To Offshore or Not To Offshore: Sourcing and Location of Commonality in Multiplant Networks

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Abstract

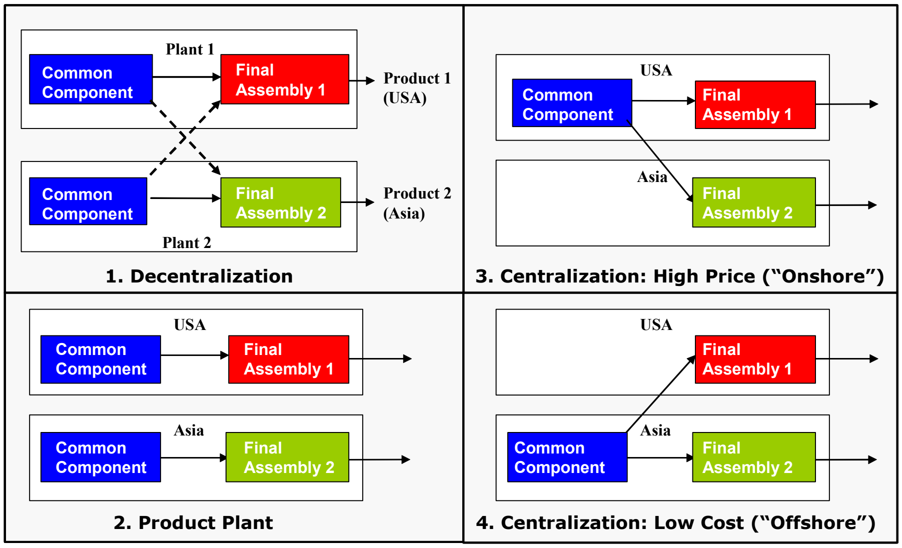
Moving production facilities to low-wage countries provides an opportunity for cost reduction, but comes with disadvantages of increased logistic costs and foreign trade barriers. This paper examines the offshoring decision from a network capacity investment perspective. We analyze a firm that manufactures two products to serve two geographically separated markets using a common component and two other product-specific components. The common part can be transported between the two markets that have different demand and financial characteristics. Two strategic network design questions arise naturally in this context: (1) Should the common part be produced centrally or in two local facilities? (2) If a centralization strategy is adopted for the common component, which market should the facility be located in? We present a transportation cost threshold that captures costs, revenues, and demand risks, and below which centralization is optimal. The optimal location of commonality crucially depends on the relative magnitude of price and manufacturing cost differentials but also on demand uncertainty. Incorporating scale economies further enlarges the centralization's optimality region. Finally, we translate our results into managerial insights for assessing the value of offshoring through direct capacity investment.

1 Introduction

Multinational manufacturing firms often have production facilities around the world that serve each major market. Moving production facilities to low-wage countries provides an opportunity for cost reduction. However, the benefit of centralized production at low-wage countries is associated with disadvantages of increased logistics costs and foreign trade barriers. In this paper we analyze a global manufacturer's offshoring decision, i.e., whether to move production to low-wage countries or to keep production onshore, from a network capacity investment perspective.

We examine the operations strategy of a firm that manufactures two products to serve two geographically separated markets using a common component and two other product-specific com-

Figure 1: Four Possible Network Configurations of a Multiplant Network



ponents. The common part can be transported between the two markets that have different demand and financial characteristics. For our 2-product, 2-market model with commonality, four supply network strategies are possible, as illustrated in Figure 1. The more developed economy, the U.S., boasts higher product price while the other market, Asia, enjoys lower manufacturing costs. A multinational car manufacturer is a good motivating example for our model when localized car models serve local markets but share a common engine. For example, Toyota recently started to produce 2.4-litre engines in its joint venture in China and plans to export two-thirds of the output to Japan and the U.S. The engines are shared by RAV4s, China- and U.S.-bound Camrys, and Japan-bound minivan Ipsums (Automotive News, March 1 & September 13, 2004). Toyota has said it plans to invest as much as $2.5 billion in China by 2010 and currently has no plans to export cars from China because of enormous local demand. We are interested in understanding how the financial and demand characteristics drive the strategic facility decision: when should engine manufacturing be offshored?

Two strategic network design questions arise naturally when deciding on offshoring: the first question is whether the common part should be produced in a single facility or in two local facilities. If a centralized facility strategy is adopted for the common component, the second question asks where such facility should be located.

The centralization decision balances the tension between transportation costs and the benefits of centralization. The transportation cost accounts for shipping the common parts (intermediate goods) between the two locations and for foreign trade barriers such as taxes and tariffs. Three benefits of centralized production are highlighted in our model: centralization benefits from scale economies in capacity investment, yields lower total demand uncertainty through risk pooling, and also has an option to maximize revenue by contingent allocation of capacity towards the higher margin product.

The localization decision balances the tension between transportation costs and the benefits of lower manufacturing costs and increased revenues from local presence. In our model, we always assume that final assembly is local, so that we always have local presence. For the common component facilities, we restrict attention to the two geographically separated markets as the only location options. This restriction highlights the key trade-offs in the location decision but precludes some interesting questions such as whether it is better for the firm to locate the commonality facility at a hub location which can economically supply both markets. We also analyze market and financial characteristics and illustrate their impacts on the optimal location decision.

We use newsvendor network methodology to analyze the model under both deterministic and stochastic demand. In the simple base case of deterministic demand, the centralization decision solely depends on the relative magnitude of transportation and manufacturing costs. If the transportation cost is higher than the manufacturing cost differential between the two locations, it is optimal to adopt the product plant configuration (strategy 2 in Figure 1) and produce common parts in both locations. Otherwise, it is optimal to adopt a process plant configuration and to centralize commonality in Asia and transship (strategy 4 in Figure 1).

In the more realistic case that demand is uncertain during network planning, centralization becomes more attractive due to the benefits of resource pooling (risk pooling and revenue maximization). Even centralization in the U.S. may emerge as the optimal configuration when the price differential is sufficiently high. This finding underscores the importance of analyzing the network design problem under stochastic demand. Not only do optimal network configurations change under stochastic demand, but the transportation cost threshold is refined to capture demand uncertainty in addition to financial characteristics.

In contrast to much of the related facility network design literature, the contributions of this paper must be found in refining network design strategies to capture uncertain demand and the revenue effect, i.e., the option to maximize revenue by contingent allocation of capacity towards the higher margin product. Strengthening the results from previous literature, we demonstrate that both process and product configurations may arise under certain conditions as boundary solutions of the general problem formulated here. Specifically, we prove that USA-based sourcing can remain optimal for high-margin, volatile products. In addition, a hybrid configuration (strategy 1 in Figure 1) may outperform both process and product configurations. We believe providing the ex-post transshipment option for both locations is important to the flexibility of the manufacturing network, as demonstrated by the concept of "chaining" in Jordan & Graves (1995). Eliminating one transshipment activity breaks the chain and can significantly lower network flexibility. Furthermore, the generality of our model encompasses all possible configurations of the multiplant network formulated here and allows for analyzing the location decision of commonality. Finally, we believe that our coverage of economies of scale through fixed capacity costs and of stochastic demand ordering are novel analytical techniques in a newsvendor network.

2 Literature Review

Configuring the right multiplant network plays an important role in a firm's operations strategy. Hayes & Wheelwright (1984) illustrate four approaches for formulating multiplant facility strategy: physical facilities analysis, geographical network analysis, functional needs and corporate philosophy analysis, and product-process focus analysis. They argue that these four approaches represent different perspectives and should be used in a proper combination. Our analysis falls under both geographical network and product-process focus approaches. The geographical network analysis is often observed when transportation costs constitute a significant portion of total production cost, or customer proximity is a key source of competitive advantage. In this paper, we will illustrate the pivoting role of transportation cost in choosing centralized versus decentralized commonality strategy. The basis of the product-process focus approach is the concept of operational focus. Firms may choose to focus their facilities according to volumes, product, process, or service. Similar in spirit, our model incorporates two products with different demand distributions, two geographically separated markets with distinct financial characteristics, and two processes with different purposes: common component manufacturing versus dedicated assembly. We will illustrate how these elements interact and drive the optimal network decisions.

Our research falls within the vast literature on facility location and supply chain network design. In particular, our paper follows the stream that deals with facility decisions in a global context. We categorize the related literature into five groups of papers according to their research methodologies: mathematical programming, stochastic programming, newsvendor network, conceptual, and empirical. Key papers in each group will be highlighted.

One of the seminal papers that formulate global manufacturing strategic planning as a mathematical programming problem is Cohen & Lee (1989). Their model is capable of capturing a large number of factors affecting resource deployment decisions in a multi-country model, such as regional demand requirements, sourcing constraints, interplant transshipments, taxation and tariffs. In a global setting, firms' manufacturing decisions are significantly affected by international trade barriers and regulations. Munson & Rosenblatt (1997) focus on the impact of local content rules on global sourcing decisions. They incorporate local content rules into the classical plant location problem and provide an efficient solution procedure. Kouvelis & Rosenblatt (2001) present a mixed integer programming model that incorporates government trade policies, such as financing subsidies, tariffs, and taxation. The solution of their model illustrates important determinants of the structure of global facility networks. For example, expensive transportation of subassemblies leads to centralized manufacturing and distribution networks while increased trade tariffs makes decentralized distribution networks more attractive. A common feature of these mathematical programming formulations is that the decision framework is deterministic, i.e., no demand, financial, production, or regulatory uncertainties.

Some papers explicitly model uncertainty in the global manufacturing environment and evaluate the benefit of operational flexibility embodied in owning international operations. Kogut & Kulatilaka (1994) treat a multinational operating network as a real option whose value depends on exchange rates. They use stochastic dynamic programming to solve the option valuation problem and conclude that high variance of exchange rates increases the value of multinational networks. Similar to Kogut & Kulatilaka (1994), Huchzermeier & Cohen (1996) develop a stochastic dynamic programming formulation for valuation of global manufacturing strategy options in the face of switching costs and correlated exchange rate processes. A more recent paper by Kazaz, Dada & Moskowitz (2005) characterizes the value of production hedging and allocation hedging in global production planning in the presence of exchange-rate uncertainty.

While some papers focus on exchange rate risk, others use newsvendor models to incorporate demand risks and specifically study the value of transshipment in reducing inventory costs. Robinson (1990) models transshipments of goods among retail outlets as recourse actions taken after demands are realized. He concludes that ex post transshipment can not only reduce controllable costs considerably but also affect ex ante optimal ordering policy. Rudi, Kapur & Pyke (2001) take a new approach to the transshipment problem by extending it to an interfirm setting and studying the impact of local vs. centralized decision making on joint profits.

Van Mieghem & Rudi (2002) present a systematic approach to study network design in a newsvendor setting. The application of their newsvendor network framework in studying network configurations can be found in Kulkarni, Magazine & Raturi (2004), which is closely related to our model. They sought to determine the better of two predetermined network configurations: process plant (corresponding to strategy 3 or 4 in Figure 1) and product plant (corresponding to strategy 2 in Figure 1) configurations for a multiplant network with commonality. One of the major distinctions that separate our work from Kulkarni et al. (2004)'s is that we let the optimal network strategy emerge from optimization.

Apart from the analytical side of the literature, important and practical managerial insights have been derived from research using conceptual and empirical approaches. A group of papers draw on extensive interviews and case studies to examine the strategies and trend in facility location selections (Schmenner (1979), Bartmess & Cerny (1993), Bartmess (1994), MacCormack, III & Rosenfield (1994)). Bartmess & Cerny (1993) emphasize the strategic impact of plant location decisions and champion capability building in the objective of facility location decisions. MacCormack et al. (1994) document the new trend of global manufacturing site location as a result of changes in production technologies, workforce sophistication, and organizational philosophies. Other papers conduct empirical studies on facility strategies of large manufacturing firms. Key characteristics that affect the attractiveness of four prevailing multiplant strategies are identified for the Fortune 500 firms (Schmenner (1982)). Similarly, Brush, Maritan & Karnani (1999) empirically investigate the determinants of multinational manufacturing firms' choices between integrated and independent plants, and between domestic and foreign plants. The novelty of their approach is to combine perspectives from international business and manufacturing, and examine the interplay of the two perspectives in shaping managers' facility decisions.

Our work contributes to the commonality literature in operations management. Commonality, as defined by Van Mieghem (2004*a*), is about assembling multiple products from common components and product-specific components. Multiproduct firms often use commonality to add flexibility to their existing production networks. As mentioned earlier, Kulkarni et al. (2004) examines the trade-offs between risk pooling and logistics cost for two extreme configurations (process vs. product) of commonality in a multiplant network. Similar to Kulkarni et al. (2004)'s work, we take commonality as given and analyze the sourcing and location decisions of commonality.

Our work is also related to dual sourcing. Our model studies the choice between single sourcing (centralization) and dual sourcing (decentralization) strategies for commonality. Anupindi & Akella (1993) study how to optimally allocate quantities between two suppliers with yield uncertainty and its effects on the buyer's inventory policies. Yazlali & Erhun (2004) examine the trade-off between responsiveness and cost in global sourcing strategies using an imbedded multi-period inventory model . Tomlin & Wang (2005) study unreliable supply chains with risk averse firms that trade-off the level of mix flexibility against risk diversification through dual sourcing. Tomlin (2005) also studies sourcing mitigation strategies in the presence of different reliable suppliers as compared to adopting inventory mitigation and contingent rerouting for managing supply chain disruptions.

Finally, our research belongs to the growing literature on offshoring. The related literature is mostly found in the economics, international business, and popular management journals. Ferdows (1997) categorizes the strategic roles of foreign plants in a manufacturing firm's facility network and suggests that firms upgrade the roles of their foreign plants over time in order to gain competitive advantage in manufacturing capability. In contrast, Markides & Berg (1988) argue that offshore manufacturing does not build long-term competitive advantages, but is rather a "short-term tactical move". Farrell (2004) and Farrell (2005) lay out a conceptual framework for firms considering offshoring and shows (based on a recent study by the McKinsey Global Institute) that firms can significantly lower their costs by moving their production to low-wage locations. She also argues that cost savings from offshoring in turn enables firms to reduce prices and attract new customers, and therefore offshoring creates enormous value for both firms and the global economy. The related economics literature focuses on the impact of offshoring on domestic labor markets (Feenstra & Hanson (1996), Baily & Lawrence (2004)).

