Some Fubini theorems on product $\sigma-$algebras for non-additive measures

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Extended Abstract

Since the pioneer paper of Paolo Ghirardato (see also Rainer Dyckerhoff) it is known that Fubini theorem for non-additive measures can hold only when considering a special class of functions, namely slice-comonotone functions. Not surprisingly we show that in case of product $\sigma-$algebras, some continuity assumptions of the capacities are also required.

Building upon a natural method for deriving Fubini theorems for capacities on product $\sigma-$algebras, through the obtention of $\sigma-$additive probability measures coinciding with given capacities on suitable chains, we first propose some "change of order of integration" versions of Fubini theorem.

It turns out that for capacities v_i $i = 1, 2$ defined on the $\sigma-$algebras $\mathcal{B}_i$ of the Borel sets of compact metric spaces Ω_i, Fubini theorem applied to a continuous slice-comonotone mapping $f : \Omega_1 \times \Omega_2 \longmapsto \mathbb{R}$ holds true by merely assuming that each v_i is either continuous from below on open sets or continuous from above on closed sets. Thus we generalize a previous result of Martin Brüning, where the v_i's were assumed to be totally monotone, continuous from below on open sets and continuous from above everywhere.

We then consider a pair of capacities v_i $i = 1, 2$ defined on $\sigma-$algebras $\mathcal{B}_i$ of subsets of nonempty sets Ω_i with v_i either convex or concave. We first show that Fubini theorem holds true for a bounded $\mathcal{B} = \mathcal{B}_1 \otimes \mathcal{B}_2$ measurable mapping $f : \Omega_1 \times \Omega_2 \longmapsto \mathbb{R}$ when assuming continuity from below at Ω_i for

v_i if v_i is convex (resp. continuity from above at the empty set $\varnothing$ for v_i if v_i is concave).Then assuming that the Ω_i 's are metric spaces, that the $\mathcal{B}_i$'s are the $\sigma-$algebras of Borel sets and that $f : \Omega_1 \times \Omega_2 \longmapsto \mathbb{R}$ is bounded and continuous, we show that $\sigma-$continuity of v_i can be relaxed to inner continuity on open sets and outer continuity on closed sets.

The paper ends by revisiting the notion of independent product of two belief functions v_1, v_2. Building upon Chateauneuf and Rébillé, where it is proved that $\sigma-$continuous belief functions defined on $\mathcal{P}(\mathbb{N})$ are characterized through non negative Möbius inverses with null masses on infinite sets, we show that on $\mathcal{P}(\mathbb{N}) \otimes \mathcal{P}(\mathbb{N})$ the independent product of $\sigma-$ continuous belief functions can be readily defined. Moreover, through the derivation of a related form of the full Fubini theorem, we obtain a natural interpretation of the independent product of two $\sigma-$continuous belief functions, as a belief function obtained through a device à la Dempster with two stochastically independent "sharp" messages.

References

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