# What does the value premium tell us about the term structures of 

equity returns?

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#### Abstract

Contrary to conventional wisdom, growth stocks (low book-to-market stocks) do not have substantially higher future cash-flow growth rates or longer cash-flow durations than value stocks, for both rebalanced and buy-and-hold portfolios. The efficiency growth (perhaps driven by decreasing returns to scale), survivorship and look-back biases, and rebalancing effect help explain the results. Using rebalanced portfolios, I find that, consistent with leading asset pricing models that feature countercyclical risk premiums or procyclical expected growth rates, there is a growth premium in the cross section of stock returns. That is, risky assets with higher expected cash-flow growth rates have higher expected returns, after controlling for cash-flow risks.


[^0]
## 1 Introduction

Leading asset pricing models such as Campbell and Cochrane (1999) and Bansal and Yaron (2004) feature countercyclical risk premiums or procyclical expected growth rates. ${ }^{1}$ These features imply that stocks are more risky, because in bad times, prices go down further, as a result of an increase of the discount rate or a decrease in the expected growth rate, than in an i.i.d. world. In simple specifications, because the effects of changes in discount rates or expected growth rates are larger in long-duration assets, these features imply that there is a growth premium in the cross section. That is, stocks with higher expected cash-flow growth rates (and therefore longer cash-flow durations) have higher expected returns, after controlling for cash-flow risks. This implication is widely viewed as being at odds with the value premium. Conventional wisdom holds that growth stocks, defined throughout this paper as stocks with low book-to-market equity, have substantially higher future cash-flow growth rates and substantially longer cash-flow durations than value stocks, and that the value premium is evidence against leading asset pricing models. This view is suggested by the naming of growth stocks, is apparently backed by empirical results, and is matched by theoretical models that try to explain the value premium. ${ }^{2}$

I argue that empirical evidence for this conventional wisdom is much weaker than commonly believed. Building on previous work, I examine growth rates of cash flows for both rebalanced and buy-and-hold portfolios. I find that annually rebalanced portfolios of growth stocks have lower growth rates than value stocks for all three cash-flow variables that I examine: earnings, accounting cash flow, and dividends. For buy-and-hold portfolios, the growth rate exhibits a Ushaped relation with the book-to-market equity, with little difference between the extreme growth stocks and the extreme value stocks. In fact, initially the earnings of value stocks grow faster

[^1]than growth stocks, although the difference does not persist beyond three years after portfolio formation. I also derive a relation between growth rates of buy-and-hold and rebalanced portfolios, and show that rebalanced growth rates should be higher than buy-and-hold growth rates for value stocks, and that the opposite is true for growth stocks, under mild conditions.

I then examine the Macaulay duration of value and growth stocks, using the weighted average of cash-flow maturities. I find that growth stocks do have longer durations. However, the conventional duration is basically the price-dividend ratio, which is not a clean variable. It can be high either because the discount rate is low, or because the expected growth rate is high. If the expected growth rate is held constant across assets, higher cash-flow risk leads to a higher discount rate, which in turn leads to a lower price-dividend ratio and shorter duration. In this case, there is a negative relation between duration and the discount rate, but this relation is not anomalous. If cash-flow risk is held constant across assets, the difference in the expected growth rate leads to a positive relation between duration and the discount rate in leading asset pricing models. To isolate the effect of the cash-flow growth profiles, I propose to measure the cash-flow duration by discounting the asset-specific cash-flow growth profiles with a common discount rate for all assets. I find that growth stocks have similar cash-flow durations with value stocks in buy-and-hold portfolios, and have shorter cash-flow durations than value stocks in rebalanced portfolios. ${ }^{3}$

Using these findings, I then test whether there is a growth premium in the cross section of stock returns. I use rebalanced portfolios to test this implication because rebalancing produces assets that have persistently different growth rates. I provide empirical evidence that the growth premium is broadly consistent with the entire cross section of stock returns. Using portfolios sorted by size, book-to-market, and momentum, I find that portfolios with higher expected cashflow growth rates do have higher future returns, after controlling for cash-flow risks.

There are at least three reasons why the conventional wisdom is widely held. First, Gordon's

[^2]formula, $\frac{P}{D}=\frac{1}{r-g}$, suggests that all else being equal, stocks with higher prices should have higher cash-flow growth rates. Second, Fama and French (1995) show that growth stocks have persistently higher returns on equity than value stocks, even five years after they are sorted into portfolios. Third, in firm-level regressions of future dividend growth rates on the book-to-market ratios, the coefficients are highly negative, even for dividend growth rates ten years in the future.

I address each of the above reasons. First, when we compare value stocks with growth stocks, all else is not equal. If we consider that value stocks have higher expected returns than growth stocks, valuation models actually imply that growth stocks have similar growth rates with value stocks in buy-and-hold portfolios, and have lower growth rates than value stocks in rebalanced portfolios. ${ }^{4}$ Second, Fama and French (1995)'s results pertain to the behavior of the return on equity, which is relevant for studying the growth rates of book equity. However, those results do not imply that cash-flow growth rates for growth stocks are higher. In fact, some back-of-theenvelope calculations suggest that Fama and French (1995)'s results imply that growth stocks have lower earnings growth rates than value stocks initially. The efficiency growth for the value stocks, perhaps driven by decreasing returns of scale, helps explain the result. Third, the dividend growth rate regression is subject to survivorship bias. After I account for survivorship bias, high book-to-market equity no longer predicts a lower future dividend growth rate.

My results help explain a number of other related puzzles in the cross section of stock returns: the momentum vs. value puzzle, the time series vs. cross section puzzle, and the good growth puzzle. To explain the momentum effect, Johnson (2002) argues that recent winners have higher expected cash-flow growth rates. He then essentially uses the growth premium, driven by procyclical expected growth rates, to explain the momentum effect. If one views value stocks as having lower growth rates, then this mechanism is at odds with the value premium. My results suggest that the growth premium is consistent with the value premium and the momentum effect at the same time. Second, in studying the aggregate time series, most authors (e.g., Menzly, Santos, and Veronesi (2004), Lettau and Ludvigson (2005), and Binsbergen and Koijen (2010)) find that the

[^3]expected return is positively related to the expected dividend growth rate. My results suggest that this positive association also exists in the cross section. Finally, Novy-Marx (2010) finds that more profitable firms (firms with good growth) have higher returns; his results are difficult to reconcile with popular models of the value premium, because more profitable firms have longer cash-flow duration. My results suggest that his are consistent with the growth premium.

A seminal paper by Lettau and Wachter (2007), motivated by the apparent inconsistency between the value premium and the growth premium, provides an explanation of the value premium. Their model has two key elements: a cash-flow process that features a countercyclical expected growth rate, and a pricing kernel with a market price of risk that itself is not priced (or "acyclical" market price of risk). Their first insight, that the correlation structure of the cash-flow process matters greatly in the equity term structure, is clearly an important contribution. However, my results call into question the second element of their model for two reasons. First, when fitting buy-and-hold portfolios, they assume that cash-flow growth rates of value stocks are substantially lower than those of growth stocks for a prolonged period. My results suggest that their assumption has understated the growth rates for value stocks relative to growth stocks. If they increase the growth rates for value stocks in their calibration, it is plausible that their model can accommodate a market price of risk that is countercyclical. Second, I find a growth premium in rebalanced portfolios. Their model does not generate a growth premium in rebalanced portfolios. Therefore, results from rebalanced portfolios support the class of models with countercyclical risk premiums or procyclical expected growth rates over theirs.

Santos and Veronesi (2010) also argue against the second element of Lettau and Wachter (2007). Their argument is mainly theoretical, as they point out that Lettau and Wachter (2007) ignore general equilibrium restrictions. My arguments are primarily based on empirical evidence from the cross section. Santos and Veronesi (2010) calibrate a habit formation model to the value premium, and find that value stocks do not have enough cash-flow risk to explain their returns (the cash-flow risk puzzle). My results suggest habit formation (or any mechanism that generates countercyclical risk premiums) does not deepen the puzzle of the value premium, because value stocks do not have shorter cash-flow durations, even in buy-and-hold portfolios. I also point out
the practical difficulty in estimating cash-flow risk in buy-and-hold portfolios, including look-back bias.

This paper builds on previous work that examines growth rates. In fact, Lakonishok, Shleifer, and Vishny (1994) already provide some results (in their Table V) that equal-weighted portfolios of extreme growth stocks have higher growth rates in cash flows from year 0 to year 2 , but often have lower growth rates from year 2 to year 5 than extreme value stocks. Part of my contribution is to extend their results to depict a complete picture. I find that their results are not driven purely by small stocks, and hold up in value-weighted portfolios as well, and that there is a U-shaped relation between cash-flow growth rates and the book-to-market equity in buy-and-hold portfolios. A contemporaneous working paper by Penman, Reggiani, Richardson, and Tuna (2011) also finds that value stocks have higher earnings growth rate than growth stocks in year 2. However, their paper does not address survivorship bias in such calculations. Evidence exists in the literature that rebalanced portfolios of value stocks have higher dividend growth rates (see Ang and Liu (2004), Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008), and Chen, Petkova, and Zhang (2008)). The first three papers do not explain why their results differ from buy-and-hold portfolios. The last paper argues that their finding is consistent with conventional wisdom, and that their results are driven by the fact that value stocks have higher capital gains. I show that cash flow growth rates of buy-and-hold portfolios differ little between growth and value stocks. Relative to all six papers, I illustrate the effect of survivorship and look-back biases. I derive an explicit relation between growth rates of buy-and-hold and rebalanced portfolios, and show that rebalanced growth rates should be higher than look-back and buy-and-hold growth rates for value stocks, and that the opposite is true for growth stocks, under mild conditions (even in the absence of the value premium); thus enriching Chen, Petkova, and Zhang (2008)'s explanation. I provide new results on cash-flow durations. Most important, I examine the implications of these growth rates on asset pricing models, and show that leading asset pricing models that feature countercyclical risk premiums or procyclical expected growth rates are broadly consistent with the cross section of stock returns.

More broadly, the evidence in this paper lends support to the effort by the literature (e.g.,

Zhang (2005)), that employs models of time-varying expected returns to help understand the cross section of stock returns. My results are also broadly consistent with the view that growth stocks have longer durations (Cornell (1999), Campbell, Polk, and Vuolteenaho (2010)), but the discount rate risk is positively priced (Campbell and Vuolteenaho (2004)).

In a recent paper, Binsbergen, Brandt, and Koijen (2010) use data from the derivative market between 1996 and 2009, and show that short-term dividend strips have higher expected returns and volatilities than the aggregate stock market. They argue that their results are inconsistent with the leading asset pricing models. To reconcile my results with theirs remains an important task for future research. ${ }^{5}$

The rest of the paper is organized as follows. I present variable definitions and data sources in Section 2. Section 3 provides evidence that cash flows of growth stocks do not grow substantially faster than those of value stocks in the future. In doing so, I also point out survivorship and look-back biases in common empirical procedures. Section 4 shows that growth stocks have similar cash-flow durations with value stocks in buy-and-hold portfolios, and have shorter cashflow durations than value stocks in rebalanced portfolios. Section 5 presents evidence on the growth premium. Section 6 concludes.

## 2 Data and variable definitions

The data I use come from CRSP and Compustat. I only include stocks with share codes 10 or 11 that are listed on NYSE, AMEX, or Nasdaq. Financials and utilities are excluded. Returns and market equity (abs(prc)*shrout) are from CRSP. Accounting variables are from Compustat fundamental file (North America). I define book equity ( $B E$ ) as stockholders' equity, plus balance sheet deferred taxes ( txdb ) and investment tax credit (itcb) (if available), minus the book value of preferred stock. Depending on availability, I use redemption (pstkrv), liquidation (pstkl), or par value (pstk), in that order, for the book value of preferred stock. I calculate stockholders' equity used in the above formula as follows. I prefer to use the stockholders' equity number reported by

[^4]Compustat (seq). If seq is not available, then I measure stockholders' equity as the book value of common equity (ceq), plus the book value of preferred stock. Note that the preferred stock is added at this stage, because it is later subtracted in the book equity formula. If common equity is not available, I compute stockholders' equity as the book value of assets (at) minus total liabilities (lt), all from Compustat.

I also obtain revenue (sale) and cost (the sum of xopr and dp) from Compustat. Earnings are defined as income before extraordinary items (ib). Accounting cash flow is defined as income before extraordinary items plus depreciation and amortization (dp). Dividends (including repurchases) are computed from CRSP, by multiplying the lagged market equity by the difference between returns with and without dividends. ${ }^{6}$ I then sum up the dividends for each firm between July and June of the following year. I use dividends constructed from CRSP because I know when they are paid out. However, I can report that the results are qualitatively the same if I simply use dividends from Compustat.

To compute per share variables, I divide most variables by Compustat variable cshpri (common shares used to calculate earnings per share - basic). For book equity and assets, I use CRSP shares outstanding (shrout/1000). For earnings per share, I use Compustat epspx directly. CRSP adjustment factor (cfacpr) is used to ensure that per share variables are comparable over time.

When forming book-to-market portfolios in June of year $t$, I sort stocks according to their book-to-market ratios. The book-to-market equity uses the book equity for the fiscal year that ends in the calendar year $t-1$. The market equity is from CRSP in December of year $t-1$. The breakpoints are computed using NYSE stocks only.

[^5]
## 3 Do cash flows of growth stocks grow faster than value stocks?

### 3.1 Two pieces of evidence that suggest growth stocks grow faster

The first piece of evidence comes from Fama and French (1995), who show that growth stocks have persistently higher returns on equity than value stocks. I update their results in Fig. 1. In each year $t$ between 1963 and 2009, I sort stocks according to their book-to-market ratios. The value, neutral, and growth portfolios consist of stocks with book-to-market equity that are in the highest $30 \%$, middle $40 \%$ and lowest $30 \%$. Once I have formed the portfolios, I then look at the return on equity for each portfolio five years before and ten years after portfolio formation. The return on equity in year $t+s$ for a portfolio formed in year $t$ is computed as: $R O E_{t+s}=\frac{E_{t+s}}{B E_{t+s-1}}$. Portfolio earnings $(E)$ and book equity $(B E)$ are the sum of firm earnings and book equity in that portfolio. I treat earnings and book equity with fiscal year ends between July of year $t+s-1$ and June of year $t+s$ as earnings and book equity in year $t+s .{ }^{7}$ I follow Fama and French (1995) and require a stock to have data for both $E_{t+s}$ and $B E_{t+s-1}$ to be included in the computation of the portfolio return on equity, although I show later that this requirement gives rise to survivorship bias. I average the portfolio return on equity across the 47 portfolio formation years 1963-2009. That is, $R O E_{s}=E\left[R O E_{t+s}\right]$, in which taking expectation means averaging over portfolio formation years $t$. Because I track the portfolio five years before its formation year, accounting information between 1957 and 2010 is used.

Fig. 1 plots the $R O E_{s}$ for $s$ between -5 and 10. Fig. 1 shows that growth stocks have persistently higher returns on equity than value stocks, even ten years after portfolio formation. This finding led Fama and French to term the stocks with low book-to-market ratios as "growth stocks". The return on equity reaches the highest value for growth stocks and the lowest value for value stocks in year 1. This pattern is the same as in Fama and French (1995).

Table 1 provides the second piece of evidence. In this table, I estimate firm-level regressions of log dividend growth rates on lagged book-to-market ratios. In particular, I estimate the following

[^6]regression in each year:
\[

$$
\begin{equation*}
\log \left(D_{i, t} / D_{i, t-1}\right)=b_{0}+b_{1} \log (B / M)_{i, t-k}+\epsilon_{i, t} . \tag{1}
\end{equation*}
$$

\]

I estimate the regression using the Fama-MacBeth procedure between 1965 and 2010, for $k$ between 1 and 10. $D_{i, t}$ is the dividend from July of year $t-1$ to June of year $t$ computed from CRSP. Variables are winsorized at $1 \%$ and $99 \%$ in each year. Table 1 reports the results. Years negative refer to the number of years in which the coefficient $b_{1}$ is negative. I report Newey-West $t$-statistics with an automatically selected number of lags.

Table 1 shows that book-to-market equity appears to strongly forecast negative dividend growth. When $k=1$, the coefficient $b_{1}$ is negative in 46 out of 46 years. The average coefficient is -0.143 and is highly statistically significant. In year 2 , the coefficient $b_{1}$ is again negative, with an average coefficient of -0.051 . It is negative in 37 out of 46 years. The coefficient is significantly negative even after nine years.

In the rest of this section, I argue that the above empirical evidence does not imply that growth stocks have higher future cash-flow rates.

### 3.2 The buy-and-hold growth rates

### 3.2.1 The back-of-the-envelope calculation

I argue that Fama and French (1995)'s results on return on equity are related to the growth rates of book equity, but not necessarily of cash flows. ${ }^{8}$ In fact, I argue that Fama and French (1995)'s results on return on equity imply that the earnings growth rates are initially higher for value stocks. Consider a back-of-the-envelope calculation for the earnings growth rates. Earnings growth rate is year $s$ is,

$$
\begin{equation*}
\frac{E_{s}}{E_{s-1}}-1=\frac{\frac{E_{s}}{B_{s-1}}}{\frac{E_{s-1}}{B_{s-2}}} \frac{B_{s-1}}{B_{s-2}}-1 . \tag{2}
\end{equation*}
$$

[^7]Assuming the clean surplus relation in year $s-1$ and a constant dividend payout ratio, po $=$ $D_{s-1} / E_{s-1}$, I show that,

$$
\begin{equation*}
\frac{E_{s}}{E_{s-1}}-1=(1-p o) R O E_{s}+\left(\frac{R O E_{s}}{R O E_{s-1}}-1\right) . \tag{3}
\end{equation*}
$$

The first term on the right-hand side of the previous equation, $(1-p o) R O E_{s}$, is commonly referred to as the sustainable growth rate. The second term, $\frac{R O E_{s}}{R O E_{s-1}}-1$, is often referred to as the efficiency growth. A standard result is that when the $R O E$ is constant, the earnings growth rate is simply equal to the sustainable growth rates. But in this case, $R O E$ exhibits clear time-varying patterns, and the efficiency growth cannot be ignored.

For value stocks, the sustainable growth rate, $(1-p o) R O E_{s}$, is lower than growth stocks, but the efficiency growth, $\frac{R O E_{s}}{R O E_{s-1}}-1$, is higher than growth stocks. It turns out that the efficiency growth dominates, at least initially. For example, assume that the payout ratio is 0.5 . In year 2, for value stocks, $R O E_{s}$ is $0.051, R O E_{s-1}$ is 0.035 , the earnings growth rate is $0.051 / 0.035+0.5 *$ $0.051-1=48.2 \%$. Note that there are rounding errors in this calculation. For growth stocks, $R O E_{s}$ is $0.186, R O E_{s-1}$ is 0.202 , the earnings growth rate is $0.186 / 0.202+0.5 * 0.186-1=1.3 \%$.

The results of the back-of-the-envelope calculations are plotted in Fig. 2. Prior to and in the first year after portfolio formation, growth stocks have higher earnings growth rates than value stocks. However, in year 2 , the earnings growth rate of value stocks ( $48.2 \%$ ) greatly exceeds that of growth stocks ( $1.3 \%$ ). In year 3, the earnings growth rate of value stocks ( $24.4 \%$ ) still exceeds that of growth stocks $(3.1 \%)$. Starting in year 4, the earnings growth rates of the three portfolios become similar. The extremely large earnings growth rate for value stocks in year 2 is clearly related to the fact that many value firms experience low or even negative earnings in year 1 . Nevertheless, this pattern does suggest that the cash flows of value stocks may not be as front loaded as commonly assumed.

I also point out that the growth rate in year 1 may not affect the future return path at all. The reason is that when we compute growth rates in year 1 , the denominator involves earnings accrued between year -1 and year 0 , at which point investors have not yet held the portfolio. Therefore, I call the growth rate in year 1 the look-back growth rate. It is possible to construct
examples in which the growth rate in year 1 affects the price in year 0 , but it does not affect the future return paths at all.

To see this, consider a stock that has paid out $D_{0}$ dividend in the last year. Next year it will pay $D_{1}$, and after that, the growth rate in dividends will be $g_{2}, g_{3}, g_{4}, \ldots$ Note that $D_{1}, g_{2}, g_{3}$, $g_{4}, \ldots$ all can be stochastic. I denote that this stock is characterized by $\left\{D_{0}, D_{1}, g_{2}, g_{3}, g_{4}, \ldots\right\}$. Now imagine another stock that is characterized by $\left\{D_{0}, 2 D_{1}, g_{2}, g_{3}, g_{4}, \ldots\right\}$. It follows immediately that these two stocks will have different price-dividend ratios, $P_{0} / D_{0}$, but their future return paths are exactly the same, although these two stocks clearly have very different growth rates in year 1. For the purpose of explaining returns, I advocate not including the look-back growth rates in both estimating the expected growth rates and cash-flow risks.

### 3.2.2 The buy-and-hold growth rates without adjusting for survivorship bias

I now examine the cash flows and book-equity growth rates before and after portfolio formation for value and growth stocks directly. I wish to compute:

$$
\begin{equation*}
g_{t+s}^{F}=\frac{F_{t+s}}{F_{t+s-1}}-1 \tag{4}
\end{equation*}
$$

where $F_{t+s}$ refers to the fundamental value (earnings, accounting cash flow, dividends, and book equity) in year $t+s$ for a portfolio that is formed in June of year $t$. The fundamental value of a portfolio is the sum of the fundamental values across firms in that portfolio.

I initially try to use Fama and French (1995)'s method. Therefore, I require that to be included in the computation of portfolio growth rates, a stock must have data for both $F_{t+s}$ and $F_{t+s-1}$.

