Future Temptation and Self-Control[*]

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Abstract

The literature on self-control has typically concentrated on immediate temptations. This paper studies a Gul and Pesendorfer (2001, 2004) style model in which decision-makers are affected by temptations that lie in the future. 'Future temptation' can induce decision makers who value commitment to postpone committing. This explains several observations about the market for financial commitment mechanisms, and suggests that a lack of demand for commitment may result from the existence, rather than the absence, of self-control problems. The model rationalizes preference reversals not explicable by hyperbolic discounting and has normative implications for addiction. It also serves as a model of procrastination.

Keywords: Self-Control, Commitment, Hyperbolic Discounting, Procrastination.

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1 Introduction

1.1 Motivation

The phenomenon of preference reversals is well documented in the psychology literature (see Ainslie (1992) for references). Preference reversals occur when subjects prefer, say, \$30 in two months to \$20 in one month, but prefer \$20 now to \$30 in one month. That is, they reverse preferences in favor of the smaller/earlier reward when it is close to the present. This suggests that subjects are tempted by opportunities of immediate gratification. This, in turn, suggests the existence of self-control problems, or the inability to fully resist temptations.

Models that incorporate self-control problems have been developed and studied by Strotz (1955), Ainslie (1992), Laibson (1997), O'Donoghue and Rabin (1999) and Gul and Pesendorfer (2001, 2004). An implication of these models is that decision-makers who are aware of their self-control problems seek commitment opportunities. To see this, consider a smoker who is deciding whether or not to commit to quitting. Denoting smoking by s and not smoking by n, his problem is to choose between facing the choice sets $\{n\}$ and $\{n, s\}$ tomorrow. In order to avoid the temptation of s in $\{n, s\}$, he chooses $\{n\}$. That is, anticipation of future temptation leads to a preference for commitment.

Given the presumption that self-control problems are common, the models predict a 'high' value for commitment. Laibson, Repetto and Tobacman (1998) estimate that the value of perfect financial commitment mechanisms is worth 36% of consumption at age twenty. However, such a high value for commitment is not consistent with the following observations.

Firstly, we observe compulsive behavior. If we agree that compulsive behavior is the result of self-control problems, we should observe addicts taking advantage of commitment opportunities.[1] Yet, in practice, there is a significant problem of noncompliance with treatment procedures among addicts (Ainslie (1992), Fuller and Roth (1979), Goldstein (2001), Hernnstein and Prelec (1992)). Thus agents with self-control problems do not necessarily demand commitment, in contrast to the prediction of the above models.

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[1] Examples of commitment opportunities include treatment involving disulfiram for alcoholics. Disulfiram leads to a reaction to ingestion of even small quantities of alcohol.

Secondly, the models imply that undersavers (those who do not save enough for retirement) do not need added incentives to participate in saving schemes such as 401(k) and IRAs. Yet such saving schemes have substantial tax benefits associated with them: all contributions are tax deductible. Furthermore, participation in these schemes is closely related to the tax benefits. For instance, in 1986, IRA contributions, which accounted for one-fifth of aggregate personal savings in the U.S., fell by 62% when the Tax Reform Act of 1986 excluded higher-income groups from tax benefits (Venti and Wise (1987), Poterba, Venti and Wise (2001)). The fall in participation took place although there was no change in the commitment aspect of IRAs (early withdrawal penalties). This suggests that the appeal of such saving vehicles is primarily the tax benefits, not their commitment value.

Thirdly, since agents who value commitment are willing to pay for it, commitment assets should cost more than the present value of their returns. However, IRAs, Christmas Clubs, etc. offer competitive rates of return comparable with those available in the market (Kocherlakota (2001)).

Lastly, it is hard to find examples of perfect commitment devices (see Gale's comments in Akerlof, Gale and Hall (1998)). For instance, 401(k)s and illiquid assets can be used as collateral for borrowing. More to the point, if agents value perfect commitment then firms and workers would write contracts that commit workers to save. Such contracts are not hard to create and yet are rarely observed.

These observations cast doubt on the claim that there is a significant demand for commitment. This may be regarded as reason not to take self-control problems seriously (Akerlof, Gale and Hall (1998), Kocherlakota (2001)). However, it is difficult to ignore the evidence on preference reversals and the normative implications of models that incorporate temptation.[2] Hence, it is worth asking the question: *is an insignificant demand for commitment compatible with the existence of self-control problems?* One way to provide a positive answer would be to argue that agents are not fully aware of their self-control problems (O'Donoghue and Rabin (1999)). This paper provides another explanation: commitment requires self-control.

To understand why commitment may require self-control, consider again the smoker who is choosing between $\{n\}$ and $\{n, s\}$. As before, anticipation

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[2] See Laibson (1997) and O'Donoghue and Rabin (1999).

of future temptation in $\{n, s\}$ leads to a preference for $\{n\}$. However, *he may be tempted by* $\{n, s\}$ *because it contains* s. In such a case, commitment is at the cost of self-control - choosing $\{n\}$ requires that he resist the temptation of $\{n, s\}$. Indeed, if the self-control cost is too high, he chooses not to commit.

The literature has typically assumed that agents experience only immediate temptations. Under such an assumption, $\{n, s\}$ cannot be tempting since tomorrow's s cannot tempt him today. Therefore, to model a smoker that does not commit, it is necessary to allow future temptations to affect him today.

1.2 Outline of the Model

Our model is based on Gul and Pesendorfer (2001, 2004), who formalize the notions of temptation and self-control. Before introducing our model it is necessary to first introduce theirs.

Gul and Pesendorfer (henceforth GP) study an infinite horizon model in GP (2004). The domain of the preference $\succsim$ is the set of infinite horizon choice problems Z. A choice problem $x \in Z$ is a set of lotteries, where each lottery μ is a measure over present consumption $c \in C$ and a continuation choice problem $y \in Z$.[3] They axiomatize 'Dynamic Self-control' (DSC) preferences which describe an agent who experiences temptation by immediate consumption. DSC preferences are represented by

$$W(x) = \max_{\mu \in x} \left(U(\mu) + V(\mu)\right) - \max_{\eta \in x} V(\eta), \qquad (1)$$

such that

$$\begin{aligned} U(\mu) &= \int_{C \times Z} \left(u(c) + \delta W(y)\right) d\mu(c, y), \\ V(\mu) &= \int_{C \times Z} v(c) d\mu(c, y). \end{aligned}$$

The function U is called commitment utility since it represents the agent's utility if he committed to a singleton x.[4] The function V represents temptation utility. Resisting temptation, that is, not choosing the V-maximizing

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[3] The domain Z is homeomorphic to $\mathcal{K}(\Delta(C \times Z))$, the set of compact sets of lotteries on $C \times Z$.

[4] If $x = \{\mu\}$ then $V(\mu) - \max_{\mu \in x} V(\mu) = 0$ and so $W(\{\mu\}) = U(\mu)$.

choice, leads to a self-control cost $|V(\mu) - \max_{\eta \in x} V(\eta)|$. The utility of the menu x is then his commitment utility net of self-control cost. Observe that maximizer in x (which represents the choice from the menu) is determined $U + V$. This suggests that when choosing from x, the agent compromises between U and V.

According to the functional form of U, commitment utility is the sum of the utility from immediate consumption, and the discounted utility of the continuation menu. Temptation utility V is a function of immediate consumption only.

In this paper, we study agents who are tempted by future consumption as well. 'DSC preferences with Future Temptation', or simply, Future Temptation (FT) preferences, have the same representation as above, except that temptation utility V is modelled as a recursive function,

$$\begin{aligned} V(\mu) &= \int_{C \times Z} \big(v(c) + \gamma \overline{V}(x)\big)\, d\mu(c,x), \\ \text{where } \overline{V}(x) &= \max_{\mu \in x} V(\mu). \end{aligned}$$

Temptation utility from current consumption is captured by v and that from continuation menus is captured by $\overline{V}$. A continuation menu is ranked by $\overline{V}$ according to the most tempting item contained in it. This structure on $\overline{V}$ implies that temptation by continuation menus is a manifestation of future temptation. DSC preferences obtain if $\gamma = 0$.

1.3 Outline of the Paper

The remainder of the paper proceeds as follows. Section 2 axiomatizes the FT model and Section 3 formally defines commitment in our model and contrasts it with the notion in GP (2004).

Section 4 studies the behavioral implications of the restriction $\gamma < \delta$. Various behaviors are shown to be equivalent under this restriction. It also discusses the ability of the FT and DSC model to rationalize a class of preference reversals that cannot be rationalized by the hyperbolic discounting approach (see Ainslie (1992) for the hyperbolic discounting approach).

Section 5 looks at four applications. First, we demonstrate that FT preferences can model the 'smoker who does not commit', and we clarify the link between self-control and demand for commitment. We also present normative implications of the FT model for addiction. Models of rational addiction suggest that there is no need to help addicts who do not seek commitment, since lack of demand for commitment is evidence of the absence of a self-control problem. In contrast, the FT model suggests that the lack of demand could in fact be due to significant self-control problems, thereby justifying intervention.

Second, the model is used to rationalize procrastination. Whereas the O'Donoghue and Rabin (1992) explanation stresses agents' lack of self-awareness, our model suggests that procrastination occurs because of a temptation to delay. As an illustration we study a life-cycle model where the agent decides when to begin saving for retirement with an IRA. The agent procrastinates early in life, but begins to save eventually because delaying ceases to be tempting as the retirement period approaches.

Thirdly, the idea that a delayed large reward can be tempting is used to resolve seemingly contradictory evidence on preference reversals. The idea is also used in the final application to model a temptation to save.

Section 6 concludes. Proofs are collected in appendices.

2 The Model

2.1 Axioms

For any compact metric space X, $\Delta(X)$ denotes the set of all probability measures on the Borel σ-algebra of X, endowed with the weak convergence topology; $\Delta(X)$ is compact and metrizable (see Parthasarathy (1970)). Let $\mathcal{K}(X)$ denote the set of all nonempty compact subsets of X. When endowed with the Hausdorff topology, $\mathcal{K}(X)$ is a compact metric space (see Brown and Pearcy (1995)).

The set C is a compact metric space that denotes possible consumption levels. The domain of preferences $\succsim$ is the set of choice problems Z. Each choice problem $z \in Z$ is a compact set of lotteries, where each lottery is a measure over current consumption and a continuation menu. Thus Z can be identified with $\mathcal{K}(\Delta(C \times Z))$. See GP (2004) for the formal definition of Z

and the homeomorphism between Z and $\mathcal{K}(\Delta(C \times Z))$. In particular, Z is compact metric.

For convenience $\Delta(C \times Z)$ is written as Δ. Generic elements of Z are x, y, z whereas generic elements of Δ are μ, η, ν. For $\alpha \in [0,1]$, $\alpha\mu + (1-\alpha)\eta \in \Delta$ is the measure that assigns $\alpha\mu(A) + (1-\alpha)\eta(A)$ to each A in the Borel $\sigma-$algebra of $C \times Z$. Similarly, $\alpha x + (1-\alpha)y \equiv \{\alpha\mu + (1-\alpha)\eta : \mu \in x, \eta \in y\} \in Z$ is a mixture of the choice problems x and y.

The axioms imposed on $\succsim$ are related to those in GP (2004) and are presented in three groups to facilitate comparison. The first set is identical to corresponding axioms in GP (2004) and the second set strengthens corresponding axioms. The last axiom is the major point of departure.

Axiom 1 *(Order)* $\succsim$ *is a complete and transitive binary relation on* Z.

Axiom 2 *(Continuity)* *The sets* $\{x : x \succsim y\}$ *and* $\{x : y \succsim x\}$ *are closed.*

Axiom 3 *(Independence)* *For any* $\alpha \in (0,1)$,

$$\{\mu\} \succ \{\eta\} \Longrightarrow \{\alpha\mu + (1-\alpha)\nu\} \succ \{\alpha\eta + (1-\alpha)\nu\}.$$

Axiom 4 *(Set-Betweenness)* $x \succsim y \Longrightarrow x \succsim x \cup y \succsim y$.

Axiom 5 *(Stationarity)* $z \succsim z' \Longleftrightarrow \{(c,z)\} \succsim \{(c,z')\}$.