3 The Model

Consider a firm that utilizes four resources to produce two products. In the capacity portfolio $K = (K_1, K_2, K_3, K_4)$, K_i captures end-product localization capacity in location $i = 1, 2$ while K_{i+2} represents production capacity of the common component in location i. In other words, resources 1 and 2 are product-dedicated while transshipment allows resources 3 and 4 to be shared by the two products with demand vector $D \in \mathbb{R}^2_+$. The firm sells each product in a geographically separated market and has the option to configure its production network as illustrated in Figure 2. The dedicated resources are exogenously chosen to be located in the market they serve. Transshipment of the common part between the two locations is available at a positive cost c_T per unit. Though it can be easily incorporated into the model, we assume that final assembly is costless for the convenience of notation. Following the convention by Van Mieghem & Rudi (2002), we define

- $p = (p_1, p_2)$ where p_i is the per unit retail price of product i.
- $c_M = (c_{M,1}, c_{M,2})$ where $c_{M,i}$ is the per unit common component manufacturing cost in location i.
- c_T : per unit transportation cost (could include taxes and tariffs).
- $c_K = (c_{K,1}, c_{K,2}, c_{K,3}, c_{K,3})$: per unit capacity investment cost. We assume common component capacity cost is the same for both locations.

Throughout the paper, we assume product 1 has a higher retail or market price and location 2 has a lower common component manufacturing cost. Accordingly we call market 1 the high-price market and market 2 the low-cost market. Formally, the assumptions are

$$p_1 \geq p_2, \tag{A1}$$

$$c_{M,1} \geq c_{M,2}. \tag{A2}$$

A1 and A2 will be our maintained assumptions and apply to all lemmas and propositions. Define two notations: the price differential Δp and the common component manufacturing cost differential Δc_M (for short, manufacturing cost differential) as follows:

$$\Delta p = p_1 - p_2, \tag{1}$$

$$\Delta c_M = c_{M,1} - c_{M,2}. \tag{2}$$

The processing network of Figure 2 has four contingent activities: x_i is the number of common parts used locally in market i, while x_{i+2} is the number of common parts transshipped from market i. The net values of the four processing activities are denoted by v where

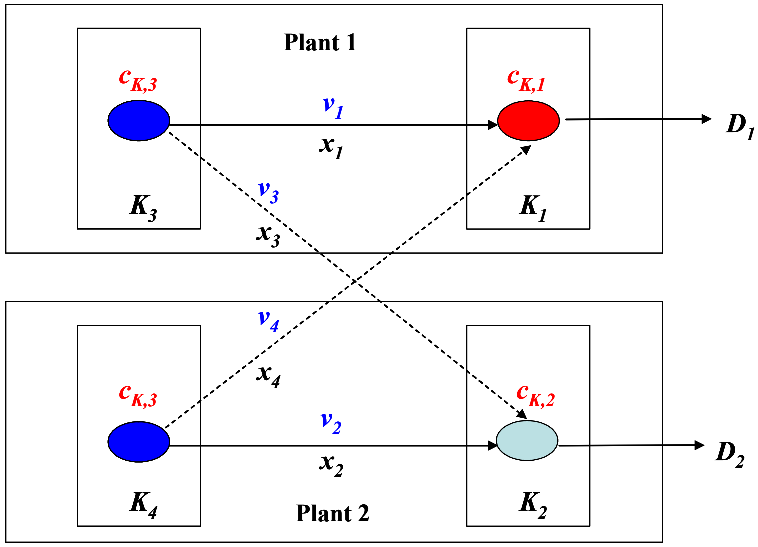
$$v_1 = p_1 - c_{M,1}, \tag{3}$$

$$v_2 = p_2 - c_{M,2}, \tag{4}$$

$$v_3 = p_2 - c_{M,1} - c_T, \tag{5}$$

$$v_4 = p_1 - c_{M,2} - c_T. \tag{6}$$

Figure 2: Newsvendor Network Decisions (K and x) and Data (c_K, v, D) of the Multiplant Network Problem



To eliminate trivialities, we assume positive net values. This means that production and transshipment is ex-post profitable. In addition, it is economically justified to produce both end products, which means investment in the dedicated resources are always positive. We also assume that transportation costs are symmetric and independent of direction. Asymmetric transportation costs due to location differences in taxes and tariffs can be accommodated at the expense of notational complexity. The transshipment process is obviously not symmetric in a real world situation as duties and tariffs are often different in each country.

The firm is a profit optimizer so that the optimal capacity strategy K and contingent activities $x(K, D)$ emerge from a two-stage optimization problem. Let $V(K)$ denote the expected firm value given capacity investment K. The optimal expected firm value is

$$V^* = \max_{K \in \mathbb{R}_+^4} \mathbb{E}\pi(K, D) - C(K) \quad (7)$$

$$\text{subject to } K \geq 0, \quad (8)$$

where the optimal operating profit is

$$\pi(K,D) = \max_{x \in \mathbb{R}^4_+} v'x \tag{9}$$

$$\text{subject to} \quad Ax \leq K, \tag{10}$$

$$R_D x \leq D, \tag{11}$$

where primes denote transposes and the consumption and output matrices are

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \qquad R_D = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}. \tag{12}$$

We first assume the capacity cost function is linear, i.e.,

$$C(K) = c_K' K, \tag{13}$$

but later add a fixed cost to study economies of scale. Finally, in order to compare the optimal value of the four network configurations as described in the introduction, we denote

- V_{dec} : optimal value of the decentralized configuration;
- V_{prod} : optimal value of the product plant configuration;
- V_{on} : optimal value of the configuration that centralizes commonality in the high-price market (i.e., onshore production);
- V_{off} : optimal value of the configuration that centralizes commonality in the low-cost market (i.e., offshore production).

Finally, only those proofs that are novel in a newsvendor network appear in the Appendix; the remainder are available in the Online Appendix.

4 Optimal Network Configurations

We analyze the capacity decision problem in two steps. First, is it optimal for the firm to adopt a centralization strategy for the common component? Second, if centralization of commonality is

Table 1: Optimal Network Configurations for Deterministic Demands

Orders	Basic Activities	Discretionary Activities	Optimal Network Configuration	Optimal Capacity Allocation K
$v_1 \geq v_4 \geq v_2 \geq v_3$ $v_1 \geq v_2 \geq v_4 \geq v_3$ $v_2 \geq v_1 \geq v_4 \geq v_3$	x_1, x_2	x_3, x_4	Product Plant	(D_1, D_2, D_1, D_2)
$v_2 \geq v_4 \geq v_1 \geq v_3$ $v_4 \geq v_1 \geq v_2 \geq v_3$ $v_4 \geq v_2 \geq v_1 \geq v_3$	x_2, x_4	x_1, x_3	Process Plant (Centralization at the Low-Cost Market)	$(D_1, D_2, 0, D_1 + D_2)$

optimal, in which market should the facility be located? To highlight the first-order cost drivers, we start with the deterministic demand setting before tackling the stochastic model.

4.1 Deterministic Demand

In a deterministic world, the investment decision boils down to evaluating the financial attractiveness of the alternative processing activities. Without imposing any assumption, there are 24 (= 4!) possible permutations of the ordering of the four activities' values. We impose the two maintained assumptions A1 and A2.

$$\text{A1 and A2} \Rightarrow \min(v_1, v_2, v_4) \geq v_3.$$

Therefore, v_3 is the smallest among the four net values, and we are left with only 6 (= 3!) possible orders, listed in Table 1. In the first three cases, activities 1 and 2 are basic leading to a product plant strategy. (Activities 3 and 4 are non-basic and thus "discretionary," as called by Van Mieghem & Rudi (2002): they are only used in stochastic settings to deal with deviations from the expected scenario.) In the last three cases in Table 1, activities 2 and 4 are basic, leading to a low-cost centralization strategy. The following proposition determines the optimality condition for the two network configurations.

Proposition 1 ***(Deterministic Demand)*** *If $c_T \leq \Delta c_M$, centralizing commonality at the low-cost market is optimal with capacity investment $K^* = (D_1, D_2, 0, D_1 + D_2)'$. Otherwise, a product plant configuration is optimal with capacity investment $K^* = (D_1, D_2, D_1, D_2)'$.*

4.2 Stochastic Demand

Uncertainty only increases the resource pooling benefits of centralization. Therefore, when centralization is optimal in the deterministic case, it remains so in the stochastic case:

Proposition 2 *If $c_T \le \Delta c_M$, centralizing commonality at the low-cost market is optimal.*

This predicts offshoring when transportation costs are small relative to manufacturing costs. While Proposition 2 is sufficient for our purposes, it also simplifies finding the specific optimal capacity levels when $c_T \le \Delta c_M$: set $K_3 = 0$ and solve the three-dimensional optimal capacity problem.

From now on, we will focus on the less obvious case where $c_T > \Delta c_M$. With uncertainty, even expensive transportation has an option value because it reduces ex-post supply-demand mismatch costs. Understanding this option value better is precisely the purpose of our analytic model and we thus proceed to analyze the optimal network configuration assuming a random demand vector that is distributed according to the continuous bivariate distribution F.

Lemma 1 *If $c_T > \Delta c_M$, the optimal capacity investment vector satisfies $K^*_{i+2} \le K^*_i$. Moreover, $K^*_{i+2} = 0$ implies $K^*_j = K^*_{j+2}$, for $i, j \in \{1, 2\}$.*

Lemma 1 says that investment in the common component is weakly less than in the dedicated component in both markets. One interpretation is that "excess" downstream capacity provides a "switching option" that requires an unbalanced capacity portfolio (Van Mieghem (2004*b*)). Viewed another way, this property manifests the resource pooling benefit brought by the ex-post transshipment option. As common parts can be shared across markets, the need to invest in the common component in both markets is reduced. Moreover, Lemma 1 provides an simplification property for analyzing the centralization configurations: equal investment in the common and dedicated components.

As illustrated in Figure 1, two centralization strategies are possible: centralization at the high-price market and at the low-cost market. Given the equal investment property stated in Lemma 1, the two boundary solutions $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$ and $\underline{K} = (\underline{K}_1, \underline{K}_2, 0, \underline{K}_2)$ represent the centralization configuration at the high-price market and at the low-cost market, respectively. In the deterministic case, we have shown that only the latter may emerge as the optimal configuration when transportation is sufficiently cheap. With uncertainty, both strategies may become optimal under certain conditions that critically hinge on the price differential and demand volatility. In the deterministic world, every unit of demand is fulfilled, and profit maximization reduces to cost minimization. In a stochastic world, however, the flexibility of being able to substitute the production of low-margin product with high-margin product generates positive value. Lemma 2 shows that there are three cases to be considered, depending on the price differential.

Lemma 2 *If $c_T > \Delta c_M$, then three cases are possible depending on the relative retail prices:*

(i) (High price differential): when $\Delta p \geq c_T$, $v_1 > v_4 \geq v_2 > v_3$.

(ii) (Medium price differential): when $\Delta c_M < \Delta p < c_T$, $v_1 > v_2 > v_4 > v_3$.

(iii) (Low price differential): when $\Delta p \leq \Delta c_M$, $v_2 \geq v_1 > v_4 \geq v_3$.

Different centralization configurations may arise for the three cases. To specify the optimal strategies for high and medium price differentials, it is useful to introduce the following transportation cost threshold defined by the "upper" boundary solution $\bar{K}$:

$$\bar{c}_T = \frac{\Delta p \bar{P}_3 - \Delta c_M - c_{K,1}}{1 - \bar{P}_3}, \tag{14}$$

where $\bar{P}_3 = \Pr(D_1 > \bar{K}_1)$.

Proposition 3 *(High & medium price differential) If $\min(\Delta p, c_T) > \Delta c_M \geq 0$, the optimal investment strategy depends on the relative cost of transportation c_T:*

(i) If $c_T < \bar{c}_T$, it is optimal to centralize commonality in the high-price market;

(ii) If $c_T \geq \bar{c}_T$, it is optimal to invest commonality in both markets;

(iii) Product plant configuration is a special case of (ii) in which activities x_3 and x_4 are identically zero, and its capacity vector $\tilde{K} = (\tilde{K}_1, \tilde{K}_2, \tilde{K}_1, \tilde{K}_2)$ is optimal if and only if

$$\begin{aligned} \Pr(D_1 &> \tilde{K}_1, D_2 > \tilde{K}_2) > \max\{a * (\frac{c_{K,1} + c_{K,3}}{v_1} - \frac{c_{K,1}}{v_4}), \frac{c_{K,2} + c_{K,3}}{v_2} - \frac{c_{K,2}}{v_3}\}, \\ \textit{where} \quad \tilde{K}_1 &= F_1^{-1}(\frac{v_1 - c_{K,1} - c_{K,3}}{v_1}), \; \tilde{K}_2 = F_2^{-1}(\frac{v_2 - c_{K,2} - c_{K,3}}{v_2}), \\ a &= \frac{v_4}{v_2} \textit{ for high } \Delta p; \; = 1 \textit{ for medium } \Delta p. \end{aligned}$$

Note that with medium to high price differentials, Proposition 3 implies that it is never optimal to centralize commonality in the low-cost market. This is in stark contrast to the deterministic base case. Further, the cases of high and medium price differentials share the same set of strategies and transportation cost threshold (though the derivations are different as shown in the proof). Moreover, product plant configuration may arise, but only as a very specific boundary solution to the multiplant network problem formulated here. The condition for the optimality of product plant configuration depends on how likely the capacity constraint $\tilde{K}$ is reached simultaneously for both products. The larger the likelihood, the less valuable the ex-post transshipment option, which is thus less likely to be used to alleviate the ex-post demand-capacity mismatch. To specify the optimal strategies when the price differential is low, we need another transportation cost threshold defined by the "lower" boundary solution $\underline{K}$:

$$\underline{c}_T = \frac{\Delta c_M - \Delta p \underline{P}_3 - c_{K,2}}{1 - \underline{P}_3}, \tag{15}$$

where $\underline{P}_3 = \Pr(D_2 > \underline{K}_2)$.

