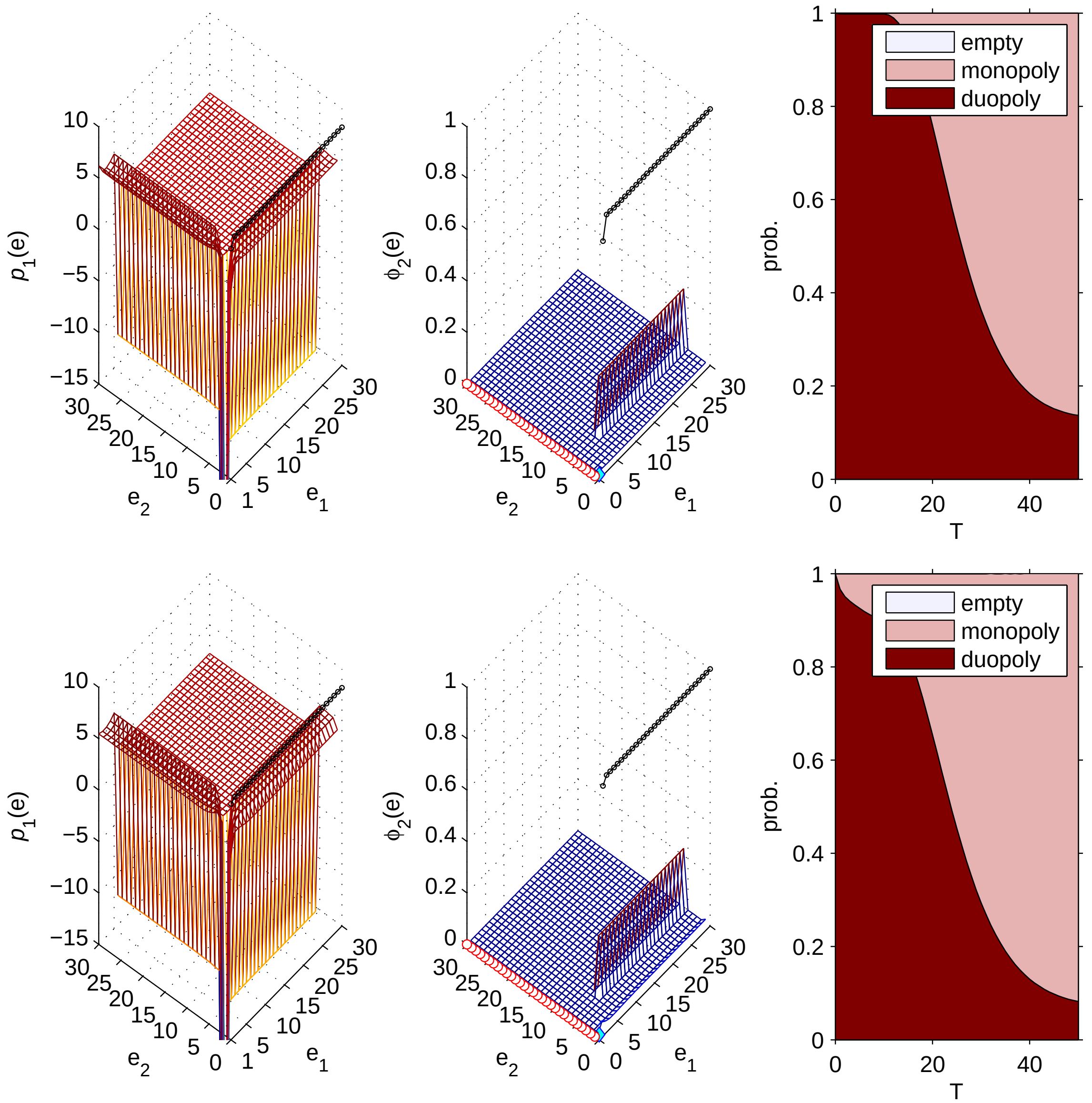

Figure A8: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $\mathbf{e} = (1, 1)$ at $T = 0$ (right panels). Equilibrium #35 (upper panels) and equilibrium #36 (lower panels) for $\rho = 0.45$ and $\sigma = 0.90$.

