Coordination of Marketing and Production for Price and Leadtime Decisions

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Abstract

We study a firm which serves customers that are sensitive to quoted price and leadtime, where pricing and leadtime decisions are made by the marketing and production departments, respectively. We analyze the inefficiencies that are due to the decentralization of price and leadtime decisions. In the decentralized setting, the total demand generated is larger, leadtimes are longer, quoted prices are lower, and the firm profits are lower as compared to the centralized setting. We show that coordination can be achieved using a transfer price contract with bonus payments. We also provide insights on the sensitivity of the optimal decisions with respect to market characteristics, sequence of decisions, and the firm's capacity level.

Key words: leadtime, pricing, marketing/production coordination.

1. Introduction

In many firms, manufacturing is evaluated as a cost center that seeks lower costs and operational efficiency, while marketing is evaluated as a revenue center with control over price and other marketing elements (Balasubramanian and Bhardwaj, 2004; Karmarkar and Lele, 2004). However, this is not necessarily an effective strategy. Otley (2002) discusses how dividing a firm into independent units for measuring performance on accounting terms would lead to misaligned incentives and suboptimal system performance. Malhotra and Sharma (2002) emphasize the need for the alignment of manufacturing and marketing incentives with the firm's goals and objectives. Hausman, Montgomery and Roth (2002) empirically demonstrate that business performance is enhanced when manufacturing and marketing work together for goal attainment.

The two prominent aspects of customer service, namely, price and leadtime, involve the decisions and actions of both departments. Shapiro (1977) identifies leadtimes and cost control as two marketing/manufacturing areas of "necessary cooperation but potential conflict" among others. Dr. Karl Kempf, Intel Fellow and Director of Decision Technologies in Intel's Technology and Manufacturing Group, reports that different divisions of large companies all too often fail to communicate on important business decisions: "I have lost count of the number of times the sales and marketing guys have made a price move on a particular product only to find that manufacturing capacity fungibility is not what they expected and to capture the increased demand for the target product required cannibalization of a number of other products - it is not uncommon for this kind of problem to have a $100M negative impact overall (prior discussion could have minimized the impact)" (Kempf, 2005). Former Kozmo.com manager John C. Wu addresses the strategic importance of coordinated marketing and operations (Wu, 2001). Kozmo.com was a web retailer that promised to deliver every order within an hour. Low prices were offered to attract customers despite the high fulfillment costs incurred as a result of their service commitment. Not surprisingly, Kozmo.com went out of business. According to AMR Research, companies on the leading edge of price management achieve their success by a centralized pricing function and the adjustment of sales incentives to include margin, not just volume (Preslan and Newmark, 2004).

In this paper, our goal is to study the impact of the decentralization of the marketing and production departments of a make-to-order (MTO) firm, where pricing decisions are made by the marketing department and leadtime decisions by the production department. Although a shorter leadtime may attract more customers and generate more demand, it puts pressure on the firm's production resources. On the other hand, customers might be willing to wait longer if they are offered lower prices. Therefore, depending on market conditions and its current workload, the firm may find it more profitable to offer customers shorter leadtimes at the expense of higher prices or vice versa. To capture the trade-off between price and leadtime, we model the demand as a function of both the price and leadtime sensitivity of the customer. Under the decentralized setting, given their incentives (objective functions) production chooses a leadtime subject to a service level constraint for reliable delivery, while marketing chooses a price. We formulate the problem as a Stackelberg game with two alternative decision making sequences, where production is the leader and marketing is the follower in the first setting, and marketing is the leader and production is the follower in the second setting. We address the following research questions:

1. What are the inefficiencies that result from the decentralization of price and leadtime decisions as quoted by marketing and production, respectively?
2. How can we design a coordination scheme that will align the incentives of marketing and production with the firm's overall objectives?
3. What is the impact of different market characteristics, decision-making sequences and capacity on the optimal decisions and overall profitability?

Our model where production and marketing/sales make the leadtime and price decisions, respectively, applies to several industries, especially to established production systems where the capacity is fixed.[1] In practice, even if the leadtime quote is communicated to the customers via sales/marketing (along with the price quote), the main input (or decision) about the leadtime quote usually comes from production. On the other hand, as the leader, marketing can influence pro-

[1] There have been some examples in practice where marketing made leadtime decisions without consultations from production, but such practices have resulted in unsatisfied customers and significant losses for the firm. For example, Kirk Drummond, chief information officer of a leading food services and products provider Sysco, reports of situations where salespeople brought in huge last minute orders for next-day fulfillment without any advance warning to operations (Johnson and Margulius, 2005).

duction's leadtime decision via its price decision based on the potential demand to be generated, before communicating the quote to the customer.

The organization of the paper is as follows. We begin by presenting a summary of the relevant literature. In Section 3, we discuss our model assumptions and introduce notation. We start our analysis with a centralized setting, where price and leadtime decisions are made by a single decision maker. We next present the decentralized settings, where marketing chooses the price and production chooses the leadtime according to their individual objectives. We compare the centralized and decentralized settings and analyze the sensitivity of the optimal decisions to problem parameters. In Section 4, we present a transfer price contract with bonus payments that provide the right incentives to both departments in order to achieve the centralized solution. In Section 5, we provide insights on including capacity as a decision variable. Section 6 concludes with a summary of insights and possible extensions for future research.

2. Literature Review

There are two streams of research related to our work: (i) due-date management, and (ii) the marketing/production interface. For an extensive review on due-date management policies, see Keskinocak and Tayur (2004). Several researchers study the due date quotation problem taking into account shop-floor congestion at the time an order is placed and consider scheduling/sequencing decisions (Elhafsi and Rolland, 1999; Elhafsi, 2000). In order to quote short and reliable leadtimes to an upcoming order, service level constraints are applied such as the percentage of orders filled on-time or job tardiness (Spearman and Zhang, 1999; Hopp and Sturgis, 2000). Some of the studies consider order selection decisions, where the probability that an arriving customer places an order decreases as the quoted leadtime increases (Duenyas and Hopp, 1995; Duenyas, 1995). Finally, a small number of papers consider price and leadtime decisions simultaneously (Easton and Moodie, 1999; Moodie and Bobrowski, 1999; Moodie, 1999; Plambeck, 2004; Wang, 2004; Celik and Maglaras, 2005; Watanapa and Techanitisawad, 2005; Charnsirisakskul, Griffin and Keskinocak, 2006).

More relevant to our work are the papers that consider price and leadtime decisions simultaneously in steady-state. Palaka, Erlebacher and Kropp (1998) study a firm, where customer demand is treated as linear in the quoted price and leadtime. Firm operations are modelled as an M/M/1 queue with first-come-first-serve (FCFS) sequencing. The objective is to maximize revenues less total variable production costs, congestion related costs and lateness penalty costs subject to a service level constraint, which specifies the minimum probability of meeting the quoted leadtime.

The authors show the impact of changing parameter values on the optimal decisions of a firm and discuss the robustness of firm profits to misestimation of the parameters. So and Song (1998) use the log-linear Cobb-Douglas demand function to model the demand in a similar setting, but do not include congestion or lateness penalty costs in the objective function. Boyaci and Ray (2003) extend the previous two models to the case of two substitutable products for which dedicated capacities are allocated. Finally, Ray and Jewkes (2004) study a variant of the linear customer demand model in Palaka et al. (1998) by treating price as a function of leadtime. These papers develop uniform delivery time guarantees as opposed to quoting a leadtime to each upcoming order with regard to the current state of the system, which constitutes a major distinction from the first set of papers mentioned.

In this study, we model customer demand as a linear function of price and leadtime as in Palaka et al. (1998), but we only consider variable production costs as in So and Song (1998). The desirable properties of the linear demand function are discussed in Palaka et al. (1998). While previous papers assume a centralized decision maker controlling price and leadtime, our main contribution is in demonstrating the inefficiencies that result when price and leadtime are quoted by two independent functions within a firm. We show that a transfer price contract with bonus payments motivates marketing and production to generate higher profits with an efficient output that matches the centralized solution. Our results for the centralized setting, which we use as a benchmark, are consistent with those found in Palaka et al. (1998). However, we provide a more detailed analysis on pricing decisions.

The second stream of research concentrates on the joint decision making of the marketing and operations functions of a firm. For an early review of marketing and production coordination, the reader is referred to Eliashberg and Steinberg (1993). Several of the papers in this stream focus on pricing and/or replenishment decisions (Eliashberg and Steinberg, 1987; Dewan and Mendelson, 1990; Mendelson and Whang, 1990; Porteus and Whang, 1991; Kouvelis and Lariviere, 2000; Kumar, Loomba and Hadjinicola, 2000; Li and Atkins, 2002; Gupta and Weerawat, 2004); however, they do not consider leadtime decisions. de Groote (1994) studies product variety versus process flexibility in the marketing/operations interface, while Balasubramanian and Bhardwaj (2004) model a duopoly in which firms with decentralized marketing and manufacturing functions with conflicting objectives compete on the basis of price and quality. Erkoc and Wu (2002), Chatterjee, Slotnick and Sobel (2002), Ho and Zheng (2004) and Slotnick and Sobel (2005) study the leadtime quotation problem within the marketing/operations interface. However, they do not consider pricing decisions. A recent paper by Liu, Parlar and Zhu (2006) considers price and leadtime decisions in a two-firm setting; a supplier and a retailer within a supply chain, where the supplier also needs to choose the transfer price. In contrast to our focus on evaluating marketing as a revenue center and production as a cost center, they focus on the inefficiencies that result from the double marginalization in the supply chain. Although they find that decentralization leads to lower profits as we do, since the decentralized structures in the two papers are different, they show that under the decentralized setting prices are higher, lead-times are shorter and demand is lower, which is exactly the opposite of what we find in our study. Moreover, they do not consider coordinating mechanisms, whereas we show that coordination in our setting can be achieved through a transfer price contract with bonus payments. In this respect, our paper is the first to study marketing and production coordination for price and leadtime decisions.

3. The Model

We consider a firm that serves customers in an MTO fashion. Capacity is assumed to be constant, while price and leadtime are decision variables. We include capacity as a decision variable in Section 5. It has been shown in the literature that for high service levels the tail of the waiting time distribution is approximated well by the exponential distribution even for a G/G/s queue (So and Song, 1998). Thus, the firm's operations are modeled as an M/M/1 queue with mean production rate, μ, and mean arrival rate, λ. We refer to the mean production rate, μ, as the capacity of the system. Subscript C denotes the centralized setting, while subscripts P and M denote the decentralized setting with production as the leader and marketing as the leader, respectively. We use the following notation throughout the text:

Parameters:

a : maximum attainable demand (market potential) corresponding to zero price and zero leadtime

b : price sensitivity of demand

c : leadtime sensitivity of demand

m : unit production cost

μ : capacity of the production system (service rate)

s : service level defined as the probability of meeting the quoted leadtime

k : used for computational simplicity, $k = \ln(1/(1-s))$

Decision Variables

p_j : price quoted by the marketing department $(j = C, P, M)$

L_j : leadtime quoted by the production department $(j = C, P, M)$

$D(p, L)$: expected demand generated by the quoted price p and quoted leadtime L

λ_j : mean arrival (demand) rate ($j = C, P, M$)

π_j : profit achieved by the firm ($j = C, P, M$)

π^{MR}, π^{PR} : profit achieved by the marketing and production departments, respectively

r : incentive per unit offered to the production department for positive demand

w : transfer price charged per unit by production to marketing

α_1, α_2 : the fraction of revenue offered to marketing and production as a bonus payment, respectively

K : unit capacity cost (to be used in Section 5).

Our model assumptions are as follows:

A1. There are no holding or lateness penalty costs; there is a variable production cost.

A2. Expected demand rate, $D(p, L)$, is linear in price, p, and leadtime, L:

$$D(p, L) = a - bp - cL$$

where $b > 0$ and $c > 0$.

A3. (**Positive Demand Assumption**) There is positive demand for the firm to provide its services when the smallest reasonable price, m, and the shortest leadtime that satisfies the service level constraint, (k/μ), are chosen: $D(m, (k/\mu)) = a - bm - ck/\mu > 0$. Note that if this assumption is not satisfied, the firm can never generate positive profits, and hence, the problem becomes trivial.

A4. All the parameters of the system are common knowledge to marketing and production.

