

**Leadership Giving in Charitable Fund-Raising:
Matching Grants or Seed Money?**

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Abstract

A benefactor's leadership gift can be packaged as seed money or a matching grant. Small donors, charities and benefactors may disagree about this choice. Small donors' preferences will depend on their utility functions, the donor base and the size of the leadership gift. For any given leadership gift, a matching scheme will raise more money and hence is preferred by both charities and benefactors. If small donors decrease their giving at higher match ratios, benefactors may prefer smaller matching gifts to the larger gifts they would make if restricted to seed money. When this means that matching raises less in total, the charity and benefactor will disagree. (JEL H41, L31)

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1: Introduction

Consider a charitable organization in the early stage of a fund-raising campaign. The organization has been approached by a wealthy benefactor interested in donating a large sum. Should the charity accept the donation in lump-sum seed money or ask the benefactor to give the money as a challenge matching gift? What will the lead donor prefer? This paper provides the answers.

A matching grant is conditional on the amount raised from smaller donors. For each dollar raised, the lead donor will match it with $\$h$ up to some maximum amount. The difference between a matching gift and lump-sum seed money lies in the effect of the lead donor's contribution on small donor's contributions: A matching gift is "multiplicative." Seed money is "additive" to the donations made by others. A recent example of a matching gift is the \$10m pledged by the Australian philanthropist David Thomas to the US-based Nature Conservancy.¹ The match ratio is one for all donations between \$10,000 and \$1 million. Organizations as diverse as Rotary International, Sara Lee, IBM and the Arkansas Humanities Council use matching grants to support small donor or employee voluntary contributions to an approved list of charities. Such a fund-raising strategy has won the endorsement of professional fund-raisers: Dove (2001, p. 21)² states that one should "*[n]ever underestimate the power of a challenge gift. North Americans love the 'buy one, get one free' concept, and the ability to leverage a gift is an irresistible temptation for many.*"

¹ See "\$10m hand raises the green stakes", *Herald Sun*, November 29, 2006.

² Dove's statement is cited in Karlan and List (2007).

Despite the popularity of matching schemes, few research papers directly examine the choice between matching grants and seed money. An exception is Karlan and List (2007) which reports a field experiment on individual giving when a lead donor provides challenge gifts with various match ratios. The study finds: (1) the size of the donor base is increased with the matching approach; (2) the per capita donation from small donors is first increasing, then decreasing in the match ratio; and (3) the total amount raised is increasing in the match ratio.³

A number of papers address the role of lump-sum seed money in fund-raising. Andreoni (1998) argues that many charitable projects involve significant fixed costs and can only be undertaken if a minimum threshold of total funds raised can be surpassed. Leadership seed money can help clear this hurdle. Another explanation for the importance of leadership gifts focuses on an information asymmetry about the quality of the projects undertaken by charitable organizations. The announcement of a leadership gift can help signal the high quality of the particular project and therefore increase donations in a fund-raising campaign; see Vesterlund (2003) and Andreoni (2006b). List and Lucking-Reiley (2002) provides experimental results showing that increased seed money increases both the participation rate of donors and the average gift received from participating donors.⁴

Our comparison of matching schemes and lump-sum seed money is based on a model of the private provision of public goods. To crystallize the difference between the two fund-raising methods, we assume that there is no information asymmetry about the quality of the provider of the public good. That is, neither the presence of a large donor nor the fund-raising method chosen signals the quality of the charity.

³ In a different experiment, Eckel and Grossman (2003) find that matching gifts are more effective in increasing individuals' donations than a rebate system, despite the functional equivalence of the two approaches. The difference in the amounts raised by the two schemes is attributed to a behavioral bias on the part of donors.

⁴ This finding is interesting, because it shows that the leadership gift can have a "crowding-in" effect. A "crowding-out" effect will occur when the leadership gift does not signal higher quality. Analysis of the crowding-out effect of government spending on the provision of public goods started with Roberts (1984) and was later extended in Bergstrom et al. (1986) and Andreoni (1988, 1989), and tested in Anderoni (1993) and Andreoni and Payne (2003).

In the first part of the analysis, the number of donors and the amount of the leadership gift are given. Our primary result is that, for a given leadership gift, the matching approach generates a higher amount of total donations than does lump-sum seed money. We then examine the relationship between small donors' contributions and the match ratio. Will an increase in the match ratio, making the public good "cheaper," lead to an increased contribution from small donors? The comparative static results show that the relation can be either positive or negative and this ambiguity is demonstrated through numerical examples. When the small donors' gifts are decreasing in the match ratio and a lead donor can choose *both* the size of his gift and the method of fund-raising, then matching is always the lead donor's preferred option. However, the charity will disagree whenever it could raise more if the lead donor were restricted to choosing an amount to be provided as seed money. A sufficient condition to rule out conflict between a charity and its benefactor is that small donors do not decrease their individual donations in response to an increase in the match ratio.

The model is then extended to include the publicity of leadership gifts and the tax deductibility of donations, respectively. We relax the assumption that the number of donors is fixed and incorporate the impact of the leadership gift on the size of the donor pool. Because the media routinely report large leadership gifts, there can be a positive relation between the size of the leadership gift and the size of the donor base. We show that each small donor will donate less when the donor base increases. Nevertheless, the overall effect of an enlarged donor base on the total amount of donations will be non-negative under either fund-raising approach.

Finally, assuming that tax receipts are not used to increase the production of the public good, the impact on individual donations of an increase in the personal tax rate is ambiguous. Intuitively, when the tax rate increases, an individual's disposable income decreases. Therefore, the *income* effect on individual donation is negative. On the other hand, donations are tax deductible, which makes the public good relatively cheap. The *substitution* effect is positive. The net result of a change in the tax rate on individual

donations is ambiguous. However, the existence of a personal income tax does not change the result that small donors contribute more under a matching scheme than under the seed money approach.

Section 2 describes the model and derives the major results when the number of donors and the size of the leadership gift are fixed. Section 3 examines the relation between the match ratio and small donor contributions, the resultant leadership gift and the total raised. In Section 4 we examine the relation between the size of the donor base and the total amount raised, emphasizing the publicity effect of a leadership gift. In Section 5 we show that matching is the preferred of lead donors who choose both the size of their gift and the method of packaging but that the charity may disagree. Section 6 uses examples of small donor utility functions to illustrate the paper's results. Section 7 focuses on small donor preferences for seed money versus matching schemes. In Section 8, we discuss the impact of an income tax on individual donations. Section 9 contains conclusions and extensions.

2: Matching versus seed money for a given donor base and leadership gift

A nonprofit organization whose budget comes from voluntary donations is assumed to produce G units of a public good. All money raised goes directly to the provision of the public good. Unlike Andreoni (1998), we do not assume that the total funds raised must exceed some minimum level before it is feasible to produce the public good.

There are two kinds of donors: a lead donor and N small followers. The public good model we adopt was introduced by Warr (1982, 1983) and Bergstrom et al. (1986).⁵ The utility U^i of a small donor i , $i = 1, \dots, N$, is a function of her private consumption x_i and the total amount of the public good G . $U^i = U^i(x_i, G)$ is continuous and strictly quasi-concave. Individuals have diminishing marginal utility with respect to both the

⁵ For a detailed account of the development and extension of the public goods model, see the survey by Andreoni (2006a).

public and private good: $U_1 > 0$, $U_2 > 0$, and $U_{11} < 0$, $U_{22} < 0$. Furthermore, we assume that increasing consumption of the public good will not reduce the utility from an additional unit of the private good; i.e., $U_{12} \geq 0$.

2.1: The seed money scheme

Assume the large donor gives g_0 . Subject to her wealth m_i , each small donor i chooses her donation g_i to maximize her utility conditional on the other donors' contributions g_j :

$$\max_{g_i} U^i \left(x_i, g_i + \sum_{j=1, j \neq i}^N g_j + g_0 \right), \quad (1)$$

subject to: $x_i + g_i = m_i$, and $x_i \geq 0$, $g_i \geq 0$.

