Saving Seats for Strategic Customers

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We consider a service provider in a market with two segments. Members of the first request a reservation ahead of service and will not patronize the firm without one. Members of the second walk in and demand service immediately. These customers have a fixed cost of reaching the firm and may behave strategically. In equilibrium, they randomize between walking in and staying home. The service provider must decide how much of a limited capacity to make available to reservation customers. When the reservation segment offers a higher per customer margin, the firm may opt to decline some reservation requests in order to bolster walk-in demand. When walk-in customers are more valuable, we have a variation of Littlewood (1972). Where Littlewood would always save some capacity for valuable late arrivals, here it is possible that the optimal policy saves *no* capacity for walk-ins. Thus, it may be better to ignore rather than pamper walk-in customers.

Key words: Revenue management; service management; reservations.

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1 Introduction

When do customers ask for service? That is a question facing many service providers. Some customers may want to reserve a spot weeks before service is needed. Others may simply walk-in. Obviously, commitments made to the former limit the provider's ability to respond to the latter and misjudging demand of either type can be costly for the firm. How to split capacity between customers that arrive at different points in time has consequently been one of the bedrock problems of revenue management.

Here we consider responding to early and late service requests when late-arriving customers must exert effort to request service, say by having to visit the provider. Further, they must request service without knowing how many seats are available or how many other walk-in customers have ventured in. From the customer's perspective, walking-in requires weighing the chance of getting a seat against the cost of asking for service. Customers must think strategically because getting a seat depends on how others act. From the provider's perspective, giving out reservations not only limits how much can be sold to walk ins, it also impacts the actual number of walk ins who request service.

Service providers ranging from hair dressers to golf courses faces this challenge. We focus on restaurants – a common service industry in which firms employ a variety of strategies to manage reservation and walk-in demand. Further, many in the industry are keenly aware of the impact offering reservations can have on walk-in traffic. Consider the experience of Little Giant, a New-York-City restaurant that has over time operated with and without reservations. When it offered reservations, there was not always sufficient walk-in demand to offset soft reservation sales or customers who failed to honor their reservations “because people without reservations figured all the tables would be occupied by people with reservations” (Bruni, 2006). In the words of a Boston restaurateur,“[a]ny successful restaurant has to hold back tables” in order to avoid alienating regulars (Arnett, 2005).

We examine these issues in a simple setting. A restaurant servers two segment who desire service at the same time but request service at different times. The first segment contacts the firm ahead of the service occasion while the second requires immediate service. Customers in the latter, walk-in segment incur a cost to request service and do not know how many seats are available relative to how many walk-in customers are in the market. In equilibrium, some walk-in customers may choose not to request service.

We consider two cases. In the first, early demand is more profitable but uncertain. Here, it may be optimal to set aside seats for walk-in customers. If a large number of seats are available via reservations, walk-in traffic may be insufficient to compensate for poor reservation demand realizations. The restaurateur commits to potentially turning away some valuable early service requests in order to assure a high level of walk-in traffic. The restaurateur is not guaranteed a full dining room. The optimal policy targets a probability of empty seats that increases with the profitability of the reservation segment and the walk-in segment’s fixed cost. This setting is related to Cachon and Kök (2007). They consider a newsvendor model in which the per-unit salvage value depends on the amount of unsold stock. Here, the salvage value is fixed but the probability that excess inventory is sold depends on the stocking decision.

Saving seats for strategic walk-in customers is intuitively appealing. Remarkably, this result does not necessarily carry over to our second setting in which walk in demand is more profitable. The firm only restricts reservations when the margin on reservation customers and the walk-in segment’s fixed cost are low. Otherwise, the restaurateur makes all capacity available for reservations. That is, it may be better to ignore the more valuable segment than to save seats for them.

Our work falls within the realm of revenue management. (See Talluri and van Ryzin, 2005, for a comprehensive survey.) Indeed, our second case is a variant of Littlewood (1972) who assumes late-arriving customers are more profitable but not strategic. His distribution of late demand is independent of the number of seats available while ours is a function of how many seats are set aside for walk ins. In Littlewood (1972), it is always optimal to save some seats for late arrivals but here it may be best not to save any seats.

Our emphasis on demand responding to availability is similar to Dana and Petruzzi (2001). They consider a retailer whose customers have random outside options. Whether an individual customer patronizes the retailer depends on that customer's option and the firm's inventory level. Our consumer model is simpler but has a similar implication: Consumer demand depends on the inventory available.

