Fee Setting Intermediaries: On Real Estate Agents, Stock Brokers, and Auction Houses*

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Abstract

Mechanisms where intermediaries charge a commission fee and let the sellers set the price are widely used in practice e.g. by real estate brokers, art galleries or auction houses. We model competition between intermediaries in a dynamic random matching model, where in every period a buyer, seller and an intermediary are randomly matched. An intermediary has a temporary monopoly until traders are rematched and designs an exchange mechanism that maximizes his own profits. Traders' valuations for the indivisible good are altered by their continuation values. The following results obtain. First, the model has an analytical solution, where in equilibrium intermediaries use a fee setting mechanism and these are intermediary optimal mechanisms. Second, when the rematching probability increases, or equivalently the period length decreases, the equilibrium fees become smaller. Third, we derive the joint distribution of the offering price and time a good stays on the market – an often investigated issue in the empirical literature on real estate brokerage. Our framework can be extended to deal with several further questions: inefficient free entry in real estate brokerage, brokers selling their own houses, and mechanisms used by used car dealers.

Keywords: Brokers, Applied Mechanism Design, linear commission fees, optimal indirect mechanisms, internet auctions, auction houses.

JEL-Classification: C72, C78, L13

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1 Introduction

Many markets are organized by intermediaries, and many of these intermediaries neither buy nor sell the goods whose exchange they enable. Instead they set percentage fees to be levied on the price, which is subsequently set by the seller. Buyers then either accept or reject the price. If the mechanism involves an auction where the seller sets a reserve price, the buyers bid in the auction, and the fee is levied on the realized price. We call such mechanisms "fee setting mechanisms".

Real estate brokers, stock brokers, art galleries, and auction houses or sites are just a few examples. Real estate brokers in the U.S. typically charge 5 to 6 percent. Commission fees by art galleries are said to be in the range of 30 to 50 percent. The auction houses Sotheby's and Christie's use a regressive fee structure and so does eBay.[1] Sotheby's and Christie's used a linear fee of 20 percent prior to being investigated by U.S. Department of Justice, convicted for collusive behavior, and induced to change the fee structure.[2] Other industries where fee setting mechanisms are frequently used include stock brokerage, share-cropping in agriculture, contracts between authors and publishing companies, and retailers that charge a percentage on the revenue a manufacturer generates with his product. Similarly, electronic payment systems and credit cards charge percentage fees, further, most governments collect substantive parts of their revenue through value added taxes without being directly involved in price setting. Percentage fees are also used in a slightly different environment in investment banking[3] and by labor market intermediaries, in particular head hunters.

As a matter of fact, industries where fee setting mechanisms are predominantly used are quite sizeable. For example, the sales generated by Sotheby's in 2007 alone exceeded

[1] The marginal rate at Sotheby's is 25 percent for items with prices up to USD 20K, 25 percent between USD 20K and USD 500K and 12 percent beyond. At eBay (ebay.com, accessed on May 5, 2008) the marginal fee on the closing price is 8.75 percent below USD 25, 3.5 percent between USD 25 and USD 1000, and 1.5 percent above USD 1000.

[2] Similarly, real estate brokerage has come under scrutiny by the U.S. Department of Justice (DOJ, 2007). There is a widespread, though rarely explicit, suspicion that in particular the almost complete invariance of broker commission fees reflects collusive behavior by intermediaries.

[3] Underwriters on initial public offerings in the U.S. charge in most cases exactly 7%, see Chen and Ritter (2000).

USD 4 billions.[4] The annual operating revenue of eBay was more than USD 7.5 billion in 2007, and Christie's annual sales in 2006 exceeded USD 4.5 billion.[5] The real estate brokerage industry in the U.S. generates annual sales beyond USD 1000 billions and commission fees of more than USD 60 billions per year.[6] Credit card companies are big business, too. For example, MasterCard's annual revenue in 2007 exceeded USD 4 billion.[7] The revenue collected through excise taxation, which consists of both value added taxes and specific (unit) taxes in the U.S. exceeds USD 100 Billion.[8]

Despite their widespread use and economic significance, fee setting mechanisms have received little attention in the theoretical economic literature. In particular, no prior analysis of the optimality of fee setting and the structure of fees from a mechanism design perspective exists.[9] The purpose of this paper is to start filling this gap and to improve economists' understanding what determines whether intermediaries set commission fees and, if they choose to do so, what determines the size and form of these fees.[10] Our main contribution is in considering the exchange mechanism as the endogenous outcome of a mechanism design problem depending on the distributions of the types of buyers and sellers, rather than exogenously given. These distributions, in turn, are endogenous outcomes of market dynamics.

[4]See Sotheby's Annual Report 2007, p.28, available at Sothebys.com. The fee structure was reported in the New York Times (2008).

[5]See www.marketwatch.com and www.sgallery.net, respectively.

[6]See Rutherford, Springer, and Yavas (2005).

[7]MasterCard Worldwide, Annual Report 2007.

[8]This includes both taxation at the federal and at the state level; see Anderson, De Palma, and Kreider (2001, p.174).

[9]The fact that, to the best of our knowledge, no name for this type of mechanism exists only goes to show how little theoretical interest these mechanisms have received. Two papers that provide explanations of when intermediaries may use percentage fees and when they set prices are Hagiu (2006) and Yavas (1992). Hagiu's argument relies on the presence and nature of network externalities, while Yavas' explanation depends on the presence and working of search markets.

[10] That said, this means also that we do not aim to explain why intermediaries emerge in equilibrium (as do e.g. Gehrig (1993); Spulber (1999); Rust and Hall (2003)). Rather, we take the existence and importance of intermediated exchange as given and ask why (some) intermediaries use fee setting mechanisms. Empirically the predominance of intermediation we presume in this paper is on solid ground. For example, real estate brokers account for approximately 80 percent of all single-family dwellings in the U.S. (see Rutherford, Springer, and Yavas, 2005). A rather simple explanation for the exclusivity of trade through intermediaries is that there is an alternative search market where traders meet directly, but search costs are prohibitively high. Alternatively, it should be possible to extend the framework by a search market as an outside option. Traders' willingness to pay an intermediary is limited by the outside option of going to the search market.

We model competition between intermediaries in a dynamic random matching model with a continuum of buyers, sellers, and intermediaries. Buyers and sellers have private information about their valuations for an indivisible homogeneous good. In every period one buyer, one seller, and one intermediary are randomly matched. Intermediaries are free to choose the trading mechanism anew in every period. We show that this model permits an analytical solution for certain cases. In a steady state equilibrium intermediaries choose a stationary fee setting mechanism. Interestingly, this equilibrium mechanism does not vary with the number of intermediaries under standard assumptions on the matching technology. The equilibrium fees become smaller as the matching frequency increases, or equivalently the period length between subsequent rematchings decreases. Moreover, we derive some empirically testable predictions, such as the implied time goods stay on the market as a function of their prices, the distribution of price on the market, and the probability a good is ever sold.

The intuition for the intermediaries' equilibrium choice of mechanism stems from a one period model, where a monopolistic intermediary is matched to one buyer and one seller. It can be shown that fee setting is optimal for the intermediary under the assumption that the buyer's and the seller's valuation distributions satisfy Myerson's regularity condition. Results are likely to carry over to the irregular case. In the dynamic model this mechanism is still intermediary optimal. We show also that if sellers' valuation is drawn from a power distribution, then the optimal fee is linear and independent of the buyer's distribution. Further, we show that with many buyers whose valuations are i.i.d. draws and one seller, a fee setting mechanism followed by an optimal auction (such as used by eBay, Sotheby's or Christie's) is intermediary optimal.

Our base line model allows for various extensions. We study inefficient free entry by intermediaries with heterogeneous opportunity costs of entry and heterogeneous levels of ability. Under the assumption that power distributions are a good approximation, the model also allows for costs of intermediation that vary with transactions. We also show that a vertically integrated intermediary (who is also a seller) charges a lower price than an independent seller in a one period model. However, in a dynamic model this may be reversed. Furthermore, we show that price posting (i.e. the intermediary himself sets a price for the seller and a price for the buyer) is not optimal for the intermediary in a static setup with one buyer and one seller, but becomes intermediary optimal in a dynamic setup if the good in question can be stored without cost.

Our paper contributes to the large and growing literature on intermediation such as Gehrig (1993), Yavas (1992), Hackett (1992), Spulber (1996, 1999, 2008), Wooders (1997) and Rust and Hall (2003) by adding a mechanism design perspective to the notion of random matching present in most of these papers. The only articles applying mechanism design (or auction theory) to intermediation we are aware of are Spulber (2006) and Matros and Zapechelnyuk (2006). We differ from the former by having multiple buyers and an indivisible good; from the latter by the private information of the seller affecting payments; and from both by having dynamic random matching, multiple competing sellers and intermediaries, and predictions on price dispersion, fee structures, and time on market. It also relates to the literature on bilateral trade initiated by Myerson and Satterthwaite (1983) and Chatterjee and Samuelson (1983).[11] That the fee setting mechanism is intermediary optimal is interesting on its own as it provides a practical counterpart to the direct, and therefore abstract, intermediary optimal mechanism derived by Myerson and Satterthwaite.

As we add intermediaries to a dynamic matching model with incomplete information similar to Satterthwaite and Shneyerov (2007, 2008) and Atakan (2006b) it also relates to this strand of literature.[12,13] Insofar as the intermediaries in our dynamic model are competing mechanism designers, the paper is related to the work of McAfee (1993),

[11] Recall that the double auction described by Chatterjee and Samuelson (1983) satisfies the social optimality condition stated in Myerson and Satterthwaite (1983, Theorem 2) for uniform distributions. We show under regularity assumptions that the fee setting mechanism described here satisfies the intermediary optimality conditions for general distributions. Further, fees are linear for general distributions of the buyer and a power distribution of the seller, of which a uniform distribution is a special case.

[12] Other papers on dynamic random matching with incomplete information are Wolinsky (1987), De Fraja and Sakovics (2001), Serrano (2002), Moreno and Wooders (2002), Lauermann (2007), Lauermann and Wolinsky (2008). Some of these papers find that the market does not converge to a Walrasian equilibrium as frictions converge to zero, others find that it does. Since our assumptions are most similar to the models that do find convergence, one may conjecture that our model also exhibits convergence as frictions go to zero and, therefore, fees charged by intermediaries converge to zero. However, our focus is a different one: we look at markets with frictions and fees bounded away from zero.

[13] For dynamic random matching with complete information see Mortensen (1982), Rubinstein and Wolinsky (1985, 1990), Gale (1986, 1987), and Mortensen and Wright (2002).

Peters and Severinov (1997, 2006), and Damianov (2005) who study mechanism design by sellers whereas in our model the mechanisms are chosen by intermediaries.[14]

In that respect, and because real estate brokerage is an industry to which our model applies, the paper also contributes to the literature on real estate economics. Most of the theoretical and empirical literature analyzing real estate brokerage remains in the principal-agent framework, where the seller (and occasionally the buyer) is the principal and the broker the agent; see e.g. Anglin and Arnott (1991), Bagnoli and Khanna (1991), Arnold (1992), Williams (1998), Lewis and Ottaviani (2008)[15] for the theoretical and Rutherford, Springer, and Yavas (2005) and Levitt and Syverson (2005) for the empirical work. The present paper offers a new perspective in that we assume that the brokers have all the (temporary) bargaining power and propose a mechanism of their choice to the traders. We think there are good reasons to depart from the principal-agent framework.

First, in the case where a broker represents a buyer, the broker typically charges 3% of the price paid by the buyer. This percentage fee cannot be explained in a principal agent framework where the buyer incentivizes the broker to find an advantageous price for him, since their interests are diametrically opposed under such a contract.

Second, even in the cases where the broker represents the seller, it is not clear why the broker typically gets 6% of the total price. If it were the seller who proposes the contract to the broker, incentive compatibility implies that he would give a much higher percentage to the broker for the *marginal* increase of price he achieves. Making the individual rationality constraint binding should lead to a lower fee on the *inframarginal* price.[16]

Third, many observations suggest that bargaining rests with the broker rather than

[14] McAfee (1993, p.1304) notes that [his] "paper falls far short of a real theory of equilibrium institutions partly because it places the design of institutions in the hands of the sellers. A more satisfactory approach requires explicit modelling of the role of intermediaries, or auctioneers, who compete among each other for both buyers and sellers."

[15] Lewis and Ottaviani have a general model of dynamic search agency with research and development as the main application. However, real estate brokerage is one of the many applications of their model.

[16] As Hsieh and Moretti (2003) point out in their empirical analysis, a 6% fee seems to be far above the costs incurred by a broker for a house selling for say USD 500,000. Especially so, since 6% is sufficient to cover the broker's costs for a house selling for USD 100,000.

the buyer or the seller: the almost complete invariance of commission fees, the concerns about collusion by real estate brokers, and the fact that brokers are long-term players with substantial benefits from reputation, whereas individual buyers and sellers trade with very low frequency with brokers. It is also unclear why competition between brokers should lead to sellers (or buyers) proposing the contract, whereas in most other industries competing firms (e.g. retailers, car manufacturers, gas stations, etc.) make take-it-or-leave-it offers (typically prices) to their clients, competition merely constrains them in what they can offer.

Fourth, empirical observations of price dispersion of houses with the same quality and the relation of the price and the time on market are difficult to explain in a principal agent framework. Our theory provides a simple explanation for such observations, e.g. that house owners with a higher loan-to-value ratio set higher prices and take longer to sell (see Genesove and Mayer, 1997).[17]

Our article also gives possible explanations for various stylized facts observed in the empirical literature.[18] Broker fees are close to invariant with respect to the number of intermediaries and the prices of houses. Further, the number of intermediaries grew proportionally to overall industry profits, so that profits per intermediary remained constant (Hsieh and Moretti, 2003). Comparable houses owned by brokers sell at a higher price than houses owned by independent sellers (Levitt and Syverson, 2005). Much of the empirical literature finds a positive correlation between the listing price and the time on market in cross-sectional data. However, correlation is negative in longitudinal data. Larsen and Park (1989) find that not considering unsold houses when analyzing time on market biases an estimation.

The remainder of this paper is structured as follows. Section 2 lays out and ana-

[17]The loan-to-value ratio can be considered as a proxy for the seller's valuation. Further, one could possibly find a rational choice explanation for the observation that house owners who bought their houses at high prices ask for higher prices when offering their houses for sale later on, even after correcting for heterogeneity of houses (see Genesove and Mayer, 2001). In our framework, even if houses were homogeneous, there is a heterogeneity between sellers: some sellers value their houses more than others. A seller who bought his house at a high price is more likely to have had a high valuation when he bought it, and hence also when he sells it. Therefore, he asks for a higher price.

[18]See e.g. Hsieh and Moretti (2003), Rutherford, Springer, and Yavas (2005), Hendel, Nevo, and Ortalo-Magné (2007) and Levitt and Syverson (2005).

lyzes the one period model with one intermediary, one buyer and one seller. Section 3 introduces and analyzes dynamic competition between intermediaries with random matching. Section 4 discusses the model's empirical implications. Section 5 extends the model in various important directions. Section 6 concludes. All proofs are banned to the Appendix.

2 Static Monopoly

We model a market with intermediaries where there is a pool of buyers, sellers, and intermediaries. In every period every trader is matched to a triple consisting of a buyer, a seller, and an intermediary. The intermediary proposes an exchange mechanism and traders can either trade or wait until the next rematching. The market is in steady state and there is a constant inflow of traders that compensates the outflow of traders that have either exchanged the good or dropped out for exogenous reasons. As we will see in Section 3, understanding a simple one period model with one buyer, one seller, and one intermediary proves to be very helpful for understanding the larger model.

But for the moment, let us postpone dealing with the dynamic random matching model and focus on the static one period problem with one intermediary, one seller, and one buyer. The seller owns one unit of a homogeneous indivisible good of known quality. The buyer has private information about his valuation of the good v which is drawn from the distribution F with strictly positive density f on the support $[\underline{v}, \bar{v}]$. For brevity, we refer to the seller's valuation of the good, or his opportunity cost of selling it, as his cost.[19] The seller has private information about his cost c, which is drawn from G with strictly positive g on $[\underline{c}, \bar{c}]$. F and G are common knowledge and independent.

All agents are risk neutral, and preferences are quasilinear, i.e. the buyer's utility is $v - p$ and the seller's payoff if $p - c$ in case of trade at price p. The seller and the buyer can only trade through the monopolistic intermediary who has all the bargaining power and can hence choose the trade mechanism. For brevity we call mechanisms that maximize the intermediary's expected profit intermediary optimal.

[19]This makes clear that the model also applies to settings where the good has to be produced by the seller at a cost.

Denote by $\Phi(v) := v-(1-F(v))/f(v)$ the buyer's virtual valuation function. Analogously let the seller's *virtual cost function* be denoted as $\Gamma(c) := c+G(c)/g(c)$. We make the assumption that Myerson (1981)'s regularity condition holds, i.e. both virtual type functions are increasing. Results are likely to carry over to cases where the regularity condition does not hold.[20]

Fee Setting Mechanisms The main focus of this paper is on the following type of indirect mechanisms, which we call *fee setting* mechanisms.

Stage 1: The intermediary first announces a fee function $\omega(.)$ that determines the amount the intermediary gets upon successful sale at price p, leaving $p-\omega(p)$ to the seller.

Stage 2: Observing $\omega(.)$ and his own cost c the seller sets a price p.

Stage 3: Observing p and her own valuation v, the buyer then accepts or rejects the offer p and the game ends. If the buyer accepts, the seller gets the net price $p-\omega(p)$ and the intermediary the fee $\omega(p)$.

Obviously in stage 3 the buyer accepts if and only if $v \geq p$. The other two stages of this one-shot game are analyzed in Section 2.2. In Section 2.4 we also study a slightly modified version of this game with multiple buyers whose valuations are independent draws from the distribution F while there is still one seller and one intermediary.

As we will show later, restricting ourselves to such fee setting mechanisms is without loss of generality, since for every optimal mechanism, there is a corresponding fee setting mechanism that gives all participants the same payoffs in interim[21] expectations. Further, at this point there is a symmetry between the buyer and the seller: the intermediary could just as well let the buyer set a price and set a fee on this price.