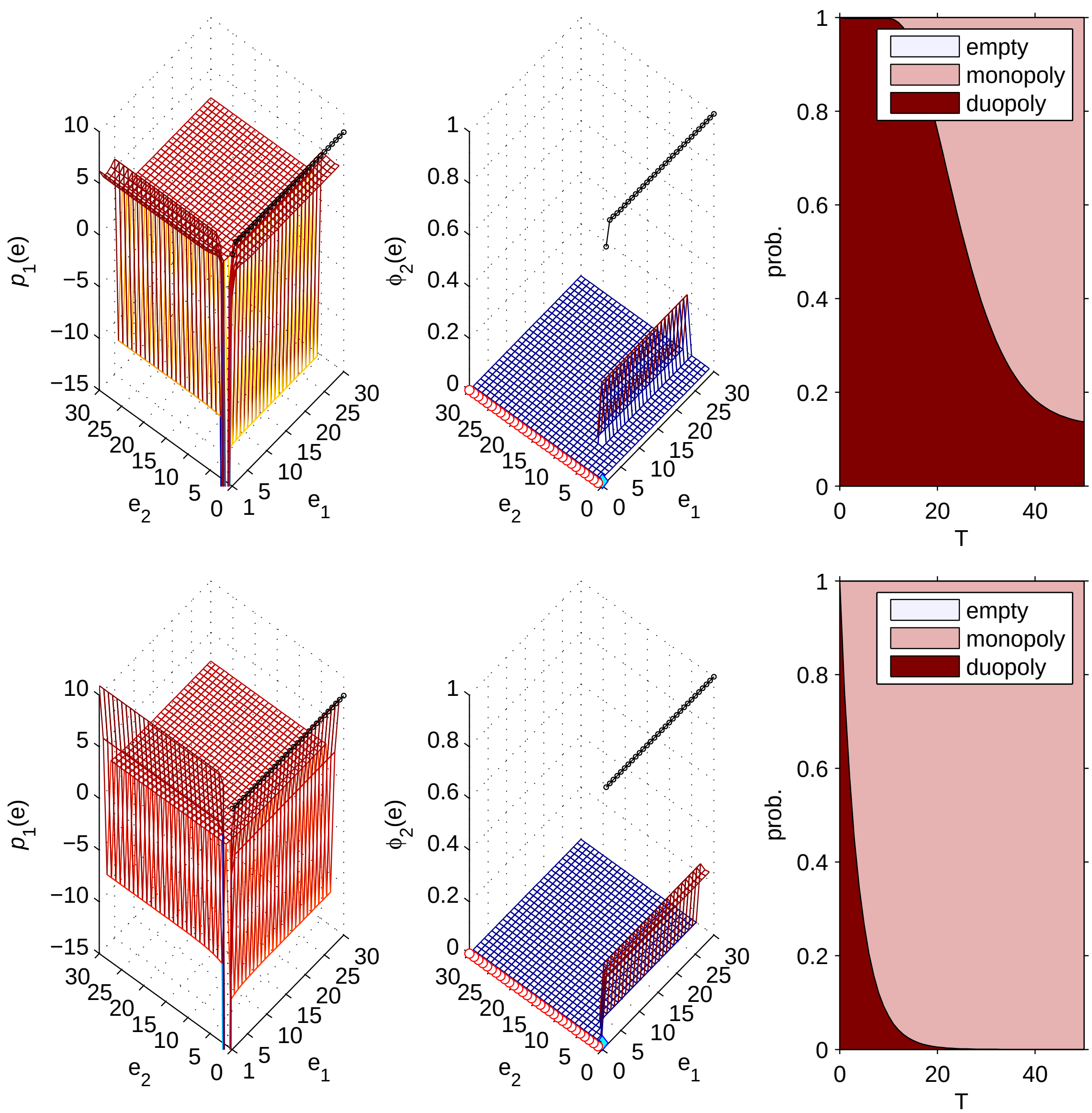

Figure A9: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $\mathbf{e} = (1, 1)$ at $T = 0$ (right panels). Equilibrium #35 (upper panels) and equilibrium #5 (lower panels) for $\rho = 0.45$ and $\sigma = 0.90$.

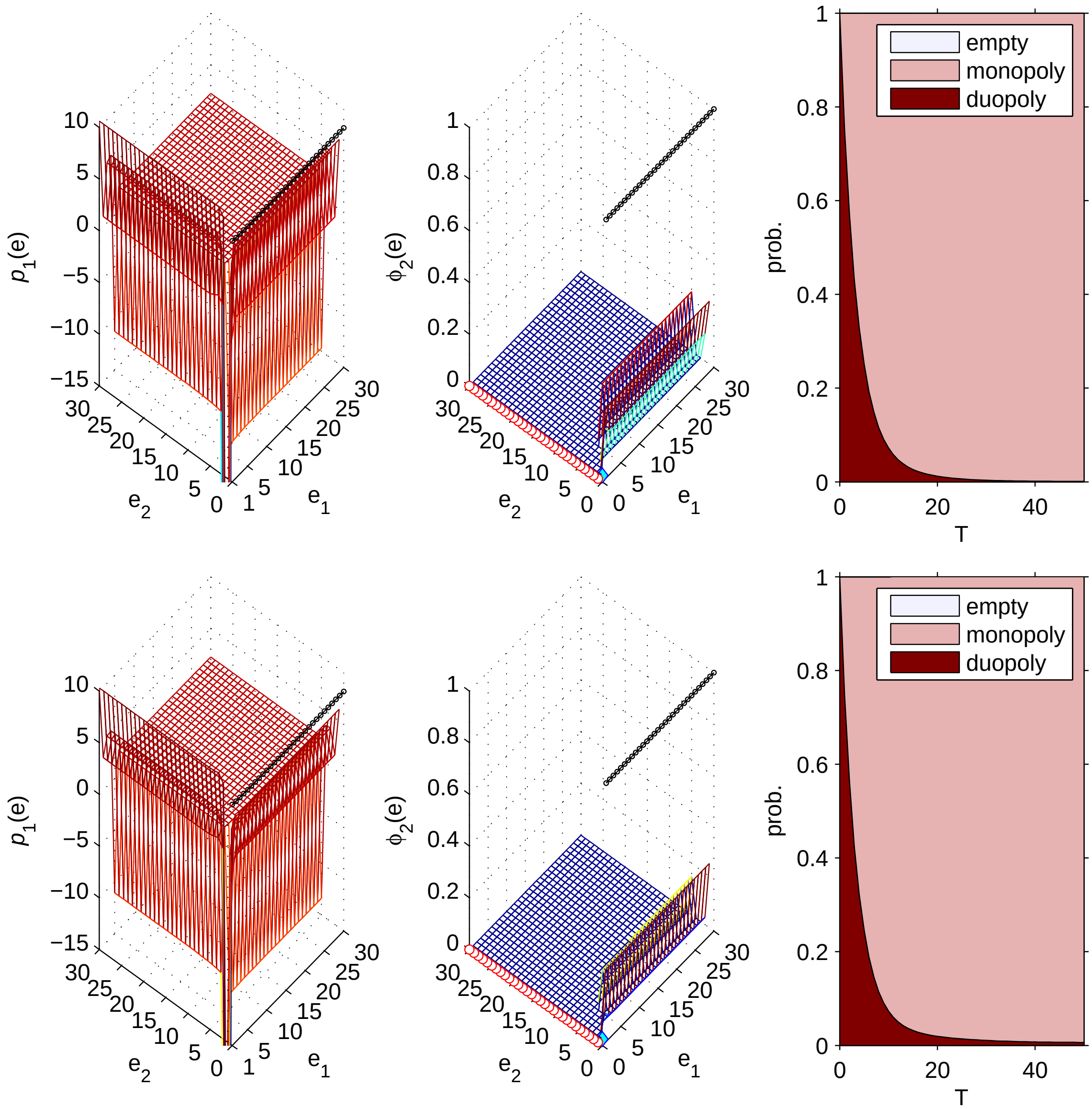

Figure A10: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $\mathbf{e} = (1, 1)$ at $T = 0$ (right panels). Equilibrium #14 (upper panels) and equilibrium #15 (lower panels) for $\rho = 0.45$ and $\sigma = 0.90$.

motives. As can be seen in the right panels, a sacrifice test based on Definitions 1 and 2 may indicate predation whereas a sacrifice test based on Definitions 3 and 4 does not.

A2.5 Counterfactual and equilibrium correspondences: Multiple counterfactuals

Figures A11, A12, and A13 show the number of counterfactuals for Definitions 2, 3, and 4 that we have identified for combinations of ρ and σ, ρ and $\overline{X}$, and σ and $\overline{X}$, respectively. Darker shades indicate more counterfactuals. As can be seen from comparing Figures A11, A12, and A13 to Figure A7, there tend to be less counterfactuals than equilibria for a given parameterization.

A2.6 Counterfactual and equilibrium correspondences and eliminated and surviving equilibria: Product differentiation and scrap value

Figures A14 and A15 complement Figure 4 in the main text. They illustrate the counterfactual correspondence for Definitions 1–4 by plotting HHI^{∞} against σ and $\overline{X}$, respectively. They superimpose the equilibrium correspondences $\mathbf{H}^{-1}(\sigma)$ and $\mathbf{H}^{-1}(\overline{X})$ from Figure 3 and distinguish between surviving and eliminated equilibria.