I then average across portfolio formation years.

$$
\begin{equation*}
g_{s}^{F}=E\left[g_{t+s}^{F}\right] \tag{5}
\end{equation*}
$$

where $E[$.$] means averaging across portfolio formation years t$.
One issue that arises is that earnings and accounting cash flow are sometimes negative, even for portfolios. To address this issue, I follow Lakonishok, Shleifer, and Vishny (1994) and first
average earnings and accounting cash flow in year $t+s$ and $t+s-1$ across portfolio formation years, then compute growth rates. ${ }^{9}$ To make the variables comparable across portfolio formation years, I scale cash flows by market capitalizations in June of year $t$. This scaling corresponds to an investment strategy that invests $\$ 1$ in each portfolio formation year. That is, I first compute,

$$
\begin{gather*}
\tilde{F}_{t+s}=\frac{F_{t+s}}{M E_{t}} .  \tag{6}\\
g_{s}^{F}=\frac{E\left[\tilde{F}_{t+s}\right]}{E\left[\tilde{F}_{t+s-1}\right]}-1, \tag{7}
\end{gather*}
$$

where the expectation is averaging across portfolio formation years $t .{ }^{10} \mathrm{I}$ compute growth rates for earnings and accounting cash flow according to Equation (7), and for dividends and book equity according to Equation (5).

Fig. 3 reports the average growth rate $g_{s}^{F}$. Panel A reports the earnings growth rates for value, neutral, and growth stocks. Prior to and in the first year after portfolio formation, growth stocks have higher earnings growth rates than value stocks. But in year 2, the earnings growth rate of value stocks $(36 \%)$ greatly exceeds that of growth stocks ( $7.6 \%$ ). In year 3, the earnings growth rate of value stocks ( $17.5 \%$ ) still exceeds that of growth stocks ( $6.8 \%$ ). Starting in year 4, the earnings growth rates of the three portfolios become similar.

Panel B plots growth rates for accounting cash flow. As with earnings, the accounting-cashflow growth rates are higher for growth stocks up to year 1 after portfolio formation. In year 2, the growth rate of value stocks ( $12.7 \%$ ) exceeds that of growth stocks ( $10 \%$ ). In year 3, growth rates are similar between value and growth stocks. Unlike earnings, starting in year 4, growth stocks appear to have higher growth rates in accounting cash flow than value stocks.

Panel C plots growth rates for dividends. As with earnings and accounting cash flow, dividend

[^8]growth rates are higher for growth stocks up to year 1 after portfolio formation. Starting in year 2 , the growth rates of dividends exhibit a non-monotonic relation with the book-to-market equity, with value stocks growing a little faster than neutral stock. Growth stocks have higher dividend growth rates than value stocks, although the difference is small.

Panel D plots growth rates of book equity. This graph looks similar to Fig. 1. The growth rate of book equity exhibits the same pattern as return of equity, consistent with the clean surplus relation.

The evidence so far suggests that growth stocks do not always grow faster after portfolio formation.

### 3.2.3 Survivorship bias in portfolio growth rates

In the procedure above, I require a firm to be alive in both years $t+s-1$ and $t+s$ to be included in the calculation for growth rates. However, when investors invest in year $t+s-1$, they do not know whether the firm will be alive in year $t+s$. Therefore, requiring the firm to have a valid data entry in year $t+s$ gives rise to a survivorship bias. Suppose that growth stocks (such as internet firms) tend to either become extremely successful (like Google), or they die. If we only look at the firms that survive, we may see a picture that is different from investors' actual experiences.

In Table 2, I report the average transition matrix for portfolios between 1963 and 2009. The number in Row $i$ and Column $j$ is the probability (average percentages) of a stock belonging to book-to-market Decile $j$ or exiting $(j=11)$ in year $t+1$, conditional on the stock belonging to Decile $i$ in year $t$. Each row sums to 100. Conditional on a firm being in the lowest book-to-market decile, there is a $9.74 \%$ chance that the firm will exit (through either merger or delisting for cause) in the next year. Conditional on a firm being in the highest book-to-market decile, there is a $12.65 \%$ chance that the firm will exit in the next year. Thus, it is a pervasive phenomenon that firms disappear (see also Chen (2011)). ${ }^{11}$ Table 2 also shows that a large fraction of firms do not stay in the same portfolio two years in a row. This finding means that when we rebalance portfolios, the portfolio composition changes significantly.

[^9]To account for survivorship bias, it is important that when computing the growth rate in year $t+s$, I do not look at just the firms that are alive in year $t+s$. Instead, I examine all firms that are alive in year $t+s-1$, and reinvest delisting proceeds in the remainder of the portfolios when firms exit in year $t+s$.

Thus, I follow a five-step procedure for the value-weighted portfolios. A similar procedure can be carried out for equal-weighted portfolios.

Step 1: I compute the fundamental-to-price ratio in year $t+s, F P_{t+s}$, for a portfolio that is formed in year $t$, as the value-weighted average of firm fundamental per share to price per share ratios, $\frac{F p s_{t+s}}{P p s_{t+s-1}} . .^{12}$ All firms that are available in year $t+s-1$, but not necessarily in $t+s$, are included. If a firm exits the portfolio in year $t+s$, its fundamental value is set to zero. In the next steps, I make sure that delisting proceeds are accounted for in the future. ${ }^{13}$

Step 2: I compute value-weighted buy-and-hold portfolio returns and returns without dividends $r e t_{t+s}$ and $r e t x_{t+s}^{*}$. The asterisk on retx indicates that it adjusts for repurchases. It is important to include delisting returns in this step. In the month of delisting, if there is no return in CRSP, I set the return (ret) and the return without dividends (retx) to the delisting return (dlret) and the delisting return without dividends (dlretx). When there is a return in the month of delisting, I compound the return and the delisting return. I also compound the retx and dlretx. For delisting returns that are missing and the delisting code is between 400 and 600 , I set dlret and dlretx to be $-30 \%$. For delisting returns that are missing and the delisting codes are either below 400 or above 600 (included), I set dlret and dlretx to be 0 .

Step 3: Once I have the return series, I compute the price series for any given amount of investment in an early year, say, $\$ 1$ investment in, year $t-7$, as follows: $P_{t-7}=1$, and

$$
\begin{equation*}
P_{t+s}=P_{t+s-1}\left(1+\text { retx }_{t+s-1}^{*}\right) . \tag{8}
\end{equation*}
$$

Step 4: I multiply $P_{t+s-1} F P_{t+s}$ to get the survivorship bias adjusted portfolio fundamental

[^10]value $F_{t+s}^{S A}$.
Step 5: For dividends and book equity, I compute $g_{t+s}^{F, S A}=\frac{F_{t+s}^{S A}}{F_{t+s-1}^{S A}}-1$, and then average across portfolio formation years as in Equation (5). For earnings and accounting cash flow, I first scale cash flows to correspond to a $\$ 1$ investment in portfolio formation year $t, \tilde{F}_{t+s}^{S A}=\frac{F_{t+s}^{S A}}{P_{t}}$. I then average across portfolio formation years before computing growth rate, as in Equation (7). ${ }^{14}$

If no firm ever exits the portfolio, then this procedure should yield the same value as the simple growth rates in Section 3.2.2. When firms do exit the portfolio, this procedure automatically accounts for survivorship bias, because it includes all firms that are alive in year $t+s-1$. It accounts for the delisting proceeds, because when computing returns, we implicitly assume that proceeds are reinvested when firms exit the portfolio.

The results for the four survivorship bias adjusted growth rates are plotted in Fig. 4. The plots are qualitatively similar to those in Fig. 3. But accounting for survivorship bias almost always decreases the growth rates of the growth portfolio, and often increases those of the value portfolio. For example, in Panel A, in year 2, without adjusting for survivorship bias, the earnings growth rates for the growth and value portfolios are $7.6 \%$ and $36 \%$, respectively. After accounting for survivorship bias, the earnings growth rates for the growth and value portfolios become 5.7\% and $54.3 \%$, respectively. In Panel B, in year 2, the accounting-cash-flow growth rates for the growth and value portfolios change from $10 \%$ and $12.7 \%$ to $7.8 \%$ and $14.5 \%$, respectively. Panels C and D are qualitatively the same as those in Fig. 3, with the levels of growth rates generally lower for all portfolios. Again, the dividend growth rate exhibits a U-shaped relation with the book-to-market equity. Both growth and value stocks grow faster than neutral stock. Growth stocks have higher dividend growth rates than value stocks, but the difference is quite small. In year 2 , the difference is $0.4 \%(8.4 \%-8.0 \%)$. If we look at the average of growth rates in year 2 through year 10 , the difference is $1.5 \%(9.5 \%-7.9 \%)$. After adjusting for survivorship bias, the book-equity growth rates of the value portfolio are still clearly lower than those of the growth portfolio.

[^11]Fig. 5 shows that even this small difference in dividend growth rates between growth stocks and value stocks is not robust, if we examine book-to-market deciles instead of terciles. Panel A of Fig. 5 plots the average dividend growth rates for book-to-market deciles in years 1, 2, and 3. In year 1, the dividend growth rate strongly decreases as book-to-market increases. In year 2, the dividend growth rate exhibits no clear relation with the book-to-market ratio. Decile $10(12.1 \%)$ grows marginally faster than Decile $1(10.8 \%)$. But in year 3 , the dividend growth rate initially decreases but then increases as book-to-market increases, with Decile 10 (19.9\%) clearly growing faster than Decile $1(13.2 \%)$. Panel B of Fig. 5 plots the dividend growth rates for Decile 1 (growth portfolio) and Decile 10 (value portfolio). The dividend growth rate for the value portfolio by and large exceeds that of the growth portfolio starting in year 2.

To sum up, so far I find the 1-year cash-flow growth rate typically exhibits a U-shaped relation with the book-to-market equity, with no clear difference between growth stocks and value stocks. In the appendix, I also examine the long horizon growth rate from year 1 for book-to-market deciles. At least for the range from year 1 to year 8 , the growth rates in all cash-flow variables exhibit a U-shaped relation against the book-to-market equity, with the extreme value portfolio growing faster than the extreme growth portfolio. The difference is large when looking at earnings and small when looking at dividends.

Some sources (e.g., Investopedia) define growth stocks as shares in a company whose earnings are expected to grow at an above-average rate relative to the market. Throughout this paper, I define growth stocks as those with low book-to-market ratios. My results show that these two definitions may contradict each other.

### 3.2.4 Decreasing returns to scale

To further investigate why growth stocks' earnings don't grow faster, I examine the growth rates of assets, revenue, and cost, all on a per-share basis. The results are plotted in Fig. 6. Panel A plots the asset growth rates for the growth, neutral, and value portfolios. The shape of the graph resembles that of the book equity. Thus, sorting on the book-to-market equity generates a clear dispersion in asset growth, suggesting a link between the book-to-market effect and the
asset growth anomaly documented by Cooper, Gulen, and Schill (2008).
Panel B plots the revenue and cost growth rates of the growth and value portfolios. Again, the growth rates look much like the growth rates in book equity and the assets. What is interesting is that after the portfolio formation, the cost grows faster than the revenue for the growth stocks, and the opposite is true for the value stocks. I also observe that shortly before the portfolio formation, the cost grows slower than the revenue for growth stocks, and the opposite is true for value stocks.

In explaining the convergence in the return on equity, Fama and French (1995) suggest the following economic story. "Sometime prior to portfolio formation low-BE/ME firms experience a demand or supply shock to increase their average return on capital, the profit-maximizing response is to expand output and investment until, at the margin, earnings on investment return to competitive equilibrium levels. Conversely, high-BE/ME firms experience a demand or supply shock that decreases their average return on capital. The profit-maximizing response is to restructure, that is, to let output and investment contract until, at the margin, earnings on investment return to competitive-equilibrium levels." Implicit in their argument is the "decreasing returns to scale". The evidence I find is consistent with their story. However, decreasing returns to scale also imply that growth stocks may grow faster in assets but slower in profits than value stocks. Decreasing returns to scale are thus one possible reason for generating the efficiency growth, shown in Section 3.2.1.

Other forces may be at work in explaining the high initial earnings growth rates in value stocks. One might point to operating leverage. Value firms are not very profitable. High costs imply that any improvement in operations leads to a large increase in profits. Or, a value firm may experience a change of management that improves performance, which in turn leads to a significant efficiency growth. Alternatively, one might argue that improving the return on equity is more feasible for a firm that generates a return on equity that is well below the average than for a firm that already outperforms. Note, however, that these stories have no predictions about asset growth. Regardless of the true reason that drives the high earnings growth rates in value stocks, my results show that the commonly made assumption that value stocks have substantially
low future cash-flow growth rates is not supported by the data.

### 3.2.5 Survivorship bias in regressions

The regression of the dividend growth rate on the book-to-market ratio in Table 1 is inherently subject to survivorship bias, because a firm has to be alive to be included in the regression. To account for survivorship bias in this regression, I include the delisting proceeds (delisting amount, abs(dlamt), multiplied by shares outstanding) as a form of liquidating dividends. I then re-estimate the regressions and report the new results in Table 3.

Table 3 shows a different picture from Table 1. Although the coefficient on the book-tomarket equity is still negative in year 1, it becomes positive starting in year 2. However, none of the positive coefficient is statistically significant. The slope coefficient generally increases over time, although the increase is not monotonic. ${ }^{15}$

The reason that adjusting for survivorship bias makes a bigger difference in the regression than in portfolio growth rates is because regressions are equal weighted in nature. Accounting for survivorship bias is more important in small firms, since large firms are less likely to exit.

### 3.3 Growth rates of annually rebalanced portfolios

Forming rebalanced portfolios has become second nature for empirical asset pricing researchers. To examine the value premium, we typically form a portfolio as of June of year $t$, and then hold the portfolio between July of year $t$ and June of year $t+1$, at which time the portfolio is rebalanced. Because the implementation of the value strategy typically involves an annually rebalanced portfolio, I study the cash-flow growth rates for rebalanced portfolios. I follow a procedure similar to that outlined in Section 3.2.3. The main difference is that when I compute price levels, I use the annually rebalanced portfolios returns.

Step 1: I compute the fundamental-to-price ratio in year $t+1$ for the portfolio formed in year $t$, $F P_{t+1}$, for a portfolio that is formed in year $t$, as the value-weighted average of firm fundamental

[^12]per share to price per share ratios, $\frac{F p s_{t+1}}{P p s_{t}}$. All firms available in year $t$ and not necessarily in $t+1$, are included. If a firm exits the portfolio in year $t+1$, its fundamental value is set to zero.

Step 2: I compute value-weighted rebalanced portfolio returns and returns without dividends $r e t_{t+1}$ and $r e t x_{t+1}^{*}$. It is important to include delisting returns in this step.

Step 3: Once I have the returns series, I compute the price series for any given amount of investment in an early year, for example, a $\$ 1$ investment in June of 1963 , as follows: $P_{1963}=1$, and

$$
\begin{equation*}
P_{t+1}=P_{t}\left(1+\operatorname{retx}_{t+1}^{*}\right) . \tag{9}
\end{equation*}
$$

Step 4: I multiply $P_{t} F P_{t+1}$ to get the rebalanced portfolio fundamental value $F_{t+1}^{R P}$.
Step 5: For dividends and book equity, I compute $g_{t+1}^{F, R P}=\frac{F_{t+1}^{R P}}{F_{t}^{R P}}-1$, and then average across years. For earnings and accounting cash flow, I first scale cash flows to correspond to a $\$ 1$ investment in portfolio formation year $t, \tilde{F}_{t+1}^{R P}=\frac{F_{t+1}^{R P}}{P_{t}}$. Then I average across portfolio formation years before computing the growth rate, $g^{F, R P}=\frac{E\left[\tilde{F}_{+1}^{R P}\right]}{E\left[\tilde{t}_{t}^{R P}\right]}-1$, analogous to Equation (7).

The results are plotted in Fig. 7. Panel A plots the average earnings growth rates for three book-to-market portfolios. I look at three portfolios because decile portfolios sometimes produce negative average earnings numbers. The growth portfolio has an average earnings per share growth rate of $6.8 \%$, while the value portfolio's growth rate is $10.2 \%$. Panel B plots the average per share growth rate for accounting cash flow, dividends, and book equity for book-to-market deciles. The average growth rate initially slightly decreases and then strongly increases with the book-to-market ratio. If we compare Decile 1 and Decile 10, Decile 10 has unambiguously higher growth rates in all three fundamental values. The growth decile has average accounting-cash-flow, dividend, and book-equity growth rates of $5.7 \%, 9.9 \%$, and $5.6 \%$, respectively, and the value decile has growth rates of $11.6 \%, 16.8 \%$, and $10.7 \%$, respectively.

### 3.4 Evidence from valuation models

Gordon's formula suggests that all else being equal, stocks with higher prices should have higher cash-flow growth rates. However, all else is not equal when we compare value stocks with growth stocks, because they differ in expected returns. I now examine whether Gordon's formula actually
implies that growth stocks have higher growth rates. To do so, I plot average dividend-price ratios, annual returns, and dividend growth rates for book-to-market deciles 1964 to 2010 in Fig. 8. The left panels plot the buy-and-hold deciles, and the right panels plot the rebalanced deciles. The top panels plot value-weighted deciles, and the bottom panels plot the equal-weighted deciles. Annual data are used. The returns and dividend growth rates are for annually rebalanced portfolios. ${ }^{16}$

For buy-and-hold portfolios, dividend-price ratios, $D_{1} / P_{0}$, are averaged across 47 portfolio formation years (1963-2009); returns are averaged across 1,128 years ( 47 years for portfolios formed in 1963, 46 years for portfolios formed in 1964, etc.); and dividend growth rates are averaged across 1,081 years (skipping the look-back growth rates in year 1, therefore 46 years for portfolios formed in 1963, 45 years for portfolios formed in 1964, etc.). Dividend growth rates are winsorized at $1 \%$ and $99 \%$ for buy-and-hold portfolios.

For buy-and-hold value-weighted portfolios, the average dividend yield (including repurchases) for the growth decile is $2.6 \%$. The dividend yield increases, although not monotonically, to about $4.8 \%$ for the value decile. The difference is $2.2 \%$. However, the average return increases from $10.3 \%$ to $12.5 \%$. The difference is also $2.2 \%$. Therefore, Gordon's formula suggests that the value portfolio's expected dividend growth rates should be the same as the growth decile. I also plot the actual average growth rates, which go from $10.5 \%$ to $13.1 \%$. The value decile actually grows by $2.6 \%$ higher, suggesting that the small-sample counterpart of Gordon's formula does not hold exactly. I also note that the relation between $B / M$ and average growth is not monotonic.

The lower left panel plots the results for the buy-and-hold equal-weighted deciles. The average dividend yield (including repurchases) for the equal-weighted growth decile is $1.6 \%$. The dividend yield increases to about $3.1 \%$ for the value decile. The difference is only $1.4 \%$ per year. The average return, however, increases from $13.3 \%$ to $16.1 \%$. The difference is $2.8 \%$. Therefore, Gordon's formula suggests that the value portfolio's expected dividend growth rate should be $1.3 \%$ higher than that of the growth decile. The actual average growth rate difference is $0 \%$ (both at $15.0 \%$ ).

For rebalanced portfolios, dividend-price ratios and returns are averaged across 47 portfolios

[^13]formation years (1963-2009), and growth rates are averaged across 46 years (1965-2010). In the upper right panel, the average dividend yield (including repurchases) for the value-weighted growth decile is $2.4 \%$. The dividend yield increases, although not monotonically, to about $4.6 \%$ for the value decile. The difference is $2.1 \% .{ }^{17}$ However, the average return increases from $10.3 \%$ to $15.3 \%$. The difference is $5 \%$. Therefore, even Gordon's formula suggests that the value portfolio's expected dividend growth rates should be $2.6 \%$ higher than the growth decile. I also plot the actual average growth rates, which go from $9.9 \%$ to almost $16.8 \%$. The difference is $6.9 \%$.

The equal-weighted portfolios exhibit even larger magnitudes. The average dividend yield (including repurchases) for the equal-weighted growth decile is $1.6 \%$. The dividend yield increases to about $2.6 \%$ for the value decile. The difference is only $1 \%$ per year. The average return, however, increases from $9 \%$ to $23.6 \%$. The difference is $14.5 \%$. Therefore, Gordon's formula suggests that the value portfolio's expected dividend growth rate should be $13.5 \%$ higher than that of the growth decile. The actual average growth rate difference is $12.7 \%$ (19.8\%-7.1\%). Overall, I conclude that valuation models suggest: growth stocks have similar future growth rates with value stocks in buy-and-hold portfolios, and have lower future growth rates than value stocks in rebalanced portfolios.

### 3.5 The relation between growth rates of rebalanced and buy-and-hold portfolios

The results so far suggest that the look-back growth rate is negatively related to the book-to-market ratio; starting in year 2, the buy-and-hold growth rate typically exhibits a U-shaped relation with the book-to-market ratio, with little difference between extreme growth and extreme value stocks. However, for rebalanced portfolios, the growth rate is clearly positively related to the book-to-market ratio. I now examine the relation between growth rates in rebalanced and buy-and-hold portfolios.

[^14]
### 3.5.1 An example

Suppose that the investment opportunity set consists of only two stocks, A and B. These two stocks are ex ante identical. Assume that in each year, one of the stocks pays a dividend of $\$ 10$ and the other one pays $\$ 20$. A coin is tossed to decide which stock pays which amount. Therefore, in any given year, there is a $50 \%$ chance that A pays a dividend of $\$ 10$, and B pays a dividend of $\$ 20$, and there is a $50 \%$ chance that A pays a dividend of $\$ 20$, and B pays a dividend of $\$ 10$.

Immediately after the dividend is paid, the two stocks are identical, and should trade at the same price. Assume that the risk-free rate (the appropriate discount rate, since coin tosses are idiosyncratic risk) is $10 \%$, so each stock should trade at $(10+20) / 2 / 10 \%=\$ 150$. Going forward, the two stocks are the same and have expected returns of $10 \%$ per year.

Next, consider two portfolio strategies. One is the high-dividend strategy, which always buys the stock that just paid a high dividend of $\$ 20$. The other, the low-dividend strategy, does exactly the opposite, always buying the stock that just paid a dividend of $\$ 10$. If we use the actual cashflow growth rate from holding the portfolios, we will find that the two portfolios generate the same expected future dividend growth rate: zero. But if we fix the stocks in the portfolios and then look at the dividend growth rates of these stocks, then the stock in the high-dividend portfolio has a negative expected dividend growth rate, i.e., $15 / 20-1$. The stock in the low-dividend portfolio has a positive expected dividend growth rate, i.e., $15 / 10-1$.