[20] Our proofs use (Myerson and Satterthwaite, 1983) theorems on bilateral trade. Myerson and Satterthwaite make the regularity assumption for the sake of expositional clarity, but note that their results carry over to the irregular case by using standard techniques described in Myerson (1981). Even though it seems likely that our results should generalize in a similar way, we prefer dealing with the regular case. Since these distributions are taken as endogenous outcomes in Section 5.4, this will mean restricting our attention to cases where the outcome is regular. We will show later that there are indeed such cases.

[21] Interim meaning that expectations of the payoff of a participant are take over the random variables not known to him, but not over his privately known information.

It is also worth mentioning that the assumption that the seller pays the fee is without loss of generality. That is, it does not matter how the intermediary's fee $\omega(p)$ is allocated between the buyer and the seller.

2.1 The Simple Economics of Optimal Intermediation

Before showing that fee setting is an optimal mechanism, we will describe what we mean by an optimal mechanism. We use the term mechanism in the sense it is used in the mechanism design literature.[22] A mechanism is intermediary optimal if there is no mechanism that gives strictly higher profits to the intermediary in expectations over the buyer's and the seller's valuation for the good.

The buyer's virtual valuation $\Phi(v)$ can be interpreted as the marginal revenue of increasing the probability of trade, the seller's virtual cost as marginal cost.[23] Therefore, the intermediary wants the seller and the buyer to trade if and only if marginal revenue exceeds marginal cost, i.e. whenever $\Phi(v) \geq \Gamma(c)$.

As Myerson and Satterthwaite (1983) show formally, this is indeed the optimal allocation rule for the intermediary.[24] Due to a notion similar to Revenue Equivalence in auctions, once the allocation rule is determined the equilibrium payoffs for all types of all players are determined up to an additive constant. It is in the intermediary's interest

[22] Since we think that our results are also of interest for an audience less familiar with the mechanism design literature, we provide a brief description of the concepts necessary to understand our analysis. A more detailed explanation of these concepts is provided e.g. in Krishna (2002, Chapter 5). A mechanism means the following. First, the mechanism designer (here the intermediary) offers a menu of possible actions to the seller and the buyer, for each combination of actions he announces the payments a participant pays or receives and whether the good is exchanged, then both seller and buyer pick an action that constitutes a Nash equilibrium. For fee setting, the seller's choice of action is the choice which price to ask. The buyer chooses between the actions of accepting or rejecting the offer. In case of acceptance the buyer pays the gross and the seller gets the net price and the good is exchanged. In case of rejection there are no payments and no trade. Another example is a price posting mechanism, which we will deal with in more detail later: the intermediary posts two prices at which buyer and seller can trade and earns the spread between the two prices in case of trade. For this mechanism, both buyer and seller have only two actions to choose from: accept or reject the offer.

[23] The reasoning is similar to Bulow and Roberts (1989)'s for optimal auctions: interpret the probability that $V \geq v$ and $C \leq c$ as quantity demanded and supplied, i.e. $q := 1-F(v)$ and $q := G(c)$. Thus the inverse demand and supply function are $v = F^{-1}(1-q)$ and $c = G^{-1}(q)$, yielding $R(q) = qF^{-1}(1-q)$ and $C(q) = qG^{-1}(q)$ as revenue and cost functions. Taking derivative w.r.t. q and substituting back in yields $R'(q) = \Phi(v)$ and $C'(q) = \Gamma(c)$.

[24] Myerson and Satterthwaite (1983) are almost exclusively cited for their impossibility results. A notable exception is Spulber (1999, Ch.7). However, he merely compares the optimal direct mechanism of Myerson and Satterthwaite with price posting by the intermediary.

to minimize this constant under the constraint that all types of buyers and sellers are willing to participate in the mechanism (individual rationality constraint). Therefore, under the intermediary optimal mechanism the worst off agents are just indifferent between participating and not, i.e. the lowest valuation buyer $\underline{v}$ and the least efficient seller $\overline{c}$ get expected payoffs of zero. See Lemma 2 in the Appendix for a summary and formalization of these results.

2.2 Optimality of Fee Setting Mechanisms

For notational ease, let us denote $P(c) := \Phi^{-1}(\Gamma(c))$, which will turn out to be the price set by the seller, and denote its inverse as P^{-1}. To simplify the analysis we also maintain the following assumption throughout the rest of the paper:

Assumption 1. $\Phi(\bar{v}) \leq \Gamma(\bar{c})$ *and* $\Phi(\underline{v}) \leq \Gamma(\underline{c})$.

Intuitively, this assumption means that under an optimal mechanism, there are sellers who are unwilling to sell for sure and there are buyers that are unwilling to buy for sure. This seems plausible for many markets, e.g. most home owners do not offer their house for sale every year. Assumption 1 simplifies the exposition, because it ensures that we do not have to deal with corner solutions and Φ and Γ are invertible everywhere.[25]

An intuitive derivation of the optimal fee setting mechanism can be obtained by taking a brief detour through a dominant strategy implementation.[26] The dominant strategy implementation is that the intermediary allows trade iff $v \geq P(c)$ (or equivalently $c \leq P^{-1}(v)$) and in case of trade the buyer pays $P(c)$ and the seller gets $P^{-1}(v)$.[27] In case of trade the seller gets $E_v[P^{-1}(v)|v \geq P(c)]$ in expectations over v. Since the seller cares

[25]Formally, the assumption ensures that a mechanism satisfying (i) of Lemma 2 in the Appendix also satisfies (ii). Dropping this assumption would make the equations unnecessarily complicated without adding any substantial insights. $\Phi(\underline{v}) > \Gamma(\underline{c})$ is not difficult to accommodate for. A seller would never set a price less than $\underline{v}$ and therefore (ii) holds. For $\Phi(\bar{v}) > \Gamma(\bar{c})$ the intermediary should be able to solve the problem by imposing a price cap: the seller is not allowed to set a price satisfying $\Phi(p) > \Gamma(\bar{c})$. Therefore, the highest cost seller would get zero profits.

[26]The idea of a dominant strategy implementation goes back to Vickrey's analysis of second price auctions. This basically means that it is a dominant strategy (i.e. optimal independently of the other agents' actions) for every participant to report their types truthfully.

[27]The buyer gets a take-it-or-leave-it offer at price $P(c)$. It is clearly a dominant strategy to accept the offer iff $v \geq P(c)$. The same applies to the seller. This dominant strategy implementation is already mentioned in Myerson and Satterthwaite (1983) after Theorem 4.

only about what he gets in expectations rather than individual realizations of v (similar to the Revenue Equivalence in auctions (see e.g. Krishna, 2002)), the intermediary could just as well pay the seller the expected value as the net price $P(c) - \omega(P(c))$. Equating the net price $P(c) - \omega(P(c))$ with the seller's expected payoff and replacing $P(c)$ with p gives the optimal fee as stated in Proposition 1. Alternatively, one can equate the seller's utility with fee setting $[P(c) - \omega(P(c)) - c][1 - F(P(c))]$ with his information rent and solve first for $\omega(P(c))$ and then for $\omega(p)$ (see the formal proof in the Appendix).

Proposition 1. *Fee setting with*

$$\omega(p) = p - E_v[P^{-1}(v) \mid v \geq p] \tag{1}$$

is an intermediary optimal mechanism, where the seller with cost c sets the price $p = P(c)$.

A few remarks are in order. [28] First, the derivative of $\omega(p)$ in (1) is

$$\omega'(p) = 1 - \frac{f(p)}{1 - F(p)} \int_p^{\bar{v}} \frac{1 - F(x)}{1 - F(p)} [P^{-1}(x)]' dx. \tag{2}$$

Therefore the marginal fee can never be higher than 100 percent, since by Myerson's regularity assumption $[P^{-1}]'$ is positive, just as the other terms, and hence $\omega'(p) < 1$. This is of course what one would expect from incentive compatibility. One can also see that in two markets with seller's distributions G_1 and G_2, F being equal in both markets, the following holds. If $(\Gamma_1^{-1}(x))' > (\Gamma_2^{-1}(x))'$ for all x, then both the overall and the marginal fee are higher for all prices in market 2, formally $\omega_2(p) > \omega_1(p)$ and $\omega_2(p))' > \omega_1(p))'$ for all p that induce trade with positive probability. The reason is that $\omega(p)$ and $\omega'(p)$ decrease with $[P^{-1}(x)]' = [\Gamma^{-1}(\Phi(x))]'$.

Second, Proposition 1 implies that the intermediary can achieve his maximal expected profit without knowing or making use of the buyer's valuation when determining payments. However, the optimal mechanism depends in general on the distribution of

[28] Note that this mechanism is, of course, revenue equivalent to the mechanism described in Myerson and Satterthwaite (1983) that follows the idea of Vickrey auctions: the allocation rule is to trade iff $\Phi(v) \geq \Gamma(c)$ and a traders pays (or gets) the minimal (maximal) reported value for which he still would have traded, i.e. $\Phi^{-1}(\Gamma(c))$ for the buyer and $\Gamma^{-1}(\Phi(v))$ for the seller.

the buyer's valuation F. It is therefore rather striking that for a certain family of distributions of seller's types, namely all those that exhibit virtual costs that are linear in c, the intermediary optimal mechanism itself is independent of F and is a linear fee.

2.3 Optimality of Linear Fee Mechanisms

To see this, assume the virtual cost function is of the form $\Gamma(c) = c/b$, where $b > 0$ is a constant. This implies $P^{-1}(p) = b\Phi(p)$. Substituting this into (1) and integrating out yields $\omega(p) = (1-b)p$. For example, for G uniform on [0,1] we have $\Gamma(c) = 2c$ and thus $b = 1/2$. The following Proposition summarizes and generalizes this.

Proposition 2. *The following are equivalent statements:*

(i) a linear fee mechanism is optimal, i.e. $\omega(p) = \xi p + \zeta$ *is intermediary optimal,*

(ii) c *is drawn from a generalized power distribution* $G(c) = \left(\frac{c-\underline{c}}{\bar{c}-\underline{c}}\right)^{\beta}$, *where* $\beta > 0$ *and* $\xi = 1/(\beta+1)$ *and* $\zeta = -\underline{c}/(1+\beta)$.

As the optimal linear fee is fully determined by the two parameters $(\beta, \underline{c})$ of the distribution of the seller's cost G, Corollary 1 follows directly from Proposition 2.

Corollary 1 (Invariance of Linear Fees)**.** *If a linear fee is intermediary optimal for some distributions* (G, F), *then it will also be optimal for* $(G, \hat{F})$, *where* $\hat{F}$ *is an arbitrary regular distribution.*

It can also be shown that the reverse implication – in some sense – of Corollary 1 holds.

Proposition 3. *If a fee function* $\omega(p)$ *is optimal for a given* G *and for an arbitrary regular* F, *then the fee has to be linear and* G *has to be a generalized power distribution.*

As optimality of linear fees implies invariance of the fees with respect to the buyer's distribution, the empirical prediction of Proposition 2 is that whenever profit maximizing intermediaries choose linear fee setting as a mechanism, these fees will be invariant. Clearly, this prediction is consistent with available empirical evidence, provided that the seller's distributions are the same across time and regions.

Of course, this raises the question whether the seller's distribution should vary if the buyer's varies as well. There are two answers. First, the upper part of the seller's cost distribution $[P^{-1}(\bar{v}), \bar{c}]$, i.e. those sellers who for sure cannot sell, is irrelevant for the intermediation problem at hand. Therefore, Proposition 2 means that a linear fee only implies a generalized power distribution in the relevant range $[\underline{c}, P^{-1}(\bar{v})]$. Above this range, G can have any shape, provided its virtual cost function is increasing.[29] Note further that the relevant range $[\underline{c}, P^{-1}(\overline{v})]$ can be say $[0; \$100K]$ in the countryside and $[0; \$1,000K]$ in a big city, but they both lead to the same fee if they have the same shape in this range.

Corollary 1 and the empirical prediction thus hold not only when the cost distribution is the same over time and across regions, but even if it has only the same shape in the relevant range. This is illustrated in Fig. 1 that shows two different distributions of the seller's cost that lead to the same fee.

The second answer relies on how the distributions of steady state dynamic types are related to the distributions of entrant static types in the dynamic game with random matching. Therefore, we defer it to the end of Section 3.4.

The widespread use of linear fees raises the question whether linear fees may perform well even when the seller's distribution does not exhibit linear virtual costs. Though a complete analysis of the performance of linear fees in such environments is beyond the scope of the present paper, we provide some numerical examples in Appendix A that suggest that linear fees are close to optimal for other distributions than power distributions. In the main text, we restrict ourself to the example $g(c) = 6c(1-c)$ and $f(v) = 1$ as depicted is Fig. 2. Even though g is far from a power distribution, choosing a linear fee (equivalent to acting as if though having a power distribution $G(c) = c^{(1-\xi)/\xi}$ with $\xi = 0.4$) gives the intermediary 99.8% of the profits he would get with the optimal mechanism. This suggests that power distributions are a useful approximation even if the seller's distribution is a different kind of distribution.

It is worth mentioning that analogous results can be obtained for mechanisms where

[29]For many markets, it is reasonable to assume that most sellers are above the relevant range, so that the sellers with a positive probability of trade are just the "tip of the iceberg". E.g. most house owners prefer staying in their houses rather than offer them for sale.

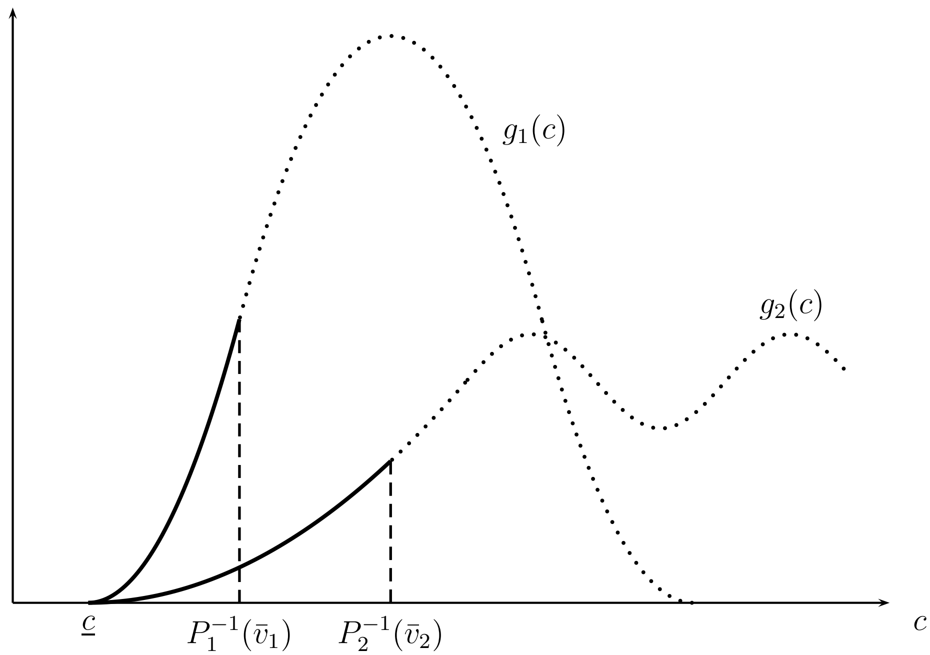

Figure 1: Two distributions of the seller's cost c leading to the same fee. The densities g_1 and g_2 have the same shape in their respective relevant ranges $[\underline{c}, P_1^{-1}(\bar{v}_1)]$ and $[\underline{c}, P_2^{-1}(\bar{v}_1)]$.

the buyer sets the price and the fee is conditioned on this price. It is for instance optimal for the intermediary to let the buyer set the price and charge a linear fee a fee $\omega_B(p) = E_c[P(c)|c \leq p] - p$, which induces the buyer to set the price $p = P^{-1}(v)$. For $F(v) = 1 - [(v - \underline{v})/(\bar{v} - \underline{v})]^{\beta_B}$ the fee will be linear and independent of the seller's distribution.

2.4 Static Monopoly with Many Buyers

The results obtained so far can be extended to a setup with one intermediary, one seller, and many buyers. As a preliminary, we first derive the intermediary optimal mechanism with many buyers and possibly many sellers. Since this is a generalization of the Myerson-Satterthwaite results on intermediary optimal mechanisms summarized in Lemma 2 above, it is of some interest on its own.[30]

Let N_B and N_S, respectively, be the number of buyers and sellers, whose valuations v_b

[30] See also Baliga and Vohra (2003).