As in the main text, the counterfactual correspondences for Definitions 3 and 4 resemble the equilibrium correspondence much more closely than those for Definitions 1 and 2. Furthermore, the stronger Definitions 1 and 2 eliminate many more equilibria that are associated with high expected long-run Herfindahl indices than the weaker Definitions 3 and 4.

A3 Definitions of predation in the literature

To complement the discussion in Section 5 of the main text, below we adapt the Ordover & Willig (1981) and Cabral & Riordan (1997) definitions of predation to our model. Both stress the advantage-building/exit motive $\Gamma_1^2(\mathbf{e})$ and the advantage-denying/exit motive $\Theta_1^2(\mathbf{e})$.

Cabral & Riordan (1997). Cabral & Riordan (1997) call "an action predatory if (1) a different action would increase the probability that rivals remain viable and (2) the different action would be more profitable under the counterfactual hypothesis that the rival's viability were unaffected" (p. 160). In the context of predatory pricing, it is natural to interpret "a different action" as a higher price $\widetilde{p}_1 > p_1(\mathbf{e})$. To port the Cabral & Riordan definition from their two-period model to our infinite-horizon dynamic stochastic game, we take the

			advantage-building					advantage-denying				sacrifice test			
												1	2	3	4
$\mathbf{e}$	$p_1(\mathbf{e})$	$c(e_1)$	$\Gamma_1^1(\mathbf{e})$	$\Gamma_1^2(\mathbf{e})$	$\Gamma_1^3(\mathbf{e})$	$\Gamma_1^4(\mathbf{e})$	$\Gamma_1^5(\mathbf{e})$	$\Theta_1^1(\mathbf{e})$	$\Theta_1^2(\mathbf{e})$	$\Theta_1^3(\mathbf{e})$	$\Theta_1^4(\mathbf{e})$	SRP	DCV	REI	REII
(1,1)	5.05	10.00	6.21	0.00	0.00	0.00	0.00	0.73	0.00	0.00	0.00	√√	√√		
(2,1)	5.34	7.50	3.74	0.00	0.00	0.00	0.00	1.96	0.00	0.00	0.00	√√	√√		
(3,1)	5.45	6.34	2.65	0.00	0.00	0.00	0.00	2.43	0.00	0.00	0.00	√√	√√		
(4,1)	5.51	5.63	2.01	0.00	0.00	0.00	0.00	2.68	0.00	0.00	0.00	√√	√√		
(5,1)	5.54	5.13	1.58	0.00	0.00	0.00	0.00	2.84	0.00	0.00	0.00	√√	√√		
(6,1)	5.56	4.75	1.27	0.00	0.00	0.00	0.00	2.95	0.00	0.00	0.00	√√	√√		
(7,1)	5.57	4.46	1.02	0.00	0.00	0.00	0.00	3.03	0.00	0.00	0.00	√√	√√		
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
(14,1)	5.59	3.34	0.09	0.00	0.00	0.00	0.00	3.17	0.00	0.00	0.00	√√	√√		
(15,1)	5.59	3.25	0.00	0.00	0.00	0.00	0.00	3.18	0.00	0.00	0.00	√√	√√		
(16,1)	5.59	3.25	0.00	0.00	0.00	0.00	0.00	3.18	0.00	0.00	0.00	√√	√√		
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
(30,1)	5.59	3.25	0.00	0.00	0.00	0.00	0.00	3.18	0.00	0.00	0.00	√√	√√		
(1,4)	6.82	10.00	4.39	0.00	0.00	0.00	0.00	0.05	0.00	0.00	0.00	√√	√		
(2,4)	6.06	7.50	2.87	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.00	√√	√		
(3,4)	5.79	6.34	2.12	0.00	0.00	0.00	0.00	0.24	0.00	0.00	0.00	√√	√		
(4,4)	5.65	5.63	1.66	0.00	0.00	0.00	0.00	0.31	0.00	0.00	0.00	√√	√		
(5,4)	5.56	5.13	1.34	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.00	√√	√		
(6,4)	5.49	4.75	1.10	0.00	0.00	0.00	0.00	0.39	0.00	0.00	0.00	√√	√		
(7,4)	5.45	4.46	0.90	0.00	0.00	0.00	0.00	0.41	0.00	0.00	0.00	√√	√		
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
(14,4)	5.32	3.34	0.09	0.00	0.00	0.00	0.00	0.47	0.00	0.00	0.00	√√	√		
(15,4)	5.32	3.25	0.00	0.00	0.00	0.00	0.00	0.47	0.00	0.00	0.00	√	√		
(16,4)	5.32	3.25	0.00	0.00	0.00	0.00	0.00	0.47	0.00	0.00	0.00	√	√		
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
(30,4)	5.32	3.25	0.00	0.00	0.00	0.00	0.00	0.47	0.00	0.00	0.00	√	√		

Table A1: Decomposed advantage-building and advantage-denying motives (left and middle panels) and sacrifice test for Definitions 1–4 (right panels). √√ means that the weighted sum of the predatory incentives is larger than 0.5, √ that the weighted sum is between 0 and 0.5, and a blank that the weighted sum smaller or equal to 0. Accommodative equilibrium.

