Security Design in a Production Economy with Flexible Information Acquisition*

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This Version: March, 2015
First Draft: June, 2012

Abstract

We offer a unified theory of the use of debt and non-debt securities in a production economy, positing that the investor can acquire costly information on the entrepreneur's project and then screen it through financing decisions. This generates a new informational friction: real production depends on information acquisition, but the two functions are performed separately by the entrepreneur and investor. Debt is optimal when the dependence is weak (the friction is not severe), and a combination of debt and equity (equivalent to participating convertible preferred stock in terms of cash flow) is optimal when the dependence is strong (the friction is severe). The optimality of the different securities in different circumstances yields new pecking orders for financing private businesses, which are consistent with the empirical facts. Flexible information acquisition allows us to work with arbitrary feasible securities over continuous states and to dispense with any distributional assumptions.

JEL: D82, D86, G24, G32, L26

*Earlier versions have been circulated under the title, "Venture Finance under Flexible Information Acquisition." We thank Malcolm Baker, John Campbell, Peter DeMarzo, Darrell Duffie, Emmanuel Farhi, Paolo Fulghieri, Mark Garmaise, Simon Gervais, Itay Goldstein, Daniel Green, Barney Hartman-Glaser, Ben Hebert, Steven Kaplan, Arvind Krishnamurthy, Josh Lerner, Deborah Lucas, Stephen Morris, Marcus Opp, Jonathan Parker, Raghu Rajan, Adriano Rampini, David Robinson, Hyun Song Shin, Andrei Shleifer, Alp Simsek, Jeremy Stein, S. Viswanathan, Michael Woodford; our conference discussants Mark Chen, Diego Garcia, Mark Loewenstein, Christian Opp, John Zhu, and seminar and conference participants at Berkeley Haas, Duke Fuqua, Harvard, Minnesota Carlson, MIT Sloan, Peking University, Stanford GSB, UNC Kenan-Flagler, University of Toronto, University of Vienna, 2013 WFA, 2013 Wharton Conference on Liquidity and Financial Crises, 2013 CEPR European Summer Symposium in Financial Market, 2013 Finance Theory Group Summer Meeting, 2013 Toulouse TIGER Forum, 2014 SFS Cavalcade, 2014 CICF, and 2014 Summer Institute of Finance for helpful comments. Ming Yang: ming.yang@duke.edu. Yao Zeng: yaozeng@fas.harvard.edu.

1 Introduction

Both debt and non-debt securities are commonly viewed as optimal financing approaches in different real-world corporate finance contexts, though it is a challenge to obtain the two as the optimal securities in different circumstances within a unified theoretical framework. This paper achieves this goal. Security design research ordinarily postulates that an entrepreneur with a project but without financial resources proposes specific contracts to an investor to get finance. The entrepreneur is often modeled as an expert who is more informed about the project. However, this common approach misses a crucial point: some investors are better able than the entrepreneur herself to acquire information and thus to assess a project's uncertain market prospects, drawing upon their industry experience. For instance, start-ups seek venture capital, and most venture capitalists are themselves former founders of successful start-ups, so they may be better able to determine whether new technologies match the market.[1] Tirole (2006) points out that one shortcoming of the classical corporate finance literature is that it overlooks this informational advantage of investors.[2] Our paper fills the gap by uncovering the interaction between entrepreneurs' security design and investos' endogenous information acquisition and screening.[3] It enables us to provide a unified theory of debt and non-debt securities and to construct new pecking orders[4] well suited for financing private businesses, which are consistent with the empirical evidence.

In a production setting, the investor's endogenous information advantage over the entrepreneur[5] leads to a new informational friction. Specifically, in our model, the investor can acquire costly but flexible information about the project's uncertain cash flow before making the financing decision. Only when the investor believes the project is good enough, will it be financed. Hence, the entrepreneur's real production depends on the investor's information acquisition, but

[1] A famous example features Peter Thiel, the first outside investor of Facebook. Thiel himself was the former founder of Paypal. In August 2004, Thiel made a $500,000 investment in Facebook in the form of convertible notes.

[2] Some exceptions have been surveyed in Bond, Edmans and Goldstein (2012). However, most of those papers focus on the role of competitive financial markets in soliciting or aggregating the information of investors or speculators (for instance, Boot and Thakor, 1993, Fulghieri and Lukin, 2001, Axelson, 2007, Garmaise, 2007, Hennessy, 2013, on security design) rather than the role of screening by individual investors. In reality, most firms are private and do not have easy access to a competitive financial market. A burgeoning security design literature highlights individual investors' endogenous information advantage directly (Dang, Gorton and Holmstrom, 2011, Yang, 2013), but these models are built to capture the asset-backed securities market as an exchange economy and not fit for the corporate finance setting with production.

[3] In our model, the terms information acquisition and screening mean the same thing. Henceforth, we use them interchangeably in verbal discussion to ease understanding. As our model does not feature entrepreneurs' private information, our notion of screening is however different from the notion of separating (different types of entrepreneurs) commonly used in the literature involving asymmetric information.

[4] Our notion of pecking order speaks to the entrepreneur's priorities of one optimal security over the other in different economic circumstances. It implies orders of optimal securities over the dimensions of certain parameters. It is more general than the classical concept in Myers and Majluf (1984), featuring the dimension of financing cost.

[5] We do not attempt to deny that entrepreneurs in reality may have private information about their technologies, which has been discussed extensively in the previous literature. Rather, we highlight the overlooked fact that investors may acquire information and become more informed about the potential match between new technologies and the market. Consequently, this also implies that our model does not feature any signaling mechanism.

these two are conducted separately, constituting the friction at the heart of our model.

Facing this friction, the entrepreneur designs a security that incentivizes the investor to acquire information in favor of the entrepreneur. Two conflicting forces arise from the friction. On the one hand, the entrepreneur wants to compensate the investor more generously, to induce her to acquire information and to screen the project more effectively, leading to a higher social surplus. This first force comes from the dependence of the entrepreneur's real production on the investor's information acquisition. The second force, on the other hand, derives from their separation: as the entrepreneur always shares the social surplus with the investor, she also wants to retain more. Therefore, the optimal security for the entrepreneur reflects the competition of the two forces.

Our model predicts standard debt and the combination of debt and equity (participating convertible preferred stock in terms of cash flow) as optimal securities in different circumstances, and constructs new pecking orders for financing entrepreneurial production. Our predictions help bridge the security design literature and the classical pecking order theory, because our pecking orders are derived from an optimization over a very general feasible security space, as opposed to monotone securities or even a given set of securities like debt and equity. Moreover, in solving for optimal securities, we do not need restrictive assumptions concerning priors or information structures. The cash flows are also modeled over continuous states and admit arbitrary distributions, as opposed to the finite states or continuous states with given distributional assumptions, which are common in the literature. Hence, our predictions are both precise and robust.

When the dependence of real production on information acquisition is weak, that is, the friction is not severe, the optimal security is debt, which does not induce information acquisition. This case corresponds to scenarios where the project's ex-ante market prospects are already good enough or the screening cost is high. This prediction is consistent with the evidence that conventional start-ups and mature private businesses rely heavily on plain-vanilla debt finance from investors who are not good at screening, such as relatives, friends, and banks (see Berger and Udell, 1998, Kerr and Nanda, 2009, Robb and Robinson, 2014). The intuition is clear: since the benefit of screening does not justify its cost, the entrepreneur finds it optimal to deter costly information acquisition by issuing debt, the least information-sensitive security. The investor thus makes the investment decision based on her prior. Interestingly, our intuition for the optimality of debt is different from the conventional wisdom, as our mechanism does not feature signaling.[6]

In contrast, when the dependence of real production on information acquisition is strong, that is, the friction is severe, the optimal security is a combination of debt and equity that induces the investor to acquire information. Regarding cash flow rights, this is equivalent to participating convertible preferred stock. This case corresponds to scenarios where the project's ex-ante market prospects are not good enough or the cost of screening is low. This prediction is

[6] Notable results regarding debt as the least information-sensitive security to mitigate adverse selection include Myers and Majluf (1984), Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), Dang, Gorton and Holmstrom (2011), Yang (2013). None of those papers considers production.

new to the security design literature, but nevertheless fits well with the empirical facts (Sahlman, 1990, Gompers, 1999). In particular, convertible preferred stocks have been used in almost all the contracts between entrepreneurs and venture investors, and nearly half of them are participating, as documented in Kaplan and Stromberg (2003). Participating convertible preferred stock is popular in particular for the early rounds of investment (Kaplan and Stromberg, 2003), when the friction is more severe.

The optimality of the combination of debt and equity is more subtle. First, the entrepreneur wants to induce the investor to screen only if the investor will screen in a potentially good project and screen out bad ones.[7] That is, any project with a higher ex-post cash flow will have a better chance to be financed ex-ante. Only when the investor's payoff is high in good states while low in bad states, the investor has the right incentive to distinguish between these different states by developing such a screening rule, because she only wants to invest when the likelihood of high payment is high. Consequently, the entrepreneur can maximally benefit from this by ensuring that her own payoff is also high when the investor's is. Therefore, an equity component with payments that are strictly increasing in the project's cash flow is offered, encouraging the investor to acquire adequate information to distinguish between any different states. However, the investor's information after screening is still imperfect, albeit perhaps with a better posterior. In other words, the investor may still end up financing a bad project after screening. Thus, downside protection is necessary to ensure that the investor will not simply reject the project without any information acquisition. This justifies the debt component. These intuitions further suggest that straight or leveraged equity alone is not optimal for financing new projects, also consistent with reality (Kaplan and Stromberg, 2003, Lerner, Leamon and Hardymon, 2012).

A new concept, flexible information acquisition, characterizes the nature of screening as described above and helps establish the predictions on the payment structure, or in visual terms, the "shape" of the securities. Differently shaped securities incentivize the investor to screen the project with varying intensity and also to allocate attention to different states of the cash flows. For instance, debt, with its flatter shape, is less likely than equity to prompt screening. Moreover, in screening, a debt holder only allocates attention to states with low cash flows, as the payments are constant over states with high cash flows so there is no point in differentiating the latter. In contrast, levered equity holders pay attention to states with high cash flows, as they benefits from the upside payments. An arbitrary security determines the investor's incentives for screening and attention allocation in this state-contingent way, and these in turn affect the entrepreneur's incentives in designing the security. The traditional approach of exogenous information asymmetry does not characterize these incentives adequately. Recent models of endogenous information acquisition also fail to capture such flexibility of incentives, since they only consider the amount

[7] Our model features continuous state, but we use the notions of good and bad at times to help develop intuitions.

or precision of information (see Veldkamp, 2011, for a review).[8] Our approach, following Yang (2012, 2013), is based on rational inattention (Sims, 2003, Woodford, 2008), but has a different focus.[9] It captures not only how much but also what kind of information the investor acquires through state-contingent attention allocation. In our setting, when screening is desirable, the optimal security encourages the investor to allocate adequate attention to all states so as to effectively distinguish potentially good from bad projects, and thus delivers the highest possible ex-ante profit to the entrepreneur. This mechanism generates the exact shape of participating convertible preferred stock, which the previous literature cannot deliver.

We map the above friction-based pecking orders of debt and convertible preferred stock to three empirical dimensions: ex-ante profitability (net present value, or NPV) of the project, its uncertainty, and the cost of screening. Comparative statics of the optimal securities over the three dimensions suggest that different projects will be endogenously financed by different securities, and potentially by different types of investors. The role of screening varies, but is still unified under the friction: the differing extent of the dependence of real production on information acquisition.

The generality of our model in unifying the use of debt and non-debt securities is derived from the identification of the costs and benefits of screening in different economies. In our production economy (a primary financial market), the aggregate cash flow depends on the financing decision. In comparison with the present paper, Yang (2013) considers a model where a seller has an asset in place and proposes an asset-backed security to a more patient buyer to raise liquidity. The buyer can also flexibly acquire information before purchase. That model features an exchange economy (a secondary financial market), as the aggregate cash flow is fixed. In such an exchange economy, social surplus depends negatively on information acquisition. Debt is the only optimal financing there, because it mitigates the buyer's adverse selection to the greatest extent. In contrast, in the present model, social surplus may depend positively on information acquisition. Adverse selection is no longer the focus, and debt may no longer be optimal when information acquisition is desirable. This contrast is reminiscent of Hirshleifer (1971), which distinguishes between information value in an exchange and in a production economy. Earlier mechanism design literature on information gathering also hints at this difference, suggesting that the contribution of information provision on liquidity would differ accordingly (Cremer and Khalil, 1992, Cremer, Khalil and Rochet, 1998a,b).[10]

Along with its sharp predictions, our parsimonious framework acommodates a variety of theoretical corporate finance contexts and real-world scenarios of financing entrepreneurial production.

[8]We call those information acquisition technologies rigid, because they impose parametric restrictions on the signals, while flexible information acquisition allows for any conditional distribution over the fundamental.

[9]The original rational inattention concept mainly captures the bounded rationality of agents: they have limited capacity of attention and cannot pay full attention to all payoff-related states or variables. Flexible information acquisition highlights the opposite: agents have the capacity to allocate attention to different states of the economy in a flexible way. This makes flexible information acquisition more suitable to study strategic interactions, while the original rational inattention works for decision problems to generate rigidity-related phenomena.

[10]But they do not focus on security design and do not predict the forms of optimal contracts in different markets.

On the one hand, we take the investor to be a screening expert. The acknowledgement of investors' screening dates back to Knight (1921) and Schumpeter (1942). Apart from extensive anecdotal evidence (see Kaplan and Lerner, 2010, Da Rin, Hellmann and Puri, 2011, for reviews), recent empirical literature (Chemmanur, Krishnan and Nandy, 2012, Kerr, Lerner and Schoar, 2014) has also identified direct screening by various investors. As the cost of screening pertains both to the project's nature and to the investor's information expertise, it also allows us to cover various investors, including family and friends, banks, and venture capitalists. On the other hand, we do highlight two particular aspects of the entrepreneur, capturing the nature of private businesses that account for most firms. First, the entrepreneur is financially constrained. Second, her human capital is inalienable, which means the investor cannot take over the project and the entrepreneur has bargaining power in designing the security. Nevertheless, relaxing these assumptions does not affect our results.[11] These settings fit the notion of entrepreneur-led financing proposed by Admati and Pfleiderer (1994), as well as the thesis, set out in Rajan (2012), that entrepreneurs' human capital is important in the early stages of firms' life cycles. This paper, to the best of our knowledge, is the first to investigate the interplay between security design and screening in a production economy and the first to deliver robust predictions consistent with the empirical evidence on contracts between real-world entrepreneurs and investors.

Related Literature. In addition to the security design literature that identifies debt as the most information-insensitive form of finance (as mentioned above), this paper is related to a series of theoretical papers that predict that non-debt securities will be optimal and potentially invalidates the classical pecking orders in various circumstances with hidden information (see Brennan and Kraus, 1987, Constantinides and Grundy, 1989, Nachman and Noe, 1994, Chemmanur and Fulghieri, 1997, Inderst and Mueller, 2006, Chakraborty and Yilmaz, 2011, Chakraborty, Gervais and Yilmaz, 2011, Fulghieri, Garcia and Hackbarth, 2013). Even closer to the present paper are Boot and Thakor (1993), Fulghieri and Lukin (2001), Axelson (2007), Garmaise (2007), and Hennessy (2013), all of which highlight the competitive financial markets' role in soliciting or aggregating investors' private information. These papers, however, do not consider individual investors' screening directly. Unlike these papers, which employ exogenous information asymmetry or rigid information acquisition, our model delivers a clear interaction between security design and screening. This allows us to unify the use of debt and non-debt securities, and to identify their optimality in different circumstances. Furthermore, previous models can only admit discrete states, continuous states with distributional assumptions,[12] or restricted sets of feasible securities.[13] With flexible information acquisition, we can model arbitrary securities over

[11] The results of optimal securities continue to hold, either if the project is transferrable or if the entrepreneur does not have full bargaining power in designing the security. See subsection 3.3 and subsection 6.2.

[12] For example, the monotone likelihood ratio property (MLRP), or various forms of stochastic dominance.

[13] Most of the security design literature only considers monotone securities, or even requires monotone residuals. We endogenize these properties.

continuous states with arbitrary distributions and information structures, and thus characterize the optimal securities in a more rigorous and robust way. Finally, our framework also allows for a new interpretation of screening in terms of moral hazard, suggesting a bridge between hidden information and hidden action, which are often addressed as separate frictions in the classical contract design literature.

A new strand of literature on the real effects of rating agencies (see Kashyap and Kovrijnykh, 2013, Opp, Opp and Harris, 2013) is also relevant. On behalf of investors, the rating agency screens the firm, which does not know its own type. Information acquisition may improve social surplus through ratings and the resulting investment decisions. Unlike this literature, we study how different shapes of securities interact with the incentives to allocate attention in acquiring information and therefore the equilibrium financing choice. Flexible information acquisition also allows us to combine the two roles of rating agencies and investors, and thus to flesh out the impact of endogenous screening on security design.

