Sample DECS Qualifying Exam

1. READ THIS PAGE BEFORE YOU BEGIN THE EXAM.
2. The exam is open book, open notes, and open Excel. You may need Excel for some of the calculations. There are no data files.
3. The exam consists of 10 questions.
4. Please attempt as many as you can. We will then evaluate your exam **both** for entering Turbo and as a waiver. Please note that the requirements for a waiver will be more stringent.
5. The time allowed for the exam is 3 hours.
6. The Kellogg Honor Code applies. In particular, the exam must be solely your own work.

**GOOD LUCK!**

Question 1 (5 points)

Let A and B be two events. Suppose P(A) = 0.2, P(B) = 0.3 and P(A ∩ B) = 0.1.

a) Compute P(A ∪ B).

b) Are A and B independent events? Explain.

c) Compute P(A | B).

Question 2 (5 points)

The finance department of a particular company has 15 employees, 6 of whom have MBA degrees. Suppose we select three employees sequentially at random from this department. Determine the probability of the following events.

a) The first employee has an MBA given that there is a total of one MBA among all three employees.

b) There are exactly two employees with an MBA given that the first employee has an MBA.

c) The first employee has an MBA given that there is at least one MBA among all three employees.

**Question 3 (5 points)**

Suppose a stock is currently priced at $100 per share. Let X represent the dollar amount by which its price will change over the next one week. X is normally distributed, with E(X)=0.10, and standard deviation of 0.50. Use the relevant EXCEL functions to answer the following.

a) What is the probability that after one week, the stock's value will be between $99.50 and $101.00?

b) Now examine the price of this stock one year from now (52 weeks). Suppose that each weekly price change of the stock is described by this distribution of X. In addition, suppose weekly

price changes are independent. Find the probability that after one year, the stock's value will be between $99 and $111.

**Question 4 (10 points)**

A fair coin is tossed 5 times. Each toss is independent of the other.

a) What is the probability of getting exactly 3 heads?

b) What is the probability of getting at least one heads?

c) What is the expected number of heads?

d) What is the variance of the number of heads?

**Question 5 (5 points)**

A major credit rating agency rates a bond of a company as CCC if the probability of default is 0.10 in any given year and defaults across years are independent.

a) What is the probability that a CCC bond defaults in the 4th year (that is, the company goes for 3 years without default, then defaults in the fourth)?

b) Suppose you own a pool of 20 CCC bonds from 20 different companies. Assume the default risks for different companies are independent. What is the probability that between 1 and 3 of these CCC bonds (that is either one or two or three CCC bonds) default in the first year?

**Question 6 (10 points)**

Unoccupied seats on flights cause airlines to lose revenues. A large airline wants to estimate its average number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected, and the number of unoccupied seats is noted for each of the flights in the sample. The sample mean is 14.5 seats and the sample standard deviation is s = 8.2 seats.

1. Provide an 80% confidence interval for the mean number of unoccupied seats per flight during the past year. **(5 points)**
2. Can you prove, at a 2% level of significance, that the average number of unoccupied seats per flight during the last year was smaller than 15.5? **(5 points)**

**Question 7 (10 points)**

You have data on prices and areas of Evanston condominiums. You use your data to estimate the dependence of price on area. The result of a linear regression is the equation :

PRICE = 8193 + 303\*AREA

PRICE is in dollars and AREA is in square feet.

1. Jorge and Gudrun both live in Evanston condos. Jorge’s unit is 150 sq. ft. larger than Gudrun’s. How much would you estimate is the difference in price between these units? **(5 points)**
2. Shirley wants to buy a condo for $300,000. What size condo would have this estimated price ? **(5 points)**

**Question 8 (5 points)**

A certain type of cold vaccine is known to be only 25% effective after a period of 2 years. That is there is a 25% probability that someone who takes the vaccine will be free of the cold virus after two years.