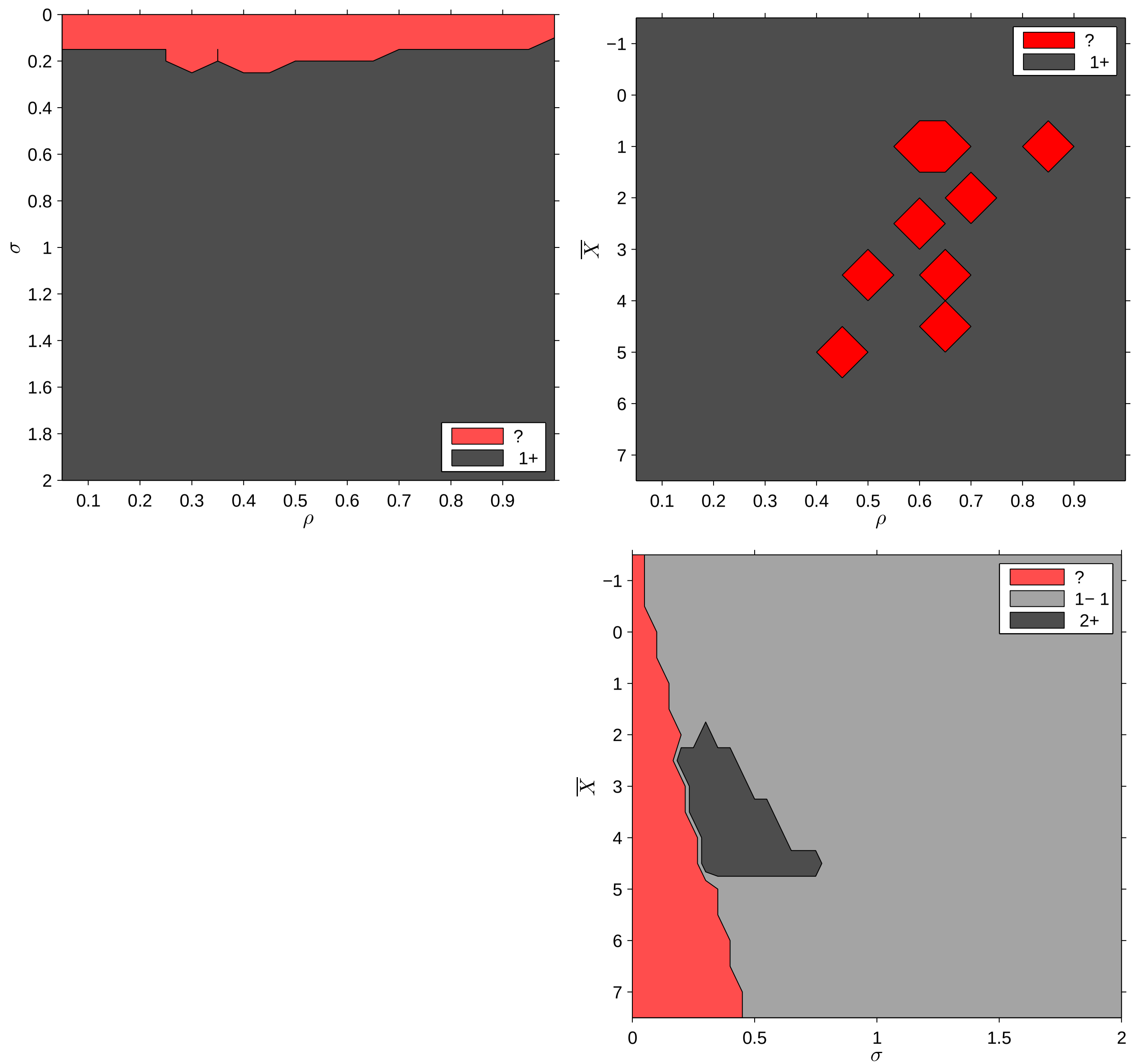

Figure A11: Number of counterfactuals for Definition 2. Counterfactual correspondence: slice along $(\rho, \sigma) \in [0, 1] \times [0, 3]$ (upper left panel), $(\rho, \overline{X}) \in [0, 1] \times [-1.5, 7.5]$ (upper right panel), and $(\sigma, \overline{X}) \in [0, 3] \times [-1.5, 7.5]$ (lower right panel). ? indicates that the homotopy algorithm crashed.

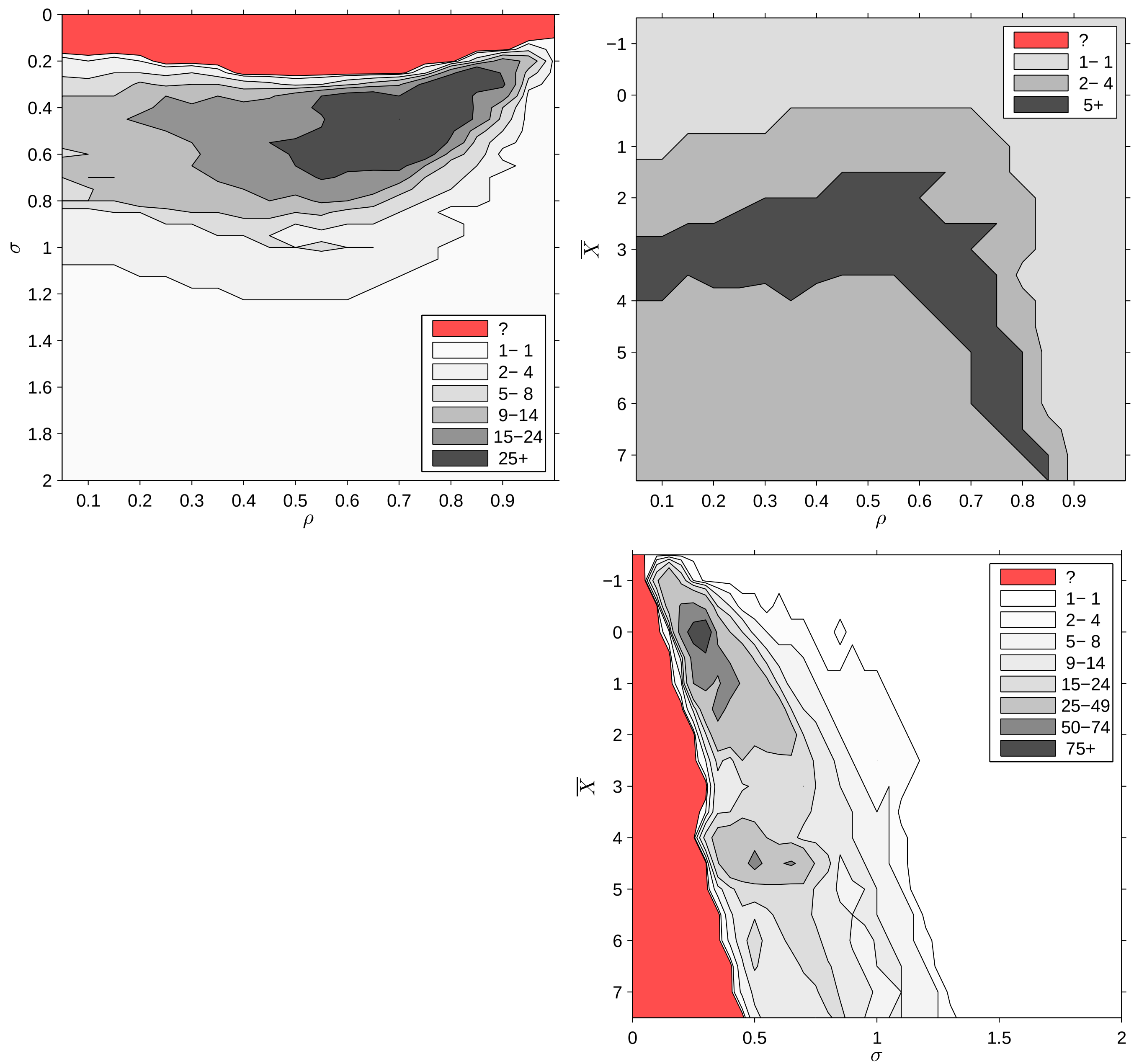

Figure A12: Number of counterfactuals for Definition 3. Counterfactual correspondence: slice along $(\rho, \sigma) \in [0,1] \times [0,3]$ (upper left panel), $(\rho, \overline{X}) \in [0,1] \times [-1.5, 7.5]$ (upper right panel), and $(\sigma, \overline{X}) \in [0,3] \times [-1.5, 7.5]$ (lower right panel). ? indicates that the homotopy algorithm crashed.