Our model also contributes to the venture contract design literature by highlighting screening. Security design is one focus of modern research in entrepreneurial finance and innovation, but the literature mostly focuses on control rights (Berglof, 1994, Hellmann, 1998, Kirilenko, 2001), monitoring (Ravid and Spiegel, 1997, Schmidt, 2003, Casamatta, 2003, Hellmann, 2006), and refinancing and staging (Admati and Pfleiderer, 1994, Bergemann and Hege, 1998, Cornelli and Yosha, 2003, Repullo and Suarez, 2004) and tends to ignore screening. Further, most of these models achieve only one class of optimal security, working with restrictive sets of feasible securities, discrete states, or restrictive distributional assumptions. In contrast, our model unifies debt and convertible preferred stock in a more general framework and provides a consistent mapping of their optimality to different real-world circumstances.

Finally, a growing behavioral literature deals with attention allocation in the interaction between firms and consumers. Using a setting similar to ours, Bordalo, Gennaioli and Shleifer (2013) argue that consumers may pay different amounts of attention to different product attributes through distorted payoff perceptions, and such attention allocation interacts with product design and competition.[14] Complementing this literature, our work focuses on attention allocation across states through information acquisition and delivers insights within the rational Bayesian updating paradigm.

The rest of the paper is organized as follows. Section 2 specifies the economy and elaborates the concept of flexible information acquisition. The optimal securities are characterized and discussed in Section 3. Section 4 characterizes our new pecking orders. Section 5 performs comparative statics on the optimal securities. Section 6 demonstrates the robustness of the results by discussing and extending the model. All proofs are attached in Appendix A.2.

[14] An implication of this mechanism in financial markets is that financial innovations may draw investors' attention to returns instead of risks, which leads to neglected risks highlighted by Gennaioli, Shleifer and Vishny (2012).

2 The Model

We present a stylized model of a production economy, focusing on the interplay between security design and flexible screening. We highlight the key friction: the dependence of real production on information acquisition and the former's simultaneous separation from the latter.

2.1 Financing Entrepreneurial Production

Consider an economy with two dates, $t = 0, 1$, and a single consumption good. There are two agents: an entrepreneur lacking financial resources and a deep-pocket investor, both risk-neutral. Their utility function is the sum of consumptions over the two dates: $u = c_0 + c_1$, where c_t denotes an agent's consumption at date t. In what follows we use subscripts E and I to indicate the entrepreneur and the investor, respectively.

We consider the financing of the entrepreneur's risky project. To initiate the project at date 0, the underlying technology requires an investment of $k > 0$. If financed, the project generates a non-negative verifiable random cash flow θ at date 1. The project cannot be initiated partially. Hence, the entrepreneur has to raise k, by selling a security to the investor at date 0. The payment of a security at date 1 is a mapping $s : \mathbb{R}_+ \to \mathbb{R}_+$ such that $s(\theta) \in [0, \theta]$ for any θ. We focus only on the cash flow of projects and securities rather than the control rights.

We specify the processes of security design and information acquisition, both at date 0. The agents have a common prior Π on the potential project's future cash flow θ, and neither party has any private information ex-ante.[15] The entrepreneur designs the security, and then proposes a take-it-or-leave-it offer to the investor at price k. Facing the offer, the investor acquires information about θ in the manner of rational inattention (Sims, 2003, Woodford, 2008, Yang, 2012, 2013), updates beliefs on θ, and then decides whether to accept the offer. We model this process through flexible information acquisition, where the information acquired is measured by reduction of entropy. The information cost per unit reduction of entropy is μ, defined as the cost of screening. We elaborate flexible information acquisition in subsection 2.2.

The assumptions implicit in the setting reflect the key features of financing entrepreneurial production, in particular the role of screening. First, the entrepreneur cannot undertake the project except by external finance. This is consistent with the empirical evidence that entrepreneurs and private firms are often financially constrained (Evans and Jovanovic, 1989, Holtz-Eakin, Joulfaian and Rosen, 1994). Even in mature firms, managers may seek outside finance where the internal capital market does not work well for risky projects (Stein, 1997, Scharfstein and Stein, 2000). Second, the investor can acquire information about the cash flow and thus screen the project

[15] We can interpret this setting as that the entrepreneur may still have some private information about the future cash flow, but she does not have any effective ways to signal that to the investor. The signaling channel has been extensively discussed in the literature and already well understood, so we choose to leave it aside for highlighting our focus on screening.

through her financing decision. This point not only accounts for the empirical evidence but also sets this model apart from most of the previous security design literature, which features the entrepreneur's exogenous information advantage. These two points together lead to the dependence and separation of real production and information acquisition, which is the key friction in our model.

It is worth noting which aspects of finance in the production economy are abstracted away, and how much they affect our work. First, to focus on screening, we set aside classical moral hazard, as it does not help deliver useful insights. To ignore moral hazard is common in the security design literature, especially when hidden information is important (see DeMarzo and Duffie, 1999, for a justification). Interestingly, in our context screening has a natural but new moral hazard interpretation (discussed in subsection 6.1). Second, we do not focus on the bargaining process and the allocation of control rights. We assume that the entrepreneur's human capital is inalienable, so that direct project transfer is impossible and the entrepreneur has the bargaining power to design the security. This notion of entrepreneur-led financing is also common in the literature (Brennan and Kraus, 1987, Constantinides and Grundy, 1989, Admati and Pfleiderer, 1994). Together with the differentiation argument in Rajan (2012), this assumption also broadly corresponds to the earlier incomplete contract literature, which suggests that ownership should go to the entrepreneur when firms are young (Aghion and Tirole, 1994). In subsection 3.3 we formally demonstrate that even if the project is transferrable, it is not optimal to transfer the project at any fixed price. Moreover, in subsection 6.2 we discuss a general allocation of bargaining power between the two agents and we show that our main results are unaffected unless the investor's bargaining power is too strong.[16] Third, we do not model the staging of finance, and we accordingly interpret the cash flow θ as already incorporating the consequences of investors' exiting. Hence, each round of investment may be mapped to our model separately with a different prior. Fourth, we do not model competition among investors. The third and fourth points pertain to the micro-structure of financial markets, which is tangential to the friction we consider. Last, risk neutrality enables us to focus on screening, not risk sharing, which is of less interest for our purpose.

2.2 Flexible Information Acquisition

We model the investor's screening by flexible information acquisition (Yang, 2013).[17] This captures the nature of screening and allows us to work with arbitrary securities over continuous states and without distributional assumptions. As a result, we are able to deliver sharp predictions. Fundamentally, the entrepreneur can design the security's payoff structure arbitrarily, which may produce arbitrary attention allocation by the investor in screening the project. This therefore

[16] The results are different if the investor's bargaining power is strong, but these results are still intuitive based on the friction point of view. See subsection 6.2 for details.

[17] For more detailed expositions of flexible information acquisition, see Woodford (2008) and Yang (2012, 2013).

calls for an equally flexible account of screening to capture the interaction between the shape of the securities and the incentives to allocate attention. This cannot be attained by the classical information acquisition technologies.

The essence of flexible information acquisition is that it captures not only how much but also which aspects of information an agent acquires. Consider an agent who chooses a binary action $a \in \{0,1\}$ and receives a payoff $u(a,\theta)$, where $\theta \in \mathbb{R}_+$ is the fundamental, distributed according to a continuous probability measure Π over $\mathbb{R}_+$. Before making a decision, the agent may acquire information through a set of binary-signal information structures, each signal corresponding to one optimal action.[18] Specifically, she may choose a measurable function $m : \mathbb{R}_+ \to [0,1]$, the probability of observing signal 1 if the true state is θ, and acquire binary signals $x \in \{0,1\}$ parameterized by $m(\theta)$; $m(\theta)$ is chosen to ensure that the agent's optimal action is 1 (or 0) when observing 1 (or 0). By choosing different functional forms of $m(\theta)$, the agent can make the signal correlate with the fundamental in any arbitrary way.[19] Intuitively, for instance, if the agent's payoff is sensitive to fluctuations of the state within some range $A \subset \mathbb{R}_+$, she would pay more attention to this range by making $m(\theta)$ co-vary more with θ in A. This gives us a desirable account to model an agent's incentive to acquire different aspects of information.

The conditional probability $m(\cdot)$ embodies a natural interpretation of screening. In our setting of financing entrepreneurial production, conditional on a cash flow θ, $m(\theta)$ is the probability of the project's being screened in and thus getting financed. It is state-contingent, capturing the investor's incentive to allocate attention in screening a project. In particular, the absolute value of the first order derivative $|dm(\theta)/d\theta|$ represents the screening intensity: when it is larger, the investor differentiates the states around θ better. Thus, in what follows we call $m(\cdot)$ a screening rule.

We then characterize the cost of information acquisition. As in Woodford (2008) and Yang (2013), the amount of information conveyed by a screening rule $m(\cdot)$ is defined as the expected reduction of uncertainty through observation of the signal, where the uncertainty associated with a distribution is measured by Shannon (1948)'s entropy. Formally, we use the concept of mutual information, which is defined as the difference between agents' prior entropy and expected posterior entropy:

$$\begin{aligned} I(m) &= H(prior) - H(posterior) \\ &= -g(\mathbb{E}[m(\theta)]) - (-\mathbb{E}[g(m(\theta))]) , \end{aligned}$$

where $g(x) = x \cdot \ln x + (1-x) \cdot \ln(1-x)$, and the expectation operator $\mathbb{E}(\cdot)$ is with respect to

[18] In general, an agent can choose any information structure. But an agent always prefers binary-signal information structures in binary decision problems. See Woodford (2008) and Yang (2012) for formal discussions.

[19] Technically, this allows agents to choose signals drawn from any conditional distribution of the fundamental, as opposed to classical information acquisition technologies that often involve restrictions on the signals to be acquired.

θ under the probability measure Π. Denote by $M = \{m \in L(\mathbb{R}_+, \Pi) : \theta \in \mathbb{R}_+, m(\theta) \in [0,1]\}$ the set of binary-signal information structures, and $c : M \to \mathbb{R}_+$ the cost of information. The cost is assumed to be proportional to the associated mutual information:

$$c(m) = \mu \cdot I(m) \ ,$$

where $\mu > 0$ is the marginal cost of information acquisition per unit of reduction of entropy.[20,21]

Built upon flexible information acquisition, the agent's problem is to choose a functional form of $m(\theta)$ to maximize expected payoff less information cost. We characterize the optimal screening rule $m(\theta)$ in the following proposition. We denote $\Delta u(\theta) = u(1,\theta) - u(0,\theta)$, which is the payoff gain of taking action 1 over action 0. We also assume that $\Pr[\Delta u(\theta) \neq 0] > 0$ to exclude the trivial case where the agent is always indifferent between the two actions. The proof is in Yang (2013) (see also Woodford, 2008, for an earlier treatment).

Proposition 1. *Given u, Π, and $\mu > 0$, let $m^*(\theta) \in M$ be an optimal screening rule and*

$$\bar{\pi}^* = \mathbb{E}[m^*(\theta)]$$

be the corresponding unconditional probability of taking action 1. Then,

i) the optimal screening rule is unique;

ii) there are three cases for the optimal screening rule:

a) $\bar{\pi}^* = 1$, i.e., $Prob[m^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \Delta u(\theta)\right)\right] \leqslant 1\,; \tag{2.1}$$

b) $\bar{\pi}^* = 0$, i.e., $Prob[m^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(\mu^{-1} \cdot \Delta u(\theta)\right)\right] \leqslant 1\,;$$

c) $0 < \bar{\pi}^* < 1$ and $Prob[0 < m^*(\theta) < 1] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(\mu^{-1} \cdot \Delta u(\theta)\right)\right] > 1 \text{ and } \mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \Delta u(\theta)\right)\right] > 1 \ ; \tag{2.2}$$

in this case, the optimal screening rule $m^*(\theta)$ is determined by the equation

$$\Delta u(\theta) = \mu \cdot \left(g'(m^*(\theta)) - g'(\bar{\pi}^*)\right) \tag{2.3}$$

[20] Although the cost $c(m)$ is linear in mutual information $I(m)$, it does not mean it is linear in information acquisition. Essentially, mutual information $I(m)$ is a non-linear functional of the screening rule $m(\cdot)$ and the prior Π, micro-founded by the information theory.

[21] The cost function following rational inattention also implies that all states are homogenous in terms of the cost of information acquisition. That is, it is equally costly to differentiate any states. See Woodford (2012) and Yang (2013) for extensive discussions on this point.

for all $\theta \in \mathbb{R}_+$, where

$$g'(x) = \ln\left(\frac{x}{1-x}\right) .$$

Proposition 1 fully characterizes the agent's possible optimal decisions of information acquisition. Case a) and Case b) correspond to the scenarios of ex-ante optimal action 1 or 0. These two cases do not involve information acquisition. They correspond to the scenarios in which the prior is extreme or the cost of information acquisition is sufficiently high. But case c), the more interesting one, involves information acquisition. In particular, the optimal screening rule $m^*(\theta)$ is not constant in this case, and neither action 1 nor 0 is optimal ex-ante. This case corresponds to the scenario where the prior is not extreme, or the cost of information acquisition is sufficiently low. In Case c) where information acquisition is involved, the agent equates the marginal benefit of information with its marginal cost. So doing, the agent chooses the shape of $m^*(\theta)$ according to the shape of payoff gain $\Delta u(\theta)$ and her prior Π.[22] In the next section we will see that the shape of $m^*(\theta)$ is crucial in characterizing the way in which the investor screens a project.

3 Security Design

Now let us consider the entrepreneur's security design problem. Denote the optimal security of the entrepreneur by $s^*(\theta)$. The entrepreneur and the investor play a dynamic Bayesian game. Concretely, the entrepreneur designs the security, and then the investor screens the project given the security designed. Hence, we apply Proposition 1 to the investor's problem, given the security, and then solve backwards for the entrepreneur's optimal security. To distinguish from the decision problem above, we denote the investor's optimal screening rule as $m_s(\theta)$, given the security $s(\theta)$; hence the investor's optimal screening rule is now denoted by $m_s^*(\theta)$.

We formally define the equilibrium as follows.

Definition **1.** *Given u, Π, k and $\mu > 0$, the sequential equilibrium is defined as a combination of the entrepreneur's optimal security $s^*(\theta)$ and the investor's optimal screening rule $m_s(\theta)$ for any generic security $s(\theta)$, such that*

i) the investor optimally acquires information at any generic information set induced by $s(\theta)$: $m_s(\theta)$ is prescribed by Proposition 1,[23] *and*

ii) the entrepreneur designs the optimal security:

$$s^*(\theta) \in \arg \max_{0 \leqslant s(\theta) \leqslant \theta} \mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] .$$

According to Proposition 1, there are three possible investor behaviors, given the entrepreneur's optimal security. First, the investor may optimally choose not to acquire information

[22] See Woodford (2008), Yang (2012, 2013) for more examples on this decision problem.

[23] The specification of belief for the investor at any generic information set is also implicitly given by Proposition 1, provided the definition of $m_s(\theta)$.

and simply accept the security as proposed. This implies that the project is certainly financed. Second, the investor may optimally acquire some information, induced by the proposed security, and then accept the entrepreneur's optimal security with a positive probability. In this case, the project is financed with a probability that is positive but less than one. Third, the investor may simply reject the security without acquiring information, which implies that the project is certainly not financed. All the three cases can be accommodated by the equilibrium definition. This third case, however, represents the outside option of the entrepreneur, who can always offer nothing to the investor and drop the project. Accordingly, we focus on the first two cases. The following lemma helps distinguish the first two types of equilibrium from the third.

Lemma **1.** *The project can be financed with a positive probability in equilibrium if and only if*

$$\mathbb{E}\left[\exp(\mu^{-1} \cdot (\theta - k))\right] > 1\,. \tag{3.1}$$

Lemma 1 is an intuitive investment criterion. It implies that the security is more likely to be accepted by the investor if the prior of the cash flow is better, if the initial investment k is smaller, or if the cost of screening μ is lower. When condition (3.1) is violated, the investor will reject the proposed security, whatever it is.

Condition (3.1) appears different from the ex-ante NPV criterion, which suggests that a project should be financed for sure when $\mathbb{E}[\theta] - k > 0$. In our model with screening, by Jensen's inequality, condition (3.1) implies that any project with positive ex-ante NPV will be financed with a positive probability. Moreover, some projects with negative ex-ante NPV may also be financed with a positive probability. This is consistent with our idea that real production depends on information acquisition. Thanks to screening, the ex-ante NPV criterion based on a fixed prior is generalized to a new information-adjusted one to admit belief updating.

The following Corollary 1 implies that the entrepreneur will never propose to concede the entire cash flow to the investor if the project is financed. This corollary is straightforward but worth stressing, in that it helps illustrate the key friction by showing that the interests of the entrepreneur and the investor are not perfectly aligned. It also helps establish some important later results. Intuitively, retaining a little bit more would still result in a positive probability of financing while also giving the entrepreneur a positive expected payoff.

Corollary **1.** *When the project can be financed with a positive probability, $s^*(\theta) = \theta$ is not an optimal security.*

In what follows, we assume that condition (3.1) is satisfied, and characterize the entrepreneur's optimal security, focusing on the first two types of equilibrium with a positive screening cost $\mu > 0$. As we will see, the entrepreneur's optimal security differs between the two cases, which implies that the investor screens the project differently. We further show that to transfer the project at a given price is always sub-optimal, which also justifies the security design approach. Finally, for additional intuitions, we consider two limiting cases, one with infinite and one with zero screening cost.