A limited number of papers have examined revenue management applications for restaurants. Kimes (2004) considers how a restaurateur can maximize revenue per available seat hour and discusses the relative advantages of offering reservations or relying on walk ins. She does not examine how offering reservations impacts walk-in demand levels. In Bertsimas and Shioda (2003), the restaurant's decisions include whether to accept a reservation request and what wait time to quote a walk-in party. They assume reservation and walk-in demand are independent. We have a single service period but allow for an interplay between reservation and walk-in demand. In Alexandrov and Lariviere (2007), a restaurant serves a single segment of strategic customers. Its sole decision is whether to make its entire capacity available via reservations. Conditions are presented under which the firm will favor offering reservations. They do not consider multiple segments or offering only part of the firm's capacity via reservations.

Below, we present the basics of the model. Sections 3 and 4 consider, respectively, having early demand and late demand be more profitable. Section 5 concludes. Proofs are in the Appendix.

2 Model fundamentals

We consider a monopolist restaurant in a market with two segments. All customers desire service at the same point in time. The segments differ in several ways including when they request service. The first segment, *reservation customers*, contacts the firm before the time service is required and requests reservations. A reservation customer only dines at the restaurant if she receives a reservation. The second segment, *walk-in customers*, requests service immediately. If, for example, the service period in question is Friday night, reservation customers call on Monday while walk-in customers simply request seats on Friday. Let M denote the number of reservation customers and N the number of walk-in customers. We assume all customers require the same amount of the firm's capacity to serve.

The segments also differ in the margins they provide the firm. The margin is π_r for reservation customers and π_w for walk-in customers. The restaurateur does not present customers with different menus depending on whether or not they have reservations. Rather, there are characteristics of the segments that lead them to spend different amounts given the same menu. For example, reservation customers may be couples out for their anniversary who are likely to spring for a bottle of wine and dessert. Reservation customers may then outspend walk-in customers (i.e., $\pi_r > \pi_w$). Alternatively, walk-in customers may be business travelers intent on taking full advantage of their expense accounts leading to $\pi_w > \pi_r$.

Finally, the segments differ in their value for service and their cost of requesting service. Reservation customers have a value $U > 0$ for receiving service and incur no cost for requesting a reservation. Consequently, every reservation customer requests a reservation. Walk-in customers value dining at the restaurant at $V > 0$ but must incur a cost T to ask for a seat. $V > T > 0$. We will refer to T as a travel cost. If a walk-in customer opts to stay home, her net utility is zero.

The restaurant can serve K customers, and its sole decision is its reservation level R, i.e., how many reservations to make available. $0 \leq R \leq K$. Both K and R are common knowledge. Reservation holders always honor their reservations and are guaranteed seats; they have priority over walk-in customers and the firm cannot overbook. Thus, giving out a reservation may prohibit the firm from seating a walk-in customer. If the number of reservation requests exceeds R, seats are rationed randomly and each customer is equally likely to receive one. For the walk-in segment, the restaurant will serve the minimum of its available seats and walk-in demand. If demand exceeds seats, the latter are rationed randomly.

Customers are assumed atomistic, i.e., each is small relative to the size of the market. As we will see below, walk-in customers may follow a mixed strategy. Atomistic customers then implies that while we cannot predict perfectly what an individual will do, we can accurately predict the aggregate outcome.

3 Uncertain but more profitable reservation demand

Here we assume that $\pi_r > \pi_w$. The number of walk-in customer N is deterministic and greater than K. The number of reservation customers M is random with continuous distribution $F(m)$ on support $\left[\underline{M}, \overline{M}\right]$. $\overline{M} > K > \underline{M} \geq 0$. $f(m)$ denotes the density of F, and $\bar{F}(m) = 1 - F(m)$. The distribution is $F(m)$ is commonly known, but only the firm observes realized reservation demand.

Given a realized reservation demand level of m, the restaurant's revenue from reservation customers when it makes R seats available for reservations is $\pi_r \min\{m, R\}$. Its expected revenue from reservation customers given R is then $\Pi_r(R) = \pi_r S(R)$ where $S(R) = \int_{\underline{M}}^{R} m f(m)\, dm + R\bar{F}(R)$. $\Pi_r(R)$ is increasing in R. Hence, if the restaurateur were certain of having more than $K - \underline{M}$ walk ins come in, he would set $K = R$, always have a full restaurant, and earn

$$\Pi_r(K) = \pi_r S(K) + \pi_w (K - S(K)). \tag{1}$$

Unfortunately, the restaurateur cannot take sufficient walk-in demand as a given. Any given walk-in customer faces a lottery. If she "spends" the travel cost T, she may win V or nothing. Winning requires getting a seat, and hence whether one walks in depends upon the chance of being seated. Let γ denote a walk-in customer's probability of getting a seat. Her expected utility is then $\gamma V - T$, and she walks in if $\gamma \geq T/V$. The value of γ, of course, depends on the firm's reservation policy, the realized reservation demand, and the behavior of walk-in customers.