We consider the Nash Equilibrium of the public good game played among the small donors. The proof that there exists a unique Nash equilibrium is in Bergstrom et al. (1986). For simplicity, we assume an identical, homogeneous population of small donors. The Nash equilibrium level of donation is then the same for each small donor.

In the Nash equilibrium, every small donor will donate g^{**} . Throughout the paper we will use a double asterisk (**) to denote a value under the seed money approach and a single asterisk (*) to denote a value under the matching approach.

The first-order condition is

$$-U_1(m - g^{**}, Ng^{**} + g_0) + U_2(m - g^{**}, Ng^{**} + g_0) \leq 0 \quad (2)$$

with equality at an interior optimum. The second-order condition for a maximum is satisfied:

$$U_{11} - 2U_{12} + U_{22} < 0.$$

2.2: The matching scheme

We initially assume that for every dollar raised from small donors, the lead donor has agreed to give h dollars. The lead donor's pockets are assumed to be deep enough. Given that small donors are identical and all other small donors contribute at the level g , a representative small donor's optimization program is:

$$\begin{aligned} \max_{g_i} \quad & U(m - g_i, [1+h][g_i + [N-1]g]), \\ \text{subject to:} \quad & x_i + g_i = m_i, \text{ and } x_i \geq 0, g_i \geq 0. \end{aligned} \quad (3)$$

We assume U is such that an interior solution is always attained and the first-order condition is

$$-U_1(m - g^*, N[1+h]g^*) + [1+h]U_2(m - g^*, N[1+h]g^*) = 0 \quad (4)$$

Relation (4) determines the reaction function $g^*(h)$ describing how individual small donors' contributions respond to the match ratio h . The second-order condition for a maximum is again satisfied:

$$U_{11} - 2[1+h]U_{12} + [1+h]^2 U_{22} < 0.$$

Given the reaction function $g^*(h)$ and the number of small donors N , the match ratio h determines the donation required from the lead donor's pledged amount, g_0 ; i.e., $g_0 = Nhg^*(h)$.

2.3: Comparison of the two schemes

Given a match ratio h , the large donor will be called upon to give $g_0 = Nhg^*(h)$. That same large gift could instead have been packaged as seed money. There is a striking difference between the small donor's optimality conditions under the two approaches. Under the matching gift approach, if the small donor reaches an interior optimum donation level then $-U_1 + [1+h]U_2 = 0$. That is, the marginal utility from an additional unit of the public good is *less* than the marginal utility of private consumption at the

optimum. Under the seed money approach, if the small donor reaches an interior optimum donation level, $-U_1 + U_2 = 0$. Intuitively, diminishing marginal utility suggests that small donors will donate more under the matching scheme.

Proposition 1: *Assume a homogenous group of small donors whose utility depends on both private consumption x and the public good G , $U = U(x, G)$, with $U_1 > 0, U_2 > 0, U_{11} < 0, U_{22} < 0, U_{12} \geq 0$. Given a leadership gift of a fixed amount, a matching system always leads to larger small donor contributions, and therefore a larger total donation.*

Proof: Suppose otherwise and $g^{**} \geq g^*$. Recall the assumption that U is such that an interior solution is always attained under a matching system. If $g^{**} \geq g^*$, then relation (2) must also hold as an equality. The equality variant of (2) and relation (4) can be rewritten as relations (5) and (6) respectively.

$$U_1(m - g^{**}, Ng^{**} + g_0) = U_2(m - g^{**}, Ng^{**} + g_0), \quad (5)$$

$$U_1(m - g^*, Ng^* + g_0) = [1 + h]U_2(m - g^*, Ng^* + g_0), \quad (6)$$

Substituting g_0 for hNg^* in (4) gives (6). Since $U_{11} < 0$ and $U_{12} \geq 0$, $g^{**} \geq g^*$ implies the left-hand-side of (5) is greater than or equal to the left-hand-side of (6). But this implies that the right-hand-side of (5) must then be no less than the right-hand-side of (6). Since $h > 0$, we have

$$U_2(m - g^{**}, Ng^{**} + g_0) \geq [1 + h]U_2(m - g^*, Ng^* + g_0) > U_2(m - g^*, Ng^* + g_0).$$

But $g^{**} \geq g^*$, $U_{22} < 0$ and $U_{12} \geq 0$ imply $U_2(m - g^{**}, Ng^{**} + g_0) < U_2(m - g^*, Ng^* + g_0)$ and we have a contradiction. **QED**

Let $G^*(h)$ denote the total amount raised under a matching scheme given a match ratio of h that satisfies $g_0 = NhG^*(h)$. As will be shown in Proposition 4, g_0 is an

increasing function of h and for notational ease we write $G^*(g_0)$. Let $G^{**}(g_0)$ denote the total amount raised under a seed money scheme and a leadership gift of size g_0 . For a fixed leadership gift, the charity always prefers a matching scheme since individual donations will be larger and hence total donations will be larger: $G^*(g_0) = Ng^* + g_0 > G^{**}(g_0) = Ng^{**} + g_0$.

Let $V = V(x_0, G)$ denote the benefactor's utility function. He splits his wealth m_0 between private consumption $x_0 = m_0 - g_0$ and a donation of g_0 to the public good. A benefactor's utility is such that $V_2 > 0$. Just like the charity itself, the benefactor will always prefer a matching scheme for a fixed g_0 .

$$V(m_0 - g_0, G^*(g_0)) > V(m_0 - g_0, G^{**}(g_0)).$$

A disagreement between the charity and the benefactor about the optimal design of the fund raising campaign can only arise when g_0 is not fixed and the benefactor determines both the design of the campaign and the amount to donate. Whether a conflict then arises depends on just how the small donors' gifts are affected by the size of the benefactor's leadership gift. As will be seen in Section 5 and Subsection 6.3, a conflict can arise when an increase in the match ratio leads to a decrease in individual donations. The following section examines the relation between the match ratio and individual donations.

3: The Relation between the Match Ratio and Donations

3.1: The relation between the match ratio and individual donations

In this subsection, we focus only on the reaction of small donors with respect to a change of match ratio h and ask whether a higher match ratio will lead to higher individual donations. We assume for the moment that for a given h a deep-pocketed benefactor provides the necessary matching amount hNg^* .

Applying the implicit function theorem to the first order condition in (4) gives the comparative static result:

$$\frac{dg^*}{dh} = \frac{Ng^* U_{12} - U_2 - [1+h]Ng^* U_{22}}{U_{11} - [N+1][1+h]U_{12} + [1+h]^2 NU_{22}} . \quad (7)$$

The denominator in (7) is negative, but the sign of the numerator is ambiguous. While it seems natural that a higher match ratio will increase individual donations it need not be so. Section 6 shows that $\frac{dg^*}{dh}$ is zero for Cobb-Douglas utility functions, positive for Square Root utility functions, but it can be negative for Mixed Power-Exponential utility functions.

Proposition 2: *Given a fixed number of small donors, individual donations to a charity may be increasing or decreasing in the match ratio depending on the small donors' utility functions.*

Although it is possible that individual contributions decline with an increase in the match ratio, the total donation (including the matching grant) is always increasing in the match ratio. A small donor can think of $1/[1+h]$ as the “price” of the public good relative to the private good. An increase in the match ratio is analogous to a decrease in the relative price of the public good, yet such a price decrease can lead to decreased individual expenditure on the good. If both goods were private goods, the assumption of a non-negative cross-partial derivative of the utility function (i.e. $U_{12} \geq 0$) would rule out Giffen goods. Here, we see an interesting contrast between a private good and a public good. In the public goods situation, an increase in the match ratio not only reduces the price for the public good, but at the same time affects the “effective” budget of small donors.