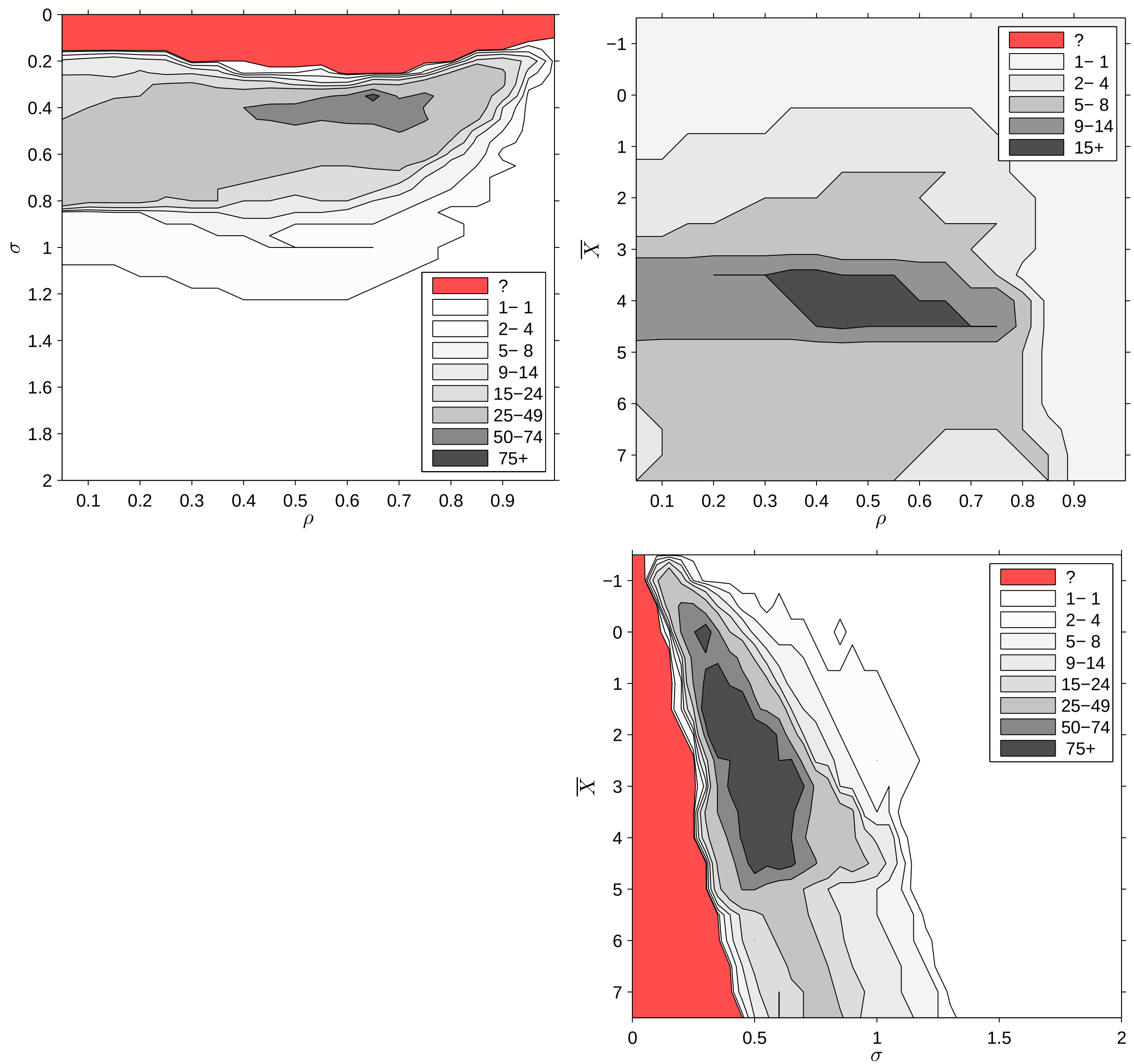

Figure A13: Number of counterfactuals for Definition 4. Counterfactual correspondence: slice along $(\rho, \sigma) \in [0, 1] \times [0, 3]$ (upper left panel), $(\rho, \overline{X}) \in [0, 1] \times [-1.5, 7.5]$ (upper right panel), and $(\sigma, \overline{X}) \in [0, 3] \times [-1.5, 7.5]$ (lower right panel). ? indicates that the homotopy algorithm crashed.