### 3.5.2 Notations

I now introduce notations. Suppose there are $N$ stocks, whose prices and dividends per share are $P_{n, t}$ and $D_{n, t}$, for $n=1,2, \ldots, N$. Prices are measured at the end of the year. Dividends are paid shortly before the end of the year. The trading strategy uses information up to year $t$ and calls for buying those stocks with a certain characteristic at the end of year $t$, and holding the stocks until the end of year $t+1$. At the end of year $t+1$, we take out and consume the dividend. We also rebalance the portfolio and use the proceeds from stock sales to buy stocks that fit the portfolio selection criteria at the end of year $t+1$, and then hold those stocks in year $t+2$. For ease of disposition, assume that there are only ten stocks, $N=10$, and our strategy calls for holding one
stock at any given point in time. Assume that the stocks selected by the strategy at the end of years $t, t+1$, and $t+2$, are stocks $i, j$, and $k$, respectively. Note that in year $t$, the identities of $j$ and $k$ are not known and may or may not be $i$. Our initial investment is $P_{i, t}$, so we can buy one share of stock $i$. Therefore, the portfolio generates a dividend of $D_{i, t+1}$ in year $t+1$. The investor is left with $P_{i, t+1}$, and then can buy $\frac{P_{i, t+1}}{P_{j, t+1}}$ shares of stock $j$. Therefore, in year $t+2$, the investor earns a dividend of $D_{j, t+2} \frac{P_{i, t+1}}{P_{j, t+1}}$. The dividend growth rate of the rebalanced portfolio in year $t+2$ is

$$
\begin{equation*}
g_{t+2}=\frac{D_{j, t+2} \frac{P_{i, t+1}}{P_{j, t+1}}}{D_{i, t+1}}-1 . \tag{10}
\end{equation*}
$$

The dividend growth rate in year $t+s$ for the buy-and-hold portfolio formed in year $t$ is:

$$
\begin{equation*}
g_{t, t+s}^{B H}=\frac{D_{i, t+s}}{D_{i, t+s-1}}-1, \text { fors } \geq 2 . \tag{11}
\end{equation*}
$$

Note that when $s \leq 1$, we have not yet bought the portfolio. Nevertheless, we can compute the growth rate of such a portfolio. When $s=1$, it is the look-back growth rate.

$$
\begin{equation*}
g_{t, t+1}^{L B}=\frac{D_{i, t+1}}{D_{i, t}}-1 \tag{12}
\end{equation*}
$$

In the above example, I note that $g_{t, t+2}^{B H}=\frac{D_{i, t+2}}{D_{i, t+1}}-1$ and $g_{t+1, t+2}^{L B}=\frac{D_{j, t+2}}{D_{j, t+1}}-1$.

### 3.5.3 The portfolio rebalancing effect

I now show that relative to the rebalanced growth rate, the look-back growth rate is necessarily lower for the value portfolio and necessarily higher for the growth portfolio. Suppose the value strategy calls for buying the stock with the highest dividend-price ratio at the end of year $t$ and then holding that stock during year $t+1$. Again, assume $N=10$, and that the stocks selected by the strategy at the end of year $t, t+1$, and $t+2$, are stocks $i, j$, and $k$, respectively.

For the value portfolio, $g_{t+2} \geq g_{t+1, t+2}^{L B}$, because

$$
\begin{equation*}
1+g_{t+2}=\frac{D_{j, t+2} \frac{P_{i, t+1}}{P_{j, t+1}}}{D_{i, t+1}}=\frac{D_{j, t+2}}{P_{j, t+1}} \frac{P_{i, t+1}}{D_{i, t+1}} \geq \frac{D_{j, t+2}}{P_{j, t+1}} \frac{P_{j, t+1}}{D_{j, t+1}}=\frac{D_{j, t+2}}{D_{j, t+1}}=1+g_{t+1, t+2}^{L B} . \tag{13}
\end{equation*}
$$

The inequality holds because we sort on dividend-price ratios and stock $j$ has the highest dividend-price ratio in year $t+1$. Similar arguments show that the look-back growth rate necessarily overstates the growth rates of the growth portfolio, that is, $g_{t+2} \leq g_{t+1, t+2}^{L B}$ for the growth portfolio.

This analysis uses dividends, but the logic works for any fundamental variable. If we sort on book-to-market ratio, then as long as the sorting preserves the ranking of the fundamental-toprice ratio in the portfolio formation year, the look-back growth rate in that fundamental value understates value investors' experiences. I will test whether sorting on book-to-market preserves the ranking of $\frac{F_{0}}{P_{0}}$.

In the equations below, I show the relation between the buy-and-hold growth rates and the rebalanced portfolio growth rate. For the value portfolio, $g_{t+2} \geq g_{t+1, t+2}^{B H}$, if

$$
\begin{equation*}
\frac{D_{j, t+2}}{P_{j, t+1}} \geq \frac{D_{i, t+2}}{P_{i, t+1}} \tag{14}
\end{equation*}
$$

This is because,

$$
\begin{equation*}
1+g_{t+2}=\frac{D_{j, t+2} \frac{P_{i, t+1}}{P_{j, t+1}}}{D_{i, t+1}}=\frac{D_{j, t+2}}{P_{j, t+1}} \frac{P_{i, t+1}}{D_{i, t+1}} \geq \frac{D_{i, t+2}}{P_{i, t+1}} \frac{P_{i, t+1}}{D_{i, t+1}}=\frac{D_{i, t+2}}{D_{i, t+1}}=1+g_{t, t+2}^{B H} . \tag{15}
\end{equation*}
$$

Thus, if we sort on book-to-market ratio, then as long as the sorting preserves the ranking of the forward-fundamental-to-price ratio, the buy-and-hold growth rate in that fundamental value understates rebalancing value investors' experiences. That is, the buy-and-hold growth rate is lower than the rebalanced portfolio growth rate if sorting on book-to-market preserves the ranking of $\frac{F_{1}}{P_{0}}$.

I test these two conditions. I plot $\frac{F_{0}}{P_{0}}$ and $\frac{F_{1}}{P_{0}}$ in Fig. 9. The plot shows that sorting on the book-to-market ratio results in a hump shape in the earnings/price ratio. But sorting on the book-
to-market ratio preserves the rankings in the accounting cash flow/price ratio, the dividend/price ratio, and of course, the book-to-market ratio. In terms of the forward fundamental to price ratio, $\frac{F_{1}}{P_{0}}$, the ranking is almost preserved for accounting cash flow and dividends, except for Deciles 9 and 10. The ranking is entirely preserved for book equity. Hence, I conclude that for the latter three variables, looking at static growth rates (both the look-back growth rate and the buy-and-hold growth rate), understates a rebalancing value investor's experiences. Further, this understatement mechanically arises when we sort on fundamental-to-price ratios.

### 3.6 Volatility of growth rates

Fig. 10 shows that the way we compute growth rates also results in different cash-flow risks. Because earnings and accounting cash flow are sometimes negative, I focus on dividends in studying cash-flow volatilities. As a robustness check, I also use book equity, although it is not a cash flow. Panel A plots the standard deviation of the dividend growth rate for the buy-and-hold portfolio in year 1 (look-back growth rate), in year 2 (buy-and-hold growth rate) and the rebalanced portfolio. The standard deviation is computed across portfolio formation years. For the look-back growth rates, the standard deviation of the dividend growth rate decreases initially and then increases as the book-to-market ratio increases. If we compare Decile 1 and Decile 10, the growth portfolio $(31.3 \%)$ is more volatile than the value portfolio $(28.5 \%)$. If we look at the buy-and-hold growth rate in year 2, the standard deviation exhibits a cleaner increasing pattern in the book-to-market equity, going from $19.4 \%$ in the growth decile to $37.4 \%$ in the value decile. If we look at the rebalanced portfolio growth rate, then the growth rate volatility increases almost monotonically from Decile 1 to Decile 10. The dash-dot line plots the cash-flow volatility for the rebalanced portfolio, almost monotonically increasing from $25.7 \%$ and $47.8 \% .^{18}$

Panel B plots the standard deviation of book-equity growth rates. If we look at the look-back growth rate, again, the volatility initially decreases and then increases as the book-to-market ratio increases. Decile $10(12.4 \%)$ is more volatile than Decile 1 ( $9.4 \%$ ). If we look at the buy-and-hold

[^15]growth rate in year 2, the pattern becomes more monotonic, although the difference between Decile 10 and Decile 1 is almost the same as those in look-back growth rates. Decile 10 is only a little more volatile than Decile 1. However, if we look at the rebalanced portfolio growth rate, then the growth rate volatility increases almost monotonically from Decile 1 to Decile 10.

In a first look at cash-flow risk in buy-and-hold portfolios, Cohen, Polk, and Vuolteenaho (2009) document a novel and important finding that value stocks have higher cash-flow risk than growth stocks, especially in long horizon cash flows. ${ }^{19}$ With the benefit of hindsight, I point out that they include the year 1 look-back growth rates, as do most other papers. My results suggest that including the look-back growth rate in estimating cash-flow risk is not only conceptually problematic, but also the effect may be empirically relevant. I refer to this problem as the lookback bias in cash-flow risk. It is possible that the increase in the explanatory power of cash-flow risk over long horizon is driven by the decreasing importance of the look-back growth rates.

More generally, this points to the difficulty in estimating cash-flow risks in buy-and-hold portfolios. Because firm characteristics in buy-and-hold portfolios systematically converge over time, it is plausible that their riskiness also converges over time. When risk is time varying, it is much harder to measure econometrically. If we measure cash-flow risks for a given portfolio formation year, then the best one can hope for is to estimate the average riskiness over the horizon. If we measure cash-flow risks across portfolio formation years, we would like to use the initial year, but doing so suffers from the look-back bias. The rebalanced portfolios do not suffer from such problems.

## 4 Do growth stocks have longer cash-flow durations?

So far I have examined cash-flow growth rates and find growth stocks have similar and lower growth rates, relative to value stocks, in buy-and-hold portfolios and rebalanced portfolios, respectively. I now directly examine the duration of portfolios. The most common notion of the duration is the Macaulay duration, which is the weighted average of cash-flow maturities. To distinguish from

[^16]the cash-flow duration (discussed later), I call this the canonical duration.
\[

$$
\begin{equation*}
\text { Dur }_{i}^{\text {canonical }}=\sum_{t=1}^{\infty} \frac{\frac{D_{i t}}{\left(1+r_{i}\right)^{t}}}{\sum_{t=1}^{\infty} \frac{D_{i t}}{\left(1+r_{i}\right)^{t}}} \times t \tag{16}
\end{equation*}
$$

\]

To implement this, I use the historical average of dividends in the first $T$ years, and assume that beyond year $T$, the cash flows are a growing perpetuity, growing at $g_{i \infty}$ per year. I report the results for $T=20$, although the results are qualitatively the same if I use $T=10$ or $T=30$.

I use $g_{i \infty}$ implied by the prices, such that

$$
\begin{equation*}
P_{i}=\sum_{t=1}^{T} \frac{D_{i t}}{\left(1+r_{i}\right)^{t}}+\frac{D_{i T}\left(1+g_{i \infty}\right)}{r_{i}-g_{i \infty}} . \tag{17}
\end{equation*}
$$

$r_{i}$ is the portfolio specific average future realized returns in Fig. 8. To facilitate the calculation, I note that the growing perpetuity beyond year $T$ has a canonical duration of $T+\frac{\left(1+r_{i}\right)}{r_{i}-g_{i \infty}}$.

The results are reported in Panel A of Table 4. I consider both buy-and-hold and rebalanced portfolios. I also consider both value-weighted and equal-weighted portfolios. The results show that growth stocks clearly have longer canonical durations than value stocks, in all four kinds of portfolios. In Panel B, I report the steady-state long-term growth rates, $g_{i \infty}$. However, $g_{i \infty}$ often increases with the book-to-market ratio.

The result can be understood as follows. Durations are related to price-dividend ratios. In a Gordon's model, the canonical duration can be shown to be

$$
\begin{equation*}
\text { Dur }{ }^{\text {canonical }}=\frac{1+r}{r-g}=\frac{P_{0}}{D_{0}} \frac{1+r}{1+g} \tag{18}
\end{equation*}
$$

Because value stocks clearly have lower price-dividend ratios, it is not surprising that they have longer durations. ${ }^{20}$

Note however, that the price-dividend ratio is not a clean variable. It can be high either because the discount rate is low, or because the expected growth rate is high. Consider two assets that have the same growth profiles, but differ in cash-flow risk. Higher cash-flow risk leads to

[^17]a higher discount rate, which in turn leads to a lower price-dividend ratio and shorter duration. In this case, there is a negative relation between duration and the discount rate even when risk premiums are countercyclical and expected growth rates are procyclical, but few would find this anomalous. However, if two assets have the same cash-flow risk, and differ in growth profiles, leading asset pricing models imply a positive relation between duration and discount rates.

Thus, it is important to separate the discount rate effect and the growth-profile effect. To do so, I propose to measure the cash-flow duration by using a common discount rate for all assets, i.e., to replace the asset-specific discount rate $r_{i}$ with a common discount rate $\bar{r}$.

$$
\begin{equation*}
\operatorname{Dur}_{i}^{\text {cashflow }}=\sum_{t=1}^{\infty} \frac{\frac{D_{i t}}{(1+\bar{r})^{t}}}{\sum_{t=1}^{\infty} \frac{D_{i t}}{(1+\bar{r})^{t}}} \times t \tag{19}
\end{equation*}
$$

In implementing the above equation, I use the same cash-flow profiles, including $D_{i 1}$ through $D_{i 20}$, and $g_{i \infty}$ imputed previously. To ensure that prices are not infinite, I use the discount rates of the value portfolio as $\bar{r} .{ }^{21}$ This assumption implies that the value portfolio has the same canonical and cash-flow durations but that other portfolios do not.

The results are presented in Panel C. In buy-and-hold value-weighted portfolios, the cashflow duration exhibits a U-shaped relation with the book-to-market equity, but little difference between the growth and the value deciles. In buy-and-hold equal-weighted portfolios, the cashflow duration increases with the book-to-market equity. In rebalanced portfolios, the cash-flow duration exhibits a clear increasing pattern with the book-to-market equity, increasing from 15.31 to 24.38 in value-weighted portfolios, and from 6.80 to 62.71 in equal-weighted portfolios. ${ }^{22}$

As a further robustness check, I compute a cash-flow duration measure similar to that in Da

[^18](2009). This measure also uses information in cash flows only and disregards discount rates. This measure computes $\sum_{s=2}^{\infty} \rho^{s} g_{i s}$ for each portfolio. As before, I use the actual dividend growth rates in the first 20 years, and growth rates beyond year 20 are assumed to be $g_{i \infty} \cdot \rho$ is assumed to be 0.95. The results in Panel D suggest that in buy-and-hold portfolios, there is little difference between growth and value stocks, but in rebalanced portfolios, value stocks clearly have longer cash-flow durations. ${ }^{23}$

The results here suggest that a common assumption that growth stocks have higher growth rates and longer cash-flow durations than value stocks has little empirical base. ${ }^{24}$

## 5 The growth premium

### 5.1 Theoretical implications of asset pricing models regarding the growth premium

In the Appendix, I analyze four affine asset pricing models. The first three are one-factor affine models with time-varying expected returns that match most of the stylized facts in the time series. The first model has time-varying market price of risk, which captures the habit formation; the second has time-varying amount of risk, which captures the conditional heteroskedasticity in the long run risk model; and the third has both time-varying market price of risk and timevarying amount of risk. When applied to assets with a constant expected growth rate, all three

[^19]models imply that the expected return is an increasing function of the expected growth rate, after controlling for cash-flow risks, if these models are to help explain the equity premium puzzle.

The economic intuition for this result is that time-varying expected returns only increase risk if expected returns are countercyclical. If expected returns are countercyclical, then in bad times, prices will go down, not only because cash flows decrease but also because expected returns increase. Thus, time-varying expected returns make stocks more risky. This mechanism also implies that longer-duration assets are more risky because the effects of changes in discount rates are larger in longer-duration assets. The opposite is true if expected returns are procyclical. In that case, in bad times, prices tend to decrease because of negative cash flow shocks, but prices also tend to go up because expected returns decrease. Thus, stocks are less risky than they would be otherwise. Longer-duration assets are less risky than short-duration assets, because procyclical expected returns essentially provide a hedge.

Fig. A1 depict these two cases in Model 1 (time-varying price of risk). In Panel A of Fig. A1, I plot the expected excess return, expected capital gain, and dividend yield when the price of risk is countercyclical. The expected excess return is equal to the expected capital gain plus dividend yield minus interest rate (assumed to be 10\%). In this case, as the expected growth rate increases, the expected capital gain increases, the dividend yield decreases, and the expected excess return increases. The expected capital gain increases in the expected dividend growth, because in the long run dividend-price ratios are stationary. Dividend-price ratios decrease because all else being equal, a higher growth rate leads to a higher price-dividend ratio and a lower dividend-price ratio. In this case, the capital gain effect dominates, and the expected return increases as the expected dividend growth rate increases.

In Panel B of Fig. A1, I plot the three quantities when the price of risk is procyclical. Again, the expected excess return is equal to the expected capital gain plus dividend yield minus interest rate ( $10 \%$ ). In this case, as the expected growth rate increases, the expected capital gain again increases, and the dividend yield again decreases. But in this case, the dividend yield effect dominates, and the expected return decreases as the expected dividend growth rate increases.

In the fourth model, I consider both time-varying market price of risk and time-varying ex-
pected growth rate. The implication on the growth premium is less clear now. As discussed in the introduction, two conditions are important in generating the growth premium. The first is whether the market price of risk is countercyclical. The second is whether the expected growth rate is procyclical. When both conditions are met, the model clearly implies a growth premium. When both conditions are violated, then the model implies a negative growth premium. However, if only one condition is satisfied (e.g., if both the market price of risk and the expected growth rate are countercyclical), then the overall relation between the expected growth rate and the expected return depends on the relative strength of these two counteracting forces. Also, in this case, the relation between the expected growth rate and the expected return may not be monotonic. As a result, requiring the equity premium to be high no longer ensures that there is a monotonic growth premium. Whether there is a growth premium is ultimately an empirical question.

### 5.2 Equity returns

I now test whether there is a growth premium in the cross section of stock returns. In principle, both buy-and-hold and rebalanced portfolios should be priced correctly and can be used as test portfolios. I choose to focus on rebalanced portfolios in this section, mainly because in buy-and-hold portfolios there is little difference between overall growth rates of growth and value stocks. Focusing on rebalanced portfolios has two additional practical benefits. First, risk clearly changes over time for buy-and-hold portfolios, while it may stay relatively constant for rebalanced portfolios; therefore, it is likely easier to measure risk for rebalanced portfolios. Second, the vast majority of asset pricing literature uses rebalanced portfolios to implement the value strategy, probably because it generates higher return dispersions.

My previous results suggest that the value premium is consistent with the growth premium, because in rebalanced portfolios value stocks clearly have high cash-flow growth rates. I now examine whether there is a growth premium in the broad cross section of stock returns. To do so, I use 3 sets of 20 portfolios, each sorted by size, book-to-market equity, and momentum (previous 11-month return, skipping one month), respectively, between 1963 Q3 and 2010 Q2. I then estimate the Fama-MacBeth regressions of returns on historical average dividend growth
rates (as a proxy of expected growth rate), while controlling for cash-flow risks.
Panel A of Table 5 reports summary statistics for quarterly real returns, historical average dividend growth rates, and cash-flow risks. The historical average dividend growth rate, $\bar{g}_{i, t-1}$, is the average annual real dividend growth rate of the rebalanced portfolio $i$, using information up to year $t-1$. $\bar{g}_{i, t-1}$ is winsorized at $1 \%$ and $99 \%$. I consider two cash-flow risk measures. The first one is the contemporaneous consumption beta of dividend growth rates, $\beta_{i}^{d c}$. It is estimated in the following equation: $\log \left(1+g_{i, t}\right)=b_{i}^{0}+\beta_{i}^{d c}\left(\log \left(1+g_{c, t}\right)\right)+\epsilon_{i, t}$, using all information between 1963 Q3 and 2010 Q2. Here $g_{i, t}$ is the quarterly real dividend growth rate, and $g_{c, t-k}$ is the quarterly real consumption growth rate. Dividends and consumptions are seasonally adjusted by summing the most recent 4 quarters. The second measure, $\gamma_{i}$, is the Bansal-Dittmar-Lundblad cash-flow risk, estimated from the regression $\log \left(1+g_{i, t}\right)=\gamma_{i}\left(\frac{1}{K} \sum_{k=1}^{K} \log \left(1+g_{c, t-k}\right)\right)+\epsilon_{i, t}$. When computing $\gamma_{i}$, both $\log \left(1+g_{i, t}\right)$ and $\log \left(\left(1+g_{c, t-k}\right)\right.$ are demeaned. As in Bansal, Dittmar, and Lundblad (2005), I primarily focus on the case $K=8$, although $K=12$ yields similar results. While $\beta_{i}^{d c}$ captures contemporaneous covariance between the realized dividend growth rate and the consumption growth rate (short-run risk), $\gamma_{i}$ captures the covariance between the expected dividend growth rate and the history of consumption growth rates (long-run risk).

Table 6 reports the results of the Fama-MacBeth regression of returns on historical average growth rates and cash-flow risks. Panel A examines value-weighted portfolios. The left-hand-side variable is the quarterly real return multiplied by four. In the first three rows of Panel A in Table 6 , I estimate univariate regressions using all 60 portfolios. I find that $\bar{g}_{i, t-1}$ is positively associated with returns, with the coefficient being 0.48 and highly statistically significant. The coefficient on cash-flow risks $\gamma_{i}$ is also positive (0.0052) and statistically significant. However, $\beta_{i}^{d c}$ appears negatively associated with returns.