Given the small donor reaction function, $g^*(h)$, each individual small donor's budget constraint $x_i + g_i = m$ can be re-written as

$$x_i + \left[\frac{G}{[1+h]} - [n-1]g^*(h) \right] = m.$$

$$x_i + p \times G = m + [n-1]g^*(h),$$

where $p \equiv \frac{1}{1+h}$ is the price of the public good in terms of the private good. An increase in the match ratio has two effects on the small donor's maximization problem. Not only is the price of the public good reduced, which given $U_{12} \geq 0$ would in itself lead to the purchase of more of the public good, the donor's "effective" wealth is altered. The amount $[n-1]g^*(h)$ is the small donor's "social income," as pointed out by Becker (1974), which augments the private income m of small donors due to the nature of a public good. In an equilibrium where small donations decline in response to a decrease in the price of the public good each small donor suffers a reduction in their "effective" wealth. The reduction in small donor "effective" income more than offsets the anticipated substitution effect.

3.2: The relation between the match ratio and total donations

Proposition 3: *Given a fixed number of small donors, the total donation to a charity is always positively related to the match ratio.*

Proof: The first order condition of the Program in (3) can be written as

$$U_1(m - g^*(h), G^*(h)) = [1+h]U_2(m - g^*(h), G^*(h)).$$

Consider two match ratios, h^A and h^B , with $h^A > h^B$ and suppose that in fact $G^*(h^A) \leq G^*(h^B)$. Since $G^*(h) = N[1+h]g^*(h)$, it must be that $g^*(h^A) \leq g^*(h^B)$.

The first order conditions corresponding to match ratio of h^A and h^B are

$$U_1\left(m - g^*(h^A), G^*(h^A)\right) = [1 + h^A]U_2\left(m - g^*(h^A), G^*(h^A)\right) \quad (8)$$

and

$$U_1\left(m - g^*(h^B), G^*(h^B)\right) = [1 + h^B]U_2\left(m - g^*(h^B), G^*(h^B)\right). \quad (9)$$

The assumption that $U_1 > 0$, $U_2 > 0$, $U_{11} < 0$, $U_{22} < 0$, $U_{12} \geq 0$ and the supposition that $G^*(h^A) \leq G^*(h^B)$ when $h^A > h^B$ has the following contradictory implications: The left-hand-side of equality (8) is no greater than the left-hand-side of equality (9), but the right-hand-side of equality (8) is strictly greater than the right-hand-side of equality (9). **QED**

3.3: The relation between the match ratio and the lead donor's gift

Given a match ratio of h the benefactor will be called upon to provide a matching grant of $g_0 = Nhg^*(h)$.

Proposition 4: *Given a fixed number of small donors, the benefactor's matching gift is always positively related to the match ratio.*

Proof: Suppose in fact that the leadership gift did decline with an increase in h . Since $g_0(h) = Nhg^*(h)$ is the product of the match ratio and the aggregate of individuals' donations, a decrease in the leadership in response to an increase in h implies that the aggregate of individuals' donations must have declined. But if in aggregate individuals' donations has declined and the benefactor's matching gift has also declined, the total donation to the charity must have declined. But such an outcome contradicts Proposition 3. Proposition 3 states that an increase in the match ratio will always increase the total donation to the charity. **QED**

Proposition 4 places a lower bound on the elasticity of individual donations with respect to the match ratio. Since $g_0(h) = Nhg^*(h)$, $\frac{dg_0}{dh} = N \left[g^* + h \frac{dg^*}{dh} \right]$. Given Proposition 4, $g^* + h \frac{dg^*}{dh} > 0$; i.e., $\frac{h}{g} \frac{dg^*}{dh} > -1$. Even when there does exist a negative relationship between each small donor's gift and the match ratio, a 1% increase in the match ratio will never lead to a greater than 1% decrease in individual giving.

3.4: The charity's desired match ratio

Proposition 3 states that the charity raises more funds whenever a benefactor is willing to fund a matching grant at a higher match ratio. Proposition 4 states that the higher the match ratio the greater the demand on the benefactor's generosity. Thus a charity will always prefer the largest feasible match ratio given the maximum leadership gift the benefactor is willing to provide. If the benefactor is willing to donate up to \bar{g} , the charity will chose h as the solution of $\bar{g} = Nhg^*(h)$.

4: The Size of the Donor Base and the Size of the Leadership Gift

In prior sections we held the number of small donors N fixed. This section first discusses the comparative static of g^* , g^{**} , G^* and G^{**} with respect to N and then consider the situation when N is linked to the size of the leadership gift and determined *endogenously*.

4.1: An exogenously determined donor base

Under the seed money approach, we can easily use relation (2) to find the comparative static result in the Nash Equilibrium:

$$\frac{\partial g^{**}}{\partial N} = \begin{cases} 0 & , \text{ if } g^{**} = 0; \\ \frac{g^{**} [U_{12} - U_{22}]}{U_{11} - [N+1]U_{12} + NU_{22}} < 0, & \text{ if } g^{**} > 0. \end{cases}$$

$$\begin{aligned} \frac{\partial G^{**}}{\partial N} &= g^{**} + N \frac{\partial g^{**}}{\partial N} \\ &= g^{**} + N \begin{cases} 0 & , \text{ if } g^{**} = 0 \\ \frac{g^{**} [U_{12} - U_{22}]}{U_{11} - [N+1]U_{12} + NU_{22}} < 0, & \text{ if } g^{**} > 0 \end{cases} \\ &= \begin{cases} 0 & , \text{ if } g^{**} = 0 \\ \frac{g^{**} [U_{11} + [1+N]U_{12}]}{U_{11} - [N+1]U_{12} + NU_{22}} > 0 & , \text{ if } g^{**} > 0. \end{cases} \end{aligned}$$

That the small donors' contributions are negatively related to the size of donor pool is not surprising: it is a manifestation of the "free-riding" nature of public goods. But the free-riding can never be so severe as to lead to a reduction in the total amount raised—a reduction in the total amount raised would mean each individual found less to free-ride on. Individual donations fall as N increases, but total donations are non-decreasing.

Similarly, under the matching gift approach, (4) implies:

$$\frac{\partial g^*}{\partial N} = \frac{[1+h]g^*[U_{12} - [1+h]U_{22}]}{U_{11} - [N+1][1+h]U_{12} + [1+h]^2 NU_{22}} < 0.$$

$$\begin{aligned} \frac{\partial G^*}{\partial N} &= [1+h]g^{**} + N[1+h] \frac{\partial g^*}{\partial N} \\ &= [1+h]g^{**} + N[1+h] \frac{[1+h]g^*[U_{12} - [1+h]U_{22}]}{U_{11} - [N+1][1+h]U_{12} + [1+h]^2 NU_{22}} \\ &= [1+h]g^{**} \left[\frac{U_{11} - [1+h]U_{12} + [1+h]hNU_{22}}{U_{11} - [N+1][1+h]U_{12} + [1+h]^2 NU_{22}} \right] > 0. \end{aligned}$$

Again, individual donations fall and total donations rise as N increases.

Proposition 5: *For a fixed gift of seed money, each small donor’s per capita contribution will be reduced as the number of small donors increases. For a matching grant system and a fixed match ratio, each small donor’s per capita contribution will also be reduced as the number of small donors increases. In both cases, total donations increase (at least weakly) as N increases.*

4.2: An endogenously determined donor base

We now consider the case where N is endogenously determined by the size of the leadership gift and assume that the relation between the two is positive. This could be attributed to a free publicity effect because the media routinely reports large donations irrespective of whether the gift takes the form of a matching grant or lump-sum seed money. An alternative signaling explanation of a positive relation is due to Andreoni (2006b), who argues that larger leadership gifts may signal a higher quality public project. Presumably large donors investigate potential recipients of their largesse and scrutinize the management of the organization carefully before leading a fund-raising campaign. Fama and Jensen (1983) point out, and Callen et al. (2003) document, that major donors play an more important role in the governance of nonprofits, often by their presence on nonprofit boards.⁶

4.2.1: A seed money scheme and an increase in the leadership gift

When a publicity effect means that the size of the potential donor base increases with the size of the leadership gift, a seed money gift affects the optimal individual donation, $g^{**} = g^{**}(N(g_0), g_0)$, through two channels. We examine how the total amount raised