Lemma 1 *Suppose the restaurant sets a reservation level of R and walk-in customers follow a symmetric equilibrium. Let $\nu(R)$ denote the number of walk-in customers who request a seat given R.*

1. If $N \leq \frac{V}{T}(K - S(R))$ all walk-in customers request a seat, i.e., $\nu(R) = N$, and the chance of getting a seat exceeds T/V.
2. Otherwise, each walk-in customer visits the restaurant with probability

$$\lambda(R) = \frac{V(K - S(R))}{TN}. \tag{2}$$

$\nu(R) = \lambda(R)\, N = \frac{V}{T}(K - S(R))$ and $\nu'(R) < 0$.

The walk-in segment cannot determine the exact number available seats, and the equilibrium is consequently based on the average number of available seats $K - S(R)$. When there are on average many seats or walk-in customers have very high net utilities (i.e., V/T is large), strategic interaction between walk-in customers is inconsequential since everyone walks in. Once seats are sufficiently limited, walk-in customers begin to ration themselves and only $\nu(R) < N$ actually walk-in.

Assuming atomistic customers implies that the realized number of customers walking in to the restaurant will be exactly $\nu(R)$. Consequently, if reservation demand is less than $K - \nu(R)$, walk-in demand will be insufficient to fill the restaurant. Let $\Pi_w(R)$ be revenue from walk-in customers given the reservation level and $\nu(R) < N$. We have

$$\Pi_w(R) = \pi_w (K - S(R)) - \pi_w \int_{\underline{M}}^{K-\nu(R)} (K - \nu(R) - m) f(m)\, dm. \tag{3}$$

Comparing (3) with (1), we see that the first term of (3) is expected sales to walk ins ignoring the strategic behavior of walk-in customers. The second term thus represents the firm's loss due to strategic customers.

The restaurateur consequently faces a trade off. Raising the number of seats available via reservations increases the expected number of (more profitable) reservation customers that the firm serves. However, because $\nu(R)$ is decreasing, a higher reservation level also decreases walk-in demand.

Proposition 1 *Suppose $N > \frac{V}{T}(K - S(K))$. Let R^* denote the optimal reservation level.*

1. If $\pi_r \geq \pi_w \left(1 + \frac{V}{T} F(K - \nu(K))\right)$, $R^* = K$.
2. If $\pi_r < \pi_w \left(1 + \frac{V}{T} F(K - \nu(K))\right)$, $R^* < K$ and is found from

$$F(K - \nu(R^*)) = \frac{T(\pi_r - \pi_w)}{V \pi_w}. \tag{4}$$

If $N < \frac{V}{T}(K - S(K))$, Lemma 1 gives that all walk-in customers attempt to get seats even if reservation customers can claim all of capacity. The naive solution of making all capacity available via reservation is then obviously optimal. Such a decision may also be optimal even if walk-in customers self-ration as long as the margin on reservation customers is sufficiently high. When π_r is low, it is optimal to hold back

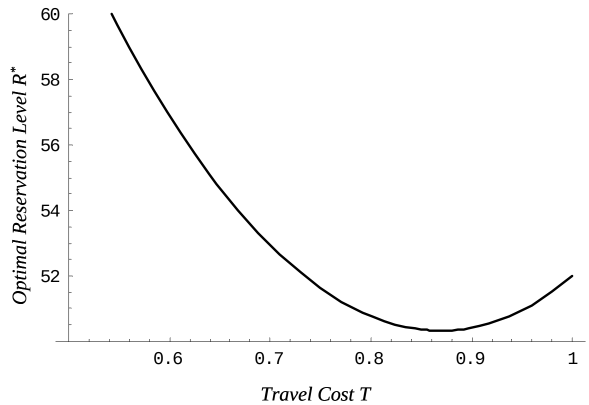

Figure 1: The impact of T on R^* for $F(m) = 1 - e^{-m/100}$, $\pi_r = 1, \pi_w = 0.75, K = 75$, and $V = 1$

some seats. Effectively, the restaurant commits to turning away reservation customers before reaching its capacity in order to assure walk-in customers a good chance of getting a seat.