In Row 4, I find that after controlling for cash-flow risk $\beta_{i}^{d c}$, the coefficient on $\bar{g}_{i, t-1}$ is almost unchanged (0.45), with a $t$-statistic of 4.76. The coefficient on $\beta_{i}^{d c}$ remains negative. In Row 5 , after controlling for cash-flow risk $\gamma_{i}$, the coefficient on $\bar{g}_{i, t-1}$ reduces to 0.30 , with a $t$-statistic of 2.80. The coefficient on $\gamma_{i}$ reduces to $0.0032(t=3.47)$. In Row 6 , after controlling for both $\beta_{i}^{d c}$ and $\gamma_{i}$, the coefficient on $\bar{g}_{i, t-1}$ is $0.30(t=3.25)$.

As a robustness check, I now use 20 portfolios sorted by one characteristic at a time. In Rows 7 through 9 , I use 20 portfolios sorted by size as test assets. After controlling for $\beta_{i}^{d c}$ and $\gamma_{i}$, the coefficient on $\bar{g}_{i, t-1}$ is 0.47 . In Rows 10 through 12, I use 20 portfolios sorted by book-to-market equity as test assets. The coefficient on $\bar{g}_{i, t-1}$ is 0.28 , after controlling for $\beta_{i}^{d c}$ and $\gamma_{i}$. When I use the momentum portfolios in Rows 13 through 15, the coefficient on $\bar{g}_{i, t-1}$ is 0.39 , after controlling for both $\beta_{i}^{d c}$ and $\gamma_{i}$. The coefficients on $\bar{g}_{i, t-1}$ are generally similar in magnitude. The results also indicate that in portfolios sorted by size and book-to-market ratio, $\beta_{i}^{d c}$ is generally positively associated with return. In momentum portfolios, the relation becomes negative. The coefficient on $\gamma_{i}$, however, is positive in all three sets of portfolios.

The results on equal-weighted portfolios are even stronger in Panel B of Table 6. In univariate regressions, I find that $\bar{g}_{i, t-1}$ is positively associated with returns, with the coefficient of 0.70 , and a $t$-statistic of 6.39 . Again, $\beta_{i}^{d c}$ appears negatively associated with returns. And the coefficient on cash-flow risks $\gamma_{i}$ is positive ( 0.0045 ) and statistically significant.

In Rows 4 through 6 of Table 6, I control for $\beta_{i}^{d c}$ alone, $\gamma_{i}$ alone, and both $\beta_{i}^{d c}$ and $\gamma_{i}$, respectively. The coefficient on $\bar{g}_{i, t-1}$ hovers around 0.72 , with a $t$-statistic of at least 5.04 . The coefficients on both cash-flow risks are statistically not significant.

In Rows 7 through 9 , I use 20 portfolios sorted by size as test assets. After controlling for both cash-flow risks, the coefficient on $\bar{g}_{i, t-1}$ is 0.69 . In Rows 10 through 12, I use 20 portfolios sorted by book-to-market equity as test assets. The coefficient on $\bar{g}_{i, t-1}$ is 0.98 , after controlling for both cash-flow risks. When I use the momentum portfolios in Rows 13 through 15, the coefficient on $\bar{g}_{i, t-1}$ is 0.39 , after controlling for $\beta_{i}^{d c}$ and $\gamma_{i}$. In sum, in equal-weighted portfolios, the coefficient on $\bar{g}_{i, t-1}$ is always statistically significant at the $10 \%$ level, with the lowest $t$-statistic being 1.93. Cash-flow risks exhibit similar patterns as those in value-weighted portfolios. They mostly lose their statistical significance after controlling for $\bar{g}_{i, t-1}$, but $\gamma_{i}$ remain positive and economically sizable.

The coefficient on the expected growth rate implies an economically important effect. In Row 6 of Panel A (value-weighted), the coefficient on the expected growth rate is 0.30 . In Row 6 of Panel B (equal-weighted), the coefficient on the expected growth rate is 0.72 . This means a one-
standard-deviation increase in the expected growth rate is associated with a $1.3 \%$ or $3.1 \%$ increase in annual expected returns in value-weighted or equal-weighted portfolios. In comparison, a one-standard-deviation increase in $\gamma_{i}$ is associated with a $1.3 \%$ or $0.35 \%$ increase in annual expected returns.

I now examine whether the coefficient on the expected growth rate is reasonable. In Panel A of Fig. A1, as the expected growth rate increases from 0 to 0.1 , the expected excess return increases approximately from 0.04 to 0.09 . These numbers suggest a slope coefficient of about 0.5 . The coefficient on growth rates is 0.30 and 0.72 , after controlling for both cash-flow risk measures, in value- and equal-weighted portfolios, respectively. Notwithstanding the fact that Panel A of Fig. A1 uses a set of parameter values that are not empirically estimated, I conclude that the magnitude of my empirical findings does not seem unreasonable.

### 5.3 Volatilities

I now examine whether the expected growth rate increases volatilities. To test this implication, I again use 3 sets of 20 value-weighted portfolios sorted by size, book-to-market equity, and momentum for the period between 1964 and 2010. I compute the annualized standard deviation in year $t$ by using the most recent 20 quarterly real returns, including year $t .{ }^{25}$ I then estimate Fama-MacBeth regressions of return standard deviations on $\bar{g}_{i, t-1}$, while controlling for cash-flow risks $\beta_{i}^{d c}$ and $\gamma_{i}$. Because volatility is total risk instead of systematic risk, I also control for the total cash-flow risk $\sigma(\Delta d)$, computed as the annualized standard deviation of the most recent 20 quarters' dividend growth rates. The summary statistics of these variables are provided in Panel B of Table 5. The summary statistics for $\beta_{i}^{d c}$ and $\gamma_{i}$ are the same as in Panel A, and therefore omitted.

The regression results are provided in Table 7. The first-stage regression is estimated each year. In the second stage, $t$-statistics are adjusted in the Newey-West procedure with automatically selected lags. Panels A and B report results on value-weighted and equal-weighted portfolios,

[^20]respectively. In Panel A, in univariate regressions, I find that $\bar{g}_{i, t-1}$ is positively associated with return volatility, with the coefficient of 0.07 and the Newey-West $t$-statistic of 1.39 . In Row 2, I find that, after controlling for cash-flow risks, the coefficient on $\bar{g}_{i, t-1}$ increases to 0.28 , with a $t$-statistic of 5.51 . In Rows 3 through 8, I find that when using portfolios sorted by size, book-to-market, and momentum, the coefficients on $\bar{g}_{i, t-1}$ are, respectively, $0.76,0.46$, and 0.28 , after controlling for all three cash-flow risks.

In Panel B of Table 7, I repeat the same exercise, except that I now use equal-weighted portfolios. The results are similar to those in Panel A. In Row 2, after controlling for cash-flow risks, the coefficient on $\bar{g}_{i, t-1}$ is 0.34 , with a $t$-statistic of 5.80 . In Rows 3 through 8, I find that when using portfolios sorted by size, book-to-market, and momentum, the coefficients on $\bar{g}_{i, t-1}$ are, respectively, $0.76,0.16$, and 0.38 , after controlling for all three cash-flow risks. The coefficients in size and momentum portfolios are statistically significant as well. In equal-weighted portfolios sorted by book-to-market, the coefficient on $\bar{g}_{i, t-1}$ is statistically not significant, but its economic magnitude, 0.16 , is on the same order of magnitude as the estimate using all 60 portfolios, 0.34 .

The evidence on the return volatility is weaker than the results on returns. Often, there is no relation between return volatility and the expected growth rate in univariate regression, and the positive association only appears after controlling for cash-flow risks. Nevertheless, the results suggest that the expected growth rate is positively associated with return volatility after controlling for cash-flow risks, i.e., there is a growth premium in return volatility as well.

### 5.4 Are the results mechanical?

I find that stocks with higher expected dividend growth rate have higher future returns. One concern is that this result is mechanical. The reason is that in the long run, if the dividend-price ratio is to stay stationary, the expected dividend growth rate should align with the expected price growth rate (capital gains).

However, I argue that this finding is not mechanical, for two reasons. ${ }^{26}$ First, I find a growth premium in return volatilities as well. The growth effect in returns ranges from 0.30 to 0.72 in

[^21]value- and equal-weighted portfolios (Row 6 in Table 6). The growth effect in return volatilities ranges from 0.28 to 0.34 (Row 2 in Table 7). These effects are on the same order of magnitude. The mechanical explanation does not explain the results in return volatilities. Second, theory suggests the opposite could happen. Recall that in Panel B of Fig. A1, I plot the expected excess return, the expected capital gain, and the dividend yield when the price of risk is procyclical and there is a negative growth premium. When the expected growth rate increases, the expected capital gain indeed increases. But in this case, the dividend yield strongly decreases with the expected growth rate. As a result, the expected return is again negatively associated with the expected growth rate. I do not observe this implication in the data.

## 6 Conclusions

Conventional wisdom holds that growth stocks, defined as low book-to-market stocks, have substantially higher future cash-flow growth rates and longer cash-flow durations than value stocks, and that leading asset pricing models imply a growth premium, inconsistent with the value premium. I find that in buy-and-hold portfolios, there is little difference between growth stocks and value stocks in cash-flow growth rates and cash-flow durations. In rebalanced portfolios, growth stocks have lower cash-flow growth rates and shorter cash-flow durations than value stocks. Using rebalanced portfolios, I find that there is a growth premium in the broad cross section of stock returns.

The growth premium is consistent with a class of asset pricing models that feature countercyclical risk premiums or procyclical expected growth rates. My results show that this class of models is not only consistent with the time series of the aggregate stock market, but also broadly consistent with the cross section of stock returns, notably the momentum effect and the value premium, all at the same time.

## References

Ang, Andrew, and Jun Liu, 2004, How to Discount Cashflows with Time-Varying Expected Returns, Journal of Finance 59, 2745-2783.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles, Journal of Finance 59, 1481-1509.

Bansal, Ravi, Robert F. Dittmar, and Christian T. Lundblad, 2005, Consumption, Dividends, and the Cross Section of Equity Returns, Journal of Finance 60, 1639-1672.
Binsbergen, Jules H. van, Michael W. Brandt, and Ralph S. J. Koijen, 2010, On the Timing and Pricing of Dividends, American Economic Review, forthcoming.
Binsbergen, Jules H. van, and Ralph S. J. Koijen, 2010, Predictive Regressions: A Present-Value Approach, Journal of Finance 65, 1439-1471.
Binsbergen, Jules H. van, and Ralph S. J. Koijen, 2011, A Note on "Dividend Strips and the Term Structure of Equity Risk Premia: A Case Study of the Limits of Arbitrage", working paper, Northwestern University/Stanford University.
Boguth, Oliver, Murray Carlson, Adlai J. Fisher, and Mikhail Simutin, 2011, Dividend Strips and the Term Structure of Equity Risk Premia: A Case Study of Limits to Arbitrage, working paper, Arizona State University.
Campbell, John Y., and John Cochrane, 1999, By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior, Journal of Political Economy 107, 205-251.
Campbell, John Y., Christopher Polk, and Tuomo Vuolteenaho, 2010, Growth or Glamour? Fundamentals and Systematic Risk in Stock Returns, Review of Financial Studies 2010, 305-344.

Campbell, John Y., and Tuomo Vuolteenaho, 2004, Bad Beta, Good Beta, American Economic Review 94, 1249-1275.
Chen, Huafeng (Jason), 2011, Firm Life Expectancy and the Heterogeneity of the Book-to-Market Effect, Journal of Financial Economics 100, 402-423.
Chen, Long, Ralitsa Petkova, and Lu Zhang, 2008, The Expected Value Premium, Journal of Financial Economics 87, 269-280.

Cochrane, John H., 2011, Presidential Address: Discount Rates, Journal of Finance 66, 1047-1108.
Cohen, Randolph B., Christopher Polk, and Tuomo Vuolteenaho, 2009. The Price Is (Almost) Right, Journal of Finance 64, 2739-2782.
Cooper, Michael J., Huseyin Gulen, and Michael J. Schill, 2008, Asset Growth and the Cross-Section of Stock Returns, Journal of Finance 63, 1609-1651.
Cornell, Bradford, 1999, Risk, Duration, and Capital Budgeting: New Evidence on Some Old Questions, Journal of Business 72, 183-200.

Croce, Mariano M., Martin Lettau, and Sydney C. Ludvigson, 2010, Investor Information, Long-Run Risk, and the Duration of Risky Cash Flows, working paper, UNC Chapel Hill.
Da, Zhi, 2009, Cash Flow, Consumption Risk, and the Cross-section of Stock Returns, Journal of Finance 64, 923-956.

Dechow, Patricia, Richard Sloan, and Mark Soliman, 2004, Implied Equity Duration: a New Measure of Equity Security Risk, Review of Accounting Studies 9, 197-228.
Fama, Eugene F., and Kenneth R. French, 1992, The Cross Section of Expected Stock Returns, Journal of Finance 47, 427-465.
Fama, Eugene F., and Kenneth R. French, 1995, Size and Book-to-Market Factors in Earnings and Returns, Journal of Finance 50, 131-155.

Fama, Eugene F., and Kenneth R. French, 2007, Migration, Financial Analyst Journal 63, 48-58.
Grullon, Gustavo, and Roni Michaely, 2002, Dividends, Share Repurchases, and the Substitution Hypothesis, Journal of Finance 57, 1649-1684.
Hansen, Lars Peter, John C. Heaton, and Nan Li, 2008, Consumption Strikes Back? Measuring Long-Run Risk, Journal of Political Economy 116, 260-302.
Johnson, Timothy C., 2002, Rational Momentum Effects, Journal of Finance 57, 585-608.
Koijen, Ralph, Hanno Lustig, and Stijn Van Nieuwerburgh, 2010, The Cross-Section and Time-Series of Stock and Bond Returns, working paper, Chicago Booth.
Lakonishok, Josef, Andrei Shleifer, and Robert W. Vishny, 1994, Contrarian Investment, Extrapolation, and Risk, Journal of Finance 49, 1541-1578.

Lettau, Martin, Sydney C. Ludvigson, 2005, Expected Returns and Expected Dividend Growth, Journal of Financial Economics 76, 583-626.

Lettau, Martin, and Jessica A. Wachter, 2007, Why is Long-Horizon Equity Less Risky? A Duration-Based Explanation of the Value Premium, Journal of Finance 62, 55-92.

Menzly, Lior, Tano Santos, and Pietro Veronesi, 2004, Understanding Predictability, Journal of Political Economy 112, 1-47.

Novy-Marx, Robert, 2010, The Other Side of Value: Good Growth and the Gross Profitability Premium, working paper, University of Rochester.
Penman, Stephen, Francesco Reggiani, Scott A. Richardson, and Irem Tuna, 2011, A Characteristic Model for Asset Pricing, working paper, Columbia University.
Santos, Tano, and Pietro Veronesi, 2010, Habit Formation, the Cross Section of Stock Returns and the Cash-flow Risk Puzzle, Journal of Financial Economics 98, 385-413.

Wang, Shubo, 2012, Incomplete Information and Asset Returns in Real Business Economies, working paper, Vanderbilt University.

Zhang, Lu, 2005, The Value Premium, Journal of Finance 60, 67-103.

Returns on equity for growth, neutral, and value portfolios


Fig. 1: Returns on equity for portfolios sorted by book-to-market ratios. In each year $t$ between 1963 and 2009, I sort stocks according to their book-to-market ratios. The growth, neutral, and value portfolios consist of stocks with book-to-market equity that are in the lowest $30 \%$, middle $40 \%$, and highest $30 \%$. The breakpoints are computed using NYSE stocks only. Portfolio return on equity is the sum of earnings (ib) over the sum of book equity. In computing the return on equity, I treat earnings and book equity with fiscal year ends between July of year $t+s-1$ and June of year $t+s$ as earnings and book equity in year $t+s$. I require a stock to have data for both $E_{t+s}$ and $B E_{t+s-1}$ to be included in the computation of the portfolio return on equity. I then average the portfolio return on equity across the 47 portfolio formation years 1963-2009.


Fig. 2: Back-of-the-envelope earnings growth rates. Back-of-the-envelope earnings growth rates are computed based on information in Fig. 1 and the following formula: $\frac{E_{s}}{E_{s-1}}-1=$ $(1-p o) R O E_{s}+\left(\frac{R O E_{s}}{R O E_{s-1}}-1\right) . E_{s}, R O E_{s}$, and po refer to earnings, return on equity and dividend payout ratio. po is assumed to be 0.5 in the back-of-the-envelope calculations. The growth, neutral, and value portfolios consist of stocks with book-to-market equity that are in the lowest $30 \%$, middle $40 \%$, and highest $30 \%$.


Fig. 3: Simple buy-and-hold growth rates not adjusted for survivorship bias. In each year $t$ between 1963 and 2009, I sort stocks according to their book-to-market ratios. The growth (blue solid line), neutral (green dashed line), and value (red dash-dot line) portfolios consist of stocks with book-to-market equity that are in the lowest $30 \%$, middle $40 \%$, and highest $30 \%$. The breakpoints are computed using NYSE stocks only. In computing the return on equity, I treat earnings, accounting cash flow, dividends, and book equity with fiscal year ends between July of year $t+s-1$ and June of year $t+s$ as those variables in year $t+s$. I require a stock to have data in both years $t+s$ and $t+s-1$ to be included in the computation of the portfolio growth rates in year $t+s$. Panels A, B, C, and D plot the average growth rate of earnings (ib), accounting cash flow (ib+dp), dividend (CRSP lagged market equity times the difference between returns with and without dividends), and book equity. For earnings and accounting cash flow, I first scale the portfolio fundamental values in years $t+s$ and $t+s-1$ by the market capitalizations in June of year $t$. I then average the scaled fundamental values across portfolio formation years before computing the portfolio growth rate. For dividends and book equity, the portfolio growth rate in year $t+s$ is the sum of the accounting variable in year $t+s$ across firms in that portfolio over the sum of that accounting variable in year $t+s-1$, minus one. I then average across portfolio formation years $t$.


Fig. 4: Buy-and-hold growth rates adjusted for survivorship bias. In each year $t$ between 1963 and 2009, I sort stocks according to their book-to-market ratios. The growth (blue solid line), neutral (green dashed line), and value (red dash-dot line) portfolios consist of stocks with book-to-market equity that are in the lowest $30 \%$, middle $40 \%$, and highest $30 \%$. The breakpoints are computed using NYSE stocks only. Earnings, accounting cash flow, and book equity in year $t+s$ refer to those variables for the fiscal year that ends between July of year $t+s-1$ and June of year $t+s$. Dividends in year $t+s$ refer to the total dividends (including repurchases) accumulated between July of year $t+s-1$ and June of year $t+s$. Survivorship bias is adjusted according to the text. Panels A, B, C, and D plot the average growth rate of earnings, accounting cash flow, dividends (based on CRSP lagged market equity times the difference between returns with and without dividends), and book equity. All growth rates are on per share bases.


Fig. 5: Buy-and-hold dividend growth rates for book-to-market deciles adjusted for survivorship bias. Panel A plots the average survivorship-bias adjusted buy-and-hold dividend growth rates in years 1, 2 and 3 after portfolio formation. The breakpoints are computed using NYSE stocks only. I average the portfolio growth rates across the 47 portfolio formation years 1963-2009. Panel B plots the average survivorship-bias adjusted buy-and-hold dividend growth rates for Deciles 1 and 10 .


Fig. 6: Buy-and-hold growth rates for assets, revenue, and cost, adjusted for survivorship bias. In each year $t$ between 1963 and 2009, I sort stocks according to their book-to-market ratios. The growth, neutral, and value portfolios consist of stocks with book-to-market equity that are in the lowest $30 \%$, middle $40 \%$, and highest $30 \%$. The breakpoints are computed using NYSE stocks only. I treat assets, revenue, and cost with fiscal year ends between July of year $t+s-1$ and June of year $t+s$ as those variables in year $t+s$. Survivorship bias is adjusted according to the text. Panel A plots the average asset growth rates for the growth (blue solid line), neutral (green dashed line), and value (red dash-dot line) portfolios. Panel B plots the average revenue growth rates for the growth (blue solid line) and value (green dashed line) portfolios, and the average cost growth rates for the growth (red dash-dot line) and value (black dotted line) portfolios. All growth rates are on per share bases.


Fig. 7: Rebalanced portfolio growth rates 1964-2010. Panel A plots the earnings (ib) growth rate for annually rebalanced portfolios. Portfolio 1,2 , and 3 refer to portfolios with the lowest $30 \%$, middle $40 \%$, and highest $30 \%$ book-to-market equity. Panel B plots rebalanced portfolios' growth rates in accounting cash flow, dividends, and book equity. I average the portfolio growth rates of dividends and book equity across the 47 portfolio formation years 1963-2009. All growth rates are on per share bases.


Fig. 8: Average dividend-price ratios, returns, and dividend growth rates for book-tomarket deciles 1964 to 2010. The left panels plot the buy-and-hold deciles and the right panels plot the rebalanced deciles. The top panels plot value-weighted deciles and the bottom panels plot the equal-weighted deciles. Annual data are used. Average dividend-price ratios are represented by blue solid lines, average returns are represented by green dashed lines, and average dividend growth rates are represented by red dash-dot lines. Average dividend-price ratios are $D_{1} / P_{0}$ average across 47 portfolio formation years (1963-2009). For buy-and-hold portfolios, returns are averaged across 1,128 years ( 47 years for portfolios formed in 1963, 46 years for portfolios formed in 1964 , etc.); dividend growth rates are averaged across 1,081 years (skipping the look-back growth rates in year 1, therefore 46 years for portfolios formed in 1963, 45 years for portfolios formed in 1964, etc.). For buy-and-hold portfolios, dividend growth rates are winsorized at $1 \%$ and $99 \%$. For rebalanced portfolios, returns are averaged across 47 portfolios formation years (1963-2009), and growth rates are averaged across 46 years (1965-2010).


Fig. 9: Fundamental-to-price ratios. This figure plots average $\frac{F_{0}}{P_{0}}$ (blue solid line) and $\frac{F_{1}}{P_{0}}$ (red dash-dot line). Ten value-weighted book-to-market portfolios are formed in June of each year. $F_{0}$ and $F_{1}$ stand for fundamentals in the formation year and the year after. Fundamental refers to earnings (Panel A), accounting cash flow (Panel B), dividend (Panel C), and book equity (Panel D). Earnings, accounting cash flow, and book equity in year $t$ are for the fiscal year that ends between July of year $t-1$ and June of year $t$. Dividends in year $t$ are the total dividends (including repurchases) accumulated between July of year $t-1$ and June of year $t . P_{0}$ is the market equity in June of portfolio formation year $t$. The ratios are averaged between 47 portfolio formation years between 1963 and 2009.