The first two axioms are standard and the third is a version of the Independence axiom applied to singleton menus. The motivation for Independence is precisely as in GP (2001, 2004). Roughly, Stationarity states that the ranking of choice problems is unchanged if the choice problems are pushed one period into the future.

To understand Set-Betweenness, begin with the stronger assumption

$$x \succsim y \Longrightarrow x \sim x \cup y \succsim y.$$

This axiom describes a standard decision-maker who evaluates a menu by its best element: if the best item in x is better than the best item in y, then $x \sim x \cup y$ since the best item in x and in $x \cup y$ are the same. The decision-maker is said to be *strategically rational* (Kreps (1998)). Such a decision-maker is not worse off with larger menus and thus does not experience temptations. Set-Betweenness is a weakening of the axiom that permits a preference for commitment, $x \succ x \cup y$.

Set-Betweenness permits the decision-maker to resist temptation. Let $\mu, \eta \in \Delta$ be such that $\{\mu\} \succ \{\mu, \eta\}$, that is, η is tempting. When $\{\mu, \eta\} \sim \{\eta\}$ holds, the indifference suggests that the agent would choose the same item when faced with $\{\mu, \eta\}$ or $\{\eta\}$. That is, choice from $\{\mu, \eta\}$ is η and so, the decision-maker succumbs to temptation. On the other hand, the ranking $\{\mu, \eta\} \succ \{\eta\}$ suggests that μ is chosen from $\{\mu, \eta\}$ and so, the decision-maker resists temptation.

The next two axioms, strengthen corresponding axioms in GP (2004).

For any lottery $\mu \in \Delta(C \times Z)$, let μ^1 denote the marginal distribution over C and μ^2 the marginal distribution over Z.

Axiom 6 *(Separability)* *If* $\mu^1 = \pi^1, \mu^2 = \pi^2, \eta^1 = \nu^1$ *and* $\eta^2 = \nu^2$, *then,*

$$\{\mu, \eta\} \backsim \{\pi, \nu\}.$$

The set of marginals on C induced by both menus in the axiom are same, and so are the induced sets of marginals on Z. Indifference between $\{\mu, \eta\}$ and $\{\pi, \nu\}$ suggests that 'only marginals matter' and hence, preferences are insensitive to correlations between current consumption and continuation menus. GP adopt the special case where

$$\begin{aligned} \mu &= \eta = \frac{1}{2}(c, z) + \frac{1}{2}(c', z') \\ \pi &= \nu = \frac{1}{2}(c, z') + \frac{1}{2}(c', z). \end{aligned}$$

In particular, their axiom imposes a restriction on singleton menus only.

Let $\Delta_s \subset \Delta$ be the set of lotteries on $C \times Z$ with finite support and $\Delta_s(Z)$ the set of lotteries on Z with finite support. Let δ_z denote the lottery degenerate at menu z. Define $\varphi : \Delta_s(Z) \longrightarrow Z$ by

$$\varphi(\sum p(x)\delta_x) = \sum p(x)x.$$

Two lotteries on Z that have the same $\varphi-$value induce the same uncertainty over continuation menus; the definition implies that for the two lotteries, the probability of *ultimately* choosing from a given continuation menu $z \in Z$ tomorrow is the same. However, the timing of the resolution of this uncertainty can be different. For example, consider lotteries μ, π such that

$$\mu^2 = \alpha\delta_z + (1-\alpha)\delta_{z'} \text{ and } \pi^2 = \delta_{\alpha z+(1-\alpha)z'}.$$

Then,

$$\varphi(\mu^2) = \varphi(\alpha\delta_z + (1-\alpha)\delta_{z'}) = \alpha z + (1-\alpha)z' = \varphi(\delta_{\alpha z+(1-\alpha)z'}) = \varphi(\pi^2).$$

Under both μ and π, the uncertainty about the continuation menu is the same: the agent chooses from z tomorrow with probability α and from z' with probability $(1-\alpha)$. However, there is early resolution of uncertainty in the former since, under μ, the uncertainty is played out today, but under π the uncertainty is resolved tomorrow.

Axiom 7 *(Indifference to Timing)* *For any* $\mu, \eta, \pi, \nu \in \Delta_s$, *if* $\mu^1 = \pi^1, \eta^1 = \nu^1, \varphi(\mu^2) = \varphi(\pi^2)$ *and* $\varphi(\eta^2) = \varphi(\nu^2)$, *then,*

$$\{\mu, \eta\} \backsim \{\pi, \nu\}.$$

According to the hypothesis, μ and π have the same first marginal and the same $\varphi-$value. That is, the two lotteries are similar, except that they may differ in how uncertainty about the continuation menu is resolved. The same is true for η and ν. Consequently, the choices available in the menus $\{\mu, \eta\}$ and $\{\pi, \nu\}$ may differ only in how uncertainty is resolved. Hence, indifference between the menus amounts to an indifference to the timing of resolution of uncertainty. GP formulate a weaker axiom restricted to singleton menus.

Our final axiom departs from GP's in a more fundamental way. GP's final axiom states

Axiom *(Temptation by Immediate Consumption)* *For any* $\mu, \eta, \nu \in \Delta$ *such that* $\eta^1 = \nu^1$, *if* $\{\mu\} \succ \{\mu, \eta\} \succ \{\eta\}$ *and* $\{\mu\} \succ \{\mu, \nu\} \succ \{\nu\}$, *then*

$$\{\mu, \eta\} \sim \{\mu, \nu\}.$$

According to the axiom, if the marginal distributions on present consumption implied by two lotteries are the same, then the lotteries are equally tempting, implying that continuation problems do not tempt the agent. Our axiom is formulated so that continuation problems may tempt the agent.

Axiom 8 *(Temptation Stationarity)* $\quad x \succ x \cup y \Longleftrightarrow \{(c,x)\} \succ \{(c,x),(c,y)\}$.

The ranking $x \succ x \cup y$ reveals that y tempts x, that is, the most tempting item in y is more tempting than that in x. Similarly, the ranking $\{(c,x)\} \succ \{(c,x),(c,y)\}$ reveals that (c,y) is more tempting than (c,x). Thus, the axiom states that y tempts x if and only if (c,y) tempts (c,x), that is, if and only if the continuation menu y tempts the continuation menu x. Note that this axiom provides a way of distinguishing an agent who experiences future temptation from one who does not. When y contains temptations, the former type of agent will strictly prefer, ex-ante, not to have the option to choose (c,y), as reflected in the ranking $\{(c,x)\} \succ \{(c,x),(c,y)\}$. An agent who does not experience future temptation would not care whether or not he has (c,y) and hence, he expresses the ranking $\{(c,x)\} \sim \{(c,x),(c,y)\}$.

2.2 Representation Result

The preference $\succsim$ is said to be *nondegenerate* if there exists $x, y \in Z$ such that $x \supset y$ and $x \succ y$.

Theorem 1 *If the nondegenerate preference $\succsim$ satisfies Axioms 1-8 then there exist $\delta \in (0,1)$, $\gamma \in (0,1)$, functions $u, v : C \longrightarrow \mathbb{R}$, $\overline{V} : Z \longrightarrow \mathbb{R}$, and $W : Z \longrightarrow \mathbb{R}$ that represents $\succsim$ such that for all $z \in Z$,*

$$\begin{aligned} W(z) &= \max_{\mu \in z} \int_{C\times Z} \big(u(c) + \delta W(x) + v(c) + \gamma \overline{V}(x)\big)\, d\mu(c,x) \\ &\quad - \max_{\eta \in z} \int_{C\times Z} \big(v(c) + \gamma \overline{V}(y)\big)\, d\eta(c,y), \qquad (2) \\ \textit{where } \overline{V}(z) &= \max_{\mu \in z} \int_{C\times Z} \big(v(c) + \gamma \overline{V}(x)\big)\, d\mu(c,x). \end{aligned}$$

Each of the functions $u, v, \overline{V}$ and W is continuous and $\overline{V}$ is linear.

Conversely, for any $\delta \in (0,1)$, $\gamma \in (0,1)$, continuous $u, v : C \longrightarrow \mathbb{R}$, there is a unique continuous function W satisfying (2), and the preference it represents satisfies Axioms 1-8.

The preferences in Theorem 1 are referred to as DSC preferences with Future Temptation, or simply Future Temptation (FT) preferences. Note that DSC preferences are not a special case of FT preferences. In the case of DSC preferences, temptation utility V has the form

$$V(\mu) = \int_C v(c) d\mu^1(c),$$

which amounts to our representation with $\gamma = 0$. For FT preferences, $\gamma > 0$.

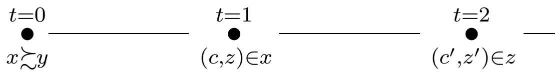
For any W as in (*2*) that represents $\succsim$, the corresponding tuple (u, v, δ, γ) is referred to as a representation of $\succsim$. Theorem 2 establishes uniqueness properties of the representation.

Theorem 2 *Suppose there exist* x *and* y *such that* $x \subset y$ *and* $x \succ y$, *and let* (u, v, δ, γ) *be a representation of* $\succsim$. *Then* $(u', v', \delta', \gamma')$ *also represents* $\succsim$ *if and only if* $\delta = \delta'$, $\gamma = \gamma'$ *and there exist* $\alpha > 0$, *and* $\beta_u, \beta_v \in \mathbb{R}$ *such that* $u' = \alpha u + \beta_u$ *and* $v' = \alpha v + \beta_v$.

3 Demand for Commitment

In the Introduction we modelled an FT agent who experiences temptation (and hence has a preference for commitment), and yet does not demand commitment. In this section we formally define ‘demand for commitment’ and explain why a preference for commitment may not imply a demand for commitment in the FT model.

Consider the following time-line:



At time 0 (the ex-ante stage), the agent chooses between menus. The chosen menu, say x, is faced at time 1, and a choice from x is made. Assuming for simplicity that objects in x are degenerate lotteries, the choice from x yields some current consumption c, and a continuation menu z that is faced at time 2. At time 2, a further choice of current consumption and continuation menu is made, and the process is repeated ad infinitum. An important feature of GP’s model is that temptation is experienced in each period except in the ex-ante stage; the ex-ante period is the period prior to the experience of temptation (see GP (2004)). Thus, while choice *from* a menu is subject to temptations, ex-ante choice *between* menus is not.

GP concentrate on commitment in the ex-ante stage, as reflected in their definition of 'preference for commitment':

Definition 1 *$\succsim$ exhibits a preference for commitment at y if there exists $x \subset y$ such that $x \succ y$. Say that $\succsim$ exhibits a preference for commitment if there exists $y \in Z$ such that $\succsim$ exhibits a preference for commitment at y.*

We wish the agent to experience temptation at the time of commitment. Since only choice *from* a menu is subject to temptations, we define commitment in terms of choice of a continuation menu.

Definition 2 *$\succsim$ exhibits a demand for commitment at y if there exists $x \subset y$ such that*

$$\{(c, x), (c, y)\} \succ \{(c, y)\},$$

for some $c \in C$. Say that $\succsim$ exhibits a demand for commitment if it exhibits a demand for commitment at some y.

The ranking $\{(c, x), (c, y)\} \succ \{(c, y)\}$ implies that (c, x) is chosen from $\{(c, x), (c, y)\}$. When there is a choice between continuation menus x and y, as in $\{(c, x), (c, y)\}$, then an agent is said to demand commitment if he chooses the smaller continuation menu x, that is, if he chooses (c, x). An assumption about the timing of commitment is implicit in the definition: commitment is chosen in the period before it is received. This implies that there is a passage of time between when commitment is chosen and when it is received; for example, the time between joining a rehabilitation clinic and actually starting treatment.

It is important to understand the relationship between the demand and preference for commitment. A demand for commitment implies a preference for commitment.[5] In what follows, we show that the converse is true for DSC preferences, but not necessarily for FT preferences.