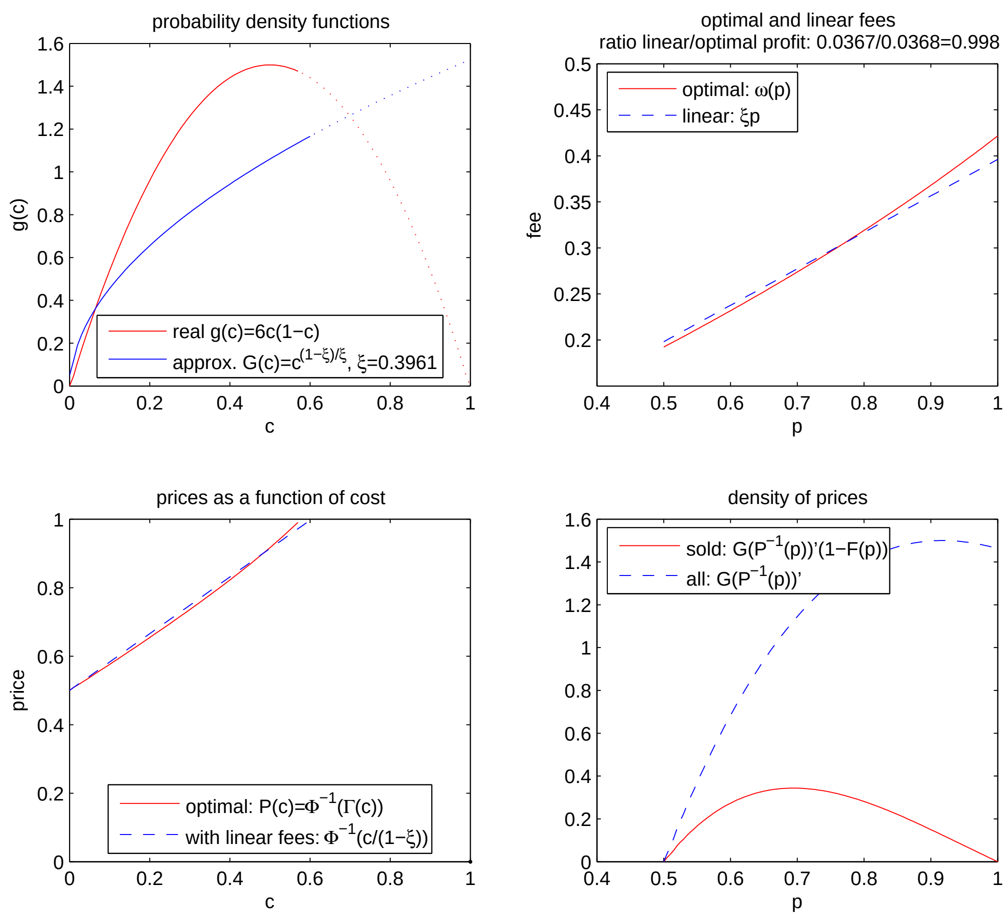

Figure 2: Optimal and linear fee for $g(c) = 6c(1-c)$ and $f(v) = 1$.

and costs c_s are independent draws from distributions F_b with densities f_b and supports $[\underline{v}_b, \overline{v}_b]$ and distributions G_s with densities g_s and supports $[\underline{c}_s, \overline{c}_s]$. As before, we consider cases where virtual valuations $\Phi_b(v_b)$ and the virtual costs $\Gamma_s(c_s)$ are strictly increasing and we use b (s) exclusively to indicate a buyer (seller). Order and relabel the realized virtual valuations in decreasing and virtual costs in increasing order, i.e. $\Phi_1 > \Phi_2 > .. > \Phi_{N_B}$ and $\Gamma_1 < \Gamma_2 < .. < \Gamma_{N_S}$. Pair buyers and sellers with equal index. The case where $N_B \neq N_S$ can be easily dealt with.[31] We define the Virtual-Walrasian allocation rule such that all pairs with $\Phi_k \geq \Gamma_k$ trade and all others do not. The Virtual-Walrasian quantity is the number of trading pairs, formally $K := \max\{k | \Phi_k \geq \Gamma_k\}$.

Lemma 1. *The intermediary optimal mechanism that respects individual rationality and incentive compatibility of buyers and sellers has a Virtual-Walrasian allocation rule and gives zero expected utility to buyers with $v_b = \underline{v}_b$ and sellers with $c_s = \overline{c}_s$.*

A sketch of the proof is in the appendix. For $N_B = N_S = 1$ the Virtual-Walrasian allocation rule reduces to the intermediary optimal allocation rule of Myerson and Satterthwaite (1983).

We now assume that there is one seller (i.e. $N_S = 1$) and that the $N_B > 1$ buyers' valuations are independently drawn from the identical distribution F with support $[\underline{v}, \overline{v}]$. It is well known from the auction literature that the reservation price is the same irrespective of the number of buyers. Hence one would suspect that results for price setting with one buyer carry over to an auction with a reservation price equal to the price one would set for one buyer. Prop. 4 shows that this is the case indeed.

Proposition 4. *Assume the intermediary faces N_B buyers whose valuations are i.i.d. draws from F and one seller whose cost is drawn from G. Then the following is an intermediary optimal mechanism. The intermediary sets the fee function $\omega(p_S) = p_S - E_v[P^{-1}(v) \mid v \geq p_S]$, where p_S is the final sale price. Then the seller sets the the reserve price $p = P(c)$, and a standard auction ensues.*

[31] If there are less buyers than sellers, fill up the ranks of buyers with fictitious buyers who do not trade for sure (i.e. $\Phi_k = -\infty$ for $N_B < k \leq N_S$). If there are less sellers, use fictitious sellers with $\Gamma_k = \infty$ for $N_S < k \leq N_B$.

Observe that the intermediary need not know the number of buyers N_B when determining the optimal mechanism. Of course, this is not surprising given that the optimal reserve price only depends on F and not on N_B.

3 Dynamic Competition with Random Matching

Two salient features of real world markets where exchange occurs predominantly via intermediaries are that agents can typically postpone trade to the future if it fails in the present and that intermediaries usually compete with one another. To account for these features that were absent in the static model of Section 2, we now extend the model as described next.

3.1 Setup

Consider a market in an infinite horizon setup as illustrated in Fig. 3. Each period mass 1 buyers and mass 1 sellers enter the market. A buyer's valuation of the good $\tilde{v}$ is drawn from the distribution $\tilde{F}_0$ with strictly positive density $\tilde{f}_0$ on the support $[\underline{\tilde{v}}_0, \bar{\tilde{v}}_0]$. A seller's cost $\tilde{c}$ is drawn from $\tilde{G}_0$ with $\tilde{g}_0 > 0$ on $[\underline{\tilde{c}}_0, \bar{\tilde{c}}_0]$. We refer to $\tilde{v}$ and $\tilde{c}$ as a buyer's and seller's static type. These static types are to be distinguished from agents' dynamic types, which are determined endogenously. Buyers enter a pool of buyers with mass σ with a distribution of their valuations $\tilde{v} \sim \tilde{F}$. Sellers enter a pool of mass σ with $\tilde{c} \sim \tilde{G}$. We assume that the market is in steady state, i.e. the traders entering the market (pool) have the same mass and distribution of valuations as those who leave. $\tilde{G}$ and $\tilde{F}$ are hence the steady state cumulative distribution functions. Their densities are denoted with $\tilde{f}$ and $\tilde{g}$. There is an infinite supply of intermediaries standing ready to offer their services. In each period each buyer, each seller, and each intermediary is uniform randomly matched[32] in a triple consisting of one member each of the three groups.

As in the one-shot game, the intermediary offers the following mechanism to the two

[32]This matching technology is essentially the same as in Atakan (2006a,b). It differs from Satterthwaite and Shneyerov (2007, 2008) who assume a seller who is matched with zero, one, or many buyers.

traders. He first announces a fee ω that is a function of the price the seller will set. Then the seller sets a price p. If the buyer accepts, he pays p, the seller gets the net price $p - \omega(p)$, and the intermediary the fee $\omega(p)$. If there is trade, the net utility of the seller with cost $\tilde{c}$ is $p - \omega(p) - \tilde{c}$, the buyer's net utility is $\tilde{v} - p$, and both traders leave the market. Intermediaries stay in the market forever.[33] Traders who do not trade stay in the market with the exogenous probability $e^{-\eta\tau}$ until the next period, where τ represents the length of a period and η a parameter of the hazard rate. With probability $1 - e^{-\eta\tau}$ a trader drops out of the market and has utility 0. Future utility is discounted with a factor $e^{-r\tau}$. As for most of the analysis only the product of these two factors matters, we define $\delta := e^{-(\eta+r)\tau}$ as the total discount factor, where τ can be interpreted as a parameter of the degree of competition: the shorter the time of a new match after a failed trade, the more competition intermediaries face.

Since in the dynamic setup agents are always given positive probability of trading in the future if trade fails in the presence, a buyer whose static type is $\tilde{v}$ will typically have a dynamic type $v < \tilde{v}$. Similarly, a seller with static type $\tilde{c}$ will have a dynamic type $c > \tilde{c}$.[34] The crucial point of the our analysis is that there is monotonic relation between static and dynamic types. Therefore, we can use the dynamic valuation of the buyer $v = B(\tilde{v})$ and the dynamic cost of the seller $c = S(\tilde{c})$ (which of course remain to be determined) to derive the endogenous distributions $F(v) = \tilde{F}(B^{-1}(v))$ and $G(c) = \tilde{G}(S^{-1}(c))$. A typical intermediary will simply use the dynamic types, or their distributions, rather than the static types and distributions to design the exchange mechanism described in the previous section. Note that from the point of view of an intermediary this is equivalent to a one-shot game, since the probability that he will meet the same buyer or the same seller in a subsequent period is 0 and he takes the mechanisms offered by other intermediaries in subsequent periods as given.

[33] This actually does not matter since they are all identical. We could just as well have intermediaries arriving each period and others dropping out.

[34] Put differently, the fact that there is a future drives a positive (negative) wedge between a buyer's (seller's) static type and his dynamic type.

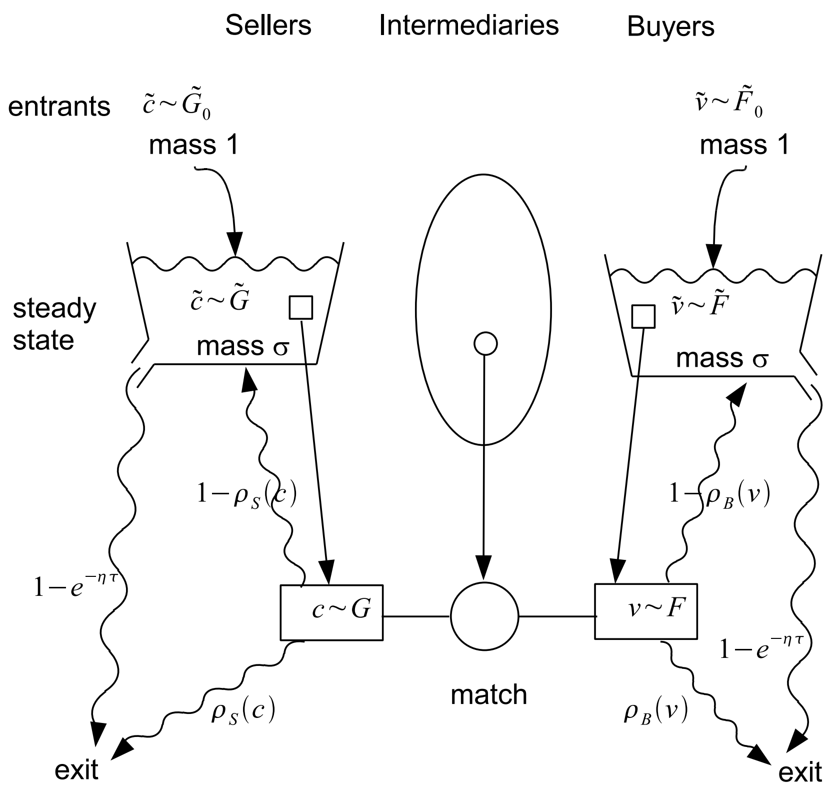

Figure 3: Market in steady state. Each period mass 1 of traders with distributions $\tilde{F}_0$ and $\tilde{G}_0$ enter the market and join pools with distributions $\tilde{F}$ and $\tilde{G}$. Traders have continuation type distributions F and G. With probabilities $\rho_B(v)$ and $\rho_S(c)$ they leave the market because they trade, with probability $1 - e^{-\eta\tau}$ they leave the market for exogenous reasons.

3.2 Dynamic Types

We have characterized what mechanism an intermediary would choose if the continuation distributions are given. Of course, these distributions in turn depend on the mechanism used by the intermediaries. We restrict our attention to steady state equilibria, i.e. equilibria where intermediaries use the same time invariant mechanisms. In the following we will determine the relation of dynamic types and static types for a given mechanism. From now on we will assume in accordance with the literature that buyers and sellers who cannot trade do not enter the market in steady state. This is equivalent to the inequalities in Assumption 1 being binding in steady state.

First, fix the mechanism used by the intermediaries. Following a similar logic as Satterthwaite and Shneyerov (2007) we first consider the discounted utility of a buyer with static valuation $\tilde{v}$ who cannot commit to reject an offer below his dynamic valuation v

$$W_B(\tilde{v}, v) = \rho_B(v)(\tilde{v} - v) + (1 - \rho_B(v))\delta W_B(\tilde{v}, v),$$

with $\rho_B(v)$ being the probability of trade as given by the mechanism chosen by the intermediaries. Rearranging yields

$$W_B(\tilde{v}, v) = (\tilde{v} - v)P_B(v),$$

where

$$P_B(v) := \frac{\rho_B(v)}{1 - \delta + \delta\rho_B(v)}$$

is "the discounted ultimate probability of trade" in Satterthwaite and Shneyerov (2007)'s terminology.

Assuming that the buyer plays a steady state strategy (i.e. the maximal price v that he is willing to accept is the same in each period), his "interim utility" is

$$W_B(\tilde{v}) = \sup_v (\tilde{v} - v)P_B(v) = (\tilde{v} - B(\tilde{v}))P_B(B(\tilde{v})).$$

By the same logic as Satterthwaite and Shneyerov (2007)'s Lemma 3 (i.e. using Milgrom and Segal (2002)'s generalized versions of the envelope theorem)

$$W_B(\tilde{v}) = W_B(\underline{\tilde{v}}) + \int_{\underline{\tilde{v}}}^{\tilde{v}} P_B(B(x))dx.$$

$W_B(\underline{\tilde{v}})$ turns out to be zero, since the lowest valuation buyer is just indifferent between participating and not. A buyer will accept an offer if the price is below his continuation valuation

$$B(\tilde{v}) = \tilde{v} - \delta W_B(\tilde{v}).$$

Combining this with the previous result we get

$$B(\tilde{v}) = \tilde{v} - \delta \int_{\underline{\tilde{v}}}^{\tilde{v}} P_B(B(x))dx, \tag{3}$$

and the differential equation

$$B'(\tilde{v}) = 1 - \delta P_B(B(\tilde{v})). \tag{4}$$

One can easily check that for infinitely long waiting times between trading opportunities $\tau \to \infty$ (and hence $\delta = e^{-(\eta+r)\tau} \to 0$, this basically means a single shot game) the dynamic valuation approaches the static valuation, i.e. $B(\tilde{v}) \to \tilde{v}$. Observe also that

$$B'(\tilde{v}) = \frac{1-\delta}{1-\delta+\delta\rho_B(v)} \in [1-\delta, \delta].$$

A similar analysis can be carried out for the seller. For expositional clarity assume for the moment that the intermediary uses the Vickrey-style mechanism described in Myerson and Satterthwaite (1983) (there is trade iff $\Phi(v) \geq \Gamma(c)$, in case of trade the buyer pays $\Phi^{-1}(\Gamma(c))$ and the seller gets $\Gamma^{-1}(\Phi(v))$). This induces a dominant strategy equilibrium and is of course revenue equivalent to our fee setting mechanism. For this mechanism clearly the same logic applies for the seller as for the buyer: he accepts any offer that is above his dynamic costs. By the same procedure we get

$$S(\tilde{c}) = \tilde{c} + \delta \int_{\tilde{c}}^{\bar{\tilde{c}}} P_S(S(x))dx, \tag{5}$$

and

$$S'(\tilde{c}) = 1 - \delta P_S(S(\tilde{c})). \tag{6}$$

The probabilities of trade are given by the optimal allocation rule (applied to the dynamic types):

$$\rho_B(v) = G(\Gamma^{-1}(\Phi(v))), \tag{7}$$

$$\rho_S(c) = 1 - F(\Phi^{-1}(\Gamma(c))). \tag{8}$$

Equations (4), (6), (7), (8), $F(v) = \tilde{F}(B^{-1}(v))$, and $G(c) = \tilde{G}(S^{-1}(c))$ characterize the dynamic types $v = B(\tilde{v})$, $c = S(\tilde{c})$ and their distributions F and G for any given steady state distributions $\tilde{F}$ and $\tilde{G}$. We will determine the relation of these distributions to the distributions of the entrants $\tilde{F}_0$ and $\tilde{G}_0$ below.

In principle, this already allows us to derive the optimal fee function ω, by obtaining F and G from the underlying steady state true distributions and substituting into (1). But closed form solutions are difficult to obtain in general. One approach is solving the problem numerically. Another approach is to solve the problem backward, starting with dynamic type distributions F and G and finding the corresponding static distributions $\tilde{F}$ and $\tilde{G}$. The second approach resembles more how one would proceed empirically since the distribution of the dynamic types is closer to what one observes empirically: if a buyer rejects a price p we know that it is above his dynamic valuation v, while his static valuation has to be inferred from the estimate of his dynamic valuation. We will use this second approach in the following.

3.3 The Relation Between Static and Dynamic Valuations

For the sake of analytical tractability we will choose continuation distributions with support $[0,1]$ such that the virtual valuation/cost functions are linear.[35] $G(c) = c^\beta$ and $F(v) = 1-(1-v)^\alpha$ is the most straightforward choice. However, we need to truncate the distributions to get rid of buyers and sellers who do not trade for sure ($\Gamma(c) > \Phi(1)$ and $\Phi(v) < \Gamma(0)$). This gives us

$$F(v) = 1 - \left(\frac{\alpha+1}{\alpha}\right)^\alpha (1-v)^\alpha \quad \text{and} \quad G(c) = \left(\frac{\beta+1}{\beta}c\right)^\beta,$$

on supports $[1/(\alpha+1), 1]$ and $[0, \beta/(\beta+1)]$, respectively. Note that truncation does not change Φ and Γ. From (7) and (8) we get with some algebra the probabilities of trade

$$\rho_B(v) = \left(\frac{\alpha+1}{\alpha}v - \frac{1}{\alpha}\right)^\beta \quad \text{and} \quad \rho_S(c) = \left(1 - \frac{\beta+1}{\beta}c\right)^\alpha.$$

The solutions for the differential equations (4) and (6) for $B(\tilde{v})$ and $S(\tilde{c})$ are given implicitly by

$$\begin{aligned} v &= B(\tilde{v}) + \frac{\delta}{1-\delta}\frac{\alpha}{\alpha+1}\frac{1}{\beta+1}\left(\frac{(\alpha+1)B(\tilde{v})}{\alpha}\right)^{\beta+1} + \text{const}, \\ c &= S(\tilde{c}) - \frac{\delta}{1-\delta}\frac{\beta}{\beta+1}\frac{1}{\alpha+1}\left(1 - \frac{(\beta+1)S(\tilde{c})}{\beta}\right)^{\alpha+1} + \text{const}, \end{aligned}$$

where the constants are such that $B(\underline{\tilde{v}}) = \underline{\tilde{v}}$ and $S(\bar{\tilde{c}}) = \bar{\tilde{c}}$. To obtain the functions B and S one needs to solve $(\beta+1)$th and $(\alpha+1)$th degree polynomials.[36] For the

[35] Of course this is with loss of generality, since by Proposition 2 this kind of distribution of the seller's cost leads to the optimality of linear fees. However, the focus of this section is to give an analytically tractable example of inferring true types rather than to determine when linear fees are optimal.