Proposition 4 *(Low price differential) If $c_T > \Delta c_M \geq \Delta p \geq 0$, the optimal investment strategy depends on the relative cost of transportation c_T:*

(i) If $c_T < \underline{c}_T$, it is optimal to centralize commonality in the low-cost market.

(ii) If $c_T \geq \underline{c}_T$, it is optimal to invest commonality in both markets.

(iii) Product plant configuration is a special case of (ii) in which activities x_3 and x_4 are identically zero, and its capacity vector $\tilde{K} = (\tilde{K}_1, \tilde{K}_2, \tilde{K}_1, \tilde{K}_2)$ is optimal if and only if

$$\begin{aligned} \Pr(D_1 &> \tilde{K}_1, D_2 > \tilde{K}_2) > \max\{\frac{c_{K,1} + c_{K,3}}{v_1} - \frac{c_{K,1}}{v_4}, \frac{c_{K,2} + c_{K,3}}{v_2} - \frac{c_{K,2}}{v_3}\}, \\ \textit{where} \quad \tilde{K}_1 &= F_1^{-1}(\frac{v_1 - c_{K,1} - c_{K,3}}{v_1}),\ \tilde{K}_2 = F_2^{-1}(\frac{v_2 - c_{K,2} - c_{K,3}}{v_2}). \end{aligned}$$

Proposition 4 shows that the optimal strategies for low price differential are similar in form to those in the deterministic base case. Propositions 3 and 4 together highlight the importance of capturing the revenue impact in stochastic network planning: *high retail price differential, low manufacturing cost differential, and high transportation costs in the presence of significant demand volatility are the key conditions to argue against offshoring.* Notice that a common feature shared by the last two propositions is that the transportation cost threshold depends on demand distributions and thus the optimality of centralization strategy depends on demand distributions. Moreover, notice that a positive transportation cost threshold requires either $\Delta p > \Delta c_M$ in the case of $\bar{c}_T$, or $\Delta p < \Delta c_M$ in the case of $\underline{c}_T$. These two opposite conditions underscore the pivoting role of the price differential in network decisions: when the transportation cost is below the threshold, it is optimal to centralize either at the high-price market or at the low-cost market, but not both.

Compared with the deterministic case, demand uncertainty makes centralization at the high-price market more attractive. When Δc_M increases, it is more likely to satisfy the centralization condition. The extreme case is that when $\Delta c_M \geq c_T$, centralization at the low-cost market is always optimal as we have shown earlier. Therefore, there is some continuity in the choice of optimal configuration. However, if Δc_M is intermediate, meaning lower than the transportation cost but higher than the price differential, centralization at the low-cost market may not be optimal and the decision crucially depends on the transportation cost threshold.

To understand why the transportation cost threshold depends on the boundary solutions $\bar{K}$ and $\underline{K}$, we illustrate the derivation of $\bar{c}_T$ for the case of medium Δp. We partition the demand space into eight regions, denoted by $\Omega_i's$ as in Figure 3, such that the shadow price of each capacity is constant for the second-stage contingent capacity allocation problem (a linear program). The first-order conditions of the capacity investment problem are given by

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ v_2 - v_3 \end{pmatrix} P(\Omega_p(K^*)) + \begin{pmatrix} 0 \\ 0 \\ v_1 - v_4 \\ 0 \end{pmatrix} P(\Omega_q(K^*)) + \begin{pmatrix} 0 \\ v_3 \\ 0 \\ v_2 - v_3 \end{pmatrix} P(\Omega_1(K^*)) + \begin{pmatrix} 0 \\ 0 \\ v_3 \\ v_2 \end{pmatrix} P(\Omega_2(K^*)$$

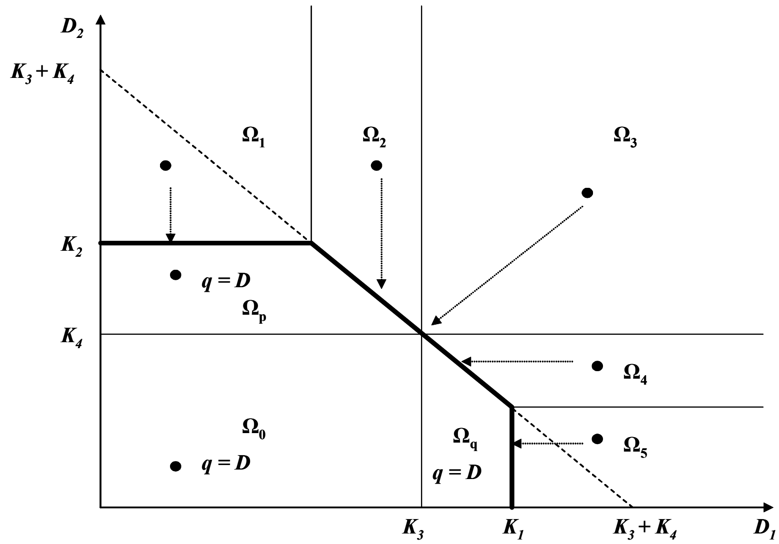
$$+ \begin{pmatrix} 0 \\ 0 \\ v_1 \\ v_2 \end{pmatrix} P(\Omega_3(K^*) + \begin{pmatrix} 0 \\ 0 \\ v_1 \\ v_4 \end{pmatrix} P(\Omega_4(K^*) + \begin{pmatrix} v_4 \\ 0 \\ v_1 - v_4 \\ 0 \end{pmatrix} P(\Omega_5(K^*) = c_K - \mu - \theta, \quad (16)$$

$$\mu' K^* = 0, \quad (17)$$

$$\theta_i(K_i^* - K_{i+2}^*) = 0, \ i \in \{1, 2\}, \quad (18)$$

where c_K, $\mu \in \mathbb{R}_+^4$ and $\theta = (\theta_1, \theta_2, -\theta_1, -\theta_2)$, $\theta_1, \theta_2 \in \mathbb{R}_+$. The economic interpretation of these conditions is that at optimality the marginal benefit of increasing one unit of capacity is equal to the marginal cost of doing that. Consider the boundary solution $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$ and simplify

Figure 3: Partitioning of the Demand Space (Assume medium Δp)



the first-order conditions to

$$v_3 P(\Omega_2(\bar{K})) + v_1 P(\Omega_3(\bar{K})) = c_{K,1} + c_{K,3}, \tag{19}$$

$$v_3 P(\Omega_1(\bar{K})) = c_{K,2}, \tag{20}$$

$$v_2 - v_3 P(\Omega_{p+1}(\bar{K})) = c_{K,3} - \mu_{4.} \tag{21}$$

If $\mu_4 > 0$, $K_4^* = 0$ and thus $\bar{K}$ is the unique solution to the general optimality equations (16)-(18). Combining equations (19)-(21), $\mu_4 > 0$ is equivalent to $c_T < \bar{c}_T$, where $\bar{c}_T$ is given by equation (14) with $\bar{P}_3 = P(\Omega_3(\bar{K})) = \Pr(D_1 > \bar{K}_1)$.

5 Impacts of Costs and Prices

5.1 How Financial Characteristics Impact Optimal Network Configuration

The previous two propositions identify the transportation cost thresholds below which centralization strategies are optimal. We are interested in how the financial characteristics, including price, manufacturing cost, and capacity investment cost, affect network decisions. The pivoting role of the price differential in network decisions is manifested in its association with the specific centralization strategy as shown earlier. There are other unanswered but interesting questions: 1) does centralization at the high-price market become more attractive when the price differential increases? 2) does centralization at the low-cost market become more attractive when the manufacturing cost differential increases? 3) how does the optimal network configuration change with the capacity investment cost? Since the transportation cost thresholds determine the optimal network configuration, we will be able to answer these questions by determining how $\bar{c}_T$ and $\underline{c}_T$ are affected by a change in any of the financial characteristics.

The transportation cost thresholds measure the attractiveness of centralization: the higher the threshold, the more attractive the centralization strategy. We will focus on $\bar{c}_T$ (the change on $\underline{c}_T$ is similar). The change of $\bar{c}_T$ w.r.t. any parameter y is given by the total derivative

$$\frac{d\bar{c}_T}{dy} = \underbrace{\frac{\partial \bar{c}_T}{\partial y}}_{\substack{\text{Direct}\\ \text{effect}}} + \underbrace{\frac{\partial \bar{c}_T}{\partial \bar{P}_3}\frac{d\bar{P}_3}{dy}}_{\substack{\text{Indirect}\\ \text{effect}}}. \tag{22}$$

Decomposing the total effect of the change in a financial parameter on the transportation cost threshold enables a better understanding of the countervailing forces that mitigate the attractiveness of centralization. The sign of $\partial\bar{c}_T/\partial y$ is straightforwardly determined. However,

$$\text{sign}\ (\frac{\partial \bar{c}_T}{\partial \bar{P}_3}\frac{d\bar{P}_3}{dy}) = \text{sign}\ (\frac{\partial \bar{c}_T}{\partial \bar{P}_3}) \times \text{sign}\ (\frac{d\bar{P}_3}{d\bar{K}_1}) \times \text{sign}\ (\frac{d\bar{K}_1}{dy})\ , \tag{23}$$

where

$$\text{sign}\ (\frac{\partial \bar{c}_T}{\partial \bar{P}_3}) = \text{sign}\ (\frac{\Delta p - \Delta c_M - c_{K,1}}{(1-\bar{P}_3)^2}) = 1, \tag{24}$$

$$\text{sign}\ (\frac{d\bar{P}_3}{d\bar{K}_1}) = \text{sign}\ (\frac{d\,\Pr(D_1 > \bar{K}_1)}{d\bar{K}_1}) = -1. \tag{25}$$

Similarly the total derivative of $\underline{c}_T$ w.r.t. y can be derived. Therefore determining the sign of the indirect effect boils down to determining the sign of $d\bar{K}_1/dy$ (and $d\underline{K}_2/dy$ for $\underline{c}_T$), which are derived analytically (in the Online Appendix). The calculation of the direct and indirect effects of all the financial parameters allows us to evaluate how $\bar{c}_T$ and $\underline{c}_T$ change when any of the financial parameters increases. The results are summarized in Table 2.

We selectively discuss the effects of some parameters. For example, the last column in Table 2 shows how the financials impact the centralization decision in Asia by considering the comparative statics of $\underline{c}_T$. An increase in the market price of the U.S. decreases $\underline{c}_T$ and thus the attractiveness of centralization in Asia. In contrast, an increase in the manufacturing cost or the investment cost of the product-specific component in the U.S., or in the investment cost of the common component

Table 2: Effects of Financial Characteristics on the Transportation Cost Thresholds

	$\bar{c}_T$			$\underline{c}_T$		
Parameter	Direct	Indirect	Total	Direct	Indirect	Total
p_1	$+$	$-$	$+/-$	$-$	$-$	$-$
p_2	$-$	$-$	$-$	$+$	$-$	$+/-$
$c_{M,1}$	$-$	$+$	$+/-$	$+$	0	$+$
$c_{M,2}$	$+$	0	$+$	$-$	$+$	$+/-$
$c_{K,1}$	$-$	$+$	$+/-$	0	$+$	$+$
$c_{K,2}$	0	$+$	$+$	$-$	$+$	$+/-$
$c_{K,3}$	0	$+$	$+$	0	$+$	$+$

enhances the attractiveness of centralization in Asia, as manifested by the increase of $\underline{c}_T$. The other three changes are ambiguous. Why do changes in the Asian market price, manufacturing cost, or product-specific capacity investment, i.e. p_2, $c_{M,2}$, and $c_{K,2}$, lead to ambiguous change in $\underline{c}_T$? We believe that the ambiguity stems from the inherent trade-offs in a centralization decision. For example, in the case of p_2, the positive direct effect is induced by the enhanced ex-post revenue maximization (because $\Delta v = v_2 - v_4 = \Delta c_M + c_T - \Delta p$ increases). The negative indirect effect is caused by the increased likelihood of invoking the ex-post transshipment option, as manifested in the decreased value of $\underline{P}_3$. This is so because the individual service level of product 1

$$\underline{SL}_1 = \underline{P}_q + \underline{P}_4 + \underline{P}_5 = 1 - \underline{P}_3, \tag{26}$$

is increased and the fact that every unit of product 1 served incurs a unit of transportation cost when commonality is centralized in Asia.

Our result on investment cost of common component is similar to those reported by Kulkarni et al. (2004) (as shown in their paper's section 4.2 and Figure 5), i.e., centralization strategy remains optimal for larger values of transportation costs when the investment cost of common component increases. There are two distinctions between our result and theirs. First, our conclusion is more general for it is based on general analytical comparative statics instead of on numerical examples assuming uniform demand distributions. Second, our result is derived for a more general model instead of from comparing process plant configuration (corresponding to our centralization strategy) with product plant configuration (which is a special case of our decentralized configuration).

5.2 How Financial Characteristics Impact Optimal Value and Capacity Investment

We study comparative statics for the stochastic demand scenario. Denote the total capacity investment in common component by K_{com}, i.e., $K_{\text{com}} = K_3 + K_4$.

Property 1 *The optimal expected firm value V^* is a non-increasing convex function of c_K with gradient $\nabla_{c_K} V^* = -(K_1^*, K_2^*, K_{com}^*)' \leq 0$. Moreover, the optimal value V^* is an increasing convex function of p with gradient $\nabla_p V^* = \mathbb{E}(x_1^* + x_4^*, x_2^* + x_3^*)' > 0$.*

The sensitivity terms of K^* on any of the financial parameters, i.e., c_K, p, and c_M, can be calculated following the approach in Van Mieghem (1998) (Calculation of those sensitivity terms are available upon request.) Here we employ a different approach to characterize how K^* depends on the financial parameters, which draws on the concept of supermodularity and increasing differences.