⁶ In Callen et. al. (2003), the definition of “major” donors is left to the discretion of the organization staff member who filled the survey. They find that major donors constitute 26% of the board on average. The greater the proportion of major donors on the board is, the lower the administrative expenses as a proportion of total expenses.

and each individual's donation responds to an increase in the benefactor's seed money gift. First consider an individual's donation.

$$\frac{dg^{**}}{dg_0} = \frac{\partial g^{**}}{\partial N} \frac{dN}{dg_0} + \frac{\partial g^{**}}{\partial g_0}. \quad (10)$$

Proposition 5 and the publicity effect of g_0 on N establish that the first term on the right-hand-side of (10) is non-positive. To determine the sign of the second term, again apply the implicit function theorem to (2).

$$\frac{\partial g^{**}}{\partial g_0} = \begin{cases} 0 & , \text{ if } g^{**} = 0; \\ \frac{U_{12} - U_{22}}{U_{11} - [1 + N]U_{12} + NU_{22}} < 0, & \text{ if } g^{**} > 0. \end{cases} \quad (11)$$

The leadership gift crowds out individual donations and the second term in (10) is also non-positive. For reinforcing reasons, a larger seed money gift means smaller individual donations. Absent a leadership gift, some number of small donors may support a charity. With a seed money gift, more people know about the charity, but no-one beside the benefactor may then support it.

This counter-intuitive result can arise when the increased publicity does not affect individuals' preferences between the private and public goods. If leadership gifts are a signal of better governance of the charity, each potential donor may become more willing to donate. For simplicity, the paper proceeds by continuing to assume that g_0 affects only the *potential donor base* N but does not affect the individuals' utility functions U . List and Lucking-Reilly (2002) document that the existence of a seed money leadership gift increases the participation rate which is consistent with the gift being a signal that the charity is of a higher "quality."

Now consider the effect of an increase in the seed money gift on the total amount raised.

$$\frac{dG^{**}}{dg_0} = \frac{\partial G^{**}}{\partial N} \frac{dN}{dg_0} + \frac{\partial G^{**}}{\partial g_0} \quad (12)$$

Proposition 5 and the publicity and/or signaling effect of g_0 on N establish that the first term on the right-hand-side of (12) is non-negative. The second term in (12) is strictly positive.

$$\frac{\partial G^{**}}{\partial g_0} = 1 + N \frac{\partial g^{**}}{\partial g_0} = \begin{cases} 1 & , \text{ if } g^{**} = 0; \\ \frac{U_{11} - U_{12}}{U_{11} - [1 + N]U_{12} + NU_{22}} > 0, & \text{ if } g^{**} > 0. \end{cases} \quad (13)$$

Thus a larger seed money gift means a larger total amount raised by the charity.

4.2.2: A matching scheme and an increase in the leadership gift

We first assume that the publicity/signaling effect relates to the announced match ratio. As will be seen, the matching grant required from the benefactor will be increasing in the match ratio, just as it is when N is fixed. Hence a larger announced h will still be synonymous with a larger announced leadership gift.

When the size of the donor base depends on the match ratio, the match ratio has two effects on the optimal individual donation: $g^* = g^*(N(h), h)$. We examine how individual donations and the total amount raised respond to an increase in the benefactor's match ratio. First consider an individual's donation.

$$\frac{dg^*}{dh} = \frac{\partial g^*}{\partial N} \frac{dN}{dh} + \frac{\partial g^*}{\partial h}. \quad (14)$$

Proposition 5 and the publicity and/or signaling effect of h on N establish that the first term on the right-hand-side of (14) is non-positive. Proposition 2 has shown that sign of the second term may be either positive or negative. Hence we are unable to uniquely sign the effect of an increase in the match ratio on the size of individual donations. But we can show that the total donations to a charity increase with an increase in the benefactor's match ratio.

Proposition 6: *When the size of the donor pool is non-decreasing in the match ratio, the total donation to a charity is always positively related to the match ratio.*

Proof: If $h^A > h^B$ and $G^*(h^A) \leq G^*(h^B)$, then since $G^*(h) = N(h)[1+h]g^*(h)$, it must be that $g^*(h^A) \leq g^*(h^B)$ if $\frac{dN}{dh} \geq 0$. The remainder of the proof mirrors that of proposition 3. **QED**

When the donor base depends on the match ratio, the benefactor will be called upon to provide a matching grant of $g_0 = Nhg^*(h)$.

Proposition 7: *When the size of the donor pool is non-decreasing in the match ratio, the benefactor's matching gift is always positively related to the match ratio.*

The proof of Proposition 7 mirrors that of Proposition 4. Propositions 6 and 7 imply that a charity will always prefer the largest feasible match ratio given the maximum leadership gift a benefactor is willing to provide. If the benefactor is willing to donate up to \bar{g} , the charity will chose h as the solution of $\bar{g} = N(h)hg^*(h)$. Propositions 6 and 7 mean that a larger matching-grant gift means a larger total amount raised by the charity.

Thus we have established that when a lead donor is prepared to contribute a larger amount, the charity will never turn it down, partially or completely out of a concern that the leadership gift will more than crowd out individual donations. And this is so irrespective of whether the leadership gift is packaged as seed money or a matching-grant. The total amount of funds raised is always increasing in the leadership gift under either approach. The charity will always prefer that any given leadership gift be packaged as a matching grant. But we have not established that a benefactor who determines the size of their leadership would want it so packaged. The next section considers the possibility of conflict between the charity and its benefactor concerning the optimal design of the fund-raising scheme.

5: The Lead Donor's Preference for Seed Money versus Matching

So far, we have discussed matching and seed money systems taking the size of the leadership gift as given. In this section, we consider the lead donor's utility maximization problem and address the issue of which system he prefers.

Let g'_0 denote the lead donor's optimal gift if the charity uses a matching system and let g''_0 denote the lead donor's optimal gift if the charity uses a seed money system. $G^*(g'_0)$ is the total amount raised when the charity uses a matching system and the lead donor donates g'_0 . $G^*(g'_0) = g'_0 + Ng^*(h(g'_0))$, where the function $h(g_0)$ satisfies $g_0 = Nh(g_0)g^*(h(g_0))$ and $g^*(h)$ is the function relating each small donor's optimal donation to the match ratio h . Recall that for a given value of g_0 , the charity never finds it optimal to choose a value of h such that $Nhg^*(h) < g_0$.

Let $G^{**}(g''_0)$ denote the total money raised by the charity when the charity uses a seed money system and the large donor donates g''_0 . $V(m_0 - g'_0, G^*(g'_0))$ denotes the utility of the lead donor who makes a gift of g'_0 to a charity that uses a matching system. $V(m_0 - g''_0, G^{**}(g''_0))$ denotes the utility of a lead donor who makes a gift of g''_0 to a charity that uses a seed money system.

Since g'_0 is chosen optimally given a matching system it must be that

$$V(m_0 - g'_0, G^*(g'_0)) \geq V(m_0 - g''_0, G^*(g''_0)). \quad (15)$$

Proposition 1 established that, for a given leadership gift, the charity always raises more money with a matching system than with a seed money system; i.e., $G^*(g_0) > G^{**}(g_0)$ for all g_0 . Thus

$$V(m_0 - g''_0, G^*(g''_0)) > V(m_0 - g''_0, G^{**}(g''_0)). \quad (16)$$

Inequalities (15) and (16) give $V(m_0 - g'_0, G^*(g'_0)) > V(m_0 - g''_0, G^{**}(g''_0))$; i.e., the lead donor always prefers that the charity uses a matching system. Thus we have established Proposition 8.

Proposition 8: *If the lead donor can choose whether the charity will use a matching or seed money system, he will always instruct the charity to use a matching system.*

A conflict of interest between a charity interested in maximizing the total donation and a lead donor interested in maximizing his own utility can arise if $G^*(g'_0) < G^{**}(g''_0)$. In this event, the total amount raised for the charity is lower under a matching system, despite it being preferred by the lead donor. Whether a conflict arises depends not on the lead donor's utility function, V , but on the small donors' utility functions, U . What matters is how the small donors react to a change in the leadership gift when the charity uses a matching system. A sufficient condition to rule out conflict between a charity and its benefactor is that the small donors do not decrease their individual donations in response to an increase in the match ratio.