If the firm turns down a reservation request, there will be sufficient walk-in customers to fill up the restaurant. This does not mean that the restaurant always has a full house. $F(K - \nu(R^*))$ is the probability that the firm has some empty seats. When there is little difference in the segments' margins (i.e., $\pi_r - \pi_w$ is small), the firm severely restricts the number of reservations it gives out in order to reduce the chance of not fully utilizing its capacity. This suggests an alternative interpretation for the first part of the proposition: if π_r is sufficiently large, the firm cannot achieve its desired probability of idle capacity even if it makes its entire capacity available to reservation customers.

The optimal reservation level is increasing in K, implying that is easier to get a reservation at a larger restaurant. This, however, does not come at the expense of walk-in customers. The optimality condition (4) requires that the difference between capacity and the number of customers actually walking in must be constant as K and R^* are adjusted. Hence, the increased reservation level does not completely offset the increase in capacity and the number of patrons walking in increases.

Other comparative statics are harder to generate since $\nu(R)$ depends on $F(m)$ as well as T and V. As Figure 1 shows, the optimal reservation level can be increasing or decreasing in T. Intuitively, two forces come in to play when the travel cost increases. A lower net utility reduces the number of walk-in customers at any R, increasing the chance the restaurant has empty seats. That suggests holding back more seats to effectively subsidize walk-in traffic. Conversely, a higher T reduces the return on saving seats, increasing the targeted probability of having empty seats. Which effect dominates depends on the parameters of the problem. In Figure 1, the reduction in walk-in demand dominates at lower values of T and the firm reduces its reservation level to boost walk-in traffic (although the chance of idle capacity still increases). At higher values of T, the reduction in walk-in traffic from just increasing T is insufficient to hit the new optimal idleness probability and R^* must also be reduced.

The optimal reservation level can also be increasing or decreasing with systematic changes in the demand distribution. For example, consider two markets A and B that are identical except for their reservation demand distributions F_A and F_B. Knowing that reservation demand in B is stochastically larger (i.e., $F_A(m) > F_B(m)$ for all m) is not enough to say whether R_A^* is smaller or larger than R_B^*. However, one can show that the market with larger reservation demand is more profitable.

4 Uncertain but more profitable walk-in demand

We now reverse the assumptions of the previous section. Fix the size of the reservation segment at $M > K$, and assume $\pi_r < \pi_w$. The number of walk-in customers N is random with continuous distribution $G(n)$ on support $\left[\underline{N}, \overline{N}\right]$. $\overline{N} > K > \underline{N} \geq 0$. $g(n)$ denotes the density and $\bar{G}(n) = 1 - G(n)$. The distribution of N is common knowledge, but a walk-in customer does not observed the realized value of N.

Now, all R seats available for reservations will be given out, but seats saved for walk-in customers may not be filled. This is true even when walk-in customers do not behave strategically (equivalently, when $T = 0$). In this case, the restaurant's profit is

$$\hat{\Pi}_L(R) = \pi_r R + \pi_w \hat{S}(K - R), \tag{5}$$

where $\hat{S}(x) = \int_{\underline{N}}^{x} n g(n)\, dn + x\bar{G}(x)$. This is the classic problem of Littlewood (1972). If $G(K) \geq \frac{\pi_w - \pi_r}{\pi_w}$, the optimal reservation level R_L is zero. Otherwise, R_L solves

$$G(K - R_L) = \frac{\pi_w - \pi_r}{\pi_w}.$$

Of course, once walk-in customers incur a cost to request a seat, the Littlewood quantity R_L may not be optimal. Walking in is still risky, and how many walk-in customers actually attempt to patronize the firm depends on the chance of getting a seat.

Lemma 2 *Suppose the restaurant sets a reservation level of R and walk-in customers follow a symmetric equilibrium. Let $\hat{Z}$ solve $\phi\left(\hat{Z}\right) = \frac{T}{V}$ where:*

$$\phi(z) = G(z) + \int_{z}^{\overline{N}} \frac{z}{n} g(n)\, dn.$$

1. If $\hat{Z} \leq K - R$, all walk-in customers request a seat, and the chance of getting a seat exceeds T/V.
2. Otherwise, each walk-in customer visits the restaurant with probability

$$\hat{\lambda}(R) = \frac{K - R}{\hat{Z}},$$

and the chance of getting a seat is T/V.

When the restaurateur saves many seats for walk ins, the distribution of customer who actually walk in is simply $G(n)$. Once the number of saved seats falls below the critical level $\hat{Z}$, the number of walk-in patrons who ask for seats falls to $\hat{\lambda}(R)\, N$. Hence, as the firm gives out more reservations, the number of walk-ins patronizing the restaurant becomes stochastically smaller.

