# Are Reservations Recommended?

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We examine the role of reservations in capacity-constrained services with a focus on restaurants. Although customer value reservations, restaurants typically neither charge for them or impose penalties for failing to honor them. However, reservations impose costs on firms offering them. We highlight ways in which reservations can increase a firm's sales by altering customer behavior. First, when demand is uncertain, reservations induce more customers to patronize the restaurant on slow nights. The firm must then trade off higher sales in a soft market with sales lost to no shows on busy nights. Competition makes reservations more attractive as long as enough customers will consider dining at either restaurant. When there are many firms in the market, it is rarely an equilibrium for none to offer reservations. Second, we show that reservations can increase sales by shifting demand from a popular peak period to a less desirable off-peak time. This is accomplished by informing diners that the peak is full. In this setting, competition may make offering reservations less attractive and a market with many firms may have no one offering reservations.

**Key words:** Service management; reservations; restaurants; capacity management.

## 1 Introduction

Restaurant reservations are a curious phenomenon. Customers value them, but restaurants give them away. Indeed, firms such as withoutreservations.biz and Weekend Epicure have stepped in to profit from the arbitrage opportunity restaurants have created. These "scalpers" reserve tables at popular spots under fictitious names and share those names with the first paying party. (Fees are on the order of \$35 to \$40.) What makes offering reservations even more remarkable is that they are costly to provide. Fischer (2005) identifies three costs to offering reservations. These include additional staff needed to take reservations and added complexity from having to balance the needs of "walk-in" customers with commitments made to reservation holders. The final consideration is "no shows." Customers can generally fail to keep reservations without penalty, but restaurants suffer if they hold capacity for customers that never come. No shows represent a real problem in the industry. Bertsimas and Shioda (2003) report a no-show rate of 3% to 15% for the restaurant they studied. More generally, rates of 20% are not unusual (Webb Pressler, 2003) and special occasions such as Valentine's Day and New Year's Eve can push rates to 40% (Martin, 2001).

Why then should restaurants offer reservations? One reason is that they provide some operational benefits. For example, reservations help manage the flow of work. By staggering seatings, a restaurateur can assure that waiters are not overwhelmed by all customers arriving at once, followed by the bartender being swamped with cocktail orders, and then the kitchen being buried with food requests. By matching the flow of customers with the system's capacity, reservations allow fast service without excessive capacity (Fischer, 2005). Reservations would then be appealing when either customers are delay sensitive or the firm's costs increase with arrival variability. Further, reservations may allow a restaurant to estimate demand and improve staffing and sourcing decision. This is particularly important when variable costs are high. Chicago-based Alinea is known for intricate, multicourse menus. It requires customers make reservations. To the extent that Alinea incurs significant expense in preparing for its guests, knowing how many are coming on a given night is crucial.

In this paper we abstract from these issues and focus on reservations' ability to influence consumer behavior and thus increase sales. We consider a capacitated restaurant whose sole decision is whether or not to offer reservations. (Our model also applies to other firms that serve both reservations and walk ins and whose charges depend on choices customers make after arrival.) At the time reservations are made (assuming they are offered), customers are uncertain how they will value the service at the time of consumption. Customers learn their valuations before traveling to the service facility. Since there is a fixed cost to accessing the facility, those with low realized values fail to honor their reservations. If reservations are not offered, customers learn their value for the service and decide whether to walk in for service. Walking in incurs a travel cost and risks incurring an additional penalty if one cannot get a seat.

We first study a base case with a known market size and a single sales period and show that reservations are never offered. We consequently introduce market-size uncertainty with two demand states. The firm can serve all customers on a slow night, but it lacks the capacity to serve everyone on a busy night. Market uncertainty reduces customer willingness to walk in. Reservations counteract this by guaranteeing all customers a seat on a slow night. The restaurant faces a trade off of selling more on slow nights but losing sales on busy nights because of no shows. Reservations are hence important to the firm in soft market but costly in a strong one. Competition can make offering reservations even more attractive. In a market with many firms, it is unlikely that no one will offer reservations.

We next split the evening into a capacitated peak and an uncapacitated off peak. All diners prefer the peak period. Absent reservations, the peak is oversubscribed. Reservations shift demand by informing some customers that they must move off-peak to receive service. Reservations thus create value for the restaurant when customers are unable to get them. If the primary reason for offering reservations is to move demand off peak, competition may limit the attractiveness of reservations. Indeed, with many small firms in the market it may now be an equilibrium for no one to offer reservations.

Below, we review the literature. §3 presents the basic model. The following two sections consider, respectively, uncertain market size and multiple sales periods. §6 concludes. Proofs are in the Appendix.

## 2 Literature review

The existing literature on reservations or advance sales emphasizes pricing or segmentation. In Png (1989), a firm with limited capacity sells to risk-averse customers who are uncertain of their need for a service at some future point. The firm offers reservations and overbooks; bumped customers receive a prespecified compensation. Reservations act as insurance and allow for higher prices than either selling outright in advance or selling in the spot market. Dana (1998) considers a competitive market with capacitated firms. Narket segments differ in their willingness to pay and the certainty with which they need service. If those with low valuations are more certain to need the service, the firms sell some capacity early at a discount but raise the price closer to the time of service delivery. In DeGarba (1995), advance selling allows the firm to capture the same revenue as first-degree price discrimination. Xie and Shugan (2001) and Shugan and Xie (2005) develop similar ideas. (See Courty, 2000, and Shugan, 2002, for reviews.) None of this work properly addresses the restaurant industry. Restaurants do not charge in advance, and menu prices are independent of holding reservations. Also, they do not publicize compensation schemes for reservation holders denied service. Further most of these papers focus on monopoly settings (Dana, 1998, and Shugan and Xie, 2005, are exceptions) while we extend our analysis to competitive environments.

Gale and Holmes (1993) show that advance-purchase discounts with limited refundability can shift demand from a popular flight to a less popular alternative. We demonstrate that reservations can profitability move some demand from a peak to an off-peak period. This is not accomplished through pricing; reservations provide information that is sufficient to induce some customers to move off peak.

Tang *et al.* (2004) consider learning market information through advance booking discounts. In their model, early sales are correlated with later sales and thus facilitate production planning. Decision variables are the production quantity and the price discount. In our model capacity is fixed, and we ignore production planning and pricing. We instead focus on how reservations alter consumer behavior and increase sales.

Dana and Petruzzi (2001) study a newsvendor model in which customers incur a cost to shop. The stocking level thus affects demand. Their results are structurally similar to our analysis of would-be diners determining whether to walk in. They focus on determining the stocking level and do not consider altering the timing of sales. We take the restaurant's capacity as exogenous but allow reservations.

Appointment systems have been widely studied. See, for example, Robinson and Chen (2003) or Savin (2006). This work generally ignores strategic consumer actions. Lariviere and Van Mieghem (2004) consider delay-sensitive consumers choosing arrival times. An appointment system allows them to pick sequentially and results in an arrival pattern that minimizes both period-to-period variation in arrivals and total delay costs. Customers are not allowed to decline seeking service and are assumed to be indifferent to when they are served. We ignore delay costs but explore how reservations can increase sales either by encouraging more customers to come in or by inducing customers to avoid crowded times.

Bertsimas and Shioda (2003) consider how many reservations a restaurant should accept and how to accommodate walk-in customers given reservation commitments. Both demand streams are exogenous. Here, offering reservations changes the number of customers patronizing the firm. Arthur (1994) considers a bar with many possible patrons. If too many show up on a given night, the bar is too crowded to be enjoyable. Customers can only learn how crowded the bar is by coming in. The analysis focuses on how customers learn over multiple evenings. We assume a single evening so there is no learning.

## 3 Model fundamentals

We consider one restaurant serving a market of known size $\theta > 0$. All customers are served in a single sales period. Customers are *a priori* homogeneous. (Our results are unaltered if there are identically sized parties as opposed to individual customers.) The net benefit from planning to stay home is normalized to zero. Each customer incurs a cost $T \geq 0$ to travel to the restaurant whether or not she holds a reservation. A customer who walks in but fails to get a seat incurs a denial cost $D \geq 0$, which represents the inconvenience of changing plans and is not paid to the firm. A strictly positive denial cost implies that the opportunities open to a customer who has been turned away are not as attractive as planning to eat at home.

While *a priori* homogeneous, customers are *ex post* differentiated by their value for dining out. Each customer draws a valuation independently from a known, continuous distribution, $F(V)$ with density $f(V)$ and support $(0, \Omega)$ for $\Omega > T$. $\bar{F}(V) = 1 - F(V)$. Customers learn their valuations before incurring the travel cost. Hence, only those whose realized valuation $V$ exceeds $T$ will even consider patronizing the restaurant. (Results are essentially the same if diners have a fixed $V$ but random $T$. See Dana and Petruzzi, 2001.) Customers are atomistic; they are sufficiently small relative to the market that aggregate uncertainty is trivial. Thus, while it is uncertain whether any specific consumer will have a positive net utility from dining out, the total number of customers interested in dining out is certain to be $\theta\bar{F}(T)$.

The firm's sole decision is whether to offer reservations. Its menu and pricing are fixed, and the expected spending is the same for each customer. The restaurant's objective is thus to maximize unit sales. The firm can serve $K$ per evening. $\theta\bar{F}(T) > K > 0$, *i.e.*, not all customers who value dining out can be seated.

Events proceed as follows. The firm announces its policy. If it offers reservations, customers simultaneously decide whether or not to ask for one. Requesting customers learn immediately if they are successful. Subsequently, each customer learns her valuation $V$. Reservation holders then decide whether to honor them while non-holders simultaneously decide whether to walk in. As customers arrive, reservation holders are seated first. If the number of walk-in customers exceeds the available capacity, seats are rationed randomly. Thus, if $M$ reservations have been given out and $\kappa > K - M$ customers walk in, the probability of any one walk-in customer being seated is $(K - M) / \kappa$. This implicitly assumes the restaurant does not realize a given reservation holder is a no show until it is too late to give her seat to a walk-in customer. If reservations are not offered, customers wait to learn their valuations and then decide whether or not to walk in. If the number of walk ins exceeds capacity, seats are rationed randomly.