Proposition 9: *If individual donations are non-decreasing in the match ratio, the charity will always be able to raise more money under the lead donor's preferred matching grant scheme than under a seed money scheme.*

Proof: Suppose otherwise and that instead $G^*(g'_0) \leq G^{**}(g''_0)$. This inequality immediately implies that $g'_0 < g''_0$. If this is not so, then $g'_0 = g''_0 + \delta$ for some $\delta \geq 0$ and

$$G^*(g'_0) = G^*(g''_0 + \delta) > G^{**}(g''_0 + \delta) \geq G^{**}(g''_0).$$

The first inequality follows from Proposition 1 and the second inequality follows from Equation (13). But this creates a contradiction.

The first-order conditions for the lead donor under the matching and seed money systems are then, respectively,

$$V_1^*(m_0 - g'_0, G^*) \geq V_2^*(m_0 - g'_0, G^*) \left[1 + N \frac{dg^*}{dh} \frac{dh}{dg_0} \right] \quad (17)$$

and

$$V_1^{**}(m_0 - g''_0, G^{**}) = V_2^*(m_0 - g''_0, G^{**}) \left[1 + N \frac{dg^{**}}{dg_0} \right]. \quad (18)$$

Equation (18) is an equality since $g''_0 > g'_0 \geq 0$ and we are considering an interior solution for the optimal leadership gift under a seed money system.

Recall from Proposition 7 that the match ratio and the leadership gift are positively related. The condition in Proposition 9 that $\frac{dg^*}{dh} \geq 0$ then implies that the term in square brackets on the right-hand-side of (17) is greater than or equal to unity. Hence

$$V_1^*(m_0 - g'_0, G^*) \geq V_2^*(m_0 - g'_0, G^*). \quad (19)$$

Given Equation (11), the term in square brackets on the right-hand-side of (18) is less than or equal to unity and hence

$$V_1^{**}(m_0 - g''_0, G^{**}) \leq V_2^*(m_0 - g''_0, G^{**}). \quad (20)$$

Recall that $V_1 > 0, V_2 > 0, V_{11} < 0, V_{22} < 0, V_{12} \geq 0$. Given $G^*(g'_0) \leq G^{**}(g''_0)$ and its immediate implication that $g'_0 < g''_0$, the left-hand-side of (19) must be less than the left-hand-side of (20). But the right-hand-side of (19) must be greater than that of (20). This creates a contradiction. Therefore, it can not be the case that $G^*(g'_0) \leq G^{**}(g''_0)$. **QED**

We turn now to a series of numerical examples that illustrate the paper's results.

6: Three Examples

In this section, we use three examples of small donor utility functions to show that individual donations can rise, fall or be unchanged by an increase in the match ratio. The first example considers Cobb-Douglas utility and shows that an individual donor's contribution is unrelated to the match ratio h . The second example considers a square root utility function and shows that an individual's donor's contribution is an increasing function of h . The final example of a mixed power and exponential utility function illustrates the possibility that an individual's donation may be at first positively and then negatively related to h . This third example fits the finding from the field experiment in Karlan and List (2007).

In the first two examples in which the small donors' optimal contribution is non-decreasing in the match ratio, the charity and its benefactor both prefer a matching system. In the third example, the small donors' optimal contribution can be decreasing in the match ratio. We use this third example to demonstrate that the charity can prefer a seed money system while the benefactor prefers a matching system.

6.1: Cobb-Douglas utility function, $U = x^\alpha G^\beta$

Under the matching gift approach with match ratio h , each small donor solves:

$$\begin{aligned} \max_{g_i} [m_i - g_i]^\alpha \left[[1+h] \left[g_i + [N-1]g^* \right] \right]^\beta \\ \text{subject to } g_i \geq 0. \end{aligned}$$

where $0 < \alpha < 1, 0 < \beta < 1$. The optimal donation from each small donor is

$$g^* = \frac{\beta m}{\alpha N + \beta} > 0. \quad (21)$$

That is, the individual donation is positively related to the income level m and negatively related to the size of the donor base N . Interestingly, g is *not* related to the matching ratio h . A higher match ratio might tempt a small donor to give more since the relative price of

the public good has fallen. But this small donor reasons that the lead donor will be giving more even if small donations remain unchanged. The resultant diminution in marginal utility from any additional unit of the public good exactly offsets his desire to give more when the price of the public good falls.

Given the lead donor pledges to provide a matching grant of up to g_0 , the match ratio is set at

$$h = \frac{g_0}{N g^*} = \frac{g_0 [\alpha N + \beta]}{N \beta m}$$

and the total amount raised is

$$G^* = g_0 + \frac{N \beta m}{\alpha N + \beta} . \quad (22)$$

Under the seed money approach, each small donor solves:

$$\begin{aligned} \max_{g_i} & [m_i - g_i]^\alpha [g_i + [N - 1] g^{**}]^\beta \\ \text{subject to} & \quad g_i \geq 0. \end{aligned}$$

The equilibrium solution is

$$g^{**} = \begin{cases} 0, & \text{if } g_0 \geq \frac{\beta}{\alpha} m; \\ \frac{\beta m - \alpha g_0}{\alpha N + \beta}, & \text{if } g_0 < \frac{\beta}{\alpha} m. \end{cases} \quad (23)$$

It is interesting to note that when g_0 is too large, small donors will not contribute to the public good at all, simply because the amount given by the large donor is deemed to be sufficient—in fact they would prefer it if the lead donor had reduced his gift to the charity and given some money to each of them. There is complete “crowding-out” of small donations. Every would-be small donor prefers instead to be a free-rider. Comparing the individual donation levels in (21) and (23), it is clear that small donors will contribute more under the matching gift approach.

The total amount raised under a seed money scheme, including the amount pledged by the lead donor, is

$$G^{**} = \begin{cases} g_0, & \text{if } g_0 \geq \frac{\beta}{\alpha}m; \\ \frac{\beta g_0}{\alpha N + \beta} + \frac{N\beta m}{\alpha N + \beta}, & \text{if } g_0 < \frac{\beta}{\alpha}m. \end{cases} \quad (24)$$

Comparing (24) with (22), we see that the total amount raised under the seed money approach will be smaller than that under the matching gift approach. Furthermore, we can see the positive impact of increasing the number of small donors on the total donation under either approach.

$$\frac{dG^*}{dN} = \frac{\beta^2 m}{(\alpha N + \beta)^2} > 0,$$

and

$$\frac{dG^{**}}{dN} = \frac{\beta(\beta m - \alpha g_0)}{(\alpha N + \beta)^2} > 0.$$

Thus, an increase in the donor base has a higher marginal impact on total funds raised under the matching gift approach than under the seed money approach.

6.2: Square-root utility function, $U = \sqrt{x} + \sqrt{G}$

Under the matching approach, small donors' preferences take the form:

$$U = \sqrt{m_i - g_i} + \sqrt{[1+h][g_i + [N-1]g^*]}$$

The first-order condition is

$$-\frac{1}{\sqrt{m - g^*}} + \frac{\sqrt{1+h}}{\sqrt{Ng^*}} = 0$$

which yields the optimal donation from a small donor as

$$g^* = \frac{[1+h]m}{N+1+h}.$$

$$\frac{dg^*}{dh} = \frac{mN}{[N+1+h]^2} > 0$$

and

$$\frac{d^2g}{dh^2} = -\frac{2mN}{[N+1+h]^3}.$$

Therefore, individual donations are an increasing concave function of h .