[36] Obtaining the inverse functions B^{-1} and S^{-1} is fairly simple. Unfortunately, since we started from the continuation types, we need B and S for our exercise to find the distributions $\tilde{F}$ and $\tilde{G}$.

uniform-uniform case ($\alpha = \beta = 1$) we can obtain the closed form solutions

$$B(\tilde{v}) = \frac{1}{2\delta}\left(2\delta - 1 + \sqrt{(1-\delta)(1-3\delta+4\tilde{v}\delta)}\right), \tag{9}$$

$$S(\tilde{c}) = \frac{1}{2\delta}\left(1 - \sqrt{(1-\delta)(1+\delta-4\tilde{c}\delta)}\right). \tag{10}$$

$S(\tilde{c})$ is plotted for different values of δ in Fig. 4 and 5. Note that for $\delta = e^{-(\eta+r)\tau} = e^{-10}$ we are already close to the one shot game continuation type $S(\tilde{c}) = \tilde{c}$. Fig. 5 illustrates that especially low cost sellers have a higher dynamic type: if they cannot sell in the current period for a high price, they are willing to wait further. Figs. 6 and 7 show $\tilde{g}(\tilde{c}) = g(S(\tilde{c}))S'(\tilde{c})$. For a large τ the distribution of the static types is nearly uniform, since static types are close to the dynamic types. For a small τ, the transformation from $\tilde{g}$ to g pulls sellers with a low valuation to the center and flattens the curve.

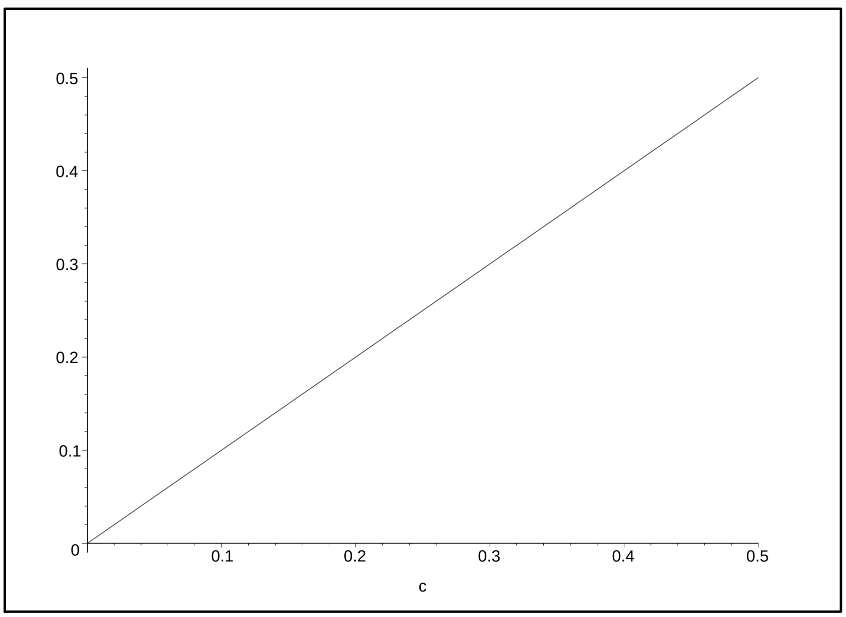

Figure 4: $S(\tilde{c})$ for $\delta = e^{-(\eta+r)\tau} = e^{-10}$ and v, c uniform

3.4 Entrant Types

Let us turn to the types of the entrants. Again, solving the problem backward turns out to be simpler than forward. Assume we already know the continuation distributions F and G and the true steady state distributions of the types in the market $\tilde{F}$ and $\tilde{G}$.

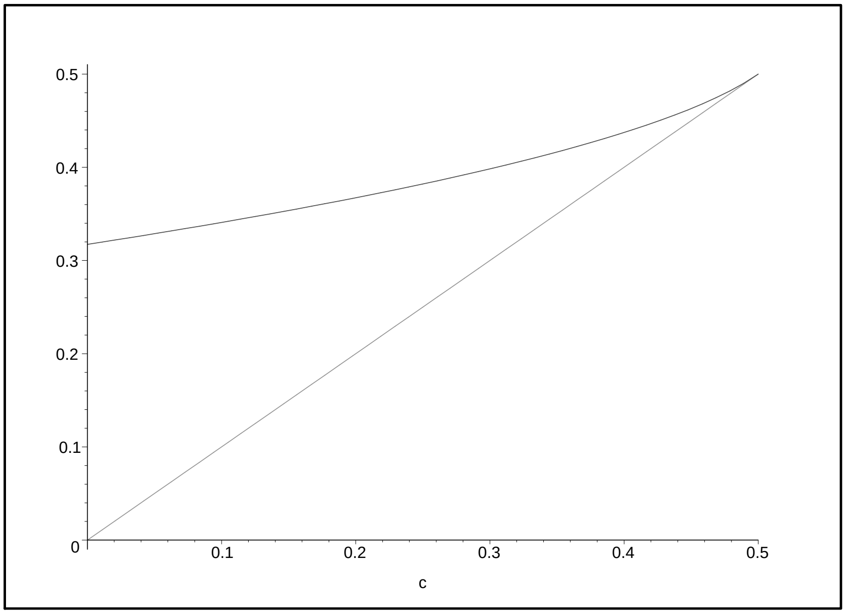

Figure 5: $S(\tilde{c})$ compared to the 45° line for $\delta = e^{-(\eta+r)\tau} = e^{-0.1}$ and v, c uniform

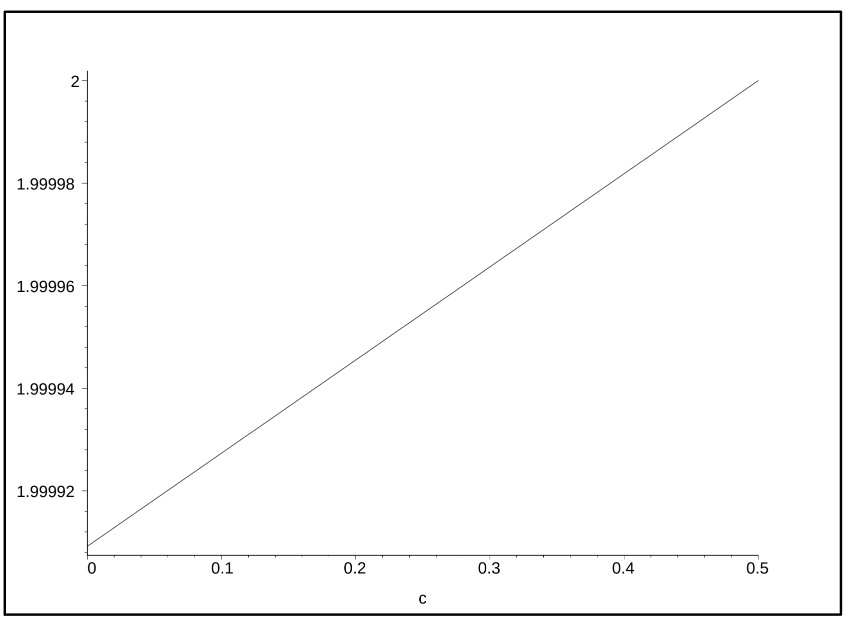

Figure 6: $\tilde{g}(\tilde{c})$ for $\delta = e^{-(\eta+r)\tau} = e^{-10}$ and v, c uniform

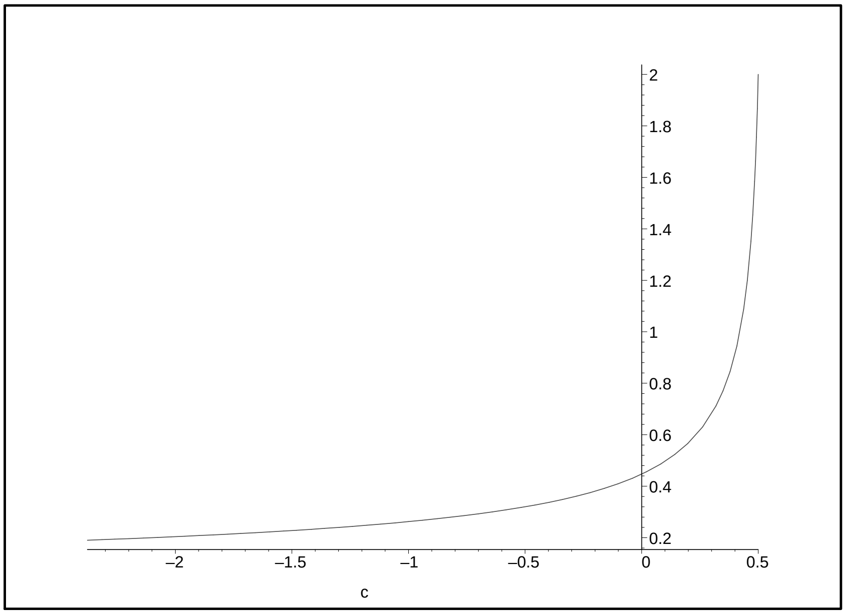

Figure 7: $\tilde{g}(\tilde{c})$ for $\delta = e^{-(\eta+r)\tau} = e^{-0.1}$ and v, c uniform

Now we want to derive the distributions $\tilde{F}_0$ and $\tilde{G}_0$ entrants must have for the market to remain in steady state.

The probability that a seller remains in the market is $(1 - \rho_S(c))e^{-\eta\tau}$. Note that here the distinction between the total discount rate $\delta = e^{-(\eta+r)\tau}$ and the probability of remaining in the market $e^{-\eta\tau}$ matters again. The mass of sellers of continuation type c leaving the market is the probability that such sellers do not remain in the market times the mass of all sellers σ. To obtain steady state we equate this with the mass of entrants of type c:

$$\sigma(1 - (1 - \rho_S(c))e^{-\eta\tau})g(c) = g_0(c), \tag{11}$$

where g_0 is the distribution of entrants' dynamic types and where σ has to be such that $\int g_0(c)dc = 1$, i.e. the total mass of entrants is equal to the total mass of sellers leaving the market. Eq. (11) shows that for every seller with continuation type c there are

$$[\sigma(1 - (1 - \rho_S(c))e^{-\eta\tau})]^{-1}$$

sellers with the same c in the market. Since this is increasing with c, this means that sellers with high continuation costs cumulate more in the market. This is intuitive, after all they have to wait longer until they can sell their good for the high price they charge.

We are now able to formulate the second answer to the question raised in Section 2 whether the seller's distribution should vary in different markets if the buyer's does. Even if the distributions of the entering buyers and sellers are the same, since high cost sellers and low valuation buyers wait longer in the market, the steady state probability density functions become steeper compared to the entrant densities with higher costs and lower valuations. This explains why buyers' continuation distributions are different from sellers'. It seems plausible (even though it has yet to be shown formally) that even if entering sellers' distributions have a somewhat different shape[37] in different geographical regions, the cumulation of high cost sellers transforms the distributions to similar distributions close to a power distribution.

So far the argument concerned distributions of dynamic types. As Eqs. (3) and (5) (and graphically Fig. 6 and 7) illustrate, the transformation from static type distributions to dynamic type distributions has an opposing effect to the cumulation of high cost sellers: since the value of continuing is higher for low cost sellers, the distribution of dynamic types is flatter than the true type distribution. In the following we illustrate in a simple example that cumulation can be a stronger effect than the continuation valuation effect. Whether this is true in general is yet to be shown.

To get the true type distribution of entrants, one has to substitute $S(\tilde{c})$ for c in (11):

$$\sigma(1 - (1 - \rho_S(S(\tilde{c})))e^{-\eta\tau})\tilde{g}(\tilde{c}) = \tilde{g}_0(\tilde{c}).$$

We will do this procedure for c, v uniform explicitly. Eq. (11) simplifies to

$$2\sigma(1 - 2ce^{-\eta\tau}) = g_0(c).$$

Using $\tilde{g}_0(\tilde{c}) = g_0(S(\tilde{c}))S'(\tilde{c})$ one gets

$$\tilde{g}_0(\tilde{c}) = 2\sigma\frac{\delta}{1-\delta}\left(e^{-\eta\tau} + \frac{1-e^{-r\tau}}{e^{-r\tau}}\sqrt{\frac{1}{1-4\tilde{c}}\frac{\delta}{1-\delta}}\right)$$

This is illustrated by Figs. 8, 9, and 10. The first case (Fig. 8) is basically the one-shot game where the continuation distribution is equal to the true market distribution which

[37]Note that the first answer to this question pointed out that if the sellers' distributions in different geographical regions are different, but have the same shape after truncation, the argument about invariance of fees still holds. Here we are only dealing with the issue of distributions having different shapes after truncation.

in turn is equal to the entrant distribution. The second case (Fig. 9) illustrates that cumulation can have a stronger effect than the transformation to dynamic costs: even if the type of entrants has a concave density, the fact that sellers with high costs wait for a long time in the market pushes up the part of the distribution on the right. Here, this transforms the concave density to a uniform one. The last case (Fig. 10) is somewhat puzzling: if one sets $\eta = 2$ and $r = 0$, the entrant distribution is flat again. The support is stretched out, however. An intuition is yet to be found for this.

This analysis should already provide a basic idea of this relation. It shows that high cost sellers (and analogously low valuation buyers) cumulate more in the market. This turns the density of the sellers to a "more convex" one that is closer to a power distribution and possibly makes differences in entrant distributions in different regions less important. This is a possible explanation for the small variance in real estate brokerage fees. The fact that the dynamics of such markets turns densities "more convex" also helps to explain the steep increase in the shape of the distribution of the sellers' dynamic types which corresponds to a fee of 6 percent: $G(c) \approx c^{16}$.

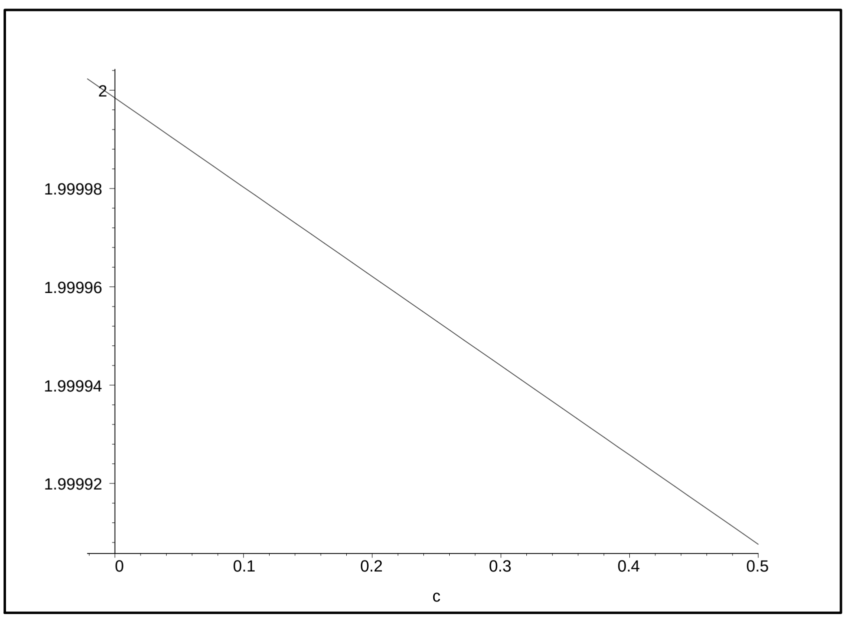

Figure 8: $\tilde{g}_0(\tilde{c})$ for $\eta = r = 1$ and $\tau = 10$ and v, c uniform

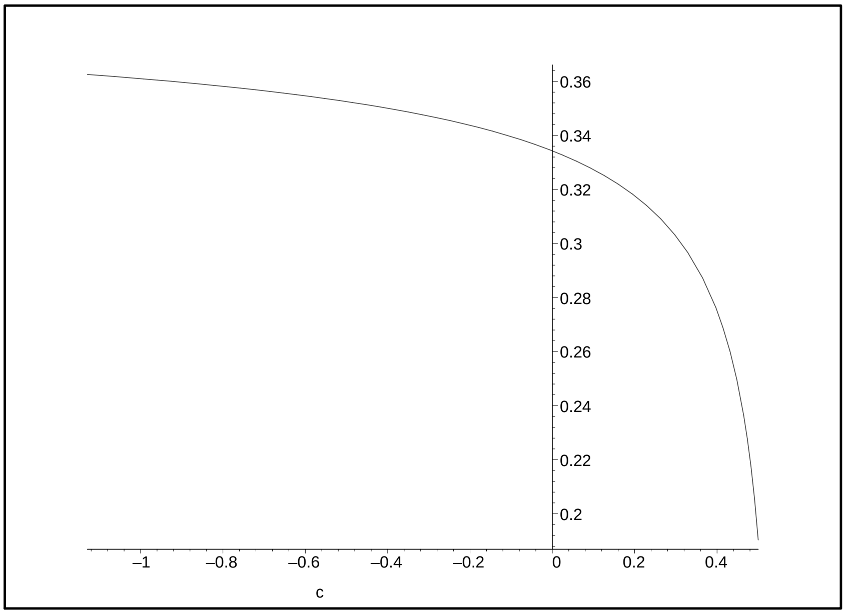

Figure 9: $\tilde{g}_0(\tilde{c})$ for $\eta = r = 1$, and $\tau = 0.1$ and v, c uniform

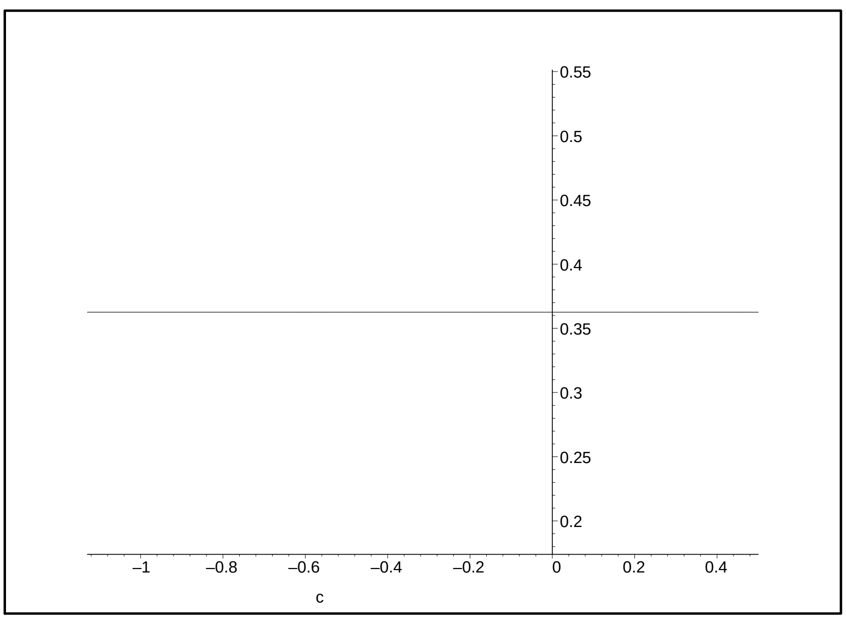

Figure 10: $\tilde{g}_0(\tilde{c})$ for $\eta = 2$ and $r = 0$, and $\tau = 0.1$ and v, c uniform

3.5 First Order Effects – Perturbation Analysis

Even though an analytical solution cannot be found in general, we can describe the effects of infinitesimally small perturbations of an analytically tractable solution.