## 3.1 The no-reservation case: A world of walk ins

When reservations are not offered, a customer with realized valuation $V'$ effectively faces a lottery. By "spending" the travel cost $T$, she either receives a benefit of $V'$ or incurs a penalty of $D$. Crucial to her decision then is her perceived probability of getting a seat. Let $\delta$ denote her anticipated probability of getting a seat. Her expected utility from walking in is $\delta V' - (1 - \delta) D - T$, and she will walk in if $\delta \geq \frac{T+D}{V'+D}$. Since all customers have the same chance of getting a seat, anyone with realized valuation $V \geq V'$ will also walk in if the customer at $V'$ walks in.

**Proposition 1** *Suppose the restaurant does not offer reservations.*

1. The unique equilibrium has all customer with valuations greater than $V'$ walking in to the restaurant, where $V'$ is found from
$$\bar{F}(V') = \frac{K}{\theta} \left( \frac{V' + D}{T + D} \right). \tag{1}$$
2. The equilibrium probability that an arriving customer gets a seat is $\frac{T+D}{V'+D}$.

In equilibrium, the marginal customer must be indifferent between walking in and staying home. Hence, the restaurant is oversubscribed (*i.e.*, $\theta \bar{F}(V') > K$). However, if $D$ becomes large, $\bar{F}(V') \approx \frac{K}{\theta}$; only the highest valuation customers attempt to dine out, and essentially all of them are seated.

It is straightforward to show that $V'$ increases with $\theta$, $T$, and $D$ but falls in $K$. Further, consider two valuation distributions, $F_0$ and $F_1$, and let $V'_0$ and $V'_1$ be the corresponding cutoffs. If $F_0(V) \geq F_1(V)$ for all $V$, $V'_0 \leq V'_1$. A higher cutoff value does not translate into a smaller crowd. The chance a walk-in customers gets a seat falls as $V'$ increases (holding $T$ constant and $D$ constant). A better restaurant (in the sense of stochastically larger valuation distribution) is more crowded. Also, as less capacity is available, fewer customers walking in does not offset the loss of seats; the chance of getting a seat falls as $K$ decreases.

## 3.2 Reservations and the restaurateur's problem

Suppose the restaurant's entire capacity is offered via reservations. (This may not be optimal; see §6.) It does not overbook, so reservation holders are guaranteed seats. If demand exceeds $K$, reservations are rationed randomly. We do not *a priori* assume reservations eliminate walk-ins; if fewer than $K$ reservations are given out, the restaurant will accept walk-in customers. However, in equilibrium, a restaurant will not serve both reservation and walk ins. To see why, let $\mathbb{V} = E\left[\max\{0, V - T\}\right].$ $\mathbb{V}$ is a representative customer's expected value for holding a reservation. Because $\mathbb{V} \geq 0$, all customer request one, and the restaurant commits all of its capacity to reservations. A customer who was unable to secure a reservation consequently does not walk in because she knows that she will not get a seat.

Everyone asks for reservations, but not all reservations are honored. With probability $F(T)$, a reservation holder becomes a no show. The firm's sales are then $K\bar{F}(T)$ with reservations but $K$ without them. Thus in market of a known size with a single sales period, it is never optimal to offer reservations.

This outcome is hardly surprising; the model is stacked against reservations. Customers make reservations before learning their valuations, opening the possibility that they fail to show. We have also limited the firm's ability to protect itself from no shows by assuming that it is unable to reassign seats and is unable to overbook. Relaxing any of these would leave the restaurant indifferent between taking reservations and accepting only walk ins. If customers learnt their valuations before making reservations, only those who plan on using a reservation would bother requesting one. If the restaurant could identify no shows in a timely manner and re-offer their seats, customers without reservations would correctly anticipate the number of no shows and generate enough walk-in business to fill up the restaurant. Similarly, the assumption of atomistic customers implies that if $M$ reservations are given out, exactly $M\bar{F}(T)$ diners will show up. Hence, overbooking with $M = K/\bar{F}(T)$ would allow the restaurant to compensate for no shows exactly.

# 4 Uncertain market size

To develop a role for reservations, we must relax some assumption. Here we introduce demand uncertainty. With probability $1 - \rho$, the market size is one. With probability $\rho$, the market size is $\theta > 1$. We assume

$1 \leq K < \theta\bar{F}(T)$; capacity is sufficient to serve all interested customers on a slow night but binds in a large market. Market uncertainty does not reflect variation over observable conditions (e.g., Fridays are busier than Tuesdays) but variation given specific circumstances. One should interpret the model as saying that given that it is Friday night, the restaurant will be busy with probability $\rho$. Customers cannot learn the market size unless they request a reservation or walk in. We first consider a monopoly and then competition.

## 4.1 The monopoly problem

We begin with the no-reservation case. Customers again use a cutoff policy, walking in only if their realized valuation is sufficiently high. The proof of the following parallels that of Proposition 1.

**Proposition 2** *Suppose the restaurant does not offer reservations.*

1. The unique equilibrium has all customer with valuations greater than $V^*$ walking in to the restaurant, where $V^*$ is found from
$$\bar{F}(V^*) = \frac{K(V^* + D)\rho}{\theta(T + \rho D - (1-\rho)V^*)}. \tag{2}$$
2. The cutoff value $V^*$ is increasing in $\rho$ and $V^* \leq V'$, where $V'$ is found from (1).
3. The equilibrium probability that an arriving customer gets a seat is $\frac{T+D}{V^*+D}$.

If the restaurant offers reservations, it has $\bar{F}(T)$ customers when the market turns out to be small but $K\bar{F}(T)$ customers when the market is large.

**Proposition 3** *The restaurant prefers offering reservations if*
$$(1-\rho)\left(\bar{F}(T) - \bar{F}(V^*)\right) \geq \rho K F(T). \tag{3}$$
*Reservations are never offered if*
$$F(T) \geq \frac{(1-\rho)\left(1 - \frac{K}{\theta}\right)}{1 - \rho + \rho K}. \tag{4}$$

Reservations pose a clear trade off for the restaurateur: In the low demand state, they increase sales (since $T < V^*$) but lower sales in the high-demand state because of no shows. The question is whether the gain on slow nights ($\bar{F}(T) - \bar{F}(V^*)$) outweighs the loss on busy nights ($KF(T)$). Reservations are consequently important on evenings in which the firm has plenty of available seats but are costly when the place is hopping. That is, the firm offers reservations not to manage demand when business is good but to entice more customers when demand would otherwise be slow.

Costly no shows are essential to our story: The trade off goes away if they are not an issue. If customers knew their valuations when making reservations, sales in the high state would be $K$, and the firm would always offer them. Alternatively, if the no show rate $F(T)$ is too high, reservations are never offered, as the bound (4) demonstrates. It is derived by considering the limit as $D$ becomes large and is useful since it is independent of $V^*$. The bound is decreasing in $K$ and $\rho$. Hence, for any given no show rate there exists a capacity level or chance of a large market sufficiently large that reservations would never be offered.

We now examine the impact of model parameters. Define $r(V) = \frac{f(V)}{F(V)}$. For distributions $F_0$ and $F_1$ with respective $r_0$ and $r_1$, $F_0$ is smaller than $F_1$ in the reversed hazard rate order if $r_0(V) \leq r_1(V)$ for all $V \geq 0$ (Shaked and Shanthikumar, 1994).

**Proposition 4** *If (3) holds for...*

1. $K_0$, then it holds for any $1 \leq K \leq K_0$.
2. $\theta_0$,then it holds for any $\theta \geq \theta_0$.
3. $D_0$, then it holds for any $D \geq D_0$.
4. $F_0$, then it holds for any valuation distribution $F_1$ larger than $F_0$ in the reversed hazard rate order.
5. $T_0$, then it holds for any $T \leq T_0$ if $F(\psi T)/F(T)$ is decreasing in $T$ for $\psi > 1$.

Offering reservations is more attractive when small capacity (*i.e.*, $K/\theta$ is low) or a high denial penalty limits walk-in demand. Similarly, when a larger valuation distribution raises the walk-in cutoff, reservations become more attractive. Together these give the empirical predictions that smaller restaurants or better quality restaurants (in the sense of higher valuation distributions) are more likely to offer reservations.

The roles of $T$ and $\rho$ are less clear. Increasing $T$ raises $V^*$, lowering walk-in sales in the low demand state, but it also increases no shows in all states. Subject to a regularity condition, the proposition shows that the latter effect dominates and that lower travel costs make reservations more attractive. The condition holds for many common distributions (e.g., the power function, gamma, Weibull, and beta for some parameter values). Numerical experimentation suggests it also holds for other distributions (e.g., the normal).

While reservation sales increase linearly with the probability of high demand, walk-in sales are not necessarily monotone in $\rho$. It is consequently difficult to generate comparative statics analytically. However, we know that sales with and without reservations are equal as $\rho$ falls to zero but relying on walk ins is always

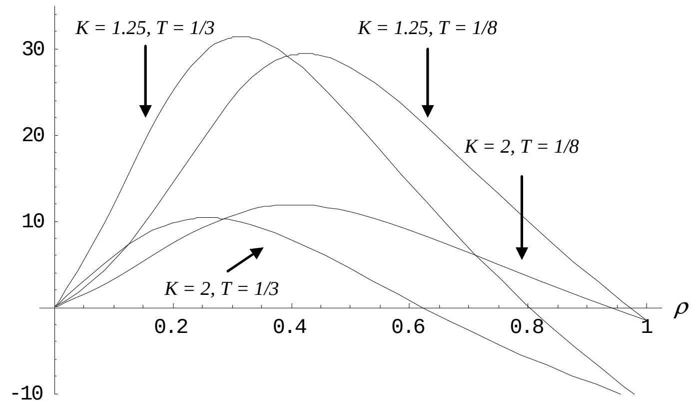


Figure 1: Percent change in sales when moving from not offering reservations to offering reservations. $F(v) = v^2$ for $0 \leq v \leq 1, D = 7/4$, and $\theta = 5$.

better for $\rho$ sufficiently high. This suggest that the gains from offering reservations are not monotone in $\rho$. Figure 1 show that this can, indeed, be the case. The percentage increase in sales peaks at intermediate values of $\rho$ while the location of the peak tends to be at lower value of $\rho$ for higher travel costs. Further, as discussed above, the gain from reservations is higher at when capacity is tight.

