

**A Measure of Inference  
in Classical and Intuitionistic Logics:  
some applications to game theory\***

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**ABSTRACT**

This paper presents a measure of inference in classical and intuitionistic logics in the Gentzen-style sequent calculi. The measure for a proof of a sequent is the width of the proof tree, that is, the number of leaves of the proof tree. Then the measure for a sequent is the minimum value of the widths of possible proofs of the sequent; if it is unprovable, the assigned value is  $+\infty$ . It counts the indispensable cases for possible proofs of a sequent. By this measure, we can separate between sequents easy to be proved and ones difficult; we can go further than provability and/or unprovability. It is motivated by some economics/game theory problem (bounded rationality). However, it would be not straightforward to obtain the exact value of this measure for a given sequent. In this paper, we will develop a method of calculating the value of the measure. We will apply our measure to various classes of problems, for example, to evaluate the difficulty of proving contradictory sequents. We also exemplify our measure with a problem of game theoretical decision making.

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