Property 2 *The expected firm value $V(K)$ is supermodular in (K_1, K_2, K_3, K_{com}) and has increasing differences in $(K_1, K_2, K_3, K_{com}, y)$, where $y = -c_K$ or p_1. Moreover, the expected firm value is supermodular in (K_1, K_2, K_4, K_{com}) and has increasing differences in $(K_1, K_2, K_4, K_{com}, y)$, where $y = -c_K$ or p_2.*

Complementarity among K_1, K_2, and K_{com} is not surprising given that the network is a connected "chain" so that a capacity increase in one type of resource should be accompanied by an increase in any other type of resource within the network. However, supermodularity property cannot be established for (K_1, K_2, K_3, K_4) because of the substitution effect between K_3 and K_4. When perturbed by a change in the financials, the system needs to be adjusted towards optimality, and the adjustment may not be monotone at all facilities. The next property describes monotonicity in some resources following changes in some financials.

Property 3 *K_1^*, K_2^*, K_3^*, and K_4^* are monotonically decreasing in any marginal capacity cost $c_{K,i}$, $i = 1, 2, 3$. K_1^*, K_2^*, K_{com}^* are monotonically increasing in p_j, $j = 1, 2$. K_3^* is monotonically increasing in p_1. K_4^* is monotonically increasing in p_2.*

The monotonicity of $K_1^*, K_2^*, K_{\text{com}}^*$ in both c_K and p are not surprising due to the complementarity property stated in Property 2. However, K_3^* and K_4^* may be increasing or decreasing in p_2 and p_1, respectively. When p_2 increases, the incentive to invest more in K_2 obviously increases.

What is less obvious is the increased incentive to invest more in K_3 because some of the capacity can be used towards the production of product 2 ex post. However, if p_2 becomes large enough so that Δp becomes less than Δc_M, centralization in Asia will become more attractive as demonstrated earlier, thus leading to a decrease in K_3.

Not only do financial characteristics impact network decisions, demand characteristics also interact with financial characteristics in deciding the optimal network configurations. We discuss how demand size and volatility change optimal network configurations in the next section.

6 Impacts of Market Size and Uncertainty

If the transportation cost is lower than the manufacturing cost differential, the optimality of low-cost centralization strategy is independent of uncertainty. Otherwise the optimal strategy depends on the demand distribution. We now analyze how demand distributions impact network configurations when transportation is relatively costly.

6.1 Demand Size

When centralizing commonality at market i is optimal for a given market size, we expect it to remain optimal when market i grows. The less obvious case is when the demand size of market $j \neq i$ is changed. The next two propositions illustrate how that change affects optimal network configurations.

Proposition 5 *Suppose product demands D_1 and D_2 are independent. Let D_i' be a stochastically larger demand than D_i, denoted $D_i' \succsim D_i$, for $i = 1, 2$.*

(i) If centralization of commonality at the high-price (or low-cost) market is optimal for (D_1, D_2), it remains optimal for (D_1', D_2) (or (D_1, D_2')).

(ii) If centralization of commonality at the high-price (or low-cost) market is optimal for (D_1, D_2') (or (D_1', D_2)), it remains optimal for (D_1, D_2).

(iii) If centralization of commonality at the high-price (or low-cost) market is optimal for (D_1, D_2), there exists a $\bar{\mu}_2$ (or $\bar{\mu}_1$) such that centralization is never optimal when $\mathbb{E}(D_2') > \bar{\mu}_2$ (or $\mathbb{E}(D_1') > \bar{\mu}_1$).

The insight from the propositions is that the size of the markets matters in network design when demands are uncertain and when transportation is relatively expensive. When demands are deterministic, every unit of demand is satisfied. Therefore, increasing market size scales up the network proportionally, and thus has no impact on the optimal network configurations. However, when demands are stochastic, the trade-off between the benefit of resource pooling and the cost of ex-post transshipment is affected by the size of the markets. Specifically, suppose centralizing commonality at market i is optimal for a given market size. When the size of market j decreases, centralization at market i remains optimal. On the other hand, only when the size of market j increases sufficiently so that its expected transportation cost outweighs the resource pooling benefit, will the localization strategy dominate the centralization strategy.

Relating the result to the context of offshoring, suppose centralization in Asia is optimal given current global demand. If U.S. demand grows larger, decentralization of commonality becomes optimal. But it is never optimal to centralize in the U.S. On the other hand, if Asian demand grows larger, centralization in Asia stays optimal. However, if we instead suppose centralization in the U.S. is optimal given the current global demand, the conclusion is changed partially. If U.S. demand grows larger, centralization in the U.S. stays optimal. But, if Asian demand grows larger, decentralization becomes optimal. Therefore, given that it is likely that growth rates in Asia will continue to surpass those in the U.S., we expect to see configurations that centralize commonality in the U.S. gradually disappear, while more configurations arise that decentralize commonality or centralize commonality in Asia.

6.2 Demand Volatility and Correlation

Our stochastic analysis highlighted that the offshoring decision not only depends on the relative market sizes, but also on their volatilities and correlation. To analyze this dependence, we assume that D is normally distributed with mean vector μ and covariance matrix Σ. (The normality assumption is a good approximation for forecasts where sales result from many customers, as justified by the central limit theorem.) It has been shown that the optimal value V^* is increasing in μ and decreasing in any variance term Σ_{ii} (Van Mieghem & Rudi (2002)). We further have established that:

Proposition 6 *Assume $\Delta p \geq c_T$ (high price differential) and D is normally distributed with mean vector μ and covariance matrix Σ. The operating profit $\pi(K, D)$ is submodular in D, and therefore,* *the optimal value V^* is decreasing in any covariance term Σ_{ij}.*

In other words, since its capacity is capped from above, demand volatility degrades the expected value of a network. In addition, increased market correlation decreases risk pooling and revenue maximization although the latter does not disappear completely when demands are perfectly positively correlated (Van Mieghem (1998)). To evaluate the impact of correlation, we further solve the model for perfect correlations without imposing the normality assumption on the demand distribution.

Proposition 7 *Let product demands be perfectly negatively correlated: $P(\{D_1+D_2 = k > 0\}) = 1$.*

(i) (High & medium price differential) If $\min(\Delta p, c_T) \geq \Delta c_M \geq 0$,

$$\bar{c}_T = c_T\{1 + \frac{(v_1 - v_3)(c_{K,2} + c_{K,3} - v_2)}{v_1 - (c_{K,1} + c_{K,2} + c_{K,3})}\}; \tag{27}$$

Moreover, it is optimal to centralize commonality in the high-price market if and only if $v_1 > c_{K,1} + c_{K,2} + c_{K,3}$ and $v_2 < c_{K,2} + c_{K,3}$.

(ii) (Low price differential) If $\Delta p \leq \Delta c_M < c_T$,

$$\underline{c}_T = c_T\{1 + \frac{(v_2 - v_4)(c_{K,1} + c_{K,3} - v_1)}{v_2 - (c_{K,1} + c_{K,2} + c_{K,3})}\}. \tag{28}$$

Moreover, it is optimal to centralize commonality in the low-cost market if and only if $v_2 > c_{K,1} + c_{K,2} + c_{K,3}$ and $v_1 < c_{K,1} + c_{K,3}$.

This proposition highlights the non-obvious role of the price and manufacturing cost differential in determining the optimal network configurations. Notice that the sufficient and necessary conditions for the optimality of the centralized configurations imply that $\Delta p - \Delta c_M > c_{K,1}$ and $\Delta c_M - \Delta p > c_{K,2}$ for (i) and (ii), respectively. The benefit of resource pooling becomes dominant only when the price advantage or the cost advantage of the centralizing market is sufficiently high. This result also provides a lower bound on the net value differential ($\Delta v > c_{K,1}$ for (i) and $\Delta v > c_{K,2}$ for (ii)) that is necessary for the optimality of centralization because resource pooling benefit is the highest when demands are perfectly negatively correlated.

When demands are perfectly positively correlated, a closed-form expression for the transportation cost thresholds is not obtainable. Nevertheless, some structural properties of the optimal capacity vector exist and are dependent on the critical fractiles of the two local processing activities:

Proposition 8 *Let product demands be perfectly positively correlated:* $P(\{D_1 = D_2\}) = 1$. *Suppose* $0 \leq \max(\Delta p, \Delta c_M) < c_T$ *(medium & low price differential). If* $\frac{c_{K,1}+c_{K,3}}{v_1} < \frac{c_{K,2}+c_{K,3}}{v_2}$, $0 \leq K_4^* \leq K_2^* < K_3^* = K_1^*$; *If* $\frac{c_{K,1}+c_{K,3}}{v_1} = \frac{c_{K,2}+c_{K,3}}{v_2}$, $K_1^* = K_2^* = K_3^* = K_4^*$; *If* $\frac{c_{K,1}+c_{K,3}}{v_1} > \frac{c_{K,2}+c_{K,3}}{v_2}$, $0 \leq K_3^* \leq K_1^* < K_4^* = K_2^*$.

The structural properties of the optimal capacity vector indicate a weakened value of ex-post transshipment. Notice that $K_j^* = K_{j+2}$ for either $j = 1, 2$ or both. It follows that all units of product j are produced locally at market j even though common component capacity at market i could be positive.

It is well known that the newsvendor critical fractile solution crucially depends on the distribution over the entire support and the optimal capacity level can be below, at, or above the mean of the demand. Hence, general results on the comparative statics of capacity on variability of the demand are hard to establish. We expect that the centralized network looses in attractiveness as total demand variability decreases. The following graphical argument, which follows from Van Mieghem & Rudi (2002), provides support for that claim. (Note that here we focus on the expensive transportation cost case, otherwise low-cost centralization strategy is optimal regardless of demand distributions as shown in Proposition 2.) Suppose the demand vector (D_1, D_2) is bivariate normal with mean (μ_1, μ_2) and covariance matrix

$$\Sigma = \gamma \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix},$$

where $\gamma \geq 0$ is a measure for the amount of variability. Consider any amount of variability $\gamma \neq 1$. Rescaling the demand space directly yields that the optimal values of $K_1 - \mathbb{E}(D_1)$, $K_2 - \mathbb{E}(D_2)$, $K_3 + K_4 - \mathbb{E}(D_1 + D_2)$ are also scaled by γ. Hence as γ decreases to zero, $K_1^* \to \mathbb{E}(D_1)$, $K_2^* \to \mathbb{E}(D_2)$, and $K_3^* + K_4^* \to \mathbb{E}(D_1)$. Thus, P_p, P_q, P_2, and P_4 all decrease to zero, resulting in a product plant network configuration.

7 The Impact of Economies of Scale

Our analysis until now has focused on the risk pooling and revenue maximization benefits of centralization. We now examine a third benefit: economies of scale (EoS). EoS is an important driver in many plant investment decisions and, similar to volatility, it increases the value of centralization. To analyze economies of scale in our model, we use the concave affine cost function specified as follows:

$$C(K) = c_0 + c_K' K. \tag{29}$$

We only add fixed cost to the common component as the capacity decision hinges on whether to invest this resource in both markets. The inclusion of fixed cost makes the cost function discontinuous at zero and complicates the first-order conditions. However, we identified a simple condition to check whether centralization is optimal.

Let $V_{\text{cen}} = \max(V_{\text{on}}, V_{\text{off}})$ and let V_{eos} denote the optimal value of the decentralized configuration with EoS. Since *a priori* it either restricts $K_3 = 0$ or $K_4 = 0$, the centralized configurations yield a lower optimal value than the decentralized one. This becomes the main result of the next lemma.

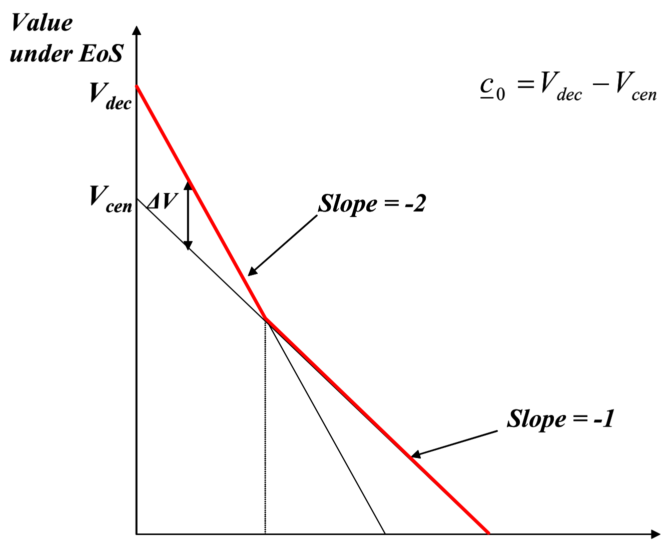
Lemma 3 *$V_{dec} \geq V_{cen}$. Moreover, $V_{dec} > V_{on}$(or V_{off}) implies K_4^*(or K_3^*) > 0, while $V_{dec} = V_{on}$(or V_{off}) implies K_4^*(or K_3^*) $= 0$.*

In addition, Lemma 3 implies that restricting attention to the centralized configuration is valid only if doing so gives rise to the same optimal solution as the decentralized configuration would. If there is no fixed cost, the optimization problem remains the same as before. When the fixed cost increases, the marginal investment decision remains unchanged for a range of fixed cost, in other words, the optimal capacity vector is determined by the same first-order conditions as before. But the optimal values of the two configurations decrease at different rates. V_{dec} decreases with slope -2 while V_{cen} decreases with slope -1 simply because the decentralized network has two common component facilities while the centralized network has only one. When the fixed cost is sufficiently high, the benefit of scale economies becomes dominant and makes centralization optimal. Figure 4 is a graphical illustration of how optimal network values change with fixed cost. The outer envelope of the two downward-sloping lines represents how the optimal network value changes with the fixed cost. It follows immediately from the graph that the fixed cost threshold, denoted by $\underline{c}_0$ (where the kink is located) is exactly the difference between V_{dec} and V_{cen}. Formally, the result is stated in the following proposition.