Fig. 10: The standard deviation of look-back growth rates with survivorship bias and rebalanced portfolio growth rates. The left panel plots the standard deviation of dividend growth rates. The right panel plots the standard deviation of book-equity growth rates. The look-back growth rates refer to the growth rates in year 1 after portfolio formation adjusted for survivorship bias. The standard deviation of the look-back growth rates is computed across portfolio formation years 1963-2009.

Table 1: Regressions of firm level dividend growth rates on lagged book-to-market ratios. $\log \left(D_{i, t} / D_{i, t-1}\right)=b_{0}+b_{1} \log (B / M)_{i, t-k}+\epsilon_{i, t}$. I follow the Fama-MacBeth procedure. NeweyWest t-statistics with an automatically selected number of lags are reported. $D_{i, t}$ is the dividend (including repurchases) from July of year $t-1$ to June of year $t$ computed from CRSP. Variables are winsorized at $1 \%$ and $99 \%$ in each year. Years negative refers to the number of years in which the coefficient $b_{1}$ is negative.

| k | $\log (B M)_{i, t-k}$ | Years negative | Number of years | years | Avg. Obs. | Adj. $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -0.143 | 46 | 46 | 1965-2010 | 1498.37 | 1.08\% |
|  | (-9.85) |  |  |  |  |  |
| 2 | -0.051 | 37 | 46 | 1965-2010 | 1427.96 | 0.20\% |
|  | (-6.16) |  |  |  |  |  |
| 3 | -0.030 | 31 | 45 | 1966-2010 | 1363.91 | 0.16\% |
|  | (-3.36) |  |  |  |  |  |
| 4 | -0.032 | 31 | 44 | 1967-2010 | 1297.50 | 0.11\% |
|  | (-4.19) |  |  |  |  |  |
| 5 | -0.032 | 31 | 43 | 1968-2010 | 1233.21 | 0.11\% |
|  | (-4.75) |  |  |  |  |  |
| 6 | -0.027 | 29 | 42 | 1969-2010 | 1172.33 | 0.08\% |
|  | (-3.87) |  |  |  |  |  |
| 7 | -0.029 | 26 | 41 | 1970-2010 | 1114.00 | 0.09\% |
|  | (-3.42) |  |  |  |  |  |
| 8 | -0.018 | 27 | 40 | 1971-2010 | 1056.60 | 0.03\% |
|  | (-1.96) |  |  |  |  |  |
| 9 | -0.018 | 26 | 39 | 1972-2010 | 1001.10 | 0.05\% |
|  | (-2.62) |  |  |  |  |  |
| 10 | -0.012 | 23 | 38 | 1973-2010 | 948.45 | 0.12\% |
|  | (-1.08) |  |  |  |  |  |

Table 2: Transition matrix. Cell $(i, j)$ in this table reports the probability (average percentages) of a stock belonging to book-to-market Decile $j$ or exiting $(j=11)$ in year $t+1$, conditionally on it belonging to Decile $i$ in year $t$. Each row adds up to 100 . I average across 47 portfolio formation years $t$ from 1963 to 2009. Book-to-market breakpoints from NYSE are used.

|  |  | Decile $_{t+1}$ |  |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Decile $_{t}$ | Growth | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Value | Exit |
| Growth | 56.24 | 16.92 | 6.79 | 3.55 | 2.02 | 1.70 | 0.98 | 0.83 | 0.61 | 0.63 | 9.74 |
| 2 | 19.05 | 30.35 | 18.76 | 10.36 | 5.28 | 3.57 | 2.20 | 1.59 | 1.12 | 0.93 | 6.78 |
| 3 | 7.79 | 17.78 | 23.03 | 18.34 | 10.38 | 6.80 | 4.16 | 2.44 | 1.51 | 0.96 | 6.81 |
| 4 | 4.11 | 8.31 | 15.56 | 20.48 | 17.46 | 11.18 | 6.57 | 4.59 | 2.75 | 1.58 | 7.41 |
| 5 | 2.52 | 4.92 | 9.15 | 14.77 | 18.37 | 17.38 | 11.98 | 7.20 | 4.28 | 2.16 | 7.26 |
| 6 | 1.98 | 3.06 | 5.25 | 8.93 | 14.51 | 18.71 | 17.93 | 12.41 | 7.20 | 3.06 | 6.97 |
| 7 | 1.36 | 1.88 | 3.13 | 5.69 | 9.42 | 14.13 | 19.71 | 19.29 | 12.36 | 5.68 | 7.36 |
| 8 | 1.17 | 1.15 | 1.96 | 3.15 | 5.29 | 8.42 | 15.84 | 23.18 | 22.18 | 10.71 | 6.96 |
| 9 | 0.93 | 0.93 | 1.25 | 1.93 | 2.79 | 4.81 | 8.38 | 16.93 | 29.17 | 24.93 | 7.95 |
| Value | 0.69 | 0.59 | 0.71 | 1.00 | 1.28 | 1.82 | 3.09 | 6.20 | 15.24 | 56.75 | 12.65 |

Table 3: Regressions of firm level dividend growth rates on lagged book-to-market ratios revisited. $\log \left(\left(D_{i, t}+d l_{i, t}\right) / D_{i, t-1}\right)=b_{0}+b_{1} \log (B / M)_{i, t-k}+\epsilon_{i, t}$. I follow the Fama-MacBeth procedure. Newey-West t-stat with automatically selected number of lags are reported. $D_{i, t}$ is the dividend (including repurchases) from July of year $t-1$ to June of year $t$ computed from CRSP. $d l_{i, t}$ is the delisting proceeds for a firm that is delisted in that year. Variables are winsorized at $1 \%$ and $99 \%$ in each year. Years negative refers to the number of years in which the coefficient $b_{1}$ is negative.

| k | $\log (B M)_{i, t-k}$ | Years negative | Number of years | years | Avg. Obs. | Adj. $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -0.111 | 42 | 46 | 1965-2010 | 1551.20 | 0.60\% |
|  | (-4.77) |  |  |  |  |  |
| 2 | 0.002 | 25 | 46 | 1965-2010 | 1478.46 | 0.11\% |
|  | (0.10) |  |  |  |  |  |
| 3 | 0.002 | 20 | 45 | 1966-2010 | 1409.40 | 0.09\% |
|  | (0.17) |  |  |  |  |  |
| 4 | 0.001 | 22 | 44 | 1967-2010 | 1338.34 | 0.01\% |
|  | (0.09) |  |  |  |  |  |
| 5 | 0.008 | 19 | 43 | 1968-2010 | 1270.63 | 0.05\% |
|  | (0.75) |  |  |  |  |  |
| 6 | 0.008 | 17 | 42 | 1969-2010 | 1205.81 | 0.03\% |
|  | (0.67) |  |  |  |  |  |
| 7 | 0.008 | 16 | 41 | 1970-2010 | 1144.93 | 0.01\% |
|  | (0.53) |  |  |  |  |  |
| 8 | 0.021 | 14 | 40 | 1971-2010 | 1085.23 | 0.06\% |
|  | (1.39) |  |  |  |  |  |
| 9 | 0.009 | 15 | 39 | 1972-2010 | 1027.62 | 0.06\% |
|  | (0.72) |  |  |  |  |  |
| 10 | 0.016 | 14 | 38 | 1973-2010 | 972.68 | 0.14\% |
|  | (0.91) |  |  |  |  |  |

Table 4: The canonical Macaulay duration and cash-flow duration. Dur canonical as the weighted average of cash-flow maturity. Dividends in the next 20 years are based on historical data. Beyond year 20, cash flows are assumed to be a growing perpetuity, in which the terminal growth rates $\left(g_{i \infty}\right)$ are the implied growth rate that equate the present value of future cash flow and the market price. Discount rates are from average future realized returns in Fig. 8. Weights are the fraction of the present value of the dividends in each year in the total market price. Dur ${ }^{\text {cashflow }}$, the cash-flow duration, is the Macaulay duration with discount rates of all assets assumed to be the same as the value portfolio. In computing, $\sum_{s=2}^{\infty} \rho^{s} g_{i s}, \rho=0.95$ and growth rates beyond year 20 are assumed to be $g_{i \infty}$. BH refers to buy-and-hold portfolios. VW and EW refer to value-weighted and equal-weighted portfolios.

|  | growth 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | value 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A: the canonical Macaulay duration, Dur $^{\text {canonical }}$ |  |  |  |  |  |  |  |  |  |  |
| BH, VW | 28.57 | 23.03 | 20.14 | 19.86 | 19.95 | 19.49 | 18.46 | 17.85 | 17.37 | 21.95 |
| BH, EW | 38.86 | 32.96 | 30.26 | 29.13 | 27.14 | 26.47 | 27.14 | 26.15 | 27.12 | 31.38 |
| Rebalanced, VW | 47.94 | 27.46 | 23.13 | 23.54 | 21.74 | 22.13 | 21.56 | 25.33 | 20.81 | 24.38 |
| Rebalanced, EW | 113.87 | 63.30 | 55.25 | 50.20 | 45.63 | 36.79 | 49.71 | 41.41 | 52.94 | 62.71 |
| Panel B: steady-state growth rate beyond year $20, g_{i \infty}(\%)$ |  |  |  |  |  |  |  |  |  |  |
| BH, VW | 5.74 | 4.85 | 3.86 | 4.46 | 5.36 | 5.40 | 5.16 | 3.97 | 4.20 | 7.26 |
| BH, EW | 9.99 | 10.27 | 10.30 | 10.53 | 10.45 | 10.78 | 11.07 | 10.74 | 11.29 | 12.62 |
| Rebalanced, VW | 8.08 | 6.28 | 5.64 | 6.40 | 5.34 | 7.78 | 7.86 | 9.47 | 9.78 | 10.73 |
| Rebalanced, EW | 8.14 | 10.46 | 12.27 | 12.59 | 14.21 | 16.31 | 15.84 | 16.29 | 19.29 | 21.79 |
| Panel C: cash-flow duration, Dur ${ }^{\text {cashflow }}$ |  |  |  |  |  |  |  |  |  |  |
| BH, VW | 20.37 | 18.03 | 17.15 | 16.40 | 17.43 | 17.28 | 17.54 | 15.52 | 15.63 | 21.95 |
| BH, EW | 22.94 | 22.50 | 22.07 | 22.28 | 21.83 | 23.06 | 23.15 | 21.97 | 24.06 | 31.38 |
| Rebalanced, VW | 15.31 | 14.46 | 14.22 | 14.16 | 11.98 | 15.27 | 15.10 | 17.55 | 20.89 | 24.38 |
| Rebalanced, EW | 6.80 | 8.62 | 9.87 | 9.90 | 11.65 | 16.40 | 12.77 | 14.60 | 22.65 | 62.71 |
| Panel D: $\sum_{s=2}^{\infty} \rho^{s} g_{i s}$ |  |  |  |  |  |  |  |  |  |  |
| BH, VW | 1.64 | 1.48 | 1.56 | 1.39 | 1.47 | 1.31 | 1.36 | 1.34 | 1.38 | 2.09 |
| BH, EW | 2.63 | 2.46 | 2.37 | 2.27 | 2.25 | 2.25 | 2.21 | 2.19 | 2.29 | 2.68 |
| Rebalanced, VW | 1.75 | 1.66 | 1.53 | 1.56 | 1.33 | 1.82 | 1.49 | 1.95 | 2.22 | 2.79 |
| Rebalanced, EW | 1.46 | 1.78 | 2.00 | 2.14 | 2.52 | 2.87 | 2.48 | 2.77 | 3.10 | 3.68 |

Table 5: Summary statistics between 1963 Q3 and 2010 Q2. 20 equity portfolios sorted by size, book-to-market, and momentum each are used. Panel A reports summary statistics for returns regressions in Table 6. qret stands for quarterly real returns. $\bar{g}_{i, t-1}$ is the average annual real dividend growth rate of the rebalanced portfolio $i$, using information between 1958 and year $t-1$. $\beta_{i}^{d c}$ is the consumption beta of quarterly dividend growth rate, estimated in $\log \left(1+g_{i, t}\right)=$ $b_{i}^{0}+\beta_{i}^{d c}\left(\log \left(1+g_{c, t}\right)\right)+\epsilon_{i, t}$, using all information between 1963 Q3 and 2010 Q2. Here $g_{i, t}$ is the quarterly real dividend growth rate, and $g_{c, t-k}$ is the quarterly real consumption growth rate. Dividends and consumptions are seasonally adjusted by summing the most recent 4 quarters. $\gamma_{i}$ is the Bansal-Dittmar-Lundblad cash-flow risk, measured from the regression $\log \left(1+g_{i, t}\right)=$ $\gamma_{i}\left(\frac{1}{8} \sum_{k=1}^{8} \log \left(1+g_{c, t-k}\right)\right)+\epsilon_{i, t}$. When computing $\gamma_{i}$, both $\log \left(1+g_{i, t}\right)$ and $\log \left(\left(1+g_{c, t-k}\right)\right.$ are demeaned. Panel B reports summary statistics for return standard deviation regressions in Table 7. $\sigma$ (qret) in year $t$ is the standard deviation of quarterly real return in the most recent 5 years, including year $t . \sigma(\Delta d)$ in year $t$ is the standard deviation of the most recent 20 quarterly log real dividend growth rates, including year $t . \bar{g}_{i, t-1}$ and $\sigma(q r e t)$ are winsorized at $1 \%$ and $99 \%$.

|  | N | Mean | Median | Standard deviation |
| :---: | :---: | :---: | :---: | :---: |
| Panel A: Return regressions |  |  |  |  |
| Value weighted |  |  |  |  |
| qret $\times 4$ | 11,280 | 0.0843 | 0.0942 | 0.4426 |
| $\bar{g}_{i, t-1}$ | 11,280 | 0.0507 | 0.0481 | 0.0432 |
| $\beta_{i}^{d c}$ | 11,280 | 2.8079 | 2.3583 | 2.9523 |
| $\gamma_{i}$ | 11,280 | 5.3205 | 5.3327 | 4.0994 |
| Equal weighted |  |  |  |  |
| qret $\times 4$ | 11,280 | 0.1169 | 0.1020 | 0.5370 |
| $\bar{g}_{i, t-1}$ | 11,280 | 0.0641 | 0.0593 | 0.0435 |
| $\beta_{i}^{d c}$ | 11,280 | 3.6957 | 3.7602 | 1.9830 |
| $\gamma_{i}$ | 11,280 | 3.9054 | 4.3141 | 2.3313 |
| Panel B: Volatility regressions |  |  |  |  |
| Value weighted |  |  |  |  |
| $\sigma($ qret $) \times 2$ | 2,820 | 0.2070 | 0.1986 | 0.0639 |
| $\sigma(\Delta d) \times 2$ | 2,820 | 0.2731 | 0.2504 | 0.1470 |
| Equal weighted |  |  |  |  |
| $\sigma($ qret $) \times 2$ | 2,820 | 0.2462 | 0.2397 | 0.0829 |
| $\sigma(\Delta d) \times 2$ | 2,820 | 0.2191 | 0.2120 | 0.0868 |

Table 6: Fama-MacBeth regressions of portfolio returns on historical average growth rates and cashflow risks between 1963 Q3 and 2010 Q2. The first-stage regressions are estimated in each quarter. 20 equity portfolios sorted by size, book-to-market, and momentum, respectively, are used. The left-hand-side variable is the quarterly real returns multiplied by $4 . \bar{g}_{i, t-1}$ is the average annual real dividend growth rate of the rebalanced portfolio $i$, using information between 1958 and year $t-1 . \bar{g}_{i, t-1}$ is winsorized at $1 \%$ and $99 \% . \beta_{i}^{d c}$ is the consumption beta of quarterly dividend growth rate, estimated in $\log \left(1+g_{i, t}\right)=$ $b_{i}^{0}+\beta_{i}^{d c}\left(\log \left(1+g_{c, t}\right)\right)+\epsilon_{i, t}$, using all information between 1963 Q 3 and 2010 Q 2 . Here $g_{i, t}$ is the quarterly real dividend growth rate, and $g_{c, t-k}$ is the quarterly real consumption growth rate. $\gamma_{i}$ is the Bansal-DittmarLundblad cash-flow risk, measured from the regression $\log \left(1+g_{i, t}\right)=\gamma_{i}\left(\frac{1}{8} \sum_{k=1}^{8} \log \left(1+g_{c, t-k}\right)\right)+\epsilon_{i, t}$. When computing $\gamma_{i}$, both $\log \left(1+g_{i, t}\right)$ and $\log \left(\left(1+g_{c, t-k}\right)\right.$ are demeaned. Panels A and B report results for value-weighted and equal-weighted portfolios, respectively. $N$ and $R^{2}$ are number of portfolios and average $R^{2}$, respectively.


Table 7: Fama-MacBeth regressions of return standard deviations on historical average growth rates between 1964 and 2010. The first-stage regressions are estimated in each year. The left-hand-side variable is $\sigma(q r e t) \times 2 . \sigma(q r e t)$ in year $t$ is the standard deviation of quarterly real return in the most recent 5 years, including year $t . \bar{g}_{i, t-1}$ is the average annual real dividend growth rate of the rebalanced portfolio $i$, using information between 1958 and year $t-1 . \sigma(\Delta d)$ in year $t$ is the standard deviation of the most recent 20 quarterly $\log$ real dividend growth rates, including year $t . \sigma(q r e t)$ and $\bar{g}_{i, t-1}$ are winsorized at $1 \%$ and $99 \% . \beta_{i}^{d c}$ is the consumption beta of quarterly dividend growth rate, estimated in $\log \left(1+g_{i, t}\right)=b_{i}^{0}+\beta_{i}^{d c}\left(\log \left(1+g_{c, t}\right)\right)+\epsilon_{i, t}$, using all information between 1963 Q3 and 2010 Q2. Here $g_{i, t}$ is the quarterly real dividend growth rate, and $g_{c, t-k}$ is the quarterly real consumption growth rate. $\gamma_{i}$ is the Bansal-Dittmar-Lundblad cashflow risk, measured from the regression $\log \left(1+g_{i, t}\right)=\gamma_{i}\left(\frac{1}{8} \sum_{k=1}^{8} \log \left(1+g_{c, t-k}\right)\right)+\epsilon_{i, t}$. When computing $\gamma_{i}$, both $\log \left(1+g_{i, t}\right)$ and $\log \left(\left(1+g_{c, t-k}\right)\right.$ are demeaned. Newey-West $t$-statistics with automatically selected lags are provided. Panels A and B report results for value-weighted and equal-weighted portfolios, respectively. $N$ and $R^{2}$ represent the number of portfolios and average $R^{2}$, respectively.


## 7 Appendix

### 7.1 Four models of time-varying expected returns and expected growth rates

I first present three simple models with time-varying expected returns. The first model has time-varying market price of risk, which is a continuous-time version of Campbell and Cochrane (1999)'s habit formation model. The second model has time-varying amount of risk, which captures the conditional heteroskedasticity in the long-run risk model. The third model has both time-varying price of risk and amount of risk.

Before presenting the three models, I reformulate the equity premium puzzle within constant expected return models. Assume that the representative agent's consumption is a simple geometric Brownian motion with a constant expected growth rate and a constant volatility.

$$
\begin{equation*}
\frac{d C_{t}}{C_{t}}=\mu_{c} d t+\sigma_{c} d B_{t}^{c} \tag{20}
\end{equation*}
$$

where $\mu_{c}$ is the constant expected growth rate, $\sigma_{c}$ is the constant volatility, and $B_{t}^{c}$ is a standard scalar Brownian motion. Further assume that the representative agent has a power utility function

$$
\begin{equation*}
U\left(C_{t}, t\right)=e^{-\delta t} \frac{\left(C_{t}\right)^{1-\gamma}}{1-\gamma} \tag{21}
\end{equation*}
$$

where $\delta$ is the intertemporal discount rate, and $\gamma$ is the relative risk-aversion parameter.
Under these assumptions, it can be shown that the marginal utility $\left(U_{c}\right)$ of the representative agent follows the process:

$$
\begin{equation*}
\frac{d U_{c}}{U_{c}}=-r d t-\gamma \sigma_{c} d B_{t}^{c} \tag{22}
\end{equation*}
$$

where $r$ is the constant interest rate.
Now assume that the market portfolio has the following dividend process:

$$
\begin{equation*}
\frac{d D_{t}}{D_{t}}=g d t+\sigma_{D}^{c} d B_{t}^{c}+\sigma_{D}^{Z} d B_{t}^{Z} \tag{23}
\end{equation*}
$$

where $g$ is the expected dividend growth rate. $B_{t}^{Z}$ is a Brownian motion that is orthogonal to $B_{t}^{c}$. For this stock, the expected return turns out to be $r+\gamma \sigma_{D}^{c} \sigma_{c}$. The equity premium puzzle, thus, is the puzzle that $\gamma \sigma_{D}^{c} \sigma_{c}$ is too low compared to the empirical equity premium.

In more general terms, let the pricing kernel $M_{t}$ (the representative agent's marginal utility $U_{c}$ ) follow a
geometric Brownian motion,

$$
\begin{equation*}
\frac{d M_{t}}{M_{t}}=-r d t-\sigma_{M} d B_{t}^{M} \tag{24}
\end{equation*}
$$

Accordingly, the dividend process can be expressed as:

$$
\begin{equation*}
\frac{d D_{t}}{D_{t}}=g d t+\sigma_{D}^{M} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z} \tag{25}
\end{equation*}
$$

Under these assumptions, the equity premium (the excess expected return for stock over the interest rate) is $\sigma_{D}^{M} \sigma_{M}$. Throughout the rest of the paper, I view the equity premium puzzle as the puzzle of $\sigma_{D}^{M} \sigma_{M}$ being too low.

In the following three simple models, the dividend process can be understood as that of the market portfolio $\left(D_{t}^{M}\right)$ or individual stocks $\left(D_{t}^{i}\right)$. For ease of exposition, the superscripts are dropped out throughout this section. To focus on time-varying market price of risk and time-varying amount of risk, I assume the interest rate to be constant.
After the first three simple models, I then consider a fourth model in which both the market price of risk and the expected growth rate are time varying.