For a DSC agent, a preference for commitment at y implies a demand for commitment at y: when Temptation by Immediate Consumption is satisfied, continuation menus are not tempting, and hence, for all x, y,

$$\{(c, x)\} \sim \{(c, x), (c, y)\}.$$

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[5] To see this, let $x \subset y$ and $\{(c, x), (c, y)\} \succ \{(c, y)\}$. Then Set-Betweenness implies $\{(c, x)\} \succ \{(c, y)\}$, which, by Stationarity implies $x \succ y$.

By Stationarity, $x \succ y$ implies $\{(c,x)\} \succ \{(c,y)\}$. Therefore, if $x \subset y$ and $x \succ y$, then

$$\{(c,x)\} \sim \{(c,x),(c,y)\} \succ \{(c,y)\},$$

That is, (c,x) is chosen from $\{(c,x),(c,y)\}$.

In the FT model, however, a preference for commitment at y may not imply a demand for commitment at y. To see this, observe that if $x \subset y$, then $x \succ y$ is equivalent to $x \succ x \cup y$, which by Temptation Stationarity is equivalent to

$$\{(c,x)\} \succ \{(c,x),(c,y)\}.$$

Hence a preference for commitment at y implies a temptation to choose (c,y). By Definition 2, the agent demands commitment when

$$\{(c,x)\} \succ \{(c,x),(c,y)\} \succ \{(c,y)\},$$

that is, when he resists the temptation to choose (c,y). However, if the self-control cost of resisting the temptation is too high, the agent submits to the temptation of (c,y),

$$\{(c,x)\} \succ \{(c,x),(c,y)\} \sim \{(c,y)\}.$$

In such a case, there is a preference for commitment at y, but no demand.

Hence future temptation severs the link between the preference and demand for commitment that exists in the DSC model. The result that a preference for commitment does not imply a choice to commit has the same paradoxical nature as the result in GP (2004), where dynamically consistent preferences may produce choices that appear dynamically inconsistent. This wedge between preferences and choices arises because temptations can make choices appear inconsistent with preferences. For instance, one may prefer $\{\mu\}$ over $\{\eta\}$, but choose η from $\{\mu,\eta\}$ due to a temptation to choose η. Future temptation serves the purpose of making certain choices tempting, namely making (c,y) tempting in $\{(c,x),(c,y)\}$. This introduces the appropriate wedge, since now a preference for commitment (which is equivalent to $\{(c,x)\} \succ \{(c,y)\}$) does not necessarily imply that (c,x) is chosen from $\{(c,x),(c,y)\}$.

4 Temptation Discounting

Two primitives of the FT model are δ and γ, the commitment and temptation discount factors respectively. In this section we define some behaviors and discuss how they depend on the relative magnitudes of δ and γ. In particular, we discuss how the demand for commitment depends on δ and γ.

4.1 Preference Reversals

Subjects of psychology experiments on preference reversals are typically asked to choose between a small reward s available at time t and a larger reward l available with a further delay of d periods (see, for instance, Kirby and Hernnstein (1995)). All choices are made at one point in time, say time 1. In most experiments, subjects choose the small reward when it is available immediately, but switch to the later reward when both rewards are delayed sufficiently. Preference reversals can be defined in our model as follows.

For some $c \in C$ and any $\mu \in \Delta$ let $\mu^{+1} = (c, \{\mu\})$ and inductively, $\mu^{+t} = (c, \{\mu^{+(t-1)}\})$. That is, μ^{+t} represents getting the reward μ with a delay of t periods, where in each of the t periods the agents receives some fixed consumption c.

Definition 3 *$\succsim$ exhibits preference reversals if $\{\mu\} \succ \{\mu, \eta\} \sim \{\eta\}$ implies the existence of t^* such that*

$$\begin{array}{rcl} \{\mu^{+t}, \eta^{+t}\} & \sim & \{\eta^{+t}\} \text{ for all } t \leq t^*, \\ \{\mu^{+t}, \eta^{+t}\} & \succ & \{\eta^{+t}\} \text{ for all } t > t^*. \end{array}$$

Identify a reward $r = s, l$ with the degenerate lottery that gives $r \in C$ consumption in the current period, and c in all subsequent periods. Suppose $s < l$. Therefore, l^{+d} is a large reward l available after a delay d, and s is an immediate small reward. The definition states that if the agent submits to the temptation of s in $\{l^{+d}, s\}$, then there exists t^* such that for any delay $t > t^*$ he reverses preferences and choose the later larger reward $l^{+(t+d)}$ from $\{l^{+(t+d)}, s^{+t}\}$.

Definition 3 is not restricted to preference reversals involving earlier rewards and later rewards. It allows for preference reversals between rewards available at the same date. An example of such a reversal is provided by

Trope and Liberman (2000). They show that subjects tend to prefer watching a noneducational but entertaining movie now to an educational but unentertaining movie now, yet they switch preferences when the movie is to be watched at some point in the future.[6] Both rewards are available at the *same* date. As we will see, the FT and DSC models rationalize such preference reversals. However, they cannot be rationalized by the hyperbolic discounting approach, for which preference reversals were a motivation (see Ainslie (1992)).

4.2 Greater Demand for Delayed Commitment

Benartzi and Thaler (2004) introduce a saving-enhancement plan, which they call the 'Save More Tomorrow' (SMT) plan. Agents are given the opportunity to commit in advance to allocating a portion of their future salary increases towards retirement savings. Benartzi and Thaler conduct an experiment and report that average saving rates for the SMT participants increased significantly (from 3.5% to 13.6%) over the course of 40 months. This constitutes evidence that the demand for delayed commitment is greater than the demand for commitment.

The formal definition of 'greater demand for delayed commitment' requires some notation. Let $B \subset C$ be a compact set of consumption levels, and B' a compact subset of B. A choice problem that allows the agent to face the same set of current consumption B in every period is[7]

$$y = B \times \{y\}.$$

A choice problem that provides commitment to the smaller set of current consumption B' in every period is

$$x = B' \times \{x\}.$$

---

[6] Such rewards can be modelled as follows. Let the consumption set C be a subset of $\mathbb{R}^2$, where the first coordinate of $(c_1, c_2) \in C$ represents the 'education rating' and the second represents the 'entertainment rating'. The movies can thus be represented by elements of C. To get 'temptation by entertaining movies', let v be a function of c_2 only, u a function of c_1 only, and u, v strictly increasing.

[7] We show that y is well-defined. Define $y^1 = B$. Given compactness of B, y^1 is compact. Inductively define $y^n = B \times \{y^{n-1}\}$ and note that each y^n is compact. Observe that $y = \times_{n=0}^{\infty} y^n$ is *consistent* in the sense of GP (2004, p 146). Hence $y \in Z$.

If $x^0 \equiv x$, and for $t \geq 1$,

$$x^{+t} = B \times \{x^{+(t-1)}\},$$

then x^{+t} provides B for t periods and commitment to x after the t periods. A choice today of whether or not to commit after $t > 0$ periods is a choice from

$$x^{+t} \cup y.$$

Suppose $x \succ y$ and the agent does not commit to x, that is,

$$x^{+1} \succ x^{+1} \cup y \sim y$$

To obtain a greater demand for delayed commitment, there must exist t^* such that for all $t > t^*$ the agent commits when commitment is delayed by t periods.

Definition 4 $\succsim$ *exhibits greater demand for delayed commitment if* $x^{+1} \succ x^{+1} \cup y \sim y$ *implies the existence of* t^* *such that*

$$\begin{aligned} x^{+t} \cup y &\sim y \text{ for all } t \leq t^*, \\ x^{+t} \cup y &\succ y \text{ for all } t > t^*, \end{aligned}$$

While Benartzi and Thaler explain their finding in terms of inertia, we interpret the finding as a preference reversal. That is, when the rewards are committing and not committing, then the agent chooses the latter, and when both rewards are delayed, the agent reverses preferences. This is another example of a preference reversal with rewards that are available at the same date, as discussed in the previous subsection.

4.3 Preference for Early Choice

When deciding about tomorrow's consumption, one might prefer to make the choice today or postpone the choice until tomorrow.

Definition 5 $\succsim$ *exhibits preference for early choice if*

$$\{\mu\} \succ \{\mu, \eta\} \succ \{\eta\} \text{ implies } \{(c,\mu),(c,\eta)\} \succ \{(c,\{\mu,\eta\})\}.$$

Under $\{(c,\mu),(c,\eta)\}$ the decision-maker has the opportunity to commit now to consuming μ or η tomorrow. Under $\{(c,\{\mu,\eta\})\}$ he has to postpone choosing tomorrow's consumption. The choice between the menus reveals his preference for when to choose tomorrow's consumption. The axiom states that the agent has a preference for early choice.

4.4 A Characterization

Theorem 3 reveals that demand for commitment, preference reversals, greater demand for delayed commitment and preference for early choice are all behavioral manifestations of large temptation discounting.

Theorem 3 *Let the FT preference* $\succsim$ *exhibit a preference for commitment. Then the following statements are equivalent.*

(a) $\gamma < \delta$.
(b) $\succsim$ *exhibits demand for commitment.*
(c) $\succsim$ *exhibits preference reversals.*
(d) $\succsim$ *exhibits greater demand for delayed commitment.*
(e) $\succsim$ *exhibits preference for early choice.*

The intuition behind the result is that discounting future temptation makes it possible to resist future temptation. There exists a demand for commitment when the agent can resist the future temptation of some menu y, and this is possible only when future temptation is discounted sufficiently. Large discounting is also responsible for preference reversals because the latter occurs when an irresistible immediate temptation is resisted when it is pushed into the future. Similarly, it is responsible for a greater demand for delayed commitment since this is essentially a preference reversal. Finally, $\{(c,\mu),(c,\eta)\}$ is strictly preferred to $\{(c,\{\mu,\eta\})\}$ (and so there is preference for early choice) if it is easier to resist the future temptation of η in $\{(c,\mu),(c,\eta)\}$ than the immediate temptation of η in $\{\mu,\eta\}$.

The restriction $\gamma < \delta$ is desirable for two reasons. Firstly, there is evidence in favor of at least three of the behaviors, namely demand for commitment (Elster (2000), Schelling (1992)), preference reversals (see references in Ainslie (1992)), and greater demand for delayed commitment (Benartzi and Thaler (2004)). Secondly, $\gamma \geq \delta$ produces behavior that is not intuitive. For instance, when $\gamma > \delta$, the following behavior arises: for any $\mu,\eta \in \Delta$, $\{\mu\} \succ \{\mu,\eta\} \succ \{\eta\}$ implies the existence of t^* such that

$$\begin{aligned}\{\mu^{+t},\eta^{+t}\} &\succ \{\eta^{+t}\} \textit{ for all } t \leq t^*,\\ \{\mu^{+t},\eta^{+t}\} &\sim \{\eta^{+t}\} \textit{ for all } t > t^*.\end{aligned}$$

That is, temptations that can be resisted when they are immediate become irresistible when they are pushed into the future. We normally expect immediate temptations to be harder to resist.

The main motivation for the FT model was to explain why agents with self-control problems may not commit. By Theorem 3 it would appear that we need to assume $\gamma \geq \delta$. But we just argued that $\gamma < \delta$ is the desirable restriction! Fortunately, Theorem 3 states that when $\gamma < \delta$, there exists *at least* one y where there is a demand for commitment, and therefore does not rule out the possibility that there are other y where there is a preference for commitment but no demand. The following table gives conditions under which a demand for commitment does or does not exist: for x, y such that $x \subset y$ and $x \succ y$,

	commits	does not commit
$\gamma = 0$	always	-
$0 < \gamma < \delta$	if $\frac{W(x)-W(y)}{\overline{V}(y)-\overline{V}(x)} > \frac{\gamma}{\delta}$	if $\frac{W(x)-W(y)}{\overline{V}(y)-\overline{V}(x)} \leq \frac{\gamma}{\delta}$
$\delta \leq \gamma$	-	always

See Appendix D for the derivation of the table.

We conclude by pointing out that the equivalence between the behaviors in the theorem holds in the DSC model as well since $\gamma = 0 < \delta$. However, greater demand for delayed commitment holds only in a vacuous sense since the hypothesis $x^{+1} \succ x^{+1} \cup y$ in Definition 4 is ruled out by Temptation by Immediate Consumption.