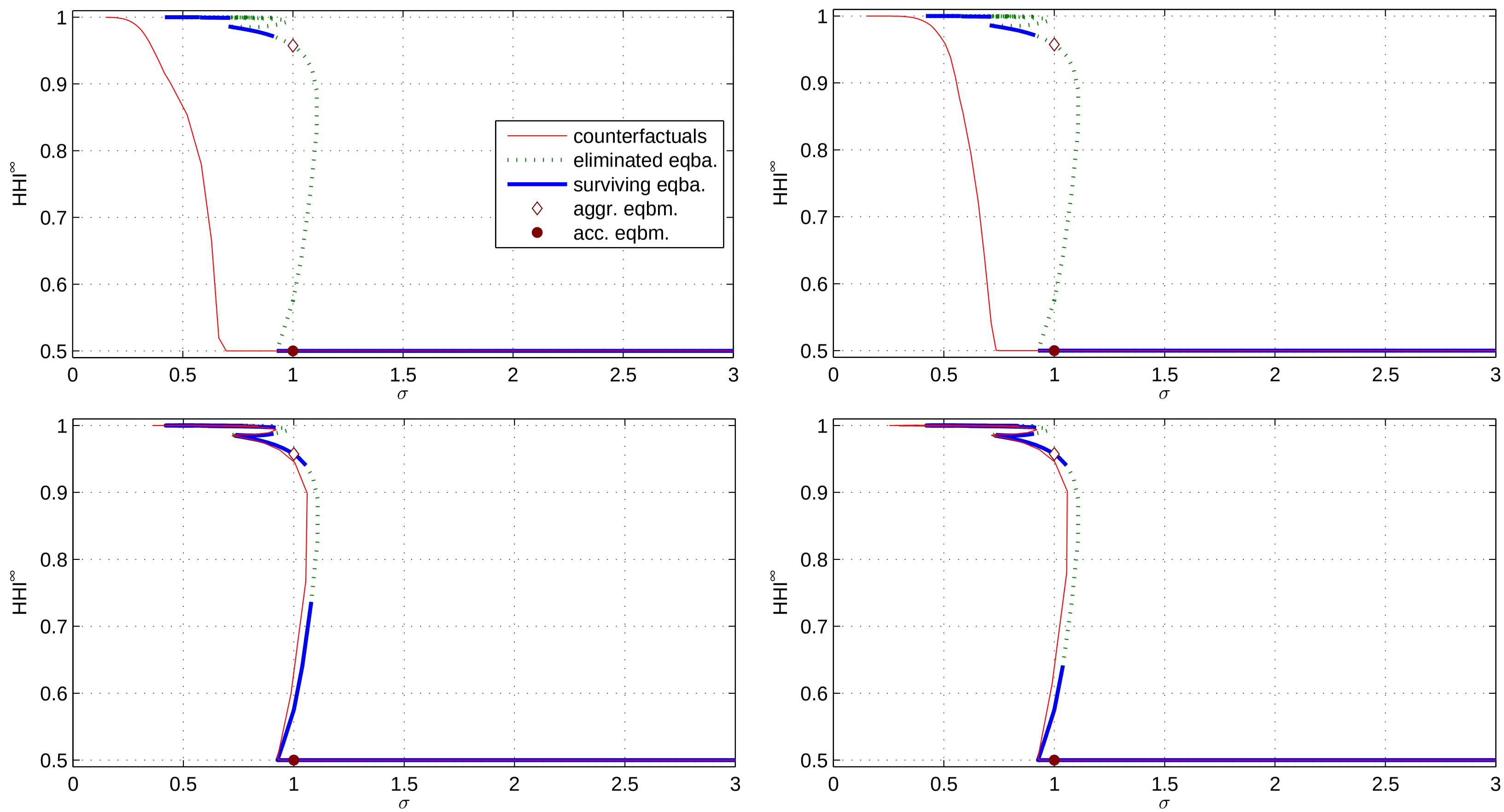

Figure A14: Expected long-run Herfindahl index. Counterfactual (solid red line) and equilibrium correspondences for Definitions 1–4 (upper left, upper right, lower left, and lower right panels) along with eliminated (dashed green line) and surviving (solid blue line) equilibria. Slice along $\sigma \in [0, 3]$.

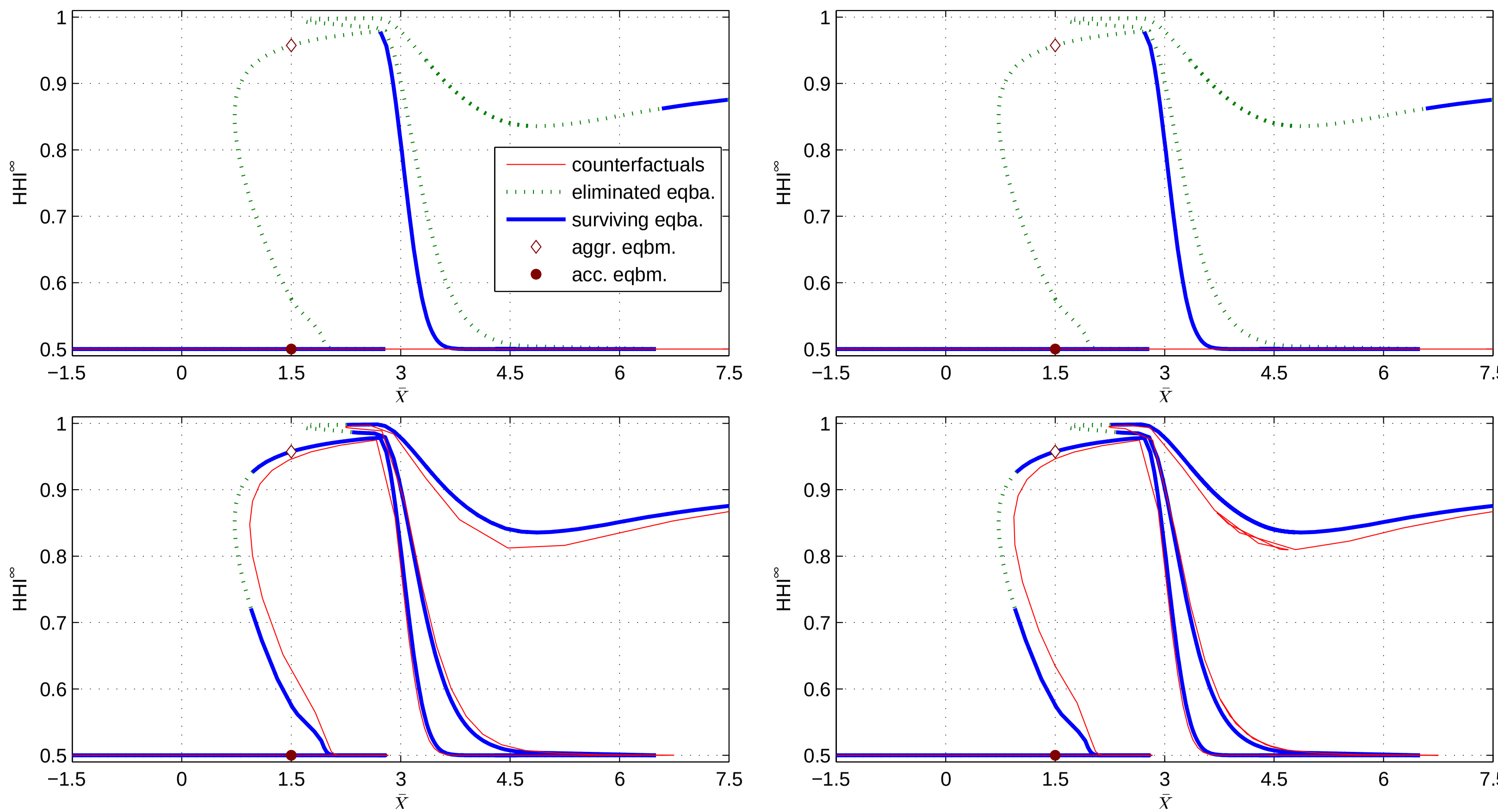

Figure A15: Expected long-run Herfindahl index. Counterfactual (solid red line) and equilibrium correspondences for Definitions 1–4 (upper left, upper right, lower left, and lower right panels) along with eliminated (dashed green line) and surviving (solid blue line) equilibria. Slice along $\overline{X} \in [-1.5, 7.5]$.

"rival's viability" to refer to the probability that the rival exits the industry in the current period. Finally, we interpret "the different action would be more profitable" in the spirit of Markov perfection to mean that by a setting a higher price in the current period but returning to equilibrium play from the subsequent period onward, the firm can affect the evolution of the state to increase its expected NPV if it believed, counterfactually, that the probability that the rival exits the industry in the current period is fixed at $\phi_2(\mathbf{e})$.