Proposition 9 *Suppose common component capacity investment incurs a positive fixed cost c_0 and $c_0 < \min(V_{dec}, V_{cen})$.*

(i) If $V_{dec} \leq V_{on} + c_0 \leq V_{off} + c_0$, it is optimal to centralize commonality in the low-cost market and $V_{eos} = V_{off} - c_0$.

Figure 4: The Impact of Economies of Scale on Optimal Network Values



(ii) *If* $V_{dec} \leq V_{off} + c_0 \leq V_{on} + c_0$, *it is optimal to centralize commonality in the high-price market and* $V_{eos} = V_{on} - c_0$.

(iii) *If* $V_{dec} \geq \max(V_{on}+c_0, V_{off}+c_0)$, *it is optimal to decentralize commonality and* $V_{eos} = V_{dec} - 2c_0$.

Given an affine concave cost structure, Proposition 9 allows us to decompose the optimization problem of a multiplant network with EoS into three optimization problems of the multiplant network, one general and the other two with predetermined centralized configurations. The latter two problems can be solved using the newsvendor network approach illustrated earlier, but with a reduced dimensionality. Further, Proposition 9 confirms our intuition that EoS only increases the optimality region of centralization: if centralization is optimal without EoS, the optimal configuration remains after EoS is incorporated. If, however, centralization is suboptimal, the decentralized configuration remains optimal as long as the fixed cost of investment is small enough (compared to the fixed cost threshold $\underline{c}_0$).

Since $\underline{c}_0$ is equal to $V_{\text{dec}} - V_{\text{cen}}$, evaluating the impact of EoS boils down to comparing the decentralized network with the centralized ones. If $V_{\text{dec}} - V_{\text{cen}}$ is strictly positive, the decentralized configuration must dominate the centralized configurations without EoS. Only under this condition should we be concerned about the impacts of EoS on the network decisions. Further, the higher the value of $\underline{c}_0$, the less likely the network decisions will be changed by EoS. Then determining the drivers of the difference of the two configurations is crucial to evaluating the impacts of EoS. Our previous analysis on financial and demand characteristics can be applied here. High transportation cost and large demand volume of the supplied market (i.e., the market that relies on the other market for the supply of common parts) make decentralization more attractive, and thus, a larger $\underline{c}_0$. Therefore, EoS is of less concern in the networks that have these characteristics.

It is important to point out that the optimal location of centralized commonality becomes dependent on demand characteristics in the presence of EoS. Recall that without EoS the optimal location of centralization is determined solely by the relative magnitude of price and manufacturing cost differentials. High and medium Δp centralize in the high-price market while low Δp puts centralization in the low-cost market, provided that transportation is inexpensive relative to the transportation cost threshold. In a more realistic situation where capacity investment involves a significant fixed cost, the performance of both centralized configurations need to be evaluated against the decentralized configuration. Since the relative ranking of V_{on} and V_{off} is impacted by demand size and uncertainty, the optimal location of centralization is impacted by those demand characteristics as well.

8 Numerical Analysis

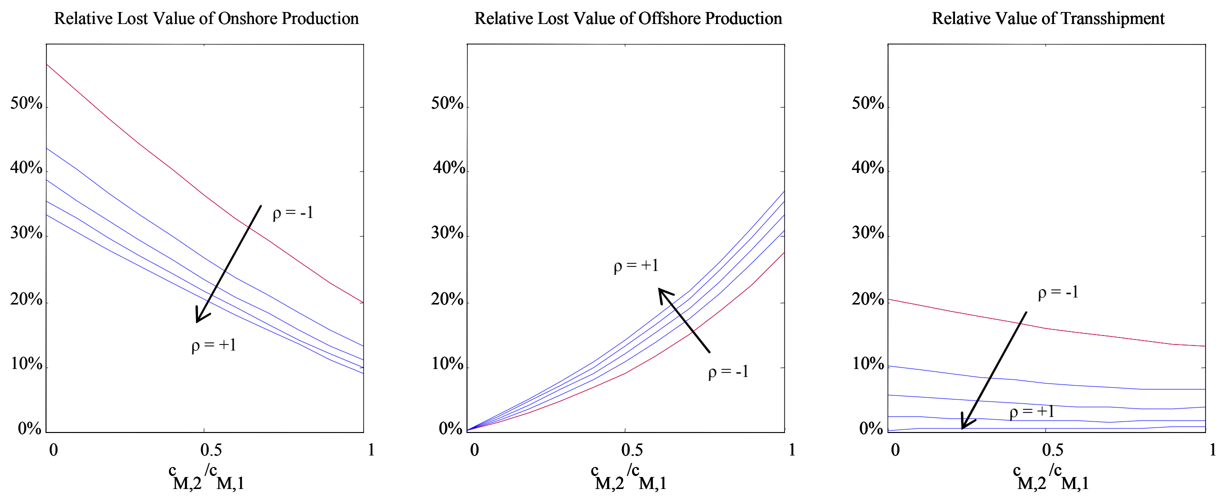
The analysis in the previous sections illustrates how optimal network configurations are impacted by financial and demand characteristics, but has two limitations. First, we are only able to determine the signs of the effects rather than the magnitude. Second, in contrast to the effect on network values, we are not able to draw a general conclusion on how demand volatility affects network configuration and capacities. To further our analysis in these two aspects, we resort to numerical examples using optimization with simulated bivariate normal demands with scaled unit means $\mu = (1, 1)$ and covariance matrix

$$\Sigma = \gamma \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix},$$

where $\sigma = (.3, .4)$, γ is the standard deviation multiplier, and ρ is the correlation. γ and ρ will be specified for each example that follows. In order to compare the performance of the four network configurations as described in the introduction, we define:

- Relative lost value of onshore production $= \frac{V_{\text{dec}} - V_{\text{on}}}{V_{\text{on}}}$;
- Relative lost value of offshore production $= \frac{V_{\text{dec}} - V_{\text{off}}}{V_{\text{off}}}$;

Figure 5: Relative lost value of onshore and offshore production, and relative value of transshipment as a function of manufacturing cost ratio, parameterized by correlation for $p = (15, 10)', c_{M,1} = 2, c_T = 2.01, c_K = (1, 1, 5, 5)', \gamma = 1$.

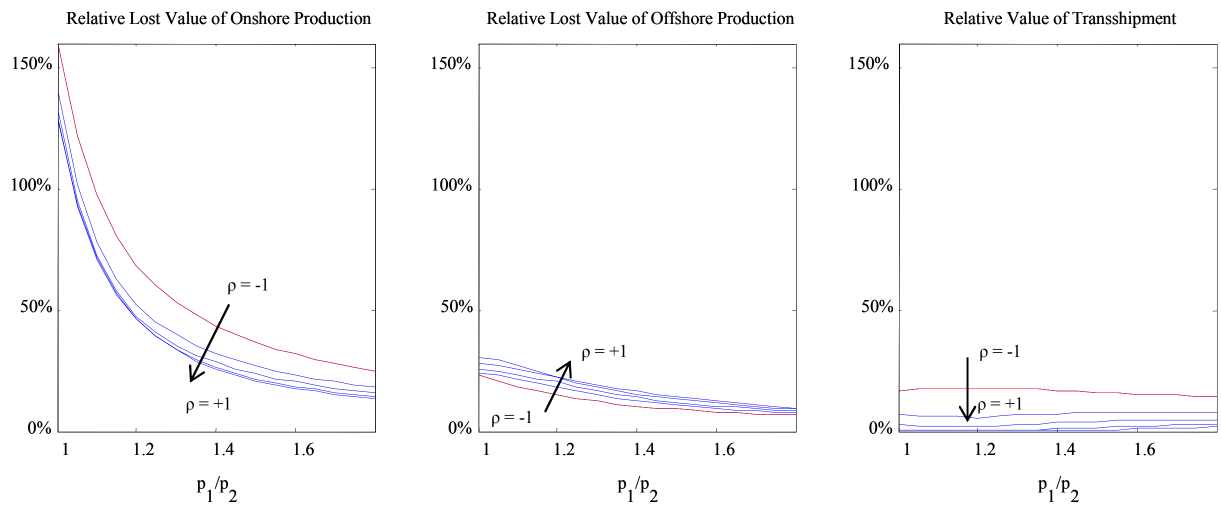


- Relative value of transshipment $= \frac{V_{\text{dec}} - V_{\text{prod}}}{V_{\text{prod}}}$.

Many offshoring decisions are driven by the significant labor cost advantages in the foreign market. In Figure 5, increased manufacturing cost in the foreign market, as measured by the ratio $c_{M,2}/c_{M,1}$, increases the attractiveness of decentralization and onshore production, but decreases the relative value of transshipment. As the cost advantage of the foreign market decreases to zero, onshore production becomes better than offshore production, though decentralized production achieves the highest network value. The impact of cost changes on the relative value of transshipment is rather flat, meaning both product plant and the decentralization configurations are impacted similarly. In this example, the fully "chained" decentralized production network (Strategy 1 in Figure 1) may outperform the other three strategies by as high as 30%. Moreover, the relative value of offshore production is rather insensitive to correlation compared to the relative value of onshore production and transshipment.

In addition to decreased cost advantage of the foreign market, increased domestic price advantage also enhances the attractiveness of onshore production, as shown in Figure 6. When the price ratio is close to 2, the \$1 cost advantage of offshoring production becomes negligible and thus making onshore production more attractive. However, when the price ratio is close to 1, slight increase in the domestic retail price sharply improves the performance of onshore production. This

Figure 6: Relative lost value of onshore and offshore production, and relative value of transshipment as a function of retail price ratio, parameterized by correlation for $p_2 = 10, c_M = (2,1)', c_T = 2.01, c_K = (1,1,5,5)', \gamma = 1$.

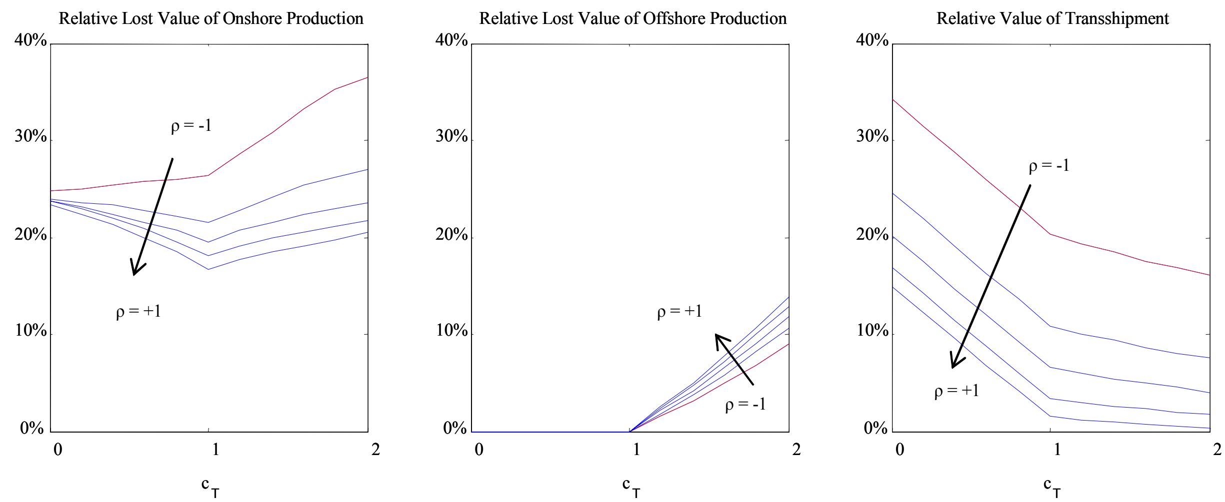


is because centralized production provides a more effective hedge when the two markets have larger net value differences.

Figure 7 shows how transportation cost impacts the relative values of the four network configurations. As expected, more expensive transportation leads to higher value of decentralization, but lower value of transshipment. Notice the kink of the curves is where transportation cost equals manufacturing cost differential, while left to the kink is where offshore production is optimal. Similar to the previous examples, the attractiveness of offshore production is rather insensitive to correlation while the relative value of transshipment is rather sensitive.

Finally, Figure 8 shows that the impact of demand volatility on network values depends on correlation in different ways. Positively correlated demands make onshore production more attractive, but make offshore production less attractive when demands become more volatile. This shows that the resource pooling benefit of the two centralized configurations are impacted rather differently by correlation. To understand this effect, we need to look at the net values closely. Notice that $\Delta v = v_1 - v_3 = 7.01$ for onshore production, but $\Delta v = v_4 - v_2 = 2.99$ for offshore production. When demands become more positively correlated, risk pooling benefit becomes less, but ex-post revenue maximization option remains and is larger when the net value differential is larger. Therefore, the difference in the resource pooling benefit between onshore and offshore production enlarges. Similar

Figure 7: Relative lost value of onshore and offshore production, and relative value of transshipment as a function of transportation cost, parameterized by correlation for $p = (15, 10)', c_M = (2, 1)', c_K = (1, 1, 5, 5)', \gamma = 1$.