The firm's profit from the walk-in segment in this range is

$$\Pi_w(R) = \pi_w (K - R) \left[\int_{\underline{N}}^{\hat{Z}} \frac{n g(n)}{\hat{Z}} dn + \bar{G}\left(\hat{Z}\right) \right] = \pi_w (K - R)\, \hat{S}\left(\hat{Z}\right) / \hat{Z}.$$

The firm's profit from walk-in customers is linear in R because the number of customers actually walking in is scaled by $\hat{\lambda}(R)$, which is linear in the reservation level. $\hat{S}\left(\hat{Z}\right)/\hat{Z}$ is always less than one. In general it can be interpreted as the fraction of units sold when $\hat{Z}$ units are stocked. Here, it is the fraction of the seats saved for walk-in customers that are filled on an average night.

The restaurant's total profit as a function of the reservation level $\hat{\Pi}(R)$ is then

$$\hat{\Pi}(R) = \begin{cases} \hat{\Pi}_L(R) & \text{if } R \leq K - \hat{Z} \\ \pi_r R + \Pi_w(R) & \text{if } R > K - \hat{Z} \end{cases}.$$

As shown in Figure 2, $\hat{\Pi}(R)$ is kinked. (In the figure, $\hat{\Pi}(R)$ is plotted as a solid line while $\hat{\Pi}_L(R)$ is plotted as a dashed line.) For reservation levels below $K - \hat{Z}$, walk-in customer do not self-ration; $\hat{\Pi}(R)$ follows

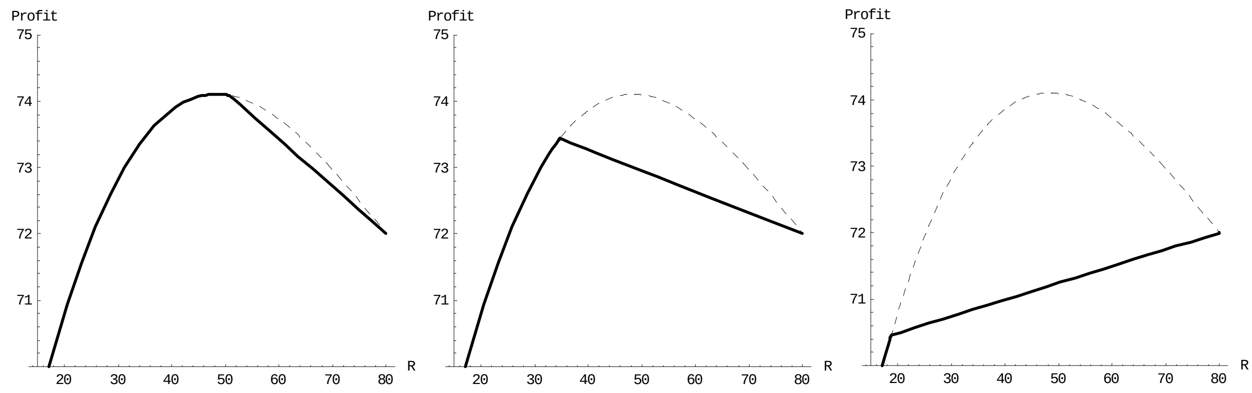

Figure 2: Profit as a function of R with both strategic (the solid line) and non-strategic customers (the dashed line). For all examples $G(n) = (n/100)^2$ for $0 \leq n \leq 100, K = 80, \pi_r = 0.9, \pi_w = 1$, and $V = 1$. For (a), $T = 0.5$. For (b), $T = 0.7$. For (c), $T = 0.85$.

the Littlewood objective and is strictly concave. Beyond $K - \hat{Z}$, not all walk ins visit the restaurant; $\hat{\Pi}(R)$ is linear and can slope up or down. The optimal reservation level then depends on two factors, the location of the Littlewood quantity R_L relative to the breakpoint and the slope beyond the breakpoint.