### 4.2 Competition

We now suppose there are two restaurants, $A$ and $B$. Travel and denial costs are the same for both firms. Restaurant $j$ has capacity $K_j$ for $j = A, B$. Let $\tilde{K} = K_A + K_B$ and $\alpha = 1 - \beta = K_A/\tilde{K}$. We assume $1 \leq K_A \leq \alpha\theta\bar{F}(T)$ and $1 \leq K_B \leq \beta\theta\bar{F}(T)$. Thus either firm could serve the entire market by itself on a slow night, but together they cannot serve all customers on a busy night even if customers are allocated in proportion to the fraction of industry capacity they control.

The sequence of events changes slightly. Both firms announce their policies. If at least one offers reservations, customers wanting one make their requests simultaneously before learning their valuations. If both offer reservations, customers seek only one reservation. (Although this is largely for simplicity, OpenTable, a reservation service, does not allow users to hold simultaneous reservations.) Once valuations are learnt, reservation holders determine whether to honor their reservation, stay home, or patronize the firm for which they do not hold a reservation. Non-reservation holders wait for their realized valuations and determine whether to walk in and if so to which firm (*i.e.*, they can only patronize $A$ or $B$). If no one offers reservations, customers choose whether to walk in (and to which firm) after learning their valuations.

It remains to specify the joint distribution of customer values. We focus on two special cases: identical values and less than perfectly correlated values.

#### 4.2.1 Identical values

In this model, each customer draws a valuation $V$ from distribution $F$ and her value for dining at $A$ is the same as her value for $B$, *i.e.*, $V_A = V_B$. We begin with the walk-in equilibrium. Consider a customer with realized value $V > T$. Since she values dining at the two restaurants the same, she goes to the firm offering a better chance of getting a seat. In equilibrium, the two restaurants must offer the same probability of successfully securing a seat. Suppose customers with valuations greater than some $\tilde{V}$ are active and go to firm $A$ with probability $\alpha$ and to $B$ with probability $\beta$. The chance of getting a seat at either firm is

$$1 - \rho + \rho \frac{\tilde{K}}{\theta \bar{F}\left(\tilde{V}\right)}.$$

It remains to find $\tilde{V}$. Since the chance of a walk-in customer getting a seat is the same as when there is a monopolist restaurant with capacity $\tilde{K}$, the cutoff can be found from (2). We take this monopoly setting as the base case against which to compare the competitive outcome. (This equilibrium is not unique; any other would have restaurant specific cutoff values.)

Now consider $A$'s problem. Suppose $B$ does not offer reservations. If $A$ also forgoes reservations, its sales are $(1-\rho)\,\alpha\bar{F}\left(\tilde{V}\right) + \rho K_A$. If it offers reservations, $A$ gets the entire market in the low demand state. Expected sales are $(1-\rho)\,\bar{F}\,(T) + \rho K_A \bar{F}\,(T)$, and reservations are preferable if

$$(1-\rho)\left(\bar{F}\,(T) - \alpha\bar{F}\left(\tilde{V}\right)\right) \geq \rho\alpha\tilde{K}F\,(T)\,. \tag{5}$$

Comparing (5) with (3), it is easy to see that if $B$ does not offer reservations, $A$ would find reservations attractive over a larger range of parameters than a monopolist with capacity $\tilde{K}$.

If $B$ offers reservations and $A$ does not, $A$'s sales are $\rho K_A$. If both offer reservations, they must offer customers the same chance of getting a reservation in equilibrium. $A$'s sales are consequently $(1-\rho)\,\alpha\bar{F}\,(T)+$

$\rho K_A \bar{F}(T)$ , and $A$ offers reservations if

$$(1-\rho)\,\alpha \bar{F}(T) \geq \rho\alpha\tilde{K}F(T)\,. \tag{6}$$

The reaction function for $B$ can be found by substituting $\beta$ for $\alpha$ in (5) and (6).

**Proposition 5** *Suppose that firm $A$ is the smaller firm so that $\alpha \leq 1/2$.*

1. The equilibrium has the following structure:
   (a) If $\rho\tilde{K}F(T) \geq \frac{1-\rho}{\alpha}\left(\bar{F}(T) - \alpha\bar{F}\left(\tilde{V}\right)\right)$, neither restaurant offers reservations.
   (b) If $\frac{1-\rho}{\alpha}\left(\bar{F}(T) - \alpha\bar{F}\left(\tilde{V}\right)\right) \geq \rho\tilde{K}F(T) > \max\left\{\frac{1-\rho}{\beta}\left(\bar{F}(T) - \beta\bar{F}\left(\tilde{V}\right)\right), (1-\rho)\,\bar{F}(T)\right\}$, $A$ offers reservations and $B$ does not.
   (c) If $\frac{1-\rho}{\beta}\left(\bar{F}(T) - \beta\bar{F}\left(\tilde{V}\right)\right) \geq \rho\tilde{K}F(T) > (1-\rho)\,\bar{F}(T)$, only one restaurant offers reservations, and it can be $A$ or $B$.
   (d) If $(1-\rho)\,\bar{F}(T) > \rho\tilde{K}F(T)$, both restaurants offer reservations.
2. If a monopolist with capacity $\tilde{K}$ would offer reservations, both restaurants to offer reservations.
3. If $\frac{(1-\rho)\left(1-\tilde{K}/\theta\right)}{1-\rho+\rho\alpha\tilde{K}} \leq F(T)$, reservations are never offered.

Competition makes reservations viable over a larger range of market parameters. This is driven by the smaller firm. The first firm to offer reservations enjoys a windfall in a soft market if its competitor does not match its policy. This slow-night windfall is larger for the small firm. Further, the smaller firm pays a smaller no-show penalty in the high-demand state. Indeed, the no-show penalty for the larger firm may be sufficiently big that it would rather forego any low demand sales than lose sales in the high demand state.

Figure 2 provides an example, showing the prevailing equilibrium for combinations of industry capacity and $A$'s share of capacity. For the given parameters, a monopolist would only offer reservations if $\tilde{K} \leq 1.61$ but both firms offers reservations for $\tilde{K} \leq 3$ (region $\Omega_1$ in the figure). In region $\Omega_2$, only one firm will offer reservations and it can be $A$ or $B$. This is the only region in which there are multiple equilibria. As the split of capacity becomes more skewed, we move from $\Omega_2$ into $\Omega_3$. Here only $A$ offers reservations as the larger firm now finds it unprofitable to match offering reservations. If capacity expands, we move to $\Omega_4$. In this region, ample capacity means that walk-in business is reasonable on slow nights and reservations lead to many no shows on busy nights. Thus, no one offers reservations.

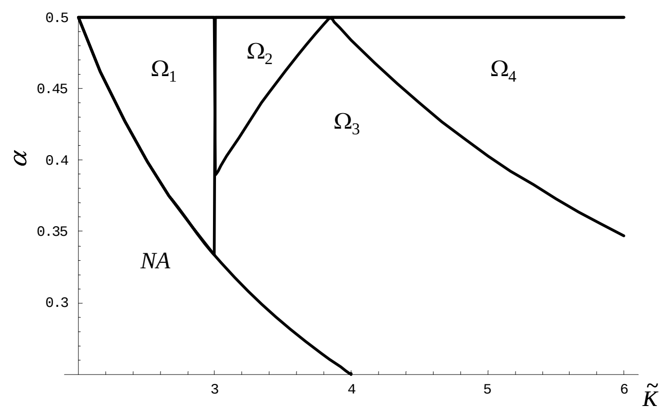


Figure 2: Equilibrium outcome with $F(V) = V$ for $V \in (0, 1)$ and parameters $\rho = 0.4$, $T = 1/3$, $D = 0.9$, and $\theta = 12$. Our analysis is not valid in region *NA* since $\alpha < 1/\tilde{K}$.

To generalize these results to $N \geq 2$ firms, we consider two cases. In the first, the restaurants are "lumpy." Each has capacity greater than one and thus can serve the entire market on a slow night. In the second case, the firms are arbitrarily small. Any one firm's capacity is inadequate for serving the entire market on a slow night, and the amount of industry capacity available via reservations $\tilde{K}_R$ can take on any value. In both settings, we assume that in any walk-in equilibrium customers first employ a common cutoff value and then split in proportion to firm capacities; the cutoff remains $\tilde{V}$ as discussed above.

**Proposition 6** *Suppose that there are $N \geq 2$ firms in the market and that $\theta\bar{F}(T) > \tilde{K} > 1$.*

1. Suppose the firms each have capacity $\tilde{K}/N > 1$. The equilibrium has the following structure:
   (a) If $\tilde{K} > N\frac{1-\rho}{\rho}\left(\frac{\bar{F}(T)}{F(T)} - \frac{\bar{F}(\tilde{V})}{NF(T)}\right)$, no firm offers reservations.
   (b) If $\frac{N}{R}\frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)} \geq \tilde{K} > \frac{N}{R+1}\frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)}$ for $R = 1, ..., N-1$, then $R$ firms offer reservations.
   (c) If $\frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)} \geq \tilde{K}$, all $N$ firms offer reservations.
2. Suppose the firms are arbitrarily small and $F(T) \leq (1-\rho)\left(1 - 1/\tilde{K}\right)$. In equilibrium:

$$\tilde{K}_R = \frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)}. \tag{7}$$

When restaurants are sizeable, the range of industry capacities in which everyone offers reservations is independent of the number firms in the market but the region in which no one offers reservations is decreasing in the number of restaurants. Thus, splitting market capacity more finely makes having at least one firm offer reservations more likely. The intuition is similar to the duopoly case. The less capacity each firm has, the less is lost to no shows in the high demand state. However, unless market capacity is severely limited, not every restaurant offers reservations. Indeed, the amount of capacity available via reservations $R\tilde{K}/N$ will be very close to $\frac{1-\rho}{\rho}\frac{\bar{F}(T)}{F(T)}$ when $N$ is large.