3.1 The Centralized Setting (Model C)

In the centralized setting, the marketing and production decisions are considered simultaneously with the objective of maximizing profit.

$$\max_{(\lambda_C, p_C, L_C) \geq 0} \quad \lambda_C (p_C - m)$$

$$s.t. \quad 1 - e^{-(\mu - \lambda_C) L_C} \geq s \tag{1}$$

$$\lambda_C \leq a - bp_C - cL_C \tag{2}$$

$$\lambda_C \leq \mu \tag{3}$$

Constraint (1) states that the probability of meeting the quoted leadtime should be at least as large as the required service level. Constraint (2) ensures that the mean demand rate served by the firm does not exceed the demand generated by the quoted price and leadtime. Constraint (3) is the stability condition.

The optimal solution is identified by the following proposition.

Proposition 1 *The optimal demand generated under the centralized setting is given by the unique root of $f_C(\lambda_C)$, i.e., $f_C(\lambda_C^*) = 0$, on the interval $[0, \mu]$, where*

$$f_C(\lambda_C^*) = (a - 2\lambda_C^* - mb)(\mu - \lambda_C^*)^2 - ck\mu \tag{4}$$

The optimal leadtime and the optimal price are then given by $L_C^* = \frac{k}{\mu - \lambda_C^*}$ and $p_C^* = (a - \lambda_C^* - cL_C^*)/b$, respectively.

These results are a special case of those found in Palaka et al. (1998) for the fixed capacity case, when there are no holding or lateness costs. [2]

3.2 The Decentralized Setting, Production Leader (Model P)

In this section, we consider the case where the production and marketing departments operate in a decentralized setting making their decisions based on individual incentives. We model the sequence of decisions as a Stackelberg game, where production moves first and chooses a leadtime that maximizes its profit subject to the service level constraint. Marketing observes this leadtime decision before quoting price with the objective of maximizing its own profit. We assume initially that no production-related costs are incurred by the marketing department. Hence, the objective function of marketing is given by the revenue of the firm.

We solve for subgame-perfect Nash equilibrium by backwards induction starting with marketing's problem.

$$\max_{p_P \geq m} \ \pi_P^{MR} = p_P(a - bp_P - cL_P)$$

We find the optimal price and demand as follows:

$$p_P^*(L_P) = \max\left\{m, \frac{a - cL_P}{2b}\right\} \text{ and } \lambda_P^*(L_P) = a - bp_P^*(L_P) - cL_P \tag{5}$$

The optimal demand, $\lambda_P^*(L_P)$ derived from marketing's best response, $p_P^*(L_P)$, to production's leadtime decision is then used in production's problem. The cost term of production's objective

[2] All proofs can be found in the appendix.

function reflects the cost incurred by the firm. However, one should note that if this was solely a cost minimization problem, production would quote the longest possible leadtime, which satisfies the service level constraint driving demand to zero. Therefore, there should be an incentive, r, given by the firm to production per unit of demand realized.

$$\max_{0 \leq L_P \leq \frac{a}{c}} \quad \pi_P^{PR} = (r - m)\lambda_P^*(L_P) \tag{6}$$

$$s.t. \quad \left(\mu - \lambda_P^*(L_P)\right) L_P \geq k \tag{7}$$

In order to generate strictly positive demand, r would range from $m \leq r \leq p_P^*$, given the linearly decreasing structure of π_P^{PR} in L_P. A reasonable incentive that we choose is $r = p_P^*(L_P)$, which turns production's problem into the firm's overall problem in the decentralized setting. Note that this objective function is consistent with the concept of creating pseudo-profit centers within the firm, i.e., associating revenues artificially with cost centers, as discussed in Otley (2002).

Proposition 2 *The optimal solution to the decentralized setting, P, is given by:*

(i) *If $a > 2mb$ and $\mu > \mu^0 = bm + \frac{ck}{a-2mb}$, then $L_P^* = \frac{(a-2\mu)+\sqrt{(2\mu-a)^2+8ck}}{2c}$ and $p_P^* = \frac{a-cL_P^*}{2b}$.*

(ii) *Otherwise, $p_P^* = m$ and $L_P^* = \frac{a-bm-\mu+\sqrt{(\mu-a+bm)^2+4ck}}{2c}$, and the firm will be selling at cost.*

Proposition 2 states that there is a minimum capacity requirement, $\mu > \mu^0$, for the production department to generate positive profits under this decentralized setting. Long leadtimes under restricted capacity motivate marketing to quote low prices in order to maximize its profit, which may not be sufficient to cover production costs. Therefore, in order for production not to drive demand to zero, the price quoted by marketing needs to be at least as large as the unit production cost. Note that, in practice, this would correspond to misaligned incentives leading to suboptimal performance.

3.3 The Decentralized Setting, Marketing Leader (Model M)

Under this setting, marketing moves first and chooses a price that maximizes its revenue. Observing this price, production quotes a lead-time with the objective of maximizing its profit subject to the service level constraint. We start with production's problem and offer the price quoted by marketing as an incentive to production as in Model P.

$$\max_{0 \leq L_M \leq \frac{a-bp_M}{c}} \quad \pi_M^{PR} = \pi_M = (p_M - m)(a - bp_M - cL_M) \tag{8}$$

$$s.t. \quad \left(\mu - (a - bp_M - cL_M)\right) L_M \geq k \tag{9}$$

As the objective function is linearly decreasing in L_M, Constraint (9) is tight at optimality, and assuming $p_M \geq m$, the optimal lead-time is found as:

$$L_M^*(p_M) = \frac{(a - bp_M - \mu) + \sqrt{(a - bp_M - \mu)^2 + 4ck}}{2c} \tag{10}$$

Marketing's problem is given by:

$$\max_{p_M \geq m} \ \pi_M^{MR} = p_M(a - bp_M - cL_M^*(p_M))$$

As long as the unique maximizer of π_M^{MR} over the interval $[0, (a\mu - ck)/b\mu]$ is at least m, then we can characterize the optimal solution by the following proposition. Otherwise, the firm will be selling at cost.

Proposition 3 *The optimal demand under decentralized setting M is given by the unique root of $f_M(\lambda_M)$ on the interval $[0, \mu]$, where*

$$f_M(\lambda_M^*) = (a - 2\lambda_M^*)(\mu - \lambda_M^*)^2 - ck\mu \tag{11}$$

The optimal leadtime and the optimal price are then given by $L_M^* = \frac{k}{\mu - \lambda_M^*}$ and $p_M^* = (a - \lambda_M^* - cL_M^*)/b$, respectively.

3.4 Comparison of the Decentralized and Centralized Settings

In this section, we discuss the inefficiencies due to decentralization of pricing and leadtime decisions and compare the centralized and decentralized settings.

Proposition 4 *The optimal decisions of the centralized setting (C) and the decentralized settings where production is the leader (P) and where marketing is the leader (M) are such that: (i) $\lambda_C^* < \lambda_M^* \leq \lambda_P^*$ (ii) $L_C^* < L_M^* \leq L_P^*$ (iii) $p_C^* > p_M^* \geq p_P^*$ (iv) $\pi_C^* > \pi_M^* \geq \pi_P^*$.*

As long as it is given a positive margin, production quotes the tightest reliable leadtime given its available capacity and the required service level. As marketing's incentive is based on revenue irrespective of the production costs, it is motivated to create more demand than the centralized firm, which requires longer leadtimes and lower prices.

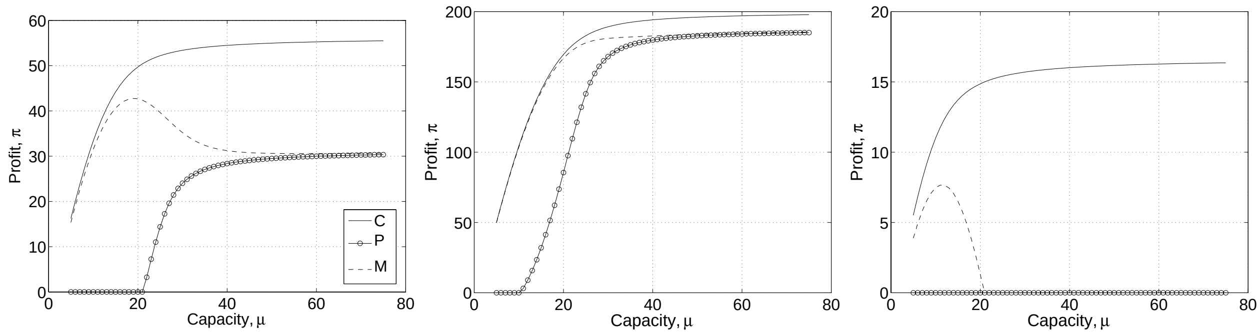
When marketing is the follower in the decentralized firm (P), it responds to longer leadtimes by decreasing prices, as it can be seen from its best response function (Equation (5)), which creates a large volume of demand but also high production costs. On the other hand, in M, marketing can anticipate production's best response as the leader, where production quotes lower leadtimes to higher prices (Equation (10)). Thus, marketing can choose a higher price to motivate production to satisfy as much demand as possible without increasing leadtimes significantly.

Proposition 5 *In a decentralized setting, highest revenues are generated when marketing is the leader, while highest volume is generated when production is the leader. The firm would prefer having marketing rather than production as the leader for generating more profits and quoting higher prices and lower leadtimes in a decentralized setting.*

The aggressive response of marketing in P results in the highest volume among all three settings. However, low prices are not sufficient to make up for the high production costs, and this setting generates the lowest profits. When marketing is the leader (M), its influence on production helps to achieve highest revenues. However, costs also increase and overall, lower profits are generated as compared to the centralized setting. Thus, our analysis shows that employing a revenue or volume-based incentive mechanism for marketing does not lead to optimal profits. Note that the dominance of the decentralized setting with marketing as leader over the one with production as the leader is consistent with the findings of Li and Atkins (2002), where marketing and production make pricing and replenishment decisions, respectively, and misalignment of incentives within the firm can be mitigated through having marketing as the dominant function. However, in our case we see that the two functions can still not achieve the centralized solution without a coordinating mechanism. Even when there are no capacity restrictions and no service level constraints, the profit difference between the centralized and decentralized solutions is $\frac{m^2 b}{4}$, i.e., constant.

The complete sensitivity analysis of the optimal decisions $L^*_{\bullet}$, $p^*_{\bullet}$ and $\lambda^*_{\bullet}$, and the optimal profit $\pi^*_{\bullet}$ with respect to problem parameters for the centralized and decentralized settings can be found in the appendix. We find that in the decentralized settings, the quoted leadtime is not affected by a change in b or m as long as production receives a positive margin, given marketing's best response in price. As marketing does not consider production costs, the optimal price is independent of a change in m. Moreover, an increase in b is met by a decrease in the quoted price. Thus, the generated demand is not affected by a change in b or m. We next discuss the cases where the behavior of the optimal price (and profit for M) changes conditionally. In all numerical demonstrations, we use $a = 50,\ b = 4$ and $m = 5$ unless otherwise stated.

Figures 1 (i) and 2 (i) show a comparison of the profits and prices, respectively, under C, P and M at different capacity levels. We observe that for low-medium capacity levels, M performs very close to C, while as the capacity increases, M deviates from C and converges to P. Under P, at low capacities production quotes long leadtimes to which marketing responds with low prices, and up to a certain capacity level, the firm sells at cost and makes no profit. As more capacity becomes available, marketing responds to shorter leadtimes by increasing prices and the profits also increase.