With these interpretations, Proposition 1 formalizes the relationship between the Cabral & Riordan definition of predation and the decomposition (10) in the main text:

Proposition 1 *Consider an industry with two incumbent firms in state* $\mathbf{e} \geq (1,1)$. *Assume* $\phi_1(\mathbf{e}) < 1$, $V_1(e_1, 0) > V_1(\mathbf{e})$, *and* $V_1(e_1+1, 0) > V_1(e_1+1, e_2)$, *i.e., exit by the firm is less than certain and the expected NPV of a monopolist exceeds that of a duopolist. (a) If* $\Gamma_1^2(\mathbf{e}) \geq 0$ *and* $\Theta_1^2(\mathbf{e}) \geq 0$, *with at least one of these inequalities being strict, and*

$$\Gamma_1^2(\mathbf{e}) + \left[\Gamma_1^3(\mathbf{e}) - \Gamma_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right] + \Upsilon(p_2(\mathbf{e}))\left[\left[\Theta_1^1(\mathbf{e}) - \Theta_1^1(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right] + \Theta_1^2(\mathbf{e}) + \left[\Theta_1^3(\mathbf{e}) - \Theta_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right]\right] > 0, \quad \text{(A3)}$$

then the firm's equilibrium price $p_1(\mathbf{e})$ *in state* $\mathbf{e}$ *is predatory according to the Cabral & Riordan (1997) definition.*[2] *(b) If* $p_1(\mathbf{e})$ *is predatory according to the Cabral & Riordan definition, then* $\Gamma_1^2(\mathbf{e}) > 0$ *or* $\Theta_1^2(\mathbf{e}) > 0$ *and inequality (A3) holds.*

Proof. The probability that firm 2 exits the industry in the current period (given $p_2(\mathbf{e})$ and $\mathbf{e}$) is

$$\Phi_2(p_1, p_2(\mathbf{e}), \mathbf{e}) = \phi_2(\mathbf{e})D_0(p_1, p_2(\mathbf{e})) + \phi_2(e_1+1, e_2)D_1(p_1, p_2(\mathbf{e})) + \phi_2(e_1, e_2+1)D_2(p_1, p_2(\mathbf{e})).$$

We say that $p_1(\mathbf{e})$ is predatory according to the Cabral & Riordan (1997) definition if there exists a price $\widetilde{p}_1 > p_1(\mathbf{e})$ such that (1) $\Phi_2(p_1(\mathbf{e}), p_2(\mathbf{e}), \mathbf{e}) > \Phi_2(\widetilde{p}_1, p_2(\mathbf{e}), \mathbf{e})$ and (2) $\Pi_1(p_1(\mathbf{e}), p_2(\mathbf{e}), \mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})} < \Pi_1(\widetilde{p}_1, p_2(\mathbf{e}), \mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}$.

Part (a): Let $\widetilde{p}_1 = \arg\max_{p_1} \Pi_1(p_1, p_2(\mathbf{e}), \mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}$. Then $\widetilde{p}_1$ is uniquely determined by

$$mr_1(\widetilde{p}_1, p_2(\mathbf{e})) - c(e_1) + \left[\Gamma_1^1(\mathbf{e}) + \Gamma_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})} + \Gamma_1^4(\mathbf{e}) + \Gamma_1^5(\mathbf{e})\right] + \Upsilon(p_2(\mathbf{e}))\left[\Theta_1^1(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})} + \Theta_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})} + \Theta_1^4(\mathbf{e})\right] = 0. \quad \text{(A4)}$$

[2] The notation $\cdot\big|_{\phi_2=\phi_2(\mathbf{e})}$ means that we evaluate the relevant term under the assumption that $\phi_2(\mathbf{e}) = \phi_2(e_1+1, e_2) = \phi_2(e_1, e_2+1)$ so that the probability that the rival exits the industry in the current period is indeed fixed at $\phi_2(\mathbf{e})$.

Subtracting equation (10) in the main text (evaluated at $p_1 = p_1(\mathbf{e})$) from equation (A4), we have

$$mr_1(\widetilde{p}_1, p_2(\mathbf{e})) - mr_1(p_1(\mathbf{e}), p_2(\mathbf{e})) = \left[\Gamma_1^2(\mathbf{e}) + \left[\Gamma_1^3(\mathbf{e}) - \Gamma_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right]\right]$$
$$+\Upsilon(p_2(\mathbf{e}))\left[\left[\Theta_1^1(\mathbf{e}) - \Theta_1^1(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right] + \Theta_1^2(\mathbf{e}) + \left[\Theta_1^3(\mathbf{e}) - \Theta_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right]\right] > 0$$

per inequality (A3). Because $mr_1(p_1, p_2(\mathbf{e}))$ is strictly increasing in p_1, it follows that $\widetilde{p}_1 > p_1(\mathbf{e})$.

Because $\Gamma_1^2(\mathbf{e}) \geq 0$ and $\Theta_1^2(\mathbf{e}) \geq 0$, with at least one of these inequalities being strict, under the maintained assumptions of Proposition 1 it follows that $\phi_2(e_1+1, e_2) - \phi_2(\mathbf{e}) \geq 0$ and $\phi_2(\mathbf{e}) - \phi_2(e_1, e_2+1) \geq 0$, with at least one of these inequalities being strict. Because $D_0(\mathbf{p}) = 1 - D_1(\mathbf{p}) - D_2(\mathbf{p})$ we thus have

$$\frac{\partial \Phi_2(p_1, p_2(\mathbf{e}), \mathbf{e})}{\partial p_1} = [\phi_2(e_1+1, e_2) - \phi_2(\mathbf{e})]\frac{\partial D_1(p_1, p_2(\mathbf{e}))}{\partial p_1} - [\phi_2(\mathbf{e}) - \phi_2(e_1, e_2+1))]\frac{\partial D_2(p_1, p_2(\mathbf{e}))}{\partial p_1} < 0$$

since $\frac{\partial D_1(p_1,p_2(\mathbf{e}))}{\partial p_1} < 0$ and $\frac{\partial D_2(p_1,p_2(\mathbf{e}))}{\partial p_1} > 0$. Thus, $\Phi_2(p_1(\mathbf{e}), p_2(\mathbf{e}), \mathbf{e}) > \Phi_2(\widetilde{p}_1, p_2(\mathbf{e}), \mathbf{e})$. This establishes part (1) of the Cabral & Riordan definition above.

To establish part (2), recall that by construction $\Pi_1(p_1(\mathbf{e}), p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})} \leq \Pi_1(\widetilde{p}_1, p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})}$. Moreover, this inequality is strict because $\Pi_1(p_1, p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})}$ is strictly quasiconcave in p_1.