Under the seed money approach, small donors' preferences take the form:

$$U = \sqrt{m_i - g_i} + \sqrt{[g_i + [N-1]g^{**}] + g_0}.$$

The first-order condition is

$$-\frac{1}{\sqrt{m - g^{**}}} + \frac{1}{\sqrt{Ng^{**} + g_0}} \leq 0$$

which yields the optimal contribution from a small donor as

$$g^{**} = \begin{cases} 0, & \text{if } g_0 \geq m; \\ \frac{m - g_0}{N+1}, & \text{if } g_0 < m. \end{cases} \quad (25)$$

Consider the match ratio such that the lead donor's donation under the matching gift system is equal to the seed money gift g_0 ; i.e., consider the unique positive solution of the quadratic in h

$$g_0 = hNg^* = hN \frac{[1+h]m}{N+1+h}.$$

The total donation, including the lead donor's donation, is then always higher under the matching gift approach. The total donation to charity under the matching gift approach is

$$G^* = Ng^* + hNg^* = N \frac{[1+h]m}{N+1+h} + g_0.$$

The total donation under the seed money approach is

$$G^{**} = \begin{cases} g_0, & \text{if } g_0 \geq m; \\ N \frac{m}{N+1} + \frac{1}{N+1} g_0, & \text{if } g_0 < m. \end{cases}$$

Since $\frac{1+h}{N+1+h} > \frac{1}{N+1}$ for any positive match ratio h , it follows immediately that $G^* > G^{**}$ for all g_0 .

6.3: Mixed Power-Exponential utility function, $U = x^\alpha - e^{-G}$

Under the matching approach, small donors' preferences take the form:

$$U(x_i, g_i) = [m_i - g_i]^\alpha - e^{-[1+h][g_i + [N-1]g^*]}.$$

Due to the rather complicated form of the utility function, there is no closed-form solution for g^* . Hence our approach is to numerically examine the comparative static of g^* with respect to h .

The first-order condition is

$$-\alpha [m - g^*]^{\alpha-1} + [1+h] e^{-[1+h]Ng^*} = 0. \quad (26)$$

Total differentiation of the first-order condition gives us

$$\begin{aligned} & \left[\alpha[\alpha-1][m-g^*]^{\alpha-2} - [1+h]^2 N e^{-[1+h]Ng^*} \right] dg^* + \\ & \left[e^{-[1+h]Ng^*} - [1+h]Ng^* e^{-[1+h]Ng^*} \right] dh = 0. \end{aligned} \quad (27)$$

Substituting (26) into the first term of (27), we have the comparative static result:

$$\frac{dg^*}{dh} = \frac{[1+h]Ng^* - 1}{[1+h] \left[(\alpha-1) - [1+h]N[m-g^*] \right]} \cdot \quad (28)$$

Since $0 < \alpha < 1$, the denominator of Equation (28) is negative. If ever total donations $G^* = N[1+h]g^* > 1$, the numerator will be positive and individuals' donations will be decreasing in the match ratio. Whenever $G^* < 1$, individuals' donations will be increasing

in the match ratio. Recall from Proposition 3 that the total donation is always increasing in h . Hence individuals' donations first increase and then decrease with increases in the match ratio. This matches the empirical relation documented in Karlan and List (2007). Figure 1 plots g^* as a function of h assuming that the small donor's income level $m = 1$, $\alpha = 0.6$, and $N = 10$.

[Insert Figure 1 Here]

Proposition 9 established that a necessary condition for there to be disagreement between a lead donor and a charity about whether to use a seed money or matching system is that there is a non-monotonic relation between small donor contribution and the match ratio. We can illustrate such disagreement in Example 3 if we specify the utility function of the lead donor as $V = [m_0 - g_0]^A - B e^{-G(g_0)}$ and assume that $A = 0.6$, $B = 2$ and $m_0 = 3$. Note that the lead donor's marginal utility from the public good is higher than that of small donors and the lead donor is wealthier than the individual small donors.

Numerical solutions for the small donor contribution g^* and the total donation G^* corresponding for various match ratios are reported in columns 2 and 5 of Table 1. Column 1 contains the match ratios themselves. Note that G^* becomes equal to unity for a match ratio somewhere between 0.6 and 0.7. The small donors' optimal contribution is increasing for $h < 0.6$ and decreasing for $h > 0.7$. Column 4 shows the leadership gift corresponding to each match ratio, namely $g_0 = N[1 + h]g^*$. The small donor contribution g^{**} and the total donation G^{**} under a seed money system given an identical leadership gift are reported in columns 3 and 6.

[Insert Table 1 Here]

Consider two particular match ratios, $h = 1.4$ and $h = 3$. At a match ratio of 1.4 the lead donor will be called on to make a leadership gift in the amount of 0.7952 if the

matching approach is adopted. Each of the ten small donors will contribute 0.0568 to the charity, and the total donation, including the lead donor's gift, will be $G^* = 1.3632$. If instead the seed money approach is adopted for the same leadership gift, then each small donor will contribute nothing, which brings the total donation to $G^{**} = 0.7952$. For the parameter values in this example, complete crowding out once the seed money leadership gift reaches around $g_0 = 0.5$.

At a match ratio of 3, the lead donor will be asked to donate 1.407 to the charity and each of the small donors will contribute 0.0459. The total donation will be $G^* = 1.876$. If instead a leadership gift of 1.407 had been used as seed money, then each small donor will contribute nothing and $G^{**} = 1.407$.

If the lead donor is free to choose between the two approaches, then he is better off by donating 0.7952 and using the matching approach with the match ratio set at $h = 1.4$. His utility, V^* , associated with each match ratio is given in column 7. His utility associated with the corresponding seed money gift, V^{**} , is given in column 8. For the match ratios reported in Table 1, V^* attains a maximum at $h = 1.4$ which requires a leadership gift of 0.7952. For the various levels of leadership gifts considered in Table 1 and a seed money system, V^{**} attains its maximum when $g_0 = 1.407$. A leadership gift of 1.407 will be required if a matching system is used and the match ratio is set equal to 3. Thus, between the two levels of leadership gift being considered here, $g_0 = 0.7952$ and $g_0 = 1.407$, the lead donor prefers a leadership gift of 0.7952 and a matching system to a leadership gift of 1.407 and a seed money system.

The key observation is that the lead donor prefers the matching approach, which allows him a higher level of utility. But the charity will prefer using the seed money approach since it leads to a larger total amount of donations. This example shows that due to the negative reaction of the small donor's contributions to an increase in the leadership

gift in the matching system, there are cases where a conflict of interest exists between the lead donor and the charity.

We now turn to an examination of small donors' preferences for seed money versus matching schemes.

7: Small Donor Preferences for Seed Money versus Matching Schemes

Each small donor contributes more under the matching gift scheme (recall Proposition 1) and her consumption of the private good is reduced by some amount Δ . The upside of this shift is that each donor will enjoy $N\Delta$ more of the public good. The tradeoff here is Δ additional units of private consumption versus $N\Delta$ additional units of the public good. She may or may not be better off with a matching system depending on her utility function, the number of small donors and the size of the leadership gift. We can show this by considering the Cobb Douglas example of Section 6.

Under a matching gift scheme, each small donor's utility is

$$U^* = \left[\frac{\alpha N m}{\alpha N + \beta} \right]^\alpha \left[N \frac{\beta m}{\alpha N + \beta} + g_0 \right]^\beta. \quad (28)$$

Under the seed money approach, each individual small donor's utility is

$$U^{**} = \left[\frac{\alpha N m + \alpha g_0}{\alpha N + \beta} \right]^\alpha \left[N \frac{[\beta m - \alpha g_0]}{\alpha N + \beta} + g_0 \right]^\beta. \quad (29)$$

We calculate a small donor's utility level under various assumptions on N and the following parameter specifications: $\alpha = 0.95$, $\beta = 0.05$, $m = 1$ and $g_0 = 0.5$. Small donors are better off under the matching gift scheme provided the donor base is sufficiently large. The result is depicted Figure 2.