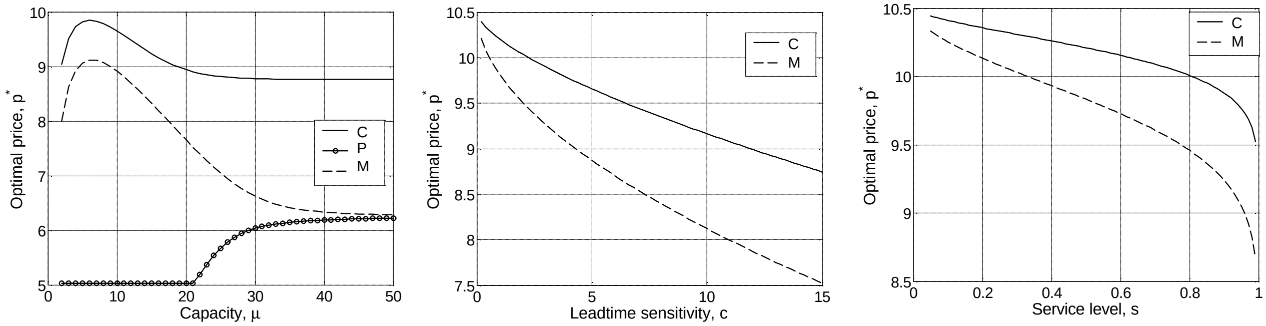
(i) $(b = 4)$ **(ii)** $(b = 2)$ **(iii)** $(b = 6)$

Figure 1: Comparison of Profits at Different Capacities $(c = 4,\ s = 0.95)$

Under M, marketing chooses a higher price as compared to P, anticipating production's response under tight capacity as the leader. As capacity increases, production can quote lower leadtimes and more demand can be met. Thus, marketing's response as the leader gets closer to its response as the follower, which results in lower prices and profits as compared to the centralized setting. From Figure 2 (i), we can see that higher capacity does not necessarily result in charging higher under C or M. Price increases in μ up to a certain point in order to quote shorter leadtimes within a tight capacity interval. As capacity increases, lower prices are quoted to increase demand. However, decentralization results in a sharper decrease in the quoted price. Note that price stabilizes as it approaches the unconstrained solution, i.e., $\frac{a+mb}{2b}$ for C and $\frac{a}{2b}$ for M under ample capacity, which is reached at $(a - mb)$ for C and a for M.

Observation 1 *Higher capacity results in higher flexibility and higher profits for a centralized firm. However, higher capacity does not necessarily result in higher profits for a decentralized firm.*

In Figures 1 (ii) and (iii), we explore the effect of b on the deviation of the decentralized profits from the centralized profit. Note that the optimality equation for λ^*_M (Equation (11)) is very similar to the optimality equation for λ^*_C (Equation (4)), but is independent of m and b. The optimal demand under P is also not affected by a change in m or b. Thus, we expect the deviation in profits to decrease as m and/or b decreases. A decrease in m alleviates the adverse effect of low prices on margin, while a decrease in b motivates marketing for less aggressive price cuts. In Figure 1(ii), we observe that at a lower b, M tracks C more closely, and the gap between the centralized and both decentralized settings decreases as the capacity increases. On the other hand, when b increases in Figure 1(iii), the firm needs to sell at cost for all capacity levels when production is the leader, while positive profit can be obtained within a tight capacity interval when marketing becomes the leader. We observe similar results when m changes rather than b. In summary, *when* *marketing is the leader, the profit difference from the centralized solution may not be significant under tight capacity and when b and/or m is low.*



(i) $(c = 4,\ s = 0.95)$ **(ii)**$(\mu = 8,\ s = 0.95)$ **(iii)**$(\mu = 8,\ c = 4)$

Figure 2: Price vs. Capacity, Leadtime Sensitivity and Service Level (Low Capacity)

The following proposition summarizes the behavior of the optimal price, p_i^*, with respect to leadtime sensitivity, c, and service level, s for $i = C,\ M$. Let $x_C = mb$ and $x_M = 0$. We define capacity levels $\mu \le \frac{a-x_i}{3}$ as low, $\mu \in \left(\frac{a-x_i}{3}, a - x_i\right)$ as medium and $\mu \ge (a - x_i)$ as high for $i = C,\ M$.

Proposition 6 *When $\mu \in \left(\frac{a-x_i}{3}, a - x_i\right)$, i.e., capacity is medium,*

(i) *p_i^* increases in c up to a threshold, c_i^0, and then decreases, where $c_i^0 = \frac{1}{k\mu}\left(\frac{a-x_i-\mu}{2}\right)\left(\frac{a-x_i-3\mu}{4}\right)^2$.*

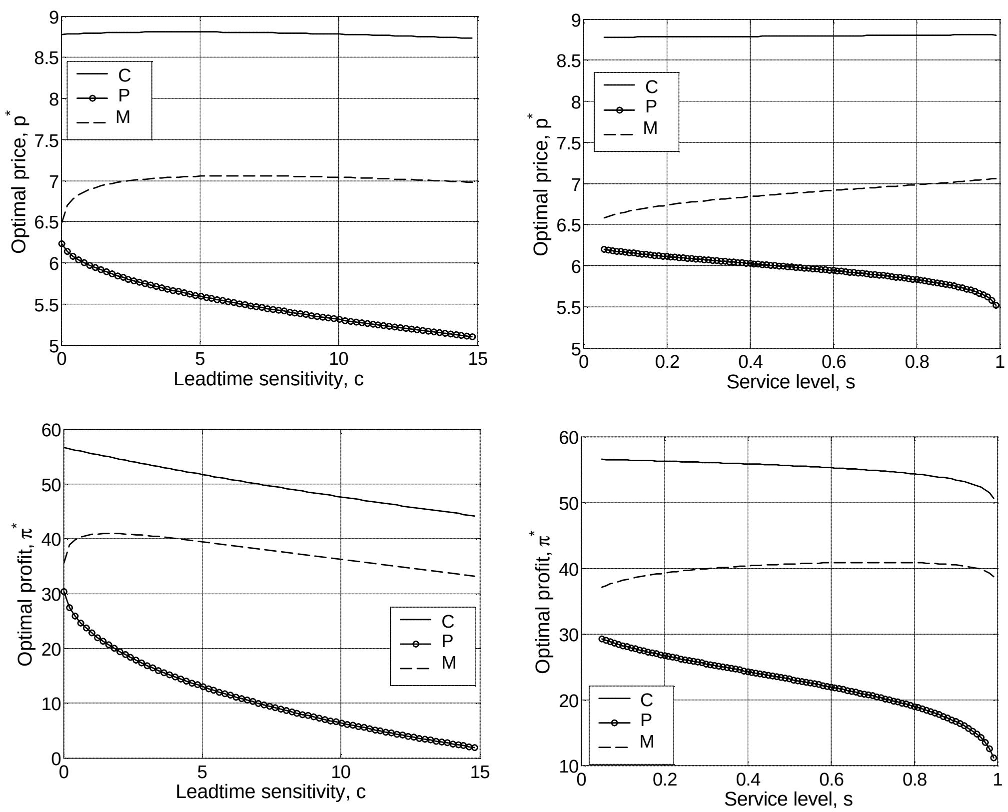
(ii) *p_i^* increases in s up to a threshold, s_i^0, and then decreases, where $s_i^0 = 1 - \frac{1}{e^{k_i^0}}$ and $k_i^0 = \frac{1}{c\mu}\left(\frac{a-x_i-\mu}{2}\right)\left(\frac{a-x_i-3\mu}{4}\right)^2$.*

When $\mu \notin \left(\frac{a-x_i}{3}, a - x_i\right)$, i.e., capacity is low or high, p_i^* decreases in c and in s.

Note that low, medium, and high capacities are defined with respect to the market potential for both C and M, but also depend only on the unit production cost and price sensitivity for C. Under high capacity, leadtimes are already low and price is decreased to capture more demand as c or s increases, although the change is relatively minor. At low capacity levels, it is not possible to shorten the leadtime further as c increases, while longer leadtimes are required as s increases. Thus, the firm needs to decrease its quoted price in order to attract customers. From Figures 2 (ii) and (iii), we can see that under "low" capacity the price decrease under M is more aggressive than under C and the gap between the two settings increases as c or s increases. At medium capacity levels, quoted price under C and M increases in c and s up to a threshold, where μ is sufficient for charging a higher price for better service. However, beyond this point, the quoted leadtime can only be decreased slightly in c, and has to be increased sharply in s, given the capacity. Thus, the quoted price decreases to attract more customers. Note that when s_C^0 or c_C^0 is beyond the feasible range for s or c, respectively, given the parameters of the system, we may only observe an increasing behavior in price. This contradicts with the result found in (So and Song, 1998) for the fixed capacity case, which states that a higher service level implies a lower price.

Observation 2 *When marketing is the leader, the decentralized firm may experience an increase in profits at medium capacity levels as the leadtime sensitivity or the service level increases.*

We demonstrate this observation in Figure 3 comparing the three settings for quoted prices and profits. For the centralized firm, prices are relatively stable. Thus, the price increase is less effective than the decrease in the generated demand as c or s increases, and the profits decrease. On the other hand, under M, as marketing is evaluated based on revenue, price increase may be more effective than the demand decrease, and the decentralized firm may benefit from higher service levels or higher leadtime sensitivity of the customer demand. Note that when production is the leader, price and demand both decrease in c and s, hence, the profits also decrease.



(i) ($s = 0.95$) **(ii)** ($c = 4$)

Figure 3: Price and Profit vs. Leadtime Sensitivity and Service Level at Medium Capacity ($\mu = 25$)

4. Transfer Price with Bonus Payments (TB) Contract

In this section, we propose a transfer price contract with bonus payments for coordinating marketing and production, where marketing pays w to production for each unit produced, and both departments receive a bonus payment as the fraction of the total revenues generated. We use subscript $i = P,\ M$ to represent production as the leader and marketing as the leader, respectively. Note that total firm profits are given by $\pi_i = \pi_i^{PR} + \pi_i^{MR}$.

We study a flexible mechanism where the fractions of revenues received by the two departments do not necessarily add to 1. Let $\alpha_1 > 0$ denote the share for marketing and $\alpha_2 \geq 0$ the share for production. We assume that $\alpha_1 + \alpha_2 \leq 1$.

4.1 Solution under the TB Contract

Marketing's problem is:

$$\max_{p_i \geq m} \ \pi_i^{MR} = (\alpha_1 p_i - w)(a - bp_i - cL_i)$$

Production's problem is:

$$\begin{aligned} \max_{L_i \geq 0} \ \pi_i^{PR} &= (\alpha_2 p_i + w - m)(a - bp_i - cL_i) \\ s.t. \quad & (\mu - (a - bp_i - cL_i))\, L_i \geq k \end{aligned} \tag{12}$$

We solve for the subgame-perfect Nash equilibrium by backwards induction starting with the follower's problem in each setting.

Proposition 7 *Decentralized setting $i = P, M$ under the TB Contract has a nontrivial optimal solution with positive profit if and only if for every α_1 fraction of revenue for marketing and an accordingly chosen transfer price, $w_i^{min}(\alpha_1) \leq w \leq \alpha_1 \left(\frac{a}{b} - \frac{ck}{\mu b}\right)$, α_2 is chosen such that $\alpha_{2i}^{min}(\alpha_1, w) \leq \alpha_2 \leq (1 - \alpha_1)$, where $\alpha_{2i}^{min}(\alpha_1, w)$ ensures that production receives a positive margin, i.e., $\alpha_2 p_i^* + w - m \geq 0$, and $w_i^{min}(\alpha_1)$ is the minimum transfer price that ensures $\alpha_2 \leq (1 - \alpha_1)$. For these (α_1, α_2, w) combinations:*

(i=P) Production Leader, Marketing Follower:

The optimal decisions are given by

$$L_P^* = \frac{a - 2\mu - wb/\alpha_1 + \sqrt{(a - 2\mu - wb/\alpha_1)^2 + 8ck}}{2c},$$

$$p_P^* = \frac{a - cL_P^*}{2b} + \frac{w}{2\alpha_1}, \quad \lambda_P^* = \frac{a - cL_P^*}{2} - \frac{wb}{2\alpha_1}.$$

(i=M) Marketing Leader, Production Follower:

The optimal demand is given by the unique root of $f_M(\lambda_M)$ *over the interval* $[0, \mu]$, *where*

$$f_M(\lambda_M^*) = (a - 2\lambda_M^* - wb/\alpha_1)(\mu - \lambda_M^*)^2 - ck\mu \tag{13}$$

The optimal leadtime and price are $L_M^* = \frac{k}{\mu - \lambda_M^*}$ *and* $p_M^* = (a - \lambda_M^* - cL_M^*)/b$.

Note that the maximum transfer price that can be charged is decreasing in service level, which seems counter-intuitive. However, this is only an upper bound to ensure that positive demand is generated. One should also note that the set of feasible contract parameters do not necessarily need to be identical for both decentralized settings.

4.2 Coordination under the TB Contract

In order to achieve coordination, we need to choose the contract parameters such that the demand generated in the decentralized settings is equal to that in the centralized setting, i.e., $\lambda_i^* = \lambda_C^*$. This demand will also guarantee the leadtime and price quoted in the centralized setting, since they are uniquely determined by the demand level. The following proposition characterizes the coordinating contract parameters for both decentralized settings.