Part (b): Because $p_1(\mathbf{e})$ is predatory according to the Cabral & Riordan definition, there exists a higher price $\widetilde{p}_1 > p_1(\mathbf{e})$ such that (1) $\Phi_2(p_1(\mathbf{e}), p_2(\mathbf{e}), \mathbf{e}) > \Phi_2(\widetilde{p}_1, p_2(\mathbf{e}), \mathbf{e})$ and (2) $\Pi_1(p_1(\mathbf{e}, p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})} < \Pi_1(\widetilde{p}_1, p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})}$. Thus we have

$$\begin{aligned} &\Phi_2(p_1(\mathbf{e}), p_2(\mathbf{e}), \mathbf{e}) - \Phi_2(\widetilde{p}_1, p_2(\mathbf{e}), \mathbf{e}) \\ &= [D_1(p_1(\mathbf{e}), p_2(\mathbf{e})) - D_1(\widetilde{p}_1, p_2(\mathbf{e}))]\,[\phi_2(e_1+1, e_2) - \phi_2(\mathbf{e})] \\ &- [D_2(p_1(\mathbf{e}), p_2(\mathbf{e})) - D_2(\widetilde{p}_1, p_2(\mathbf{e}))]\,[\phi_2(\mathbf{e}) - \phi_2(e_1, e_2+1)] > 0. \end{aligned} \tag{A5}$$

Because $\frac{\partial D_1(p_1,p_2(\mathbf{e}))}{\partial p_1} < 0$ and $\frac{\partial D_2(p_1,p_2(\mathbf{e}))}{\partial p_1} > 0$, $D_1(p_1(\mathbf{e}), p_2(\mathbf{e})) - D_1(\widetilde{p}_1, p_2(\mathbf{e})) > 0$ and $D_2(p_1(\mathbf{e}), p_2(\mathbf{e})) - D_2(\widetilde{p}_1, p_2(\mathbf{e})) < 0$. The only way for inequality (A5) to hold is thus that $\phi_2(e_1+1, e_2) - \phi_2(\mathbf{e}) > 0$ or $\phi_2(\mathbf{e}) - \phi_2(e_1, e_2+1) > 0$ which, in turn, implies $\Gamma_1^2(\mathbf{e}) > 0$ or $\Theta_1^2(\mathbf{e}) > 0$.

Because $\Pi_1(p_1, p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})}$ is strictly quasiconcave in p_1, it follows from $\widetilde{p}_1 > p_1(\mathbf{e})$

and $\Pi_1(p_1(\mathbf{e}), p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})} < \Pi_1(\widetilde{p}_1, p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})}$ that

$$\begin{aligned}\frac{\partial\, \Pi_1(p_1(\mathbf{e}), p_2(\mathbf{e}), \mathbf{e})|_{\phi_2=\phi_2(\mathbf{e})}}{\partial p_1} &= mr_1(p_1(\mathbf{e}), p_2(\mathbf{e})) - c(e_1) \\ &+ \left[\Gamma_1^1(\mathbf{e}) + \Gamma_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})} + \Gamma_1^4(\mathbf{e}) + \Gamma_1^5(\mathbf{e})\right] \\ +\Upsilon(p_2(\mathbf{e}))\, &\left[\Theta_1^1(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})} + \Theta_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})} + \Theta_1^4(\mathbf{e})\right] < 0. \end{aligned} \tag{A6}$$

Subtracting inequality (A6) from equation (10) in the main text (evaluated at $p_1 = p_1(\mathbf{e})$) then yields

$$\begin{aligned} &\Gamma_1^2(\mathbf{e}) + \left[\Gamma_1^3(\mathbf{e}) - \Gamma_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right] \\ +\Upsilon(p_2(\mathbf{e}))&\left[\left[\Theta_1^1(\mathbf{e}) - \Theta_1^1(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right] + \Theta_1^2(\mathbf{e}) + \left[\Theta_1^3(\mathbf{e}) - \Theta_1^3(\mathbf{e})\big|_{\phi_2=\phi_2(\mathbf{e})}\right]\right] > 0. \end{aligned}$$

■

Ordover & Willig (1981). According to Ordover & Willig (1981), "[p]redatory behavior is a response to a rival that sacrifices part of the profit that could be earned under competitive circumstances were the rival to remain viable, in order to induce exit and gain consequent additional monopoly profit" (pp. 9–10). As Cabral & Riordan (1997) observe, the premise in the Ordover & Willig definition is that the rival is viable with certainty.[3] We have:

Proposition 2 *Consider an industry with two incumbent firms in state* $\mathbf{e} \geq (1,1)$*. Assume* $\phi_1(\mathbf{e}) < 1$*,* $V_1(e_1, 0) > V_1(\mathbf{e})$*, and* $V_1(e_1+1, 0) > V_1(e_1+1, e_2)$*, i.e., exit by the firm is less than certain and the expected NPV of a monopolist exceeds that of a duopolist. (a) If* $\Gamma_1^2(\mathbf{e}) \geq 0$ *and* $\Theta_1^2(\mathbf{e}) \geq 0$*, with at least one of these inequalities being strict, and*

$$\begin{aligned} &\Gamma_1^2(\mathbf{e}) + \left[\Gamma_1^3(\mathbf{e}) - \Gamma_1^3(\mathbf{e})\big|_{\phi_2=0}\right] + \Gamma_1^5(\mathbf{e}) \\ +\Upsilon(p_2(\mathbf{e}))&\left[\left[\Theta_1^1(\mathbf{e}) - \Theta_1^1(\mathbf{e})\big|_{\phi_2=0}\right] + \Theta_1^2(\mathbf{e}) + \left[\Theta_1^3(\mathbf{e}) - \Theta_1^3(\mathbf{e})\big|_{\phi_2=0}\right]\right] > 0, \end{aligned} \tag{A7}$$

then the firm's equilibrium price $p_1(\mathbf{e})$ *in state* $\mathbf{e}$ *is predatory according to the Ordover & Willig (1981) definition. (b) If* $p_1(\mathbf{e})$ *is predatory according to the Ordover & Willig definition, then* $\Gamma_1^2(\mathbf{e}) > 0$ *or* $\Theta_1^2(\mathbf{e}) > 0$ *and inequality (A7) holds.*

Proof. Omitted as it follows the same logic as the proof of Proposition 1. ■

[3]This observation indeed motivates Cabral & Riordan (1997) to propose their own definition: "Is the appropriate counterfactual hypothesis that firm B remain viable with probability one? We don't think so. Taking into account that firm B exits for exogenous reasons (i.e. a high realization of [the scrap value]) hardly means that firm A intends to drive firm B from the market" (p. 160).

References

Cabral, L. & Riordan, M. (1997), 'The learning curve, predation, antitrust, and welfare', *Journal of Industrial Economics* **45**(2), 155–169.

Ordover, J. & Willig, R. (1981), 'An economic definition of predation: Pricing and product innovation', *Yale Law Journal* **91**, 8–53.