### 7.1.1 Model 1: Time-varying price of risk

Assume that the pricing kernel follows:

$$
\begin{equation*}
\frac{d M_{t}}{M_{t}}=-r d t-x_{t} d B_{t}^{M} \tag{26}
\end{equation*}
$$

where $r$ is the constant interest rate, and $x_{t}$ is the time-varying market price of risk. $x_{t}$ can be roughly thought of as the time-varying risk-aversion coefficient for the representative agent. Without formally writing down a utility function, I assume that the market price of risk follows a mean-reverting process:

$$
\begin{equation*}
d x_{t}=\phi_{x}\left(\bar{x}-x_{t}\right) d t+\sigma_{x} d B_{t}^{M} \tag{27}
\end{equation*}
$$

where $\phi_{x}$ measures the convergence speed of $x_{t}$ to the long-run average price of risk $\bar{x}$. Without loss of generality, I assume that the innovation to $x_{t}$ is perfectly correlated with innovations to the pricing kernel process.

The dividend process is assumed to be:

$$
\begin{equation*}
\frac{d D_{t}}{D_{t}}=g d t+\sigma_{D}^{M} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z} \tag{28}
\end{equation*}
$$

where $g$ is the expected dividend growth rate. $\sigma_{D}^{M} d B_{t}^{M}$ denotes the systematic component of the dividend process, where $\sigma_{D}^{Z} d B_{t}^{Z}$ is the idiosyncratic component of the dividend process. Under these assumptions, the stock price has the following semi closed-form solution that involves an integral of closed-form quantities. Under these conditions, the price of the stock is

$$
\begin{equation*}
P_{t}=D_{t} \int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N, \tag{29}
\end{equation*}
$$

where $A(N)$ and $B(N)$ are defined in Section 7.2.
The instantaneous expected excess return is:

$$
\begin{equation*}
\mu_{R}=x_{t}\left(\sigma_{D}^{M}+\frac{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} B(N) \sigma_{x} d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N}\right), \tag{30}
\end{equation*}
$$

and

$$
\begin{align*}
\sigma_{R}^{2}= & \left(\sigma_{D}^{M}\right)^{2}+2 \sigma_{D}^{M} \sigma_{x} \frac{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} B(N) d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N} \\
& +\sigma_{x}^{2}\left(\frac{\int_{0}^{+\infty} \exp \left(g N+A(N)+B(N) x_{t}\right) B(N) d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N}\right)^{2}+\left(\sigma_{D}^{Z}\right)^{2} . \tag{31}
\end{align*}
$$

I now define a very-short-duration claim. In this economy, consider the following instrument $I_{T}$ that pays a liquidating dividend $Y_{T}$. The expectation of $Y_{T}$ at time is denoted as $Y_{t}$. Further assume that the cash-flow expectation evolves as follows $\frac{d Y_{t}}{Y_{t}}=\sigma_{D}^{M} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}$. Then the expected instantaneous return of this instrument is $x_{t}\left(\sigma_{D}^{M}+B(N) \sigma_{x}\right)$, where $N=T-t$, and $B(N)$ is defined in Section 7.2. The very-shortduration claim is the limiting case of such $I_{T}$ as $T \rightarrow t$. The expected return on the very-short-duration claim is:

$$
\begin{equation*}
\lim _{N \rightarrow 0} x_{t}\left(\sigma_{D}^{M}+B(N) \sigma_{x}\right)=x_{t} \sigma \sigma_{D}^{M} \tag{32}
\end{equation*}
$$

the expected return in a model with constant cash-flow risk $\sigma_{D}^{M}$ and constant market price of risk equal to $x_{t}$.
In loose terms, a model of constant expected returns dictates that the expected return is the product of risk aversion and dividend risk. In a model with time-varying risk aversion, the average expected return is not
equal to the average risk aversion multiplied by average dividend risk. When risk aversion is countercyclical, the long-run average return is greater than the product of average risk aversion and average dividend risk. This inequality is reversed when the risk aversion is procyclical.
Formally, I prove the following proposition.
Proposition 1. This model of time-varying market price of risk does not change the expected return of the very-short-duration claim. The necessary and sufficient condition for the model to generate a higher equity premium than $\sigma_{M} \sigma_{D}^{M}$ is $\sigma_{x}<0$; that is, the market price of risk is countercyclical. Under the above condition, stocks with higher expected growth rate have higher expected returns, all else being equal.
Proof: See Section 7.2.
Panel A of Fig. A1 plots the expected excess returns of stocks that only differ in their dividend growth rate $g$. The parameters used in this plot are $x_{t}=0.3, \sigma_{D}^{M}=0.005, \phi_{x}=5.05, \bar{x}=0.3$ and the interest rate is $10 \%$. A high interest rate of $10 \%$ is chosen only for computational convenience. But the shape of the curve is not affected by a particular choice of interest rate as long as the expected growth rate is not so high that the price is infinite. As the expected growth rate $g$ increases from 0 to $10 \%$, the expected excess stock return increases from approximately $4 \%$ to $9 \%$. These parameters are chosen to illustrate the working of time-varying market price of risk on the equity premium. If the market price of risk $x_{t}$ is constant (equal to $\bar{x}$, then that stock's equity premium would be $\sigma_{D}^{M} * \bar{x}=0.005 * 0.3=0.15 \%$. With time-varying price of risk $x_{t}$, the model can deliver a much higher equity premium. ${ }^{27}$

### 7.1.2 Model 2: Time-varying amount of risk

I consider another model with time-varying amount of risk. In this model, the market price of risk is constant. However, the amount of risk is time varying. This model also generates time-varying expected returns and the cross-sectional implication on growth rates.

In this model, I assume that the pricing kernel follows:

$$
\begin{equation*}
\frac{d M_{t}}{M_{t}}=-r d t-\sigma_{M} d B_{t}^{M} \tag{33}
\end{equation*}
$$

where $r$ is the constant interest rate and $\sigma_{M}$ is the constant market price of risk.

[^22]The dividend process is assumed to be:

$$
\begin{equation*}
\frac{d D_{t}}{D_{t}}=g d t+x_{t} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}, \tag{34}
\end{equation*}
$$

where $g$ is the constant expected growth rate. $x_{t} d B_{t}^{M}$ is the systematic component of the dividend process, and $\sigma_{D}^{Z} d B_{t}^{Z}$ is the idiosyncratic component of the dividend process. The time-varying amount of risk is assumed to follow a mean-reverting process:

$$
\begin{equation*}
d x_{t}=\phi_{x}\left(\bar{x}-x_{t}\right) d t+\sigma_{x} d B_{t}^{M}, \tag{35}
\end{equation*}
$$

where $\phi_{x}$ measures the speed of convergence of $x_{t}$ to the long-run amount of risk $\bar{x}$. $\sigma_{x}$ is the volatility of the $x_{t}$ process. For convenience, I assume that $x_{t}$ is perfectly correlated with the innovations of the pricing kernel.
The stock price, expected return, and volatility have semi-closed-form solutions that involve the integral of closed-form quantities. The solutions can be found in Section 7.2. In this economy, the very-short-duration claim as the limit of instrument $I_{T}$ that pays a liquidating dividend of $Y_{T}$, the expectation of which follows the process: $\frac{d Y_{t}}{Y_{t}}=x_{t} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}$, where $x_{t}$ follows the process as in equation (35). With this, I prove the following proposition.
Proposition 2. This model of time-varying amount of risk does not change the expected return of the very-short-duration claim. The necessary and sufficient condition for the average expected return of the stock to be greater than $r+\bar{x} \sigma_{M}$ is that $\sigma_{x}<0$; that is, the cash flow is more risky when times are bad. In this case, after controlling for cash flow risk $x_{t}$, the expected return is increasing in $g$.
Proof: See Section 7.2.

### 7.1.3 Model 3: Time-varying price of risk and amount of risk

I now consider the third model, with both time-varying price of risk and amount of risk. Although this model involves both time-varying market price of risk and time-varying cash-flow risk, it does not nest the above two models. This model generates time-varying expected returns and the cross-sectional implication on growth rates.
In this model, I assume the pricing kernel to follow:

$$
\begin{equation*}
\frac{d M_{t}}{M_{t}}=-r d t-\sqrt{x_{t}} d B_{t}^{M} \tag{36}
\end{equation*}
$$

where $r$ is the constant interest rate, and $\sqrt{x_{t}}$ is the market price of risk. To obtain semi closed-form solutions, I assume that the process $x_{t}$ follows the square-root process of

$$
\begin{equation*}
d x_{t}=\phi_{x}\left(\bar{x}-x_{t}\right) d t+\sigma_{x} \sqrt{x_{t}} d B_{t}^{M} \tag{37}
\end{equation*}
$$

where $\phi_{x}$ measures the speed of convergence of $x_{t}$ to the long-run value $\bar{x} . \sigma_{x} \sqrt{x_{t}}$ is the volatility of the $x_{t}$ process. I assume that $x_{t}$ is perfectly correlated with the innovations of the pricing kernel.

I assume that the dividend process also follows a square-root process:

$$
\begin{equation*}
\frac{d D_{t}}{D_{t}}=g d t+\sigma_{D}^{M} \sqrt{x_{t}} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z} \tag{38}
\end{equation*}
$$

where $g$ is the constant expected growth rate. $\sigma_{D}^{M} \sqrt{x_{t}} d B_{t}^{M}$ is the systematic component of the dividend process, and $\sigma_{D}^{Z} d B_{t}^{Z}$ is the idiosyncratic component of the dividend process.
The very-short-duration claim in this economy is the limit of instrument $I_{T}$ that pays a liquidating dividend of $Y_{T}$, the expectation of which follows the process: $\frac{d Y_{t}}{Y_{t}}=\sigma_{D}^{M} \sqrt{x_{t}} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}$. This model also implies that stocks with higher growth rates have higher expected returns, all else being equal.

Proposition 3. This model of time-varying price of risk and amount of risk does not change the expected return of the very-short-duration claim. The necessary and sufficient condition for the unconditional expected return of the stock to be greater than $r+\bar{x} \sigma_{x}$ is that $\sigma_{x}<0$; that is, the market price of risk is countercyclical and the cash flow is more risky when times are bad. In this case, controlling for cash-flow risk $\sigma_{D}^{M}$, the expected return is increasing in $g$.

Proof: See Section 7.2.
To recap, the two robust predictions from all three models are: (1) Models of time-varying expected returns do not change the expected return on a very-short-duration claim. (2) In the cross section, the expected return increases in the expected dividend growth rate, controlling for cash flow risk. I test the second prediction in the next section.

### 7.1.4 Model 4: Time-varying market price of risk and expected growth rate

Assume that the pricing kernel follows:

$$
\begin{equation*}
\frac{d M_{t}}{M_{t}}=-r d t-x_{t} d B_{t}^{M} \tag{39}
\end{equation*}
$$

where $r$ is the constant interest rate, and $x_{t}$ is the time-varying market price of risk. $x_{t}$ can be roughly thought of as the time-varying risk-aversion coefficient for the representative agent. Without formally writing down a utility function, I assume that the market price of risk follows a mean-reverting process:

$$
\begin{equation*}
d x_{t}=\phi_{x}\left(\bar{x}-x_{t}\right) d t+\sigma_{x} d B_{t}^{x}, \tag{40}
\end{equation*}
$$

where $\phi_{x}$ measures the convergence speed of $x_{t}$ to the long-run average price of risk $\bar{x}$. Without loss of generality, I assume that the innovation to $x_{t}$ is perfectly correlated with innovations to the pricing kernel process.

The dividend process is assumed to be:

$$
\begin{equation*}
\frac{d D_{t}}{D_{t}}=g_{t} d t+\sigma_{D} d B_{t}^{D} \tag{41}
\end{equation*}
$$

where $g_{t}$ is the expected dividend growth rate.
The expected growth rate follows the process:

$$
\begin{equation*}
d g_{t}=\phi_{g}\left(\bar{g}-g_{t}\right) d t+\sigma_{g} d B_{t}^{g} \tag{42}
\end{equation*}
$$

The correlation between $B_{t}^{M}, B_{t}^{x}, B_{t}^{D}$, and $B_{t}^{g}$ is assumed to be $\rho_{x M}, \rho_{D M}, \rho_{g M}, \rho_{x D}, \rho_{x g}$, and $\rho_{D g}$. Under these conditions, the price of the $t+N$ dividend strip at time $t$ is:

$$
\begin{equation*}
P_{t}=D_{t} e^{A(N)+B(N) x_{t}+C(N) g_{t}} \tag{43}
\end{equation*}
$$

where $A(N), B(N)$, and $C(N)$ are defined in Section 7.2.
The instantaneous expected excess return is:

$$
\begin{equation*}
\mu_{R}=x_{t}\left(\sigma_{D} \rho_{D M}+B(N) \sigma_{x} \rho_{x M}+C(N) \sigma_{g} \rho_{g M}\right) \tag{44}
\end{equation*}
$$

and
The return process is:

$$
\begin{equation*}
\frac{d P_{t}}{P_{t}}=\left(r+\mu_{R}\right) d t+\sigma_{D} d B_{t}^{D}+B(N) \sigma_{x} d B_{t}^{x}+C(N) \sigma_{g} d B_{t}^{g} \tag{45}
\end{equation*}
$$

which implies that the return volatilities

$$
\begin{equation*}
\sigma_{R}^{2}=\sigma_{D}^{2}+B(N)^{2} \sigma_{x}^{2}+C(N)^{2} \sigma_{g}^{2}+2 B(N) \sigma_{D} \sigma_{x} \rho_{x D}+2 \sigma_{D} C(N) \sigma_{g} \rho_{D g}+2 B(N) C(N) \sigma_{x} \sigma_{g} \rho_{x g} \tag{46}
\end{equation*}
$$

Proposition 4. This model of time-varying price of risk and expected growth does not change the expected return of the very-short-duration claim. The term structure of the expected return depends on both $\rho_{x M}$ and $\rho_{g M}$, among other parameters. Under the assumption that $\rho_{x M}<0$, as well as the regularity conditions that $\phi_{x}+\rho_{x M} \sigma_{x}>0$, 1. If $\rho_{g M}>0$, then the term structure of equity return is largely upward sloping. 2. If $\rho_{g M}<0$ and $\rho_{g M} \sigma_{g}+\phi_{g} \rho_{D} M \sigma_{D}<0$, then the term structure is largely downward sloping. 3. If $\rho_{g M}<0$ and $\rho_{g M} \sigma_{g}+\phi_{g} \rho_{D} M \sigma_{D}>0$, then the term structure is largely upward sloping iff $-\frac{\rho_{g M} \sigma_{g}+\phi_{g} \rho_{D M} \sigma_{D}}{\phi_{g}\left(\phi_{x}+\rho_{x M} \sigma_{x}\right)} \sigma_{x} \rho_{x M}+$ $\frac{\sigma_{g} \rho_{g M}}{\phi_{g}}>0$.
"Largely" means comparing $N=0$ and $N=+\infty$. In general, the term structure of the expected return does not have to be monotonic.

Proof: See Section 7.2.

### 7.2 Proofs

Proposition 1 stated in the text follows from the following expanded exposition.

The price of the stock is

$$
\begin{equation*}
P_{t}=D_{t} \int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N \tag{47}
\end{equation*}
$$

where

$$
\begin{gather*}
B(N)=-\frac{\sigma_{D}^{M}\left(1-e^{-\left(\phi_{x}+\sigma_{x}\right) N}\right)}{\phi_{x}+\sigma_{x}}  \tag{48}\\
A(N)=A_{1} B(N)^{2}+A_{2}\left[\sigma_{D}^{M} N+B(N)\right]-r N \tag{49}
\end{gather*}
$$

$A_{1}=-\frac{\sigma_{x}^{2}}{4\left(\phi_{x}+\sigma_{x}\right)}$, and $A_{2}=-\frac{\phi_{x} \bar{x}+\sigma_{D}^{M} \sigma_{x}}{\phi_{x}+\sigma_{x}}+\frac{\sigma_{D}^{M} \sigma_{x}^{2}}{2\left(\phi_{x}+\sigma_{x}\right)^{2}}$.
There are some regularity conditions that have to be satisfied. These are: 1). $\sigma_{D}^{M}>0$ (positive cashflow risk); 2) $x_{t}>0$ (positive market price of risk); 3) $\phi_{x}+\sigma_{x}>0$ and $g+A_{2} \sigma_{D}^{M}-r<0$ (conditions for price of the stock to be finite). Under the above conditions, the following holds for the stock that pays a continuous stream of dividends according to $\frac{d D_{t}}{D_{t}}=g d t+\sigma_{D}^{M} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}$.

The price-dividend ratio is increasing in $g$ and decreasing in $\sigma_{D}^{M}$ (cash-flow risk). The volatility is increasing in $\sigma_{D}^{M}$ (cash-flow risk).

1) When $\sigma_{x}<0$ (countercyclical market price of risk), the unconditional expected return of the stock
is greater than $r+\bar{x} \sigma_{D}^{M}$, thus helping to produce a higher equity premium. Controlling for cash-flow risk $\sigma_{D}^{M}$ the expected return is increasing in g .
2) When $\sigma_{x}=0$ (constant discount rate), the volatility and expected returns are only determined by cash-flow risk and do not depend on g . The expected return is $r+x_{t} \sigma_{M}=r+\bar{x} \sigma_{M}$.
3) When $\sigma_{x}>0$ (procyclical market price of risk), the unconditional expected return of a stock is less than $r+\bar{x} \sigma_{D}^{M}$, thus, in this case, the time-varying market price of risk does not help us with the equity premium puzzle. In the cross section, the expected return is decreasing in $g$. The volatility decreases in $g$ at first and eventually increases in $g$.

To show the above, first consider an instrument that pays a liquidating dividend $D_{T}$ at time T . With abuse of notation, let $D_{t}=E\left[D_{T} \mid F_{t}\right]$ denote the expectation of $D_{T}$ at time $t$. By the law of iterated expectations, $D_{t}$ is a Martingale. Suppose the expectation evolves according to $\frac{d D_{t}}{D_{t}}=\sigma_{D}^{M} d B_{t}^{M}+\sigma_{D}^{z} d B_{t}^{z}$. The price of this instrument $S_{t}$ has to satisfy the following condition:

$$
\begin{equation*}
E\left[\frac{d S_{t}}{S_{t}}\right]-r d t=-E_{t}\left[\frac{d S_{t}}{S_{t}} \frac{d M_{t}}{M_{t}}\right] \tag{50}
\end{equation*}
$$

by definition of the pricing kernel. I posit that $S$ is a function of $D_{t}, x_{t}$ and $N=T-t$, time to expiration. Then $S$ has to satisfy the following Partial Differential Equation:

$$
\begin{gather*}
\frac{\partial S}{\partial x} \phi_{x}\left(\bar{x}-x_{t}\right)+\frac{1}{2} \frac{\partial^{2} S}{\partial x^{2}} \sigma_{x}^{2}+\frac{\partial^{2} S}{\partial x \partial D} D_{t} \sigma_{x} \sigma_{D}^{M}+\frac{1}{2} \frac{\partial^{2} S}{\partial D^{2}} D_{t}^{2}\left[\left(\sigma_{D}^{M}\right)^{2}+\left(\sigma_{D}^{z}\right)^{2}\right]-\frac{\partial S}{\partial N}-r S \\
=\frac{\partial S}{\partial x} \sigma_{x} x_{t}+\frac{\partial S}{\partial D} \sigma_{D}^{M} x_{t} D_{t} \tag{51}
\end{gather*}
$$

with the boundary condition $S(D, x, 0)=D$. It can be verified that the solution to the above equation is $S_{t}=D_{t} e^{A(N)+B(N) x_{t}}$ where $A(N)$ and $B(N)$ are specified as in the text.

Now suppose the expectation evolves according to $d D_{t} / D_{t}=g d t+\sigma_{D}^{M} d B_{t}^{M}+\sigma_{D}^{z} d B_{t}^{z}$ (so it is not a rational expectation). Redo the PDE ; it is easy to find that the price of this instrument is now $S_{t}=$ $D_{t} e^{g N+A(N)+B(N) x_{t}}$. The price of the stock, which has a real dividend that evolves according to: $\frac{d D_{t}}{D_{t}}=$ $g d t+\sigma_{D}^{M} d B_{t}^{M}+\sigma_{D}^{z} d B_{t}^{z}$, is the sum of all $S$, i.e. $P_{t}=D_{t} \int_{0}^{\infty} e^{g N+A(N)+B(N) x_{t}} d N$.

The instantaneous return $\frac{d P_{t}+D_{t} d t}{P_{t}}$ can be derived with a straightforward application of Ito's Lemma. The expected excess return is $x_{t}\left(\sigma_{D}^{M}+\frac{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} B(N) \sigma_{x} d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N}\right)$. Since $\mathrm{B}(\mathrm{N})$ is negative (immediate from the functional form of $\mathrm{B}(\mathrm{N}))$, and $x_{t}, \sigma_{D}^{M}, \exp \left(g N+A(N)+B(N) x_{t}\right)$ are assumed to be positive, the necessary and sufficient condition for the expected return to be greater than $x_{t} \sigma_{D}^{M}$ is that $\sigma_{x}<0$. Noting that the unconditional expectation of $x_{t}$ is $\bar{x}$, we show that the necessary and sufficient condition for the unconditional expected return to be greater than $\bar{x} \sigma_{D}^{M}$ is $\sigma_{x}<0$.