Proposition 8 *Under the TB contract, there exists a unique coordinating transfer price* $w_P^* = \alpha_1 \left(\frac{a - 2\lambda_C^* - \frac{ck}{\mu - \lambda_C^*}}{b} \right)$ *under* P *and* $w_M^* = \alpha_1 m$ *under* M *for a given* $0 < \alpha_1 \leq 1$ *fraction of the revenue for marketing and* $\alpha_2 \in \left[\alpha_{2i}^{min}(\alpha_1, w_i^*), (1 - \alpha_1)\right]$ *fraction of the revenue for production.*

It can be observed from Proposition 8 that the coordinating transfer price only depends on α_1 and m when marketing is the leader, while it depends on all problem parameters except m and requires knowledge of the centralized demand level when production is the leader. Moreover, $w_P^* > \alpha_1 m = w_M^*$, i.e., marketing needs to pay a larger transfer price to production as the follower than as the leader. One should also note that the coordinating transfer price, w_i^*, increases linearly in α_1 under both decentralized settings. In other words, as marketing gets a higher fraction of the revenue, it needs to pay a higher per unit transfer price to production.

Under coordination, marketing receives α_1 fraction of the centralized revenue. Proposition 9 indicates that the fraction of the centralized profit it achieves is not the same under P and M.

Proposition 9 *The fraction of the centralized profit that is realized by marketing is* α_1 *when it is the leader and less than* α_1 *when it is the follower.*

Note that when production is the leader, the fraction of the centralized profit realized by marketing increases with capacity. Moreover, as the profit achieved by marketing reduces to $\alpha_1 \frac{(\lambda_C^*)^2}{b}$ and λ_C^* is increasing in μ, the absolute profit achieved by marketing is also increasing in μ. Hence, marketing becomes better off with higher capacity.

Observation 3 *For a given α_1 and the coordinating transfer price as described in Proposition 8, production prefers $\alpha_2 = (1-\alpha_1)$ to generate higher profits under both decentralized settings. In this case, the fraction of the centralized profit that is realized by production is equal to $(1-\alpha_1)$ when it is the follower and greater than $(1-\alpha_1)$ when it is the leader.*

The TB contract with parameters as described in Observation 3, where $\alpha_1 + \alpha_2 = 1$, has been frequently discussed in the literature as the "Revenue Sharing Contract". This contract provides flexible allocation of profits between marketing and production, where the fraction of profit marketing achieves ranges from 0 to $\frac{\lambda_C^*}{(p_C^* - m)b}$ under P and anywhere from 0 to 1 under M.

Another special case of the TB contract is the transfer price-only contract with $\alpha_1 = 1,\ \alpha_2 = 0$, which has been addressed in the literature as the "Wholesale Price Contract". From Proposition 8, we find that the unique coordinating transfer price is given by $w_P^* = \left(\frac{a - 2\lambda_C^* - \frac{ck}{\mu - \lambda_C^*}}{b} \right)$ for P and $w_M^* = m$ for M. Note that when marketing is the leader, production breaks even, i.e., generates zero profit under this contract. However, we see that replacing the objective function of marketing with the firm profit rather than the firm revenue will achieve the centralized solution. Therefore, the proposed contract is in line with the view of industry experts on adjusting the sales incentives to include margin (Preslan and Newmark, 2004).

Finally, under a TB contract with no transfer price ($w = 0$), coordination cannot be achieved under either of the decentralized settings. In fact, such a contract generates the same solution as the original decentralized settings, and thus, it is not possible to coordinate the two departments. For an extensive review of supply chain contracts, the reader may refer to Cachon (2004).

4.3 Robustness of the TB Contract

In this section, we examine the robustness (in terms of percent profit loss) of the TB contract to estimation errors in the price sensitivity and leadtime sensitivity of the customers. Note that when marketing is the leader, the coordinating transfer price is independent of the problem parameters, except m. Thus, misestimation of the demand parameters may only affect the feasibility of α_2 for a given α_1, which would not constitute a problem under a Revenue Sharing Contract with $\alpha_2 = 1 - \alpha_1$. However, when production is the leader, misestimating the demand parameters may have a significant effect on choosing the correct coordinating transfer price. Thus, we investigate the robustness of the TB contract for decentralized setting P and we use a transfer price-only contract with $\alpha_1 = 1$ and $\alpha_2 = 0$ for demonstration purposes. As the general contract setting offers more flexibility, its robustness cannot be expected to be worse than this contract.

We perform the analysis as follows: (i) Calculate the transfer price that coordinates production and marketing at an estimated value of the parameter, and (ii) calculate the optimal decisions of the two parties under the contract with the true parameter value using the decentralized setting, P. Figure 4 shows the percent profit loss caused by the estimation errors in b and c. We observe that *underestimation of b or c leads to higher profit losses as compared to overestimation.* When b (c) is underestimated, the chosen transfer price motivates a higher price (a longer leadtime) than the customer is willing to accept, which results in a sharp decrease in demand and a high profit loss. When b is overestimated, the offered price is lower than the customer is willing to pay, and the generated demand becomes higher than optimal, which also results in lower profits. Although a similar reasoning follows for c, we observe that large overestimation errors result in higher profit losses than underestimation. As capacity increases, the gap between the decentralized and centralized solutions decreases and the contract becomes more robust to estimation errors in b. Interestingly, we observe the highest profit loss at medium capacity levels for c as compared to low and high capacities, since there is more room to make errors at medium capacity levels, given the service level constraint. We finally note that *estimation errors in price sensitivity are much costlier than those in leadtime sensitivity.*

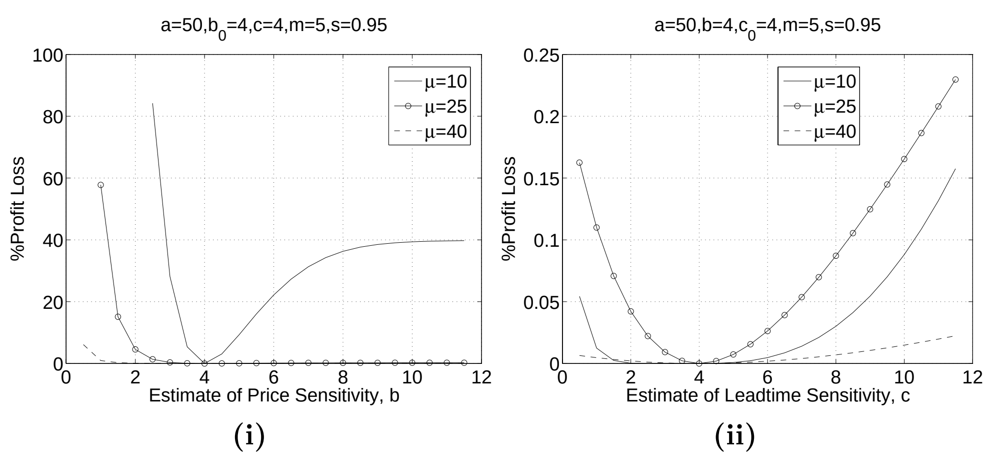


Figure 4: Percent Profit Loss when Price Sensitivity or Leadtime Sensitivity is misestimated

5. Capacity Decision

In this section, we include capacity as a decision variable, and compare the decentralized settings P, M and the centralized setting C. Let K denote the unit capacity cost. In the centralized setting, the firm aims to maximize profit, which is given by revenue minus production and capacity costs.

$$\begin{aligned} \max_{(\lambda_C, L_C, \mu_C)\geq 0} \quad & \pi_C = (a - cL_C - \lambda_C)\lambda_C/b - m\lambda_C - K\mu_C \\ s.t. \quad & (\mu_C - \lambda_C)L_C \geq k \end{aligned}$$

As one unit of production will require at least one unit of capacity, the minimum cost incurred per unit will be $(m+K)$. Thus, we can revise Assumption A3 to $a-b(m+K)-ck/\mu > 0$, and constrain the capacity decision to $\mu_C > ck/(a-(m+K)b)$ in order to satisfy it. We also restrict our attention to values of $K \leq (a-mb)/b$ for non-triviality. In the decentralized setting P, where production is the leader, production first chooses a capacity and a leadtime and marketing then chooses a price. The best response of marketing to a given leadtime and capacity is given by Equation (5). The following proposition describes the optimal solution for C and P.

Proposition 10 *The optimal leadtime decisions under C and P are equal, i.e., $L_P^* = L_C^* = L$, and satisfy*

$$c^2L^3 - c(a-(m+K)b)L^2 + 2Kkb = 0 \tag{14}$$

(i) *for $K \leq \min(K^1, \bar{K})$ under C,*

(ii) *for $K \leq \min(K^0, \bar{K})$ under P,*

where K^1 and K^0 are the minimum values of K for which $\pi_C(L_C) = 0$ and $\pi_P(L_P) = 0$, respectively, and $\bar{K}$ is the K value beyond which Equation (14) does not have a real root on $\left[0, \frac{a-(m+K)b}{c}\right]$. Then, the optimal decisions will be given by

$$\lambda_C^*(L_C^*) = \frac{a - cL_C^*}{2} - \frac{(m+K)b}{2}, \;\; p_C^*(L_C^*) = \frac{a - cL_C^*}{2b} + \frac{m+K}{2}, \;\; \mu_C^* = \lambda_C^*(L_C^*) + \frac{k}{L_C^*} \tag{15}$$

$$\lambda_P^*(L_P^*) = \frac{a - cL_P^*}{2}, \;\; p_P^*(L_P^*) = \frac{a - cL_P^*}{2b}, \;\; \mu_P^* = \lambda_P^*(L_P^*) + \frac{k}{L_P^*} \tag{16}$$

Otherwise, capacity becomes too costly to generate positive profit.

It can be seen from Proposition 10 that the optimal capacity decision is given by the optimal generated demand plus an adjustment amount to meet the service level.

Corollary 1 *The difference in the optimal demand, capacity, price and profit between C and P is as follows: (i)* $(\lambda^*_P - \lambda^*_C) = (\mu^*_P - \mu^*_C) = (m + K)b/2 > 0$ *(ii)* $p^*_C - p^*_P = (m + K)/2 > 0$ *(iii)* $\pi^*_C - \pi^*_P = (m + K)^2 b/4 > 0$.

The inefficiencies caused by the decentralization of price, leadtime and capacity decisions can be clearly seen from Corollary 1. Although the leadtime decision is the same under both settings, there is no markup in price for the average cost per unit under the decentralized setting since marketing's performance is solely based on revenue. On the other hand, for the centralized setting, the markup is given by (m+K)/2. This discrepancy also results in an associated amount of extra demand and capacity under the decentralized setting, which lowers the profit.