3.1 Optimal Security without Inducing Information Acquisition

In this subsection, we consider the case in which the entrepreneur's optimal security is accepted by the investor without information acquisition. In other words, the entrepreneur finds screening not worthwhile and wants to design a security to deter it. Concretely, this means $Pr\left[m_s(\theta)=1\right]=1$. We first consider the investor's problem of screening, given the entrepreneur's security, then characterize the optimal security.

Given a security $s(\theta)$, the investor's payoff gain from accepting rather than rejecting the security is

$$\Delta u_I(\theta)=u_I(1,\theta)-u_I(0,\theta)=s\left(\theta\right)-k\,. \tag{3.2}$$

According to Proposition 1 and conditions (2.1) and (3.2), any security $s(\theta)$ that is accepted by the investor without information acquisition must satisfy

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot\left(s\left(\theta\right)-k\right)\right)\right]\leqslant 1\,. \tag{3.3}$$

If the left-hand side of inequality (3.3) is strictly less than one, the entrepreneur could lower $s(\theta)$ to some extent to increase her expected payoff gain, without affecting the investor's incentives. Hence, condition (3.3) always holds as an equality in equilibrium.

By backward induction, the entrepreneur's problem is to choose a security $s(\theta)$ to maximize her expected payoff

$$u_E(s(\cdot))=\mathbb{E}\left[\theta-s(\theta)\right]$$

subject to the investor's information acquisition constraint

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot\left(s\left(\theta\right)-k\right)\right)\right]=1\,,$$

and the feasibility condition $0\leqslant s(\theta)\leqslant\theta$.[24]

In this case, the entrepreneur's optimal security is a debt. We characterize this optimal security by the following proposition, along with its graphical illustration in Figure 1. It is easy to see that the face value of the debt is unique in this case.

[24]With this feasibility condition, the entrepreneur's individual rationality constraint $\mathbb{E}\left[\theta-s(\theta)\right]\geqslant 0$ is automatically satisfied, which is also true for the later case with information acquisition. This comes from the fact that the entrepreneur has no endowment. It also implies that the entrepreneur always prefers to undertake the project, which is consistent with real-world practices. However, it is not correct to interpret this as that the entrepreneur would like to contract with any investor, as we do not model the competition among different investors.

Proposition 2. *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to accept the security without acquiring information in equilibrium, then it takes the form of a debt:*

$$s^*(\theta) = \min(\theta, D^*)$$

where the unique face value D^* is determined by

$$D^* = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu) > k\,,$$

in which λ is a positive constant determined in equilibrium.[25] *Also, the expected payment of the optimal debt satisfies*

$$\mathbb{E}[s^*(\theta)] > k\,.$$

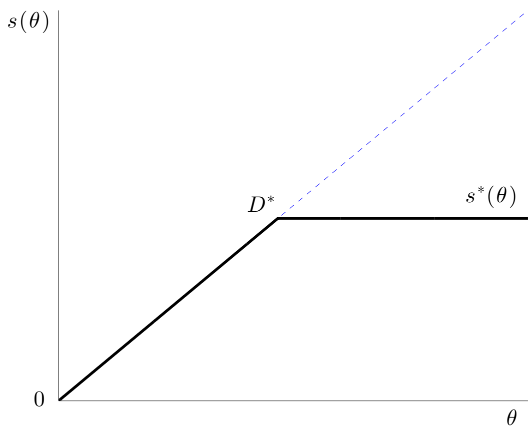

Figure 1: The Unique Optimal Security without Information Acquisition

It is intuitive that debt is the optimal means of finance when the entrepreneur finds it optimal not to induce information acquisition. Since screening is not worthwhile, and the entrepreneur wants to design a security to deter it, debt is the least information-sensitive form that provides the entrepreneur's desired expected payoff. From another perspective, the optimal security enables the investor to break even between acquiring and not acquiring information. Hence, thanks to flexible information acquisition, any mean-preserving spread of the optimal security, which gives the entrepreneur the same expected payoff, would induce the investor to acquire unnecessary information. This implies that the optimal security should be as flat as possible when the limited liability constraint is not binding, which leads to debt.

Interestingly, although in this case the investor only provides material investment (rather than acquire costly information), the expected payment $\mathbb{E}[s^*(\theta)]$ exceeds the investment requirement

[25] Our focus is the qualitative nature of the optimal security, instead of quantities. Thus, we do not solve for the face value in closed form, which is less tractable and does not help deliver insights. This also applies to the other optimal security discussed in subsection 3.2.

k. The extra payment exceeding k works as a premium to make the investor comfortable with accepting the offer with certainty, as otherwise, without screening she might worry about financing a potentially bad project.

Debt accounts for the real-world scenarios in which new projects are financed by fixed-income securities. On the one hand, when a project's market prospects are good and thus not much extra information is needed, it is optimal to deter or mitigate investor's costly information acquisition by resorting to a debt security, which is the least information-sensitive. Interestingly, the rationale for debt in our model does not feature adverse selection, but rather a cost-benefit trade-off of screening. On the other hand, empirical evidence suggests that many conventional businesses and less revolutionary start-ups relying heavily on plain vanilla debt finance from investors who are not good at screening, such as relatives, friends, and traditional banks (for example, Berger and Udell, 1998, Kerr and Nanda, 2009, Robb and Robinson, 2014), as opposed to more sophisticated financial contracts with venture capital or buyout funds.

3.2 Optimal Security Inducing Information Acquisition

Here we characterize the entrepreneur's optimal security that does induce the investor to acquire information and to accept the security with positive probability but not certainty. In other words, the entrepreneur finds screening desirable in this case and designs a security to incentivize it. According to Proposition 1, this means $Prob\left[0 < m_s(\theta) < 1\right] = 1$.

Again, according to Proposition 1 and conditions (2.2) and (3.2), any generic security $s(\theta)$ that induces the investor to acquire information must satisfy

$$\mathbb{E}\left[\exp\left(\mu^{-1}\left(s\left(\theta\right)-k\right)\right)\right] > 1 \tag{3.4}$$

and

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\left(s\left(\theta\right)-k\right)\right)\right] > 1\ , \tag{3.5}$$

Given such a security $s(\theta)$, Proposition 1 and condition (2.3) also prescribe that the investor's optimal screening rule $m_s(\theta)$ is uniquely characterized by

$$s\left(\theta\right)-k=\mu\cdot\left(g'\left(m_s\left(\theta\right)\right)-g'\left(\overline{\pi}_s\right)\right)\ , \tag{3.6}$$

where

$$\overline{\pi}_s=\mathbb{E}\left[m_s\left(\theta\right)\right]$$

is the investor's unconditional probability of accepting the security and does not depend on θ. In what follows, we denote by $\overline{\pi}_s^*$ the unconditional probability induced by the entrepreneur's optimal security $s^*(\theta)$.

We derive the entrepreneur's optimal security backwards. Taking account of investor's response $m_s(\theta)$, the entrepreneur chooses a security $s(\theta)$ to maximize

$$u_E(s(\cdot)) = \mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \tag{3.7}$$

subject to (3.4), (3.5),[26] (3.6), and the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$.[27]

To illustrate the idea, we first offer an intuitive roadmap to investigate the optimal security and the associated optimal screening rule, highlighting their key properties. Then we follow with a formal proposition to characterize the optimal security and discuss its implications. The detailed derivation of the optimal security is presented in Appendix A.1.[28]

First, the investor's optimal screening rule $m_s^*(\theta)$, induced by the optimal security $s^*(\theta)$, must increase in θ. When the entrepreneur finds it optimal to induce information acquisition, screening by the investor benefits the entrepreneur. Effective screening makes sense only if the investor screens in a potentially good project and screens out bad ones; otherwise it lowers the total social surplus. Under flexible information acquisition, this implies that $m_s^*(\theta)$ should be more likely to generate a good signal and to result in a successful finance when the cash flow θ is higher, while more likely to generate a bad signal and a rejection when θ is lower. Therefore, $m_s^*(\theta)$ should be increasing in θ. As we will see, the monotonicity of $m_s^*(\theta)$ and the shape of $s^*(\theta)$ are closely interrelated.

To induce an increasing optimal screening rule $m_s^*(\theta)$, the optimal security $s^*(\theta)$ must be increasing in θ as well, according to the first order condition of information acquisition (3.6). Intuitively, this monotonicity reflects the dependence of real production on information acquisition: the entrepreneur is willing to compensate the investor more in the event of higher cash flow to encourage effective screening. Unlike the classical security design literature, which often restricts the feasible set to non-decreasing securities (for example, Innes, 1990, DeMarzo and Duffie, 1999, DeMarzo, 2005, among others), our model does not need such constraints to predict an increasing optimal security.

We also argue that the non-negative constraint $s(\theta) \geqslant 0$ is not binding for the optimal security $s^*(\theta)$ for any $\theta > 0$. Suppose $s^*(\widetilde{\theta}) = 0$ for some $\widetilde{\theta} > 0$. Since $s^*(\theta)$ is increasing in θ, for all $0 \leqslant \theta \leqslant \widetilde{\theta}$ we must have $s^*(\theta) = 0$. This violates the foregoing argument that $s^*(\theta)$ must be increasing in θ. Intuitively, zero payment in states with low cash flows gives the investor too little incentive to acquire information, which is not optimal for the entrepreneur. The security with zero payment in states with low cash flows looks closest to levered common stock, which is the least commonly used security between entrepreneurs and investors (Kaplan and Stromberg, 2003,

[26] According to Proposition 1, both conditions (3.4) and (3.5) should not be binding for the optimal security; otherwise the investor would not acquire information.

[27] Again, the entrepreneur's individual rationality constraint $\mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \geqslant 0$ is automatically satisfied.

[28] To facilitate understanding, the intuitive investigation of the optimal security is not organized in the same order as the derivation goes in the Appendix, but all the claims in the main text are guaranteed by the formal proofs.

Kaplan and Lerner, 2010, Lerner, Leamon and Hardymon, 2012).

For closer examination of the optimal security, a perturbation argument on the security design problem gives the entrepreneur's first order condition. Specifically, denote by $r^*(\theta)$ the marginal contribution to the entrepreneur's expected payoff $u_E(s(\cdot))$ of any feasible perturbation to the optimal security $s^*(\theta)$.[29] As $s^*(\theta) > 0$ for any $\theta > 0$, it is intuitive to show that for any $\theta > 0$:

$$r^*(\theta) \begin{cases} = 0 & \text{if} \quad 0 < s^*(\theta) < \theta \\ \geqslant 0 & \text{if} \quad \quad s^*(\theta) = \theta \end{cases},$$

which is further shown to be equivalent to

$$(1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} = \mu & \text{if} \quad 0 < s^*(\theta) < \theta \\ \geqslant \mu & \text{if} \quad \quad s^*(\theta) = \theta \end{cases}, \tag{3.8}$$

where w^* is a constant determined in equilibrium.

We argue that the optimal security $s^*(\theta)$ follows the 45° line in states with low cash flows and then increases in θ with some smaller slope in states with high cash flows. That is, the residual of the optimal security, $\theta - s^*(\theta)$, also increases in θ in states with high cash flows. According to the entrepreneur's first order condition (3.8) and the monotonicity of $m_s^*(\theta)$, if $s^*(\widehat{\theta}) = \widehat{\theta}$ for some $\widehat{\theta} > 0$, it must be $s^*(\theta) = \theta$ for any $0 < \theta < \widehat{\theta}$. Similarly, if $s^*(\widehat{\theta}) < \widehat{\theta}$ for some $\widehat{\theta} > 0$, then for any $\theta > \widehat{\theta}$ it must be $s^*(\theta) < \theta$. In addition, Corollary 1 rules out $s^*(\theta) = \theta$ for all $\theta > 0$ as an optimal security. Thus, since $s^*(\theta)$ is increasing in θ, the limited liability constraint can only be binding in states with low cash flows.[30] Importantly, given condition (3.8) and, again the monotonicity of $m_s^*(\theta)$, when the limited liability constraint is not binding in states with high cash flows, not only $s^*(\theta)$ but also $\theta - s^*(\theta)$ are increasing in θ. In other words, $s^*(\theta)$ is dual monotone when it deviates from the 45° line in states with high cash flows.

The shape of the optimal security $s^*(\theta)$ reflects the friction of the economy. Recall that the monotonicity of $s^*(\theta)$ reflects the dependence real production on information. The monotonicity of $\theta - s^*(\theta)$, however, reflects their separation: the entrepreneur wants to retain as much as possible even when incentivizing the investor to screen the project. Specifically, the area between $s^*(\theta)$ and the 45° line not only captures the entrepreneur's retained benefit, but also reflects the degree to which the allocation of resources is inefficient when screening is desirable. This is intuitive: dependence implies that the investor should get all the resources, but separation precludes proposing such a deal, as shown in Corollary 1. The competition of the two forces is alleviated in a most efficient way: rewarding the investor more but also retaining more in better

[29] Formally, $r^*(\theta)$ is the Frechet derivative, the functional derivative used in the calculus of variations, of $u_E(s(\cdot))$ at $s^*(\theta)$. It is analogical to the commonly used derivative of a real-valued function of a single real variable but generalized to accommodate functions on Banach spaces.

[30] In the formal proofs we further show that the limited liability constraint must be binding for some states $(0, \widehat{\theta})$ with $\widehat{\theta} > 0$.

states. In this sense, our prediction of dual monotonicity derives endogenously from the friction of the economy, whereas in the previous literature (for example, Nachman and Noe, 1994, Biais and Mariotti, 2005, Garmaise, 2007, among many others) it must be posited by assumptions.

Formally, the following proposition characterizes the optimal security $s^*(\theta)$ that induces the investor to acquire information. We interpret it as a participating convertible preferred stock.

Proposition **3.** *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to acquire information in equilibrium, then it takes the form of a combination of debt and equity. In terms of cash flow, it is equivalent to a participating convertible preferred stock with a face value $\widehat{\theta} > 0$:*

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta \leqslant \widehat{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta > \widehat{\theta} \end{cases},$$

where $\widehat{\theta}$ is determined in equilibrium and the unconstrained part $\widehat{s}(\theta)$ satisfies:

i) $\widehat{\theta} < \widehat{s}(\theta) < \theta$;

ii) $0 < d\widehat{s}(\theta)/d\theta < 1$.

Finally, the corresponding optimal screening rule satisfies $dm_s^*(\theta)/d\theta > 0$.

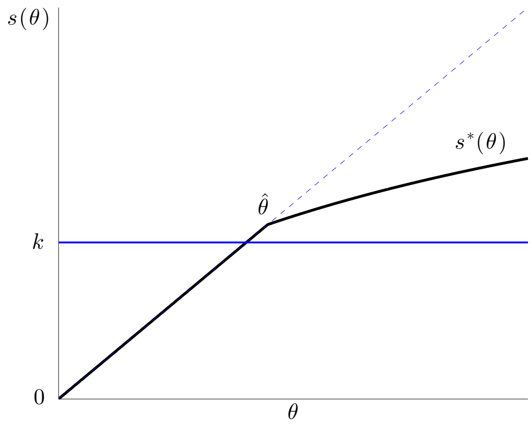

Figure 2: The Unique Optimal Security with Information Acquisition

Proposition 3 offers a clear prediction on the entrepreneur's optimal security when screening is desirable. The form of this security most closely resembles participating convertible preferred stock, with $d\widehat{s}(\theta)/d\theta$ as the converting rate,[31] which grants holders the right to receive both the face value and their equity participation as if it was converted, in the real-world event of a public offering or sale. The payoff structure shown in Figure 2 may be also interpreted as debt

[31] In Appendix A.1, we provide the implicit function that determines $d\widehat{s}(\theta)/d\theta$. We have further shown that $d^2\widehat{s}(\theta)/d\theta^2 < 0$, which implies that the unconstrained part is concave, as illustrated in Figure 2. We interpret this as a state-contingent converting rate, with which the entrepreneur retains more shares in better states. It is consistent with the use of contingent contracts in venture finance and private equity buyouts when screening is relevant, also documented in Kaplan and Stromberg (2003).

plus equity (common stock),[32] or participating convertible debt.[33] This prediction is consistent with the empirical evidence of venture contracts documented in Kaplan and Stromberg (2003), who find that 94.4% of all financing contracts are convertible preferred stock,[34] among which 40.8% are participating, and the participating feature is especially frequent in the earlier rounds of investment. Kaplan and Stromberg (2003) suggest that participating is preferable even to straight convertible preferred stock for screening purpose. Our model implies the same: straight convertible preferred stock is better than debt but still not optimal, because its flat payoffs in intermediate states do not provide enough incentive for the investor to differentiate these states. Our prediction also fits in line with earlier evidence (Sahlman, 1990, Bergemann and Hege, 1998, Gompers, 1999) on the popularity of participating convertible preferred stock and the combination of debt and equity in financing young firms and new projects. For brevity, in what follows we refer to the optimal security in this case as convertible preferred stock.

Flexible information acquisition plays an important role in predicting the shape of convertible preferred stock. When screening is desirable, on the one hand, a globally increasing security incentivizes the investor to pay sufficient attention to all states so as to discriminate between potentially good and bad projects. On the other hand, a higher converting rate $d\widehat{s}(\theta)/d\theta$ induces the investor to screen the underlying project more intensively, at a higher cost to the entrepreneur.[35] Hence, the entrepreneur weighs the benefit of screening against its cost by choosing the optimal converting rates on a state-contingent basis to ensure the highest possible ex-ante profit. By this mechanism, we are also able to work with arbitrary securities on continuous states with arbitrary distributions, which the models in the previous literature cannot do.