5 Applications

5.1 Addiction

In the Introduction it was noted that addicts do not necessarily make use of treatments that provide commitment. For example, a treatment for alcoholics involving disulfiram is known to be of limited effectiveness, primarily because patients do not comply with the disulfiram regimen. Fuller and Roth (1979) state that "willingness to take the drug may not in itself be sufficient to achieve abstinence - receiving the drug is probably necessary."

The FT model explains why addicts do not necessarily seek commitment: if drug consumption is tempting, then there is a temptation to continue drug usage. In the model, a lack of self-control causes a lack of demand for commitment. We demonstrate this for the 'smoker who does not commit' referred to in the Introduction.

There are two periods, and the problem is to choose between smoking s and not smoking n today, and also whether or not to commit to n in the next period. The choice problem is

$$x_1 = \{n, s\} \times \{\{n\}, \{n, s\}\}.$$

Preferences are represented by

$$\begin{aligned} W_1(x_1) &= \max_{(c, x_2) \in x_1} \left(u(c) + \delta W_2(x_2) + \lambda v(c) + \gamma \overline{V}_2(x_2) \right) \\ &\quad - \max_{(c', x_2') \in x_1} \left(\lambda v(c') + \gamma \overline{V}_2(x_2') \right), \\ \text{where } W_2(x_1) &= \max_{(c, x_2) \in x_1} \left(u(c) + \lambda v(c) \right) - \max_{(c', x_2') \in x_1} \lambda v(c') \text{ and } 0 < \gamma < \delta. \end{aligned} \tag{3}$$

The parameter λ is a measure of 'instantaneous self-control' (see GP (2004)). Roughly, it measures the ability to resist immediate temptations - a high λ represents a larger weight on temptation utility from immediate consumption.

Assume that the agent prefers to commit to n $(u(n) > u(s))$, and that he is tempted by s $(\lambda v(s) > \lambda v(n))$.[8] Since these are FT preferences,

$$\overline{V}_2(x_2) = \max_{c \in x_2} \lambda v(c), \tag{4}$$

that is, the future temptation utility of a menu is the maximum temptation utility that can be achieved by it. Hence the assumption $\lambda v(s) > \lambda v(n)$ implies that the agent is tempted by the continuation menu $\{n, s\}$, that is, $\overline{V}_2(\{n, s\}) > \overline{V}_2(\{n\})$.

By the representation, the choice of current consumption from $\{n, s\}$ solves

$$\max_{c \in \{n, s\}} \left(u(c) + \lambda v(c) \right), \tag{5}$$

and the choice of continuation menu from $\{\{n\}, \{n, s\}\}$ solves

$$\max_{x_2 \in \{\{n\}, \{n, s\}\}} \left(\delta W_2(x_2) + \gamma \overline{V}_2(x_2) \right). \tag{6}$$

---

[8] This assumption, expressed as a restriction on preference, is $\{(c, \{n\})\} \succ \{(c, \{n, s\})\}$.

By (3), (4), (5) and (6), the assumptions imply that the smoker chooses immediate consumption s from $\{n, s\}$ if and only if

$$\frac{u(n) - u(s)}{\lambda\,(v(s) - v(n))} \leq 1,$$

and he chooses continuation menu $\{n, s\}$ from $\{\{n\}, \{n, s\}\}$ if and only if

$$\frac{u(n) - u(s)}{\lambda\,(v(s) - v(n))} \leq \frac{\gamma}{\delta}.$$

Presumably, addicts have a high λ. For sufficiently high λ, both above inequalities are satisfied, and so the smoker submits to the temptation of s in $\{n, s\}$ and the temptation of $\{n, s\}$ in $\{\{n\}, \{n, s\}\}$. This illustrates our claim that in the FT model, a sufficient lack of self-control implies a lack of demand for commitment. Note that for DSC preferences ($\gamma = 0$), the smoker never submits to the temptation of continuation menu $\{n, s\}$ and hence commits to not smoking in the second period.

The normative implications of an FT-type model of addiction are different from those of models stemming from Becker and Murphy (1988). These latter models suggest that addiction is a rational choice, and so the government has no role in regulating the addict's choices, except to the extent that his choices impose an externality on others. On the other hand, models that incorporate self-control problems, such as GP (2003), imply that, along with externalities on others, an addict imposes negative internalities on himself and hence there is a role for the government to intervene. However, by predicting a demand for commitment, these models suggest that addicts that don't seek treatment are 'happy addicts'. That is, they are addicted because it is optimal for them, and so they do not need help.

In contrast, the FT model suggests that an addict that does not seek commitment may be a very unhappy addict. He does not seek commitment because of a lack of self-control. Hence, there is a role for intervention.

5.2 Procrastination

Procrastination occurs when an agent delays doing a task which, in some sense, is preferable to do now rather than later (Akerlof (1991)). O'Donoghue and Rabin (1999, 2001) develop a theory of procrastination: given some task that is worth doing now rather than later, they suggest that a decision-maker procrastinates because, firstly, he incorrectly believes that he will do the task tomorrow if he delays today, and secondly, given the incorrect belief, it is optimal to delay. The naivete in expectations is an important ingredient in their theory.

We suggest an alternative explanation. Procrastination occurs because delaying is tempting. Thus, it is possible for a fully self-aware, rational agent to find it optimal to delay doing a task, despite preferring to commit to doing the task now. We illustrate how the FT model can be used to produce procrastination in the context of saving for retirement through a commitment asset.

There are T periods. A finite horizon choice problem x_t is an element of Z_t, where $Z_T = \mathcal{K}(\Delta(C))$ and for $t < T$, $Z_t = \mathcal{K}(\Delta(C \times Z_{t+1}))$. Time t preferences are represented by

$$\begin{aligned} W_t(x_t) &= \max_{(c_t, x_{t+1}) \in x_t} \big(u(c) + \delta W_{t+1}(x_{t+1}) + v(c) + \gamma \overline{V}_{t+1}(x_{t+1})\big) \\ &\quad - \max_{(c'_t, x'_{t+1}) \in x_t} \big(v(c') + \gamma \overline{V}_{t+1}(x'_{t+1})\big), \\ \text{where } W_T(x_T) &= \max_{(c_T) \in x_T} (u(c) + v(c)) - \max_{(c'_T) \in x_T} v(c'). \end{aligned}$$

Let $0 < \gamma < \delta$ and $u = v$. The agent receives an endowment $\omega_t > 0$ every period $t < T$, but in the last period he has no endowment. In each period, besides choosing consumption, he decides whether or not to commit d units of consumption to an IRA. Contributions to the IRA can be consumed only in the last period, and there are no other saving opportunities. Formally, the choice problem is[9]

$$\begin{aligned} x_0(w_0) &= \{(c_0, x_1(w_1 - d_1)) : c_0 \le w_0 \text{ and } d_1 = 0 \text{ or } d\}, \\ x_t(w_t - d_t) &= \{(c_t, x_{t+1}(w_{t+1} - d_{t+1})) : c_t \le w_t - d_t \text{ and } d_{t+1} = 0 \text{ or } d\},\ 0 < t < T, \\ x_T(w_T) &= \{c_T : c_T \le w_T \text{ and } w_T = \sum_{t=0}^{T-1} d_t\}. \end{aligned}$$

---

[9] Recall from Section 3 that in the FT model, commitment is chosen in the period prior to when it is received. This is evident in the way the choice problems are defined. For instance, at $t = 0$ the agent chooses d_1.

He enters period 0 with resources w_0, chooses current consumption $c_0 \leq w_0$, and the continuation menu $x_1(w_1 - d_1)$. The choice of continuation menu is made by choosing $d_1 \in \{0, d\}$, that is, by deciding whether or not to contribute d in the asset. The same kind of choices are made in every period $t < T$. In the final period T, his resources are $\sum_{t=0}^{T-1} d_t$, the quantity contributed to the IRA over his life, and his only choice is to decide how much of this to consume.

Assume that life-cycle commitment utility is maximized if he begins saving for retirement from the first period and that life-cycle temptation utility is maximized if he begins saving by some period $t^* > 0$. That is, commitment utility is maximized when $d_t = d$ for all $t < T$, and temptation utility is maximized when $d_t = 0$ for all $t < t^*$ and $d_t = d$ for all $t \geq t^*$. Given $\gamma < \delta$, this assumption is satisfied if the time horizon is sufficiently long. Note that there is a temptation to delay until t^*, so that any commitment before t^* is at the cost of self-control. For appropriate parameter values (for instance, a small d), the benefit of committing (that is, the discounted utility of gaining d units of consumption in the last period) is smaller than the self-control cost for all $t < t^*$. For $t \geq t^*$, there is no self-control cost of committing, and so he commits. That is, the agent rationally procrastinates on saving for retirement until t^*. The reason he eventually starts saving is that delaying ceases to be tempting once the deadline is close enough.

5.3 Decreasing vs Increasing Impatience

Preference reversals are understood to be evidence of *decreasing impatience*, that is, greater impatience when choosing between rewards that are close to the present. Some authors (for instance, Holcomb and Nelson (1992), Read (2001) and Rubinstein (2003)) provide experimental evidence that is inconsistent with decreasing impatience. We describe some of the evidence in Read (2001).

Consider a preference $\succeq$ over $(money, time)$ pairs. Read's experiment 1 (see Table 1 in his paper) aims at finding specifications of s and l such that $(s, t) \sim (l, t + d)$, for given d and different t. Under standard assumptions[10]

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[10] Specifically, assume that subjects prefer larger rewards (that is $(s, t) \prec (l, t)$) and earlier rewards (that is, $(m, t) \succ (m, t + d)$ for $m > 0$).

we can deduce that his subjects would have the following preferences (the experiment took place in feb 2000):

$$
\begin{aligned}
(500, feb\ 2000) &\succ (700, oct\ 2000) \qquad (7)\\
(500, jun\ 2001) &\prec (700, feb\ 2002) \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
(338, feb\ 2000) &\prec (500, oct\ 2000) \qquad (9)\\
(338, jun\ 2001) &\succ (500, feb\ 2002). \qquad (10)
\end{aligned}
$$

That is, with $d = 8$ months, Read's subjects exhibit decreasing impatience (DI) when $s = 500$ and $l = 700$ but the opposite, *increasing impatience* (II), when $s = 338$ and $l = 500$. This suggests that agents can simultaneously exhibit both decreasing and increasing impatience. This is an anomaly because these are apparently opposite behaviors.

Existing models cannot give a satisfactory account of the above choices. In the β-δ model (see Laibson (1997)), agents can exhibit either DI (when $\beta < 1$) or II (when $\beta > 1$) but not both simultaneously. DSC preferences do not give a satisfactory account because DSC agents exhibit (9) and (10) only when they find 0 more tempting than 338.[11] Hence DSC agents can either exhibit II, or find higher consumption more tempting, but not both. In what follows we restrict attention to the case where both commitment and temptation utility increase with current consumption.

While DI is understood in terms of a temptation to choose the earlier reward, II can be understood in terms of a temptation to choose the later reward. It is intuitive for a delayed reward to be tempting *if the reward is large enough*. Temptation by a delayed reward, however, requires agents to be tempted by future consumption and so such an explanation cannot be provided by DSC preferences.

That DI and II can be simultaneously explained by FT preferences is clear from Theorem 3. We simply required that there exist two pairs of rewards, one in which an immediate small reward is irresistibly tempting, and the

---

[11] To see why, note that (10) is equivalent to $u(338) > \delta^9 u(500)$ and that (9) is equivalent to $u(338) + v(350) < \delta^9 u(500) + v(0)$. These choices can not be produced by DSC preferences if we assume that $v(338) \geq v(0)$.

other in which the delayed large reward is irresistibly tempting. That is, in the notation of Section 5.1, there must exist $s_1, s_2, l_1, l_2, d_1, d_2$ such that $s_i < l_i$ for $i = 1, 2$ and

$$\begin{aligned} \{l_1^{+d_1}\} &\succ \{l_1^{+d_1}, s_1\} \sim \{s_1\} \\ \{s_2\} &\succ \{l_2^{+d_2}, s_2\} \sim \{l_2^{+d_2}\}. \end{aligned}$$

Necessary conditions for this are $u \neq v$ and $0 < \gamma$. By Theorem 3, if $\gamma < \delta$, we get the appropriate reversals. The following example confirms the model's ability to reconcile this seemingly contradictory evidence on decreasing impatience.