[Insert Figure 2 Here]

Proposition 10: *A small donors' preference for seed money versus matching depends on her utility function, the number of small donors and the size of the leadership gift.*

If small donors were to give nothing under a matching system, no money would be raised. Provided the marginal utility from the first unit of the public good, $U_2(m,0)$, exceeds the marginal utility from the last unit of the private consumption, $U_1(m,0)$, small donors will participate under the matching gift approach. In contrast, under the seed money approach would-be small donors can chose to free-ride on the benefactor's leadership gift g_0 . Small donors will give nothing if

$$U_2(m, g_0) < U_1(m, g_0);$$

i.e., it is easier to get corner solutions to the utility maximization program under the seed money approach. This can be seen in Equation (23) of the Cobb Douglas example; in Equation (25) of the Square Root example; and in Table 1's numerical solution for the Mixed Power-Exponential example. Heterogeneity of participation is easily incorporated into the model by recognizing different wealth levels and/or different small donor utility functions. List and Luckling-Reilly (2002) and Karlan and List (2007) document that the existence of leadership gift increases the participation rate for both seed money and matching systems.

8: The Effect of the Tax Rate on Donations and the Fund-raising Method

Suppose individual donations are tax-deductible. Income is taxed at the rate τ . Tax receipts are not used to increase the production of the public good.⁷ We first assume that there is no leadership gift. An individual with before tax income of m who spends x_i on the private good and donates g_i faces the following budget constraint:

⁷ Pittel and Rübhelke (2006) discusses the case where the government uses tax receipts to finance public goods while maintains a balanced budget.

$$x_i + g_i = m[1 - \tau] + \tau g_i.$$

$$x_i = [m - g_i][1 - \tau].$$

Each of N identical individuals solves:

$$\max_{x_i, G} U(x_i, G)$$

$$\text{subject to: } x_i + [1 - \tau]G = [m + [N - 1]g^{***}][1 - \tau].$$

where g^{***} is the optimal individual donation in equilibrium with taxes.

The first order condition is

$$-[1 - \tau]U_1\left([m - g^{***}][1 - \tau], Ng^{***}\right) + U_2\left([m - g^{***}][1 - \tau], Ng^{***}\right) = 0. \quad (30)$$

Applying the implicit function theorem gives:

$$\frac{\partial g^{***}}{\partial \tau} = \frac{-U_1 - [1 - \tau][m - g^{***}][U_{11} - U_{12}]}{[1 - \tau]^2 U_{11} - [1 - \tau]NU_{12} - [1 - \tau]U_{21} + NU_{22}} \quad (31)$$

The denominator in (31) is always negative. The numerator may be positive or negative. Intuitively, when the tax rate increases, the individual disposable income decreases. Therefore, the income effect on individual donation is negative. On the other hand, donations are tax deductible, which makes the public good relatively cheap. The substitution effect is positive. The net result of a change in tax rate on the individual donation is ambiguous.

We can now reexamine the problem of individual donations in the presence of a leadership gift. Under the scheme with a match ratio h , the total donation is

$$G^* = [1 + h][[N - 1]g^* + g_i].$$

The individual small donor solves

$$\max_{g_i} U\left([m - g_i][1 - \tau], [1 + h][[N - 1]g^*(h) + g_i]\right)$$

The first order condition is:

$$-[1-\tau]U_1\left(\left[m-g^*\right][1-\tau], [1+h]Ng^*\right) + [1+h]U_2\left(\left[m-g^*\right][1-\tau], [1+h]Ng^*\right) = 0. \quad (32)$$

Under the seed money approach, the total donation is

$$G = g_0 + [N-1]g^{**} + g_i.$$

The individual small donor solves

$$\max_{g_i} U\left(\left[m-g_i\right][1-\tau], g_0 + [N-1]g^{**} + g_i\right).$$

The first order condition is:

$$-[1-\tau]U_1\left(\left[m-g^{**}\right][1-\tau], Ng^{**} + g_0\right) + U_2\left(\left[m-g^{**}\right][1-\tau], Ng^{**} + g_0\right) = 0. \quad (33)$$

Applying the implicit function theorem to (32) and (33) to derive the comparative statics of individual donations with respect to a change of tax rate shows that the sign of the relation is ambiguous, This is similar to the result in equation (31) and the detailed calculations are omitted.

Proposition 11: *Individual donations may either fall or rise in response to a change in the tax rate, with or without a leadership gift.*

Recall that in the proof of Proposition 1, we compared individual donations under matching grant schemes and seed money schemes. The first order conditions in (32) and (33) with a tax rate of τ correspond to the first order conditions (6) and (5), respectively. The size of the leadership gift is set at the same value under the two approaches. Following the steps in the proof of Proposition 1, establishes that given a personal income tax, individual small donors contribute more under the matching system.

Proposition 12: *The result that, for a given leadership gift, small donors donate more to the charity under the matching scheme than under the seed money approach is unaffected by the tax deductibility of donations.*

9: Conclusions and Extensions

We have compared two approaches to fund-raising when there is a large donor: a matching gift versus seed money. We have assumed that the nonprofit organization has a perfect efficiency ratio: every dollar of donations is spent on the provision of the public good. Under both approaches the total amount raised is increasing in the size of the leadership gift; i.e., an increased leadership gift never crowds out more in small donations than it adds directly. This means that the charity should never turn down a major leadership gift, partially or fully, because of a concern about crowding out.

The paper's primary result is that, for any *given* leadership gift, a matching approach will always raise more in total than a seed money approach. Whether the matching gift approach make small donors better off than an equal gift of seed money depends on small donors' utility functions, the size of the donor base and the size of the leadership gift. A publicity-related link between the size of the potential donor base and the size the leadership gift has no affect on the paper's results. But if the leadership gift is a signal of the quality of the charity, a larger leadership gift may lead more small donors to participate and to increase their contributions.

The relationship between an individual's voluntary contribution and the match ratio is ambiguous. But the larger the match ratio, the larger the amount the lead donor will be called up to contribute. Even if each small donor's contribution does decrease, the decline in the amount to be matched is always more than offset by the increase in the match ratio. When small donors' contributions are negatively related to the match ratio, there can be instances where the lead donor strictly prefers the matching approach, while the charity prefers the seed approach. The charity will prefer seed money provided the benefactor's optimal donation is sufficiently larger under a seed money approach that the charity raises more in total. Thus, when small donors' contributions are negatively related to the match ratio, there is the potential for conflict between the charity and its benefactor.

One direction for future research is to examine efficiency ratios and the governance of non-profits. Efficiency ratios are not one. The amount spent on the public good is less than the charity's total donations by the amount of its fund-raising and administrative expenses. A charity's management may seek to maximize the net amount raised, the gross amount raised or the difference. Management who seek to maximize the production of the public good will recognize that administrative and fund-raising expenses should be chosen as optimal inputs to the production of donations. A management team more interested in the perceived status associated with managing a bigger organization may seek to maximize donations per se and will overspend on fund-raising. A corrupt management team will simply seek to maximize the fund-raising and administrative "expenses." In the latter two cases, donors act as a discipline mechanism since their decisions are based on the net amount raised as modeled in this paper.

Another direction is to apply this paper's results on matching gifts to corporate employee matching grant schemes and their potential conflict with shareholder value maximization. A socially responsible company can fund an employee matching grant scheme through a reduction in the wages paid to its philanthropic employees. The company's shareholders bear no cost. Why then is the company's board feted for its "generosity"? Because their adoption of a corporate matching grants scheme allows the employees to co-ordinate and achieve an equilibrium combination of private and public good consumption that they prefer to that attained when they are paid more but make their donations without a match. The charity is able to raise more than it could in the equilibrium of individual donations from better-paid employees. In doing so, the board has implemented a Pareto improvement and demonstrated that it is "socially responsible" without reducing investors' profits