We will start out with a case where we have an analytical solution: the static model. If traders discount the future with factor $\delta = e^{-(r+\eta)\tau} \to 0$ (equivalent to $r \to \infty$) and their probability of staying in the market $\epsilon := e^{-\eta\tau} \to 0$ (equivalent to $\eta \to \infty$), the solution of the dynamic game trivially coincides with the static game, we have $c = S(\tilde{c}) = \tilde{c}$ and $g = \tilde{g} = \tilde{g}_0$. As a next step we increase the probability of staying in the market ϵ infinitesimally, but keep the discount rate δ constant. This has the effect that true types and dynamic types still coincide, but entrant and steady state distributions become different since sellers accumulate. We will also assume that the change of ϵ only affects G and not F, because only the drop out rate of sellers changes. Another possible reason is that we are close to a power distribution, so that a change of F hardly has any effect.

The following analysis uses perturbation analysis[38] to perturb a function infinitesimally and look at the first order effect of this perturbation on the system of differential equations. As long as second order effects are sufficiently small, this is a good approximation of the exact solution.

To simplify the exposition, we will denote in this subsection the entrant true type distributions F and G instead of $\tilde{F}_0$ and $\tilde{G}_0$; the distributions of the steady state dynamic types (which coincide with the true types) will be denoted by $\hat{F}$ and $\hat{G}$.

Now recall from (11)

$$\sigma(1 - (1 - \rho_S(c))\epsilon)\hat{g}(c) = g(c), \tag{12}$$

where $\rho_S(c) = 1 - F(\Phi^{-1}(\Gamma(c)))$ and σ is a constant such that the density function $\hat{g}$ adds up to one. In the following, we want to have a function γ that infinitesimally perturbs $\hat{g}$

$$\hat{g}(c) = (1 + \epsilon\gamma(c))g(c)$$

and ensures that $\hat{g}$ adds up to one, i.e. $\int_{\underline{c}}^{\overline{c}} \gamma g = 0$.

Increasing ϵ infinitesimally has the following first order effects.

[38] xxx

Proposition 5. *The first-order effects of an increase of ϵ from $\epsilon = 0$ are the following:*

(i) $(\ln \hat{G})'$ *increases,*

(ii) $\hat{\Gamma}$ *decreases and* $\hat{\Gamma}^{-1}$ *increases,*

(iii) the sign of the change of $(\hat{\Gamma}^{-1})'$ *is ambiguous.*

Since $\hat{G}$ enters $\hat{\omega}$ only through $\hat{\Gamma}^{-1}$ and $\hat{\omega}'$ only through $(\hat{\Gamma}^{-1})'$ (see equations (1) and (2)), this leads us immediately to the following Corollary.

Corollary 2. *As the waiting time between rematchings decreases (starting from an infinite waiting time and considering first-order effects)*

(i) the overall fee $\omega(p)$ *becomes lower,*

(ii) the sign of the change of the marginal fee $\omega'(p)$ *is ambiguous.*

That the overall fee is lower with more frequent rematching could be suspected by intuition for two reasons. First, a more frequent rematching resembles more competition and should therefore drive down fees. Second, we have seen for the case of uniform dynamic cost distributions that more frequent rematching makes the seller's cost distribution more convex, which corresponds for a power distribution to β increasing. A higher β means a lower fee.

But a formal derivation of the first order effects shows that the intuition does not carry over further. An increase of β for a power distribution also means a lower marginal fee. However, the first order effect for the marginal fee is ambiguous.

4 Empirical Implications

Sections 2.4 and 5.3 have given the basic idea of why the working of auction houses such as Sotheby's and Christie's (or also eBay) can be seen in a very similar way as sales through real estate agents. The auction house sets a fee (which happens to be a percentage fee of 20% for Sotheby's and Christie's and which used to be a percentage fee of 5% for eBay) and the seller sets a reservation price rather than a take-it-or-leave-it price. The reservation price coincides with the price the seller would set for one buyer.[39] This similarity allows us to give an interpretation of the different percentage fees observed for different intermediation markets. In a market where it takes a long time until participants are rematched (i.e. τ is large) there is less cumulation of high cost sellers (for $\tau \to \infty$ there is no cumulation at all since all sellers leave the market with probability $1 - e^{-\eta\tau} \to 1$) and hence the probability density function is less steep. The lower β in a power distribution c^β implies a higher fee $\xi = 1/(1 + \beta)$. By the same logic a short period τ between consecutive rematchings implies a lower fee. For the provided examples a fee of 6% in real estate agency corresponds to a distribution $G(c) = c^\beta$ with $\beta \approx 16$, the 5% formerly charged by eBay to $\beta \approx 19$, and the 20% charged by Sotheby's and Christie's to $\beta = 4$. This result seems intuitive, a shorter time between consecutive matches is similar to more competition between intermediaries and hence leads to lower fees.

Of course, in our model we have abstracted away from several, potentially important issues arising in these markets. Whereas for real estate brokerage a large number of intermediaries is arguably a good approximation, for Sotheby's, Christie's and eBay it is not. This has the effect that the intermediary will take into account that agents not trading today will be matched with him in the future with a positive probability. Hence he will choose a mechanism that leads to no trade occurring more often. However, for the auction houses it has to be noted that there are two effects that (partially) offset this one. First, since in an auction the probability that no buyer bids more than the reserve price is smaller, it is less likely that the seller will return to the intermediary. Second, goods auctioned off are likely to be further away from the assumption of homogeneity we have made. Hence, a buyer not buying the good now is less likely to bid for a sufficiently similar good in the next period.

A further issue we have abstracted away from is collusion. This is relevant since the Department of Justice uncovered commission fixing between Sotheby's and Christie's in

[39] This is a result well known from the auction literature for the case without an intermediary. The auctioneer sets the same reservation price irrespective of the number of buyers.

the 1990s and is currently investigating whether real estate agents are colluding (DOJ, 2007). Investigating how collusion would look like in such a market is therefore of particular interest and relevance. For the two extreme cases for τ a simple answer can be found to this question. For $\tau \to \infty$ the probability of being rematched is zero, therefore, collusion (i.e. joint profit maximization) looks exactly the same like the imperfect competition described in our model. For $\tau \to 0$ bilateral matching essentially becomes a multilateral matching of all buyers and all sellers at once. For this case it can be shown that one optimal mechanism for a monopolist (or for colluding firms) is to post two prices at which sellers are allowed to sell and buyers are allowed to buy. For intermediate values of τ the intermediary has to solve a dynamic mechanism design problem where the technological constraint is that only matched buyers and sellers can trade in a certain period. Results have yet to be obtained for such a setup.

4.1 Quality Adjusted Price

There is a large empirical literature dealing with the relation between the quality adjusted listing price and the time on market.[40] For example, Rutherford, Springer, and Yavas (2005) consider the degree of overpricing (DOP) which is similar to a quality adjusted price. Assuming that everyone agrees that the objective value of a certain house is θ, an individual trader's valuation for the house is the product of θ and the trader's subjective valuation, i.e. θv and θc for the buyer and the seller, respectively. The observed price $\hat{p}$ is given by $\hat{P}(c) = \theta P(c)$, where $P(c)$ is the quality adjusted price. The degree of overpricing is the percentage which the listing price of the house is above the objective value of the house, $\text{DOP} = (\hat{P}(c) - \theta)/\theta$. The quality adjusted price would hence be $P(c) = \text{DOP} + 1$. The price $P(c)$ in our model can be interpreted as a quality adjusted price.[41] Therefore, our model can be interpreted e.g. as consisting of several separate submarkets that differ only in their θ, which is publicly observable, the distributions of

[40] See e.g. Hendel, Nevo, and Ortalo-Magné (2007), Rutherford, Springer, and Yavas (2005), Genesove and Mayer (1997, 2001), Kang and Gardner (1989), Yavas and Yang (1995) and Larsen and Park (1989)

[41] Since Rutherford, Springer, and Yavas (2005) estimate DOP as the average listing price a house with certain characteristics has, this means that DOP has mean 0. This corresponds to a quality adjusted price normalized to have mean 1.

v and c being the same. If the submarkets were not separate, it would be an issue that a trader's dynamic valuation depends on the distribution of θ he expects to get matched with in the next period.

4.2 Time On Market

A very relevant issue in practice in real estate brokerage is the time an object is on the market. We now derive predictions on the relation between the quality adjusted price and the time a house is on the market before being taken off the list, which happens either because it is sold or because the seller leaves the market without selling. Interestingly, the baseline model predicts that the average time on market is the same for sold and unsold houses.

Homogeneous Market First, consider a market with only one homogeneous good. Consider only the subsample of houses offered for a certain price p. Then in each period a constant fraction $1 - F(p)$ of houses leaves the market because they are sold and a constant fraction $\epsilon F(p) = e^{-\eta\tau} F(p)$ drops out for exogenous reasons. Since in every period the ratio of those selling and of those dropping out is constant, the distribution of the time on market is the same (geometric) distribution for sold houses and for houses that drop out for exogenous reasons. The continuous time approximation of the discrete time geometric distribution is described in Prop. 6.[42]

Proposition 6. *For homogeneous houses the time on market of sold and unsold houses has the same distribution. The continuous time approximation of this distribution is exponential with the cumulative distribution function* $1 - \exp(-(\phi(p) + \eta)t)$ *and mean*

$$T(p) = \frac{1}{\phi(p) + \eta}, \tag{13}$$

where ϕ *is defined as* $e^{-\phi(p)\tau} := F(p)$*. For a price* p *the ratio of houses ever sold, denoted* $1 - F_\infty(p)$*, is*

$$1 - F_\infty(p) = \frac{\phi(p)}{\phi(p) + \eta}. \tag{14}$$

[42] We provide the continuous time version of the distribution since it is more convenient for empirical purposes. The discrete time version is derived in the formal proof in the Appendix.

Consistent with our model the empirical literature (see e.g. Hendel, Nevo, and Ortalo-Magné, 2007) finds that the quality adjusted listing price and the time on market are positively correlated in cross-sectional data. One can also easily find an explanation in our framework for the negative correlation observed in longitudinal data.[43]

In theory, observing the relation between listing price p and the time on market $T(p)$ on the one hand and the ratio of houses sold, $1 - F_\infty(p)$, on the other hand would be sufficient to estimate the steady state dynamic distribution F. The most straightforward approach is solving (13) and (14) for $\phi(p)$ and η which results in $\phi(p) = (1 - F_\infty(p))/T(p)$ and $\eta = F_\infty(p)/T(p)$.

An empirical analysis would of course have to overcome further issues not present in our theoretical framework. Some could be dealt with by straightforward extensions of our framework or standard econometric tools, such as seasonal differences, the fact that it takes some (random) time until advertisements for a house appear and potential buyers first view the house,[44] or truncation of data. Others will require further thought.

We provide a numerical example of the prediction our model would give for dynamic type distributions $F(v) = v$ and $G(c) = c^{16}$ in Fig. 11. Note that the two subfigures on the left are our assumptions on F and G, the other four subfigures represent our predictions of the empirically observable functions.

Heterogeneous Submarkets Typically a data set includes heterogeneous submarkets. Difficulties arise when drawing borders between submarkets, some of which are similar to the well-known difficulties in antitrust economics of defining the relevant market.[45] A simple approach to the heterogeneity of submarkets is to assume that they

[43] I.e. during a boom period houses are sold faster and at higher prices than during a recession. In our framework this means that F and G change in booms such that both the average price $\int_{\underline{c}}^{P^{-1}(\bar{v})} P(c)dG(c)$ increases and the overall average time on market $\int_{\underline{c}}^{P^{-1}(\bar{v})} T(P(c))dG(c)$ decreases.

[44] This leads to the time on market not being exponentially distributed, but having a peak for some $t > 0$.

[45] For example, is the joint group of one and two bedroom apartments one submarket or are one bedroom apartments in Evanston with central air conditioning a separate submarket? Other difficulties may simply stem from the choice of narrow submarkets leading to too few data points within a submarket.

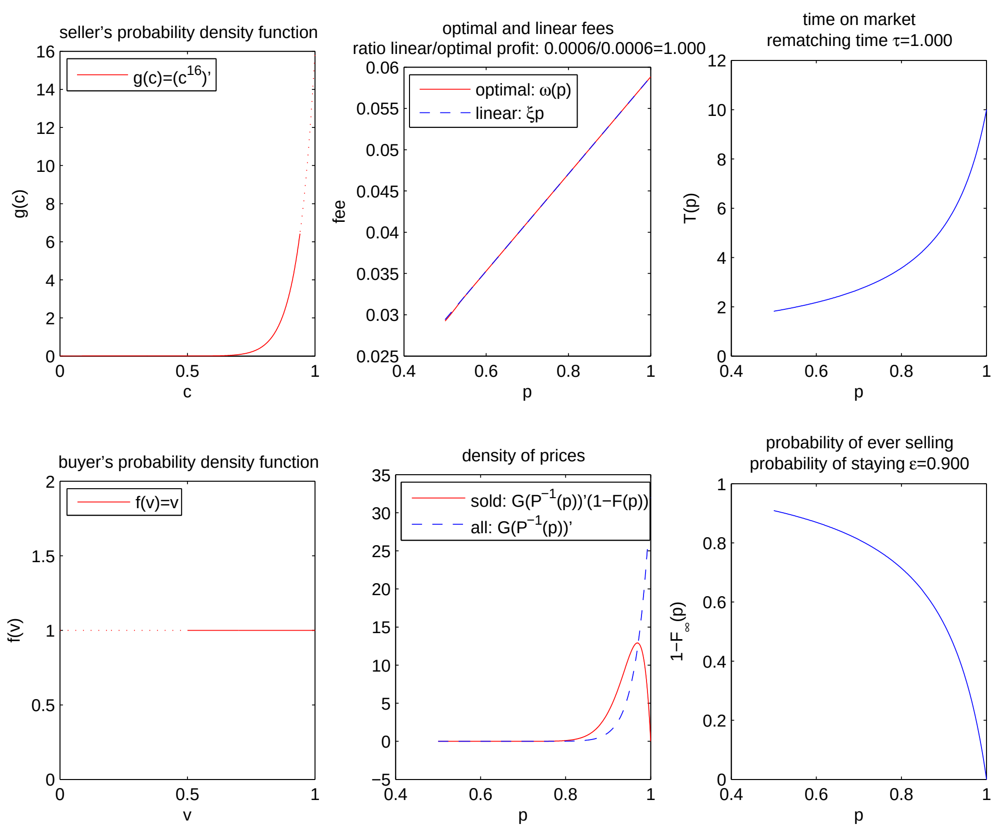

Figure 11: Predicted fee, density of prices, time on market, and probability of ever selling for $F(v) = v$ and $G(c) = c^{16}$.

differ only by a constant multiplier θ, e.g. the distributions of valuations and costs of one bedroom apartments and mansions have the same shape, but mansions are worth, say, four times more to everyone. Our previous results carry over without any change to such forms of heterogeneity.

However, for other kinds of heterogeneity, observations from several submarkets with different shapes of distributions showing up in the data would lead to average times on market that are lower for sold than for unsold houses. Intuitively, houses that have a higher probability of selling at the same price have a shorter time on market and are also relatively overrepresented in the set of sold houses. This is formally shown in Prop. 7.

Proposition 7. *If heterogeneous submarkets are in the observed sample, the time on market is lower for sold than unsold houses.*

This result is consistent with Larsen and Park (1989)'s remark in their empirical analysis that failing to include unsold houses may lead to a bias in the estimation of time on market. The analysis above suggests that such a bias stems from heterogeneity. Results in Prop. 7 naturally carry over to the case where times on market are estimated as averages over all prices rather than for a specific price.

5 Extensions

5.1 Inefficient Free Entry

Inefficient free entry of real estate agents is studied by Hsieh and Moretti (2003). Their empirical analysis also includes a reduced form model of inefficient free entry, similar in spirit to Salop's (1979) and Mankiw and Whinston's (1986) theoretical work. Their model takes fees as exogenously given and hence independent of the number of intermediaries. Our framework can be easily extended to allow for endogenous entry of intermediaries We can show that, under a condition we maintained in the dynamic model above and that seems empirically warranted, the mass of active intermediaries does not affect the equilibrium mechanism. Our model can hence also serve as a microfoundation for Hsieh and Moretti's assumption of exogenously given fees.

Homogeneous Intermediaries Assume first that intermediaries are homogeneous and have the per period opportunity cost K of participating in the market. Denote an intermediary's expected profit during a match as $\Pi := \int_{\underline{v}}^{\overline{v}} \int_{\underline{c}}^{\overline{c}} \max\{0, \Phi(v) - \Gamma(c)\} f(v) g(c) dc dv$ and the mass of intermediaries in the market as ι.