Example 1 *Let* $u(c) = (c+1)^{\frac{1}{2}}$ *and* $v(c) = c^2$ *where* c *belongs to a compact set in* $\mathbb{R}_+$ *that includes* 0*. Let* $\delta = 0.8, \gamma = 0.1, d = 1$*. For* $s = 0.1$ *and* $l = 0.7$ *the agent exhibits II and for* $s = 2$ *and* $l = 4.5$ *he exhibits DI.*

5.4 Temptation to Save

Krussel, Kuruscu and Smith (2002) consider a generalization of DSC preferences. The preferences they study are represented by

$$\begin{aligned} W(z) &= \max_{\mu \in z} \int_{C \times Z} (u(c) + \delta W(x) + v(c) + \gamma W)\, d\mu(c, x) \\ &\quad - \max_{\eta \in z} \int_{C \times Z} (v(c) + \gamma W)\, d\eta(c, y). \end{aligned}$$

In particular, temptation utility is of the form

$$V(\mu) = \int_{C \times Z} (v(c) + \gamma W)\, d\mu(c, x).$$

The authors use these preferences to explain the equity premium puzzle, risk free rate puzzle and the income distribution in the U.S. A central feature of their model is that some agents (who eventually become the wealthiest in the economy) have a temptation to save. A temptation to save arises when $\gamma > 1$.

By making use of 'temptation by later larger rewards' discussed in the previous subsection, a temptation to save can be produced in the FT model. Furthermore, this can be done without having to deviate from $\gamma < \delta$. To illustrate, assume that v is linear so that temptation preferences do not care about intertemporal consumption smoothing, and also that commitment preferences desire some positive consumption in the current period. Then, for any r such that

$$1 + r > \frac{1}{\gamma},$$

the agent is tempted to save his entire endowment. This holds even though $\gamma < \delta$. Note that the agent switches between being tempted to save and tempted to spend as $1 + r$ fluctuates around $\frac{1}{\gamma}$.

6 Conclusion

While the literature on self-control problems has concentrated on implications of immediate temptation, this paper explores the implications of future temptation. The model unifies certain behaviors and is relevant to the literature on preference reversals, addiction and procrastination. It also provides an explanation for why agents who are aware of their self-control problem may not behave in a sophisticated manner (in the sense of Strotz (1955)): the possibility of indulging temptations in the future is itself a source of temptation, and strategies such as commitment require this future temptation to be resisted. That is, it takes self-control to be sophisticated, and so self-awareness does not necessarily lead to sophistication.

A Appendix: Proof of Theorem 1

$\Longleftarrow$: Given a representation W, necessity of the axioms is straightforward to establish, and hence omitted. To show that W is unique and well defined, consider the mappings S and T, from the space of all continuous, real valued functions on Z (endowed with the sup norm) to itself, defined below:

$$\begin{aligned} S\overline{V}(z) &= \max_{\eta \in z} \int v(c) + \gamma \overline{V}(y) d\eta, \\ TW(z) &= \max_{\mu \in z} \int u(c) + \delta W(x) + v(c) + \gamma \overline{V}(x) d\mu \\ &\quad - \max_{\eta \in z} \int v(c) + \gamma \overline{V}(y) d\eta. \end{aligned}$$

For any v and γ as defined in Theorem 1, Blackwell's Theorem (see Aliprantis and Border (1994)) gives a well-defined and unique $\overline{V}$ that is a fixed point of S. Using this $\overline{V}$ in the definition of T and invoking Blackwell's Theorem again yields that for u, v, δ and γ as defined in Theorem 1, there is a well-defined and unique W that is the fixed point of T.

$\Longrightarrow$: By Independence, Stationarity and Indifference to Timing, $\succsim$ satisfies the stronger version of Independence:

$$x \succ y, \alpha \in (0,1) \Longrightarrow \alpha x + (1-\alpha)z \succ \alpha y + (1-\alpha)z. \tag{11}$$

By Theorem 1 of GP (2001), $\succsim$ satisfies Order, Continuity, Set-Betweenness and (11) if and only if there exist linear and continuous functions $U, V : \Delta(C) \longrightarrow \mathbb{R}$ and the function $W : \mathcal{K}(\Delta(C)) \longrightarrow \mathbb{R}$ defined by

$$W(z) = \max_{\mu \in z} \left(U(\mu) + V(\mu)\right) - \max_{\eta \in z} V(\eta), \tag{12}$$

that represents $\succsim$. Lemma 1 establishes some facts about (12) that will be used throughout the Appendix. Define $\overline{V} : Z \longrightarrow \mathbb{R}$ and $\overline{U+V} : Z \longrightarrow \mathbb{R}$ by

$$\begin{aligned} \overline{V}(x) &= \max_{\eta \in x} V(\eta), \\ (\overline{U+V})(x) &= \max_{\eta \in x} \left(U(\eta) + V(\eta)\right). \end{aligned}$$

Lemma 1 *For all x, y,*
(a) $x \succ x \cup y \Longleftrightarrow \overline{V}(y) > \overline{V}(x)$ and $W(x) > W(y)$.
(b) $x \cup y \succ y \Longleftrightarrow (\overline{U+V})(x) > (\overline{U+V})(y)$ and $W(x) > W(y)$.
(c) $x \succ x \cup y \succ y \Longleftrightarrow (\overline{U+V})(x) > (\overline{U+V})(y)$ and $\overline{V}(y) > \overline{V}(x)$.

Proof. (a) $\Longrightarrow$: Set-Betweenness implies that $x \succ y$, and hence $W(x) > W(y)$. Suppose by way of contradiction, $\overline{V}(y) \leq \overline{V}(x)$. Then

$$\begin{aligned} W(x) &= (\overline{U+V})(x) - \overline{V}(x) \\ W(x \cup y) &= (\overline{U+V})(x \cup y) - \overline{V}(x). \end{aligned}$$

Because $x \subset x \cup y$, $(\overline{U+V})(x \cup y) \geq (\overline{U+V})(x)$, and so $W(x \cup y) \geq W(x)$, a contradiction.

$\Longleftarrow$: Let $\overline{V}(y) > \overline{V}(x)$ and $W(x) > W(y)$. There are two cases to consider. If $x \cup y \sim y$, then $W(x) > W(y)$ implies that $x \succ x \cup y$. If $x \cup y \succ y$, then

$$\begin{array}{rcl} W(x \cup y) & = & \overline{(U+V)}(x \cup y) - \overline{V}(y) \\ & > & \overline{(U+V)}(y) - \overline{V}(y) = W(y), \end{array}$$

and so, $\overline{(U+V)}(x \cup y) > \overline{(U+V)}(y)$. It follows that $\overline{(U+V)}(x) = \overline{(U+V)}(x \cup y)$, and so,

$$\begin{array}{rcl} W(x) & = & \overline{(U+V)}(x) - \overline{V}(x) \\ & > & \overline{(U+V)}(x \cup y) - \overline{V}(y) = W(x \cup y). \end{array}$$

That is, $x \succ x \cup y$.

(b) $\Longrightarrow$: Set-Betweenness implies that $x \succ y$, and hence $W(x) > W(y)$. Suppose by way of contradiction that $\overline{(U+V)}(x) \leq \overline{(U+V)}(y)$. Then $\overline{(U+V)}(y) = \overline{(U+V)}(x \cup y)$, and

$$\begin{array}{rcl} W(x \cup y) & = & \overline{(U+V)}(y) - \overline{V}(x \cup y) \\ W(y) & = & \overline{(U+V)}(y) - \overline{V}(y). \end{array}$$

Because $y \subset x \cup y$, $\overline{V}(x \cup y) \geq \overline{V}(y)$, and so $W(y) \geq W(x \cup y)$, a contradiction.

$\Longleftarrow$: There are two cases to consider. First, $\overline{V}(x) \geq \overline{V}(y)$. Then

$$W(x) = \overline{(U+V)}(x) - \overline{V}(x) = W(x \cup y)$$

and since by hypothesis $W(x) > W(y)$, Set-Betweenness implies $x \cup y \succ y$. Second, $\overline{V}(x) < \overline{V}(y)$. Then

$$\begin{array}{rcl} W(x \cup y) & = & \overline{(U+V)}(x) - \overline{V}(y) \\ & > & \overline{(U+V)}(y) - \overline{V}(y) = W(y), \end{array}$$

as desired.

(c) This follows from (a) and (b). ■

Separability and Indifference to Timing imply GP's axioms 5 and 7, respectively. Hence our axioms 1-7 imply GP's axioms 1-7. The proof of Theorem 1 in GP (2004) yields that $U(\cdot)$ can be written as

$$U(\mu) = \int_{C \times Z} (u(c) + \delta W(z))\, d\mu(c, z),$$

for some $u : C \longrightarrow \mathbb{R}$ and $\delta \in (0,1)$. We want to show that

$$V(\mu) = \int_{C \times Z} \big(v(c) + \gamma \overline{V}(y)\big)\, d\mu(c,z),$$

where $\overline{V}(x) = \max_{\mu \in x} \int_{C \times Z} v(c) + \gamma \overline{V}(y) d\mu(c,y)$. Consider two possibilities:

Case (1) **V is constant or U is a positive affine transformations of V.**

In either case, we can take $U' = (U + V)$ and $V' = 0$ and U', V' yield the representation with $v(.) = 0$ and any γ. In particular, the representation holds for $0 < \gamma < 1$.

Case (2) **V is not constant and U is not a positive affine transformations of V.**

It is not possible for there to exist $\alpha \leq -1$ such that $V = \alpha U + \beta$, $\beta \in \mathbb{R}$, since that would contradict nondegeneracy. Therefore consider the case that $\alpha \in (-1, 0)$ or U and V are not affine transformations of each other. The remainder of the proof will establish the result in this case. Let $\Delta_s \subset \Delta$ represent the set of measures on $C \times Z$ with finite support.

Lemma 2 *Under Case (2), there exists* $\overline{\mu}, \underline{\mu} \in \Delta_s$ *such that*

$$\{\overline{\mu}\} \succ \{\overline{\mu}, \underline{\mu}\} \succ \{\underline{\mu}\}.$$

Furthermore, for any finite $L \subset \Delta$, *there exists* $\alpha \in (0,1]$ *such that* $\forall \nu \in L$,

$$\{\overline{\mu}\} \succ \{\overline{\mu}, \nu\alpha\underline{\mu}\} \succ \{\nu\alpha\underline{\mu}\}.$$

Proof. In the proof of Theorem 1 in GP (2004), GP establish that under the conditions of Case (2), there exist $\overline{\nu}, \underline{\nu}$ such that $U(\overline{\nu}) + V(\overline{\nu}) - U(\underline{\nu}) - V(\underline{\nu}) > 0 > V(\overline{\nu}) - V(\underline{\nu})$. By Lemma 1(c),

$$\{\overline{\nu}\} \succ \{\overline{\nu}, \underline{\nu}\} \succ \{\underline{\nu}\}.$$

Since Δ_s is dense in Δ, and U, V are continuous, there exist $\overline{\mu}, \underline{\mu} \in \Delta_s$ such that $U(\overline{\mu}) + V(\overline{\mu}) - U(\underline{\mu}) - V(\underline{\mu}) > 0 > V(\overline{\mu}) - V(\underline{\mu})$, and so,

$$\{\overline{\mu}\} \succ \{\overline{\mu}, \underline{\mu}\} \succ \{\underline{\mu}\}.$$

To prove the second part of the Lemma, take any finite $L \subset \Delta$. By continuity of $\succsim$, for every $\eta \in L$ there exists some $\alpha_\eta \in (0,1)$ such that for all $\alpha'_\eta \in (0, \alpha_\eta]$,

$$\{\overline{\mu}\} \succ \{\overline{\mu}, \eta\alpha'_\eta\underline{\mu}\} \succ \{\eta\alpha'_\eta\underline{\mu}\}.$$

Taking $\alpha = \min\{\alpha_\eta\}_{\eta \in L}$ establishes the result. ■

The next two Lemmas establish Separability of V.