Our prediction of the multiple of convertible preferred stock, defined as the ratio of the face value $\widehat{\theta}$ to the investor's initial investment k, is also consistent with the empirical evidence (Kaplan and Stromberg, 2003, Lerner, Leamon and Hardymon, 2012). The multiple is a key characteristic of convertible preferred stock, sometimes taken as analogous to investment returns.

[32] In terms of cash flow rights, the package of redeemable preferred stock (can be viewed as a form of debt) and common stock is used as equivalent to participating convertible preferred stock in practice. But the package is less popular in practice, since it is harder to assign reasonable value to each component of the package, for the following reasons not fully modeled in our work. On the one hand, an expensive common stock component leaves too much downside risk to the investor and perhaps too little tax deferral benefit as well. On the other hand, with a cheap common stock component and an expensive redeemable preferred stock component, the expected large payout to the preferred stock holder at the public offering or sale may cause significant adverse effect on the project's public market value. In contrast, the mandatory conversion feature of participating convertible preferred stock helps overcome the dilemma in reality. Although the entrepreneur still pays back the face value to the investor, it is included in the value of the converted common stocks, and thus the investor do not take it out of the market value. See Lerner, Leamon and Hardymon (2012) for more detailed discussion on this point. Also see Cornelli and Yosha (2003) for a theoretical exposition on the advantage of convertible securities over a combination of debt and equity when characteristics other than the cash flow rights are taken into account.

[33] Compared to equity (common stock), debt and preferred stock are identical in our model as it only features two tranches and no dividends.

[34] If we include convertible debt and the combination of debt and equity, this number increases to 98.1%.

[35] In the Appendix A.1, we formally show that the screening intensity $|d\widehat{m}(\theta)/d\theta|$ in the converting region is increasing in $d\widehat{s}(\theta)/d\theta$.

Corollary **2.** *The multiple of convertible preferred stock, as the optimal security $s^*(\theta)$ that induces information acquisition in equilibrium, is greater than one. In other words, $\widehat{\theta} > k$.*

Like the debt case, this property again derives from the fact that the entrepreneur should offer the investor a premium for accepting the offer with positive probability. Even after screening, the investor's information is still imperfect. Thus, this premium makes the investor comfortable with not rejecting the offer, because even with information acquisition the investor may still end up financing a potentially bad project.

Finally, comparison of the production with an exchange economy helps show why our model can predict both debt and non-debt securities. In a production economy (a primary financial market), costly information contributes to the output, whereas in an exchange economy (a secondary financial market) it only helps reallocate existing resources. Yang (2013) considers security design and information acquisition in a comparable exchange economy. In that model, a seller has an asset in place and proposes a security to a less patient buyer to raise liquidity. The buyer can acquire information about the asset's cash flow before purchasing. In this model debt is uniquely optimal because it offers the greatest mitigation of the buyer's adverse selection. In that exchange economy, information is socially wasteful for the two reasons. First, the information acquired makes the buyer better off at the expense of the seller through endogenous adverse selection and results in illiquidity. Second, information is costly per se. As a result, to discourage information acquisition is desirable. In the present paper, however, the entrepreneur and the investor jointly tap the project's cash flow if the investor accepts the proposed security, not if the security is rejected. Thus, the present model features a production economy in which the social surplus may depend positively on costly information. In this case, adverse selection is no longer the focus. Instead, the entrepreneur may want to design a security that encourages the investor to acquire information favorable to the entrepreneur. That is, debt may no longer be optimal when information acquisition is desirable.

3.3 Project Transfer

This subsection considers the potential for transferring the project at a fixed non-negative price and shows that it is not optimal even though agents are risk neutral. Equivalently, it can be seen as tantamount a scenario in which the entrepreneur works for the investor for a fixed wage, as there is no moral hazard on the entrepreneur's side. According to Rajan (2012), entrepreneurs' human capital is often inalienable because of the great differentiation of young firms, which justifies our security design approach. Here, we further argue that even if the project is transferrable, the entrepreneur still finds it not to be optimal.

The key to understand this idea is to posit project transfer as one of the feasible securities, as in Figure 3, and show that this security is not optimal. When the entrepreneur proposes a project transfer to the investor at a fixed price $p \geqslant 0$, it is equivalent to proposing a security $s(\theta) = \theta - p$ without the non-negative constraint $s(\theta) \geqslant 0$. To see why, notice that if the investor accept the offer of transfer and undertake the project, she gets the entire cash flow θ and pay p as an upfront cost. This interpretation allows us to analyze project transfer in our security design framework.

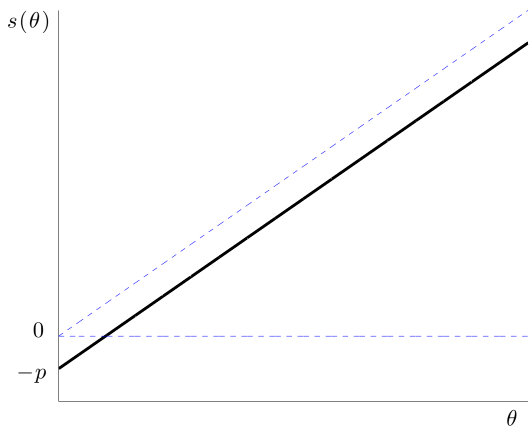

Figure 3: Project Transfer as A Security

To see why the equivalent security $s(\theta) = \theta - p$ is feasible but not optimal, observe that the non-negative constraint $s(\theta) \geqslant 0$ is not binding in either case of security design. Hence, we may also consider a larger set of feasible securities, which is still restricted by the limited liability constraint $s(\theta) \leqslant \theta$ but allows negative payoffs to the investor. As debt and convertible preferred stock are still the only two optimal securities in this generalized problem, and $s(\theta) = \theta - p$ (project transfer) is feasible, we conclude that project transfer is not optimal to the entrepreneur at any transfer price p. Intuitively, project transfer is not optimal because it does not follow the least costly way to compensate the investor, whether or not information acquisition is induced.

PROPOSITION **4.** *When the project can be financed with positive probability, transferring the project at a fixed price $p \geqslant 0$ is not optimal for the entrepreneur.*

The timeline and sequence of moves in the economy is important for Proposition 4. Consider an alternative sequence in which the investor acquires information only after the transfer. In this economy friction is no longer present, because real production and information acquisition are both performed by the investor after the transfer. With this sequence, project transfer is optimal to the entrepreneur through setting a price that is equivalent to the expected profit of the investor. In this case, the entrepreneur's bargaining power is too strong in the sense that she can prevent the investor from acquiring information when proposing the transfer deal, which essentially removes the friction from the economy. In practice, however, it is common (and reasonable) for investors to have the option of acquiring information about the project before the transfer. This justifies our sequence of moves and suggests that transfer is not optimal when screening is important.

3.4 Optimal Security When Cost of Screening is Infinite or Nil

This subsection considers the optimal securities in two limiting cases, i.e., when the cost of screening approaches infinity or zero, offering another perspective for understanding the interaction of security design and screening.[36] When the cost of screening μ is infinity, the investor is unable to acquire information. Thus, both the entrepreneur and the investor make decisions based on the common prior. The investor accepts the entrepreneur's offer if and only if the expected payment $\mathbb{E}[s(\theta)]$ exceeds the investment requirement k, regardless of the shape of the security. On the contrary, when μ is zero, the investor always acquires information, and specifically learns the true state of the cash flow θ. Thus, when θ is lower than k, the investor rejects the offer regardless of its payments. Instead, when θ exceeds k, the investor accepts the offer as long as the entrepreneur promises a payment no less than k. Taking the entrepreneur's optimization into account, we immediately have the following results.

Proposition **5.** *For any project,*

i) if $\mu = \infty$ and $\mathbb{E}[\theta] \geqslant k$, the optimal security $s^*(\theta)$ satisfies $\mathbb{E}[s^*(\theta)] = k$ and $0 \leqslant s^*(\theta) \leqslant \theta$ for any $\theta \in \Theta$;

ii) if $\mu = 0$, the optimal security $s^*(\theta)$ satisfies

$$\begin{cases} 0 \leqslant s^*(\theta) \leqslant \theta & \text{if} \quad 0 \leqslant \theta < k \\ s^*(\theta) = k & \text{if} \quad \theta \geqslant k \end{cases}.$$

It is helpful, to gain a clearer intuition, to to associate the two limiting cases with the two types of generic equilibrium discussed in subsection 3.1 and subsection 3.2 according to their respective information acquisition status. In particular, in the first case we focus on the optimal debt $s^*_\infty(\theta)$ and in the second the optimal (degenerate) convertible preferred stock $s^*_0(\theta)$, among the infinitely many optimal securities in these limiting cases.

Corollary **3.** *For any project,*

i) if $\mu = \infty$ and $\mathbb{E}[\theta] \geqslant k$, the optimal debt $s^*_\infty(\theta)$ satisfies

$$s^*_\infty(\theta) = \min(\theta, D^*_\infty),$$

where $D^*_\infty > k$ is chosen to satisfy $\mathbb{E}[s^*_\infty(\theta)] = k$;

ii) if $\mu = 0$, the optimal (degenerate) convertible preferred stock $s^*_0(\theta)$ satisfies

$$s^*_0(\theta) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta \leqslant k \\ \widehat{\theta}_0 = k & \text{if} \quad \theta > k \end{cases}.$$

[36] In the two limiting cases, Proposition 1 and Definition 1 are not directly applicable. However, as the two cases are both straightforward, we omit a formal equilibrium definition for brevity.

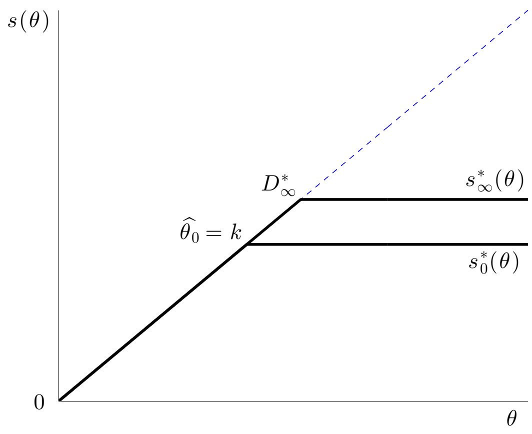

Figure 4: Optimal Securities When the Cost of Screening is Infinity or Zero

Two observations reveal the interaction of security design and screening in these two limiting cases, enabling us to better understand the role of screening. First, compared with the situation when optimal debt does not induce information acquisition in the cases with a positive screening cost μ, the optimal debt $s_\infty^*(\theta)$ has the lowest face value D_∞^*, *ceteris paribus*. This is evident from Proposition 2, which suggests that the face value D^* in those generic cases satisfies $\mathbb{E}[s^*(\theta)] > k$, the extra payment exceeding k as a premium to ensure that the investor will not acquire information, while D_∞^* is chosen to satisfy $\mathbb{E}[s_\infty^*(\theta)] = k$. The intuition is as follows. When $\mu = \infty$, the investor simply cannot acquire information, so there is no need for the entrepreneur to offer the premium. In addition, any security satisfying $\mathbb{E}[s^*(\theta)] = k$ is equivalent to the least expensive debt $s_\infty^*(\theta)$, since the shape of securities is no longer relevant when screening is absent.

Similarly, compared with the situation when convertible preferred stock is optimal, which induces information acquisition in cases with a positive screening cost μ, the optimal (degenerate) convertible preferred stock $s_0^*(\theta)$ has both the lowest face value $\widehat{\theta}_0 = k$ and the lowest (degenerate) converting rate zero, *ceteris paribus*. This is seen from Corollary 2 and Proposition 3, which indicate a multiple larger than one and a positive converting rate. But the intuition here for the lowest face value is subtly different from the case of debt. Since the investor knows the true state and can make investment decisions in a state-contingent way, there is no need to worry about making a bad investment. Thus, there is no need for the entrepreneur to offer the premium exceeding the investment k as in Corollary 2. The intuition for the lowest converting rate is also different. When $\mu = 0$, even though the investor acquires the maximum information, it is completely costless. So again there is no need for the entrepreneur to offer compensation for it. In all, the debt-like convertible preferred stock $s_0^*(\theta)$ represents the least expensive way for the entrepreneur to encourage the investor to acquire information.[37]

[37]Although the optimal security $s_0^*(\theta)$ looks like debt, there are two reasons why we interpret is as convertible preferred stock. First, in our theoretical framework, debt does not induce information acquisition. Second, the

4 Pecking Orders of the Optimal Security

A natural question is: in our production economy, when is debt optimal, and when is convertible preferred stock is optimal? Having characterized the optimal securities with and without induction of screening, we take them together and determine the optimal security given the characteristics of the production economy. It also helps generalize the pecking order theory (Myers and Majluf, 1984): the entrepreneur chooses different optimal securities and thus different capital structures in different circumstances. The new pecking orders help unify the use of debt and convertible preferred stock, which are often viewed as distinct securities in many respects. Our approach also links the security design literature and the classical pecking order theory, since our new pecking orders derive from security design over a general space of feasible securities, rather than a set of commonly observed securities such as debt and equity.

Our new pecking orders are presented in respect of two dimensions: ex-ante NPV and efficiency. The first offers an intuitive perspective for investigating the optimality of the two optimal securities in different circumstances, and highlights two different roles of screening-in and screening-out. The second disclose the relationship between the friction in the production economy and the optimal security in a more fundamental manner. Whether convertible preferred stock or debt is optimal depends on whether or not the entrepreneur wants to encourage screening, which in turn depends on the degree of dependence of real production on information acquisition. If the dependence is strong, the friction in the economy is severe, and screening is accordingly more valuable, so that the entrepreneur finds it more worthwhile to induce screening and proposes convertible preferred stock. Otherwise, the friction is not severe and inducing costly screening is not economic, so debt is optimal.

4.1 NPV Dimension for the Pecking Order

We first benchmark our new pecking order to the ex-ante NPV dimension.

PROPOSITION **6.** *When the project is financed with a positive probability:*

i). If $\mathbb{E}[\theta] \leqslant k$, the optimal security $s^*(\theta)$ is convertible preferred stock; and

ii). If $\mathbb{E}[\theta] > k$, $s^*(\theta)$ may be either convertible preferred stock or debt.[38]

Intuitively, a negative NPV project can only be financed by convertible preferred stock: only through screening could it be found to be potentially good and worth financing. This is consistent

face value of debt should be larger than k to ensure an expected payment no less than k. In an equilibrium with information acquisition, as μ approaches zero, we conjecture that the optimal convertible preferred stock converges to $s_0^*(\theta)$ pointwisely. To formally show this is beyond the interest and scope of this paper, but we provide numerical examples in Section 5 to support this point.

[38] We do not give explicit conditions to distinguish between the two optimal securities when the ex-ante NPV is positive. Technically, doing this requires restrictions on the prior, which does not help deliver insights in general. However, we fully separate the two optimal securities over the efficiency dimension.

with the conventional wisdom that a negative NPV project can never be financed by debt with a given, fixed belief.

Interestingly, convertible preferred stock may be optimal for financing both negative- and positive-NPV projects, but the underlying mechanisms of screening are subtly different. In both cases, the dependence of real production on information acquisition is strong.

When the project has a zero or negative NPV, convertible preferred stock is used to encourage the investor to screen in a potentially good project. The investor will never finance the project without screening it, because it incurs an expected loss even if the entrepreneur promises the entire cash flow. Thus, if it is to be financed, the only way is to use convertible preferred stock to encourage screening. This implies that the dependence of real production on information acquisition is strong due to the relatively poor prior, and thus the friction is accordingly severe. When the investor acquires information, she may expect either a good signal, which leads to a deal, or a bad signal, which results in a rejection, but the ex-ante probability of financing the project becomes positive since a potentially good project can be screened in. Hence, the entrepreneur is better off by proposing convertible preferred stock.

In contrast, when the project has a positive NPV, convertible preferred stock may still be used, but now the aim is to encourage the investor to screen out a potentially bad project. Here, the dependence of real production on information acquisition is still strong due to a relatively mediocre prior. In the status quo where the investor is unable to screen the project, the entrepreneur can finance the positive-NPV project certainly by proposing debt with a high face value. However, when the investor can acquire information, such certain financing might be too expensive or even impossible, because it leaves too little for the entrepreneur. Instead, the entrepreneur could retain more by offering convertible preferred stock, less generous, and invite the investor to screen the project. Although this results in less than certain finance for the project, the entrepreneur's total expected profit is higher, since a potentially bad project may be screened out, which justifies convertible preferred stock as optimal.

Finally, debt may be optimal for some positive-NPV projects. When the prior is sufficiently good, the dependence of real production on information acquisition is weak, and thus the benefit from screening does not justify the cost. In this case, it is optimal for the entrepreneur to propose debt to deter costly screening while still retaining enough profit.