To prove the expected return increases in the expected growth rate, it suffices to show that $f=$ $\frac{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} B(N) \sigma_{x} d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N}$ is an increasing function of $g$ when $\sigma_{x}<0$. For ease of exposition, define $Z=e^{g N+A(N)+B(N) x_{t}}$. Therefore,

$$
\begin{align*}
\frac{\partial f}{\partial N} & =\frac{\int_{0}^{+\infty} e^{Z} N B(N) \sigma_{x} d N \int_{0}^{+\infty} e^{Z} d N-\int_{0}^{+\infty} e^{Z} B(N) \sigma_{x} d N \int_{0}^{+\infty} e^{Z} N d N}{\left(\int_{0}^{+\infty} e^{Z} d N\right)^{2}} \\
& =E^{g}\left[N B(N) \sigma_{x}\right]-E^{g}\left[B(N) \sigma_{x}\right] E^{g}[N] \\
& =\operatorname{Cov}^{g}\left[N, B(N) \sigma_{x}\right], \tag{52}
\end{align*}
$$

where $E^{g}$ is defined on the probability density over $N: \frac{e^{Z}}{\int_{0}^{+\infty} e^{Z} d N}$. Because $B(N)$ is monotonically decreasing in $N, \operatorname{Cov}^{g}\left[N, B(N) \sigma_{x}\right]>0$, iff $\sigma_{x}<0$.

To prove that

$$
\begin{equation*}
\lim _{N \rightarrow 0} x_{t}\left(\sigma_{D}^{M}+B(N) \sigma_{x}\right)=x_{t} \sigma_{D}^{M} \tag{53}
\end{equation*}
$$

plug in the functional form of $\mathrm{B}(\mathrm{N})$, and it follows immediately.
For Model 2, consider the following instrument that pays a liquidating dividend $Y_{T}$. The expectation of $Y_{T}$ at time is denoted as $Y_{t}$. Further assume that the cash-flow expectation follows: $\frac{d Y_{t}}{Y_{t}}=x_{t} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}$, where $x_{t}$ follows the same process as above. Then the expected instantaneous return of this instrument is $\sigma_{M}\left(x_{t}+B(N) \sigma_{x}\right)$, where $N=T-t$. Furthermore,

$$
\begin{equation*}
\lim _{N \rightarrow 0} \sigma_{M}\left(x_{t}+B(N) \sigma_{x}\right)=\sigma_{M} x_{t} \tag{54}
\end{equation*}
$$

the expected return that would be in a model with constant cash-flow risk that equals to $x_{t}$ and constant market price of risk $\sigma_{M}$.

For the stock that pays a continuous stream of dividend according to: $\frac{d D_{t}}{D_{t}}=g d t+x_{t} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}$, the price of the stock is

$$
\begin{equation*}
P_{t}=D_{t} \int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N \tag{55}
\end{equation*}
$$

where

$$
\begin{gather*}
B(N)=\frac{\sigma_{M}\left(1-e^{\left(\sigma_{x}-\phi_{x}\right) N}\right)}{\sigma_{x}-\phi_{x}}  \tag{56}\\
A(N)=A_{1} B(N)^{2}+A_{2}\left[\sigma_{M} N+B(N)\right]-r N  \tag{57}\\
A_{1}=\frac{\sigma_{x}^{2}}{4\left(\sigma_{x}-\phi_{x}\right)} \tag{58}
\end{gather*}
$$

and

$$
\begin{equation*}
A_{2}=\frac{\phi_{x} \bar{x}-\sigma_{M} \sigma_{x}}{\sigma_{x}-\phi_{x}}+\frac{\sigma_{M} \sigma_{x}^{2}}{2\left(\sigma_{x}-\phi_{x}\right)^{2}} \tag{59}
\end{equation*}
$$

The instantaneous return of the stock is

$$
\begin{align*}
\frac{d P_{t}+D_{t} d t}{P_{t}}= & \left(r+\sigma_{M}\left(x_{t}+\frac{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} B(N) \sigma_{x} d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N}\right)\right) d t \\
& +\left(x_{t}+\frac{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} B(N) \sigma_{x} d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N}\right) d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z} \tag{60}
\end{align*}
$$

For Model 3, consider the following instrument that pays a liquidating dividend $Y_{T}$. The expectation of $Y_{T}$ at time is denoted as $Y_{t}$. Further assume that the cash-flow expectation follows information structure that is $\frac{d Y_{t}}{Y_{t}}=\sigma_{D}^{M} \sqrt{x_{t}} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}$, where $x_{t}$ follows the same process as above. Then the expected instantaneous return of this instrument is $\sqrt{x_{t}}\left(\sigma_{D}^{M} \sqrt{x_{t}}+B(N) \sigma_{x}\right)$, where $N=T-t$. See below for the functional form of $\mathrm{B}(\mathrm{N})$. Furthermore,

$$
\begin{equation*}
\lim _{N \rightarrow 0} \sqrt{x_{t}}\left(\sigma_{D}^{M} \sqrt{x_{t}}+B(N) \sigma_{x}\right)=\sigma_{D}^{M} x_{t} \tag{61}
\end{equation*}
$$

the expected return that would be in a model with constant cash-flow risk that equals to $\sigma_{D}^{M} \sqrt{x_{t}}$ and constant market price of risk $\sqrt{x_{t}}$.

Now for the stock that pays a continuous stream of dividend according to $\frac{d D_{t}}{D_{t}}=g d t+\sigma_{D}^{M} \sqrt{x_{t}} d B_{t}^{M}+$ $\sigma_{D}^{Z} d B_{t}^{Z}$, the price of the stock is then

$$
\begin{equation*}
P_{t}=D_{t} \int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N \tag{62}
\end{equation*}
$$

where

$$
\begin{gather*}
B(N)=-\frac{2 \sigma_{D}^{M}\left(e^{\delta N}-1\right)}{\left(\delta+\sigma_{D}^{M} \sigma_{x}-\phi_{x}-\sigma_{x}\right)\left(e^{\delta N}-1\right)+2 \delta}  \tag{63}\\
A(N)=\frac{\phi_{x} \sigma_{D}^{M} \bar{x}}{\sigma_{x}^{2}}\left(2 \log \left(\frac{2 \delta}{\left(\sigma_{D}^{M} \sigma_{x}-\phi_{x}-\sigma_{x}+\delta\right)\left(e^{\delta N}-1\right)+2 \delta}\right)+\left(\sigma_{D}^{M} \sigma_{x}-\phi_{x}-\sigma_{x}+\delta\right) N\right)-r N \tag{64}
\end{gather*}
$$

and

$$
\begin{equation*}
\delta=\sqrt{\left(\sigma_{D}^{M} \sigma_{x}-\phi_{x}-\sigma_{x}\right)^{2}+2 \sigma_{x}^{2}} \tag{65}
\end{equation*}
$$

The instantaneous return of the stock is

$$
\begin{align*}
& \frac{d P_{t}+D_{t} d t}{P_{t}}=\left(r+\sqrt{x_{t}}\left(\sigma_{D}^{M} \sqrt{x_{t}}+\frac{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} B(N) \sigma_{x} d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N}\right)\right) d t  \tag{66}\\
& \quad+\left(\sigma_{D}^{M} \sqrt{x_{t}}+\frac{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} B(N) \sigma_{x} d N}{\int_{0}^{+\infty} e^{g N+A(N)+B(N) x_{t}} d N}\right) d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z} \tag{67}
\end{align*}
$$

To see Proposition 4, note that $P_{t}=D_{t} e^{A(N)+B(N) x_{t}+C(N) g_{t}}$ implies that $\frac{\partial P_{t}}{\partial D_{t}}=\frac{P_{t}}{D_{t}}, \frac{\partial P_{t}}{\partial x_{t}}=P_{t} B(N)$, $\frac{\partial P_{t}}{\partial g_{t}}=P_{t} C(N), \frac{\partial P_{t}}{\partial N}=P_{t}\left(A^{\prime}(N)+B^{\prime}(N) x_{t}+C^{\prime}(N) g_{t}\right), \frac{\partial^{2} P_{t}}{\partial D_{t}^{2}}=0, \frac{\partial^{2} P_{t}}{\partial x_{t}^{2}}=P_{t} B^{2}(N), \frac{\partial^{2} P_{t}}{\partial g_{t}^{2}}=P_{t} C^{2}(N)$, $\frac{\partial^{2} P_{t}}{\partial D_{t} \partial x_{t}}=\frac{P_{t}}{D_{t}} B(N), \frac{\partial^{2} P_{t}}{\partial D_{t} \partial g_{t}}=\frac{P_{t}}{D_{t}} C(N), \frac{\partial^{2} P_{t}}{\partial x_{t} \partial g_{t}}=P_{t} B(N) C(N)$. The pricing kernel implies that,

$$
\begin{equation*}
E\left[\frac{d P_{t}}{P_{t}}\right]-r d t=-E_{t}\left[\frac{d P_{t}}{P_{t}} \frac{d M_{t}}{M_{t}}\right] \tag{68}
\end{equation*}
$$

Using Ito's lemma, I can derive that, $g_{t}-\left(A^{\prime}(N)+B^{\prime}(N) x_{t}+C^{\prime}(N) g_{t}\right)+B(N) \phi_{x}\left(\bar{x}-x_{t}\right)+C(N) \phi_{g}(\bar{g}-$ $\left.g_{t}\right)+\frac{1}{2} B^{2}(N) \sigma_{x}^{2}+\frac{1}{2} C^{2}(N) \sigma_{g}^{2}+B(N) \sigma_{D} \sigma_{x} \rho_{D x}+C(N) \sigma_{D} \sigma_{g} \rho_{D g}+B(N) C(N) \sigma_{x} \sigma_{g} \rho_{x g}=r+x_{t}\left(\sigma_{D} \rho_{D M}+\right.$ $\left.B(N) \sigma_{x} \rho_{x M}+C(N) \sigma_{g} \rho_{g M}\right)$.

This implies a set of PDEs. $1-C^{\prime}(N)-C(N) \phi_{g}=0,-B^{\prime}(N)-B(N) \phi_{x}=\sigma_{D} \rho_{D M}+B(N) \sigma_{x} \rho_{x M}+$ $C(N) \sigma_{g} \rho_{g M}$, and $-A^{\prime}(N)+B(N) \phi_{x} \bar{x}+C(N) \phi_{g} \bar{g}+\frac{1}{2} B^{2}(N) \sigma_{x}^{2}+\frac{1}{2} C^{2}(N) \sigma_{g}^{2}+B(N) \sigma_{D} \sigma_{x} \rho_{D x}+C(N) \sigma_{D} \sigma_{g} \rho_{D g}+$ $B(N) C(N) \sigma_{x} \sigma_{g} \rho_{x g}=r$. The initial conditions are $A(0)=B(0)=C(0)=0$.

The solutions are: $C(N)=\frac{1-e^{-\phi_{g} N}}{\phi_{g}}, B(N)=d_{1}+d_{2} e^{-\phi_{g} N}+d_{3} e^{-\phi_{x}^{*} N}$, where $\phi_{x}^{*}=\phi_{x}+\rho_{x M} \sigma_{x}, d_{1}=$ $-\frac{\rho_{g M} \sigma_{g}+\phi_{g} \rho_{D M} \sigma_{D}}{\phi_{g} \phi_{x}^{*}}, d_{2}=\frac{\rho_{g M} \sigma_{g}}{\phi_{g}\left(\phi_{x}^{*}-\phi_{g}\right)}$, and $d_{3}=\frac{-\rho_{g M} \sigma_{g}+\rho_{D M} \sigma_{D}\left(\phi_{x}^{*}-\phi_{g}\right)}{\phi_{x}^{*}\left(\phi_{x}^{*}-\phi_{g}\right)}$. Also, $A(N)=a_{0}+a_{1} N+a_{2} e^{-\phi_{g} N}+$ $a_{3} e^{-\phi_{x}^{*} N}+a_{4} e^{-2 \phi_{g} N}+a_{5} e^{-\left(\phi_{g}+\phi_{x}^{*}\right) N}+a_{6} e^{-2 \phi_{x}^{*} N}$, where $a_{1}=-r+\bar{g}+\frac{\sigma_{g}^{2}}{2 \phi_{g}^{2}}+\frac{\sigma_{D} \sigma_{g} \rho_{D g}}{\phi_{g}}+d_{1}\left(\phi_{x} \bar{x}+\sigma_{D} \sigma_{x} \rho_{D x}+\right.$ $\left.\frac{\sigma_{x} \sigma_{g} \rho_{x g}}{\phi_{g}}\right)+\frac{1}{2} d_{1}^{2} \sigma_{x}^{2}, a_{2}=\frac{1}{\phi_{g}}\left(\bar{g}-d_{2} \phi_{x} \bar{x}-d_{1} d_{2} \sigma_{x}^{2}-d_{2} \sigma_{D} \sigma_{x} \rho_{D x}\right)+\frac{1}{\phi_{g}^{2}}\left(\sigma_{D} \sigma_{g} \rho_{D g}+\left(d_{1}-d_{2}\right) \sigma_{x} \sigma_{g} \rho_{x g}\right)+\frac{1}{\phi_{g}^{3}} \sigma_{g}^{2}$, $a_{3}=-\frac{d_{3}}{\phi_{x}^{*}}\left(\phi_{x} \bar{x}+d_{1} \sigma_{x}^{2}+\sigma_{D} \sigma_{x} \rho_{D x}+\frac{\sigma_{x} \sigma_{g} \rho_{x g}}{\phi_{g}}\right), a_{4}=\frac{1}{4 \phi_{g}}\left(\frac{2 d_{2} \sigma_{x} \sigma_{g} \rho_{x g}}{\phi_{g}}-d_{2}^{2} \sigma_{x}^{2}-\frac{\sigma_{g}^{2}}{\phi_{g}^{2}}\right), a_{5}=\frac{d_{3}}{\phi_{g}+\phi_{x}^{*}}\left(\frac{\sigma_{x} \sigma_{g} \rho_{x g}}{\phi_{g}}-d_{2} \sigma_{x}^{2}\right)$, $a_{6}=-\frac{d_{3}^{2} \sigma_{x}^{2}}{4 \phi_{x}^{*}}$, and $a_{0}=-\left(a_{2}+a_{3}+a_{4}+a_{5}+a_{6}\right)$. It is useful to note that $d_{1}+d_{2}+d_{3}=0$. To prove the proposition, note that when $N=0$, the expected return is $\mu_{R}=x_{t} \sigma_{D} \rho_{D M}$. When $N=+\infty$, $\mu_{R}=x_{t}\left(\sigma_{D} \rho_{D M}-\frac{\rho_{g M} \sigma_{g}+\phi_{g} \rho_{D M} \sigma_{D}}{\phi_{g} \phi_{x}^{*}} \sigma_{x} \rho_{x M}+\frac{\sigma_{g} \rho_{g M}}{\phi_{g}}\right)$. The regularity conditions include $\phi_{x}^{*}>0, \phi_{g}>0$, $\sigma_{D}>0, \sigma_{x}>0, \sigma_{g}>0$, and $\rho_{D M}>0$.

### 7.3 Long horizon growth rates

In the main text, I have examined 1-year growth rates. I now examine long horizon growth rates. To provide a benchmark for horizons, I first examine the profitability of the value strategy across different horizons. Specifically, in June of each year $t$, I use available information to construct the book-to-market ratio. I then examine the profits of the value strategy from July of year $t$ to June of year $t+1$ (year 1),
and from July of year $t+1$ to June of year $t+2$ (year 2), up to year 20. To measure the profitability, I use both the portfolio approach (constructing High minus Low portfolio) and the regression approach (Fama-MacBeth regression of returns on $\log$ book-to-market equity). I also use both equal-weighted and value-weighted constructions.

The results are provided in Table A1. HML is the average monthly return of the value portfolio (highest $30 \%$ book-to-market equity) minus that of the growth portfolio (lowest $30 \%$ book-to-market equity). Both equal-weighted and value-weighted HMLs decline over the holding horizon. I also report the Fama-MacBeth slope coefficient. In the column $\mathrm{FM}(\mathrm{vw})$, the first stage regression is estimated using the weighted least squares in which the weights are the lagged market capitalizations. The Fama-MacBeth slope coefficients also decline over the holding horizon. Based on this table, I determine that the value strategy is profitable in 8 years. Within 8 years, the profitability of the equal-weighted strategy is always positive and statistically significant. The profitability of the equal-weighted strategy is always positive and mostly statistically significant. Beyond year 9 , the profitability of the trading strategy is never statistically significant and often negative.

With this finding, I now examine the cumulative growth rate from year 0 to year 1 (short term, look-back growth rate), year 1 to year 8 (medium term), and also from year 8 to year 20 (long term). The results are reported in Table A2. In each panel, I first report the dollar amounts of cash flows corresponding to a $\$ 100$ investment in each book-to-market sorted portfolio and then the cumulative growth rates. Panel A, B, and C report results on earnings, accounting cash flow, and dividends (including repurchases), respectively.

For all three variables, the short-term look-back growth rate decreases in the book-to-market ratio. The long-term growth rate also largely decreases in the book-to-market ratio (although for earnings, and accounting cash flow, this relation is not monotonic.

The growth rate from year 1 to year 8 exhibits a U-shaped relation against book-to-market equity, with the extreme value portfolio growing faster than the extreme growth portfolio. For earnings, the value portfolio ( $187 \%$ ) grows faster than the growth portfolio ( $55 \%$ ), which in turn grows faster than the neutral portfolio ( $50 \%$ ). For accounting cash flow, the extreme value portfolio (Decile 10, 167\%) grows faster than the extreme growth portfolio ( $83 \%$ ), which in turn grows faster than Decile $7(50 \%)$. For dividends, the extreme value portfolio (Decile 10, 123\%) grows faster than the extreme growth portfolio ( $110 \%$ ), which in turn grows faster than Decile $7(51 \%) .{ }^{28}$

[^23]When I examine the growth rates from year 1 to year 20, there is still a U-shaped relation against the book-to-market equity for all three variables. However, whether the extreme value portfolio grows faster than the extreme growth portfolio depends on the variable. For earnings and accounting cash flow, the extreme value portfolio ( $534 \%$ and $557 \%$ ) grows faster than the extreme growth portfolio ( $308 \%$ and $423 \%$ ). For dividends, the extreme value portfolio ( $291 \%$ ) grows slower than the extreme growth portfolio ( $594 \%$ ).

For completeness, I also report the growth rates from year 0 to year 8 , and from year 0 to year 20 . Using year 0 as the basis, as opposed to year 1, generally increases the growth rates for growth stocks and decreases the growth rates for value stocks. The growth rates from year 0 to year 8 typically exhibit a U-shaped relation against the book-to-market, with the extreme value portfolio growing faster (slower) for earnings (accounting cash flow and dividends) than the extreme growth portfolio. For earnings and accounting cash, the growth rates from year 0 to year 20 exhibit a U-shaped relation against the book-tomarket, with the extreme value portfolio growing slower for earnings than the extreme growth portfolio. For earnings, however, the differential is very small ( $352 \%$ for the value portfolio vs. $361 \%$ for the growth portfolio). For dividends, the growth rate from year 0 to year 20 almost monotonically declines with the book-to-market equity.

Fig. A2 plots the entire cash-flow paths. I plot $E\left(F_{s}\right)$ for each fundamental variable: earnings, accounting cash flow, dividends, and book equity, for the growth, neutral, and value portfolios 30 years out. For ease of comparison, the fundamental variable is normalized to be $\$ 1$ in year 1 for each portfolio. For earnings, the growth, neutral, and value portfolios consist of stocks with book-to-market equity that are in the lowest $30 \%$, middle $40 \%$, and highest $30 \%$. For the other three variables, the growth, neutral, and value portfolios consist of stocks with book-to-market equity that are in the lowest $10 \%, 6$ th decile, and highest $10 \%$. The first three cash-flow variables show that both growth portfolios and value portfolios tend to outgrow the neutral portfolios from year 1 to year 30 (a U-shape relation, if one were to plot the growth rate against the book-to-market ratio). For earnings and accounting cash flow, the value portfolio grows faster than the growth portfolio, while the opposite is true for dividends. Even for dividends, the difference between the growth portfolio and the value portfolio is small, amounting to about $1.7 \%$ a year in growth rate. Furthermore, valuation models actually suggest that beyond year 30 , the value stocks are expected to grow faster than growth stocks. For book equity, there is a monotonic negative relation between the book-to-market ratio and the book-equity growth rate.


Fig. A1: Plot of the expected excess return (blue solid line), the expected capital gain (green dashed line), and the dividend yield (red dash-dot line) against the expected dividend growth rate $g$ under Model 1. In Model 1, the pricing kernel $M_{t}$ follows $\frac{d M_{t}}{M_{t}}=-r d t-x_{t} d B_{t}^{M}$, where $x_{t}$, the market price of risk, is mean reverting and follows $\frac{d x_{t}}{x_{t}}=\phi_{x}\left(\bar{x}-x_{t}\right) d t-\sigma_{x} d B_{t}^{M}$. The stock's dividend process is assumed to follow $\frac{d D_{t}}{D_{t}}=g d t+$ $\sigma_{D}^{M} d B_{t}^{M}+\sigma_{D}^{Z} d B_{t}^{Z}$, where $B_{t}^{Z}$ is orthogonal to $B_{t}^{M}$. In Panel A, the market price of risk is countercyclical and there is a positive growth premium. The parameter values used here are: $x_{t}=0.3, \sigma_{D}^{M}=0.005, \phi_{x}=5.05, \sigma_{x}=-5, \bar{x}=0.3, r=0.10$. In Panel B, the market price of risk is procyclical and there is a negative growth premium. The parameter values used here are: $x_{t}=0.6, \sigma_{D}^{M}=0.2, \phi_{x}=0.01, \sigma_{x}=0.2, \bar{x}=0.6, r=0.10$.


Fig. A2: Paths of fundamental variables for buy-and-hold portfolios. This figure plots the paths of average fundamentals for an investment that generates $\$ 1$ in each fundamental variable in each portfolio in year 1. In each year $t$ between 1963 and 2009, I sort stocks according to their book-to-market ratios. For Panel A, the growth (blue solid line), neutral (green dashed line), and value (red dash-dot line) portfolios consist of stocks with book-to-market equity that are in the lowest $30 \%$, middle $40 \%$, and highest $30 \%$. For Panels B through D, the growth (blue solid line), neutral (green dashed line), and value (red dash-dot line) portfolios consist of stocks with book-to-market equity that are in the lowest $10 \%, 6$ th decile, and highest $10 \%$. The breakpoints are computed using NYSE stocks only. Earnings, accounting cash flow, and book equity in year $t+s$ refer to those variables for the fiscal year that ends between July of year $t+s-1$ and June of year $t+s$. Dividends in year $t+s$ refer to the total dividends (including repurchases) accumulated between July of year $t+s-1$ and June of year $t+s$. Survivorship bias is adjusted according to the text. Panels A, B, C, and D plot the expected values of earnings, accounting cash flow, dividends, and book equity. All variables are on per share bases.