When firms are small, this holds exactly. A market with many small restaurants will consequently never have all firms following the same policy. Instead, some will offer reservations and have sales levels that do not vary with the realized number of customers. Those that do not offer reservations will make do with boom-and-bust sales. On busy nights, they are turning customers away; on slow nights they are empty. Any given restaurateur, however, is indifferent between maintaining her current policy and changing it.

#### 4.2.2 Less than perfectly correlated values

We now consider the possibility that customers may *ex post* have different opinions about the firms. To facilitate the analysis, we impose a simplified, binary value distribution. In the monopoly case, a given customer's valuation for dining out $V$ equals $\nu > T$ with probability $1 - \tau$ and zero otherwise. For the duopoly case, we consider the following joint valuation distribution for $0 \leq \gamma \leq 1/2$:

| $\{V_A, V_B\}$ | Probability |
|---|---|
| $\{0, 0\}$ | $\tau$ |
| $\{\nu, 0\}$ | $\gamma(1-\tau)$ |
| $\{0, \nu\}$ | $\gamma(1-\tau)$ |
| $\{\nu, \nu\}$ | $(1-2\gamma)(1-\tau)$ |

The fraction of customers with a positive value for dining out remains $1 - \tau$, and the probability a customer has no interest in dining out is $\tau$. For active customers (*i.e.*, those for whom $\max\{V_A, V_B\} = \nu$), $\gamma$ manipulates the level of correlation between $V_A$ and $V_B$. $\gamma = 0$ yields the binary analog of the identical values case while $\gamma = 1/2$ results in perfect negative correlation conditional on being active. If $\gamma = \frac{\sqrt{\tau}}{1+\sqrt{\tau}}$, the values are independent. Note that even though *ex post* customers may prefer one firm to the other, all are *ex ante* indifferent between the firms.

We now examine a monopoly with capacity $\tilde{K}$ for $1 < \tilde{K} < (1-\tau)\theta$ and set $D = 0$. The equilibrium probability of getting a seat in a walk-in equilibrium must then be at least $T/\nu$. If the chance of the getting a seat exceeds $T/\nu$, all customers walk in. The following lemma is then immediate.

**Lemma 1** *In equilibrium, an active customer walks in with probability* $\lambda^* = \min\{1, \lambda'\}$ *where*

$$\lambda' = \frac{\tilde{K}}{\theta(1-\tau)} \frac{\rho\nu}{\rho\nu - (\nu - T)}. \tag{8}$$

When valuations are continuous, a cutoff equilibrium appropriately rations seats as those whose valuations barely exceed the travel cost stay home. With binary valuation, all active customers have the same net expected value from walking in. Hence, all will walk in if the net utility from dining out $\nu - T$ is sufficiently high. In particular, everyone will walk in if $1 - \rho > T/\nu$. When the net value to dining out is lower, a mixed strategy equilibrium results with active customers randomizing between walking in and staying home.

Obviously, the monopolist never offers reservations when all active customers walk in. To see when reservations are valuable, note that they will increase slow-night sales from $(1-\tau)\lambda^*$ to $(1-\tau)$ but lower busy-night sales by $\tilde{K}\tau$. This will be worthwhile when $\lambda^*$ is sufficiently small, *i.e.*, if $\lambda^* < 1 - \frac{\rho\tilde{K}\tau}{(1-\rho)(1-\tau)}$.

For the duopoly setting, we assume $K_A = K_B > 1$. (Asymmetric capacities make the smaller firm more likely to offer reservations.) Two factors differentiate this setting from the previous formulation. First, when a restaurant gives out reservations, there are two sources of no shows. A reservation holder may fail to show up because she simply has no positive utility from dining out. Alternatively, she may want to dine out but only at the restaurant for which she does not hold a reservation. The second difference concerns these consumers holding the "wrong" reservation; they may merely stay home or they may walk into the other restaurant. Thus, there is the possibility that a firm both offers reservations and has walk-in traffic.

Suppose no one offers reservations. How many customers should entertain walking into $A$? Clearly, those who value only $A$ should. Suppose, in addition, that half of those who value both restaurants are assigned to $A$. On a busy night, the number of customers considering $A$ will be $\theta(1-\tau)\left(\gamma + \frac{1-2\gamma}{2}\right) = \theta(1-\tau)/2$. Because, all customers have the same chance of getting a seat and have the same value for dining at $A$, they must walk in with the same probability $\lambda^*$ making the chance of getting a seat:

$$1 - \rho + \rho\frac{\tilde{K}/2}{\theta(1-\tau)\lambda^*/2} = 1 - \rho + \rho\frac{\tilde{K}}{\theta(1-\tau)\lambda^*},$$

which is the probability of getting a seat when there is a monopoly with capacity $\tilde{K}$ and active customers randomize with probability $\lambda^*$. Consequently, there exists an equilibrium in which customers who value both restaurants are split evenly between $A$ and $B$ and all active customers walk into their assigned restaurant with probability $\lambda^*$. This equilibrium results in a profit of

$$(1-\rho)\frac{(1-\tau)\lambda^*}{2} + \rho\tilde{K}/2. \tag{9}$$

Now suppose that only $A$ offers reservations. A reservation holder will show up if either she values only $A$ (which occurs with probability $\gamma(1-\tau)$) or if she values either firm (probability $(1-2\gamma)(1-\tau)$). $A$ thus yields $(1-\tau)(1-\gamma)$ customers for every reservation given out. The firm's profit is then:

$$(1-\rho)(1-\tau)(1-\gamma) + \rho\tilde{K}(1-\tau)(1-\gamma)/2. \tag{10}$$

Comparing (9) and (10), we see that $A$ will prefer to offer reservations (assuming $B$ does not) if:

$$\lambda^* < 2(1-\gamma) - \frac{\rho\tilde{K}(\tau+\gamma(1-\tau))}{(1-\rho)(1-\tau)} \tag{11}$$

If $\gamma = 0$, we have results similar to Proposition 5, and reservations are going to be more attractive for the duopolist than the monopolist. In particular, one of the duopolists may offer reservations even if $\lambda^* = 1$ and all customers are already walking in. However, as $\gamma$ increases, the right-hand side of (11) falls; we have the possibility that neither duopolist offers reservations when a monopolist would.

Now consider the profit of $B$ when it does not offer reservations but $A$ does. Customers holding reservations at $A$ who ultimately only value $B$ will all walk in to $B$. Thus, unlike the setting above, a firm not offering reservations sees some traffic on a slow night while still selling out completely on busy nights. The resulting profit for $B$ is $(1-\rho)(1-\tau)\gamma + \rho\tilde{K}/2$.

If $B$ instead matches $A$ and offers reservations, there are two cases depending on the behavior of customers who end up holding reservations at one restaurant but only are interested in eating at the other. Getting a reservation is not independent of demand, allowing reservation holders to update their beliefs about the market. The revised probability that demand is high is

$$\hat{\rho} = \frac{\rho\tilde{K}}{\rho\tilde{K} + (1-\rho)\theta}.$$

A customer holding a reservation for the wrong restaurant bases her decision on whether to try her preferred firm on $\hat{\rho}$. Given that both firms are offering reservations, the only way a walk-in customer can get a seat is if it is a slow night. All customers who prefer the firm for which they do not hold reservations will try walking in to their preferred restaurant if $1-\hat{\rho} \geq T/\nu$. If this holds, a restaurant's sales when both firms offer reservations is $(1-\rho)(1-\tau)/2+\rho\tilde{K}(1-\tau)(1-\gamma)/2$. If $1-\hat{\rho} < T/\nu$, the firms see no walk-in traffic on top of their reservation business and sales are $(1-\rho)(1-\tau)(1-\gamma)/2+\rho\tilde{K}(1-\tau)(1-\gamma)/2$.

**Proposition 7** *Let* $\Lambda = \frac{\tilde{K}}{2}(\tau+\gamma(1-\tau))$ *be the expected number of no shows on a busy night.*

1. Suppose that $1-\hat{\rho} \geq T/\nu$.
   (a) If $\rho\Lambda > (1-\rho)(1-\tau)(1-\gamma-\lambda^*/2)$, neither firm offers reservations.
   (b) If $(1-\rho)(1-\tau)(1-\gamma-\lambda^*/2) \geq \rho\Lambda > (1-\rho)(1-\tau)(1/2-\gamma)$, only one firm offers reservations.
   (c) If $(1-\rho)(1-\tau)(1/2-\gamma) \geq \rho\Lambda$, both firms offer reservations.
2. Suppose that $1-\hat{\rho} < T/\nu$.
   (a) If $\rho\Lambda > (1-\rho)(1-\tau)(1-\gamma-\lambda^*/2)$, neither firm offers reservations.
   (b) If $(1-\rho)(1-\tau)(1-\gamma-\lambda^*/2) \geq \rho\Lambda > (1-\rho)(1-\tau)(1-3\gamma)/2$, only one firm offers reservations.
   (c) If $(1-\rho)(1-\tau)(1-3\gamma)/2 \geq \rho\Lambda$, both firms offer reservations.

The results are illustrated in Figure 3. A monopolist offers reservations if $\tilde{K} \leq 3.5$. For low $\gamma$ and moderate capacity levels, region $D_2$ produces results similar to those above: Both duopolists offer reservations even though a monopolist would not. As industry capacity increases further, we enter $D_1$ and only one firm offers reservation. However, we can also move into $D_1$ by increasing the chance of *ex post* differentiation. For example, at $\tilde{K} = 4$, both firms offer reservations if $\gamma < 0.011$, but only one firm does for $0.011 < \gamma < 0.086$. For higher values of $\gamma$, neither duopolist offers reservations (region $N$).