In the decentralized setting M, production first chooses a capacity, and the rest of the game is exactly the same as the original marketing Stackelberg game. As a result of the analytical intractability of this problem, we compare the optimal decisions under different settings numerically. Figure 5 shows the optimal profit and decisions with respect to the capacity cost, K. We do not include P beyond the K value where positive profit cannot be generated. We can summarize our

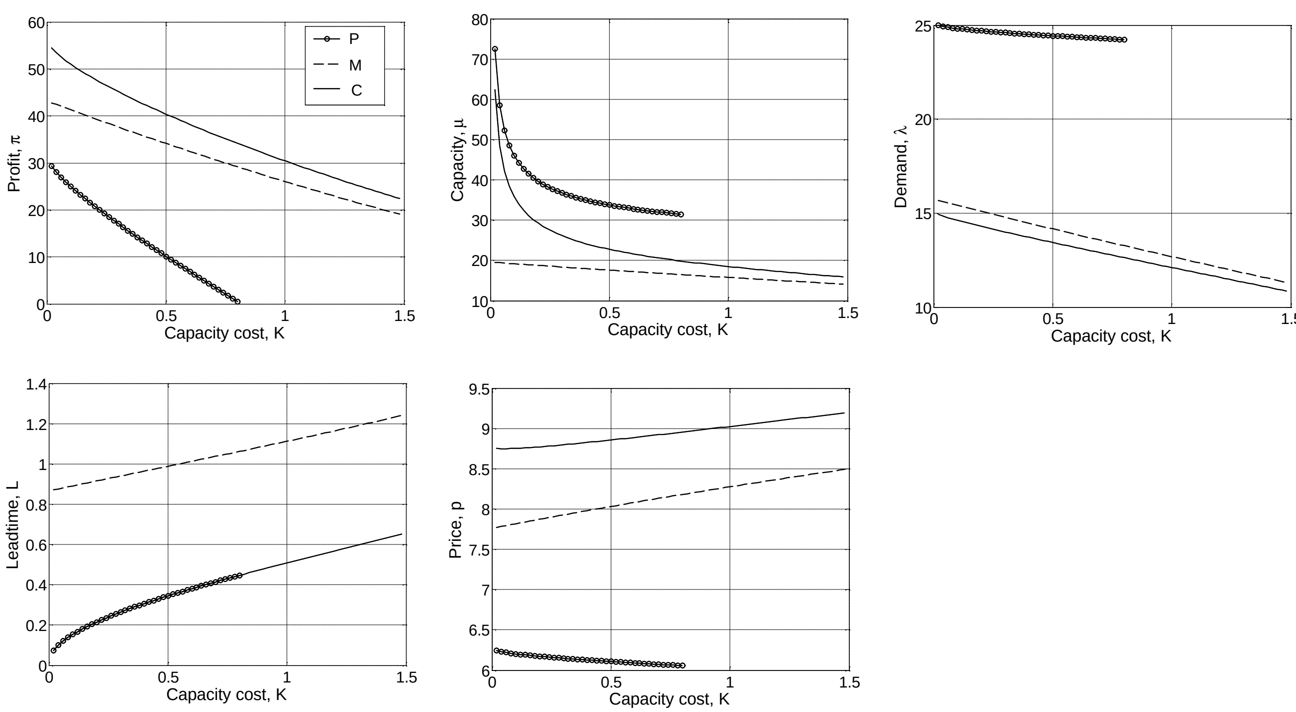


Figure 5: Comparison of Settings with Capacity Decision ($a = 50,\ b = 4,\ c = 4,\ s = 0.95,\ m = 5$)

observations as follows:

- When capacity is included as a decision variable, the ordering of optimal decisions and profit under different settings is the same as in Proposition 4 at given capacity costs with the exception that $L^*_P = L^*_C$.

- The decentralized setting M prefers a lower capacity than the centralized setting, while the decentralized setting P prefers a higher one. This result is consistent with our previous findings, where M generated higher profits and was closer to C at low capacity levels. When marketing is the leader, a lower capacity is sufficient, while production needs to choose a high capacity to meet the generated demand as the leader, which becomes too costly beyond a certain point.

- When production is the leader, as K increases, capacity decreases and leadtime increases. Thus, marketing lowers price in response. When marketing is the leader, it responds to a lower capacity level with a higher price. Therefore, in Figure 5, we observe that as K increases, p^*_M increases while p^*_P decreases. Moreover, the profit generated under M approaches that under C, while the profit generated under P deviates.

6. Conclusion and Future Work

We studied a firm with two independent functions, marketing and production, which serves customer demand that is sensitive to both price and leadtime. Price and leadtime decisions are made by marketing and production, respectively. Production needs to satisfy a certain percentage of orders on time under limited capacity. We analyzed the types of inefficiencies that result from the decentralization of these two functions.

In order to achieve coordination, we proposed a transfer price contract with bonus payments, where marketing pays production a transfer price per unit produced, and both departments receive a fraction of the total revenues generated as a bonus payment. We showed the existence of a unique transfer price for a given fraction of total revenues offered to marketing, α_1, that achieves coordination as long as production receives a satisfactory incentive as a fraction of total revenues. Finally, we analyzed the optimal decisions and profit when production can choose the capacity level.

Our key findings are as follows:

Decentralized Settings

- *Leadtimes are longer, prices are lower, demand is larger and profits are lower as compared to the centralized setting.*

- *The leadtime decision is independent of the changes in price sensitivity or production cost as long as production receives a positive margin. In this case, the price decision is not affected by a change in the unit production cost.*

- *The unique production Stackelberg equilibrium is dominated by the unique marketing Stackelberg equilibrium. The dominance becomes more significant for firms having tight capacity.*

- *Higher capacity results in higher profits under the centralized setting and the decentralized setting, where production is the leader. However, higher capacity may result in a decrease of profits when marketing is the leader.*

- *When capacity can be chosen by production and has a constant unit cost, the dominance of the marketing Stackelberg game over the production Stackelberg game becomes more significant at higher capacity costs. As compared to the centralized setting, the optimal capacity level is higher when production is the leader and lower when marketing is the leader.*

Under Coordination:

- *Higher capacity does not necessarily lead to charging higher for better service.*

- *Under a Revenue-Sharing Contract, estimation errors in price sensitivity are much costlier than those in leadtime sensitivity when production is the leader. On the other hand, contract parameters are independent of the demand parameters when marketing is the leader.*

We are currently working on extending this work to a competitive setting, where two firms with decentralized marketing and production functions compete on the basis of their price and lead-time decisions (Pekgün, Griffin and Keskinocak, 2006). Another possible extension of this work would be generalizations to other queueing settings (though in all likelihood it would be analytically intractable). It would also be interesting to study a Nash bargaining framework rather than a Stackelberg framework. Finally, extensions to operational settings, such as a multi-period model, would be of interest, as the steady-state results (e.g., the service level constraint) would not always hold in the day-to-day operations of a firm.

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Appendix

PROOF OF PROPOSITION 1: As in Palaka et al. (1998), Constraint (2) is tight at optimality. Thus, it is sufficient to treat two of the three variables, p_C, L_C, and λ_C as decision variables and determine the other variable via the equality, $\lambda_C = D(p_C, L_C)$. We choose to eliminate price from this formulation for computational simplicity. Moreover, since Constraint (1) can be rewritten as $(\mu - \lambda_C)L_C \geq \ln(1/(1-s))$, denoting $\ln(1/(1-s))$ by k we get the following formulation:

$$\max_{(\lambda_C, L_C)\geq 0} \quad \pi_C = \lambda_C(a - cL_C - \lambda_C - mb)/b$$
$$s.t. \quad (\mu - \lambda_C)L_C \geq k$$

Note that the stability condition $\lambda_C \leq \mu$ is implied by the service constraint since $k \geq 0$ and $L_C \geq 0$. The service constraint must be binding at optimality as in Palaka et al. (1998) and So and Song (1998), since the objective function is linearly decreasing in L_C, and the minimum possible leadtime is defined by the boundary of the service constraint. Hence the optimal leadtime is:

$$L_C^* = \frac{k}{\mu - \lambda_C^*}$$

Substituting L_C^* into π_C gives an unconstrained optimization problem. First order conditions (FOC) provide Equation (4) for λ_C^*. To show uniqueness, note that

$$\frac{\partial f_C(\lambda_C)}{\partial \lambda_C} = -2(\mu - \lambda_C)^2 - 2(a - 2\lambda_C - mb)(\mu - \lambda_C) < 0$$

since $(\mu - \lambda_C) > 0$ on the interval $[0, \mu]$ and $(a - 2\lambda_C - mb) > 0$ by Equation (4) for the objective function to have a maximizer. This implies that $f_C(\lambda_C)$ is decreasing. Thus, if $f_C(\lambda_C)$ has a root on the interval $[0, \mu]$, then it will give the unique optimal arrival rate, λ_C^* for the centralized problem. Noting that $f_C(\mu) = -ck\mu < 0$, we also need $f_C(0) = (a - mb)\mu^2 - ck\mu > 0$ for the existence of a root on $[0, \mu]$, which holds due to Assumption A3.

Second order conditions ensure that the root of Equation (4) is a global maximum, since the objective function is concave in λ_C. The optimal price p_C^* can then be obtained by rearranging terms in $\lambda_C^* = D(p_C^*, L_C^*)$. ■

PROOF OF PROPOSITION 2: In order for $p^*_P(L_P) = \frac{a-cL_P}{2b}$, we should have $\frac{a-cL_P}{2b} \geq m$ or equivalently, $L_P \leq \frac{a-2mb}{c}$. Thus, we first need $a - 2mb > 0$. Let's assume that $p^*_P(L_P) = \frac{a-cL_P}{2b}$. Then, let y^1_P and y^2_P be the roots of Constraint (7) when it is binding, i.e., $\left(\mu - \frac{a-cL_P}{2}\right) L_P = k$. The positive root, y^1_P is given by

$$y^1_P = \frac{(a-2\mu) + \sqrt{(2\mu - a)^2 + 8ck}}{2c}$$

Hence, in a feasible solution, $L_P \in [y^1_P, \frac{a}{c}]$ since demand is downward sloping in L_P, and is zero at $\frac{a}{c}$. Note that we have $y^1_P \leq \frac{a}{c}$ if and only if $\mu \geq \frac{ck}{a}$, which holds under Assumption A3. Since the objective function is convex, the optimal solution lies on the boundaries of the interval $[y^1_P, \frac{a}{c}]$. Let x^1_P and x^2_P be the roots of the objective function, π^{PR}_P, where $x^1_P = \frac{a}{c}$ and $x^2_P = \frac{a-2mb}{c}$. Figure 6 shows an illustration of π^{PR}_P for $a - 2mb > 0$. We need $y^1_P < x^2_P$ for generating positive profits, which only holds if $\mu > bm + \frac{ck}{a-2mb}$. Note that if $a - 2mb \leq 0$ and/or $x^2_P \leq y^1_P \leq x^1_P$, then, y^1_P generates zero or negative profits, so the optimal price would be constrained to the unit production cost, m, and the optimal lead-time, which lies on the boundary of Constraint (7), would be given by $\frac{a-bm-\mu+\sqrt{(\mu-a+bm)^2+4ck}}{2c}$. ■

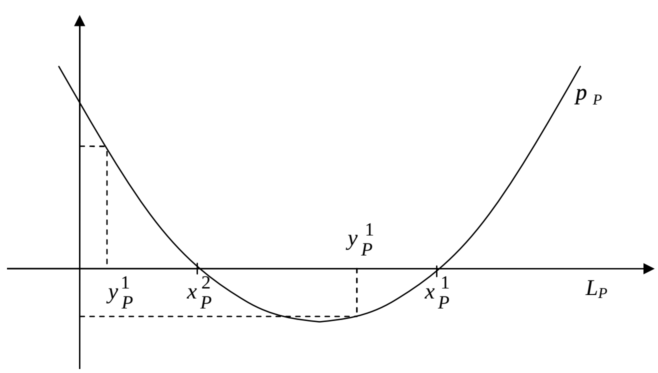

Figure 6: Illustration of case-(ii)

PROOF OF PROPOSITION 3: FOC for π^{MR}_M give:

$$g(p_M) = \frac{\partial \pi^{MR}_M}{\partial p_M} = \frac{1}{2}\left[a + \mu - 2bp^0_M - \frac{(a - bp^0_M - \mu)(a - 2bp^0_M - \mu) + 4ck}{\sqrt{(a - bp^0_M - \mu)^2 + 4ck}}\right] = 0$$

The optimal price is given by $p^*_M = \max\{p^0_M, m\}$, where p^0_M is the unique maximizer of π^{MR}_M over the interval $p_{_M} \in [0, (a\mu - ck)/b\mu]$. Concavity of $\pi_{_M}^{^{MR}}$ can be shown from the second order conditions:

$$\frac{\partial^2 \pi_{_M}^{^{MR}}}{\partial p_{_M}^2} = -b\left[1 - \frac{(a - bp_{_M}^0 - \mu)}{\sqrt{(a - bp_{_M}^0 - \mu)^2 + 4ck}} + \frac{2ckbp_{_M}^0}{\sqrt{(a - bp_{_M}^0 - \mu)^2 + 4ck}}\right] < 0$$

Moreover, $g(0) = \frac{1}{2}\left[a + \mu - \sqrt{(a - \mu)^2 + 4ck}\right] > 0$ and $g(\frac{a\mu - ck}{b\mu}) = -\mu\left[\frac{a\mu - ck}{ck + \mu^2}\right] < 0$ as $a > ck/\mu$ from Assumption A3. Therefore, $p_{_M}^0$ is the unique maximizer of $\pi_{_M}^{^{MR}}$ over $[0, \frac{a\mu - ck}{b\mu}]$. FOC for $\pi_{_M}^{^{MR}}$ with a change of variables give Equation (11). ■

PROOF OF PROPOSITION 4: $f_{_M}(\mathbf{.})$ is decreasing and convex on the interval $[0, \mu]$. Since $f_{_M}(\lambda_{_P}^*) = -\frac{1}{4}ck\left(a + 2\mu - \sqrt{(a - 2\mu)^2 + 8ck}\right) < 0$, it follows that $\lambda_{_P}^* > \lambda_{_M}^*$ except when $\max\{p_{_P}^*, m\} = m$ and $\max\{p_{_M}^*, m\} = m$ in which case $\lambda_{_P}^* = \lambda_{_M}^*$. For the centralized setting, combining Equations (4) and (11), we can write

$$\frac{a - 2\lambda_{_C}^* - mb}{a - 2\lambda_{_M}^*} = \frac{(\mu - \lambda_{_M}^*)^2}{(\mu - \lambda_{_C}^*)^2}$$

which gives $\lambda_{_C}^* < \lambda_{_M}^*$. The results for leadtimes and prices follow from the fact that the former increases and the latter decreases in the demand generated as $L_{\mathbf{.}}^* = \frac{k}{\mu - \lambda_{\mathbf{.}}^*}$ and $p_{\mathbf{.}} = \frac{a - \lambda_{\mathbf{.}} - \frac{ck}{\mu - \lambda_{\mathbf{.}}}}{b}$. Note that since $p_{_C}^* > m$, which holds through Assumption A3, we do not have equalities between the decisions of the centralized and the decentralized settings.