4.2 The Efficiency Dimension for the Pecking Order

We then benchmark our pecking order to a more fundamental efficiency dimension. To understand how the optimal security evolves with the severity of friction, we consider a frictionless centralized economy in which real production and information acquisition are aligned. We define a new efficiency dimension with help of this centralized economy. If and only if the friction in the decentralized economy is not severe in the sense that an optimal security can achieve efficiency, the optimal security is debt and screening is not induced in equilibrium. If and only if the friction is severe in the sense that not even an optimal security can achieve efficiency, the optimal security is convertible preferred stock and screening is induced. This dichotomy again highlights the close interconnection of the shape of the optimal security, the role of screening, and the extent of friction in the production economy.

First let us define the expected social surplus and the efficiency dimension. In the decentralized economy, the expected surplus is the difference between the expected profit of the project and the cost of screening, both of which are functions of the screening rule. Thus, an optimal security achieves efficiency if the induced optimal screening rule maximizes expected social surplus in equilibrium.

Definition **2.** *An optimal security in the decentralized economy achieves efficiency if and only if the associated optimal screening rule $m_s^*(\theta)$ satisfies:*

$$m_s^*(\theta) \in \arg \max_{0 \leqslant m(\theta) \leqslant 1} \mathbb{E}[m(\theta) \cdot (\theta - k)] - \mu \cdot I(m(\theta)) \,. \tag{4.9}$$

To facilitate discussion, we characterize a frictionless centralized economy to help benchmark the friction in the corresponding decentralized economy. In the centralized economy, u, Θ, Π, k and μ are given as the same. However, we assume that the entrepreneur has sufficient initial wealth and can also screen the project. Thus, real production still depends on information acquisition, but the two are aligned. In this economy, security design is irrelevant. The entrepreneur's problem is to decide whether to undertake the project directly, to screen it, or to abandon it. The entrepreneur's payoff gain from undertaking the project rather than abandoning it is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = \theta - k \,.$$

We denote the screening rule in the centralized economy by $m_c(\theta)$ and the optimal screening rule by $m_c^*(\theta)$. Thus, the entrepreneur's problem in the centralized economy is

$$\max_{0 \leqslant m_c(\theta) \leqslant 1} \mathbb{E}[m_c(\theta) \cdot (\theta - k)] - \mu \cdot I(m_c(\theta)) \,. \tag{4.10}$$

By construction, the entrepreneur's objective (4.10) in the centralized economy is just the expected social surplus in the decentralized economy. It is convenient to examine the centralized economy to analyze the efficiency of equilibria in the corresponding decentralized economy.

Since the optimal screening rules are unique for both the centralized and the decentralized economy, efficiency is achieved if and only if information is acquired in the same manner in both.

Lemma **2.** *An optimal security in the decentralized economy achieves efficiency if and only if the* *associated optimal screening rule $m_s^*(\theta)$ satisfies*

$$Prob[m_s^*(\theta) = m_c^*(\theta)] = 1\,,$$

where $m_c^*(\theta)$ is the optimal screening rule in the corresponding centralized economy.

The efficiency concept in Lemma 2 demonstrates the role of screening in the production economy and provides a natural measure of the severity of friction in the decentralized economy. Fundamentally, we may view the expected social surplus (4.10) or (4.9) in our production economy as a production function, with information characterized by the screening rule $m_c(\theta)$ or $m(\theta)$ as the sole input. This again fits with the idea that real production depends on information acquisition. Consequently, efficiency is achieved if and only if the optimal security in the decentralized economy delivers the same equilibrium allocation of input, information acquisition, as the centralized economy. If the optimal security does this, friction in the decentralized economy is not severe, as it can be effectively removed by optimal security design. Otherwise, friction is severe, in that it cannot be completely removed even by an optimal security.

Thanks to the efficiency concept in Definition 2 and Lemma 2, we can characterize our pecking order of optimal securities over the efficiency dimension, as follows.

Proposition **7.** *In the decentralized production economy, when the project is financed with a positive probability:*

i) the optimal security $s^*(\theta)$ is debt if and only if friction in the decentralized economy is not severe, i.e., an optimal security achieves efficiency; and

ii) $s^*(\theta)$ is convertible preferred stock if and only if the friction is severe, i.e., not even an optimal security can achieve efficiency.

This pecking order along the efficiency dimension is important not only because it distinguishes between the two optimal securities given the characteristics of the production economy, but also because it summarizes the effects of optimal securities and of screening in reducing friction. In the decentralized economy, real production is performed by the entrepreneur while information acquisition by the investor. This separation is always present, unchanged despite the different exogenous characteristics of the economy. Hence, the severity of the friction is reflected in the extent to which real production depends on information acquisition. If the friction is severe, the dependence is strong, which makes screening worthwhile and makes convertible preferred stock optimal. Similarly, if the friction is not severe, the dependence is weak, screening does not justify its cost, and debt becomes optimal.

Our pecking orders help unify the empirical evidence. They are particularly well suited for private businesses. In practice, debt financing is popular for conventional projects and for investors who have less expertise in screening, i.e., when the informational friction is not severe. Instead, financing with convertible preferred stock (or a combination of debt and equity) is common for innovative projects, especially in the early rounds, which need more screening and for the investors with better screening capacity, i.e., when the friction is severe.

5 Comparative Statics of the Optimal Security

For additional intuitions, we look at numerical comparative statics on the shape of the optimal securities with respect to three empirical dimensions: the profitability of the project, its uncertainty, and the cost of screening.[39] When the environment varies, the role of screening changes, and the way in which the entrepreneur incentivizes screening changes accordingly, producing different shapes of optimal securities.

5.1 Project Profitability

First, we consider the effects of variations in the project's profitability on the shape of the optimal security $s^*(\theta)$, holding constant the project's market prospects, the prior distribution of the cash flow θ, and the cost of screening, μ. Thus, a decrease in the investment requirement k implies that the project is more profitable ex-ante.

We show the results in Figure 5. The investment k takes three increasing values: 0.4, 0.475, and 0.525. When $k = 0.4$, the optimal security is debt; for two other projects with larger k, one with positive and one with negative ex-ante NPV, it is convertible preferred stock. In particular, the face value $\widehat{\theta}$ and the converting rates $d\widehat{s}(\theta)/d\theta$ of the convertible preferred stock are both increasing in k. For the prior of the cash flow θ, we take a normal distribution with mean 0.5 and standard deviation 0.125, and then truncate and normalize this distribution to the interval $[0, 1]$. The screening cost μ is fixed at 0.2.

The comparative statics with respect to the profitability of the project serve as a detailed illustration of Proposition 6. When the project is sufficiently profitable ex-ante, the friction is not severe and the project will be financed by debt without inducing screening. When the project looks mediocre in terms of its profitability but still has a positive ex-ante NPV, the friction becomes severe, and information acquisition becomes worthwhile to screen bad projects out. When the project is not profitable in the sense that its NPV is negative, the friction is more severe, and the only way for the entrepreneur to obtain financing is to propose convertible preferred stock and expect a potentially good project to be screened in. For this type of project, in particular, screening is more valuable, and hence the entrepreneur is willing to compensate the investor more

[39] We are not aware of any analytical comparative statics pertaining to functionals. An analytical comparative statics requires a total order, which is not applicable for our security space. Even for some ordered characteristics of the optimal security, for instance, the face value, analytical comparative statics are not achievable. Thus, we rely on numerical results to deliver intuitions and leave analytical work to future research. Numerical analysis in our framework is tractable but intensively technical, and the algorithm and codes are available upon request.

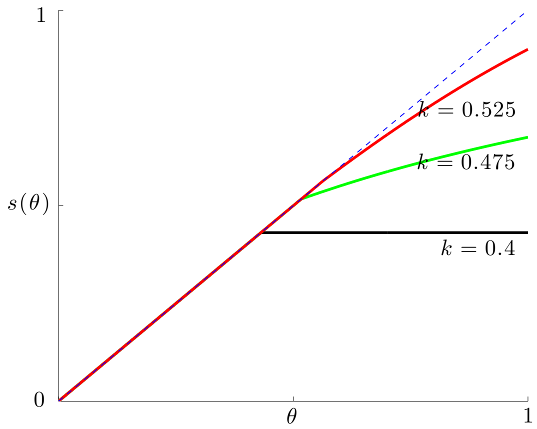

Figure 5: Change of Investment with $\mathbb{E}[\theta] = 0.5, \mu = 0.2$

generously to induce more effective screening.

5.2 Project Uncertainty

We consider the effects on the optimal security $s^*(\theta)$ of varying the degree of the project's uncertainty. Concretely, we consider different prior distributions of the cash flow θ with the same mean, ranked by second order stochastic dominance.[40] We also hold constant the investment requirement k and the cost of screening μ. Note that, the effect of varying uncertainty cannot be accounted for by any argument involving risks, because both the entrepreneur and the investor are risk-neutral. Instead, we still focus on friction and the role of screening to explain these effects.

Interestingly, the comparative statics with respect to uncertainty depend on the sign of the project's ex-ante NPV. As in Proposition 6, the role of screening differs when these signs differ. This further leads to different patterns of comparative statics when the degree of uncertainty is variable.

First, we consider projects with positive ex-ante NPV and increasing uncertainty. The results are shown in Figure 6, the left panel illustrating the priors of the cash flow θ and the right panel the evolution of the optimal security. When the project is the least uncertain, the optimal security is debt. For more uncertain projects convertible preferred stock becomes optimal, while the face value $\widehat{\theta}$ and the converting rates $d\widehat{s}(\theta)/d\theta$ are both increasing in uncertainty. For the priors, we take normal distributions with mean 0.5 and standard deviations 0.125 and 0.25, and then truncate and normalize them to the interval $[0, 1]$. We also construct a third distribution, in which the project is so uncertain that the cash flow has a greater probability of taking extreme values in $[0, 1]$. The investment is $k = 0.4$, and the cost of screening is $\mu = 0.2$.

[40]There are other ways to measure the project's uncertainty. For comparative statics, our idea is to find a partial order of uncertainty over the space of distributions, while to keep the project's ex-ante NPV constant. Thus, second order stochastic dominance is a natural choice.

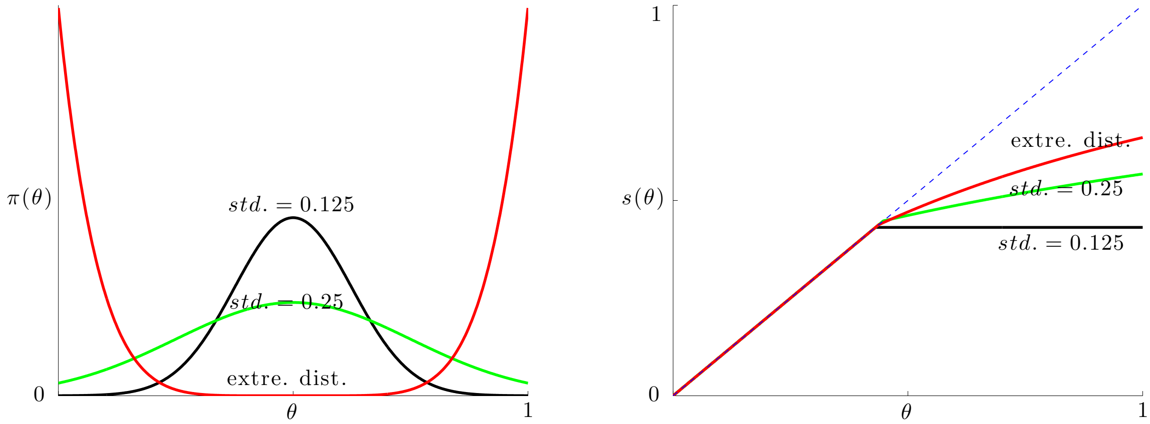

Figure 6: Change of Uncertainty: $k = 0.4 < \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The comparative statics in this case demonstrate how varying uncertainty affects the screening-out of bad projects, given positive ex-ante NPV. When the project is least uncertain, it is least likely to be bad, which implies that screening-out is least relevant and debt financing is accordingly optimal. When uncertainty increases, the project is more likely to be bad, and screening-out becomes more valuable. Hence, the entrepreneur finds it optimal to propose a more generous convertible preferred stock to induce screening-out.

Next we consider projects with negative ex-ante NPV, focusing on the those that may be financed with a positive probability, thanks to screening-in through convertible preferred stock. The results are shown in Figure 7: both the face value $\widehat{\theta}$ and the converting rate $d\widehat{s}(\theta)/d\theta$ of the convertible preferred stock are decreasing in uncertainty. The priors are generated as we did in Figure 6. The investment is $k = 0.525$ and the cost of screening is $\mu = 0.2$.

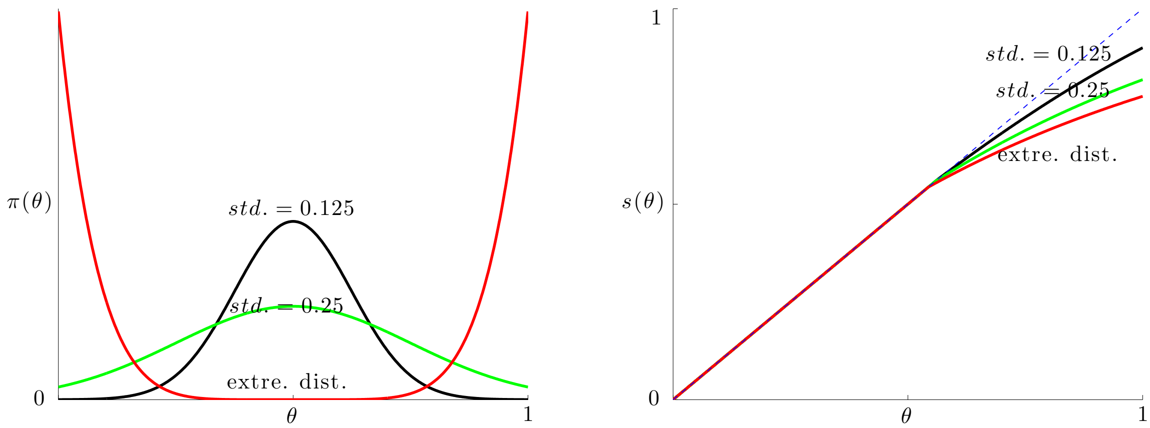

Figure 7: Change of Uncertainty: $k = 0.525 > \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The comparative statics in this case are also intuitive, thanks to the role of screening-in.

Given negative ex-ante NPV, the investor screens in potentially good projects. In contrast to the positive-NPV case, here the increase in uncertainty means that the negative-NPV project is more likely to be good. Thus, to acquire costly information to screen in a potentially good project becomes less necessary. Therefore, the entrepreneur wants to propose a less generous convertible preferred stock for less costly screening. Not surprisingly, the resulting convertible preferred stock moves away from the 45° line when the project is more uncertain.

5.3 Screening Cost

We consider the effects on the optimal security $s^*(\theta)$ of changing the screening cost μ, with the prior of the cash flow θ and the investment requirement k constant. The comparative statics again depend on the sign of the project's ex-ante NPV, and fundamentally, on the different roles of screening-in and screening-out.

First, we consider an ex-ante positive-NPV project with increasing cost of screening. The results are shown in Figure 8. The cost of screening μ takes five values: 0, 0.2, 0.4, 1, and infinity.[41] When μ is low, convertible preferred stock is the optimal financing instrument because the friction is severe and screening-out is called for;[42] debt is instead optimal for high values of μ, when the friction is not severe. The investment is $k = 0.4$. The prior distribution of the cash flow θ is also fixed: we take a normal distribution with mean 0.5 and standard deviation 2, and then truncate and normalize it to the interval $[0, 1]$.

Interestingly, both the face value $\widehat{\theta}$ or D^* and the converting rates $d\widehat{s}(\theta)/d\theta$ exhibit non-monotonic patterns not found in the other comparative statics exercises. On the one hand, $\widehat{\theta}$ is first increasing in the screening cost μ when the optimal security is convertible preferred stock. But then D^* is decreasing in μ when the optimal means of finance becomes debt with high values of μ. On the other hand, $d\widehat{s}(\theta)/d\theta$ first increases in μ from zero (degenerate convertible preferred stock) and then falls back to zero (debt).

The intuitive explanation of the non-monotonic patterns of the face value and the converting rate is more nuanced. First consider the face value. When the screening cost μ is high values and the optimal security is debt, the investor does not acquire information and enjoys a premium for accepting the offer with certainty. In this case, increasing μ diminishes the investor's ability to acquire information even if she is willing to do so. Thus, the entrepreneur can offer a lower premium to ensure that the investor will not acquire information as μ increases. In contrast, when μ is low and the optimal security is convertible preferred stock, the investor still enjoys a premium for financing a potentially bad project by chance. In this case, increasing μ makes it harder to screen a bad project out, so that the investor would demand a higher premium for financing a bad project with higher probability and still accepting the offer with positive probability.

[41]The limiting cases have been characterized in subsection 3.4. We plot them in a separate panel for clarity.

[42]When $\mu = 0$, the optimal convertible preferred stock is degenerate like a debt.