Proposition 2 *Let $\hat{R}$ denote the optimal reservation level.*

1. If $R_L \leq K - \hat{Z}$, $\hat{R} = R_L$.
2. If $R_L > K - \hat{Z}$,

$$\hat{R} = \begin{cases} K - \hat{Z} & \text{if } \pi_r/\pi_w \leq \hat{S}\left(\hat{Z}\right)/\hat{Z} \\ K & \text{if } \pi_r/\pi_w > \hat{S}\left(\hat{Z}\right)/\hat{Z} \end{cases}.$$

When reservation demand was uncertain but more profitable, the restaurateur could disregard strategic walk-in behavior of as long as walk-in demand was sufficiently robust. Otherwise, reservations were limited. Part of that logic carries over. If all walk-in customers come in over a large range of available seats, the firm effectively lives in Littlewood's world and can simply choose R_L. See Figure 2(a). This requires that $\hat{Z}$ and R_L be relatively small. That, in turn, implies that walk-in customers have a high net utility (since $\hat{Z}$ is increasing in T/V) and that walk-in customers are significantly more profitable than reservation customers (since R_L is decreasing in $\pi_w - \pi_r$).

Once walk-in customers are rationing themselves, the firm does not necessarily save extra seats for them. This happens when walk ins are significantly more profitable than reservation customers. A seat saved for a walk-in garners π_w when it sells but the average return is only $\pi_w \hat{S}\left(\hat{Z}\right)/\hat{Z}$. As the the margin on reservation customers increases, the optimal decision eventually tips to committing all capacity to reservation customers. Thus although Littlewood (1972) always holds back seats for the more profitable segment, it may be best to forego doing any business with this segment if they behave strategically.

Changes in the customers' utility may also lead to the abandonment of the walk-in market. This is shown in Figures 2(b) and (c). These examples differ only in their travel cost. As T/V increases, walk-in demand shrinks until it is not worth saving seats for them. More formally, $\hat{Z}$ is increasing in T/V while $\hat{S}\left(\hat{Z}\right)/\hat{Z}$ decreases in $\hat{Z}$, which leads to the following corollary.

Corollary 1 *If $R_L > K - \hat{Z}$, there exists a ρ such that if $T/V \leq \rho$, $\hat{R} = K - \hat{Z}$ and $\hat{R} = K$ otherwise.*

The firm's capacity has a limited impact on whether reservations are plentiful or scarce. Changing capacity may move one from the regime in which the Littlewood quantity is optimal to the regime in which customers ration themselves. Once the restaurateur deviates from R_L, capacity does not play a role in determining whether only $K - \hat{Z}$ reservations or K reservations are offered.

Proposition 2 has two notable implications. First, it may be optimal to save fewer seats in a market with strategic customers than in a market with non-strategic customers. Second, when customers ration themselves, the firm need only consider two reservation levels. We now examine whether these depend on the utility structure. Suppose there is a random variable ε with distribution $H(\varepsilon)$ on support $[0, \bar{\varepsilon}]$. Each customer independently draws a ε and then has value $V(\varepsilon) = T + \varepsilon$ for dining out.

This changes the nature of the walk-in equilibrium since not all customers can be indifferent between walking in and staying home. In equilibrium, there would exist a critical value $\hat{\varepsilon}$ such that all customer for whom $\varepsilon < \hat{\varepsilon}$ stay home while those for whom $\varepsilon \geq \hat{\varepsilon}$ walk-in. As long as there is a positive probability that some walk-in customer will be left without a seat, not everyone will walk in. Thus, we immediately lose the existence of a range of parameters over which the restaurateur can safely ignore strategic customer behavior. One can show that the firm's profit is now always lower than in a market with non-strategic customers. This is demonstrated in Figure 3. Note that in this example, profit is concave in R and that

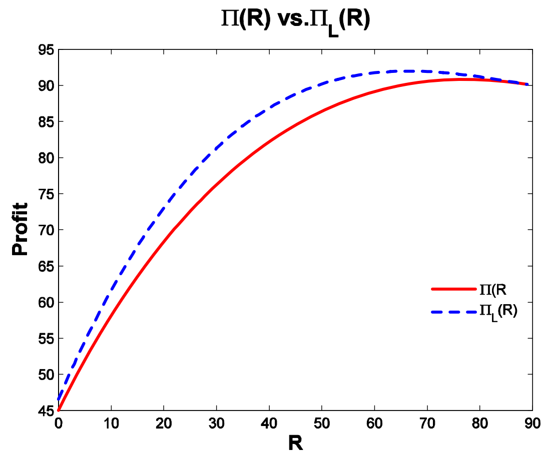

Figure 3: Firm profit with and without strategic customers. $H(\varepsilon) = \varepsilon/10$ for $0 \le \varepsilon \le 10$, $G(n) = (n/100)^2$ for $0 \le n \le 100, K = 90, \pi_r = 1, \pi_w = 4/3$, and $T = 50$.

the optimal reservation level exceeds the best choice without strategic customers. Hence, the "bang-bang" optimal reservation levels of Proposition 2 depend on walk-in customers having a common value for dining out, but the possibility of saving fewer seats for strategic customers does not.