As for the comparison of profits, we find the profit difference between C and M as:

$$\begin{aligned}
\pi_{_C}^* - \pi_{_M}^* &= \left(p_{_C}^* - m\right)\lambda_{_C}^* - \left(p_{_M}^* - m\right)\lambda_{_M}^* \\
&= \frac{a}{b}\left(\lambda_{_C}^* - \lambda_{_M}^*\right) - \frac{1}{b}\left((\lambda_{_C}^*)^2 - (\lambda_{_M}^*)^2\right) - \frac{1}{b}ck\mu\frac{\left(\lambda_{_C}^* - \lambda_{_M}^*\right)}{\left(\mu - \lambda_{_C}^*\right)\left(\mu - \lambda_{_M}^*\right)} - m\left(\lambda_{_C}^* - \lambda_{_M}^*\right)
\end{aligned}$$

Since $\lambda_{_M}^* > \lambda_{_C}^*$, $\mu > \lambda_{_M}^*$ and $\mu > \lambda_{_C}^*$, the first term of this equation is negative, while the other three terms are positive. Therefore, we need to examine the relation between these terms in more detail. From Equation (4), if we replace $ck\mu$ with $\left(a - 2\lambda_{_C}^* - mb\right)\left(\mu - \lambda_{_C}^*\right)^2$, after simplification, the profit difference becomes:

$$\begin{aligned}
\pi_{_C}^* - \pi_{_M}^* = \frac{a}{b}\left(\lambda_{_C}^* - \lambda_{_M}^*\right) - \frac{1}{b}\left((\lambda_{_C}^*)^2 - (\lambda_{_M}^*)^2\right) - m\left(\lambda_{_C}^* - \lambda_{_M}^*\right)& \\
-\frac{1}{b}\left(a - 2\lambda_{_C}^* - mb\right)\left(\mu - \lambda_{_C}^*\right)\frac{\left(\lambda_{_C}^* - \lambda_{_M}^*\right)}{\left(\mu - \lambda_{_M}^*\right)}& \\
= \frac{\left(\lambda_{_C}^* - \lambda_{_M}^*\right)}{b}\left[\left(a - (\lambda_{_C}^* + \lambda_{_M}^*) - mb\right) - \left(a - 2\lambda_{_C}^* - mb\right)\frac{\left(\mu - \lambda_{_C}^*\right)}{\left(\mu - \lambda_{_M}^*\right)}\right]& \qquad \text{(A.1)}
\end{aligned}$$

The first term of Equation (A.1) is negative, while the term in brackets is negative since $\big(a - (\lambda^*_C + \lambda^*_M) - mb\big) < \big(a - 2\lambda^*_C - mb\big)$, while $\big(\mu - \lambda^*_C\big) > \big(\mu - \lambda^*_M\big)$. Hence, we conclude that $\pi^*_C - \pi^*_M > 0$. The profit difference between M and P is:

$$
\begin{aligned}
\pi^*_M - \pi^*_P &= \big(p^*_M - m\big)\,\lambda^*_M - \big(p^*_P - m\big)\,\lambda^*_P \\
&= \frac{a}{b}\left(\lambda^*_M - \lambda^*_P\right) - \frac{1}{b}\left((\lambda^*_M)^2 - (\lambda^*_P)^2\right) - \frac{1}{b}ck\mu\frac{\left(\lambda^*_M - \lambda^*_P\right)}{\left(\mu - \lambda^*_M\right)\left(\mu - \lambda^*_P\right)} - m\left(\lambda^*_M - \lambda^*_P\right)
\end{aligned}
$$

From Equation (11) we replace $ck\mu$ with $\left(a - 2\lambda^*_M\right)\left(\mu - \lambda^*_M\right)^2$:

$$
\begin{aligned}
\pi^*_M - \pi^*_P &= \frac{a}{b}\left(\lambda^*_M - \lambda^*_P\right) - \frac{1}{b}\left((\lambda^*_M)^2 - (\lambda^*_P)^2\right) - m\left(\lambda^*_M - \lambda^*_P\right) \\
&\qquad - \frac{1}{b}\left(a - 2\lambda^*_M\right)\left(\mu - \lambda^*_M\right)\frac{\left(\lambda^*_M - \lambda^*_P\right)}{\left(\mu - \lambda^*_P\right)} \\
&= \frac{\left(\lambda^*_M - \lambda^*_P\right)}{b}\left[\big(a - (\lambda^*_M + \lambda^*_P) - mb\big) - \left(a - 2\lambda^*_M\right)\frac{\left(\mu - \lambda^*_M\right)}{\left(\mu - \lambda^*_P\right)}\right] \qquad \text{(A.2)}
\end{aligned}
$$

The first term of Equation (A.2) is non-positive, while the term in brackets is negative since $\big(a - (\lambda^*_M + \lambda^*_P) - mb\big) < \big(a - 2\lambda^*_M\big)$, while $\big(\mu - \lambda^*_M\big) \geq \big(\mu - \lambda^*_P\big)$. Hence, we conclude that $\pi^*_M - \pi^*_P \geq 0$. ■

PROOF OF PROPOSITION 5: Let $R(.)$ denote revenue. The positive revenue difference between M and P $(R(M) - R(P) > 0)$ directly follows from the proof of Proposition 4, where $\pi^*_M - \pi^*_P \geq 0$ with $m = 0$. As for the comparison of revenues under C and M, we change the proof for profits slightly:

$$
\begin{aligned}
R(C) - R(M) &= p^*_C\lambda^*_C - p^*_M\lambda^*_M \\
&= \frac{a}{b}\left(\lambda^*_C - \lambda^*_M\right) - \frac{1}{b}\left((\lambda^*_C)^2 - (\lambda^*_M)^2\right) - \frac{1}{b}ck\mu\frac{\left(\lambda^*_C - \lambda^*_M\right)}{\left(\mu - \lambda^*_C\right)\left(\mu - \lambda^*_M\right)}
\end{aligned}
$$

From Equation (11), if we replace $ck\mu$ with $\left(a - 2\lambda^*_M\right)\left(\mu - \lambda^*_M\right)^2$, after simplification, the revenue difference becomes:

$$
\begin{aligned}
R(C) - R(M) &= \frac{a}{b}\left(\lambda^*_C - \lambda^*_M\right) - \frac{1}{b}\left((\lambda^*_C)^2 - (\lambda^*_M)^2\right) - \frac{1}{b}\left(a - 2\lambda^*_M\right)\left(\mu - \lambda^*_M\right)\frac{\left(\lambda^*_C - \lambda^*_M\right)}{\left(\mu - \lambda^*_C\right)} \\
&= \frac{\left(\lambda^*_C - \lambda^*_M\right)}{b}\left[\big(a - (\lambda^*_C + \lambda^*_M)\big) - \left(a - 2\lambda^*_M\right)\frac{\left(\mu - \lambda^*_M\right)}{\left(\mu - \lambda^*_C\right)}\right] \qquad \text{(A.3)}
\end{aligned}
$$

The first term of Equation (A.3) is negative, while the term in brackets is positive as $\big(a - (\lambda^*_C + \lambda^*_M)\big) > 0$ and $\frac{(a-(\lambda^*_C+\lambda^*_M))}{(a-2\lambda^*_M)} > 1$, while $\frac{(\mu-\lambda^*_M)}{(\mu-\lambda^*_C)} < 1$. Hence, we conclude that $R(M) - R(C) > 0$. ■

SENSITIVITY ANALYSIS FOR OPTIMAL DECISIONS AND PROFIT: Sensitivity of the optimal decisions $L^*_{\cdot}$, $p^*_{\cdot}$ and $\lambda^*_{\cdot}$, and the optimal profit $\pi^*_{\cdot}$ to problem parameters for the centralized and decentralized settings is given in Table 1 assuming that the problem parameters are such that $p^*_{_M} > m$ and $p^*_{_P} > m$. For most cases, it can be seen from Table 1 that the direction of change in the optimal decisions and profit is independent of the decision-making paradigm (i.e., centralized or decentralized). The cases where the behavior of the optimal price (and profit for M) changes conditionally are indicated by a "?". We demonstrate these conditions analytically or numerically in the text.

Table 1: Sensitivity analysis on optimal decisions and profit

	Centralized				**Decentralized,** P				**Decentralized,** M			
	$\lambda^*_{\mathbf{C}}$	$\mathbf{L}^*_{\mathbf{C}}$	$\mathbf{p}^*_{\mathbf{C}}$	$\pi^*_{\mathbf{C}}$	$\lambda^*_{\mathbf{P}}$	$\mathbf{L}^*_{\mathbf{P}}$	$\mathbf{p}^*_{\mathbf{P}}$	$\pi^*_{\mathbf{P}}$	$\lambda^*_{\mathbf{M}}$	$\mathbf{L}^*_{\mathbf{M}}$	$\mathbf{p}^*_{\mathbf{M}}$	$\pi^*_{\mathbf{M}}$
$a \nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$
$b \nearrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	-	-	$\searrow$	$\searrow$	-	-	$\searrow$	$\searrow$
$c \nearrow$	$\searrow$	$\searrow$	?	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	?	?
$m \nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	-	-	-	$\searrow$	-	-	-	$\searrow$
$s \nearrow$	$\searrow$	$\nearrow$	?	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\nearrow$	?	?
$\mu \nearrow$	$\nearrow$	$\searrow$	?	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$	?	?

We first provide an example for the centralized setting, namely the derivation of the change in $\lambda^*_{_C}$ with respect to μ.

$$\frac{\partial \lambda^*_{_C}}{\partial \mu} = -\frac{ck - 2(\mu - \lambda^*_{_C})(a - mb - 2\lambda^*_{_C})}{2(\mu - \lambda^*_{_C})[(\mu - \lambda^*_{_C}) + (a - mb - 2\lambda^*_{_C})]} \tag{A.4}$$

It can be seen from Equation (4) that $(a-2\lambda^*_{_C}-mb) \geq 0$ since the right hand side and $(\mu-\lambda^*_{_C})^2$ are positive. We also know that $(\mu-\lambda^*_{_C}) \geq 0$ for the stability condition to hold. Hence, the denominator of Equation (A.4) is positive. From Equation (4) it follows that $(a - 2\lambda^*_{_C} - mb) = \frac{ck\mu}{(\mu-\lambda^*_{_C})^2}$, and we obtain:

$$\begin{aligned} \frac{\partial \lambda^*_{_C}}{\partial \mu} &= \frac{-\left(ck - 2(\mu - \lambda^*_{_C})\frac{ck\mu}{(\mu-\lambda^*_{_C})^2}\right)}{2(\mu - \lambda^*_{_C})[(\mu - \lambda^*_{_C}) + (a - mb - 2\lambda^*_{_C})]} \\ &= \frac{-ck\left((\mu - \lambda^*_{_C}) - 2\mu\right)}{2(\mu - \lambda^*_{_C})^2[(\mu - \lambda^*_{_C}) + (a - mb - 2\lambda^*_{_C})]} \geq 0 \ \ (\text{since } (-\lambda^*_{_C} - \mu) \leq 0) \end{aligned}$$

Therefore, the optimal demand rate for the centralized system is increasing in capacity. In order to see the effect of capacity on the optimal leadtime, we use that $L^*_{_C} = \frac{k}{\mu-\lambda^*_{_C}}$.

$$\begin{aligned} \frac{\partial L^*_{_C}}{\partial \mu} &= -\frac{k}{\left(\mu - \lambda^*_{_C}\right)^2}\left(1 - \frac{\partial \lambda^*_{_C}}{\partial \mu}\right) \\ &= -\frac{2k\left(\mu - \lambda^*_{_C}\right)^2 + ck^2}{2\left(\mu - \lambda^*_{_C}\right)^3\left[\left(\mu - \lambda^*_{_C}\right) + \left(a - mb - 2\lambda^*_{_C}\right)\right]} \leq 0 \end{aligned}$$

So, optimal leadtime for the centralized system is decreasing in capacity.