First, consider the case where there are more intermediaries than traders in the market ($\iota > \sigma$). A trader will always be matched and his probability of trade will be independent of ι. Therefore, his dynamic valuation and the equilibrium mechanism intermediaries use will be independent of ι and so will, therefore, be Π. An intermediary, on the other hand, is matched with probability σ/ι. If entry is free, intermediaries will enter until expected profits equal opportunity costs, i.e. $(\sigma/\iota^*)\Pi = K$, which gives us the equilibrium mass ι^* of active intermediaries in the market.

For $\Pi < K$ excess supply of intermediaries ($\iota > \sigma$) is not possible. In this case there is a scarcity of intermediaries, therefore, buyers and sellers will have to take into account that their matching probability ι/σ is less than 1. Hence ι will determine traders' dynamic valuations, an intermediary's equilibrium mechanism, and his profit per transaction Π. The equilibrium number of intermediaries is then given by $\hat{\Pi}(\iota) = K$, where $\hat{\Pi}(\iota)$ is some (decreasing) function that determines equilibrium per transaction profits depending on the number of intermediaries in the market.

Which of the two scenarios ($\iota > \sigma$ or $\iota < \sigma$) is relevant is, of course, an empirical question. Hsieh and Moretti find that the number of transactions per year per intermediary decreased over the time horizon where real estate was booming and attracted a lot of entry. This means that after the increase in housing prices and the wave of entry associated with it there was an excess supply of intermediaries ($\iota > \sigma$). Further, the number of intermediaries ι has increased proportionally to the overall profits made in real estate brokerage ($\sigma\Pi$). This means that there was an excess supply of intermediaries already before new intermediaries entered. Therefore, the empirically relevant case seems indeed to be the one with $\iota > \sigma$ we focus on.[46]

[46] Besides, the number of transactions (only including sales, not rentals) changed from 6 to 3 per year in the region investigated by Hsieh and Moretti. It is unlikely that intermediaries had to turn away potential traders because they did not have capacity to take care of them.

Heterogeneous Intermediaries Let us now assume that intermediaries are heterogeneous with respect to their opportunity costs and their productivity.[47] This is more in line with the empirical observation of a small number of "star brokers" who account for a large fraction of profits in the industry. Index intermediaries by $\iota \geq 0$ with ι increasing with their costs $K(\iota)$ (hence $K'(\iota) > 0$). We model an intermediary's productivity by his relative probability of being matched $r(\iota)$. This means that if the set I of intermediaries is active, an intermediary's matching probability is $r(\iota)/R(I)$ with $R(I) := \int_{\iota \in I} r(\iota) d\iota$ and his expected profit $(\sigma r(\iota)/R(I))\Pi$ in case of excess entry.[48]

Consider the simple case with $r'(\iota) \leq 0$, i.e. agents with higher opportunity costs are (weakly) less talented as brokers. This means the lower ι the more willing to participate a broker is. Hence brokers in $I^* = [0, \iota^*]$ participate in equilibrium for some marginal broker ι^*. I^* and ι^* are given by $(\sigma r(\iota^*)/R(I^*))\Pi = K(\iota^*)$. Since r is decreasing and K increasing, an increase of Π will clearly increase per year profits of the marginal intermediary ι^*. Hence, average industry profits will increase even more.

For other slopes of r we can get e.g. the case described in Hsieh and Moretti Appendix A: brokers in $I^* = [\iota^*_{\min}, \iota^*_{\max}] \subset (0, \infty)$ participate, where $\iota^*_{\min}$ and $\iota^*_{\max}$ are the marginal indifferent brokers. This is Hsieh and Moretti's story of the "middle class" doing real estate brokerage: people with very low skills would not earn enough as brokers, people with very high skills earn more in other jobs. If r is strongly increasing in the upper part of the interval I^* of active brokers, we have the case of a few "star brokers" that get a large share of the transactions and the profits in the industry.

The shapes of r and K determine whether the active set of intermediaries may consist of multiple non-connected intervals, whether there is multiplicity of equilibria, the sign of the effect of an increase of per transaction profits Π on average industry profits

[47] This relates to Hsieh and Moretti's Appendix A, where they study differently talented brokers with heterogeneous participation costs.

[48] If entry is not excessive, i.e. the mass of intermediaries $\int_{\iota \in I} d\iota$ is less than σ, profits per transaction will be some $\hat{\Pi}(I)$.

$(\sigma/\int_{\iota\in I} d\iota)\Pi$,[49] and the welfare effect of entry tests for brokers.[50] Empirical observations of the distribution of per year profits among intermediaries should allow to make inferences about the slopes of r and K.[51]

5.2 Per Transaction Costs of Intermediation

Instead of or in addition to assuming a fixed cost of intermediation, let us now briefly consider now the case where intermediation services involve a variable (as well). We show that under the assumption of a power distribution with $\underline{c} > 0$ of the seller's cost, all the previous results go through rather nicely.

Recall first that a power distribution G with $\underline{c} > 0$ implies a negative fee $\xi < 0$, i.e. a subsidy from the intermediary to the seller which we typically do not observe. However, intermediaries also provide services (advertising, showing the house to potential buyers, legal advice) we do not account and that are costly. Thus, the free of charge provision of these services can be interpreted as a subsidy from the intermediary to the seller paid in resources.

This is also consistent with the following observation. In the U.K. real estate brokers typically charge a lower fee (2.5 percent) than in the U.S. but they do also provide less services. Recall that for a power distribution a larger β implies a lower percentage fee ... but also a smaller subsidy. Thus, assuming that the seller side both in the U.S. and the UK are characterized by power distributions , this is consistent with $\beta_{UK} > \beta_{US}$ which

[49] That average industry profits can increase with per transaction profits has been already shown for r weakly decreasing and K increasing. That they can decrease can be illustrated with the following example. Let initial per transaction profits be Π_0. Assume $r(\iota) = 1$ for $\iota < 1$ and $r(\iota) = 2$ for $\iota \geq 1$. Assume $K(\iota) = \sigma\Pi_0/2 + \epsilon$ for $\iota < 2$ with some small ϵ and $K(\iota) = \infty$ else. Initially, only agents in the interval $I^* = [1, 2]$ participate. Per year profits are $\sigma\Pi_0$. An increase to $\Pi_1 = (3/2)\Pi_0 - \epsilon$ results in the entry of low ability agents with $\iota \in [0, 1]$. Average per year per intermediary profits fall to $\sigma(3/4)\Pi_0$.

[50] The National Realtor Association and individual states require candidates to pass an exam to be licensed as a broker. Besides the slopes of r and K the effect would also depend on the details of the test. E.g. for r weakly decreasing a test that excludes brokers with the highest opportunity costs would be clearly welfare enhancing, a test that excludes the lowest opportunity cost intermediaries welfare reducing. Otherwise the effect is ambiguous.

[51] If the cumulative distribution function H of per year profits among intermediaries is observable, the following reasoning applies. Under the assumption that r is weakly increasing and $I^* = [\iota^*_{\min}, \iota^*_{\max}]$, the inverse of the cumulative distribution function is equal to $H^{-1}(x) = \sigma r((\iota^*_{\max} - \iota^*_{\min})x + \iota^*_{\min})/R(I^*)\Pi$, which should allow to recover r for the domain I^*. For the opportunity cost K one can only infer that it is weakly less than per year profits in I^* and equal at the boundaries of the interval, $\iota^*_{\min}$ and $\iota^*_{\max}$. A change of I^* over time (e.g. because of a change of Π) would allow for further estimates.

in turn implies $\xi_{US} < \xi_{UK} < 0$, which is consistent with the difference in service levels provided by brokers.

5.3 Many Buyers in the Dynamic Game

"Non-Discriminating Mechanisms" One can also consider a market with more buyers than sellers in the pool of current potential traders[52], where a seller and an intermediary are matched with a predetermined number of buyers in each period. Our results relatively easily generalize to such a setup if we assume that the intermediary is retricted to a certain class of mechanisms. Define "non-discriminating mechanisms" as mechanisms that cannot distinguish between buyers. An intermediary restricted to non-discriminating mechanisms basically treats all buyers as one "representative buyer" whose valuation is drawn from a distribution F, F being the distribution of the highest order statistic of the buyers.[53] Our results carry over to such a setup if we adapt the continuation valuations of the buyers adequately. This implies in particular that whenever linear fees are an optimal non-discriminating mechanism, they are independent of the distributions of the buyers' valuations and, therefore, also of the number of buyers.

An advantage of looking at non-discriminating mechanisms is that they are a superset of mechanisms that do not condition transfers on the buyers' valuations and, therefore, our optimality results carry over a fortiori to these mechanisms. Not conditioning transfers on buyers' valuations reduces communication costs: imagine a real estate agent posting prices of houses on sale outside of his office. Passers-by see the prices and only walk into the office if they are willing to buy at the posted price.

This allows us to interpret our model as equivalent to a model that takes into account a feature of real estate brokerage markets: buyers are rematched more frequently with an intermediary than sellers, a seller typically has a real estate agent show his house to several buyers before he looks for another real estate agent. Consider the following alternative description of what happens in a period in which a seller and an intermediary stay together. n buyers with continuation distributions F_1, F_2, ..., F_n arrive sequentially

[52] This can be the case because the mass of entering buyers is larger then that of entering sellers or because the exit rates of sellers are higher.

[53] One can interpret $F(p)$ as the probability that no buyer is willing to buy at price p.

in a random order. Assume that time 0 passes between their arrivals, so that discounting and exogenous exit do not matter. Then the intermediary and the seller can choose the fee and the price as if though there were only one buyer with distribution F equal to the highest order statistic of the distributions $\{F_i\}_{i=1}^n$. The first buyer who is willing to accept the price gets the house.[54]

Auctions An alternative view of many buyers being matched with one intermediary in each period is that all buyers arrive at the same time. Then, as argued in Section 2.4, the optimal mechanism for the intermediary is – provided the seller's cost is drawn from a power distribution – to charge a percentage fee and let the seller auction off the good. Even though this part of the analysis is yet to be formally investigated, results from the static problem with many buyers should carry over.[55]

5.4 Price Posting

Static Setup Consider now an alternative mechanism, called *price posting*, which is widely used e.g. by stock markets, used car dealers, and currency exchange offices: the intermediary sets an ask (or buyer) price p^B and a bid (or seller) price p^S. If both seller and buyer are willing to trade at these prices, then the intermediary earns the bid-ask spread. Otherwise, there is no trade. The following proposition characterizes the optimal bid and ask price and shows under somewhat stricter regularity conditions that price posting is never optimal for the intermediary in a static setup.[56] This is puzzling at first sight since we observe price posting in many real world examples. We will show later on how this puzzle is resolved in a dynamic setup.

Proposition 8. *Assume the inverse hazard rate* $(1 - F(v))/f(v)$ *is decreasing and*

[54] This will clearly be optimal in the class of non-discriminating mechanisms. We should compare this setup to Riley and Zeckhauser (1983) to have a clearer view of optimality in a general class of mechanisms under the assumption that a buyer who didn't get the house when he arrived will never return.

[55] One has to take into account that a buyer's continuation value changes for two reasons. First, the probability that he will trade in a period is lower. Second, the price he pays will be higher.

[56] These conditions ensure global concavity of the profit function and one can hence work with a unique solution of the first order conditions. However, without these assumptions, most of the analysis should carry over since our non-optimality result does not rely on uniqueness.

$G(c)/g(c)$ is increasing and that there is one buyer and one seller. Then the optimal ask price p^B and bid price p^S are given by the equations $p^B = \Gamma(p^S)$ and $p^S = \Phi(p^B)$. Further, price posting is never optimal for the intermediary.

This rather general non-optimality result of price posting hinges critically on Assumption 1. If one assumes instead that distributions do not overlap and that there is a sufficiently large gap between the two distributions (more precisely $\Phi(\underline{v}) > \Gamma(\bar{c})$). Then by Lemma 2 the intermediary always allows trade to occur and sets prices $p^B = \underline{v}$ and $p^S = \bar{c}$.

Dynamic Setup Proposition 8 stands in contrast with the empirical observation that many intermediaries do use price posting mechanisms and trade does not occur each time a potential buyer or seller meets an intermediary. However, a closer look reveals that in all of the examples of price posting intermediaries there is a crucial difference to real estate brokers: an intermediary has the possibility of storing the good. Hence demand and supply need not equal in every point in time, but only on average. A possible explanation of why real estate agents cannot do this are liquidity constraints and other costs of storage. Interestingly, as we show next, if storing the good is possible, price posting becomes optimal.[57]

For simplicity, let us model the ability to store the good in the following way. An intermediary can store the good between subsequent rematchings. Assume he has no liquidity constraints, no storage constraints, his discount factor is 1, he can have short positions of the good, and storage costs are zero. His only constraint is that he has to buy and sell with the same probability.[58] Therefore we can just as well consider a mechanism

[57]Riley and Zeckhauser (1983) and Peck (1996) also look at the optimality of price posting mechanisms, but in different setups and without intermediaries. The former relies on buyers' ability to wait without costs for a cheaper offer by the same seller, the latter assumes buyers with identical valuations for the good choosing simultaneously between offers from multiple sellers.

[58] Our assumption is essentially the same as Garman's (1976) assumption of an intermediary with an infinite inventory of cash and stock and Glosten and Milgrom's (1985) assumption of "zero costs associated with all short positions in cash and stock". Both papers look at intermediaries facing sequentially arriving traders (in stock markets), but in a very different setup and with a very different focus: both assume the price posting mechanism as exogenously given, the former considers non-strategic traders arriving in continuous time, the latter the bid-ask spread set by zero profit intermediaries in the presence of insider trading. In Garman (1976) an intermediary also faces the constraint that his probability

design problem where all buyers and sellers that will be matched with the intermediary at any point in time arrive at once. In the following we will solve this mechanism design problem.

Assume that there are N buyers and N sellers with i.i.d. draws $v_i \sim F$ and $c_i \sim G$. In a static setup where all buyers and sellers arrive at the same time, v_i and c_i are valuations and costs. For the dynamic setup, v_i and c_i are interpreted as steady state continuation valuations and costs. Our analysis relies on N converging to infinity. To simplify exposition index the realized valuations in decreasing and costs in increasing order. To avoid dealing separately with the special case where all buyers and sellers trade, add a fictitious buyer who will never trade with $v_{N+1} = -\infty$ and a fictitious seller with $c_{N+1} = \infty$.[59] Denote the Virtual-Walrasian quantity as defined before Lemma 1 as $K := \max\{i|\Phi(v_i) \geq \Gamma(c_i)\}$. The following Proposition shows that posting two prices maximizes the intermediary's profits as the number of buyers and sellers goes to infinity. The basic idea is that for an infinite number of traders the intermediary knows the distribution of the realized types. Hence he knows the Virtual-Walrasian quantity without any reports by the traders, he only has to make sure that traders within the Virtual-Walrasian quantity trade.

Proposition 9. *Consider a price posting mechanism with N buyers and sellers, where prices are given by the equation system $\Phi(p^B) = \Gamma(p^S)$ and $1 - F(p^B) = G(p^S)$. As N converges to infinity*

(i) the optimal mechanism converges to a price posting mechanism with these prices,

(ii) the price posting mechanism with these prices converges to optimality.

By the argument at the beginning of this subsection, this leads us to the following corollary.

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of buying and selling must equal. We could depart from our assumptions and the equal probability constraint by using a more complicated setup where the intermediary has a limited storage capacity and no possibility of short positions. In the long run, he would still have to buy and sell with the same probability on average, but in the short run the probabilities would be different. E.g. if the intermediary is about to run out of stocks, he will increase prices to have a higher probability of trading with a seller than with a buyer. However, one would conjecture that as storage capacity approaches infinity, the mechanism used in such a setup should approach our mechanism.

[59] The number of buyers and sellers being unequal could be dealt with in a similar fashion. If there are e.g. less sellers than buyers, the missing sellers can be filled up with fictitious sellers.

Corollary 3. *If the good is storable, price posting is the optimal mechanism.*

The intuition for why for non-storable goods fee setting is the optimal mechanism and for storable goods price posting is fairly clear. The intermediary is more likely to make the payments contingent on the seller's (or the buyer's) report for non-storable than for storable goods, since for non-storable goods the type of an individual trader is more important. For storable goods, the intermediary's problem of simultaneously trading with a buyer and a seller can be separated to two separate problem, that of selling a good and that of buying a good.

One seller – infinite number of buyers Another result concerns the equivalence between, and optimality of, price setting by the intermediary and linear fee setting when the number of buyers N_B approaches infinity while the number of sellers N_S is kept fixed at one. For simplicity, assume that buyers' valuations are identically and, as before, independently distributed according to the distribution F on $[\underline{v}, \overline{v}]$. The seller's cost is distributed according to G on $[\underline{c}, \overline{c}]$ with $\underline{c} < \overline{v}$ and as in Proposition 8, we assume that $G(c)/g(c)$ is increasing. Moreover, as in the rest of the paper, we assume that F and G satisfy Assumption 1.

Proposition 10. *As N_B approaches infinity, intermediary price setting and fee setting are equivalent. Moreover, both mechanisms are intermediary optimal if F is regular.*

Note that for an infinite number of buyers and fee setting any ω satisfying $\omega(\bar{v}) = \bar{v} - \Gamma^{-1}(\bar{v})$ and incentive compatibility is profit maximizing for the intermediary. For instance the intermediary could just as well charge a fixed fee $\bar{v} - \Gamma^{-1}(\bar{v})$.

5.5 Vertically Integrated Intermediary-Seller

Levitt and Syverson (2005) and Rutherford, Springer, and Yavas (2005) observe that houses owned by brokers yield on average a higher price than comparable houses of independent sellers.

We can use our framework to investigate the selling behavior of intermediaries who are also sellers. Suppose the intermediary owns the good (i.e. the seller and the intermediary can be considered as a vertically integrated firm), so that he knows its cost $\tilde{c}$. Then price posting will be optimal for this vertically integrated seller.[60] Not surprisingly, the profit of the vertically integrated seller will exceed the joint profits of the stand alone seller and stand alone intermediary.