5 Conclusion and extensions

We have presented a simple model of a firm selling to two segments. The first attempts to reserve capacity before the time service is required. The other demands service at the time it is required. The key assumptions are that the second, walk-in segment incurs a cost to request service but cannot verify how much capacity is available relative to market demand. Our model thus fits a restaurant setting for which walk-in customers must be physically present to request a seat but cannot verify whether the restaurant is full before going to the restaurant. Walking in is then a risky proposition; a customer may expend the cost of getting to the restaurant but be unable to get a seat. A customer's decision to walk-in then depends on how much capacity the firm holds back and the actions of other walk-in customers.

We consider two cases. In the first, reservation customers are more profitable but the size of that segment is uncertain. The second is a variant of a classical revenue management problem (Littlewood, 1972); the late arriving segment is of uncertain size but more profitable. In either setting, when sufficiently few seats are likely available, walk-in customers randomize between walking in and staying home. As fewer and fewer seats are available, a smaller and smaller share of walk-in customers actually attempts to patronize the restaurant. As a Yankee great might have predicted, no one goes because it is too crowded.

This problem is mitigated when the firm has a large capacity. In either setting, a restaurateur with sufficient capacity can ignore any strategic behavior. For the first case, this implies making all capacity available for reservations and counting on walk-in traffic to fill any empty seats. In the second, it implies solving a newsvendor problem to determine how many seats to hold back.

The restaurateur deviates from these policies when walk-in customers limit their patronage. When reservation demand is more profitable but random, the optimal response is to lower the reservation level. The firm commits to potentially turning away some high-value reservation customers in order to bolster walk-in demand. The firm may have second thoughts on this policy if reservation demand is strong, but it pays off if reservation demand is low by assuring greater walk-in traffic.

The policy of saving extra seats for walk ins is intuitively appealing and may carry over to the setting with more profitable but random walk-in demand. But it is not always the best policy. It is only optimal when the margin on reservation customers and the travel cost of walk-in customers are sufficiently low. When these are high, it is optimal to deal only with the reservation segment. Thus just because it is impossible to dine at restaurant without calling weeks in advance does not mean that the restaurateur is behaving suboptimally.

We have imposed a number of simplifying assumptions. For example, it is assumed that asking for a reservation is costless. If there is a fixed cost τ to requesting a reservation, a reservation customer must weigh the cost of asking with the likelihood of success. (We have in mind a non-monetary, hassle cost as opposed to a fee paid to the restaurant that would raise the its margin.) When reservation demand is random, the analysis would parallel Lemma 2. If τ is sufficiently low relative to the segment's value of dining out U, everyone will still request a reservation and our results are unchanged. When the cost of requesting a reservation is high, the problem would be significantly harder.

If reservation demand is deterministic, the analysis would mimic Lemma 1 but with a fixed number of seats R instead of an expected number of seats $K - S(R)$. Reservation customers would randomize between asking for a reservation and staying home. Assuming staying home has a value of zero, they ask for a reservation with probability $\min\left\{1, \frac{UR}{\tau M}\right\}$ and the number of reservation requests would exceed R. As all reservations will be given out, our analysis is unchanged. Comparing this outcome with that of Section 3 shows that uncertainty in the number of available seats is a key driver of the results in Proposition 1. If walk-in customers could always verify the number of available seats, they would generate enough demand to fill them all.

We have assumed that demand from the lower value segment is always deterministic. In our first setting, this implies that a walk-in customer is uncertain of the number of available seats but knows how many other walk-in customers are in the market. In the second setting, the reverse holds. If the size of both segments were random, a walk-in customer would be blind to both the number of available seats and how many other customers she was competing with. This would complicate the analysis substantially but not change the basic outcome. When seats are limited, not all members of the walk-in segment would in fact venture in and the restaurateur would have to balance sales to reservation customers and the amount of walk-in traffic.

Reservation customers are assumed to always honor their reservations. This is rarely true in practice. The impact of no shows depends on how the restaurant is able to respond. If the seats of no shows can be re-offered to walk-in customers, walk-in demand will increase at any reservation level leading to more seats being made available for reservations. Conversely if no shows translate into lost sales, the expected margin on reservations falls, leading to fewer seats being offered to the segment.

There are some obvious ways in which our model can be extended. First, competition is a natural generalization. Consider a symmetric duopoly in which customers are indifferent between the two firms. When reservation demand is more profitable, competition increases the industry reservation level for a sufficiently high value of M. That is, two firms each with capacity K make more seats available to the reservation segment than a monopolist with capacity $2K$ would. This echoes the results of Cachon (2003) who considers competing newsvendors in which demand is assumed to be allocated in proportion to the stocking level.