Lemma 3

$$V(\frac{1}{2}(c,z) + \frac{1}{2}(c',z')) = V(\frac{1}{2}(c,z') + \frac{1}{2}(c',z)).$$

Proof. Take $\nu_1 = \frac{1}{2}(c,z) + \frac{1}{2}(c',z')$ and $\nu_2 = \frac{1}{2}(c,z') + \frac{1}{2}(c',z)$. By Lemma 2, there is $\overline{\mu}, \underline{\mu}$ and α such that

$$\{\overline{\mu}\} \succ \{\overline{\mu}, \nu_1\alpha\underline{\mu}\} \succ \{\nu_1\alpha\underline{\mu}\} \text{ and } \{\overline{\mu}\} \succ \{\overline{\mu}, \nu_2\alpha\underline{\mu}\} \succ \{\nu_2\alpha\underline{\mu}\}.$$

By Separability,

$$\{\overline{\mu}, \nu_1\alpha\underline{\mu}\} \sim \{\overline{\mu}, \nu_2\alpha\underline{\mu}\}.$$

By the representation (12), $\{\overline{\mu}, \nu_1\alpha\underline{\mu}\} \sim \{\overline{\mu}, \nu_2\alpha\underline{\mu}\}$
$\iff U(\overline{\mu}) + V(\overline{\mu}) - V(\alpha\nu + (1-\alpha)\mu') = U(\overline{\mu}) + V(\overline{\mu}) - V(\alpha\eta + (1-\alpha)\mu')$
$\iff V(\nu_1\alpha\underline{\mu}) = V(\nu_2\alpha\underline{\mu})$
$\iff \alpha V(\nu_1) + (1-\alpha)V(\underline{\mu}) = \alpha V(\nu_2) + (1-\alpha)V(\underline{\mu})$ by linearity of V
$\iff V(\nu_1) = V(\nu_2)$.
That is, $V(\frac{1}{2}(c,z) + \frac{1}{2}(c',z')) = V(\frac{1}{2}(c,z') + \frac{1}{2}(c',z))$. ■

Lemma 4 *There exists continuous functions* $v : C \longrightarrow \mathbb{R}$, $\widehat{V} : Z \longrightarrow \mathbb{R}$ *such that* $\forall \mu \in \Delta$,

$$V(\mu) = \int_{C \times Z} v(c) + \widehat{V}(x) d\mu$$

Proof. Since V is linear and continuous, there exists continuous $\overline{v} : C \times Z \longrightarrow \mathbb{R}$ such that $V(\mu) = \int \overline{v}(c,x) d\mu$ for all $\mu \in \Delta$. By the previous Lemma,

$$V(\frac{1}{2}(c,x) + \frac{1}{2}(\overline{c},\overline{x})) = V(\frac{1}{2}(c,\overline{x}) + \frac{1}{2}(\overline{c},x)).$$

Then
$V(\frac{1}{2}(c,x) + \frac{1}{2}(\overline{c},\overline{x})) = V(\frac{1}{2}(c,\overline{x}) + \frac{1}{2}(\overline{c},x))$

$\Longrightarrow V(c,x)+V(\bar{c},\bar{x})=V(c,\bar{x})+V(\bar{c},x)$
$\Longrightarrow \overline{v}(c,x)+\overline{v}(\bar{c},\bar{x})=\overline{v}(c,\bar{x})+\overline{v}(\bar{c},x)$
$\Longrightarrow \overline{v}(c,x)=\overline{v}(c,\bar{x})-\overline{v}(\bar{c},\bar{x})+\overline{v}(\bar{c},x)$.
Define $v(c)\equiv\overline{v}(c,\bar{x})-\overline{v}(\bar{c},\bar{x})$ and $\widehat{V}(x)\equiv\overline{v}(\bar{c},x)$. We can then write $\overline{v}(c,x)=v(c)+\widehat{V}(x)$. Therefore $V(\mu)=\int v(c)+\widehat{V}(x)d\mu$ for all $\mu\in\Delta$. ■

The next two Lemmas establish the linearity of $\widehat{V}$.

Lemma 5

$$V(\alpha(c,z)+(1-\alpha)(c,z'))=V((c,\alpha z+(1-\alpha)z')).$$

Proof. Take $\nu_1=\alpha(c,z)+(1-\alpha)(c,z')$ and $\nu_2=(c,\alpha z+(1-\alpha)z')$. By Lemma 2, there exists $\overline{\mu},\underline{\mu}\in\Delta_s$ and β such that

$$\{\overline{\mu}\}\succ\{\overline{\mu},\nu_1\beta\underline{\mu}\}\succ\{\nu_1\beta\underline{\mu}\}\text{ and }\{\overline{\mu}\}\succ\{\overline{\mu},\nu_2\beta\underline{\mu}\}\succ\{\nu_2\beta\underline{\mu}\}.$$

Observe that the first marginals of $\nu_1\beta\underline{\mu}$ and $\nu_2\beta\underline{\mu}$ are the same, the second marginals have finite support, and $\varphi(\nu_1\beta\underline{\mu})=\varphi(\nu_2\beta\underline{\mu})$. Hence by Indifference to Timing,

$$\{\overline{\mu},\nu_1\alpha\underline{\mu}\}\sim\{\overline{\mu},\nu_2\alpha\underline{\mu}\}.$$

Arguing as in Lemma 3,

$$V(\nu_1)=V(\nu_2),$$

that is, $V(\alpha(c,z)+(1-\alpha)(c,z'))=V((c,\alpha z+(1-\alpha)z'))$. ■

Lemma 6 $\widehat{V}$ *is linear.*

Proof. By the previous lemma, $V((c,\alpha x+(1-\alpha)y))=V(\alpha(c,x)+(1-\alpha)(c,y))$. But,
$V((c,\alpha x+(1-\alpha)y))=V(\alpha(c,x)+(1-\alpha)(c,y))$
$\Longrightarrow V((c,\alpha x+(1-\alpha)y))=\alpha V(c,x)+(1-\alpha)V(c,y)$ by linearity of V
$\Longrightarrow v(c)+\widehat{V}(\alpha x+(1-\alpha)y)=\alpha[v(c)+\widehat{V}(x)]+(1-\alpha)[v(c)+\widehat{V}(y)]$ by Lemma 4
$\Longrightarrow v(c)+\widehat{V}(\alpha x+(1-\alpha)y)=v(c)+\alpha\widehat{V}(x)+(1-\alpha)\widehat{V}(y)$
$\Longrightarrow \widehat{V}(\alpha x+(1-\alpha)y)=\alpha\widehat{V}(x)+(1-\alpha)\widehat{V}(y)$
$\Longrightarrow \widehat{V}$ is linear. ■

Recall the function $\overline{V}:Z\longrightarrow\mathbb{R}$ defined by

$$\overline{V}(x)=\max_{\eta\in x}V(\eta).$$

The next two Lemmas establish linearity and continuity of $\overline{V}$.

Lemma 7 $\overline{V}$ *is linear.*

Proof. The linearity of V implies $\overline{V}(\alpha x+(1-\alpha)y)=\alpha\overline{V}(x)+(1-\alpha)\overline{V}(y)$. ■

Lemma 8 $\overline{V}$ *is continuous.*

Proof. Since $V:\Delta\longrightarrow\mathbb{R}$ is continuous, and the correspondence $\Phi:Z\rightsquigarrow\Delta$ defined by $\Phi(x)=x$ is compact-valued and continuous, the Maximum Theorem delivers continuous $\overline{V}(x)=\max_{\eta\in\Phi(x)}V(\eta)$. ■

The next two Lemmas establish the ordinal equivalence between $\overline{V}$ and $\widehat{V}$.

Lemma 9 *If* $x\succ y$, *then,*

$$\overline{V}(y)>\overline{V}(x)\Longleftrightarrow\widehat{V}(y)>\widehat{V}(x).$$

Proof. If $x\succ y$, then $W(x)>W(y)$, and Stationarity implies $W(c,x)>W(c,y)$. Then by Lemma 1(a), $x\succ y$ implies

$$\begin{aligned} x &\succ x\cup y\Longleftrightarrow\overline{V}(y)>\overline{V}(x),\\ \{(c,x)\} &\succ \{(c,x),(c,y)\}\Longleftrightarrow V(c,y)>V(c,x). \end{aligned}$$

By Temptation Stationarity,

$$x\succ x\cup y\Longleftrightarrow\{(c,x)\}\succ\{(c,x),(c,y)\},$$

and so $\overline{V}(y)>\overline{V}(x)\Longleftrightarrow V(c,y)>V(c,x)$. But, by Lemma 4, $V(c,y)>V(c,x)\Longleftrightarrow\widehat{V}(x)>\widehat{V}(y)$ and we are done. ■

Lemma 10 *For all* x,y,

$$\overline{V}(y)>\overline{V}(x)\Longleftrightarrow\widehat{V}(y)>\widehat{V}(x).$$

Proof. We show that the conclusion of Lemma 9 also holds when its hypothesis is negated. So suppose $y\succsim x$. To establish '$\Longleftarrow$', suppose $\overline{V}(x)\geq\overline{V}(y)$. By Lemma 2, there exists $w,z\in Z$ such that $w\succ z$ and $\overline{V}(w)<\overline{V}(z)$.[12] Linearity of W and V implies $y\alpha w\succ x\alpha z$ and $\overline{V}(x\alpha z)>\overline{V}(y\alpha w)$ for all

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[12] Take $w=\{\overline{\mu}\}$ and $z=\{\overline{\mu},\underline{\mu}\}$.

$\alpha \in (0,1)$. Lemma 9 implies $\widehat{V}(x\alpha z) > \widehat{V}(y\alpha w)$ for all $\alpha \in (0,1)$ and continuity of $\widehat{V}$ implies $\widehat{V}(x) \geq \widehat{V}(y)$, as desired.

To establish '$\Longrightarrow$', let $\widehat{V}(x) \geq \widehat{V}(y)$. As above, there is $w, z \in Z$ such that $w \succ z$ and $\overline{V}(w) < \overline{V}(z)$. By Lemma 9, $\widehat{V}(w) < \widehat{V}(z)$. Therefore, $y\alpha w \succ x\alpha z$ and $\widehat{V}(x\alpha z) > \widehat{V}(y\alpha w)$ for all $\alpha \in (0,1)$. Lemma 9 implies $\overline{V}(x\alpha z) > \overline{V}(y\alpha w)$ for all $\alpha \in (0,1)$ and continuity of $\overline{V}$ implies $\overline{V}(x) \geq \overline{V}(y)$, as desired. ■

Lemma 11 *There exists $\gamma > 0$ and $\theta \in \mathbb{R}$ such that $\widehat{V}(x) = \gamma\overline{V}(x) + \theta$ for all $x \in Z$.*

Proof. By Lemma 10, $\widehat{V}$ and $\overline{V}$ are ordinally equivalent. By Lemmas 4, 6, 7 and 8, they are also continuous and linear. Moreover, under the conditions of Case 2, both are nonconstant.[13] It follows that $\widehat{V}$ and $\overline{V}$ are cardinally equivalent (the proof is analogous to Step 2 of Lemma 9 in GP (2004)). ■

Lemma 12 $\gamma < 1$.