For regions $M_0$, $M_1$, and $M_2$, a monopolist would offer reservations but both duopolists follow suit only in $M_2$. As $\gamma$ increases, only one restaurant offers reservations in $M_1$, and neither does in $M_0$. Our earlier statements must consequently be tempered. Competition makes reservations more attractive as long as most customers are *ex post* indifferent between the firms. When it is likely that many customers will

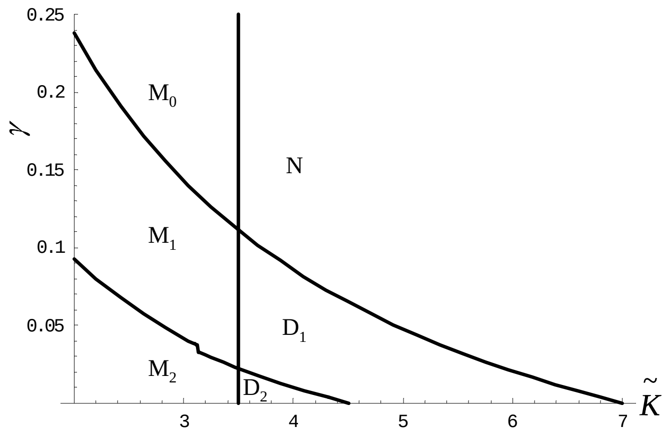


Figure 3: Equilibrium outcome with $\nu = 5/4, T = 1, \rho = 2/3, \tau = 1/10$, and $\theta = 25$.

strictly favor one firm over the other, the attractiveness of reservations falls. Further, we can no longer infer that reservations will be part of a competitive equilibrium just because a monopolist would offer them.

Moving from the first to the second part of the proposition highlights the informational role of reservations. When capacity is very tight, getting a reservation is informative and a reservation holder is inclined to put a high probability on the night being slow. Hence, the reservation holder perceives little risk in abandoning a sure seat to walk into her preferred restaurant. Larger values of $\tilde{K}$ make getting a reservation generally easier and consequently less informative. Matching a competitor's policy of offering reservations becomes less attractive since customers holding reservations at a restaurant at which they have no desire to eat now stay home. The jump in the boundary between $M_1$ and $M_2$ at $\tilde{K} = 3.125$ illustrates this switch. It is this informational role that drives us to use a binary distribution of preferences. If customers drew their values from a continuous distribution, customers holding a reservation at the wrong restaurant would base their decision to walk into the other firm on a cutoff value that varied continuously with capacity.

## 5 A two-period evening

We now return to a fixed market size but split the evening into peak and off-peak periods. Dining off peak lowers any customer's net utility. This is reasonable when the off-peak period is unfashionably early or annoyingly late. We implement the drop in net utility through period-specific travel costs. The peak cost is $T_p \geq 0$ while the off-peak cost is $T_o > T_p$. A customer dining off-peak thus has her net utility reduced by

$T_o - T_p$. We assume $\Omega > T_o$ so dining off peak is feasible for some customers.

Events proceed as before except walk-in customers must choose a period. The choices are mutually exclusive. A walk-in customer unable to get a peak-period seat cannot switch to the off-peak period.

## 5.1 The monopoly problem

We now interpret $K$ as the number of patrons the monopolist restaurant can serve during the peak period and assume $K < \theta \bar{F}(T_p)$. However, the firm has unlimited capacity during the off-peak period. In a walk-in equilibrium, customers with valuations greater than $T_o$ face a choice: Walking in during the peak offers the *possibly* of receiving $V - T_p$ but walking in off peak delivers the *certainty* of receiving $V - T_o$. When $\Omega$ is finite and $K$ is large, no one may choose to walk in off peak. Specifically, if

$$K > \theta \bar{F}(V_\Omega)\left(1 - \frac{T_o - T_p}{\Omega + D}\right), \tag{12}$$

where $V_\Omega$ is found from $\frac{K}{\theta \bar{F}(V_\Omega)} = \frac{D+T_p}{V_\Omega + D}$, all customers with realized values greater than $V_\Omega$ walk in during the peak; those with lower values stay home. When (12) fails, some dine off peak.

**Proposition 8** *Suppose* (12) *fails. The unique equilibrium is defined by* $\underline{V}$ *and* $\overline{V}$ *such that* $T_p < \underline{V} \leq T_o < \overline{V}$. *Customers for whom* $V \leq \underline{V}$ *stay home. Those for whom* $\underline{V} < V \leq \overline{V}$ *walk in during the peak. Those for whom* $V > \overline{V}$ *walk in off peak.* $\underline{V}$ *and* $\overline{V}$ *are found from*

$$\frac{K}{\theta\left(F\left(\overline{V}\right) - F\left(\underline{V}\right)\right)} = \frac{T_p + D}{\underline{V} + D} \tag{13}$$

$$\frac{K}{\theta\left(F\left(\overline{V}\right) - F\left(\underline{V}\right)\right)} = 1 - \frac{T_o - T_p}{\overline{V} + D}. \tag{14}$$

Walking in during the peak is risky and is traded off against two certain options. At low valuations, customers weigh staying home versus walking in during the peak. Customers with extremely high valuations, in contrast, are unwilling to chance not getting a seat and opt to dine off peak.

The restaurant's sales in the walk-in equilibrium are $\Pi_W = K + \theta \bar{F}\left(\overline{V}\right)$. Unlike uncertain market size model of Section 4, walk-in sales may not increase with capacity. Expanding capacity entices more low-value customers to venture in but also results in high-value customers switching from off-peak to peak patronage. When $F(V) = V/\Omega$ for $0 \leq V \leq \Omega$, the equilibrium for $K < \theta - \frac{\theta T_o}{\Omega}$ is given by

$$\underline{V} = \frac{K\Omega T_p + \theta T_o (D + T_p)}{K\Omega + \theta (D + T_p)} \qquad \overline{V} = \frac{K\Omega}{\theta} + T_o.$$

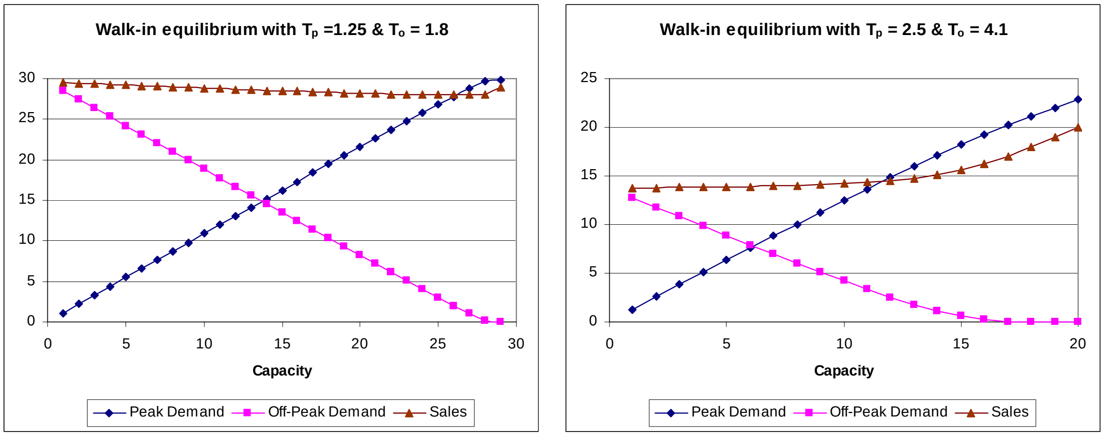


Figure 4: Walk-in equilibrium demand with $D = 3$, $\theta = 30$ and valuations normally distributed with a mean of $4$ and a standard deviation of $1$.

Off-peak sales are $\theta - K - \frac{\theta T_o}{\Omega}$, and total sales are fixed at $\theta - \frac{\theta T_o}{\Omega}$. For other distributions, total sales can be either increasing or decreasing depending on both the distribution and the cost parameters. See Figure 4. Even when sales increase, they may not rise very quickly because of the drop in off-peak sales.

Because the peak is oversubscribed, the firm may want to shift some demand off peak. Reservations accomplish this. We suppose all customers request peak-period reservations. If they are unsuccessful, we may assume they either take an off-peak reservation or decline an off-peak reservation but determine whether or not to walk in off peak after learning their value for dining out. Since the off-peak period is uncapacitated, these are equivalent, leading to sales of $\Pi_R = K\bar{F}(T_p) + (\theta - K)\bar{F}(T_o)$.

**Proposition 9** *Suppose (12) fails. The restaurant prefers offering reservations if*

$$\frac{K}{\theta} \leq \frac{F(\overline{V}) - F(T_o)}{1 - F(T_o) + F(T_p)}.$$

*The restaurant never offers reservations if:*

$$F(T_p) \geq \frac{\theta - K}{K}\bar{F}(T_o). \tag{15}$$

*If (12) holds, (15) is both necessary and sufficient for reservations not to be offered.*

Reservations increase sales by providing information. Those unable to secure a reservation know the peak is full and only consider off-peak service. Stated another way, customers value receiving reservations because they guarantee peak-period seats, but the firm values customers being denied reservations because

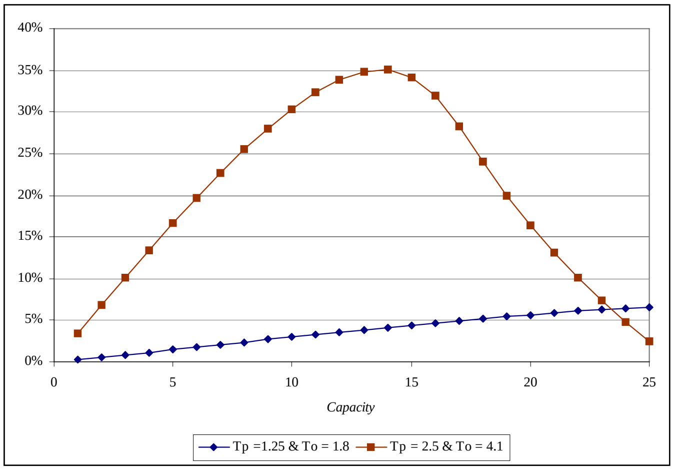


Figure 5: Percent change in sales from offering reservations with $D = 1, \theta = 30$, and valuations normally distributed with a mean of $4$ and a standard deviation of $1$.

this moves demand off peak. Again the price the firm pays for reservation is lower capacity utilization in some settings due to no shows. Before no shows lowered sales on busy nights. Now they reduce peak-period sales. If the no-show rate $F(T_p)$ is sufficiently high, reservation are never offered. In particular, when there are relatively few customers left to serve off peak (*i.e.*, $\theta - K$ is small), reservations are not offered.