To determine if a new, more expensive vaccine is superior in providing protection against the cold virus for a longer period of time, 200 people are chosen at random and inoculated with the new vaccine. If fewer than 120 of those receiving the new vaccine contract the virus within a two year period, the new vaccine will be considered superior to the present one.

a) Suppose the new vaccine has exactly the same effectiveness as the present one. What is the chance that the outcome of the test with the 200 subjects will conclude that the new vaccine is superior to the present one?

b) Suppose the new vaccine is in fact twice as effective as the present one. That is 50% of those who receive the vaccine are protected for 2 years or more. What is the probability that our test will declare the new vaccine NOT superior to the present one?

Question 9 (10 points)

A company sells an insurance policy against losses incurred during travel for $15.

40,000 customers buy the insurance policy. The policy pays the full amount claimed up till $5,000. Most policy holders claim nothing, the rest either $500 or $5,000.

Each customer has the following probabilities of claiming a certain amount for damages,

independent of the other customers.

|  |  |  |  |
| --- | --- | --- | --- |
| Claim amount | $0 | $500 | $5,000 |
| Probability | 99% | 0.8% | 0.2% |

Use the central limit theorem to estimate the probability that the company makes a positive net profit on these contracts.

Question 10 (10 points)

There are n investors labeled n=1, 2, ….. Each investor must decide whether to invest one dollar (action I) or not (action nI). Investors are risk neutral. The payoff received by an investor depends on the action taken and the underlying state (S or W), and are given in the following table:

|  |  |  |
| --- | --- | --- |
|  | S | W |
| I | 1 | -1 |
| nI | 0 | 0 |

In the absence of any information each investor believes that Prob(S)=0.55 and Prob(W)=0.45.

The investors make their decisions sequentially.

* Individual n receives a signal sn, which can take one of two possible values, g or b.
* Given the state, the signals are independent and identically distributed across individuals. Specifically, Prob(sn=gn|S)= Prob(sn=bn|W)=0.65.
* Individual n makes a decision.
* Individual n’s information consists of his private signal sn and the *actions* a1, …, an-1 taken by individuals 1, 2, … n-1.

Signals are not free. Rather, investor n must pay $0.01 (or 1% of his investment) to see the value of his signal sn. Interpret this as the cost of marketing research, internal resources needed to compile public and privately acquired information, etc.

Investor n observes allpast decisions made by other investors (i.e. whether they gather information or not). (Investor 1 has no predecessor, so no such information is available). In addition, investor n has the option to: (a) pay nothing, obtain no new information and proceed to make his investment decision based on available information about the decisions of previous investors; or (b) pay 0.01 to acquire new information in the form of finding out whether sn=g or b, then proceed to make his investment decision. The content of the signal itself, i.e. whether sn=g or b, is known only to investor n; it is never revealed to others.

1. Should investor 1 pay 0.01 to learn the value of his signal? Draw a decision tree for this investor as follows: he first chooses whether or not to pay the 0.01 for the signal; if he decides not to pay, he makes the investment decision based on his prior belief since he learns nothing new. In this case, the payoff to investing is simply P(S)-P(W). If he pays to acquire information, then the 0.01 is sunk, and he proceeds to making his invest/not invest decision based on whether s1=g or b. In doing so, he updates his beliefs about S/W using Bayes rule (for example, if he observes g, then his payoff is P(S|g)-P(W|g)-0.01).

Carefully label all relevant parts of the decision tree, and fold it back to find the optimal decision. **[3points]**

1. Consider now investor 2. Suppose he just observed that investor 1 paid to acquire information (that is, he sees that 0.01 was paid, but not whether s1=b or g) and that investor 1 invested. Based on this information, should investor 2 pay the 0.01 to find out whether his signal is b or g? Would he choose to invest or not? Draw a decision tree and fold it back to find the optimal decision. **[3points]**

Hint: use the same structure of the tree as part 1; adjust the probabilities to reflect investor 2’s information about investor 1’s actions.

1. Given your answers to 1 and 2 above, should investor 3 pay to acquire information? Does he invest? **[4points]**