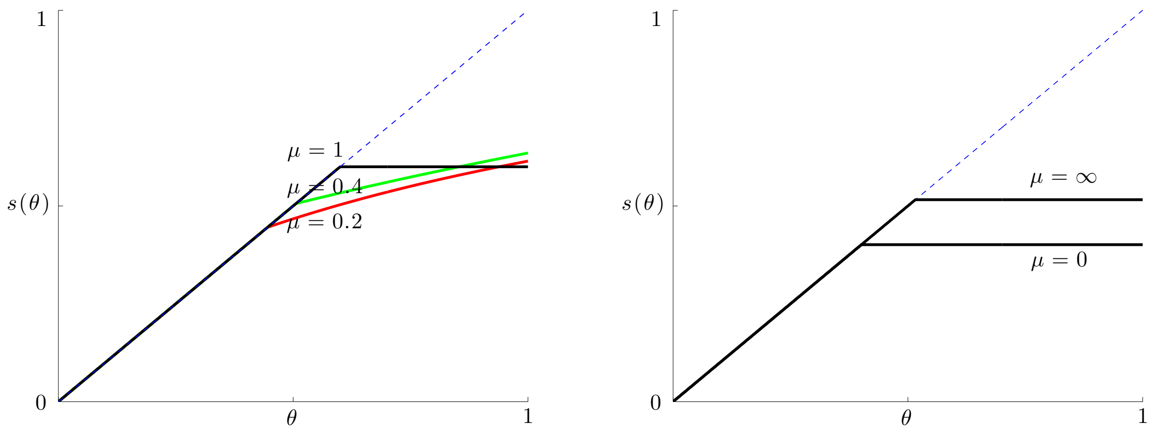

Figure 8: Change of Screening Cost: $k = 0.4 < \mathbb{E}[\theta] = 0.5$

Regarding the converting rates $d\widehat{s}(\theta)/d\theta$, non-monotonicity again stems from the interaction of security design with screening-out. When the screening cost μ is low, then as μ increases, it becomes necessary for the entrepreneur to compensate the investor more generously for the higher screening cost. Thus, the entrepreneur proposes higher converting rates. But when μ is higher, the entrepreneur may find incentivizing screening-out too costly. As μ increases, she instead proposes lower converting rates and a much higher face value (a higher premium), to make the investor screen less intensively but still feel comfortable in accepting the offer. When μ is too high, the entrepreneur just gives up and uses debt with a sufficiently high face value to achieve certain financing without inducing any screening-out.

Finally, consider a negative-NPV project with increasing cost of screening, in which case only convertible preferred stock is optimal and screening-in is called for. The results are shown in Figure 9. The screening cost μ takes three values: 0.075, 0.125, and 0.225. As shown, both the face value $\widehat{\theta}$ and the converting rate $d\widehat{s}(\theta)/d\theta$ of convertible preferred stock are increasing in μ. The investment is $k = 0.6$. The prior is the same as in the previous case: a normal distribution with mean 0.5 and standard deviation 2, and then truncated and normalized to $[0, 1]$.

The intuition is again straightforward under the unified explanation of screening. When financing the negative-NPV project, the investor acquires information to screen in a potentially good project. As screening-in through convertible preferred stock is the only way to finance a project with negative ex-ante NPV, the entrepreneur has no other choices but to compensate the investor more when the cost of screening is higher. This results in a more generous convertible preferred stock.

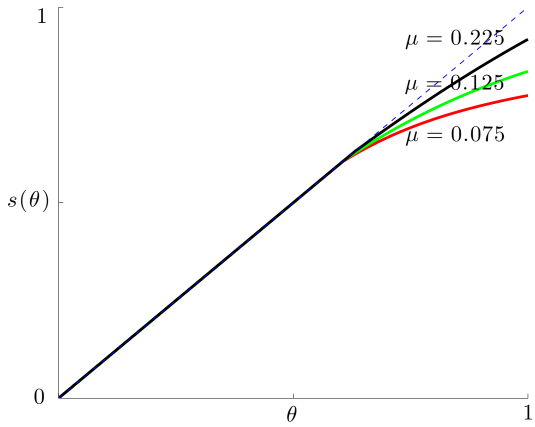

Figure 9: Change of Screening Cost: $k = 0.6 > \mathbb{E}[\theta] = 0.5$

6 Discussion and Extension

In this section, we demonstrate the generality and robustness of our framework by reinterpreting and extending the baseline model.

6.1 The Moral Hazard Interpretation of Screening

This subsection reinterprets investor's screening as a hidden effort, by mapping our setting to the standard principal-agent moral hazard setting. We show that our framework admits a new bridge between hidden information and hidden effort, which the contract design literature often addresses as separate frictions. This reinterpretation also provides an easy way to understand the pecking order over the efficiency dimension as in Proposition 7.

First, it is instructive to interpret information acquisition, captured by the screening rule $m(\theta)$, as the sole effort in the centralized production economy discussed in subsection 4.2. We can see this from the production function in the entrepreneur's problem (4.10):

$$\mathbb{E}[m(\theta) \cdot (\theta - k)] - \mu \cdot I(m(\theta)),$$

where $m(\theta)$ is the only choice variable (functional). The entrepreneur chooses the optimal screening rule $m_c^*(\theta)$ and achieves the first-best allocation. It corresponds to the first-best allocation in typical moral hazard models if effort is observable.

In contrast, in our baseline model as a decentralized production economy, we may interpret the investor as a worker, whose hidden effort is to acquire information. This effort is not observable and non-verifiable, but it determines production, which is observable and verifiable. Parallel to the standard hidden effort models, the entrepreneur (the principal) designs an optimal security (contract) $s^*(\theta)$ to induce a screening rule (hidden effort) $m^*(\theta)$ by the investor (the agent). This reinterpretation is consistent with our Definition 1 of the equilibrium. It is also consistent with the welfare implications of standard hidden effort models, where the first-best can only be achieved if the optimal effort level is zero. As Proposition 7 indicates, first-best is achieved if and only if the optimal screening rule satisfies $Prob[m^*(\theta) = 1] = 1$, i.e., if in equilibrium the investor does not acquire information (zero effort). In this case, the optimal security is debt. On the contrary, when the optimal screening rule satisfies $0 < Prob[m^*(\theta) = 1] < 1$, implying information acquisition (positive effort) in equilibrium, the economy only achieves second-best. In this case, the optimal security is convertible preferred stock.

6.2 General Allocation of Bargaining Powers

This subsection extends our baseline model to a more general setting that allows for the arbitrary allocation of bargaining power between the entrepreneur and the investor. It demonstrates that our framework and qualitative results are robust to the allocation of bargain power.

Without loss of generality, let the entrepreneur's bargaining power in security design be $1 - \alpha$ and the investor's α. Suppose a third party in the economy knows α, designs the security and proposes it to the investor. The investor acquires information according to the security and decides whether or not to accept this offer. The third party's objective function is an average of the entrepreneur's and the investor's utilities, weighted according to the bargaining power of each. When $\alpha = 0$, this reduces to our baseline model. The derivations for the results are the same as in the baseline model.

In this setting, the third-party's objective function, i.e., the payoff gain, is

$$u_T(s(\theta)) = \alpha \cdot (\mathbb{E}[(s(\theta) - k) \cdot m(\theta)] - \mu \cdot I(m)) + (1 - \alpha) \cdot \mathbb{E}[(\theta - s(\theta)) \cdot m(\theta)] \,.$$

We can show that, with information acquisition, the equation that governs information acquisition is still the same as condition (3.6):

$$s\left(\theta\right) - k = \mu \cdot \left(g'\left(m_s\left(\theta\right)\right) - g'\left(\overline{\pi}_s\right)\right) \;,$$

while the equation that characterizes the optimality of the unconstrained optimal security becomes

$$r(\theta) = (2\alpha - 1) \cdot m(\theta) + (1 - \alpha) \cdot \mu^{-1} \cdot m(\theta) \cdot (1 - m(\theta)) \cdot (\theta - s(\theta) + w) \,.$$

The following two propositions characterize the optimal security in the general setting.[43]

Proposition 8. *When $0 \leqslant \alpha < 1/2$ and information acquisition happens in equilibrium, the*

[43] The proofs for the extended model follow those for the benchmark model closely, so we do not repeat them in the appendix.

unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$ satisfy

$$\frac{d\widehat{s}\,(\theta)}{d\theta} = \frac{1-\widehat{m}_s\,(\theta)}{1-\frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} \in (0,1)$$

and

$$\frac{d\widehat{m}_s\,(\theta)}{d\theta} = \frac{\mu^{-1}\cdot \widehat{m}_s\,(\theta)\cdot (1-\widehat{m}_s\,(\theta))^2}{1-\frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} > 0\ .$$

Also, all the results from Lemma 5 to Lemma 8 and from Proposition 1 to Proposition 7 still hold.

Proposition **9.** *When $1/2 \leqslant \alpha \leqslant 1$, the optimal security features $s^*(\theta) = \theta$.*

Our generalized results show that when the investor has some bargaining power, but not too much, all the qualitative results remain unchanged. But if the investor's bargaining power is strong, the optimal security is most favorable to the investor: a complete takeover. This is intuitive considering the situation from the standpoint of friction. If the entrepreneur dominates, she will still plays a considerable role in real production, which depends on the investor's information acquisition. Thus, the presence of friction still calls for a meaningful security design that follows our interaction between the shape of the securities and the incentive for screening. On the contrary, if the investor dominates, she may take over the project and effectively eliminate the friction. In this case, real production and information acquisition are joined and security design is not important. This corresponds to the empirical fact that buyouts and takeovers are common for mature companies, where the role of entrepreneurs and founders is no longer inalienable, a point also highlighted in Rajan (2012) and Lerner, Leamon and Hardymon (2012).

7 Conclusion

This paper posits a new type of informational friction to investigate security design in a production economy. Real production depends on information acquisition, but these two functions are performed separately by entrepreneur and investor. New pecking orders of optimal securities are predicted: debt, which does not induce screening, is optimal when the dependence is weak and the friction is therefore not severe, whereas convertible preferred stock, which does induce screening, is optimal when the dependence is strong and the friction accordingly severe. The findings on both optimal securities and pecking orders are supported by the empirical evidence.

This paper contributes to the security design literature in several respects, as well as to the broader corporate finance and contract design literature. Information-insensitive and information-sensitive securities are brought together in a new, single origin: in financing different projects with different degrees of friction, the investor's expertise in screening is required in different ways, which further influences the shape of the optimal security. This origin is emphasized by a comparison between the production economy and an exchange economy. Thanks to the new concept of flexible information acquisition, we can identify the nature of screening, and work with arbitrary securities over continuous states while dispensing with distributional assumptions.

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A Appendix

A.1 Derivation of Convertible Preferred Stock as the Optimal Security

This appendix derives the optimal security $s^*(\theta)$ when it induces information acquisition. To make the intuition clearer, we proceed by two steps.

First, we solve for an "unconstrained" optimal security without the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$.[44] We denote the solution by $\widehat{s}(\theta)$. We also denote the corresponding screening rule by $\widehat{m}_s(\theta)$. The unconstrained optimal security recovers the unconstrained part $\widehat{s}(\theta)$ of the eventual optimal security in Proposition 3. After that, we resume the feasibility condition and characterize the optimal security $s^*(\theta)$. This two-step approach streamlines our presentation.

Lemma **3.** *In an equilibrium with information acquisition, the unconstrained optimal security $\widehat{s}(\theta)$ and its corresponding screening rule $\widehat{m}_s(\theta)$ are determined by*

$$\widehat{s}\,(\theta) - k = \mu \cdot \left(g'\left(\widehat{m}_s\,(\theta)\right) - g'\left(\overline{\pi}_s^*\right)\right) \ , \tag{A.1}$$

where

$$\overline{\pi}_s^* = \mathbb{E}\left[m_s^*(\theta)\right] \ ,$$

and

$$(1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) = \mu \,, \tag{A.2}$$

where

$$w^* = \mathbb{E}\left[(\theta - s^*\,(\theta))\,\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\left(1 - \mathbb{E}\left[\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\right)^{-1} \ ,$$

in which $\overline{p}_s^*$ and w^* are two constants determined in equilibrium, and $s^*(\theta)$ and $m_s^*(\theta)$ are the solutions of the original constrained problem.

Lemma 3 exhibits the relationship between the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$. Condition (A.1) specifies how the investor responses to the unconstrained optimal security by adjusting her screening rule. On the other hand, condition (A.2) is derived from the entrepreneur's optimization problem. It indicates the entrepreneur's optimal choices of payments across states, given the investor's screening rule. In equilibrium, $\widehat{s}(\theta)$ and $\widehat{m}_s(\theta)$ are jointly determined.

Although it is not tractable to solve the system of equations (A.1) and (A.2), we are able to deliver important analytical characteristics of the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$.

[44] More precisely, $\widehat{m}_s(\theta)$ is not the solution to the entrepreneur's unconstrained problem (without the feasibility condition), but is a translation of that solution. This will be seen clearer in the statement of Lemma 3.

Lemma 4. *In an equilibrium with information acquisition, the unconstrained optimal security* $\widehat{s}(\theta)$ *and the corresponding screening rule* $\widehat{m}_s(\theta)$ *satisfy*

$$\frac{\partial \widehat{m}_s(\theta)}{\partial \theta} = \mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 > 0 \ , \tag{A.3}$$

and

$$\frac{\partial \widehat{s}(\theta)}{\partial \theta} = 1 - \widehat{m}_s(\theta) \in (0, 1) \, . \tag{A.4}$$

We have several interesting observations from Lemma 4. First, condition (A.3) implies that the unconstrained optimal screening rule $\widehat{m}_s(\theta)$ is strictly increasing. Second, condition (A.4) implies that the unconstrained optimal security $\widehat{s}(\theta)$ is also strictly increasing. These are because, according to Proposition 1, we have $Prob[0 < \widehat{m}_s(\theta) < 1] = 1$ in this case, and thus the right hand sides of (A.3) and (A.4) are positive. It follows immediately that the residual of the unconstrained optimal security, $\theta - \widehat{s}(\theta)$, is also strictly increasing. Last, the unconstrained optimal security $\widehat{s}(\theta)$ is strictly concave. This is because conditions (A.3) and (A.4) imply

$$\frac{\partial^2 \widehat{s}(\theta)}{\partial \theta^2} = -\mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 < 0 \ .$$

Therefore, the unconstrained optimal security $\widehat{s}(\theta)$ is an increasing concave function of θ.

The monotonicity of the unconstrained screening rule $\widehat{m}_s(\theta)$ as shown in (A.3) is intuitive. Screening makes sense only when the investor screens in a potentially good project and screens out bad ones. In other words, a better cash flow would be more likely to generate a good signal and result in a successful finance, while a worse cash flow would to more likely to generate a bad signal and result in a rejection. This implies an increasing $\widehat{m}_s(\theta)$.

Moreover, the monotonicity of $\widehat{m}_s(\theta)$ has two important implications on the shape of the unconstrained optimal security $\widehat{s}(\theta)$. On the one hand, according to condition (A.1), $\widehat{s}(\theta)$ is increasing because $\widehat{m}_s(\theta)$ is increasing. This reflects the dependence of real production on information acquisition: the entrepreneur is willing to compensate the investor more in an event of higher cash flow to encourage screening. On the other hand, according to condition (A.2), $\theta - \widehat{s}(\theta)$ is also increasing because $\widehat{m}_s(\theta)$ is increasing. This however reflects the separation of real production and information acquisition: the entrepreneur wants to retain as much as possible. Again as $\widehat{m}_s(\theta)$ is increasing, the competition of the two effects above implies that the least costly way for the entrepreneur to encourage the investor to acquire information is to reward the investor more but also retain more in better states.

Given Lemma 4, it is instructive to have the following lemma to illustrate the possible relative positions between the unconstrained security and the feasibility constraints.

Lemma 5. *Three possible relative positions between the unconstrained optimal security* $\widehat{s}(\theta)$ *and the feasibility constraints* $0 \leqslant s(\theta) \leqslant \theta$ *may occur in equilibrium, in the* $\theta \sim s$ *space:*

i) $\widehat{s}(\theta)$ intersects with the $45°$ line $s = \theta$ at $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, and does not intersect with the horizontal axis $s = 0$;

ii) $\widehat{s}(\theta)$ goes through the origin $(0,0)$, and does not intersect with either the $45°$ line $s = \theta$ or the horizontal axis $s = 0$ for any $\theta \neq 0$;

iii) $\widehat{s}(\theta)$ intersects with the horizontal axis $s = 0$ at $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, and does not intersect with the $45°$ line $s = \theta$.

In the three different cases, it is easy to imagine that the actual optimal security $s^*(\theta)$ will be constrained by the feasibility condition in different ways. For example, $s^*(\theta)$ will be constrained by the $45°$ line $s = \theta$ in Case i) while by the horizontal axis $s = 0$ in Case iii). By imposing the feasibility conditions, we have the following characterization for $s^*(\theta)$:

LEMMA **6.** *In an equilibrium with information acquisition, the corresponding optimal security $s^*(\theta)$ satisfies*

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad \widehat{s}(\theta) > \theta \\ \widehat{s}(\theta) & \text{if} \quad 0 \leqslant \widehat{s}(\theta) \leqslant \theta \\ 0 & \text{if} \quad \widehat{s}(\theta) < 0 \end{cases},$$

where $\widehat{s}(\theta)$ is the corresponding unconstrained optimal security.