Proof. Define $z^c = \{(c, z^c)\}$. Since V is nonconstant (Case 2), we can find η such that $V(\eta) \neq V(z^c)$, where we have identified z^c with (c, z^c). Let $y^1 = \{(c, \eta)\}$ and define y^n inductively as $y^n = \{(c, y^{n-1})\}$. Note that $y^n \longrightarrow z^c$. We shall identify y^n with (c, y^{n-1}) below. By Lemma 2 there exists α such that

$$\{\overline{\mu}\} \succ \{\overline{\mu}, \underline{\mu}\alpha z^c\} \succ \{\overline{\mu}\alpha z^c\}.$$

Since $y^n \longrightarrow z^c$ implies $\{\overline{\mu}, \underline{\mu}\alpha y^n\} \longrightarrow \{\overline{\mu}, \underline{\mu}\alpha z^c\}$ and $\{\underline{\mu}\alpha y^n\} \longrightarrow \{\underline{\mu}\alpha z^c\}$, Continuity of W implies $W(\{\overline{\mu}, \underline{\mu}\alpha y^n\}) \longrightarrow W(\{\overline{\mu}, \underline{\mu}\alpha z^c\})$ and $W(\{\underline{\mu}\alpha y^n\}) \longrightarrow W(\{\underline{\mu}\alpha z^c\})$. Then, there exists N^* such that for all $n \geq N^*$,

$$\{\overline{\mu}\} \succ \{\overline{\mu}, \underline{\mu}\alpha y^n\} \succ \{\overline{\mu}\alpha y^n\}.$$

Without loss of generality, $N^* = 1$. But then,

$W(\{\overline{\mu}, \underline{\mu}\alpha y^n\}) - W(\{\overline{\mu}, \underline{\mu}\alpha z^c\}) \longrightarrow 0$

$\Longrightarrow V(\underline{\mu}\alpha y^n) - V(\underline{\mu}\alpha z^c) \longrightarrow 0$ by representation (12),

$\Longrightarrow V(y^n) - V(z^c) \longrightarrow 0$

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[13] To see that $\widehat{V}$ is nonconstant, note that by Temptation Stationarity, $\{\overline{\mu}\} \succ \{\overline{\mu}, \underline{\mu}\} \Longleftrightarrow \{(c, \overline{\mu})\} \succ \{(c, \overline{\mu}), (c, \underline{\mu})\}$, where $\overline{\mu}, \underline{\mu}$ are as in Lemma 2. By Lemma 1(a), $V(c, \overline{\mu}) \neq V(c, \underline{\mu})$, which implies that $\widehat{V}$ is nonconstant.

$\Longrightarrow \gamma^n[V(\eta) - V(z^c)] \longrightarrow 0.$
Since $V(\eta) \neq V(z^c)$ by construction, it follows that $\gamma < 1$. ■

Lemmas 4 and 11 establish that $V(\mu) = \int(v(c) + \gamma\overline{V}(x) + \theta)d\mu$. By Theorem 4 in GP (2001), we can write $V(\mu) = \int v(c) + \gamma\overline{V}(x)d\mu$.[14] It is also established that $0 < \gamma < 1$. Hence, we are done.

B Appendix: Proof of Theorem 2

First establish that for any (U, V) representing the preference $\succsim$, U and V are not constant, and U is not an affine transformation of V. By hypothesis, $\succsim$ exhibits a preference for commitment, and so, by Lemma 1(a) it is clear that U and V are not constant and that U cannot be a positive affine transformation of V. Suppose by way of contradiction that U is a negative affine transformation of V. Then, $U(c, x) = -\alpha V(c, x) + \beta$ for some $\alpha > 0$, and so, by the functional form of U and V,

$$W(x) = -\alpha\overline{V}(x) + \xi(c), \tag{13}$$

where $\xi(c)$ is some function of c. Observe that by definition of $\overline{V}$, for any x, y,

$$\overline{V}(x) \geq \overline{V}(y) \Longrightarrow \overline{V}(x) = \overline{V}(x \cup y) \geq \overline{V}(y). \tag{14}$$

But by Lemma 2, there is w, z such that

$$W(w) > W(w \cup z) > W(z).$$

It follows by (13) that

$$\overline{V}(z) > \overline{V}(w) \text{ and } \overline{V}(z) > \overline{V}(z \cup w) > \overline{V}(w),$$

contradicting (14). Hence, for any (U, V) representing $\succsim$, U is not an affine transformation of V.

The proof of Theorem 3 in GP (2004) establishes that $(u', v', \delta', \gamma')$ represents $\succsim$ if and only if $\delta = \delta'$ and there exist $\alpha > 0, \beta_u, \beta_v \in \mathbb{R}$ such that $u' = \alpha u + \beta_u$ and $V' = \alpha V + \beta_v$. We show that $\gamma = \gamma'$ and $v' = \alpha v + (1-\gamma)\beta_v$ if and only if $V' = \alpha V + \beta_v$. Begin by noting that, by hypothesis, there is

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[14] To see that U is not an affine transformation of V, argue as in Appendix B.

a preference for commitment at some x and hence V is nonconstant. Therefore v is nonconstant as well. Also note that $V' = \alpha V + \beta_v$ if and only if $\overline{V}' = \alpha\overline{V} + \beta_v$. Now, observe that,

$$\begin{aligned}
&V'(c,x) = \alpha V(c,x) + \beta_v \\
&\Leftrightarrow v'(c) + \gamma'\overline{V}'(x) = \alpha[v(c) + \gamma\overline{V}(x)] + \beta_v \\
&\Leftrightarrow v'(c) + \gamma'\overline{V}'(x) = \alpha v(c) + (1-\gamma)\beta_v + \gamma[\alpha\overline{V}(x) + \beta_v] \\
&\Leftrightarrow v'(c) + \gamma'\overline{V}'(x) = \alpha v(c) + (1-\gamma)\beta_v + \gamma\overline{V}'(x) \\
&\Leftrightarrow v'(c) - [\alpha v(c) + (1-\gamma)\beta_v] = (\gamma - \gamma')\overline{V}'(x) \\
&\Leftrightarrow \gamma = \gamma' \text{ and } v' = \alpha v + (1-\gamma)\beta_v.
\end{aligned}$$

The last equivalence holds because v is nonconstant.

C Appendix: Proof of Theorem 3

First, two preliminary lemmas.

Lemma 13 *(a)* $x \subset y \Longrightarrow \overline{V}(y) \geq \overline{V}(x)$ *and* $W(y) + \overline{V}(y) \geq W(x) + \overline{V}(x)$.
(b) $x \subset y$ *and* $y \succsim x \Longrightarrow \{(c,x),(c,y)\} \sim \{(c,y)\}$.

Proof. (a) If $x \subset y$, the definition of $\overline{V}$ implies

$$\overline{V}(y) \geq \overline{V}(x).$$

To see $W(y) + \overline{V}(y) \geq W(x) + \overline{V}(x)$, note that $x \subset y$ implies

$$\max_{\mu \in y}(U + V) \geq \max_{\mu \in x}(U + V). \tag{15}$$

The result then follows from the fact that for any $z \in Z$,

$$W(z) + \overline{V}(z) = \left(\max_{\mu \in z} U + V - \overline{V}(z)\right) + \overline{V}(z) = \max_{\mu \in z} U + V.$$

(b) By Stationarity and Set-Betweenness, if $y \sim x$ then $\{(c,x),(c,y)\} \sim \{(c,y)\}$. Next, if $y \succ x$, then by Stationarity, $\{(c,y)\} \succ \{(c,x)\}$ and so $U(c,y) > U(c,x)$. Furthermore, since $x \subset y$, $\overline{V}(y) \geq \overline{V}(x)$ and so $V(c,y) \geq V(c,x)$. But then by Lemma 1(a), $\{(c,y)\} \precsim \{(c,x),(c,y)\}$. Since $\{(c,y)\} \succ \{(c,x)\}$, Set-Betweenness implies that $\{(c,x),(c,y)\} \sim \{(c,y)\}$. ■

Lemma 14 *If* $\{\mu\} \succ \{\mu,\eta\} \succ \{\eta\}$ *and* $\{(c,\mu)\} \succ \{(c,\mu),(c,\eta)\} \succ \{(c,\eta)\}$, *then*

$$\{(c,\mu),(c,\eta)\} \succsim \{(c,\{\mu,\eta\})\} \Longleftrightarrow \gamma \leq \delta.$$

Proof. If the hypothesis holds, then

$$\begin{aligned}
&\{(c,\mu),(c,\eta)\} \succsim \{(c,\{\mu,\eta\})\}\\
&\iff W(\{(c,\mu),(c,\eta)\}) \geq W(\{(c,\{\mu,\eta\})\\
&\iff u(c)+\delta W(\mu)+v(c)+\gamma\overline{V}(\mu)-v(c)-\gamma\overline{V}(\eta) \geq u(c)+W(\{\mu,\eta\})\\
&\iff u(c)+\delta U(\mu)+\gamma[V(\mu)-V(\eta)] \geq u(c)+\delta(U(\mu)+V(\mu)-V(\eta))\\
&\iff u(c)+\delta U(\mu)+\gamma[V(\mu)-V(\eta)] \geq u(c)+\delta U(\mu)+\delta[V(\mu)-V(\eta)]\\
&\iff \gamma[V(\mu)-V(\eta)] \geq \delta[V(\mu)-V(\eta)]\\
&\iff \gamma \leq \delta \text{ since } V(\mu)-V(\eta)<0 \text{ by } \{\mu\} \succ \{\mu,\eta\} \text{ and Lemma 1(a).} \quad \blacksquare
\end{aligned}$$

Proof of $(a) \iff (b)$:

$\Longrightarrow$: We prove the contrapositive, that is, $\gamma \geq \delta$ implies $\{(c,x),(c,y)\} \precsim \{(c,y)\}$ for all x,y such that $x \subset y$. By Lemma 13(b), it suffices to prove that $\gamma \geq \delta$ implies

$$\{(c,x),(c,y)\} \precsim \{(c,y)\} \text{ for all } x,y \text{ such that } x \subset y \text{ and } x \succ y.$$

So let $\gamma \geq \delta$ and take any x,y such that $x \subset y$ and $x \succ y$. By Lemma 13(a),

$$\begin{aligned}
W(y)+\overline{V}(y) &\geq W(x)+\overline{V}(x),\\
\text{and } \overline{V}(y) &\geq \overline{V}(x).
\end{aligned}$$

Since $\frac{\gamma}{\delta} \geq 1$, it follows that

$$W(y)+\frac{\gamma}{\delta}\overline{V}(y) \geq W(x)+\frac{\gamma}{\delta}\overline{V}(x),$$

and so

$$\delta W(y)+\gamma\overline{V}(y) \geq \delta W(x)+\gamma\overline{V}(x).$$

Adding $u(c)+v(c)$ to both sides gives $U(c,y)+V(c,y) \geq U(c,x)+V(c,x)$. By Stationarity, $x \succ y$ implies $U(c,x) > U(c,y)$, and so it follows from Lemma 1(b) that

$$\{(c,x),(c,y)\} \precsim \{(c,y)\},$$

as desired.

$\Longleftarrow$: By hypothesis, there is x,y such that $x \subset y$ and $x \succ y$. Since $y = x \cup y$, $x \succ x \cup y$. By Lemma 1(a), this implies that V is nonconstant and U is not a positive affine transformation of V. Hence, by Lemma 2, there is $\mu,\eta \in \Delta$ such that $\{\mu\} \succ \{\mu,\eta\} \succ \{\eta\}$.