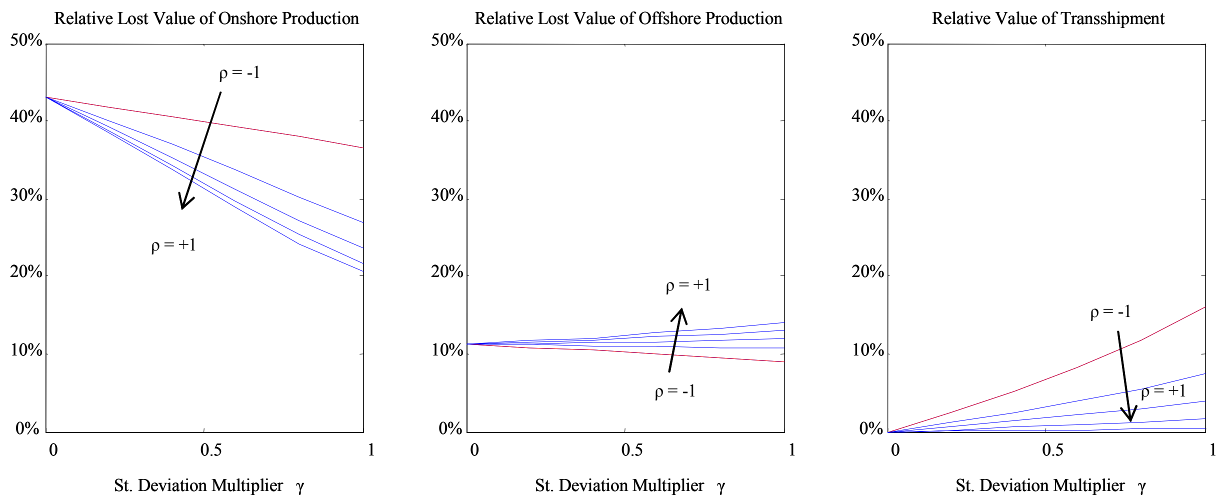
to onshore production, transshipment becomes less attractive when demands become more volatile, which is due to increased risk pooling benefit.

9 Conclusions and Managerial Insights

We present an analytical model to study sourcing and location decisions of commonality in multi-plant networks. As a special case, the model allows us to analyze offshoring from a network capacity investment perspective. The model in this paper belongs to the broad class of newsvendor networks but has a distinct focus on network design in the presence of inter-plant transshipment. The model is kept simple for analytical tractability, yet it captures three key factors in global capacity planning: cost, revenue, and demand. Our main objective is to show how uncertainty may change the operations network strategy relative to simpler deterministic thinking based on cost only. We demonstrate that onshore production can remain optimal under certain conditions.

Our key findings are: (1) When the manufacturing cost advantage of offshoring outweighs the transportation cost, centralizing common component production in low-wage countries is always optimal, as we expect. Otherwise, when transportation is costly, the optimal network strategy is much more complex and depends on price ratios (revenue impact) as well as cost, market size and uncertainty. (2) Centralizing common component onshore may be more attractive when the domestic

Figure 8: Relative lost value of onshore and offshore production, and relative value of transshipment as a function of volatility, parameterized by correlation for $p = (15, 10)', c_M = (2, 1)', c_T = 2.01, c_K = (1, 1, 5, 5)'$.



price advantage outweighs the manufacturing cost disadvantage and when demands are uncertain, i.e., high-margin products with volatile demands. (3) Demand volatility affects the centralization vs. decentralization decision, but the location of centralization, without EoS, is independent of the demand characteristics, but rather restricted by the relative magnitude of the price and the manufacturing cost differential. However, with EoS, both decisions will be dependent on demand and financial characteristics of the two markets. (4) Demand characteristics change optimal network configurations only when transportation is costly, i.e., when its cost outweighs the manufacturing cost differential. We provide the transportation cost thresholds that can serve as an indicator of the attractiveness of centralization. Comparative statics analysis on the transportation cost thresholds also shed light on how financial and demand characteristics impact network decisions.

As telecommunication and transportation costs decreased and international trade barriers became lifted over the last decade, many companies in the developed countries have moved production to low-wage countries. Our study leads to managerial insights that can guide global manufacturing managers in evaluating the cost and benefit of offshore production. The key take-away is that it is crucial to incorporate the revenue effect of resource pooling and the global demand characteristics into the decision framework, in addition to understanding the cost structure of the global manufacturing network.

The limitation of our work lies in three aspects. First, in contrast to our assumption, common component capacity costs could differ by locations (i.e., $c_{K,3} \neq c_{K,4}$). We believe this will drive the solution in obvious ways: the low capacity cost location would be favored. Second, lead time is not modeled as our model is not dynamic. Lastly, transshipment of end product is not considered. We believe this is not relevant as our model assumes that the localized end products are not substitutable.

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Appendix

Property 2 Proof. The following proof applies to medium and low Δp cases (the high Δp case can be proved similarly). We proceed to prove the first half of the property. In the proof of Proposition 3 (see Online Appendix), we have shown the Hessian matrix of $V(K)$ is

$$H = \begin{pmatrix} -a_1 - a_2 & 0 & a_1 & a_1 \\ 0 & -a_3 - a_4 & a_4 & a_4 \\ a_1 & a_4 & -a_1 - a_4 - a_5 - a_6 & -a_1 - a_4 - a_5 \\ a_1 & a_4 & -a_1 - a_4 - a_5 & -a_1 - a_4 - a_5 - a_7 \end{pmatrix}, \tag{30}$$

where $a_i \geq 0, i = 1, ..., 7$. With a change of variable, let $\tilde{V}(\tilde{K}) = \tilde{V}(K_1, K_2, K_3, K_{\text{com}}) = V(K_1, K_2, K_3, K_{\text{com}} - K_3)$. Calculating the cross partial derivatives, we get

$$\frac{\partial^2 \tilde{V}}{\partial K_1 \partial K_2} = \frac{\partial^2 V}{\partial K_1 \partial K_2} = 0 \tag{31}$$

$$\frac{\partial^2 \tilde{V}}{\partial K_1 \partial K_3} = \frac{\partial^2 V}{\partial K_1 \partial K_3} - \frac{\partial^2 V}{\partial K_1 \partial (K_{\text{com}} - K_3)} = 0 \tag{32}$$

$$\frac{\partial^2 \tilde{V}}{\partial K_1 \partial K_{\text{com}}} = \frac{\partial^2 V}{\partial K_1 \partial (K_{\text{com}} - K_3)} = a_1 \tag{33}$$

$$\frac{\partial^2 \tilde{V}}{\partial K_2 \partial K_3} = \frac{\partial^2 V}{\partial K_2 \partial K_3} - \frac{\partial^2 V}{\partial K_2 \partial (K_{\text{com}} - K_3)} = 0 \tag{34}$$

$$\frac{\partial^2 \tilde{V}}{\partial K_2 \partial K_{\text{com}}} = \frac{\partial^2 V}{\partial K_2 \partial (K_{\text{com}} - K_3)} = a_4 \tag{35}$$

$$\frac{\partial^2 \tilde{V}}{\partial K_3 \partial K_{\text{com}}} = \frac{\partial^2 V}{\partial K_3 \partial (K_{\text{com}} - K_3)} - \frac{\partial^2 V}{\partial (K_{\text{com}} - K_3)^2} = a_7. \tag{36}$$

Notice that all cross partials are nonnegative, satisfying the condition for supermodularity. Further,

$$\frac{\partial \tilde{V}}{\partial K_1} = \frac{\partial V}{\partial K_1} = \mathbb{E}\lambda_1(K) - c_{K,1} + \theta_1, \tag{37}$$

$$\frac{\partial \tilde{V}}{\partial K_2} = \frac{\partial V}{\partial K_2} = \mathbb{E}\lambda_2(K) - c_{K,2} + \theta_2, \tag{38}$$

$$\frac{\partial \tilde{V}}{\partial K_3} = \frac{\partial V}{\partial K_3} - \frac{\partial V}{\partial K_4} = \mathbb{E}\lambda_3(K) - \mathbb{E}\lambda_4(K) - \theta_1 + \theta_2, \tag{39}$$

$$\frac{\partial \tilde{V}}{\partial K_{\text{com}}} = \frac{\partial V}{\partial K_4} = \mathbb{E}\lambda_4(K) - c_{K,3} - \theta_2. \tag{40}$$

Taking partial derivative w.r.t. $c_{K,i}$ gives that all the cross partial terms are nonpositive, proving the increasing differences result. Given that

$$\mathbb{E}\lambda_1(K) = v_4 P(\Omega_5(K)) = (p_1 - c_{M,2} - c_T) P(\Omega_5(K)) \tag{41}$$

$$\mathbb{E}\lambda_2(K) = v_3 P(\Omega_1(K)) = (p_2 - c_{M,1} - c_T) P(\Omega_1(K)) \tag{42}$$

$$\begin{aligned} \mathbb{E}\lambda_3(K) &= (v_1 - v_4) P(\Omega_{q+5}(K)) + v_3 P(\Omega_2(K)) + v_1 P(\Omega_{3+4}(K)) \\ &= (c_{M,2} + c_T - c_{M,1}) P(\Omega_{q+5}(K)) + (p_2 - c_{M,1} - c_T) P(\Omega_2(K)) \\ &\quad + (p_1 - c_{M,1}) P(\Omega_{3+4}(K)) \end{aligned} \tag{43}$$

$$\begin{aligned} \mathbb{E}\lambda_4(K) &= (v_2 - v_3) P(\Omega_{p+1}(K)) + v_2 P(\Omega_{2+3}(K)) + v_4 P(\Omega_4(K)) \\ &= (c_{M,1} + c_T - c_{M,2}) P(\Omega_{p+1}(K)) + (p_2 - c_{M,2}) P(\Omega_{2+3}(K)) \\ &\quad + (p_1 - c_{M,2} - c_T) P(\Omega_4(K)) \end{aligned} \tag{44}$$

Calculating the cross partial derivatives w.r.t. p_1 yields

$$\frac{\partial^2 \tilde{V}}{\partial K_1 \partial p_1} = \frac{\partial \mathbb{E}\lambda_1(K)}{\partial p_1} = P(\Omega_5(K)) \tag{45}$$

$$\frac{\partial^2 \tilde{V}}{\partial K_2 \partial p_1} = \frac{\partial \mathbb{E}\lambda_2(K)}{\partial p_1} = 0 \tag{46}$$

$$\frac{\partial^2 \tilde{V}}{\partial K_3 \partial p_1} = \frac{\partial \mathbb{E}\lambda_3(K)}{\partial p_1} - \frac{\partial \mathbb{E}\lambda_4(K)}{\partial p_1} = P(\Omega_3(K)) \tag{47}$$

$$\frac{\partial^2 \tilde{V}}{\partial K_{\text{com}} \partial p_1} = \frac{\partial \mathbb{E}\lambda_4(K)}{\partial p_1} = P(\Omega_4(K)) \tag{48}$$

The nonnegativitity of all cross partial terms yields the increasing differences property. The second half of the property can be proved similarly with a different change of variable: $\hat{V}(K_1, K_2, K_4, K_{\text{com}}) = V(K_1, K_2, K_{\text{com}} - K_4, K_4)$. ■

Property 3 Proof. We have already established supermodularity and increasing differences property and the result is an application of Topkis' monotonicity theorem. ■

Proposition 5 Proof. We will show the proof for the high-price case (the low-cost case stated in the bracket follows a similar logic). We proceed to prove part (ii) and (iii) (part (i) can be proved similarly). Let $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$ and $\bar{K}' = (\bar{K}_1', \bar{K}_2', \bar{K}_1', 0)$ be the boundary solution for (D_1, D_2) and (D_1, D_2'), respectively.

Claim: $\bar{K}_2' > \bar{K}_2$. Suppose to the contrary that $\bar{K}_2' \leq \bar{K}_2$. Recall the optimality condition for the boundary solution $\bar{K}$ is

$$\begin{aligned} v_3 P(\Omega_1(\bar{K})) &= c_2, \\ v_3 P(\Omega_2(\bar{K})) + v_1 P(\Omega_3(\bar{K})) &= c_1 + c_3, \\ (v_2 - v_3)(P(\Omega_{p+1}(\bar{K}))) + v_2(P(\Omega_{2+3}(\bar{K}))) &= c_3 - \boldsymbol{\mu}_4. \end{aligned}$$

To keep $P(\Omega_1(K)) = P(D_1 < K_1 - K_2)P(D_2 > K_2)$ unchanged as we change the demands from (D_1, D_2) to (D_1, D_2'), we must have

$$\bar{K}_1' - \bar{K}_2' < \bar{K}_1 - \bar{K}_2, \tag{49}$$

which follows from that D_2' first-order stochastic dominates D_2 and the supposition that $\bar{K}_2' \leq \bar{K}_2$. Thus, $\bar{K}_1' < \bar{K}_1$,

$$P(\Omega_{2+3}(\bar{K}')) = P(D_1 > \bar{K}_1' - \bar{K}_2', D_1 + D_2 > \bar{K}_1') > P(\Omega_{2+3}(\bar{K})), \tag{50}$$

and

$$P(\Omega_3(\bar{K}')) = P(D_1 > \bar{K}_1') > P(\Omega_3(\bar{K})). \tag{51}$$

It follows that

$$
\begin{aligned}
v_3 P(\Omega_2(\bar{K}')) + v_1 P(\Omega_3(\bar{K}')) &= v_3 P(\Omega_{2+3}(\bar{K}')) + (v_1 - v_3) P(\Omega_3(\bar{K}')) && (52)\\
&> v_3 P(\Omega_{2+3}(\bar{K})) + (v_1 - v_3) P(\Omega_3(\bar{K})) && (53)\\
&= v_3 P(\Omega_2(\bar{K})) + v_1 P(\Omega_3(\bar{K})) && (54)\\
&= c_1 + c_3, && (55)
\end{aligned}
$$

contradicting the optimality condition. Hence, $\bar{K}_2' > \bar{K}_2$. It follows that $\bar{K}_1' > \bar{K}_1$ in order to satisfy the second optimality condition. It follows that

$$
\begin{aligned}
\text{sign}\ (\frac{\partial \bar{c}_T}{\partial D_2}) &= \text{sign}\ (\frac{\partial \bar{c}_T}{\partial \bar{P}_3}) \times \text{sign}\ (\frac{\partial \bar{P}_3}{\partial \bar{K}_1}) \times \text{sign}\ (\frac{\partial \bar{K}_1}{\partial D_2})\\
&= 1 \times (-1) \times 1\\
&= -1, && (56)
\end{aligned}
$$

where ∂D_2 is in the sense of first-order stochastic dominance. Hence $\bar{c}_T > \bar{c}_T'$, which proves part (ii). Since it is monotonically decreasing in D_2 in the sense of first-order stochastic dominance, $\bar{c}_T$ becomes negative when D_2 is large enough, which will happen when $\mathbb{E}(D_2)$ is large enough. This proves part (iii). ■

Online Appendix

Lemma 1 Proof. Suppose to the contrary that $K_3^* > K_1^*$. Let $\varepsilon = K_3^* - K_1^*$, then ε can only be utilized by activity x_3. Moving this ε capacity to K_4^* gives a higher expected profit because $v_2 \geq v_3$, contradicting the assumed optimality. Similarly, we can prove that $K_4^* \leq K_2^*$. ■

Proposition 1 Proof. Since demand is deterministic, choosing the most profitable processing activity for each product yields the optimal configuration and capacities. If $c_T \leq c_M$, the bottom three cases of Table 1 arise. Otherwise, the top three cases arise. ■

Proposition 2 Proof. If $c_T \leq c_M$, the basic activities of both products (i.e., x_2 and x_4) are provided solely by the low-cost market. Since the basic activities yield higher net values than the discretionary activities, optimal investment has to set $K_3^* = 0$ because it only provides the discretionary activities. ■

Lemma 2 Proof. The ordering follows immediately from the definition of the $v_i's$ and the assumptions. ■

Proposition 3 Proof. The proof for part (i) and (ii) proceeds in three steps. First, we establish the strict concavity of the optimization problem, i.e., the uniqueness of the optimal solution. Second, we provide the optimality conditions. Third, we derive the transportation cost threshold. We separate the proof for the medium and high Δp cases.