Lemma 6 is helpful because it tells us how to construct an optimal security $s^*(\theta)$ from its corresponding unconstrained optimal security $\widehat{s}(\theta)$. Concretely, $s^*(\theta)$ will follow $\widehat{s}(\theta)$ when the latter is within the feasible region $0 \leqslant s \leqslant \theta$. When $\widehat{s}(\theta)$ goes out of the feasible region, the resulting optimal security will follow one of the feasibility constraints that is binding.

We apply Lemma 6 to the three cases of the unconstrained optimal security $\widehat{s}(\theta)$ described in Lemma 5. This gives the three potential cases of the optimal security $s^*(\theta)$, respectively.

LEMMA **7.** *In an equilibrium with information acquisition, the optimal security $s^*(\theta)$ may take one of the following three forms:*

i) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ intersects with the $45°$ line $s = \theta$ at $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, we have

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta < \widehat{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta \geqslant \widehat{\theta} \end{cases};$$

ii) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ goes through the origin $(0,0)$, we have $s^*(\theta) = \widehat{s}(\theta)$ for $\theta \in \mathbb{R}_+$;

iii) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ intersects with the horizontal axis $s = 0$ at $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, we have

$$s^*(\theta) = \begin{cases} 0 & \text{if} \quad 0 \leqslant \theta < \widetilde{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta \geqslant \widetilde{\theta} \end{cases}.$$

The optimal security $s^*(\theta)$ takes different shapes in the three potential cases. In Case i), $s^*(\theta)$ follows a debt in states with low cash flows but increases in states with high cash flows. In Case iii), $s^*(\theta)$ has zero payment in states with low cash flows, while is an increasing function in states with high cash flows. Case ii) lies in between as a cut-off case, in which $s^*(\theta)$ is an increasing function.

We proceed by determining whether these three potential cases are valid solutions to the entrepreneur's problem in an equilibrium with information acquisition. Interestingly, not all the three cases can occur in equilibrium.

LEMMA **8.** *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to acquire information in equilibrium, then it must follow Case i) in Lemma 7, which corresponds to a participating convertible preferred stock with a face value $\widehat{\theta} > 0$.*

Together with the lemmas already established, Lemma 8 immediately leads to Proposition 3. Intuitively, Case ii) and Case iii) in Lemma 7 cannot sustain an equilibrium with information acquisition because the investor is underpaid. Recall that the investor provides two types of inputs. The first is the investment required to initiate the project, and the second is the costly information to screen the project. As a result, the entrepreneur wants to make sure that the investor is sufficiently compensated for both inputs to be willing to accept the security. This argument is further strengthened by Corollary 2, which suggested that $\widehat{\theta}$ should be larger than the investment requirement k.

A.2 Proofs

This appendix provides all proofs omitted above.

PROOF OF LEMMA 1. We first prove the "only if" part. Suppose that

$$\mathbb{E}\left[\exp(\mu^{-1}(\theta - k))\right] \leqslant 1\,.$$

According to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the investor would reject the offer without acquiring information. Since $s(\theta) \leqslant \theta$, the project cannot be initiated in this case.

Then we prove the "if" part. Let $t \in (0, 1)$. Since $\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right]$ is continuous in t, there exists $t < 1$ such that

$$\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right] > 1\,.$$

Hence, according to Proposition 1, the security $s_t(\theta) = t \cdot \theta$ would be accepted by the investor with a positive probability. Moreover, let $m_t(\theta)$ be the corresponding screening rule. As $s_t(\theta)$ would be accepted with a positive probability, $m_t(\theta)$ cannot be always zero. Hence, the entrepreneur's expected payoff is $\mathbb{E}[(1 - t) \cdot \theta \cdot m_t(\theta)]$, which is strictly positive.

Note that the security $s_t(\theta)$ is a feasible security. Hence, the optimal security $s^*(\theta)$ would also be accepted with a positive probability and delivers positive expected payoff to the entrepreneur. This concludes the proof. □

PROOF OF COROLLARY 1. The proof is straightforward following the above proof of Lemma 1. Proposing $s^*(\theta) = \theta$ gives the entrepreneur a zero payoff, while proposing $s_t(\theta) = t \cdot \theta$ constructed in the proof of Lemma 1 gives her a strictly positive expected payoff. This suggests $s^*(\theta) = \theta$ is not optimal. □

PROOF OF PROPOSITION 2. The Lagrangian of the entrepreneur's problem is

$$\mathscr{L} = \mathbb{E}\left[\theta - s(\theta) + \lambda \cdot \left(1 - \exp\left(\mu^{-1} \cdot (k - s(\theta))\right)\right) + \eta_1(\theta) \cdot s(\theta) + \eta_2(\theta) \cdot (\theta - s(\theta))\right],$$

where λ, $\eta_1(\theta)$ and $\eta_2(\theta)$ are multipliers.

The first order condition is

$$\frac{d\mathscr{L}}{ds(\theta)} = -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (q - s(\theta))\right) + \eta_1(\theta) - \eta_2(\theta) = 0. \tag{A.5}$$

We first consider a special case that is helpful for us to solve the optimal security. If $0 < s(\theta) < \theta$, the two feasibility conditions are not binding. Thus $\eta_1(\theta) = \eta_2(\theta) = 0$, and the first order condition is simplified as

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - s(\theta))\right) = 0.$$

By rearrangement, we get

$$s(\theta) = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu). \tag{A.6}$$

We denote the right hand side of (A.6), which is irrelevant of θ, as D^*. By definition, we have $D^* > 0$. Also, it is straightforward to have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) = 0. \tag{A.7}$$

In what follows, we characterize the optimal solution $s^*(\theta)$ for different regions of θ.

First, we consider the region of $\theta > D^*$. We show that $0 < s^*(\theta) < \theta$ in this region by contradiction.

If $s^*(\theta) = \theta > D^*$, we have $\eta_1(\theta) = 0$ and $\eta_2(\theta) \geqslant 0$. From the first order condition (A.5) we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - \theta)\right) = \eta_2(\theta) \geqslant 0. \tag{A.8}$$

On the other hand, as $\theta > D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - \theta)\right) . \tag{A.9}$$

Conditions (A.7), (A.8), and (A.9) construct a contradiction. So we must have $s^*(\theta) < \theta$ if $\theta > D^*$.

Similarly, if $s^*(\theta) = 0$, we have $\eta_1(\theta) \geqslant 0$ and $\eta_2(\theta) = 0$. Again from the first order condition (A.5) we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot k\right) = -\eta_1(\theta) \leqslant 0 . \tag{A.10}$$

On the other hand, as $D^* > 0$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) < -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot k\right) . \tag{A.11}$$

Conditions (A.7), (A.10), and (A.11) construct another contradiction. So we must have $s^*(\theta) > 0$ if $\theta > D^*$.

Therefore, we have shown that $0 < s^*(\theta) < \theta$ for $\theta > D^*$. From the discussion above for this specific case, we conclude that $s^*(\theta) = D^*$ for $\theta > D^*$.

We then consider the region of $\theta < D^*$. We show that $s^*(\theta) = \theta$ in this region.

Since $s^*(\theta) \leqslant \theta < D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - s^*(\theta))\right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) . \tag{A.12}$$

From condition (A.7), the right hand side of this inequality (A.12) is zero. Together with the first order condition (A.5), the inequality (A.12) implies that $\eta_1(\theta) = 0$ and $\eta_2(\theta) > 0$. Therefore, we have $s^*(\theta) = \theta$ in this region.

Also, from the first order condition (A.5) and the condition (A.7), it is obvious that $s^*(D^*) = D^*$.

To sum up, the entrepreneur's optimal security without inducing the investor to acquire information features a debt with face value D^* determined by condition (A.6).

We need to check that there exists $D^* > 0$ and the corresponding multiplier $\lambda > 0$ such that

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot (\min(\theta, D^*) - k)\right)\right] = 1 , \tag{A.13}$$

where D^* is determined by condition (A.6).

Consider the left hand side of condition (A.13). Clearly, it is continuous and monotonically decreasing in D^*. When D^* is sufficiently large, the left hand side of (A.13) approaches $\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)\right]$, a number less than one, which is guaranteed by the condition (3.3) as well as the feasibility condition $s(\theta) \leqslant \theta$. On the other hand, when $D^* = 0$, it approaches

$\exp\left(\mu^{-1} \cdot k\right)$, which is strictly greater than one. Hence, there exists $D^* > 0$ such that the condition (A.13) holds.

Moreover, from the condition (A.6), we also know that D^* is continuous and monotonically increasing in λ. When λ approaches zero, D^* approaches negative infinity, while when λ approaches positive infinity, D^* approaches positive infinity as well. Hence, for any $D^* > 0$ there exists a corresponding multiplier $\lambda > 0$.

Last, suppose $D^* \leqslant k$. It is easy to see that this debt would be rejected by the investor due to Proposition 1, a contradiction.

Finally, by condition (3.3) again, since the optimal security $s^*(\theta)$ satisfies

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \left(s^*\left(\theta\right) - k\right)\right)\right] = 1\,,$$

Jensen's inequality implies that $\mathbb{E}[s^*(\theta)] > k$ given $\mu > 0$. This concludes the proof. $\square$

Proof of Lemma 3. We derive the entrepreneur's optimal security $s^*\left(\theta\right)$ and the corresponding unconstrained optimal security $\widehat{s}(\theta)$ through variational methods. Specifically, we characterize how the entrepreneur's expected payoff responds to the perturbation of her optimal security.

Let $s\left(\theta\right) = s^*\left(\theta\right) + \alpha \cdot \varepsilon\left(\theta\right)$ be an arbitrary perturbation of the optimal security $s^*\left(\theta\right)$. Note that the investor's optimal screening rule $m_s\left(\theta\right)$ appears in the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (3.7), and it is implicitly determined by the proposed security $s\left(\theta\right)$ through the functional equation (3.6). Hence, we need to first characterize how $m_s\left(\theta\right)$ varies with respect to the perturbation of $s^*\left(\theta\right)$. Taking derivative with respect to α at $\alpha = 0$ for both sides of (3.6) leads to

$$\begin{aligned} \mu^{-1}\varepsilon\left(\theta\right) &= g''\left(m_s^*\left(\theta\right)\right) \cdot \left.\frac{\partial m_s\left(\theta\right)}{\partial \alpha}\right|_{\alpha=0} \\ &\quad - g''\left(\overline{\pi}_s^*\right) \cdot \mathbb{E}\left.\frac{\partial m_s\left(\theta\right)}{\partial \alpha}\right|_{\alpha=0}. \end{aligned}$$

Take expectation of both sides and we get

$$\begin{aligned} & \mathbb{E}\left[\left.\frac{\partial m_s\left(\theta\right)}{\partial \alpha}\right|_{\alpha=0}\right] \\ = \; & \mu^{-1} \cdot \left(1 - \mathbb{E}\left[\left(g''\left(m_s^*\left(\theta\right)\right)\right)^{-1}\right] \cdot g''\left(\overline{\pi}_s^*\right)\right)^{-1} \cdot \mathbb{E}\left[\left(g''\left(m_s^*\left(\theta\right)\right)\right)^{-1} \varepsilon\left(\theta\right)\right]. \end{aligned}$$

Combining the above two equations, for any perturbation $s\left(\theta\right) = s^*\left(\theta\right) + \alpha \cdot \varepsilon\left(\theta\right)$, the investor's screening rule $m_s(\cdot)$ is characterized by

$$\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} = \mu^{-1}\cdot\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\varepsilon(\theta) + \frac{\mu^{-1}\cdot\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\cdot\mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\varepsilon(\theta)\right]}{\left(g''\left(\overline{\pi}_s^*\right)\right)^{-1}-\mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\right]}. \tag{A.14}$$

We interpret condition (A.14). The first term of the right hand side of (A.14) is the investor's local response to $\varepsilon(\theta)$. It is of the same sign as the perturbation $\varepsilon(\theta)$. When the payment of the security increases at state θ, the investor is more likely to accept the security at this state. The second term measures the investor's average response to perturbation $\varepsilon(\theta)$ over all states. It is straightforward to verify that the denominator of the second term is positive due to Jensen's inequality. As a result, if the perturbation increases the investor's payment on average over all states, she is more likely to accept the security as well.

Now we can calculate the variation of the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (3.7). Taking derivative of $u_E(s(\cdot))$ with respect to α at $\alpha=0$ leads to

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} = \mathbb{E}\left[\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0}(\theta - s(\theta))\right] - \mathbb{E}\left[m_s^*(\theta)\cdot\varepsilon(\theta)\right]. \tag{A.15}$$

Substitute (A.14) into (A.15) and we get

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} = \mathbb{E}\left[r(\theta)\cdot\varepsilon(\theta)\right], \tag{A.16}$$

where

$$r(\theta) = -m_s^*(\theta) + \mu^{-1}\cdot\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\cdot\left(\theta - s^*(\theta) + w^*\right) \tag{A.17}$$

and

$$w^* = \mathbb{E}\left[\left(\theta - s^*(\theta)\right)\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\left(1-\mathbb{E}\left[\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\right)^{-1}.$$

Note that w^* is a constant that does not depend on θ and will be endogenously determined in the equilibrium. Besides, $r(\theta)$ is the Frechet derivative of the entrepreneur's expected payoff $u_E(s(\cdot))$ at $s^*(\theta)$, which measures the marginal contribution of any perturbation to the entrepreneur's expected payoff when the security is optimal. Specifically, the first term of (A.17) is the direct contribution of perturbing $s^*(\theta)$ disregarding the variation of $m_s^*(\theta)$, and the second term measures the indirect contribution through the variation of $m_s^*(\theta)$. This Frechet derivative $r(\theta)$ plays an important role in shaping the entrepreneur's optimal security.

To further characterize the optimal security, we discuss the Frechet derivative $r(\theta)$ in detail. Recall that the optimal security would be restricted by the feasibility condition $0 \leqslant s^*(\theta) \leqslant \theta$.

Let

$$A_0 = \{\theta \in \Theta : \theta \neq 0, s^*(\theta) = 0\} \ ,$$

$$A_1 = \{\theta \in \Theta : \theta \neq 0, 0 < s^*(\theta) < \theta\} \ ,$$

and

$$A_2 = \{\theta \in \Theta : \theta \neq 0, s^*(\theta) = \theta\} \ .$$

Clearly, $\{A_0, A_1, A_2\}$ is a partition of $\Theta \backslash \{0\}$. Since $s^*(\theta)$ is the optimal security, we have

$$\left. \frac{\partial u_E(s(\cdot))}{\partial \alpha} \right|_{\alpha=0} \leqslant 0$$

for any feasible perturbation $\varepsilon(\theta)$.[45] Hence, condition (A.16) implies

$$r(\theta) \begin{cases} \leqslant 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geqslant 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.18}$$

According to Proposition 1, when the optimal security $s^*(\theta)$ induces the investor to acquire information, we have $0 < m_s^*(\theta) < 1$ for all $\theta \in \Theta$. Hence, condition (A.18) can be rearranged as

$$\frac{r(\theta)}{m_s^*(\theta)} = -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} \leqslant 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geqslant 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.19}$$

Recall condition (3.6), given the optimal security $s^*(\theta)$, the investor's optimal screening rule $m_s^*(\theta)$ is characterized by

$$s^*(\theta) - k = \mu \cdot \left(g'(m_s^*(\theta)) - g'(\overline{\pi}_s^*) \right) \ , \tag{A.20}$$

where

$$\overline{\pi}_s^* = \mathbb{E}[m_s^*(\theta)]$$

is the investor's unconditional probability of accepting the optimal security $s^*(\theta)$. Conditions (A.19) and (A.20) as a system of functional equations jointly determine the optimal security $s^*(\theta)$ when it induces the investor's information acquisition.

Finally, when we focus on the unconstrained optimal security $\widehat{s}(\theta)$, note that is would not be restricted by the feasibility condition. Hence, the corresponding Frechet derivative $r(\theta)$ would be always zero at the optimum. On the other hand, the investor's optimal screening rule would not

[45] A perturbation $\varepsilon(\theta)$ is feasible with respect to $s^*(\theta)$ if there exists $\alpha > 0$ such that for any $\theta \in \Theta$, $s^*(\theta) + \alpha \cdot \varepsilon(\theta) \in [0, \theta]$.

be affected. As a result, the conditions (A.20) and (A.19) become

$$\widehat{s}(\theta) - k = \mu \cdot \left(g'\left(\widehat{m}_s(\theta)\right) - g'\left(\overline{\pi}_s^*\right)\right) \ ,$$

where

$$\overline{p}_s^* = \mathbb{E}\left[m_s^*(\theta)\right] \ ,$$

and

$$(1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) = \mu \,,$$

where

$$w^* = \mathbb{E}\left[(\theta - s^*(\theta)) \frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right] \left(1 - \mathbb{E}\left[\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\right)^{-1} \ ,$$

in which $\overline{p}_s^*$ and w^* are two constants that do not depend on θ. This concludes the proof. □

Proof of Lemma 4. From Lemma 3, $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfies the two equations (A.1) and (A.2). By condition (A.2), we get

$$\widehat{m}_s(\theta) = 1 - \frac{\mu}{\theta - \widehat{s}(\theta) + w^*} \ . \tag{A.21}$$

Substituting (A.21) into (A.1) leads to

$$\mu^{-1}\left(\widehat{s}(\theta) - k\right) = g'\left(\frac{\mu}{\theta - \widehat{s}(\theta) + w^*}\right) - g'\left(\overline{\pi}_s^*\right) \ .$$

Taking derivatives of both sides of the above equation with respect to θ leads to

$$\begin{aligned} \mu^{-1} \cdot \frac{d\widehat{s}(\theta)}{d\theta} &= g''\left(\widehat{m}_s(\theta)\right) \cdot \frac{d\widehat{m}_s(\theta)}{d\theta} \\ &= g''\left(\widehat{m}_s(\theta)\right) \cdot \frac{\mu \cdot \left(1 - \frac{d\widehat{s}(\theta)}{d\theta}\right)}{\left(\theta - \widehat{s}(\theta) + w^*\right)^2} \\ &= \frac{1 - \frac{d\widehat{s}(\theta)}{d\theta}}{\theta - \widehat{s}(\theta) + w^* - \mu} \ , \end{aligned}$$

where we use

$$g''(x) = \frac{1}{x(1-x)}$$

while deriving the third equality. Rearrange the above equation, and we get

$$\begin{aligned} \frac{d\widehat{s}(\theta)}{d\theta} &= \frac{\mu}{\theta - \widehat{s}(\theta) + w^*} \\ &= 1 - \widehat{m}_s(\theta) \ , \end{aligned}$$

where the last equality follows (A.21).