There are parallels between the uncertain market size and two-period models, but the role of capacity is different. In the former, reservations are most valued when capacity is tight, implying that small restaurants are more likely to offer reservations. Here, the returns from reservations can be increasing in available capacity. With uniform valuations, reservation sales are $\theta - \frac{\theta T_o}{\Omega} + \frac{K}{\Omega}(T_o - T_p)$. Reservations are always preferred, and the gain from offering them is increasing in $K$. Given that reservation sales are strictly increasing in capacity, it is not surprising that they are valuable when walk-in sales are flat or decreasing. However, they can offer a higher return even when walk-in sales increase with capacity. See Figure 5. The empirical prediction is now that if restaurants are relying on reservations to shift some demand off peak, reservations are more valuable at medium to high capacity levels.

## 5.2 Competition

We now examine competition using the identical values model. $\alpha$ denotes the fraction of industry peak-period capacity $\tilde{K}$ controlled by $A$. $\beta = 1 - \alpha$ is the fraction controlled by $B$. An important issue is how off-peak demand is split. For example, supposing all off-peak demand goes to $A$ when neither firm offers reservations but goes to $B$ when it does impose a bias on the model. We will assume off-peak demand is split in proportion to peak-period capacity. This allows for a simple comparison to the monopoly case.

If no one offers reservations, the restaurants must provide the same chance of a peak-period seat in equilibrium. This can be accomplished by having customers randomize between $A$ and $B$ in proportion to their peak-period capacities and then employ the cutoffs found from (13) and (14) assuming the monopolist has capacity $\tilde{K}$. Let $\Pi_W^M$ denote the profit in the walk-in equilibrium of a monopolist with capacity $\tilde{K}$; the comparable profit for $A$ [$B$] is $\alpha\Pi_W^M$ [$\beta\Pi_W^M$]. If the monopolist earns $\Pi_R^M$ from making her full capacity available via reservations, $A$ and $B$'s profits when both offer reservations are $\alpha\Pi_R^M$ and $\beta\Pi_R^M$, respectively.

Now suppose only $A$ offers reservations. To establish a benchmark, consider a monopolist with capacity $\tilde{K}$ who makes $\alpha\tilde{K}$ seats available via reservations. A non-reservation holding customer whose realized value for dining out $V$ is greater than $T_p$ will walk-in during the peak period if there is a sufficiently high chance of getting a seat. The relevant cutoffs for customers are again found from (13) and (14) but with $(1-\alpha)\tilde{K}$ and $\theta - \alpha\tilde{K}$ replacing $K$ and $\theta$, respectively. Let $\underline{V}_\alpha$ and $\overline{V}_\alpha$ denote the equilibrium cutoff values. Note that the former is increasing in $\alpha$ while the latter decreases. As $\alpha$ goes to one, both go to $T_o$.

Defining $g(\alpha) = \left(\theta - \alpha\tilde{K}\right)\bar{F}\left(\overline{V}_\alpha\right)$, the monopolist's profit is $\Pi^M(\alpha) = \alpha\tilde{K}\bar{F}(T_p) + (1-\alpha)\tilde{K} + g(\alpha)$. In a duopoly, if only $A$ offers reservations, she earns $\Pi_A(\alpha) = \alpha\left(\tilde{K}\bar{F}(T_p) + g(\alpha)\right)$, and $B$ receives $\Pi_B(\alpha) = \beta\left(\tilde{K} + g(\alpha)\right)$. Thus while $A$ incurs all of the cost in offering reservations (*i.e.*, no shows), it captures only part of the benefit. Reducing the number of peak-period seats available to walk-in customers induces a greater fraction of customers to patronize the off-peak period, but some will go to $B$.

**Proposition 10** *Suppose that $g(\alpha)$ is increasing in $\alpha$ and assume $A$ has less capacity than $B$ (i.e., $\alpha \leq \frac{1}{2}$).*

1. If $g(\alpha) - g(0) \geq g(1) - g(\beta)$, the equilibrium has the following structure:
   (a) If $\tilde{K}F(T_p) > g(\beta) - g(0)$, no firm offers reservations.
   (b) If $g(\beta) - g(0) \geq \tilde{K}F(T_p) > g(\alpha) - g(0)$, $B$ offers reservations and $A$ does not.
   (c) If $g(\alpha) - g(0) \geq \tilde{K}F(T_p) > g(1) - g(\beta)$, either $A$ or $B$ offers reservations.
   (d) If $g(1) - g(\beta) \geq \tilde{K}F(T_p)$, both firms offer reservations.
2. If $g(\alpha) - g(0) < g(1) - g(\beta)$, the structure is similar unless $g(1) - g(\beta) > \tilde{K}F(T_p) > g(\beta) - g(0)$ in which case there are two equilibria. In one, both offer reservations; in the other, no one does.

The results here are distinct from those of the uncertain market size model in several ways. First, now it is the larger firm that is more likely to offer reservations. Before the smaller firm found reservations more attractive because it captured a larger windfall in a soft market. Now the bigger restaurant takes the lead because it captures the lion's share of the benefits. Reservations create value by providing information, but a duopolist offering reservations cannot control how customers use that information. The larger firm, however, can be assured of seeing a bigger increase in off-peak business than the small firm.

Next, since $g(\alpha)$ is increasing, a monopolist would make her full capacity available via reservations, but competition may result in no seats being offered via reservations. If $F(V) = V/\Omega$ for $0 \leq V \leq \Omega$,

$$g(\alpha) = \theta\left(1 - \frac{T_o}{\Omega}\right) - K + \frac{\alpha \tilde{K} T_o}{\Omega},$$

and neither $A$ or $B$ offers reservations if $T_p > \beta T_o$. Only $B$ offers reservations if $\beta T_o \geq T_p > \alpha T_o$. The monopoly outcome is only replicated if $\alpha T_o \geq T_p$.

Assuming that $g(\alpha)$ is increasing is obviously restrictive. The condition is likely to fail when $\tilde{K}/\theta$ is large. When valuations are normally distributed, $g(\alpha)$ can be either increasing or U-shaped. In the latter case, the monopolist favors making all or none of her capacity available. Numerical examples show that the equilibrium structure still holds with the larger firm taking the lead in offering reservations. Also, there are examples in which having everyone or no one offer reservations are both equilibria.

Finally, increasing the number of restaurants in the uncertain market size model made it more likely that the equilibrium would involve at least some firms offering reservations. With a two-part evening, small firms find reservations less attractive. Intuitively, this suggests that splitting industry capacity more finely should make reservations less prevalent. The following proposition shows that for uniformly distributed values this is so. Indeed, given enough restaurants in the market, no one offers reservations.

**Proposition 11** *Suppose $F(V) = V/\Omega$ for $0 \leq V \leq \Omega$ and that there are $N$ firms in the market each with capacity $\tilde{K}/N$. If $N > T_o/T_p$, no one offers reservations. Otherwise all $N$ firms offer reservations.*

## 6 Discussion and extensions

We have examined whether a restaurant should offer reservations. Reservations are never employed in a market of a known size with a single sales period: Either the firm is never sold out so customers know they may simply walk in or it is always at capacity so reservations result in no shows and lower sales. Reservations can be profitable if the market size is uncertain or the evening is divided into peak and off-peak periods. With uncertain demand, slow nights are exceptionally slow when reservations are not offered because customers are uniformed of the market state and fear being denied a seat. Reservations address this problem by guaranteeing potential diners seats, generating more demand in a slack market. The downside is sales lost to no shows in a large market. Thus, reservations are valuable to the restaurant when business is slow but costly when the joint is hopping. In contrast, customers would not value reservations if they knew the market were small but would prize them highly on a busy night. Trading off lost sales on busy nights for increased sales on slow nights carries over to competitive environments. With positively correlated values, competition expands the range of parameters over which reservations are offered. This is particularly true for a smaller firm, and a market with many small firms almost certainly has some firms offering reservations. However, reservation become less attractive when customers are likely to favor just one restaurant.

When the evening is split in two, reservations move customers from an oversubscribed peak period to available seats off peak. In this setting reservations work by providing information. Those unable to get a reservation learn that they must come off peak in order to be served. Thus while a customer values the certainty a reservation provides, the firm values customers being denied reservations because this shifts demand. In contrast to the uncertain market size variant of the model, reservations here are more valuable to moderate size firms as opposed to small firms. Further, competition in this setting may reduce the attractiveness of offering reservations. In a competitive environment, larger firms are more likely to offer reservations, and no one may offer reservations in a market with many small firms.

All models have limitations, and ours is no exception. There are assumptions that are essential to our results and several factors that are ignored. We now discuss a number of these.

- **Other benefits of reservations.** We have purposefully suppressed reservations' operational benefits to emphasize their impact on consumers. To the extent that the firm can use reservations to manage the flow of work or forecast demand, we have systematically underestimated their value.

- **No show mitigation.** Key to our model is that no shows are costly. The firm would clearly benefit from reducing them. One possibility may be altering how customers make their reservations. The reservation service OpenTable tracks customers and closes accounts of users who fail to be keep four reservations in a calendar year. The no-show rate for reservations made via OpenTable is just 4%. However, this adds cost; OpenTable charges a restaurant a dollar per reservation (Webb Pressler, 2003). Alternatively the firm could call customers to confirm reservations. Chicago-based Tru pre-emptively cancels any reservation they are unable to confirm by the scheduled date. While this may weed out no shows, it adds to the cost of offering reservations.

Finally, the restaurant could impose a no-show fee. Another Chicago restaurant Alinea charges $75 per party member if a reservation is not kept. Such a fee, say, $P$ would induce a customer to keep her reservations if $V - T \geq -P$, reducing the no-show rate to $F(T - P)$ . For $P \geq T$, no shows are eliminated. Such a fee makes reservations more attractive since it both lowers no shows and offers some compensation for those that do occur. How large a fee to charge depends on the travel cost $T$ and the firm's margin per customer $\pi$. If $\pi \geq T$, it is optimal to eliminate no shows. Otherwise, we have $T > P > \pi$, and the restaurant profits when reservation holders fail to show.