However, because the fee setting mechanism that is optimal for the non-integrated intermediary imposes an upward distortion in the seller's effective cost, the price set by the non-integrated seller should be larger than the price set by the integrated intermediary-seller in a static setup. Formally, $\Phi^{-1}(\Gamma(c)) = P(c) > P_I(c) := \Phi^{-1}(c)$, since $\Gamma(c) > c$ and Φ is increasing, where P_I is the price set by the intermediary-seller. Accordingly, the welfare of the buyer with an integrated intermediary-seller should be larger than when the seller and the intermediary are independent. This prediction of lower prices under vertical integration contrasts with the empirical findings of Levitt and Syverson (2005) and Rutherford, Springer, and Yavas (2005).

Quite interestingly, dynamics may reverse this result, as we show next. The dynamic costs satisfy $S_I(\tilde{c}) > S(\tilde{c})$, where S_I is the dynamic cost of the intermediary-seller. So the intermediary-sellers with true cost $\tilde{c}$, having a higher dynamic cost than the independent seller, will optimally set a higher price than the independent seller would, absent the fee charged by an intermediary. Therefore, dynamics tend to mitigate and in some cases overturn the price difference between vertically integrated and stand alone sellers. To see this, consider the following example where continuation valuations are uniformly distributed with $F(v) = 2v - 1$ for $v \in [1/2, 1]$ and $G(c) = 2c$ for $c \in [0, 1/2]$. Consequently in equilibrium the independent seller with dynamic cost c sets the price $P(c) = c + 1/2$. Since $c = S(\tilde{c})$ as given in (10), the seller with the static cost $\tilde{c}$ sets the price $P(\tilde{c}) = S(\tilde{c}) + \frac{1}{2} = \frac{1}{2\delta}\left(1 + \delta - \sqrt{(1-\delta)(1+\delta-4\tilde{c}\delta)}\right)$.

A simple way of introducing integrated sellers is to assume that there are few of them (i.e. they are of measure zero) so that their behavior does not affect the distribution of continuation valuations. Assume also that an integrated seller leaves the market after

[60] If the intermediary and the seller are independent agents, then the intermediary will offer $p^S = c$ to the seller and the seller's profit is zero. Whether the intermediary and the seller are vertically integrated or not, the optimal price charged to the buyer will be $p^B = \Phi^{-1}(c)$. That price posting is optimal with one buyer follows from the theory of optimal selling mechanisms; see e.g. Myerson (1981).

Figure 12: In the static setup trade with the integrated seller takes place iff $v \geq 1/2 + c/2$ (blue) for uniform(s) on [0,1].

successful sale just like stand alone sellers do and are matched to a buyer with probability one in every period. Then, we have the following:

Proposition 11. *An integrated seller with static cost $\tilde{c}$ may set a higher price in equilibrium than a stand alone seller with the same static cost.*

An analysis yet to be done is to assume that intermediary-sellers have the same entrant true cost distribution as other sellers and see whether the average price set by intermediaries is higher.

6 Conclusions

The widespread use of linear fee setting mechanisms is empirically well documented and so is the fact that these fees exhibit very little variance across different regional markets (Hsieh and Moretti, 2003; Hendel, Nevo, and Ortalo-Magné, 2007). In the present paper, we have shown that linear fee setting is optimal for the intermediary, i.e. satisfies the intermediary optimality conditions of Myerson and Satterthwaite (1983), if and only if the seller's cost is drawn from a generalized power distribution. Whenever such a mechanism is optimal, the fee structure is also independent of the distribution of buyers' valuations. Moreover, we have generalized the intermediary optimal mechanism of Myerson and Satterthwaite (1983) to a setup with many buyers and possibly many sellers. We have further shown that any standard auction format with a reserve price set by the seller and a linear fee is intermediary optimal if the seller's cost is drawn from a power distribution under the assumption that the buyers' valuations are identically and independently distributed.

Further, we have shown how results from the static mechanism design problem of the intermediary can be used as part of a larger model where intermediaries, sellers, and buyers are randomly rematched each period. The intermediary faces the same mechanism design problem, however, he has to use the traders' continuation type rather than their true type. Our preliminary results on the random matching markets point to the direction that the dynamics of such markets cause the sellers' distributions to become more similar across regions. The dynamics of such markets also form the shape of the steady state distributions. The shorter the time between two consecutive rematchings, the steeper the density of the seller's cost distribution, and hence the lower the fee charged by the intermediary.

Appendix

A Robustness of Linear Fees

In the following, we will make a semi-formal argument in favor of the robustness of linear fees, i.e. that they perform well even if the seller's cost is not drawn from a power distribution. Alternatively, this can be interpreted as arguing in favor of using a power distribution as an approximation of another distribution G. For analytical convenience, we assume that the buyer's valuation is drawn from the uniform on [0,1], so that given the linear fee ξ the seller's optimal price is $p(\xi, c) = 1/2 + c/(2\xi)$ independently of the distribution G_k. We consider the following four different distributions of c on [0,1]:

(i) $G_H(c) = c^2(3-2c)$ with $g_H(c) = 6c(1-c)$ and $\Gamma_H(c) = \frac{c}{1-c}\frac{9-8c}{6}$

(ii) $G_T(c) = 2c - c^2$ with $g_T(c) = 2(1-c)$ and $\Gamma_T(c) = \frac{c}{1-c}\frac{4-2c}{2}$

(iii) $G_C(c) = 3(c - c^3/3)/2$ with $g_C(c) = 3(1-c^2)/2$ and $\Gamma_C(c) = \frac{2c(3-2c^2}{1-c^2}$

(iv) $G_O(c) = 2c^{3/2}/3 - 2c^{5/2}/5$ with $g_O(c) = 15c^{1/2}(1-c)/4$ and $\Gamma_O(c) = \frac{c}{1-c}\frac{25-21c}{15}$.

Observe that all of these satisfy increasing virtual costs. Denote by $\Pi^*_{G_k} := E_{v,c}[\Phi - \Gamma_k \mid \Phi - \Gamma_k > 0]$ the intermediary's expected profit under the optimal mechanism when the distribution is G_k and by $\Pi^L_{G_k} = \max_\xi (1-\xi) \int_0^\xi p(\xi, c)(1 - p(\xi, c)) g_k(c) dc$ the intermediary's expected profit under the same distribution when using the linear fee ξ optimally with $k \in \{H, T, C, O\}$. Quite interestingly, for all examples $\Pi^L_{G_k} > 0.9979\Pi^*_{G_K}$.

For these examples, and it seems reasonable to conjecture also for others, even if the intermediary merely uses an optimal linear mechanism he only loses very little of the profit he could achieve using an optimal mechanism. Though farther analysis is certainly warranted, we find this result quite remarkable.

B Proofs

Since we will refer to the properties of intermediary optimal mechanisms often, we summarize Myerson and Satterthwaite (1983)'s Theorems 3 and 4 on intermediation in the following lemma.

Lemma 2 (Myerson-Satterthwaite)**.** *An incentive compatible, interim individually rational mechanism is intermediary optimal if and only if it is such that (i) the good is transferred iff $\Phi(v) \geq \Gamma(c)$ and (ii) the seller with the highest cost $\bar{c}$ and the buyer with the lowest valuation $\underline{v}$ both have zero expected utility.*

Proposition 1: Optimal fee.

Proof of Proposition 1. Note that an indirect mechanism that lets the seller set the price corresponds to a direct mechanism with the properties that there are no payments if the good is not exchanged and payments in case of exchange can only be conditioned on the seller's report but not on the buyer's. Therefore, by the revelation principle we can focus our attention to direct mechanisms with these properties. We will first derive the optimal direct mechanism. Denote the probability that the good is exchanged depending on reported cost c and reported valuation v as $Q(c, v)$. For the class of mechanisms we consider it is clearly a weakly dominant strategy for the buyer to accept whenever the price is less than or equal to his valuation. Therefore, the seller's expected probability of exchange is $q(c) := E_v[Q(c, v)] = 1 - F(P(c))$ and consequently, trade occurs iff the buyer accepts the offer, i.e. iff $v \geq P(c)$. Combining this with the optimality condition (i) of Lemma 2 and the monotone increasingness of Φ, we get that for an optimal mechanism trade occurs iff $\Phi(v) \geq \Phi(P(c)) = \Gamma(c)$. This gives us $p = P(c)$. Because of Assumption 1 property (i) of Lemma 2 implies (ii).

Denote a truthfully reporting seller's utility as $U(c) := (P(c) - \omega(P(c)) - c)q(c)$. By standard arguments (see e.g. the argument leading up to equation (4) in the proof of Theorem 1 in Myerson and Satterthwaite (1983)) incentive compatibility implies

$$U(c) = U(\bar{c}) + \int_c^{\bar{c}} q(t)dt. \tag{15}$$

This expression for $U(c)$ can be interpreted as the seller's information rent. We already know that the highest cost seller is not going to sell and hence $U(\bar{c}) = 0$. This is also true for other sellers with sufficiently high cost, namely $P(c) \geq \bar{v}$ or $c \geq P^{-1}(\bar{v}) := \Gamma^{-1}(\Phi(\bar{v}))$. Therefore, the upper limit of the integral can be written as $P^{-1}(\bar{v})$. Equating (15) with $U(c) = (P(c) - \omega(P(c)) - c)q(c)$ from its definition and rearranging yields

$$\omega(P(c)) = P(c) - c - \int_c^{P^{-1}(\bar{v})} \frac{q(t)}{q(c)} dt = P(c) - c - \int_c^{P^{-1}(\bar{v})} \frac{1 - F(P(t))}{1 - F(P(c))} dt. \tag{16}$$

This function gives the fee the intermediary earns when the seller with cost c sets the price $p = P(c)$. Ultimately, however, we want to know the intermediary's fee as a function of the seller's price p since this is the empirically relevant concept. To that end, substitute $p = P(c)$ into (16) and integrate using this substitution to get

$$\omega(p) = p - P^{-1}(p) - \frac{\int_p^{\bar{v}} (1 - F(v))[P^{-1}(v)]' dv}{1 - F(p)}. \tag{17}$$

Integrating by parts and simplifying reveals that the righthand side in (17) equals $p - E_v[P^{-1}(v) \mid v \geq p] = p - (\int_p^{\bar{v}} f(v) P^{-1}(v) dv)/(1 - F(p))$, where the second term is the expectation of $P^{-1}(v)$ taken with respect to the distribution F conditional on v being larger than p. □

Proposition 2: Linear fee.

Proof of Proposition 2. By the same standard arguments leading to (15) we also get $U'(c) = -q(c)$ almost everywhere because of incentive compatibility. Equating this with the derivative obtained from the definition $U'(c) = [(P(c) - \omega(P(c)) - c)q(c)]'$ and rearranging yields

$$\Phi(P(c)) = P(c) - \frac{P(c) - \omega(P(c)) - c}{1 - \omega'(P(c))}. \tag{18}$$

(i) implies (ii) Take $\omega(p) = \xi p + \zeta$. Then the right hand side of (18) becomes $(c + \zeta)/(1 - \xi)$. Equating this with $\Gamma(c)$ in order to achieve optimality according to Lemma 2 (i) gives the differential equation $g(c) = G(c)(1-\xi)/(\xi c + \zeta)$. With the condition $G(\underline{c}) = 0$ one obtains the expression in part (ii) of the proposition with $\beta = (1 - \xi)/\xi$ and $\underline{c} = -\zeta/\xi$. The upper bound of the support $\bar{c}$ remains arbitrary.

(ii) implies (i) Observe that with the distribution G specified in part (ii) one has $\Gamma^{-1}(p) = (1-\hat{\xi})p - \hat{\zeta}$ with $\hat{\xi} := 1/(\beta+1)$ and $\hat{\zeta} := -\underline{c}/(\beta+1)$ and, therefore, $P^{-1}(p) = \Gamma^{-1}(\Phi(p)) = (1-\hat{\xi})\Phi(p) - \hat{\zeta}$. Take (18) and replace $P(c)$ with p, c with $P^{-1}(p)$, and Φ by its definition. Rearranging leads to

$$(1-F(p))(\kappa'(p) - (1-\hat{\xi})) - f(p)(p - \omega(p) - ((1-\hat{\xi})p - \hat{\zeta})) = 0. \tag{19}$$

Defining $l(p) := p - \omega(p) - ((1-\hat{\xi})p - \hat{\zeta})$ equation (19) leads to $[l(p)(1-F(p))]' = 0$. From part (ii) of Proposition 2 follows that $p - \omega(p)$ is not singular at $p = \bar{v}$ (actually $\omega(\bar{v}) = \bar{v} - P^{-1}(\bar{v})$). Since $1 - F(\bar{v}) = 0$ it follows that $l(p) \equiv 0$, i.e. $\omega(p) = \xi p + \zeta$ as in part (i) Proposition 2 is satisfied with $\xi = \hat{\xi}$ and $\zeta = \hat{\zeta}$. □

Proposition 3: Invariance and linearity of fees.

Proof of Proposition 3. The optimality condition (i) of Lemma 2 implies $\Phi(P(c)) = \Gamma(c)$. If we want optimality to hold for arbitrary distributions F, and hence for arbitrary functions $P(c)$, equating the right hand side of (18) and $\Gamma(c)$ yields $\Gamma(c) = p - (p - \omega(p) - c)/(1 - \omega'(p))$ for arbitrary p. This differential equation in ω has the solution

$$\omega(p) = p - (1-\xi)(p - \Gamma(c)) - c \tag{20}$$

defined up to a constant $1-\xi$. If we want this to hold for any c we need $c-(1-\xi)\Gamma(c) = -\zeta$ for some constant ζ, and hence $\Gamma(c) = (c+\zeta)/(1-\xi)$. Substituting this back to (20) results in $\omega(p) = \xi p + \zeta$, i.e. a linear fee. This also implies a generalized power distribution G by Proposition 2. □

Lemma 1: Optimal mechanism with many buyers and sellers.

Sketch of the Proof of Lemma 1. A direct mechanism asks buyers and sellers to report their valuations and costs. Denoting by $(\boldsymbol{v}, \boldsymbol{c})$ a collection of such reports with $\boldsymbol{v} = (v_1, .., v_{N_B})$ and $\boldsymbol{c} = (c_1, .., c_{N_S})$, the direct mechanism is then characterized by the probability $Q_b(\boldsymbol{v}, \boldsymbol{c})$ that b gets a unit of the good and $Q_s(\boldsymbol{c}, \boldsymbol{v})$ that s produces a unit of the good for $b = 1, .., N_B$ and $s = 1, .., N_S$ and by the payments $M_b(\boldsymbol{v}, \boldsymbol{c})$ it asks from buyers and the payments $M_s(\boldsymbol{c}, \boldsymbol{v})$ it makes to sellers. Clearly, a mechanism is only feasible if for all $(\boldsymbol{v}, \boldsymbol{c})$, $\sum_{b=1}^{N_B} Q_b(\boldsymbol{v}, \boldsymbol{c}) \leq \sum_{s=1}^{N_S} Q_s(\boldsymbol{c}, \boldsymbol{v})$. Let $\boldsymbol{Q}$ be the collection of these probabilities. We refer to $\boldsymbol{Q}$ as the allocation rule of the mechanism.

We only sketch the proof, a fully detailed version of which is available upon request. Lengthy, though completely standard arguments (see e.g. Krishna, 2002) can be applied to show that a revenue (or payoff) equivalence theorem holds. Formally, $m_b(v_b) = m_b(\underline{v}_b) + q_b(v_b)v_b - \int_{\underline{v}_b}^{v_b} q_b(t)dt$ and $m_s(c_s) = m_s(\underline{c}_s) + q_s(c_s)c_s - \int_{c_s}^{\bar{c}_s} q_s(t)dt$ for all c, v, lower case functions standing for expectations about all others' valuations and costs (e.g. $m_b(v_b) := E_{\boldsymbol{v}_{-b},\boldsymbol{c}}[M_b(\boldsymbol{v}, \boldsymbol{c})]$). Again, by standard arguments, this implies $E[m_b(v_b)] = m_b(\underline{v}_b) + E[\Phi_b(v_b)q_b(v_b)]$ and $E[m_s(c_s)] = m_s(\bar{c}_s) + E[\Gamma_s(c_s)q_s(c_s)]$. A profit maximizing intermediary will make the individual rationality constraint just binding, therefore, his expected profit $\sum_{b=1}^{N_B} E[m_b(v_b)] - \sum_{s=1}^{N_S} E[m_s(c_s)]$ is

$$\int_X \left\{ \sum_{b=1}^{N_B} \Phi_b(v_b) Q_b(\boldsymbol{v}, \boldsymbol{c}) - \sum_{s=1}^{N_S} \Gamma_s(c_s) Q_s(\boldsymbol{c}, \boldsymbol{v}) \right\} f(\boldsymbol{v}) g(\boldsymbol{c}) d\boldsymbol{v} d\boldsymbol{c}, \tag{21}$$

where $f(\boldsymbol{v})$ and $g(\boldsymbol{c})$ are the joint densities of all buyers and sellers, respectively, and X is the product set containing all $(\boldsymbol{v}, \boldsymbol{c})$. Inspection of the term in curly brackets reveals that the profit can be maximized point by point by implementing the Virtual-Walrasian allocation for each realization $(\boldsymbol{v}, \boldsymbol{c})$. □

Proposition 4: Intermediary optimal auction.

Proof of Proposition 4. It is sufficient to prove our statement for a second price auction, since by the revenue equivalence theorem it then also holds for any standard auction.[61] So consider a second price auction where the seller faces the fee function $\omega(p_S)$ levied on the sale price p_S. The seller reports his cost as $\hat{c}$ and the intermediary sets the reservation

[61] It follows from Lemma 3 in Myerson (1981) that all standard auction formats will have the same expected revenue and indeed the same reserve price. See also Milgrom (2004, Ch.3) and Jehle and Reny (2001, Th.9.9 and Ex.9.20).

price $P(\hat{c})$. This seller's expected profit is

$$N_B\Big\{(P(\hat{c})-\omega(P(\hat{c})))(1-F(P(\hat{c})))F(P(\hat{c}))^{N_B-1}$$
$$+\int_{P(\hat{c})}^{\overline{v}}(y-\omega(y))(1-F(y))(N_B-1)F(y)^{N_B-2}f(y)dy\Big\}+cF(P(\hat{c}))^{N_B}$$

because if the reserve price $P(\hat{c})$ is binding, the sale price is $p_S = P(\hat{c})$, which explains the first $\omega(.)$ term. If the reserve is not binding, the sale price is the second highest bid y, and this explains the second $\omega(.)$; see also Krishna (2002, p.25). Note that the good is sold to the buyer with the largest virtual valuation, provided this is larger than the reserve p.