(1) Medium $\Delta p : \Delta c_M < \Delta p < c_T$

Step 1. There exists a unique K^* that solves the capacity investment problem.

Let F be the joint distribution function of $D_1$and D_2 and f be the density function. The greedy solution for the stage-2 contingent capacity allocation problem is:

$$x_1(K, D) = \min\{D_1, K_1, K_3\}, \tag{57}$$

$$x_2(K, D) = \min\{D_2, K_2, K_4\}, \tag{58}$$

$$x_3(K, D) = \min\{D_2 - x_2, K_2 - x_2, K_3 - x_1\}, \tag{59}$$

$$x_4(K, D) = \min\{D_1 - x_1, K_1 - x_1, K_4 - x_2\}. \tag{60}$$

Partition the demand space as in Figure 3. Let H denote the Hessian matrix of $V(K; c_K)$. For the interior solution $K^* = (K_1^*, K_2^*, K_3^*, K_4^*)$,

$$H = D_K^2 V(K; c_K) = D_K^2 \mathbb{E}\pi(K, D) = D_K \mathbb{E}(\nabla_K \pi(K, D)) = D_K \mathbb{E}\lambda, \tag{61}$$

$$\mathbb{E}\lambda = \Lambda P \tag{62}$$

where

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & v_4 \\ 0 & 0 & 0 & v_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_1 - v_4 & 0 & v_3 & v_1 & v_1 & v_1 - v_4 \\ 0 & v_2 - v_3 & 0 & v_2 - v_3 & v_2 & v_2 & v_4 & 0 \end{pmatrix}, \tag{63}$$

$$P = (P(\Omega_0(K^*)), P(\Omega_p(K^*)), P(\Omega_q(K^*)), P(\Omega_1(K^*)), \tag{64}$$

$$P(\Omega_2(K^*)), P(\Omega_3(K^*)), P(\Omega_4(K^*)), P(\Omega_5(K^*)))'. \tag{65}$$

Hence,

$$H = \Lambda D_K P = \Lambda \begin{pmatrix} 0 & 0 & I_5 & I_3 \\ 0 & I_1 & I_2 & I_2 - I_3 \\ I_{10} & 0 & I_8 - I_5 & I_8 \\ 0 & -I_1 - I_6 & I_6 & I_6 \\ 0 & I_6 & I_7 - I_2 - I_6 & -I_2 - I_6 \\ 0 & 0 & -I_7 & -I_4 \\ I_9 & 0 & -I_8 - I_9 & I_4 - I_8 - I_9 \\ -I_9 - I_{10} & 0 & I_9 & I_9 \end{pmatrix} \tag{66}$$

where

$$I_1 = \int_0^{K_1+K_4-K_2} f(x, K_2)dx, \quad I_2 = \int_{K_1+K_4-K_2}^{K_1} f(x, K_1 + K_4 - x)dx, \tag{67}$$

$$I_3 = \int_0^{K_3} f(x, K_4)dx, \quad I_4 = \int_{K_3}^{\infty} f(x, K_4)dx, \tag{68}$$

$$I_5 = \int_0^{K_4} f(K_3, x)dx, \quad I_6 = \int_{K_2}^{\infty} f(K_1 + K_4 - K_2, x)dx, \tag{69}$$

$$I_7 = \int_{K_4}^{\infty} f(K_3, x)dx, \quad I_8 = \int_{K_3}^{K_1} f(x, K_3 + K_4 - x)dx, \tag{70}$$

$$I_9 = \int_{K_1}^{\infty} f(x, K_3 + K_4 - K_1)dx, \quad I_{10} = \int_0^{K_3+K_4-K_1} f(K_1, x)dx. \tag{71}$$

Thus

$$H = \begin{pmatrix} -a_1 - a_2 & 0 & a_1 & a_1 \\ 0 & -a_3 - a_4 & a_4 & a_4 \\ a_1 & a_4 & -a_1 - a_4 - a_5 - a_6 & -a_1 - a_4 - a_5 \\ a_1 & a_4 & -a_1 - a_4 - a_5 & -a_1 - a_4 - a_5 - a_7 \end{pmatrix}, \tag{72}$$

where

$$a_1 = v_4 I_9, \quad a_2 = v_4 I_{10}, \tag{73}$$

$$a_3 = v_3 I_1, \quad a_4 = v_3 I_6, \tag{74}$$

$$a_5 = v_3 I_2 + v_4 I_8, \quad a_6 = (v_1 - v_3) I_7 + (v_1 - v_4) I_5, \tag{75}$$

$$a_7 = (v_2 - v_3) I_3 + (v_2 - v_4) I_4. \tag{76}$$

H is negative definite because it is symmetric and has a negative and dominant diagonal (To see the diagonal is dominant, let $\eta = (1, 1, 1/2, 1/2)$. We then have $|\eta_i H_{ii}| > \sum_{j \neq i} |\eta_j H_{ij}|$.) The uniqueness of the optimal solution follows from the negative definiteness of H.

Step 2. The optimality conditions follow from

$$\begin{aligned} \nabla_K V(K^*; c_K) &= \nabla_K \{\mathbb{E}\pi(K^*, D) - c_K' K^* + \mu' K^* + \theta_1 (K_1^* - K_3^*) + \theta_2 (K_2^* - K_4^*)\} && (77) \\ &= \mathbb{E}\lambda - c_K + \mu + \theta = 0. && (78) \end{aligned}$$

See equation (16)-(18).

Step 3. Derivation of $\bar{c}_T$. See the derivation of $\bar{c}_T$ in the main text that follows Proposition 4.

Proof for part (iii). The product plant configuration is optimal iff $P(\Omega_p(K^*)) = P(\Omega_q(K^*)) = P(\Omega_2(K^*)) = P(\Omega_4(K^*)) = 0$. The first-order condition simplifies to

$$\begin{pmatrix} 0 \\ v_3 \\ 0 \\ v_2 - v_3 \end{pmatrix} P(\Omega_1(\tilde{K})) + \begin{pmatrix} 0 \\ 0 \\ v_1 \\ v_2 \end{pmatrix} P(\Omega_3(\tilde{K})) + \begin{pmatrix} v_4 \\ 0 \\ v_1 - v_4 \\ 0 \end{pmatrix} P(\Omega_5(\tilde{K})) = \begin{pmatrix} c_{K,1} - \theta_1 \\ c_{K,2} - \theta_2 \\ c_{K,3} + \theta_1 \\ c_{K,3} + \theta_2 \end{pmatrix}. \tag{79}$$

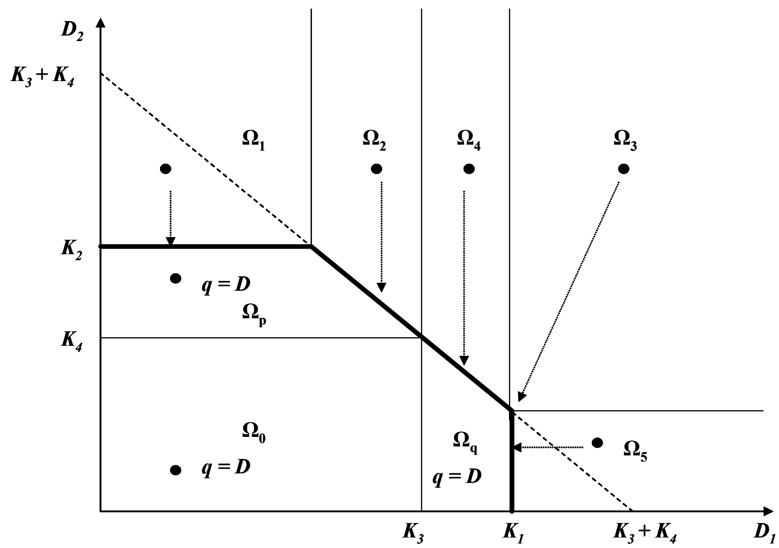
Solving it yields

$$\begin{aligned} P(D_1 &> \tilde{K}_1) = \frac{c_{K,1} + c_{K,3}}{v_1}, && (80) \\ P(D_2 &> \tilde{K}_2) = \frac{c_{K,2} + c_{K,3}}{v_2}. && (81) \end{aligned}$$

By complementary slackness, the optimality of $\tilde{K}$ requires $\theta_i > 0$, for $i = 1, 2$. It follows from the above first-order conditions that

$$\begin{aligned} P(\Omega_1(\tilde{K})) &< \frac{c_{K,2}}{v_3}, && (82) \\ P(\Omega_5(\tilde{K})) &< \frac{c_{K,1}}{v_4}, && (83) \end{aligned}$$

Figure 9: Partitioning of the Demand Space (assuming high Δp)



which are equivalent to

$$P(\Omega_3(\tilde{K})) > \min(\frac{c_{K,1}+c_{K,3}}{v_1} - \frac{c_{K,1}}{v_4}, \frac{c_{K,2}+c_{K,3}}{v_2} - \frac{c_{K,2}}{v_3}). \tag{84}$$

(2) High $\Delta p : \Delta p \geq c_T > \Delta c_M$

The proof follows a similar logic as in the case of medium Δp. Here we only provide the details for the steps that are different. The greedy solution for the stage-2 contingent capacity allocation problem is:

$$x_1(K, D) = \min\{D_1, K_1, K_3\}, \tag{85}$$

$$x_4(K, D) = \min\{D_1 - x_1, K_1 - x_1, K_4\}, \tag{86}$$

$$x_2(K, D) = \min\{D_2, K_2, K_4 - x_4\}, \tag{87}$$

$$x_3(K, D) = \min\{D_2 - x_2, K_2 - x_2, K_3 - x_1\}. \tag{88}$$

Partition the demand space as in Figure 9. A capacity investment vector $K^* \in R^4_+$ is optimal if and only if

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ v_2 - v_3 \end{pmatrix} P(\Omega_p(K^*)) + \begin{pmatrix} 0 \\ 0 \\ v_1 - v_4 \\ 0 \end{pmatrix} P(\Omega_q(K^*)) + \begin{pmatrix} 0 \\ v_3 \\ 0 \\ v_2 - v_3 \end{pmatrix} P(\Omega_1(K^*))$$

$$+\begin{pmatrix} 0 \\ 0 \\ v_3 \\ v_2 \end{pmatrix} P(\Omega_2(K^*) + \begin{pmatrix} v_4 - v_2 \\ 0 \\ v_1 + v_2 - v_4 \\ v_2 \end{pmatrix} P(\Omega_3(K^*) + \begin{pmatrix} 0 \\ 0 \\ v_1 + v_2 - v_4 \\ v_2 \end{pmatrix} P(\Omega_4(K^*)$$

$$+\begin{pmatrix} v_4 \\ 0 \\ v_1 - v_4 \\ 0 \end{pmatrix} P(\Omega_5(K^*) \quad = \quad \tilde{c}_K - \mu - \theta, \tag{89}$$

$$\mu' K^* \quad = \quad 0, \tag{90}$$

$$\theta_i(K_i^* - K_{3,i}^*) \quad = \quad 0, \ i \in \{1, 2\}, \tag{91}$$

where $\mu \in R_+^4$ and $\theta = (\theta_1, \theta_2, -\theta_1, -\theta_2), \theta_1, \theta_2 \in R_+$. Consider the boundary solution $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$ and the first-order conditions simplify to equation (19)-(21). The rest of the proof is similar to the medium Δp case. ■

Proposition 4 Proof. The proof is similar to the medium Δp case because both the demand partition and greedy solutions are identical. The modification is that we consider boundary solution $\underline{K} = (\underline{K}_1, \underline{K}_2, 0, \underline{K}_2)$ instead because boundary solution $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$ does not exist for this case. This is evident from the expression of $\bar{c}_T$ derived previously. Since $\Delta p \leq \Delta c_M$, the numerator of $\bar{c}_T$ is strictly negative. Hence, $c_T < \bar{c}_T$ can never be satisfied. ■

Table 2 Proof. We prove the comparative statics for $\bar{c}_T$, the proof for $\underline{c}_T$ is omitted due to similarity. As mentioned in the main text, the proof boils down to determining the sign of $d\bar{K}_1/dy$. For boundary solution $\bar{K} = (\bar{K}_1, \bar{K}_2, \bar{K}_1, 0)$, the optimization problem is reduced to two dimensional. Thus, the Hessian matrix becomes

$$\bar{H} \quad = \quad \bar{\Lambda} D_K \bar{P} = \begin{pmatrix} 0 & 0 & v_3 & v_1 \\ 0 & v_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} I_2 & I_2 \\ I_6 & -I_1 - I_6 \\ -I_2 - I_6 + I_7 & I_6 \\ -I_7 & 0 \end{pmatrix} \tag{92}$$

$$= \quad \begin{pmatrix} -v_3(I_2 + I_6) - (v_1 - v_3) I_7 & v_3 I_6 \\ v_3 I_6 & -v_3(I_1 + I_6) \end{pmatrix}. \tag{93}$$