When walk ins are more profitable, an equilibrium exists in which the duopolists serve the same number of reservation customers as a monopolist with industry capacity. Here, walk ins allocate themselves between the firms in proportion to the number of seats available. Increasing the number of saved seats also draws in more walk-in customers into the market. The net effect is to leave each firm's profit from the walk-in segment proportional to the number of seats it offers the walk-in market and independent of its competitor's actions.

Alternatively, one can more fully explore the consequences of disregarding strategic behavior. This is most pertinent when walk ins are the more profitable segment. Ignoring strategic behavior, the restaurateur would solve a newsvendor model but then observe a pattern of demand that is stochastically smaller than he had anticipated. In a repeated environment, one would have system similar to Cooper et al. (2006) in which a decision maker repeatedly recalibrates a mis-specified model ignoring how his action impacts the observed data. If we assume that the restaurateur starts with a correct estimate of walk-in demand, he would begin with the Littlewood quantity R_L but then pick higher and higher reservation levels as walk-in customer increasingly ration themselves. In the limit, all seats would be available via reservations. This may, in fact, be the optimal action but it could also be diametrically opposite to the best decision.

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Appendix: Proofs

*Proof of Lemma 1:*Following Dana and Petruzzi (2001), the chance of getting a seat is given by the expected fill rate. If walk ins randomizes with probability $\lambda(R)$, the chance any one customer gets a seat is $\gamma(R) = \frac{K-S(R)}{\lambda(R)N}$. If $N < \frac{V}{T}(K - S(R))$ and $\lambda(R) = 1$, $\gamma(R) > T/V$ and every customer has a positive expected utility from walking in; $\lambda(R) = 1$ is an equilibrium. If $N > \frac{V}{T}(K - S(R))$, $\gamma(R) \leq T/V$ if $\lambda(R) = 1$, and it cannot be an equilibrium for all customers to enter. For customers to randomize between walking in and staying home, we must have $\gamma(R) = T/V$, which yields 2. $\nu'(R) < 0$ follows from $S'(R) > 0$. $\square$

*Proof of Proposition 1:*The firm's profit given R is

$$\Pi(R) = \pi_r S(R) + \pi_w (K - S(R)) - \pi_w \int_{\underline{M}}^{K-\nu(R)} (K - \nu(R) - m) f(m)\, dm.$$

We then have

$$\Pi'(R) = S'(R)\left[\pi_r - \pi_w - \pi_w \frac{V}{T} F(K - \nu(R))\right], \tag{6}$$

Equation (6) yields (4). It is straightforward to show that second order conditions are satisfied. Note that $R^* \leq K$. Hence, if $F(K - \nu(K)) < \frac{\pi_r - \pi_w}{\pi_w}\left(\frac{T}{V}\right)$, it is optimal to have $R^* = K$. $\square$

Proof of Lemma 2: Suppose a given walk-in customer actually visits the restaurant with probability $\hat{\lambda}(R)$. If the realized size of the walk-in segment is less than $\frac{K-R}{\hat{\lambda}(R)}$, everyone who visits the restaurant gets a seat. If the realized segment size n exceeds $\frac{K-R}{\hat{\lambda}(R)}$, the chance of getting is $\frac{K-R}{\hat{\lambda}(R)n}$. The expected chance of getting a seat is then $\phi\left(\frac{K-R}{\hat{\lambda}(R)}\right)$. $\phi'(z) \geq 0$. If $K - R \geq \hat{Z}$, the chance of getting a seat exceeds T/V, and all walk-in customers request one. If the number of available seats is less than $\hat{Z}$, walk-in customers randomize between walking in and staying home. For indifference, we need $\frac{K-R}{\hat{\lambda}(R)} = \hat{Z}$. $\square$

Proof of Proposition 2: For $R_L \leq K - \hat{Z}$, the firm's profit is maximized by R_L. Beyond R_L profit is either decreasing or maximized at $\pi_r R$ which is less than $\hat{\Pi}(R_L)$ by the optimality of R_L. When $R_L > K - \hat{Z}$, the firm's profit is increasing for $R < K - \hat{Z}$. For R between $K - \hat{Z}$ and K, $\hat{\Pi}'(R)$=$\pi_r - \pi_w \hat{S}\left(\hat{Z}\right)/\hat{Z}$, which yields the second part of the proposition.

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