We show that $\gamma \geq \delta$ is implied by $\{(c,x),(c,y)\} \precsim \{(c,y)\}$ for all x, y such that $x \subset y$. So suppose the latter claim. Then $\{\mu\} \subset \{\mu,\eta\}$ and $\{\mu\} \succ \{\mu,\eta\}$ implies

$$\{(c,\mu),(c,\{\mu,\eta\})\} \sim \{(c,\{\mu,\eta\})\}. \tag{16}$$

By Stationarity, $\{\mu\} \succ \{\mu,\eta\}$ implies $\{(c,\mu)\} \succ \{(c,\{\mu,\eta\})$. Then, by Set-Betweenness and (16),

$$\{(c,\mu)\} \succ \{(c,\mu),(c,\{\mu,\eta\})\} \sim \{(c,\{\mu,\eta\})\}.$$

It follows from Lemma 1 that $U(c,\{\mu,\eta\}) + V(c,\{\mu,\eta\}) \geq U(c,\mu) + V(c,\mu)$ and $V(c,\{\mu,\eta\}) > V(c,\mu)$. But,

$U(c,\{\mu,\eta\}) + V(c,\{\mu,\eta\}) \geq U(c,\mu) + V(c,\mu)$
$\Longrightarrow \delta W(\{\mu,\eta\}) + \gamma \overline{V}(\{\mu,\eta\}) \geq \delta W(\{\mu\}) + \gamma \overline{V}(\{\mu\})$
$\Longrightarrow W(\{\mu,\eta\}) + \frac{\gamma}{\delta}\overline{V}(\{\mu,\eta\}) \geq W(\{\mu\}) + \frac{\gamma}{\delta}\overline{V}(\{\mu\})$
$\Longrightarrow U(\mu) + V(\mu) - V(\eta) + \frac{\gamma}{\delta}V(\eta) \geq U(\mu) + \frac{\gamma}{\delta}V(\mu)$
$\Longrightarrow \frac{\gamma}{\delta}(V(\eta) - V(\mu)) \geq V(\eta) - V(\mu)$
$\Longrightarrow \frac{\gamma}{\delta} \geq 1.$
The last implication holds because $V(\eta) - V(\mu) > 0$.[15]

Proof of $(a) \Longleftrightarrow (c)$ **:**

$\Longrightarrow$: Let $\gamma < \delta$. By Lemma 1, the hypothesis $\{\mu\} \succ \{\mu,\eta\} \sim \{\eta\}$ implies,

$$\begin{aligned} V(\eta) - V(\mu) &> 0 & (17)\\ U(\mu) - U(\eta) &> 0 & (18)\\ \frac{U(\mu) - U(\eta)}{V(\eta) - V(\mu)} &\leq 1 & (19) \end{aligned}$$

By repeated application of Stationarity, $\{\mu\} \succ \{\eta\}$ implies $\{\mu^{+t}\} \succ \{\eta^{+t}\}$. Then by Lemma 1(b), $\{\mu^{+t},\eta^{+t}\} \succ \{\eta^{+t}\}$ if and only if $U(\mu^{+t}) + V(\mu^{+t}) > U(\eta^{+t}) + V(\eta^{+t})$ if and only if

$$(\frac{\gamma}{\delta})^t < \frac{U(\mu) - U(\eta)}{V(\eta) - V(\mu)}.$$

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[15] From $V(c,\{\mu,\eta\}) > V(c,\mu)$ it follows that $\overline{V}(\{\mu,\eta\}) > \overline{V}(\mu)$, and thus $V(\eta) - V(\mu) > 0$.

By (17), (18) and (19), we have $0 < \frac{U(\mu)-U(\eta)}{V(\eta)-V(\mu)} \leq 1$. Define $\phi(t) \equiv (\frac{\gamma}{\delta})^t$ and note that ϕ is a continuous, monotone decreasing function with $\phi(0) = 1$ and $\phi(\infty) = 0$. Hence,

$$\phi(\infty) < \frac{U(\mu) - U(\eta)}{V(\eta) - V(\mu)} \leq \phi(0).$$

By the intermediate value theorem, there exists $t^* > 0$ such that for all $t \leq t^*$, $\{\mu^{+t}, \eta^{+t}\} \sim \{\eta^{+t}\}$, and for all $t > t^*$, $\{\mu^{+t}, \eta^{+t}\} \succ \{\eta^{+t}\}$. Uniqueness of t^* follows from the monotonicity of $\phi(\tau)$.

$\Longleftarrow$: Suppose $\{\mu\} \succ \{\mu, \eta\} \sim \{\eta\}$. Recall from the above that $\{\mu^{+t}, \eta^{+t}\} \succ \{\eta^{+t}\}$ if and only if

$$U(\mu) - U(\eta) > (\frac{\gamma}{\delta})^t (V(\eta) - V(\mu)), \tag{20}$$

and that for $t = 0$, $U(\mu) - U(\eta) < V(\eta) - V(\mu)$. Suppose by way of contradiction that $\gamma \geq \delta$. It follows that (20) never holds for any t, that is, there is no t^* such that for all $t > t^*$, $\{\mu^{+t}, \eta^{+t}\} \succ \{\eta^{+t}\}$, contradicting the hypothesis that $\succsim$ exhibits preference reversals. Hence $\gamma < \delta$.

Proof of $(a) \Longleftrightarrow (d)$:

Since $x^{+(t+1)} \cup y = B \times \{x^{+t}, y\}$, it follows from the representation that choice of continuation menu is according to

$$\max_{\{x, y^t\}} \delta W + \gamma \overline{V}.$$

Let $c_y \in \arg\max_B (u + v)$, $c_x \in \arg\max_{B'} (u + v)$, $b_y \in \arg\max_B v$, and $b_x \in \arg\max_{B'} v$. Observe that

$$\begin{aligned}
W(y) &= \frac{1}{1-\delta}\left(u(c_y) + v(c_y) - \frac{1}{1-\gamma} v(b_y)\right), \quad \overline{V}(y) = \frac{1}{1-\gamma} v(b_y), \\
W(x) &= \frac{1}{1-\delta}\left(u(c_x) + v(c_x) - \frac{1}{1-\gamma} v(b_x)\right), \quad \overline{V}(x^t) = \sum_{\tau=0}^{t-1} \gamma^\tau v(b_y) + \frac{\gamma^t}{1-\gamma} v(b_x), \\
W(x^t) &= \sum_{\tau=0}^{t-1} \delta^\tau \left(u(c_y) + v(c_y) - \sum_{\tau=0}^{t-1} \gamma^\tau v(b_y) - \frac{\gamma^t}{1-\gamma} v(b_x)\right) \\
&\quad + \frac{\delta^t}{1-\delta}\left(u(c_x) + v(c_x) - \frac{1}{1-\gamma} v(b_x)\right).
\end{aligned}$$

Using the above expressions we get

$$\begin{aligned}\delta W(x^{+t})+\gamma\overline{V}(x^{+t}) &\geq \delta W(y)+\gamma\overline{V}(y)\\ &\Longleftrightarrow \frac{1}{\delta^t(1-\delta)(1-\gamma\delta)}+\frac{N_2}{(1-\delta)N_1}\geq\frac{\gamma}{\delta(1-\gamma)}(\frac{\gamma}{\delta})^t+\frac{\gamma^t}{1-\delta\gamma},\end{aligned}$$

where

$$\begin{aligned}N_1 &= v(b_x)-v(b_y)>0\\ N_2 &= u(c_y)+v(c_y)-u(c_x)-v(c_x)+\frac{1}{1-\gamma}N_1>0.\end{aligned}$$

Since $\frac{\gamma}{\delta}<1$, the right hand side in the following inequality

$$\frac{1}{\delta^t(1-\delta)(1-\gamma\delta)}+\frac{N_2}{(1-\delta)N_1}\geq\frac{\gamma}{\delta(1-\gamma)}(\frac{\gamma}{\delta})^t+\frac{\gamma^t}{1-\delta\gamma},$$

goes to 0 monotonically and the left hand side monotonically goes to infinity as t goes to infinity. Hence, there exists $t^*>0$ such that for all $t\geq t^*$,

$$x^{t+}\cup y\succ y,$$

that is, the agent chooses x^{+t}, as desired.

Proof of $(a)\Longleftrightarrow(e)$:

$\Longleftarrow$: By hypothesis there exists a preference for commitment, and so we are in Case 2 (see proof of Theorem 1). Then by Lemma 2, there exists μ,η such that $\{\mu\}\succ\{\mu,\eta\}\succ\{\eta\}$. We show that Preference for Early Choice implies

$$\{(c,\mu)\}\succ\{(c,\mu),(c,\eta)\}\succ\{(c,\eta)\}.$$

The result then follows from Lemma 14.

By Stationarity and Preference for Early Choice, $\{\mu,\eta\}\succ\{\eta\}$ implies

$$\{(c,\mu),(c,\eta)\}\succsim(c,\{\mu,\eta\})\succ\{(c,\eta)\},$$

that is, $\{(c,\mu),(c,\eta)\}\succ\{(c,\eta)\}$. Also, by Temptation Stationarity, $\{\mu\}\succ\{\mu,\eta\}$ implies

$$\{(c,\mu)\}\succ\{(c,\mu),(c,\eta)\}.$$

Therefore, $\{\mu\} \succ \{\mu, \eta\} \succ \{\eta\}$ implies

$$\{(c,\mu)\} \succ \{(c,\mu),(c,\eta)\} \succ \{(c,\eta)\}.$$

$\Longrightarrow$: Take $\mu, \eta \in \Delta$ such that $\{\mu\} \succ \{\mu, \eta\} \succ \{\eta\}$. We show that $\gamma \leq \delta$ implies

$$\{(c,\mu)\} \succ \{(c,\mu),(c,\eta)\} \succ \{(c,\eta)\},$$

and then the result follows from Lemma 14.

By Temptation Stationarity, $\{\mu\} \succ \{\mu, \eta\}$ implies

$$\{(c,\mu)\} \succ \{(c,\mu),(c,\eta)\}.$$

To show that $\{(c,\mu),(c,\eta)\} \succ \{(c,\eta)\}$, by Lemma 1(b) it suffices to show that $U(c,\mu) + V(c,\mu) > U(c,\eta) + V(c,\eta)$.[16] By hypothesis, $\{\mu, \eta\} \succ \{\eta\}$, and so by Lemma 1(b), $U(\mu) + V(\mu) > U(\eta) + V(\eta)$. Also, $\{\mu\} \succ \{\mu, \eta\}$ implies (by Lemma 1(a)) that $V(\mu) < V(\eta)$. Observe that $U(\mu) + V(\mu) > U(\eta) + V(\eta)$ implies

$$U(\mu) - U(\eta) > V(\eta) - V(\mu),$$

and $V(\eta) > V(\mu)$ and $\gamma \leq \delta$ implies

$$V(\eta) - V(\mu) \geq \frac{\gamma}{\delta}\left(V(\eta) - V(\mu)\right).$$

Therefore, $U(\mu) - U(\eta) > \frac{\gamma}{\delta}\left(V(\eta) - V(\mu)\right)$. But

$U(\mu) - U(\eta) > \frac{\gamma}{\delta}\left(V(\eta) - V(\mu)\right)$
$\Longrightarrow \delta U(\mu) - \delta U(\eta) > \gamma V(\eta) - \gamma V(\mu)$
$\Longrightarrow \delta U(\mu) + \gamma V(\eta) > \delta U(\eta) + \gamma V(\mu)$
$\Longrightarrow u(c) + \delta U(\mu) + v(c) + \gamma V(\eta) > u(c) + \delta U(\eta) + v(c) + \gamma V(\mu)$
$\Longrightarrow U(c,\mu) + V(c,\mu) > U(c,\eta) + V(c,\eta)$,

and hence by Lemma 1(b),

$$\{(c,\mu),(c,\eta)\} \succ \{(c,\eta)\},$$

as was to be shown.

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[16] It suffices because by Set-Betweenness $\{(c,\mu)\} \succ \{(c,\mu),(c,\eta)\}$ implies $\{(c,\mu)\} \succ \{(c,\eta)\}$, and so $U(c,\mu) > U(c,\eta)$ already holds.

D Appendix: Derivation of Table in Section 5.4

A choice between continuation menus x and y is a choice from

$$z = \{(c,x),(c,y)\},$$

and choice from z is determined by $\max_{\mu \in z}(U(\mu)+V(\mu))$. By the functional forms for U and V, choice of continuation menu is determined by

$$\max_{\{x,y\}}(W(\cdot)+\frac{\gamma}{\delta}\overline{V}(\cdot)).$$

Therefore, the agent chooses to commit if $\frac{W(x)-W(y)}{\overline{V}(y)-\overline{V}(x)} > \frac{\gamma}{\delta}$. Note that by Lemma 1(a), the hypotheses $x \subset y$ and $x \succ y$ imply $\frac{W(x)-W(y)}{\overline{V}(y)-\overline{V}(x)} > 0$. This explains the first two rows of the table.

To see why the agent never commits when $\gamma \geq \delta$, note that for any menu w,
$W(w) + \frac{\gamma}{\delta}\overline{V}(w)$
$= \max_{\mu \in w} U + V - \max_{\eta \in w} V + \frac{\gamma}{\delta}\overline{V}(w)$
$= \max_{\mu \in w} U + V + k \max_{\eta \in w} V$, where $k = \frac{\gamma}{\delta} - 1 > 0$.
Hence, $x \subset y$ implies that $\frac{W(x)-W(y)}{\overline{V}(y)-\overline{V}(x)} \leq \frac{\gamma}{\delta}$.

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