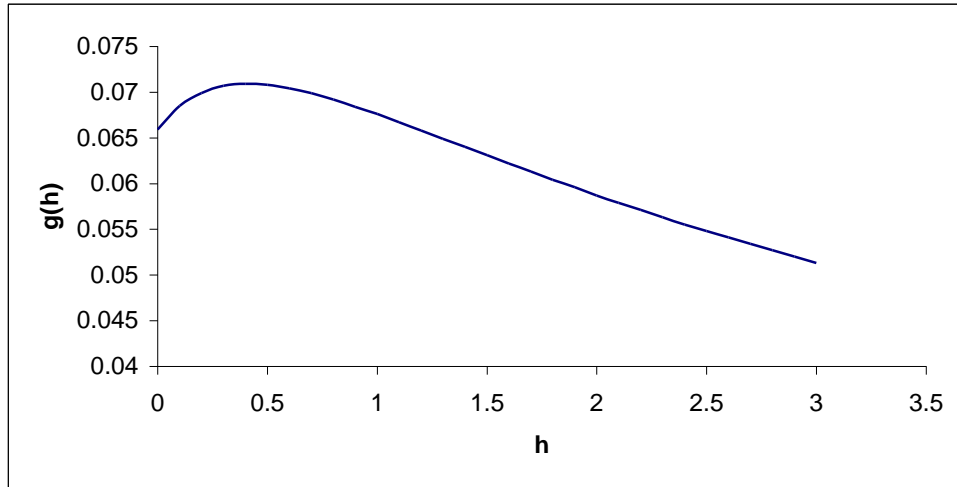
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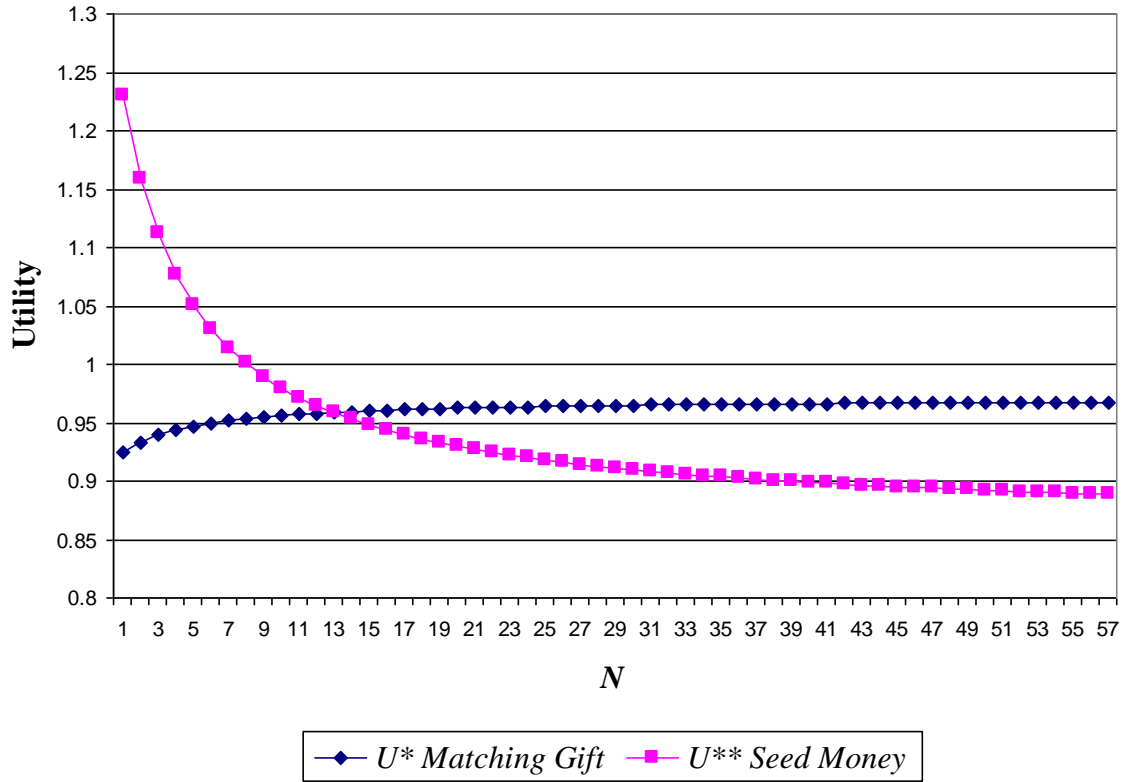
Figure 1

Each small donor's optimal gift, $g^*(h)$, as a function of the match ratio, h



Each of 10 small donors has wealth level $m = 1$ and utility of a mixed power-exponential form: $U = [1 - g]^{0.6} - e^{-G}$.

Figure 2
Small Donor Utility as a Function of the Donor Base



The figure depicts each small donor's utility given a matching system, U^* , and each small donor's utility given a seed money approach, U^{**} , given the same leadership gift. Each small donor has Cobb-Douglas Utility, $U = x^{0.95} G^{0.05}$ and wealth $m=1$. The leadership gift is $g_0=0.5$.

Table 1

This table lists the detailed results of calculations for the Mixed Power-Exponential example of Section 6. Both the small donor's utility $U(x,G)=x^{0.6}-e^{-G}$ and the lead donor's utility, $V(x,G)$ take the form $x^{0.6}-2e^{-G}$. The income level for a small donor is 1, while the income level for the lead donor is 3.

h	g^*	g^{**}	g_0	G^*	G^{**}	V^*	V^{**}
0	0.0491	0.0491	0	0.491	0.491	0.709154	0.709154
0.1	0.0531	0.044	0.0531	0.5841	0.4931	0.797363	0.691118
0.2	0.0558	0.0384	0.1116	0.6696	0.4956	0.86588	0.671296
0.3	0.0576	0.0325	0.1728	0.7488	0.4978	0.919713	0.649847
0.4	0.0588	0.0265	0.2352	0.8232	0.5002	0.962715	0.627946
0.5	0.0595	0.0205	0.2975	0.8925	0.5025	0.996503	0.605732
0.6	0.0598	0.0146	0.3588	0.9568	0.5048	1.022699	0.583686
0.7	0.0598	0.0089	0.4186	1.0166	0.5076	1.042852	0.562621
0.8	0.0597	0.0032	0.4776	1.0746	0.5096	1.059293	0.54069
0.9	0.0594	0	0.5346	1.1286	0.5346	1.071461	0.546625
1	0.059	0	0.59	1.18	0.59	1.080601	0.586504
1.1	0.0585	0	0.6435	1.2285	0.6435	1.087016	0.621578
1.2	0.058	0	0.696	1.276	0.696	1.091718	0.65287
1.3	0.0574	0	0.7462	1.3202	0.7462	1.094192	0.680026
1.4	0.0568	0	0.7952	1.3632	0.7952	1.095339	0.704039
1.5	0.0562	0	0.843	1.405	0.843	1.095291	0.725191
1.6	0.0555	0	0.888	1.443	0.888	1.093653	0.743133
1.7	0.0549	0	0.9333	1.4823	0.9333	1.091617	0.759339
1.8	0.0542	0	0.9756	1.5176	0.9756	1.08831	0.772852
1.9	0.0536	0	1.0184	1.5544	1.0184	1.084702	0.78499
2	0.0529	0	1.058	1.587	1.058	1.080111	0.794889
2.1	0.0523	0	1.0983	1.6213	1.0983	1.075285	0.803693
2.2	0.0516	0	1.1352	1.6512	1.1352	1.069741	0.810665
2.3	0.051	0	1.173	1.683	1.173	1.064001	0.816758
2.4	0.0504	0	1.2096	1.7136	1.2096	1.057875	0.821673
2.5	0.0498	0	1.245	1.743	1.245	1.051424	0.825532
2.6	0.0492	0	1.2792	1.7712	1.2792	1.044706	0.828444
2.7	0.0486	0	1.3122	1.7982	1.3122	1.037772	0.830512
2.8	0.0481	0	1.3468	1.8278	1.3468	1.030524	0.831915
2.9	0.0475	0	1.3775	1.8525	1.3775	1.023248	0.83252
3	0.0469	0	1.407	1.876	1.407	1.015895	0.832545
3.1	0.0464	0	1.4384	1.9024	1.4384	1.008178	0.831983
3.2	0.0459	0	1.4688	1.9278	1.4688	1.00034	0.830873
3.3	0.0454	0	1.4982	1.9522	1.4982	0.99242	0.829278
3.4	0.0449	0	1.5266	1.9756	1.5266	0.984449	0.827259
3.5	0.0444	0	1.554	1.998	1.554	0.976461	0.824872