Taking derivative w.r.t. $\tilde{c}_K = (c_{K,1}, c_{K,2}, c_{K,3})$ on both sides of the first-order condition and applying chain rule gives

$$D_K^2 \mathbb{E}\pi(K,D) D_{\tilde{c}_K}\bar{K} = D_{\tilde{c}_K} c_K = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \bar{H} D_{\tilde{c}_K}\bar{K}. \tag{94}$$

Hence,

$$\begin{aligned} D_{\tilde{c}_K}\bar{K} &= \bar{H}^{-1}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} && (95) \\ &= \frac{1}{|\bar{H}|}\begin{pmatrix} -v_3(I_1+I_6) & -v_3 I_6 & -v_3(I_1+I_6) \\ -v_3 I_6 & -v_3(I_2+I_6)-(v_1-v_3)I_7 & -v_3 I_6 \end{pmatrix}. && (96) \end{aligned}$$

All elements of $D_{\tilde{c}_K}\bar{K}$ are negative, which follows from the negative definiteness of $\bar{H}$ (thus $|\bar{H}| > 0$). Therefore

$$\text{sign}(\frac{d\bar{K}_1}{dc_{K,i}}) = -1, \ \ i = 1,2,3. \tag{97}$$

Now we prove the part for $p_i's$ and $c'_{M,i}s$. From the first-order condition, we have

$$\bar{\Lambda}\bar{P} = c_K - \mu - \theta. \tag{98}$$

Taking derivative w.r.t. to $y (y = p_i$ or $c_{M,i})$,

$$\frac{\partial\bar{\Lambda}}{\partial y}\bar{P} + \bar{\Lambda}\frac{\partial\bar{P}}{\partial y} = \frac{\partial\bar{\Lambda}}{\partial y}\bar{P} + \bar{\Lambda}\nabla_K\bar{P}\frac{\partial\bar{K}}{\partial y} = 0. \tag{99}$$

Since

$$\begin{aligned} \bar{H} &= \bar{\Lambda} D_K \bar{P}, && (100) \\ \bar{\Lambda} &= \begin{pmatrix} 0 & 0 & v_3 & v_1 \\ 0 & v_3 & 0 & 0 \end{pmatrix}, && (101) \\ \frac{\partial\bar{K}}{\partial y} &= -\bar{H}^{-1}\frac{\partial\bar{\Lambda}}{\partial y}\bar{P}, && (102) \end{aligned}$$

it follows that,

$$\frac{\partial \bar{K}}{\partial p_1} = -\bar{H}^{-1}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\bar{P} = \frac{1}{|\bar{H}|}\begin{pmatrix} \bar{P}_3 v_3(I_1+I_6) \\ \bar{P}_3 v_3 I_6 \end{pmatrix} >> 0, \tag{103}$$

$$\frac{\partial \bar{K}}{\partial p_1} = -\bar{H}^{-1}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\bar{P} \tag{104}$$

$$= \frac{1}{|\bar{H}|}\begin{pmatrix} \bar{P}_1 v_3 I_6 + \bar{P}_2 v_3(I_1+I_6) \\ \bar{P}_1 v_3(I_2+I_6) + \bar{P}_1(v_1-v_3)I_7 + \bar{P}_2 v_3 I_6 \end{pmatrix} >> 0, \tag{105}$$

$$\frac{\partial \bar{K}}{\partial c_{M,1}} = -\bar{H}^{-1}\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 0 \end{pmatrix}\bar{P} \tag{106}$$

$$= \frac{1}{|\bar{H}|}\begin{pmatrix} -\bar{P}_1 v_3 I_6 - \bar{P}_2 v_3(I_1+I_6) - \bar{P}_3 v_3(I_1+I_6) \\ -\bar{P}_1 v_3(I_2+I_6) - \bar{P}_1(v_1-v_3)I_7 - \bar{P}_2 v_3 I_6 - \bar{P}_3 v_3 I_6 \end{pmatrix} << 0, \tag{107}$$

$$\frac{\partial \bar{K}}{\partial c_{M,2}} = -\bar{H}^{-1}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\bar{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{108}$$

■

Property 1 Proof. It follows from a similar proof as in Proposition 3 and 4 of Van Mieghem (1998). ■

Proposition 6 Proof. By Proposition 3 in Van Mieghem & Rudi (2002), it is sufficient to show π is submodular in D. This boils down to show $\partial\pi/\partial D_1 = v'\partial x/\partial D_1$ is decreasing in D_2. Consider three scenarios (see Figure 9): (i) if $0 < D_1 < K_3 + K_4 - K_2$ or $D_1 > K_1$, then, $v'\partial x/\partial D_1$ remains unchanged for any value of D_2; (ii) if $K_3 + K_4 - K_2 < D_1 < K_3$, then as D_2 increases, $v'\partial x/\partial D_1$ remains constant throughout Ω_0, Ω_p, and Ω_2,but decreases from v_1 to $v_1 - v_3$ when crossing the border of Ω_p and Ω_2; (iii) if $K_3 < D_1 < K_1$, then as D_2 increases, $v'\partial x/\partial D_1$ remains constant throughout Ω_q and Ω_4,but decreases from $v_1 + v_4$ to $v_1 + v_4 - v_3$ when crossing the border of Ω_q and Ω_2. ■

Proposition 7 Proof. (i) High and medium Δp. First notice that the total demand $k \geq \bar{K}_1$. Otherwise, both $\bar{P}_2$ and $\bar{P}_3$ are zero, violating the first order condition of the boundary solution. $k < \bar{K}_1$ then implies the following set of equations that determine the boundary solution $\bar{K}$.

$$v_3\bar{P}_2 + v_1\bar{P}_3 = c_{K,1} + c_{K,3}, \tag{109}$$

$$v_3\bar{P}_1 = c_{K,2}, \tag{110}$$

$$\bar{P}_1 + \bar{P}_2 + \bar{P}_3 = 1. \tag{111}$$

Solving the equation gives

$$\bar{P}_1 = \frac{c_{K,2}}{v_3}, \tag{112}$$

$$\bar{P}_2 = \frac{v_1(v_3 - c_{K,2}) - v_3(c_{K,1} + c_{K,3})}{v_3(v_1 - v_3)}, \tag{113}$$

$$\bar{P}_3 = \frac{c_{K,1} + c_{K,2} + c_{K,3} - v_3}{v_1 - v_3}. \tag{114}$$

It follows that

$$\begin{aligned} \bar{c}_T - c_T &= \frac{\Delta p \bar{P}_3 - \Delta c_M - c_{K,1}}{1 - \bar{P}_3} - c_T && (115) \\ &= \frac{(\Delta p + c_T)(c_{M,2} + c_{K,2} + c_{K,3} - p_2)}{p_1 - c_{M,1} - (c_{K,1} + c_{K,2} + c_{K,3})} && (116) \\ &= \frac{(v_1 - v_3)(c_{K,2} + c_{K,3} - v_2)}{v_1 - (c_{K,1} + c_{K,2} + c_{K,3})}. && (117) \end{aligned}$$

In order to have a feasible boundary solution $\bar{K}$, we must have $\bar{P}_1 > 0$, $\bar{P}_2 > 0$, and $\bar{P}_3 \geq 0$. It follows from the expression of $\bar{P}_i's$ that

$$v_3 \leq c_{K,1} + c_{K,2} + c_{K,3} \tag{118}$$

and

$$\frac{c_{K,1} + c_{K,3}}{v_1} + \frac{c_{K,2}}{v_3} < 1. \tag{119}$$

These two conditions further imply that

$$v_1 > c_{K,1} + c_{K,2} + c_{K,3}. \tag{120}$$

Suppose to the contrary that $v_1 \leq c_{K,1} + c_{K,2} + c_{K,3}$, then

$$\begin{aligned} \frac{c_{K,1} + c_{K,3}}{v_1} + \frac{c_{K,2}}{v_3} &\geq \frac{c_{K,1} + c_{K,3}}{c_{K,1} + c_{K,2} + c_{K,3}} + \frac{c_{K,2}}{v_3} && (121) \\ &\geq \frac{c_{K,1} + c_{K,3}}{c_{K,1} + c_{K,2} + c_{K,3}} + \frac{c_{K,2}}{c_{K,1} + c_{K,2} + c_{K,3}} = 1, && (122) \end{aligned}$$

contradicting the second condition above. Finally, centralization in the high-price market is optimal if and only if $\bar{c}_T > c_T$, which requires $v_2 < c_{K,2} + c_{K,3}$ given that $v_1 > c_{K,1} + c_{K,2} + c_{K,3}$ and $v_1 > v_3$. The proof for part (ii) is similar to (i). ■

Proposition 8 Proof. (i) Medium Δp. As the demand curve is reduced to a 45-degree line starting from the origin, there are four cases to consider depending on where in the demand partition space the demand curve passes through. Case(1): $K_2^* < K_3^* + K_4^* - K_2^*$, i.e. $P(\Omega_q) = P(\Omega_4) =$

$P(\Omega_5) = 0$. The first order conditions reduce to

$$0 = c_{K,1} - \theta_1, \quad (123)$$

$$v_3 P(\Omega_1) = c_{K,2} - \theta_2, \quad (124)$$

$$v_3 P(\Omega_2) + v_1 P(\Omega_3) = c_{K,3} + \theta_1 - \mu_3, \quad (125)$$

$$(v_2 - v_3)P(\Omega_p) + (v_2 - v_3)P(\Omega_1) + v_2(P(\Omega_2) + P(\Omega_3)) = c_{K,3} + \theta_2 - \mu_4. \quad (126)$$

$\theta_1 = c_{K,1} > 0$ implies that $K_1^* = K_3^*$ and thus $P(\Omega_p) = P(\Omega_4) = 0$. $(123) + (125)$ and $(124) + (126)$ and rearranging give

$$\frac{v_3}{v_1}P(\Omega_2) + P(\Omega_3) + \frac{\mu_3}{v_1} = \frac{c_{K,1} + c_{K,3}}{v_1}, \quad (127)$$

$$1 - P(\Omega_0) - \frac{v_3}{v_2}P(\Omega_p) + \frac{\mu_4}{v_2} = \frac{c_{K,2} + c_{K,3}}{v_2}. \quad (128)$$

Since $\frac{v_3}{v1}P(\Omega_2) + P(\Omega_3) < 1 - P(\Omega_0) - \frac{v_3}{v_2}P(\Omega_p)$, this case is feasible only if $\frac{c_{K,1}+c_{K,3}}{v_1} < \frac{c_{K,2}+c_{K,3}}{v_2}$. Case (2): $K_2^* \geq K_3^* + K_4^* - K_2^*$ and $K_3^* \geq K_4^*$, i.e. $P(\Omega_q) = P(\Omega_1) = P(\Omega_4) = P(\Omega_5) = 0$. It follows from the FOCs above that $\theta_1 = c_{K,1}$ and $\theta_2 = c_{K,2}$. Hence, $P(\Omega_p) = P(\Omega_2) = 0$ and $K_1^* = K_3^* = K_2^* = K_4^*$. The FOCs simplify to

$$v_1 P(\Omega_3) = c_{K,1} + c_{K,3} - \mu_3, \quad (129)$$

$$v_2 P(\Omega_3) = c_{K,2} + c_{K,3} - \mu_4. \quad (130)$$

Since $\mu_3 = \mu_4 = 0$, this case is feasible only if $\frac{c_{K,1}+c_{K,3}}{v_1} = \frac{c_{K,2}+c_{K,3}}{v_2}$. Case (3): $K_1^* \geq K_3^* + K_4^* - K_1^*$ and $K_4^* \geq K_3^*$, i.e., $P(\Omega_p) = P(\Omega_1) = P(\Omega_2) = P(\Omega_5) = 0$. Similar to Case (2), this case gives $K_1^* = K_3^* = K_2^* = K_4^*$ and it is feasible only if $\frac{c_{K,1}+c_{K,3}}{v_1} = \frac{c_{K,2}+c_{K,3}}{v_2}$. Case (4): $K_1^* < K_3^* + K_4^* - K_1^*$, i.e., $P(\Omega_p) = P(\Omega_1) = P(\Omega_2) = 0$. This case is a mirror image of Case (1). The simplified FOCs become

$$1 - P(\Omega_0) - \frac{v_4}{v_1}P(\Omega_q) + \frac{\mu_3}{v_1} = \frac{c_{K,1} + c_{K,3}}{v_1}, \quad (131)$$

$$P(\Omega_3) + \frac{v_4}{v_2}P(\Omega_4) + + \frac{\mu_4}{v_2} = \frac{c_{K,2} + c_{K,3}}{v_2}. \quad (132)$$

This case gives $K_2^* = K_4^*$ and is feasible only if $\frac{c_{K,1}+c_{K,3}}{v_1} > \frac{c_{K,2}+c_{K,3}}{v_2}$. ■

Lemma 3 Proof. The optimal solutions to the centralized configurations is a feasible solution to the decentralized configuration, which implies $V_{\text{dec}} \geq V_{\text{cen}}$. If $V_{\text{dec}} > V_{\text{on}}$, we must have $K_4^* > 0$. Suppose not, then $K_4^* = 0$, which implies the optimal solution is also a solution to the centralized configuration problem. This contradicts the fact that $V_{\text{dec}} > V_{\text{on}}$. If $V_{\text{dec}} = V_{\text{on}}$, the uniqueness of the optimal solution to the decentralized configuration problem yields $K_4^* = 0$. Similarly for $V_{\text{off.}}$ ■

Proposition 9 Proof. (i) Since $V_{\text{dec}} - 2c_0 \leq V_{\text{on}} - c_0 \leq V_{\text{off}} - c_0$, offshore production leads to the highest network value. Similarly for (ii) and (iii). ■