We next give an example for the decentralized setting, P. For simplicity, we choose the change with respect to the price sensitivity of demand, b.

$$\begin{aligned} L_P^* &= \frac{(a-2\mu)+\sqrt{(2\mu-a)^2+8ck}}{2c} \quad \Rightarrow \quad \frac{\partial L_P^*}{\partial b} = 0 \\ p_P^* &= \frac{a-cL_P^*}{2b} \quad \Rightarrow \quad \frac{\partial p_P^*}{\partial b} = -\frac{a}{2b^2} \leq 0 \\ \lambda_P^* &= \frac{a-cL_P^*}{2} \quad \Rightarrow \quad \frac{\partial \lambda_P^*}{\partial b} = 0 \end{aligned}$$

For this case, optimal price is decreasing in price sensitivity, while optimal leadtime and optimal demand rate are not affected. The other entries of Table 1 have been developed similarly. ■

PROOF OF PROPOSITION 6: Let $x_C = mb$ and $x_M = 0$, and $i = C,\ M$. Then, the following analysis will hold for both settings. The change in p_i^* with respect to c is:

$$\frac{\partial p_i^*}{\partial c} = -\frac{k}{2b} \frac{(a-x_i-4\lambda_i^*+\mu)}{(\mu-\lambda_i^*)[(\mu-\lambda_i^*)+(a-x_i-2\lambda_i^*)]}$$

If $(a-x_i-4\lambda_i^*+\mu)<0$, then p_i^* is increasing in c. If $(a-x_i-4\lambda_i^*+\mu)>0$, then $\frac{\partial p_i^*}{\partial c}<0$ and p_i^* is decreasing in c, which is satisfied when $f_i\left(\frac{1}{4}(a-x_i+\mu)\right)<0$, since $f_i(\lambda_i)$ is decreasing in λ_i on $[0,\mu]$. Note that if $\frac{1}{4}(a-x_i+\mu)>\mu$, then since $\lambda_i^*\leq\mu$, p_i^* will be decreasing in c. Thus, c_i^0 is given as the c, which sets Equation (A.5) to 0.

$$f_i\left(\frac{1}{4}(a-x_i+\mu)\right) = \left(\frac{a-x_i-\mu}{2}\right)\left(\frac{a-x_i-3\mu}{4}\right)^2 - ck\mu \tag{A.5}$$

Note that $f_i(\centerdot)<0$ when $\mu>a-x_i$ as seen from Equation (A.5). Similarly, for the change in p_i^* with respect to s:

$$\frac{\partial p_i^*}{\partial k} = -\frac{c}{2b} \frac{(a-x_i-4\lambda_i^*+\mu)}{(\mu-\lambda_i^*)[(\mu-\lambda_i^*)+(a-x_i-2\lambda_i^*)]} \tag{A.6}$$

The sign of this equation also depends on $(a-x_i-4\lambda_i^*+\mu)$. Therefore, when $\lambda_i^*<\frac{1}{4}(a-x_i+\mu)$, we obtain the same capacity interval for each related setting, and s_i^0 and k_i^0 are given by the k, which sets Equation (A.6) to 0. ■

PROOF OF PROPOSITION 7: We first analyze the problems of marketing and production under P. We start with marketing's problem and find its best response as:

$$p_P^*(L_P) = \left[\frac{a-cL_P}{2b}+\frac{w}{2\alpha_1}\right]^+ \text{ and } \lambda_P^*(L_P) = \left[\frac{a-cL_P}{2}-\frac{wb}{2\alpha_1}\right]^+$$

Production's problem is:

$$\max_{(0 \le L_P \le \frac{a}{c} - \frac{wb}{c\alpha_1})} \quad \pi_P^{PR} = (w-m)\lambda_P^*(L_P) + \alpha_2 p_P^*(L_P)\lambda_P^*(L_P) \tag{A.7}$$

$$= (w-m)\left(\frac{a-cL_P}{2} - \frac{wb}{2\alpha_1}\right) + \alpha_2 b\left(\left(\frac{a-cL_P}{2b}\right)^2 - \frac{w^2}{4(\alpha_1)^2}\right)$$

$$s.t. \quad \left(\mu - \frac{a-cL_P}{2} + \frac{wb}{2\alpha_1}\right) L_P \ge k \tag{A.8}$$

Note that $L_P \le \frac{a}{c} - \frac{wb}{c\alpha_1}$ ensures positive margin for marketing and positive demand. Let y_P^1 be the positive root of Constraint (A.8) when it is binding:

$$y_P^1 = \frac{a - 2\mu - wb/\alpha_1 + \sqrt{(a-2\mu-wb/\alpha_1)^2 + 8ck}}{2c}$$

The two roots of π_P^{PR} are given by $x_P^1 = \left(\frac{a}{c} - \frac{wb}{c\alpha_1}\right)$ and $x_P^2 = \frac{a}{c} + \frac{wb}{c\alpha_1} - 2\frac{(m-w)b}{c\alpha_2}$. The feasible region is defined by $[y_P^1, x_P^1]$, and for feasibility, it should hold that $y_P^1 \le x_P^1 = \frac{a}{c} - \frac{wb}{c\alpha_1}$, which is satisfied if $w \le \alpha_1\left(\frac{a}{b} - \frac{ck}{\mu b}\right)$. As long as $\alpha_2 \ge 0$, π_P^{PR} is convex, and the optimal solution lies on the boundaries of $[y_P^1, x_P^1]$. The optimal leadtime is determined as follows:

(i) $x_P^1 > x_P^2 \Leftrightarrow w < \frac{\alpha_1 m}{\alpha_1+\alpha_2}$: If $y_P^1 < x_P^2$, then $L_P^* = y_P^1$ and $\pi_P^{PR} > 0$; otherwise, $L_P^* = x_P^1$ and $\pi_P^{PR} = 0$.

(ii) $x_P^1 \le x_P^2 \Leftrightarrow w \ge \frac{\alpha_1 m}{\alpha_1+\alpha_2}$: $L_P^* = y_P^1$ and $\pi_P^{PR} > 0$.

Case (i) is developed similar to the proof of Proposition 2, and case (ii) is trivial. If $w \ge m$, production will have a positive margin, i.e., $\alpha_2 p_P^* + w - m \ge 0$, for any $\alpha_2 \ge 0$. In other words, production does not require a fraction of the revenues as long as the transfer price paid by marketing covers the production costs. Otherwise, the minimum α_2 that achieves $L_P^* = y_P^1$ is given by the value which satisfies $y_M^1 < x_M^2$ in case (i):

$$\alpha_{2P}^{min}(\alpha_1, w) = \max\left\{\frac{4b(m-w)}{a + 2\mu + 3wb/\alpha_1 - \sqrt{(a-2\mu-wb/\alpha_1)^2 + 8ck}}, 0\right\}$$

As the maximum fraction of revenue that can be offered to production is $1-\alpha_1$, we should guarantee $\alpha_{2P}^{min}(\alpha_1, w) \le (1-\alpha_1)$ by choosing a transfer price, $w \ge w_P^{min}$:

$$w_P^{min} = \max\left\{\frac{\alpha_1\left(\alpha_1(\alpha_1\mu + a + bm) + 3bm - \mu - a + (1-\alpha_1)\sqrt{(\mu\alpha_1 - a + bm + \mu)^2 + 4ck(1+\alpha_1)}\right)}{2b(1+\alpha_1)}, 0\right\}$$

For the (α_1, α_2, w) combinations stated in Proposition 7, the optimal solution for Model P is given by $L_P^* = y_P^1$, $p_P^* = p_P^*(L_P^*)$ and $\lambda_P^* = \lambda_P^*(L_P^*)$.

We next analyze the Marketing-Stackelberg game, M. As long as $\alpha_2 p_M^* + w - m \geq 0$, the service level constraint will be tight at optimality and the best response of production will be given by Equation (10):

$$L_M^*(p_M) = \frac{(a - bp_M - \mu) + \sqrt{(a - bp_M - \mu)^2 + 4ck}}{2c}$$

We prefer to solve marketing's problem in terms of λ_M^* and employ a change of variables. In this case, $L_M^*(\lambda_M) = k/(\mu - \lambda_M)$. Marketing's problem is then given by

$$\max_{(0 \leq \lambda_M \leq \mu)} \pi_M^{MR} = \left(\alpha_1 \frac{a - \lambda_M - \frac{ck}{\mu - \lambda_M}}{b} - w \right) \lambda_M$$

FOC give Equation (13) after rearranging terms. As $f_M(\mu) = -ck\mu < 0$, we desire to have $f_M(0) = (a - wb/\alpha_1)\mu^2 - ck\mu > 0$, and thus, we restrict our attention to $w \leq \alpha_1 \left(\frac{a}{b} - \frac{ck}{\mu b} \right)$. Now that $f_M(\cdot)$ has a root over $[0, \mu]$, it should hold that $(a - 2\lambda_M^* - wb/\alpha_1) > 0$. Uniqueness is guaranteed as $f_M(\cdot)$ is decreasing over $[0, \mu]$:

$$\frac{\partial f_M(\lambda_M)}{\partial \lambda_M} = -2(\mu - \lambda_M)^2 - 2(a - 2\lambda_M - wb/\alpha_1)(\mu - \lambda_M) < 0$$

We can check that the margin for marketing is positive, i.e., $\alpha_1 p_M^* - w \geq 0$ from the optimality equation.

$$a - \lambda_M^* - wb/\alpha_1 > a - 2\lambda_M^* - wb/\alpha_1 = \frac{ck\mu}{(\mu - \lambda_M^*)^2} > \frac{ck}{\mu - \lambda_M^*}$$

$$\Rightarrow a - \lambda_M^* - \frac{ck}{\mu - \lambda_M^*} > wb/\alpha_1 \Rightarrow p_M^* > w/\alpha_1 \quad \surd$$

We also need to provide a positive margin to production for optimality, i.e., $\alpha_2 p_M + w - m \geq 0$. Note that this margin is positive for all $\alpha_2 \geq 0$ if $w \geq m$. When $w < m$, we need to solve $\alpha_2(a - \lambda_M - \frac{ck}{\mu - \lambda_M}) + (w - m)b = 0$ for λ_M. The root that falls in $[0, \mu]$ is given by:

$$\lambda_M^0(\alpha_2) = \frac{1}{2\alpha_2} \left(\alpha_2(\mu + a) - (m - w)b - \sqrt{(-\alpha_2(\mu - a) - b(m - w))^2 + 4\alpha_2^2 ck} \right) \tag{A.9}$$

Thus, $\alpha_{2M}^{min}(\alpha_1, w)$ is equal to the α_2 value that sets $f_M(\lambda_M^0(\alpha_2)) = 0$ for $w < m$ and 0 for $w \geq m$. We also need to find the minimum transfer price for a given α_1 that will ensure $\alpha_2 \leq 1 - \alpha_1$. If we solve $(1 - \alpha_1)p_M + w - m = 0$ for λ_M, we will obtain Equation (A.9) with $\alpha_2 = 1 - \alpha_1$, i.e., $\lambda_M^0(w)$. Then, $w_M^{min}(\alpha_1)$ will be equal to the maximum of 0 and the w value that sets $f_M(\lambda_M^0(w)) = 0$. ■