Any of these changes would lower the cost of reservations and make the more attractive. They would not eliminate the impact on consumer behavior that we have identified.

- **Overbooking.** Overbooking does not reduce the no-show rate but provide hedge against unreliable customers. Giving out $K/\bar{F}(T)$ reservations results in exactly $K$ customers showing up. Unfortunately, overbooking is not so simple in practice. Our atomistic customers allow the firm to give out reservations above its capacity without ever worrying about turning away customers. In reality, overbooking means risking having more customers than can be accommodated. Such logic has lead industry experts to advise that "[t]he potential damage from turning away guests who have reservations is too great. It is better to book the dining room to capacity and replace no-shows with last-minute calls or spur-of-the moment arrivals." (The Culinary Institute of America, 2001, page 58.)

- **Limiting reservations.** Our analysis assumes a restaurant offering reservations makes its entire capacity available. This is clearly suboptimal when demand is uncertain. Giving out more reservations than customers in the market on a slow night does nothing to raise sales when demand is soft but increases no shows on a busy night. It would be better to cap the number of reservations given out at one or less; limiting reservations increases the range of parameters over which the firm is willing to offer reservations. Also, limiting reservations provides information to customers since those denied a seat learn that it is in fact a busy night. Thus, although the firm uses all of its unreserved capacity on a busy night, it turns away fewer walk-in customers than when it does not offer reservations.

- **Segmentation.** A restaurant may use reservations to attract customers with different characteristics. For example, one might suppose the existence of a segment that will only dine out if they have a reservation. More subtly, in our model, segments could differ in their travel or denial costs. If a restaurant does not offer reservations, high-cost customers will be under-represented among its customers. Reservations move the firm's sales mix closer to the underlying market mix. Tweaking the sale mix would be worthwhile if the segments differ in both their costs and spending proclivities. If high-cost customers are more likely to run up large tabs, reservations would be warranted if the gain in the average bill is sufficient to compensate for the resulting no shows. Note that the motivations for reservations we have presented increase unit sales and do not depend on spending differences.

- **Parties of multiple sizes.** We have assumed that all parties consume the same capacity. In reality, restaurants serve parties of varying sizes. Large parties generally take longer to serve and may be difficult to accommodate if small tables must be moved together. Reservations (even if only for large parties) may then allow the restaurant to make more efficient table assignments (Fischer, 2005).

- **Selling reservations.** Restaurants generally do not charge for reservations. This may well be due to historical considerations; without broad credit card penetration, selling reservations is simply impractical. However, suppose the firm charges $\phi$ per reservation. This is an additional charge and not a pre-payment of the meal. It would seem that a monopolist could charge $\mathbb{V}$, a customer's expected value from holding a reservation. One limit on doing so is that for $\phi$ sufficiently high, multiple equilibria exist. In one equilibrium, customers randomize between requesting a reservation and waiting to learn their valuation. A high fee may then cause reservation demand to collapse.

Competition will also limit reservation charges. In modeling competition, we assumed all customers requested reservations simultaneously. If instead we allow sequential arrivals, a customer may first request a reservation from $A$ and then $B$ if she was unsuccessful. Consider the setting with market-size uncertainty and assume that customers are *a priori* indifferent between the firms. Suppose that without reservation charges both firms would want to offer reservations. If $\phi_A < \phi_B$, arriving customers would first ask $A$ for a reservation and then ask $B$, allowing $A$ to take the entire market on a slow night while $B$ would still have no shows on a busy night. $B$ would thus have an incentive to undercut $\phi_A$; Bertrand-like competition would result until $\phi_A = \phi_B = 0$.

There are a number of ways to extend this work. In particular, one could empirically evaluate the model variants since they lead to distinct predictions. For example, if reservations are driven by market-size uncertainty, they should be more common at low values of capacity. If shifting demand between peak and off-peak times is the principal driver, reservations should be more common at higher values of capacity. The impact of competition also varies across the variants we have developed. In addition, higher quality restaurants (in the sense of a larger valuation distribution) are more likely to offer reservations.

# Appendix: Proofs

*Proof of Proposition 1:* If only those with values greater than $V'$ walk in, the chance an arriving customer gets a seat is $K/\theta\bar{F}(V')$ and (1) follows. For uniqueness, note that the left-hand side of (1) is decreasing in $V'$ while the right-hand side is increasing. $\square$

*Proof of Proposition 3:* Reservations are preferable if they lead to higher sales:

$$(1-\rho)\bar{F}(T)+\rho K\bar{F}(T)\geq(1-\rho)\bar{F}(V^*)+\rho K\Longrightarrow(1-\rho)\left[\bar{F}(T)-\bar{F}(V^*)\right]\geq\rho K\left[1-\bar{F}(T)\right].$$

For the bound on $F(T)$, $\bar{F}(V^*)\geq\frac{K}{\theta}$ so $(1-\rho)\frac{K}{\theta}+\rho K$ underestimates walk-in sales. Reservations are never offered if $\bar{F}(T)(1-\rho+\rho K)\leq(1-\rho)\frac{K}{\theta}+\rho K$,which give the bound. $\square$

*Proof of Proposition 4:* For the first part, as $K$ falls, $V^*$ increase. Hence, the left-hand side of (3) increases and the right-hand side falls as $K$ decreases. Results for $\theta$ and $D$ follow similarly. Next let $V_j^*$ be the equilibrium cutoff for $F_j$ for $j=0,1$. If $F_1$ is larger than $F_0$ in the reversed hazard rate order, we have that $F_1(t)\leq F_0(t)$ for all $t$ and $F_0(t)/F_1(t)$ is decreasing in $t$ (Shaked and Shanthikumar, 1994). The former implies that $V_1^*\geq V_0^*$; the latter give $F_0(T)/F_1(T)\geq F_0(V_0^*)/F_1(V_0^*)$. We then have

$$\frac{F_0(V_0^*)}{F_0(T)}\leq\frac{F_1(V_0^*)}{F_1(T)}\leq\frac{F_1(V_1^*)}{F_1(T)}.$$

For the travel cost, consider $T_1<T_0$ and define $\psi_j=V_j^*/T_{j.}$ We first show that $\psi_1\geq\psi_0$. Since $T_0>T_1$, it must be that $V_0^*>V_1^*$ and $\frac{T_0+D}{V_0^*+D}>\frac{T_1+D}{V_1^*+D}$ because a higher cutoff value means fewer customers walking in which implies a higher probability of getting a seat. If $\psi_1<\psi_0$, it must be

$$\frac{T_1+D}{\psi_1T_1+D}>\frac{T_1+D}{\psi_0T_1+D}>\frac{T_0+D}{\psi_0T_0+D},$$

which implies $\frac{T_1+D}{V_1'+D}>\frac{T_0+D}{V_0'+D}$. Given that $\psi_1\geq\psi_0$. We then have

$$F(V_0^*)/F(T_0)=F(\psi_0T_0)/F(T_0)\leq F(\psi_0T_1)/F(T_1)\leq F(\psi_1T_1)/F(T_1)=F(V_1^*)/F(T_1).\ \square$$

*Proof of Proposition 5:* First, because $\bar{F}(T)-\alpha\bar{F}\left(\tilde{V}\right)\geq\alpha\bar{F}(T)$, (6) implies (5). Next, the conditions for $B$ to offer reservations are

$$(1-\rho)\left(\bar{F}(T)-\beta\bar{F}\left(\tilde{V}\right)\right)\quad\geq\quad\rho\beta\tilde{K}F(T)\tag{16}$$

$$(1-\rho)\beta\bar{F}(T)\quad\geq\quad\rho\beta\tilde{K}F(T).\tag{17}$$

Comparing (6) and (16), it is evident that the former always holds if the latter does because $\beta \geq \alpha$. Comparing (5) and (17), one sees that they are scalings of each other. Hence, the firms would have same best responses to a competitor offering reservations.

Now suppose (5) fails (as in part 1a of the proposition), it must also be the case that $B$ does not want to be the first to offer reservations. Also, $A$ (and hence $B$) would not respond to a competitor offering reservations by also offering reservations. Thus, no one offering reservations is the only outcome. If, however, (5) holds while (6) and (16) fail (part 1b), $A$ is willing to offer reservations if $B$ does not, and $B$ never is willing to offer reservations. If (16) holds but (6) does not, both firms are willing to offer reservations if the other does not do likewise. Since (6) fails, neither will match the others offering reservations. Hence, one firm will offer reservations and it can be either $A$ or $B$. If (5) and (6) both hold, offering reservations is a dominant strategy for $A$ and $B$'s best response is to offer reservations (part 1d).

For part 2, (3) immediately implies (6) and both offer reservations. The proof of the final is similar to the corresponding part of Proposition 3. $\square$

*Proof of Proposition 6:* With lumpy firms and no one offering reservations, a firm earns $\pi_0 = (1-\rho)\frac{\bar{F}(\tilde{V})}{N} + \rho\frac{\tilde{K}}{N}$. If one firm offers reservations, it makes $\pi_1 = (1-\rho)\bar{F}(T) + \rho\frac{\tilde{K}}{N}\bar{F}(T)$. $\pi_0 > \pi_1$ requires $\tilde{K} > N\frac{1-\rho}{\rho}\left(\frac{\bar{F}(T)}{F(T)} - \frac{\bar{F}(\tilde{V})}{NF(T)}\right)$. If $R \geq 1$ firms offer reservations, a firm offering reservations earns $\pi_R = \frac{(1-\rho)\bar{F}(T)}{R} + \rho\frac{\tilde{K}}{N}\bar{F}(T)$. A firm without reservations makes $\pi_{NoR} = \rho\frac{\tilde{K}}{N}$. $\pi_R$ decreases monotonically with $R$ while $\pi_{NoR}$ is independent of $R$. For $R$ firms to offer reservations in equilibrium, it must be the case that $\pi_R \geq \pi_{NoR} \geq \pi_{R+1}$, which leads to (1b). For (1c), one compares $\pi_N$ and $\pi_{NoR}$.