Again, taking derivatives of both sides of the above equation with respect to θ leads to

$$\begin{aligned}
\mu^{-1} \cdot \frac{d\widehat{s}\,(\theta)}{d\theta} &= g''\left(\widehat{m}_s\,(\theta)\right) \cdot \frac{d\widehat{m}_s\,(\theta)}{d\theta} \\
&= \frac{1}{\widehat{m}_s\,(\theta)\left(1-\widehat{m}_s\,(\theta)\right)} \cdot \frac{d\widehat{m}_s\,(\theta)}{d\theta}\,.
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{d\widehat{m}_s\,(\theta)}{d\theta} &= \mu^{-1} \cdot \widehat{m}_s\,(\theta) \cdot \left(1-\widehat{m}_s\,(\theta)\right) \cdot \frac{d\widehat{s}\,(\theta)}{d\theta} \\
&= \mu^{-1} \cdot \widehat{m}_s\,(\theta) \cdot \left(1-\widehat{m}_s\,(\theta)\right)^2 .
\end{aligned}$$

This completes the proof. □

Proof of Lemma 5. From Lemma 4, it is easy to see that the slope of $\widehat{s}(\theta)$ is always less than one. Hence, Lemma 5 is straightforward. □

Proof of Lemma 6. We proceed by discussing three cases.

Case 1: We show that $\widehat{s}(\theta) > \theta$ would imply $s^*(\theta) = \theta$.

Suppose $s^*(\theta) < \theta$. Then we have $s^*(\theta) < \widehat{s}(\theta)$. Since both $(s^*(\theta), m_s^*(\theta))$ and $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfy condition (3.6), we must have $m_s^*(\theta) < \widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned}
\frac{r(\theta)}{m_s^*(\theta)} &= -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&> -1 + \mu^{-1} \cdot (1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) \\
&= 0\,,
\end{aligned}$$

which implies $s^*(\theta) = \theta$, a contradiction.

Note that, the logic for the inequality above is as follows. Since $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfies condition (A.2), we must have $\theta - \widehat{\theta} + w^* > 0$. Hence, $\widehat{s}(\theta) > s^*(\theta)$ implies that

$$\theta - s^*(\theta) + w^* > \theta - \widehat{s}(\theta) + w^* > 0\,.$$

Also by noting that

$$1 - m_s^*(\theta) > 1 - \widehat{m}_s(\theta) > 0\,,$$

we get the inequality above.

Hence, we have $s^*(\theta) = \theta$ in this case.

Case 2: We show that $\widehat{s}(\theta) < 0$ would imply $s^*(\theta) = 0$.

Suppose $s^*(\theta) > 0$. Then we have $s^*(\theta) > \widehat{s}(\theta)$. By similar argument we know that $m_s^*(\theta) >$

$\widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned}
\frac{r(\theta)}{m_s^*(\theta)} &= -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&< -1 + \mu^{-1} \cdot (1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) \\
&= 0 \,,
\end{aligned}$$

which implies $s^*(\theta) = 0$. This is a contradiction. Hence, we have $s^*(\theta) = 0$ in this case.

Case 3: We show that $0 \leqslant \widehat{s}(\theta) \leqslant \theta$ would imply $s^*(\theta) = \widehat{s}(\theta)$.

Suppose $\widehat{s}(\theta) < s^*(\theta)$. Then similar argument suggests $r(\theta)/m_s^*(\theta) < 0$, which implies $s^*(\theta) = 0 < \widehat{s}(\theta)$. This is a contradiction.

Similarly, suppose $s^*(\theta) < \widehat{s}(\theta)$. Similar argument suggests that $r(\theta)/m_s^*(\theta) > 0$, which implies $s^*(\theta) = \theta > \widehat{s}(\theta)$. This is, again, a contradiction. Hence, we have $s^*(\theta) = \widehat{s}(\theta)$ in this case.

This concludes the proof. □

Proof of Lemma 7. Apply Lemma 5 to Lemma 6, then Lemma 7 is straightforward. □

Proof of Lemma 8. We prove by contradiction. Suppose that the last two cases in Lemma 7 can occur in equilibrium. Hence, there exists a $\widetilde{\theta} \geqslant 0$, such that $s^*(\theta) = 0$ when $0 \leqslant \theta \leqslant \widetilde{\theta}$ and $s^*(\theta) = \widehat{s}(\theta)$ when $\theta > \widetilde{\theta}$.

Note that, $s^*(\theta)$ is strictly increasing when $\theta > \widetilde{\theta}$. Also, since we focus on the equilibrium with information acquisition, there must exist a θ'' such that $s^*(\theta'') > k$; otherwise the optimal security would be rejected without information acquisition. Therefore, there exists a $\theta' > \widetilde{\theta}$ such that $s^*(\theta') = \widehat{s}(\theta') = k$. Recall condition (A.1), we have

$$m_s^*(\theta') = \bar{\pi}_s^* \,.$$

Moreover, notice that we have $s^*(\theta') \in (0, \theta')$, we have

$$\begin{aligned}
0 = r(\theta') &= -m_s^*(\theta') + \mu^{-1} \cdot m_s^*(\theta') \cdot (1 - m_s^*(\theta')) \cdot (\theta' - s^*(\theta') + w^*) \\
&= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k + w^*) \\
&= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) + \mathbb{E}[r(\theta)] \,,
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}[r(\theta)] &= -\bar{\pi}_s^* + \mu^{-1}\left(\mathbb{E}\left[\frac{(\theta - s(\theta))\cdot g''(\bar{\pi}_s^*))}{g''(m(\theta))}\right]/g''(\bar{\pi}_s^*) + w^*\mathbb{E}\left[\frac{1}{g''(m(\theta))}\right]\right) \\
&= -\bar{\pi}_s^* + \mu^{-1}\left(w^*\cdot\left(1-\mathbb{E}\left[\frac{g''(\bar{\pi}_s^*)}{g''(m(\theta))}\right]\right)/g''(\bar{\pi}_s^*) + w^*\mathbb{E}\left[\frac{1}{g''(m(\theta))}\right]\right) \\
&= -\bar{\pi}_s^* + \frac{\mu^{-1}w^*}{g''(\bar{\pi}_s^*)} \\
&= -\bar{\pi}_s^* + \mu^{-1}\cdot\bar{\pi}_s^*\cdot(1-\bar{\pi}_s^*)\cdot w^* \ .
\end{aligned}$$

We can express the expectation term $\mathbb{E}[r(\theta)]$ in another way. Note that, for any $\theta \in [0, \widetilde{\theta}]$, by definition we have

$$\begin{aligned}
r(\theta) &= -m_s^*(\theta) + \mu^{-1}\cdot m_s^*(\theta)\cdot(1-m_s^*(\theta))\cdot(\theta - s^*(\theta) + w^*) \\
&= -\widehat{m}_s(\widetilde{\theta}) + \mu^{-1}\cdot\widehat{m}_s(\widetilde{\theta})\cdot(1-\widehat{m}_s(\widetilde{\theta}))\cdot(\theta - 0 - \theta^* + \theta^* + w^*) \\
&= r(\widetilde{\theta}) - \mu^{-1}\cdot\widehat{m}_s(\widetilde{\theta})\cdot(1-\widehat{m}_s(\widetilde{\theta}))\cdot(\widetilde{\theta}-\theta) \\
&= -\mu^{-1}\cdot\widehat{m}_s(\widetilde{\theta})\cdot(1-\widehat{m}_s(\widetilde{\theta}))\cdot(\widetilde{\theta}-\theta) \ .
\end{aligned}$$

Also, as $s^*(\theta) = \widehat{s}(\theta)$ for any $\theta > \widetilde{\theta}$, we have $r(\theta) = 0$ for all $\theta > \widetilde{\theta}$. Hence,

$$\mathbb{E}[r(\theta)] = -\mu^{-1}\cdot\widehat{m}_s(\widetilde{\theta})\cdot(1-\widehat{m}_s(\widetilde{\theta}))\int_0^{\widetilde{\theta}}(\widetilde{\theta}-\theta)d\Pi(\theta) \ .$$

Therefore, we have

$$\begin{aligned}
\mu^{-1}\cdot\bar{\pi}_s^*\cdot(1-\bar{\pi}_s^*)\cdot(\theta'-k) &= -\mathbb{E}[r(\theta)] && \text{(A.22)} \\
&= \mu^{-1}\cdot\widehat{m}_s(\widetilde{\theta})\cdot(1-\widehat{m}_s(\widetilde{\theta}))\int_0^{\widetilde{\theta}}(\widetilde{\theta}-\theta)d\Pi(\theta) \ . && \text{(A.23)}
\end{aligned}$$

Now we take the tangent line of $s^*(\theta)$ at $\theta = \widetilde{\theta}$. The tangent line intersects $s = k$ at $\widetilde{\theta}'$, which is given by

$$\frac{k}{\widetilde{\theta}'-\theta'} = \left.\frac{ds^*(\theta)}{d\theta}\right|_{\theta'} = 1 - \widehat{m}_s(\widetilde{\theta})\,.$$

Hence, we have

$$\widetilde{\theta}' = \widetilde{\theta} + \frac{k}{1-\widehat{m}_s(\widetilde{\theta})}\ .$$

Also, note that we have shown that for any $\theta \geqslant \widetilde{\theta}$, we have

$$\frac{ds^*(\theta)}{d\theta} = \frac{d\widehat{s}(\theta)}{d\theta} = 1 - \widehat{m}_s(\theta) = 1 - m_s^*(\theta)\,.$$

Hence,

$$\frac{d^2 s^*(\theta)}{d\theta^2} = -\mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta))^2 < 0\,.$$

Therefore, $s^*(\theta)$ is strictly concave for $\theta \geqslant \widetilde{\theta}$, and consequently, we also have $\widetilde{\theta}' < \theta'$. As a result, from conditions (A.22) and (A.23), we have

$$\begin{aligned}
\bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot (\widetilde{\theta}' - k) \quad &< \quad \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \\
&= \quad \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1 - \widehat{m}_s(\widetilde{\theta})} \cdot k \right)\,.
\end{aligned}$$

By Jensen's inequality, we get

$$\bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) > \mathbb{E}\left[m_s^*(\theta) \cdot (1 - m^*(\theta))\right]\,.$$

Therefore, we have

$$\begin{aligned}
\widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \quad &> \quad \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1 - \widehat{m}_s(\widetilde{\theta})} \cdot k \right) \\
&> \quad \mathbb{E}\left[m_s^*(\theta) \cdot (1 - m^*(\theta))\right] \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1 - \widehat{m}_s(\widetilde{\theta})} \cdot k \right)\,.
\end{aligned}$$

Expand the expectation term above and rearrange, we get

$$\begin{aligned}
&\widehat{m}_s(\widetilde{\theta})^2 \cdot k \cdot Prob[\theta \leqslant \widetilde{\theta}] + \int_{\widetilde{\theta}}^{+\infty} m_s^*(\theta) \cdot (1 - m_s^*(\theta)) d\Pi(\theta) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1 - \widehat{m}_s(\widetilde{\theta})} \cdot k \right) \\
< \quad &\widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (-\theta) d\Pi(\theta) \\
\leqslant \quad &0\,.
\end{aligned}$$

Nevertheless, the left hand side of the above inequality should be positive, which is a contradiction. This concludes the proof. □

Proof of Proposition 4. We first consider the case with a positive transfer price $p > 0$. Suppose the corresponding security $s(\theta) = \theta - p$ is optimal in a generalized security design problem without the non-negative constraint. However, this security can be accommodated by neither Proposition 2 or Proposition 3, which two exclusively characterize the optimal security in the generalized security design problem, a contradiction.

By Corollary 1, we know that the security $s(\theta) = \theta$ that represents transfer with a zero price is not optimal. This concludes the proof. □

Proof of Corollary 2. First, note that $s^*(\theta)$ is strictly increasing and continuous. Also, note that there exists a θ'' such that $s^*(\theta'') > k$; otherwise, the offer will be rejected without information acquisition.

Therefore, there exists an unique θ' such that $s^*(\theta') = k$, which ensures that $m_s^*(\theta') = \bar{\pi}_s^*$, and

$$\begin{aligned} r(\theta') &= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta') + w^*) \\ &= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta')) + \mathbb{E}[r(\theta)] \, . \end{aligned}$$

Note that $\mathbb{E}[r(\theta)] > 0$ and $\theta' - s^*(\theta') \geqslant 0$, we have $\theta' < \widehat{\theta}$. As $\theta' = s^*(\theta') = k$, it follows that $\widehat{\theta} > \theta' = k$. This concludes the proof. □

Proof of Proposition 6. When we have $\mathbb{E}[\theta] \leqslant k$ and

$$\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right] > 1 \, ,$$

according to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the security would induce the investor to acquire information and accept it with positive (but less than one) probability. The only optimal security for this case is convertible preferred stock. This concludes the proof. □

Proof of Lemma 2. The "if" part is straightforward, following the definition of efficiency. The "only if" part is ensured by the fact that the optimal screening rule is always unique given a security, established in Proposition 1. □

Proof of Proposition 7. We state a useful lemma to begin. It allow us to focus on the first two types of equilibrium for welfare analysis.

Lemma **9.** *A project is initiated with a positive probability in the decentralized economy if and only if it is initiated with a positive probability in the corresponding centralized economy.*

Proof of Lemma 9. With the objective function (4.10) in the centralized economy, the entrepreneur's optimal screening rule $m_c^*(\theta)$ is characterized by Proposition 1. Specifically, the investor will initiate the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] \leqslant 1 \, ,$$

will skip the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] \leqslant 1 \, ,$$

and will initiate the project with probability $0 < \bar{\pi}_c^* < 1$, $\bar{\pi}_c^* = \mathbb{E}[m_c^*(\theta)]$, if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] > 1 \text{ and } \mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1\,,$$

in which $m_c^*(\theta)$ is determined by

$$\theta - k = \mu \cdot (g'(m_c^*(\theta)) - g'(\bar{\pi}_c^*))\,.$$

It is straightforward to observe that, the project is initiated with a positive probability in the frictionless centralized economy if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1\,. \tag{A.24}$$

Note that, condition (A.24) is the same as condition (3.1) in Lemma 1 that gives the investment criterion in a corresponding decentralized economy. This concludes the proof. □

Thanks to Lemma 9, once we prove the "only if" parts of both cases of debt and convertible preferred stock, the "if" parts would follow.

First, consider the case when $s^*(\theta)$ is debt. In this case, we have $Prob[m_s^*(\theta) = 1] = 1$, and

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)] \leqslant 1\,,$$

both from Proposition 1. Since $s^*(\theta) < \theta$ when $\theta > D^*$, it follows that

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] \leqslant 1\,,$$

which implies that $Prob[m_c^*(\theta) = 1] = 1$, also by Proposition 1. Hence, we know that

$$Prob[m_s^*(\theta) = m_c^*(\theta)] = 1\,,$$

which suggests that $s^*(\theta)$, as debt, achieves efficiency, according to Lemma 2.

Second, consider the case when $s^*(\theta)$ is convertible preferred stock that induces information acquisition. In this case, we have $Prob[0 < m_s^*(\theta) < 1] = 1$, and

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)] > 1\,,$$

again both from Proposition 1. Since $s^*(\theta) < \theta$ when $\theta > \widehat{\theta}$, the relationship between $\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)]$ and 1 is ambiguous. If

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] \leqslant 1\,,$$

we have $Prob[m_c^*(\theta) = 1] = 1$, and information acquisition is not induced in the centralized economy. It follows that

$$Prob[m_s^*(\theta) = m_c^*(\theta)] \neq 1 \,.$$

Otherwise, if

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] > 1 \,,$$

suppose we also have $Prob[m_s^*(\theta) = m_c^*(\theta)] = 1$, then according to condition (A.20), we have

$$Prob[s^*(\theta) = \theta] = 1 \,,$$

which violates Corollary 1. A contradiction. As a result, from Lemma 2, we know that $s^*(\theta)$, as convertible preferred stock, cannot achieve efficiency. This concludes the proof. □