PROOF OF PROPOSITION 8: For the (α_1, α_2, w) combinations stated in Proposition 7, the service level constraints will be tight at optimality for all settings. Then, choosing the contract parameters such that $\lambda_i^* = \lambda_C^*,\ i = P, M$ is sufficient to achieve coordination as $L_\cdot^* = \frac{k}{\mu - \lambda_\cdot^*}$ and $p_\cdot^* = \frac{a - \lambda_\cdot^* - \frac{ck}{\mu - \lambda_\cdot^*}}{b}$. Thus, we only need to ensure that w_i^* lies in the feasible range for each decentralized setting as defined in Proposition 7. We start with the Production-Stackelberg game, P. First, we show that w_P^* lies in the interval $\left(\alpha_1 m, \alpha_1\left(\frac{a}{b} - \frac{ck}{\mu b}\right)\right]$.

$$\left(a - mb - 2\lambda_C^*\right)\left(\mu - \lambda_C^*\right)^2 = ck\mu > ck\left(\mu - \lambda_C^*\right)$$

$$\frac{a - 2\lambda_C^* - \frac{ck}{\mu - \lambda_C^*}}{b} > m \Rightarrow w_P^* = \alpha_1\left(\frac{a - 2\lambda_C^* - \frac{ck}{\mu - \lambda_C^*}}{b}\right) > \alpha_1 m \quad \checkmark$$

Note that $w_P^* = \alpha_1 m$ only when $\lambda_C^* = 0$. For $w_P^* \le \alpha_1\left(\frac{a}{b} - \frac{ck}{\mu b}\right)$, it should hold that $2\lambda_C^* \ge \frac{ck}{\mu} - \frac{ck}{\mu - \lambda_C^*}$, which reduces to $2\lambda_C^* \ge 0 \ge -\lambda_C^* \frac{ck}{\mu(\mu - \lambda_C^*)}$. According to Proposition 7, it should also hold that $w_P^* \ge w_P^{min}$. Note that it is sufficient to show $\alpha_1 m \ge w_P^{min}$, as $w_P^* \ge \alpha_1 m$. Let $g(m) = w_P^{min} - \alpha_1 m$. If we solve $g(m_0) = 0$ for m_0, we find the root to be $m_0 = -\frac{ck}{b\mu} + \frac{a}{b}$. We know from Assumption A3 that $m < -\frac{ck}{b\mu} + \frac{a}{b}$.

$$\frac{\partial g}{\partial m} = \frac{\alpha_1\left(1 - \alpha_1\right)\left[\left(\alpha_1\mu - a + bm + \mu\right) + \sqrt{\left(\alpha_1\mu - a + bm + \mu\right)^2 + 4ck(1 + \alpha_1)}\right]}{2\left(1 + \alpha_1\right)\sqrt{\left(\alpha_1\mu - a + bm + \mu\right)^2 + 4ck(1 + \alpha_1)}} \ge 0$$

As the function g is nondecreasing in m, $g(m) \le 0$ and $w_P^{min} \le \alpha_1 m$. Finally, Proposition 1 states that λ_C^* is unique as long as Assumption A3 is satisfied. Hence, w_P^* is also unique as $w_P^* = \frac{a - 2\lambda_C^* - \frac{ck}{\mu - \lambda_C^*}}{b}$.

For the Marketing-Stackelberg game, M, we know that $w_M^* = \alpha_1 m \le \alpha_1\left(\frac{a}{b} - \frac{ck}{\mu b}\right)$ by Assumption A3. Since $(1 - \alpha_1)p_C^* + w_M^* - m = (1 - \alpha_1)p_C^* + \alpha_1 m - m = (1 - \alpha_1)(p_C^* - m) \ge 0$, it also holds that $w_M^* \ge w_M^{min}$. ■

PROOF OF PROPOSITION 9: The fraction of the centralized profit that marketing achieves is less than α_1 under P, since

$$\pi_P^{MR} = (\alpha_1 p_C^* - w_P^*)\lambda_C^* < \alpha_1(p_C^* - m)\lambda_C^* = \alpha_1 \pi_C^* \quad (w_P^* > \alpha_1 m)$$

while

$$\pi_M^{MR} = (\alpha_1 p_C^* - w_M^*)\lambda_C^* = \alpha_1(p_C^* - m)\lambda_C^* = \alpha_1 \pi_C^* \quad (w_M^* = \alpha_1 m) \quad .■$$

ANALYSIS OF THE TB CONTRACT WITH NO TRANSFER PRICE: The contract parameters of the revenue-sharing contract and the transfer price-only contract satisfy the conditions described in Proposition 8. However, when $w = 0$, we demonstrate that coordination cannot be achieved. Under P, the feasible region of production's problem is the same as in the original production Stackelberg game, and hence, this contract cannot perform better. The second root of $\pi_{_P}^{PR}$ is given by $x_{_P}^2 = \left(\frac{a}{c} - \frac{2mb}{(1-\alpha_1)c}\right) < \frac{a}{c} = x_{_P}^1$. For $x_{_P}^2 > 0$, the fraction of revenue offered to marketing, α_1, should not be greater than $1 - \frac{2mb}{a}$, which is positive if $a > 2mb$. Therefore, if $a \leq 2mb$, production will drive demand to zero by quoting $L_{_P}^* = x_{_P}^1$. In order to generate positive profits, the firm should choose α_1 such that $y_{_P}^1 < x_{_P}^2$, as long as $a > 2mb$. Similarly, we can easily show that under the TB contract with $w = 0$, FOC on $\pi_{_M}^{MR}$ give Equation (11). In order to generate positive profit, the firm needs to choose α_2 such that production receives positive margin, i.e., $\alpha_2 p_{_M}^* - m \geq 0$. ■

PROOF OF PROPOSITION 10: As $\pi_{_C}$ is decreasing in $\mu_{_C}$, the service level constraint is tight at optimality, and $\mu_{_C} = \lambda_{_C} + k/L_{_C}$. When we plug $\mu_{_C}$ in $\pi_{_C}$, we get $\pi_{_C}(\lambda_{_C}, L_{_C}) = (a - cL_{_C} - \lambda_{_C})\lambda_{_C}/b - (m+K)\lambda_{_C} - Kk/L_{_C}$. First and second order conditions give:

$$\begin{aligned}
\frac{\partial \pi_{_C}(\lambda_{_C}, L_{_C})}{\partial \lambda_{_C}} &= \frac{a - cL_{_C} - 2\lambda_{_C}}{b} - (m+K) = 0 \Rightarrow \lambda_{_C}(L_{_C}) = \frac{a - cL_{_C}}{2} - \frac{(m+K)b}{2} \\
\frac{\partial^2 \pi_{_C}(\lambda_{_C}, L_{_C})}{\partial \lambda_{_C}^2} &= -\frac{2}{b} < 0 \Rightarrow \text{Concave}
\end{aligned}$$

Next, we plug $\lambda_{_C}(L_{_C})$ in $\pi_{_C}(\lambda_{_C}, L_{_C})$, and we get

$$\pi_{_C}(L_{_C}) = \frac{(a - cL_{_C} - (m+K)b)^2}{4b} - \frac{Kk}{L_{_C}}$$

First and second order conditions give:

$$\frac{\partial \pi_{_C}(L_{_C})}{\partial L_{_C}} = \frac{c^2 L_{_C}^3 - c(a - (m+K)b)L_{_C}^2 + 2Kkb}{2bL_{_C}^2} = 0 \tag{A.10}$$

$$\frac{\partial^2 \pi_{_C}(L_{_C})}{\partial L_{_C}^2} = \frac{c^2 L_{_C}^3 - 4Kkb}{2bL_{_C}^3} \tag{A.11}$$

Thus, the maximizer of $\pi_{_C}(L_{_C})$, $L_{_C}^*$, will satisfy Equation (A.10) as long as $L_{_C}^* \leq L^0 = \left(\frac{4Kkb}{c^2}\right)^{\frac{1}{3}}$, where $\pi_{_C}(L_{_C})$ is concave (Equation (A.11)). Here are some observations:

- $L_{_C}$ should not be greater than $(a - (m+K)b)/c$ for $\lambda_{_C} \geq 0$.
- As $L_{_C} \to 0$, $\frac{\partial \pi_{_C}(L_{_C})}{\partial L_{_C}} \to +\infty$, and $\left.\frac{\partial \pi_{_C}(L_{_C})}{\partial L_{_C}}\right|_{L_{_C} = (a-(m+K)b)/c} = \left(\frac{Kkc}{a-(m+K)b}\right)^2 > 0$.
- As $L_{_C} \to 0$, $\pi_{_C}(L_{_C}) \to -\infty$, and $\pi_{_C}\left(L_{_C} = (a - (m+K)b)/c\right) = -\frac{Kkc}{a-(m+K)b} < 0$

Thus, for Equation (A.10) to have a real root on $\left(0, \frac{a-(m+K)b}{c}\right]$, it should hold that $\left.\frac{\partial \pi_C(L_C)}{\partial L_C}\right|_{L_C=L^0} = -\frac{c2^{\frac{1}{3}}(a-(m+K)b)-3c^{\frac{4}{3}}(Kkb)^{\frac{1}{3}}}{4^{\frac{2}{3}}b} \leq 0$. Let $g(K)$ denote this equation as a function of K. Then,

$$g(0) = -\frac{c(a-mb)}{2b} < 0$$

$$g\left(\frac{a-mb}{b}\right) = \frac{3c^{\frac{4}{3}}((a-mb)k)^{\frac{1}{3}}}{4^{\frac{2}{3}}b} > 0$$

$$\frac{\partial g(K)}{\partial K} = \frac{c\left(2^{\frac{1}{3}}(Kbk)^{\frac{2}{3}} + c^{\frac{1}{3}}k\right)}{(4Kbk)^{\frac{2}{3}}} > 0$$

and $g(K)$ has a unique root, $\bar{K}$, on $K \in \left[0, \frac{a-mb}{b}\right]$. Thus, for all $K \leq \bar{K}$, $\pi_C(L_C)$ will be concave and its maximizer will be given by the root of Equation (A.10). Moreover, if $\bar{K} < K^1$, the firm will generate positive profit. For $K > \bar{K}$, $\frac{\partial \pi_C(L_C)}{\partial L_C} > 0$ for all L_C, and $\pi_C(L_C)$ is increasing, which gives $L_C^* = (a-(m+K)b)/c$. However, $\pi_C(L_C^*) < 0$, and the firm cannot generate positive profit. Similarly, for K values which make $L^0 > (a-(m+K)b)/c$, the firm can also not generate positive profit.

A change of variables gives the following set of equations at optimality for C, which is consistent with the findings of Palaka et al. (1998):

$$\left(a - 2\lambda_C^* - (m+K)b\right)^2 \lambda_C^* = ckbK$$

$$\mu_C^* = \lambda_C^* + \sqrt{\frac{c\lambda_C^* k}{Kb}} \qquad L_C^* = \frac{k}{\mu_C^* - \lambda_C^*} \qquad p_C^* = \frac{a - cL_C^* - \lambda_C^*}{b}$$

For P, as π_P is decreasing in μ_P, the service level constraint is tight at optimality, and $\mu_P = \frac{a-cL_P}{2} + \frac{k}{L_P}$. When we plug μ_P in π_P, we get

$$\pi_P(L_P) = \frac{(a-cL_P)(a-cL_P-2(m+K)b)}{4b} - \frac{Kk}{L_P}$$

FOC give:

$$\frac{\partial \pi_P(L_P)}{\partial L_P} = \frac{c^2L_P^3 - c(a-(m+K)b)L_P^2 + 2Kkb}{2bL_P^2} = 0$$

which is the same equation as Equation (A.10). Note that production will require the margin per unit that it gets to be positive, i.e., $p_P \geq (m+K)$, which gives $L_P \leq (a-(m+K)b)/c$. Moreover, K^1 and K^0 do not need to equal each other, as the objective functions values are different for different K values under the two settings.

In the decentralized setting M, the optimal capacity can be determined by FOC for the objective function with optimal leadtime and demand given as in the original decentralized setting M:

$$\frac{\partial \pi_M}{\partial \mu_M} = \frac{\partial p_M}{\partial \mu_M}\lambda_M + (p_M - m)\frac{\partial \lambda_M}{\partial \mu_M} - K = 0 \quad .\blacksquare$$