Table A1: The profitability of the value strategy across different horizons. The growth, neutral, and value portfolios consist of stocks with book-to-market equity that are in the lowest $30 \%$, middle $40 \%$, and highest $30 \%$. The breakpoints are computed using NYSE stocks only. Monthly returns are used. HML in year $s$ refers to the difference between the monthly returns of the value and the growth portfolios from July of year $t+s-1$ to June of year $t+s$, averaged across portfolio formation years $t$. Ew and vw refer to equal weighted and value weighted, respectively. FM refers to the average slope coefficient $b_{1}$ in the following Fama-MacBeth regressions: ret $i_{i, t}=$ $b_{0}+b_{1} \log (B / M)_{i, t-s}+\epsilon_{i, t}$. In the column $\mathrm{FM}(\mathrm{vw})$, the first stage regression is estimated using the weighted least squares in which the weights are the lagged market capitalizations. For the Fama-MacBeth regressions, the book-to-market ratio is winsorized at $1 \%$ and $99 \%$ in each year.

| year | HML(ew) | $t$-stat | HML(vw) | $t$-stat | FM(ew) | $t$-stat | FM(vw) | $t$-stat | formation years |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $0.79 \%$ | $(6.21)$ | $0.33 \%$ | $(2.39)$ | 0.0041 | $(6.50)$ | 0.0012 | $(1.61)$ | $1963-2009$ |
| 2 | $0.70 \%$ | $(6.03)$ | $0.39 \%$ | $(3.10)$ | 0.0039 | $(6.74)$ | 0.0018 | $(2.46)$ | $1963-2008$ |
| 3 | $0.53 \%$ | $(5.03)$ | $0.26 \%$ | $(2.09)$ | 0.0030 | $(5.38)$ | 0.0014 | $(1.99)$ | $1963-2007$ |
| 4 | $0.42 \%$ | $(4.13)$ | $0.20 \%$ | $(1.64)$ | 0.0023 | $(4.24)$ | 0.0011 | $(1.42)$ | $1963-2006$ |
| 5 | $0.34 \%$ | $(3.43)$ | $0.13 \%$ | $(1.04)$ | 0.0018 | $(3.33)$ | 0.0007 | $(0.93)$ | $1963-2005$ |
| 6 | $0.25 \%$ | $(2.67)$ | $0.25 \%$ | $(1.84)$ | 0.0014 | $(2.73)$ | 0.0014 | $(1.82)$ | $1963-2004$ |
| 7 | $0.19 \%$ | $(2.16)$ | $0.14 \%$ | $(1.01)$ | 0.0011 | $(2.27)$ | 0.0009 | $(1.22)$ | $1963-2003$ |
| 8 | $0.19 \%$ | $(2.20)$ | $0.26 \%$ | $(1.82)$ | 0.0010 | $(2.08)$ | 0.0014 | $(1.88)$ | $1963-2002$ |
| 9 | $0.10 \%$ | $(1.20)$ | $0.15 \%$ | $(1.05)$ | 0.0007 | $(1.45)$ | 0.0012 | $(1.46)$ | $1963-2001$ |
| 10 | $0.05 \%$ | $(0.70)$ | $0.19 \%$ | $(1.31)$ | 0.0004 | $(0.89)$ | 0.0010 | $(1.17)$ | $1963-2000$ |
| 11 | $-0.02 \%$ | $(-0.24)$ | $0.12 \%$ | $(0.78)$ | 0.0002 | $(0.36)$ | 0.0010 | $(1.13)$ | $1963-1999$ |
| 12 | $-0.05 \%$ | $(-0.59)$ | $0.13 \%$ | $(0.88)$ | -0.0003 | $(-0.72)$ | 0.0005 | $(0.58)$ | $1963-1998$ |
| 13 | $-0.03 \%$ | $(-0.39)$ | $0.05 \%$ | $(0.36)$ | -0.0002 | $(-0.40)$ | 0.0003 | $(0.42)$ | $1963-1997$ |
| 14 | $-0.08 \%$ | $(-1.02)$ | $0.00 \%$ | $(0.01)$ | -0.0005 | $(-0.97)$ | 0.0003 | $(0.30)$ | $1963-1996$ |
| 15 | $-0.14 \%$ | $(-1.61)$ | $-0.11 \%$ | $(-0.74)$ | -0.0006 | $(-1.10)$ | -0.0001 | $(-0.09)$ | $1963-1995$ |
| 16 | $-0.11 \%$ | $(-1.07)$ | $-0.02 \%$ | $(-0.12)$ | -0.0006 | $(-0.99)$ | -0.0001 | $(-0.09)$ | $1963-1994$ |
| 17 | $-0.11 \%$ | $(-0.99)$ | $-0.13 \%$ | $(-0.82)$ | -0.0006 | $(-0.98)$ | -0.0002 | $(-0.23)$ | $1963-1993$ |
| 18 | $-0.04 \%$ | $(-0.40)$ | $-0.11 \%$ | $(-0.67)$ | -0.0002 | $(-0.33)$ | -0.0006 | $(-0.60)$ | $1963-1992$ |
| 19 | $-0.06 \%$ | $(-0.58)$ | $-0.02 \%$ | $(-0.13)$ | -0.0001 | $(-0.23)$ | -0.0004 | $(-0.47)$ | $1963-1991$ |
| 20 | $0.00 \%$ | $(0.02)$ | $0.03 \%$ | $(0.20)$ | 0.0001 | $(0.18)$ | -0.0004 | $(-0.37)$ | $1963-1990$ |

Table A2: Long horizon growth rates for cash flows. In each panel, I first report the dollar amounts of cash flows corresponding to a $\$ 100$ investment in each book-to-market sorted portfolio, and then report the cumulative growth rates. Panel A, B, and C report results on earnings, accounting cash flow, and dividends (including repurchases), respectively. Book-to-market breakpoints are based on stocks on NYSE only. Survivorship bias is adjusted according to the text. Years refer to the year relative to portfolio formation. In Panels A and B, the cumulative growth rates from year $s 1$ to year $s 2$ are computed as $E\left(F_{s 2}\right) / E\left(F_{s 1}\right)-1$, in which $E()$ is averaging across portfolio years, and $F$ refers to earnings or accounting cash flow. In Panel C, the cumulative growth rates are computed as $E\left(D_{s 2} / D_{s 1}\right)-1$.

| Panel A: Earnings |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | Growth | Neutral | Value |  |  |  |  |  |  |  |
| 0 | 4.78 | 7.25 | 5.48 |  |  |  |  |  |  |  |
| 1 | 5.39 | 7.58 | 3.91 |  |  |  |  |  |  |  |
| 2 | 5.68 | 7.91 | 6.29 |  |  |  |  |  |  |  |
| 8 | 8.37 | 11.34 | 11.22 |  |  |  |  |  |  |  |
| 20 | 22.03 | 21.92 | 24.80 |  |  |  |  |  |  |  |
| Cumulative growth |  |  |  |  |  |  |  |  |  |  |
| Year 1/Year 0 | 13\% | 5\% | -29\% |  |  |  |  |  |  |  |
| Year 8/Year 1 | 55\% | 50\% | 187\% |  |  |  |  |  |  |  |
| Year 20/Year 8 | 163\% | 93\% | 121\% |  |  |  |  |  |  |  |
| Year 20/Year 1 | 308\% | 189\% | $534 \%$ |  |  |  |  |  |  |  |
| Year 8/Year 0 | 75\% | $56 \%$ | 105\% |  |  |  |  |  |  |  |
| Year 20/Year 0 | $361 \%$ | 203\% | $352 \%$ |  |  |  |  |  |  |  |
| Panel B: Accounting cash flow |  |  |  |  |  |  |  |  |  |  |
| Year | Growth | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Value |
| 0 | 5.56 | 8.38 | 10.50 | 12.08 | 13.61 | 15.72 | 16.80 | 18.00 | 19.17 | 19.24 |
| 1 | 6.47 | 9.39 | 11.47 | 13.17 | 14.61 | 16.31 | 17.45 | 18.45 | 18.40 | 13.10 |
| 2 | 7.01 | 10.07 | 12.42 | 13.91 | 15.50 | 17.21 | 18.22 | 20.22 | 20.47 | 19.50 |
| 8 | 11.81 | 15.58 | 19.83 | 21.81 | 22.87 | 25.41 | 26.17 | 28.23 | 31.07 | 35.05 |
| 20 | 33.84 | 39.28 | 48.23 | 47.48 | 45.08 | 51.12 | 57.88 | 59.20 | 68.74 | 86.10 |
| Cumulative growth |  |  |  |  |  |  |  |  |  |  |
| Year 1/Year 0 | 16\% | 12\% | 9\% | 9\% | 7\% | 4\% | 4\% | 3\% | -4\% | -32\% |
| Year 8/Year 1 | 83\% | 66\% | 73\% | 66\% | 57\% | 56\% | 50\% | 53\% | 69\% | 167\% |
| Year 20/Year 8 | 187\% | 152\% | 143\% | 118\% | 97\% | 101\% | 121\% | 110\% | 121\% | 146\% |
| Year 20/Year 1 | 423\% | 318\% | $321 \%$ | 261\% | 209\% | 213\% | 232\% | 221\% | 274\% | 557\% |
| Year 8/Year 0 | 113\% | 86\% | 89\% | 81\% | 68\% | 62\% | $56 \%$ | 57\% | 62\% | 82\% |
| Year 20/Year 0 | 509\% | 369\% | $359 \%$ | 293\% | 231\% | 225\% | 245\% | 229\% | 259\% | $347 \%$ |
| Panel C: Dividends |  |  |  |  |  |  |  |  |  |  |
| Year | Growth | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Value |
| 0 | 2.34 | 2.94 | 3.68 | 3.94 | 4.13 | 4.68 | 4.98 | 5.12 | 5.22 | 6.03 |
| 1 | 2.62 | 3.49 | 3.92 | 4.28 | 4.76 | 5.02 | 5.25 | 5.20 | 5.30 | 4.80 |
| 2 | 2.80 | 3.72 | 4.14 | 4.84 | 4.98 | 5.32 | 5.57 | 5.76 | 5.54 | 5.14 |
| 8 | 5.00 | 6.26 | 7.22 | 7.15 | 8.02 | 8.50 | 8.13 | 8.20 | 8.25 | 10.36 |
| 20 | 16.47 | 18.24 | 22.14 | 18.22 | 18.70 | 18.71 | 21.83 | 19.53 | 19.67 | 20.55 |
| Cumulative growth |  |  |  |  |  |  |  |  |  |  |
| Year 1/Year 0 | 14\% | 20\% | 10\% | 10\% | 18\% | 10\% | 8\% | 3\% | 5\% | -13\% |
| Year 8/Year 1 | 110\% | 91\% | 90\% | 68\% | 65\% | 63\% | 51\% | 56\% | 55\% | 123\% |
| Year 20/Year 8 | $234 \%$ | 201\% | 211\% | 166\% | 153\% | 129\% | 188\% | 141\% | 138\% | 132\% |
| Year 20/Year 1 | 594\% | 468\% | 461\% | 302\% | 282\% | 255\% | $286 \%$ | 235\% | 239\% | 291\% |
| Year 8/Year 0 | 131\% | 121\% | 108\% | 83\% | 91\% | 75\% | 58\% | 56\% | 54\% | 83\% |
| Year 20/Year 0 | 688\% | 567\% | 525\% | $350 \%$ | $344 \%$ | 285\% | $328 \%$ | 260\% | 259\% | $259 \%$ |


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[^1]:    ${ }^{1}$ To be precise, cycles are defined by the pricing kernel. The procyclical expected growth rate is closely related to the long-run risk.
    ${ }^{2}$ A number of authors, including myself in the past, have expressed views in line with the conventional wisdom. Cornell (1999) speculates growth stocks have longer durations. Dechow, Sloan, and Soliman (2004) and Da (2009) find, that growth stocks have longer cash-flow durations. In the theoretical literature, the most prominent example is Lettau and Wachter (2007), which also led to work in Croce, Lettau, and Ludvigson (2010) and Wang (2012). For a classic paper on the value premium, see Fama and French (1992).

[^2]:    ${ }^{3}$ Da (2009) and Santos and Veronesi (2010) also make a distinction between duration and cash-flow duration. Santos and Veronesi (2010) also distinguish between variation in the price-dividend ratio driven by cash-flow risk and the expected growth rate. I also revisit Dechow, Sloan and Soliman (2004) and Da (2009)'s cash-flow duration results. Dechow, Sloan, and Soliman (2004)'s measure is conceptually biased towards finding a longer cash-flow duration for growth stocks. Da (2009)'s measure is conceptually fine but his empirical results are driven by unrealistic assumptions about the terminal $R O E$ s.

[^3]:    ${ }^{4}$ Interestingly, in studying the time series of the aggregate stock market, most authors (see references in Cochrane (2011)) find that the dividend-price ratio does not predict the future dividend growth rate. My finding provides the cross-sectional counterpart.

[^4]:    ${ }^{5}$ Boguth, Carlson, Fisher, and Simutin (2011) argue that the results in Binsbergen, Brandt, and Koijen (2010) are partially driven by microstructure issues. Binsbergen and Koijen (2011) dispute the implication of Boguth, Carlson, Fisher, and Simutin (2011).

[^5]:    ${ }^{6}$ Grullon and Michaely (2002) find that firms have gradually substituted repurchases for dividends. Throughout the paper, unless stated otherwise, I use dividends to refer to dividends plus repurchases. This amounts to redefine $\operatorname{retx}_{t}^{*}=(1+\operatorname{retx}) \min \left(n_{t} / n_{t-1}, 1\right)-1$, following Bansal, Dittmar, and Lundblad (2005). $n_{t}$ is the number of shares after adjusting for splits, etc., using the CRSP share adjustment factor. All results are similar, if I do not consider repurchases.

[^6]:    ${ }^{7}$ When looking at return on equity, Fama and French (1995) treat earnings and book equity with fiscal year ends in calendar year $t+s$ as earnings and book equity in year $t+s$. Because most firms have December fiscal year ends, their year 0 roughly corresponds to my year 1.

[^7]:    ${ }^{8}$ The clean surplus relation holds that $B E_{t}=B E_{t-1}+E_{t}-D_{t}$. When dividends are proportional to earnings, the return on equity is proportional to book-equity growth rates.

[^8]:    ${ }^{9}$ There is a difference between the growth rate of expected earnings and the expectation of earnings growth rates, due to Jensen's inequality. However, accounting for Jensen's inequality would recover the nonsensical growth rates when the denominator is negative. Penman, Reggiani, Richardson, and Tuna (2011) computes $\frac{\Delta F_{t+2}}{\left(\left|F_{t+2}\right|+\left|F_{t+1}\right|\right) / 2}$ to deal with negative earnings in the denominator.
    ${ }^{10} \mathrm{I}$ include all stocks in computing the market capitalization of the portfolio in year $t$. But in computing the portfolio cash flows in year $t+s$ and $t+s-1$, a stock must have cash flows in both year $t+s$ and $t+s-1$.

[^9]:    ${ }^{11}$ Fama and French (2007) report the transition matrix for six size and book-to-market sorted portfolios between 1926 and 2005.

[^10]:    ${ }^{12}$ In unreported results, I do not make the per-share adjustment, that is, I simply compute $F P_{t+s}$ as the ratio of the sum of fundamental values across firms to the sum of market values in that portfolio. Adjustment on a per-share basis somewhat decreases the growth rates, but does not change the cross-sectional ranking.
    ${ }^{13}$ One could also take the delisting proceeds out as a form of dividends. The results are qualitatively the same, but the growth rates of portfolios are more volatile due to outliers.

[^11]:    ${ }^{14}$ Throughout the paper, I do not need to go through the first four steps to compute dividends. Monthly dividends (value-weighted or equal-weighted) are computed using $D_{t+s}=P_{t+s-1}\left(\operatorname{ret}_{t+s}-r e t x_{t+s}^{*}\right)$ and $P_{t+s}=$ $P_{t+s-1}\left(1+r e t x_{t+s-1}^{*}\right)$. Annual dividends are the sum of monthly dividends from July to the following June.

[^12]:    ${ }^{15}$ If I do not include repurchases in the dividends, I find that the coefficient on book-to-market is negative in years 1 and 2. But it becomes positive starting in year 2. Starting in year 5, each coefficient is statistically significant at the $10 \%$ level.

[^13]:    ${ }^{16}$ See also Cochrane (2011) for results on rebalanced value-weighted portfolios.

[^14]:    ${ }^{17}$ Results are qualitatively the same if I use the dividend yield in the beginning of the sample period.

[^15]:    ${ }^{18}$ Koijen, Lustig, and Van Nieuwerburgh also find that realized dividend growth rates are more volatile for value stocks in rebalanced portfolios. Much of my results on growth rates are also consistent with the mispricing hypothesis or the luck hypothesis (i.e., value stocks have simply been lucky and their high realized returns were not expected returns). These hypotheses, however, do not predict value stocks have riskier cash flows.

[^16]:    ${ }^{19}$ Santos and Veronesi (2010) benchmark on Cohen, Polk, and Vuolteenaho (2009)'s estimates of cash-flow risk, and find that they are not large enough to match the value premium quantitatively.

[^17]:    ${ }^{20}$ Another notion of duration relates to the sensitivity of prices to long-term discount rates. This duration is similar to the Macaulay duration and price/dividend ratios: $D u r^{\text {sensitivity }}=-\frac{\partial P / P}{\partial r}=\frac{P_{0}}{D_{0}(1+g)}$, in Gordon's model.

[^18]:    ${ }^{21}$ Results are qualitatively the same if I use the average discount rates as $\bar{r}$. However, this leads to infinite prices in rebalanced equal-weighted portfolios.
    ${ }^{22}$ In an original paper, Dechow, Sloan, and Soliman (2004) for the first time compute equity duration systematically. Their duration is not a pure cash-flow duration, despite their claim. The reason is that they use a common cost of equity of $12 \%$ for all assets, but they also use the actual market price to decide the relative weight of the terminal perpetuity. Their duration is biased towards finding a longer cash-flow duration for growth stocks with lower discount rates. To see this, recall that their duration measure is $D u r^{D S S}=$ $\frac{\sum_{t=1}^{T} t \times C F_{t} /(1+\bar{r})^{t}}{\sum_{t=1}^{T} C F_{t} /(1+\bar{r})^{t}} \times \frac{\sum_{t=1}^{T} C F_{t} /(1+\bar{r})^{t}}{P}+\left(T+\frac{1+\bar{r}}{\bar{r}}\right) \times\left(1-\frac{\sum_{t=1}^{T} C F_{t} /(1+\bar{r})^{t}}{P}\right)$. Now consider a value stock and a growth stock that have the same cash-flow profiles $\left\{C F_{t}\right\}$, but differ in discount rates. Because the growth stock has a lower discount rate, it has a higher price, $P$, than the value stock. For the growth stock, Dur ${ }^{D S S}$ would put more weight on the longer-duration terminal perpetuity, relative to the value stock. Thus, Dur ${ }^{D S S}$ would be higher for the growth stock, even with exactly the same cash-flow profiles.

[^19]:    ${ }^{23}$ In a seminal paper, Da (2009) proposes to measure a pure cash-flow based duration as this infinite sum of dividend growth rates. To compute it, he first uses a log linearization to transform this cash-flow duration into the difference between an infinite sum of $R O E$ s and the log dividend-to-book ratio. His finding that growth stocks have longer cash-flow durations is primarily driven by his assumption on the terminal ROEs. He assumes that beyond year $7, R O E$ is equal to the average $R O E$ during the first 7 years. Given Fig. 1 shows clear convergence of $R O E$ over time, this assumption is biased towards finding longer cash-flow durations for growth stocks. He reports that the difference between the cash-flow durations of the growth and value deciles is 1.34 . If we were to use actual average $R O E$ s between year 8 and year 20 , and assume that $R O E$ stays constant after year 20 , this reduces the difference by 1.10 , by my calculation. There are three other minor differences between our measures: 1). Da (2009) computes $\sum_{s=1}^{\infty} \rho^{s} g_{i s}$, while I exclude the first year look-back growth rates, which reduces the difference by a further 0.23 . 2). I use simple dividend growth rates, while Da (2009) uses log dividend growth rates. 3). Mine is adjusted for survivorship bias while his is not.
    ${ }^{24}$ For example, Lettau and Wachter (2007) assume that cash flows of growth stocks grow at $20 \%$ per year faster than those of the market, and cash flows of value stocks grow at $20 \%$ per year slower than those of the market, for the next 25 years. Then the cycle reverses and repeats itself. These assumptions imply extreme differences in cash-flow durations between growth and value stocks.

[^20]:    ${ }^{25}$ Results are qualitatively the same if I use standard deviations of monthly returns or annual returns. However, the quantitative magnitude is larger when using low frequency (annual) returns, and smaller when using high frequency (monthly) returns.

[^21]:    ${ }^{26}$ I have also been careful to use the historical growth rates as the proxy for the expected growth rate.

[^22]:    ${ }^{27}$ This model matches the facts about the equity premium, the volatility puzzle, and predictability. The advantage of this model is that we do not have to model the heteroskedasticity of the dividend in order to get closed-form solutions. However, with a countercyclical market price of risk, it cannot replicate the leverage effects. Thus, this model is not suitable for drawing inferences about the time-series behavior of stock return volatility. The other drawback of this model is that sometimes the market price of risk can be negative.

[^23]:    ${ }^{28}$ Note that in Panels A and B, the cumulative growth rates from year $s 1$ to year $s 2$ are computed as $E\left(F_{s 2}\right) / E\left(F_{s 1}\right)-1$, in which $E()$ is averaging across portfolio years, and $F$ refers to earnings or accounting cash flow. In Panel C, the cumulative growth rates are computed as $E\left(D_{s 2} / D_{s 1}\right)-1$, which is not the exactly the same as $E\left(D_{s 2}\right) / E\left(D_{s 1}\right)-1$.