For the case of small restaurants, we first show that given $F(T) \leq (1-\rho)\left(1 - 1/\tilde{K}\right)$ the equilibrium must involve reservations. Suppose that no firm offered reservations in equilibrium. Letting $k$ be the capacity of a representative firm, this would require:

$$(1-\rho)\frac{k}{\tilde{K}}\bar{F}\left(\tilde{V}\right) + \rho k \geq k(1-\rho)\bar{F}(T) + \rho k\bar{F}(T) = k\bar{F}(T),$$

which implies $F(T) \geq (1-\rho)\left(1 - \bar{F}\left(\tilde{V}\right)/\tilde{K}\right)$ but $(1-\rho)\left(1 - \bar{F}\left(\tilde{V}\right)/\tilde{K}\right) > (1-\rho)\left(1 - 1/\tilde{K}\right)$. Next, we show that $\tilde{K}_R$ cannot be less than 1. For $1 > \tilde{K}_R > 0$, the profit of a firm offering reservations is $k\bar{F}(T)$ and is constant in $\tilde{K}_R$. The profit of a firm not offering reservations is $\pi\left(\tilde{K}_R\right) = (1-\rho)\left(1-\tilde{K}_R\right)\frac{k}{\tilde{K}-\tilde{K}_R}\bar{F}(V_R)+\rho k$, where $V_R$ is the equilibrium appropriate cutoff value for those customers who were not able to get a reservation. One can easily show that $\pi'\left(\tilde{K}_R\right) < 0$. Hence, as more firms offer reservations but $\tilde{K}_R$ remains less than one, those not offering reservations have an increasing gain from offering reservations. Consequently, $\tilde{K}_R < 1$ cannot be an equilibrium. In equilibrium, the two policies must offer the same returns, which leads to (7). □

*Proof of Proposition 7:* If one restaurant offers reservations and the other firm does not, the one offering reservations has an expected increase in sales of $\Delta_a = 1-\tau-(1-\tau)\gamma-(1-\tau)\lambda^*/2 = (1-\tau)(1-\gamma-\lambda^*/2)$. $(1-\rho)\Delta_a < \rho\Lambda$ implies the slow night gain does not exceed busy night losses. If the reverse holds, at least one firm will offer reservations. Suppose the other firm does not offer reservations; its slow-night sales are $(1-\tau)\gamma$. If it offers reservations and $1-\hat{\rho} \geq [<]T/V$, its slow-night sales are $1/2(1-\tau)[1/2(1-\tau)(1-\gamma)]$. The gain in slow-night sales is then $\Delta_{1b} = (1-\tau)(1/2-\gamma)$ if $1-\hat{\rho} \geq T/V$ and $\Delta_{2b} = (1-\tau)(1-3\gamma)/2$ otherwise. Since $\lambda^* \leq 1$, it is straightforward to show that $\Delta_a \geq \Delta_{1b} \geq \Delta_{2b}$. Hence, when $\rho\Lambda$ is greater than the relevant value of $(1-\rho)\Delta_{jb}$, the second firm does not offer reservations. If $\rho\Lambda \leq (1-\rho)\Delta_{jb}$, both firms offer reservations. □

*Proof of Proposition 8:* First, if a customer at $\underline{V}$ is indifferent between walking in during the peak period and staying home, anyone with $V \leq \underline{V}$ must stay home. Similarly, if a valuation $\overline{V}$ implies indifference between walking in during the peak and walking in off-peak, anyone with $V \geq \overline{V}$ must come off peak. $\frac{K}{\theta\left(F(\overline{V})-F(\underline{V})\right)}$ is then the probability of getting a seat in the peak period. (13) and (14) follow from finding the indifferent customers. We now show uniqueness. There cannot be another equilibrium that offers the same probability of getting a peak period seat. If there were, there would be a customer who would be walking in during the peak period in one equilibrium but not in the other despite having the same utility for coming in the peak period in both. Now suppose there are two equilibria $\left(\underline{V}_1, \overline{V}_1\right)$ and $\left(\underline{V}_2, \overline{V}_2\right)$ that offer different chances of getting peak period seats. Suppose $\left(\underline{V}_1, \overline{V}_1\right)$ offers customers a higher probability of getting a seat. It must then be the case $\underline{V}_1 < \underline{V}_2$ and $\overline{V}_1 > \overline{V}_2$. However, this would give a better chance of getting a seat under the second equilibrium. Finally, $T_p < \underline{V}$ and $T_o < \overline{V}$ are obvious. A customer at $\underline{V}$ has zero net utility from walking in during the peak. If $\underline{V}$ were greater than $T_o$, she would have a positive utility from dining off peak. Hence, $\underline{V} \leq T_o$. $\square$

*Proof of Proposition 9:* For the first part, compare $\Pi_W$ and $\Pi_R$. For the second part, reservations will never be offered if they result in sales less than $K$ (which equals sales when (12) holds), *i.e.*, if

$$K \geq K\bar{F}(T_p) + (\theta - K)\bar{F}(T_o) \Longrightarrow KF(T_p) \geq (\theta - K)\bar{F}(T_o). \square$$

*Proof of Proposition 10:* $B$ prefers being the only firm offering reservations to having no one offer reservations if $\tilde{K}\bar{F}(T_p) + g(\beta) > \tilde{K} + g(\beta)$. Since $g(\alpha) \leq g(\beta)$, if $B$ is unwilling to be the first to offer reservations, $A$ will similarly prefer to not offer reservations. If $B$ [$A$] offers reservations, $A$ prefers not to offer reservations if $\tilde{K}\bar{F}(T_p) + g(1) < \tilde{K} + g(\beta)$ [$\tilde{K}\bar{F}(T_p) + g(1) < \tilde{K} + g(\alpha)$]. Since $g(\beta) \geq g(\alpha)$, if $A$ prefers to match $B$ in offering reservations as opposed to being the only firm not offering reservation, $B$ also prefers that both offer reservations. The results for $g(\alpha) - g(0) < g(1) - g(\beta)$ are similar. $\square$

*Proof of Proposition 11:* Suppose there are $N$ firms of which $j$ offer reservations. A firm that does not offer reservations has a profit of $\pi_{NoR}(j|N) = \frac{\theta}{N} + \frac{T_o\left(N\theta - j\tilde{K}\right)}{N^2\Omega}$. A firm offering reservations earns $\pi_R(j,R) = \frac{\theta}{N}\left(1 - \frac{T_o}{\Omega}\right) + \frac{\tilde{K}(jT_o - NT_p)}{N^2\Omega}$. If a firm moves from not offering reservations to offering, the change in its profit is $\pi_R(j+1,R) - \pi_{NoR}(j|N) = \frac{\tilde{K}(T_o - NT_p)}{N^2\Omega}$, which is positive if $N \leq T_o/T_p$. Note that the change in profit is independent of $j$. Hence, if $N \leq T_o/T_p$, all firms offer reservations. Otherwise, no one does. $\square$

# References


Arthur, W. B. 1994. Inductive Reasoning and Bounded Rationality. *American Economic Review*. **84** 406 - 411.

Bertsimas, D. and R. Shioda. 2003. Restaurant Revenue Management. *Operations Research*. **51** 472 - 486.

Courty, P. 2000. An Economic Guide to Ticket Pricing in the Entertainment Industry. *Louvain Economic Review*. **66** 167 - 192.

The Culinary Institute of America. 2001. *Remarkable Service*, Hoboken, NJ: John Wiley & Sons.

Dana, J. D. 1998. Advance-Purchase Discounts and Price Discrimination is Competitive Markets. *Journal of Political Economy*. **106** 395 - 422.

Dana, J. D. and N. C. Petruzzi. 2001. Note: The Newsvendor Model with Endogenous Demand. *Management Science*. **47** 1488-1497.

DeGarba, P. 1995. Buying Frenzies and Seller-Induced Excess Demand. *RAND Journal of Economics*. **26** 331 - 342.

Fischer, J. W. 2005. *At Service Your Service*, Hoboken, NJ: John Wiley & Sons.

Gale. I. L. and T. J. Holmes. 1993. Advance-Purchase Discounts and Monopoly Allocation of Capacity. *American Economic Review*. **83** 135 - 146.

Lariviere, M. A. and J. A. Van Mieghem. 2004. Strategically Seeking Service: How Competition Can Generate Poisson Arrivals. *Manufacturing & Service Operations Management*. **6** 23 - 40.

Martin, M. 2001. Side Dish. *Riverfront Time*. May 2.

Png, I. L. 1989. Reservations: Customer Insurance in the Marketing of Capacity. *Marketing Science*. **8** 248 - 264.

Robinson, L. W. and R. R. Chen. 2003. Scheduling Doctors' Appointments: Optimal and Empirically-Based Heuristic Policies. *IIE Transactions*, **35** 295 - 307.

Savin, S. 2006. Managing Patient Appointments in Primary Care in Randolph W. Hall, editor, *Patient Flow: Reducing Delay in Healthcare Delivery*, New York, NY: Springer.

Shaked, M. and J. G. Shanthikumar. 1994. *Stochastic Orders and Their Applications*, San Diego, CA: Academic Press.

Shugan, S. 2002. Service Marketing and Management: Capacity as a Strategic Marketing Variable in Barton A. Weitz and Robin Wensley, editors, *The Handbook of Marketing*, Thousand Oaks, CA: Sage Publications.

Shugan, S., and J. Xie. 2005. Advance-Selling as a Competitive Marketing Tool. *International Journal of*

*Research in Marketing*. 22, 351-373.

Tang, C. S., K. Rajaram, A. Alptekinoglu, and J. Ou, The Benefits of Advance Booking Discount Programs: Model and Analysis. *Management Science*. **50** 465-478.

Webb Pressler, M. 2003. Wonder Why You're Simmering? Turnover, Atmosphere Shape Policies on Making Diners Wait. *The Washington Post*. July 27. F5.

Xie J., and S. Shugan. 2001. Electronic Tickets, Smart Cards, and Online Payments: When and How to Advance Sell. *Marketing Science*. **20** 219 - 243.