For truth telling to be an equilibrium, the first order condition with respect to $\hat{c}$ has to be satisfied at $\hat{c} = c$. With some algebra, the first order condition can be rearranged to

$$(1-\omega'(P(c)))(1-F(P(c)))-(P(c)-\omega(P(c))-c)f(P(c))=0.$$

As (16) is the solution to this differential equation, it follows from the proof of Proposition 1 that the fee structure with the fee function $\omega(p_S) = p - E_v[P^{-1}(v) \mid v \geq p_S]$ induces the seller to set the intermediary the reserve in the intermediary optimal way. Thus, the mechanism described in Proposition 4 is the intermediary optimal allocation rule. □

Proposition 5: Effect of an infinitesimal perturbation.

Proof of Proposition 5. Since we only care about first-order effects, $\hat{g}(c) = (1-\epsilon\gamma(c))g(c)$ can be rewritten as

$$g = \frac{1}{1+\epsilon\gamma}\hat{g} = (1-\epsilon\gamma + O(\epsilon^2))\hat{g}, \tag{22}$$

where $O(\epsilon^2)$ stands for the second order effect. Taking a constant α with $1+\alpha\epsilon = \sigma$, this has to be equal to

$$(1+\alpha\epsilon)(1-\epsilon(1-\rho_S(c)))\hat{g}(c) = (1-\epsilon[(1-\rho_S(c))-\alpha]+O(\epsilon^2))\hat{g}(c). \tag{23}$$

α has to be chosen as $\alpha = \int(1-\rho_S)\hat{g}$ so that the density functions add up to one. Hence equating the right hand sides of (22) and (23) results in

$$\gamma(c) = F(\Phi^{-1}(\Gamma(c))) - \int_{\underline{c}}^{\overline{c}} F(\Phi^{-1}(\Gamma(t)))g(t)dt. \tag{24}$$

We know that γ is increasing, γ and g are orthogonal ($\int \gamma g = 0$), $\gamma(\underline{c}) < 0$, and $\gamma(\overline{c}) > 0$.

(i) We will first show that $(\ln G(c))'$ increases if ϵ increases:

$$\frac{\partial^2}{\partial\epsilon\partial c} \ln \hat{G} > 0. \tag{25}$$

Using

$$\frac{\partial}{\partial\epsilon}\hat{G} = \frac{\partial}{\partial\epsilon}\int(1+\epsilon\gamma)g = \int \gamma g, \tag{26}$$

we get

$$\frac{\partial}{\partial\epsilon} \ln \hat{G}\bigg|_{\epsilon=0} = \frac{1}{G}\int_{\underline{c}}^{c} g(c')\gamma(c')dc'. \tag{27}$$

Taking the derivative with respect to c yields

$$\frac{\partial}{\partial c}\frac{\partial}{\partial\epsilon} \ln \hat{G}\bigg|_{\epsilon=0} = \frac{g}{G^2}\int_{\underline{c}}^{c} g(c')\gamma(c')dc' + \frac{1}{G}g(c)\gamma(c),$$

the sign of which is to be determined. Multiplying by the positive expression G^2/g we get

$$G(c)\gamma(c) - \int_{\underline{c}}^{c} g(c')\gamma(c')dc' = \int_{\underline{c}}^{c} g(c')[\gamma(c)-\gamma(c')]dc'. \tag{28}$$

The expression in the brackets is positive since $\gamma'(c) > 0$ and $c > c'$, therefore, the whole expression is positive, which proves the statement

$$\frac{\partial^2}{\partial\epsilon\partial c} \ln G > 0. \tag{29}$$

(ii) As next we will prove that Γ is decreasing and Γ^{-1} increasing with ϵ. The following analysis can be simplified by defining a further function ψ, such that

$$\hat{G}(c) = (1-\epsilon\psi(c))G(c), \tag{30}$$

The relation between ψ and γ is the following

$$g\gamma = -(G\psi)' \tag{31}$$

or

$$\psi = -\frac{1}{G}\int g\gamma \tag{32}$$

which is equal to the negative of the right hand side of (27). Therefore, $\psi' < 0$ by the argument in the previous section. We also know $\psi(\bar{c}) = 0$, $\psi \geq 0$.

The derivative is

$$\hat{g} = g - \epsilon(g\psi + G\psi') \tag{33}$$

By definition

$$\hat{\Gamma} \stackrel{\text{def}}{=} c + \frac{\hat{G}}{\hat{g}} = c + \frac{[1-\epsilon\psi]G}{[(1-\epsilon\psi) - \epsilon(G/g)\psi']g} = c + \frac{(G/g)[1-\epsilon\psi]}{g - \epsilon(\psi + (G/g)\psi')}. \tag{34}$$

The Taylor expansion is

$$c + \frac{G}{g}[1-\epsilon\psi]\left[1 + \epsilon(\psi + \frac{G}{g}\psi')\right] + O(\epsilon^2) = c + \frac{G}{g}\left[1 + \epsilon\frac{G}{g}\psi'\right] + O(\epsilon^2) \tag{35}$$

Using the definition of $\hat{\Gamma}$ this gives us

$$\hat{\Gamma} = \Gamma + \epsilon\left(\frac{G}{g}\right)^2 \psi' + O(\epsilon^2). \tag{36}$$

Since ψ' is negative $\hat{\Gamma}$ is decreasing with ϵ. The inverse of $\hat{\Gamma}$ is

$$\hat{\Gamma}^{-1} = \Gamma^{-1} - \epsilon\frac{(G/g)^2\psi'}{\Gamma'} + O(\epsilon^2). \tag{37}$$

The fraction is negative since $\psi' < 0$ and Γ is increasing by Myerson's regularity assumption. Therefore, for a perturbation with $\epsilon > 0$ we have $\hat{\Gamma}^{-1} > \Gamma^{-1}$.

(iii) Next, we will look at the change of $[\Gamma^{-1}]'$. Taking the derivative of (37) gives us

$$(\hat{\Gamma}^{-1})' = (\Gamma^{-1})' - \epsilon\left[\frac{(G/g)^2\psi'}{\Gamma'}\right]' + O(\epsilon^2) \tag{38}$$

$$= (\Gamma^{-1})' - \epsilon\frac{[2(G/g)\frac{g^2-Gg'}{g^2}\psi' - (\frac{G}{g})^2\psi'']\Gamma' - (\frac{G}{g})^2\psi'\Gamma''}{(\Gamma')^2} + O(\epsilon^2) \tag{39}$$

The sign of the multiplier of ϵ is ambiguous. For instance, since $\psi' < 0$, for Γ'' sufficiently negative, $(\hat{\Gamma}^{-1})'$ is increasing with ϵ. However, for Γ'' sufficiently large, we have the opposite effect.

We can also make the analysis for $\Gamma'' = 0$ (or close to zero), which means that we have a power distribution and linear fees (or are close to it). After some algebra the expression in brackets in (39) can be transformed to

$$-3G\gamma(g^2 - Gg') + 3g\int g\gamma - 2\frac{Gg'}{g}\int g\gamma + G^2 g\gamma' - Gg\gamma. \tag{40}$$

If this is negative then $[\Gamma^{-1}]'$ will be larger if ϵ increases, that is we have a flatter fee.

However, one can find examples of power distributions where this condition is not satisfied. Take $G = c^\beta$, $F = 1 - (1-v)^\alpha$, which results in linear virtual valuation functions Γ and Φ. E.g. for $\alpha = 3$ and $\beta = 3$, the condition is not satisfied for certain values of c, the sign of

$$\frac{1}{4860}\,\frac{-225 + 1280\,c^4 - 3840\,c^3 - 363\,c^2 + 1920\,c}{c^2} \tag{41}$$

is different for different values of c as depicted in Fig. 13.

Figure 13: For $\alpha = 2$ and $\beta = 3$ the expression in Eq. (40) has different signs for different values of c.

$\square$

Proposition 6: Time on market with one homogeneous good.

Proof of Proposition 6. Discrete Time. Consider first a cohort of sellers, who entered the market at some point t, normalized to $t = 0$, and offered their houses for some price p. Label the number of rematchings since $t = 0$ with $k := t/\tau$ and the expected number of sellers in the cohort staying in the market at the beginning with N_0 and in subsequent periods with N_k. The probability that a seller stays in the market until the next rematching is the probability that he cannot sell times the probability that he does not drop out for exogenous reasons, i.e. $\epsilon F(p)$ with $\epsilon := e^{-\eta\tau}$. The number of sellers in period k is hence $N_k = (\epsilon F(p))^k N_0$. Time on market for the total population of both sold and unsold houses follows hence a geometric distribution with the cumulative distribution function $1 - (\epsilon F(p))^{t/\tau}$ and mean $T(p) = \tau/(1 - \epsilon F(p))$. Denote the number of sellers who leave the market in period k because they sell as N_k^s and those who leave with unsold houses as N_k^u. Clearly, $N_k^s = (1 - F(p))N_k$ and $N_k^u = \epsilon N_k$. Therefore, the ratio of sellers able to sell is $(1 - F(p))/(1 - \epsilon F(p))$. Now consider only the subsample of sellers who managed to sell their houses. Since N_k^s is just a constant factor smaller than N_k, the distribution of time on market of this subsample is the same as for the total population. Hence the cumulative distribution function is also $1 - (\epsilon F(p))^k$ and the mean time on market for sold houses is $T^s(p) = 1/(1 - \epsilon F(p))$. The same reasoning applies for sellers who did not sell their houses, so that $T^u(p) = 1/(1 - \epsilon F(p))$ is the mean time on market for unsold houses as well. Since we are looking at a market in a stationary equilibrium, in every period the same number of N_0 sellers enters and the previous argument carries over to a setup where cohorts of sellers enter every period rather than only one cohort entering at $t = 0$.

Continuous Time. The same logic applies to the continuous time approximation of the distribution. Denote the mass of sellers in the cohort at period $t = 0$ as $N(0)$. The number of sellers remaining in the market in period t is $N(t) = N(0)e^{-(\phi+\eta)t}$ dropping the argument p in $\phi(p)$. In each period $dN^s(t) = N(t)\phi dt$ houses are sold and $dN^u(t) = N(t)\eta dt$ drop out unsold. Cumulatively, we have $N^s(t) = \int_0^t dN^s(t') = (\phi/(\phi + \eta))\,[N(0) - N(t)]$ and $N^u(t) = \int_0^t dN^u(t') = (\eta/(\phi + \eta))\,[N(0) - N(t)]$. After infinitely many periods, fraction $1 - F_\infty := N^s(\infty)/N(0) = \phi/(\phi + \eta)$ of houses have been sold. The average time on market for sold houses is

$$T^s = \frac{\int_0^\infty t dN^s(t)}{\int_0^\infty dN^s(t)} = -\frac{\partial}{\partial\phi}\ln\int_0^\infty e^{-(\phi+\eta)t}dt = -\frac{\partial}{\partial\phi}\ln\frac{1}{\phi+\eta} = \frac{1}{\phi+\eta}.$$

By the same logic, the average time on market of unsold houses is $T^u = 1/(\phi + \eta)$.

□

Proposition 7: Time on market in heterogeneous submarkets.

Proof of Proposition 7. Consider multiple submarkets, indexed by i, with different probabilities of sale $\phi_i(p)$. Houses of each submarket are represented with weight w_i in the total sample. Taking averages over submarkets, the mean time on market for sold $T^s(p)$

and unsold $T^u(p)$ houses is

$$T^s = \left(\sum_i w_i \frac{\phi_i}{\phi_i+\eta}\frac{1}{\phi_i+\eta}\right)\left(\sum_i w_i \frac{\phi_i}{\phi_i+\eta}\right)^{-1},$$
$$T^u = \left(\sum_i w_i \frac{\eta}{\phi_i+\eta}\frac{1}{\phi_i+\eta}\right)\left(\sum_i w_i \frac{\eta}{\phi_i+\eta}\right)^{-1},$$

the parameter p being dropped. The ratio of the two means is

$$\frac{T^u}{T^s} = \frac{\sum_i w_i \frac{\eta}{(\phi_i+\eta)^2}\sum_j w_j \frac{\phi_j}{\phi_j+\eta}}{\sum_i w_i \frac{\phi_i}{(\phi_i+\eta)^2}\sum_j w_j \frac{\eta}{\phi_j+\eta}} =: \frac{N}{D}\,.$$

The difference between the numerator N and the denominator D is

$$\begin{aligned} N - D &= \eta \sum_{ij} w_i w_j \frac{\phi_j - \phi_i}{(\phi_i+\eta)^2(\phi_j+\eta)}, \\ &= -\eta \sum_{ij} w_i w_j \frac{\phi_j - \phi_i}{(\phi_i+\eta)(\phi_j+\eta)^2}\,, \end{aligned}$$

where the second equation comes from interchanging the summation variables. Adding the two expressions for $N - D$ one gets

$$2(N-D) = \eta \sum_{ij} w_i w_j \frac{(\phi_j-\phi_i)^2}{(\phi_i+\eta)^2(\phi_j+\eta)^2} \geq 0\,,$$

hence $T^u \geq T^s$. The inequality is strict for heterogeneous submarkets. □

Proposition 8: Non-optimality of price posting mechanisms.

Proof of Proposition 8. The intermediary's expected profit with price setting is $(p^B - p^S)(1-F(p^B))G(p^S)$. The assumptions about the inverse hazard rates ensure concavity of the profit function. Therefore, the unique maximum is given by the first order conditions. Taking derivatives with respect to p^B and p^S yields $p^S = \Phi(p^B)$ and $p^B = \Gamma(p^S)$. We complete the proof by showing that trade with price setting neither implies nor is implied by trade in the intermediary optimal mechanism of Myerson and Satterthwaite for arbitrary distributions F and G.

Trade with price posting, no trade with the intermediary optimal mechanism. Take a buyer and a seller for whom trade just occurs with price setting, i.e. valuation p^B

and cost p^S. We know that a profit maximizing intermediary will always set $p^B > p^S$. Combining this with the first order conditions we get $\Phi(p^B) = p^S < p^B = \Gamma(p^S)$. This implies by Lemma 2 (i) that no trade occurs with the optimal mechanism for valuation p^B and cost p^S.

Trade with the intermediary optimal mechanism, no trade with price posting. Take the lowest cost seller with cost $\underline{c}$ and a buyer with valuation v' such that trade just occurs with the optimal mechanism, i.e. $\Phi(v') = \Gamma(\underline{c})$. As $p^S > \underline{c}$ must hold for positive probabilities of trade with price setting, we have $\Phi(v') = \Gamma(\underline{c}) = \underline{c} < p^S = \Phi(p^B)$. This implies $v' < p^B$ and hence no trade with price setting. □

Proposition 9: Optimality of price posting with infinite number of buyers and sellers.

Proof of Proposition 9. (i) We consider the case where $\text{plim}_{N\to\infty} K/N < 1$, i.e. not all buyers and sellers trade in the limit. For $\text{plim}_{N\to\infty} K/N = 1$ the proof is similar and therefore omitted.

It can be easily shown that for a finite number of buyers and sellers a dominant strategy implementation of the Virtual-Walrasian allocation rule is optimal: everyone reports their valuations/costs, a buyer pays the minimal valuation which would have been sufficient for him to get the good, the seller gets analogously the maximal cost. Formally, a buyer pays $\max\{v_{K+1}, \Phi^{-1}(\Gamma(c_K))\}$ and a seller gets $\min\{c_{K+1}, \Gamma^{-1}(\Phi(v_K))\}$.

The valuation of the marginal trading and non-trading buyers and the marginal seller's cost plus the spread charged by the intermediary converge in probability to the same value, which we denote as p^B:

$$\underset{N\to\infty}{\text{plim}}\, v_K = \underset{N\to\infty}{\text{plim}}\, v_{K+1} = \underset{N\to\infty}{\text{plim}}\, \Phi^{-1}(\Gamma(c_K)) =: p^B. \tag{42}$$

Similarly,

$$\underset{N\to\infty}{\text{plim}}\, c_K = \underset{N\to\infty}{\text{plim}}\, c_{K+1} = \underset{N\to\infty}{\text{plim}}\, \Gamma^{-1}(\Phi(v_K)) =: p^S. \tag{43}$$

For the fraction of buyers and sellers who trade we have

$$\underset{N\to\infty}{\text{plim}}\, \frac{K}{N} = \underset{N\to\infty}{\text{plim}}\, \frac{\max\{i|v_i \geq p^B\}}{N} = 1 - F(p^B), \tag{44}$$

$$\underset{N\to\infty}{\text{plim}}\, \frac{K}{N} = \underset{N\to\infty}{\text{plim}}\, \frac{\max\{i|c_i \geq p^S\}}{N} = G(p^S). \tag{45}$$

(42), (43), (44), and (45) imply that the optimal mechanism converges to price posting with p^B and p^S that satisfy $\Phi(p^B) = \Gamma(p^S)$ and $1 - F(p^B) = G(p^S)$.

(ii) Define the number of buyers willing to trade as $k_b := \max\{i|v_i \geq p^B\}$, and for the sellers $k_s := \max\{i|c_i \leq p^S\}$. By $\Phi(p^B) = \Gamma(p^S)$

$$\operatorname*{plim}_{N\to\infty} \frac{k_b}{N} = 1 - F(p^B) = G(p^S) = \operatorname*{plim}_{N\to\infty} \frac{k_s}{N}.$$

By $1 - F(p^B) = G(p^S)$ we have $\Phi(v_{k_b}) \geq \Phi(p^B) = \Gamma(p^S) \geq \Gamma(c_{k_s})$ and by analogy $\Phi(v_{k_b+1}) < \Gamma(c_{k_s+1})$. Therefore, the fraction of traded quantity is in the limit

$$\operatorname*{plim}_{N\to\infty} \frac{\min\{k_b, k_s\}}{N} = \frac{\max\{i|\Phi(v_i) \geq \Gamma(c_i)}{N} \stackrel{\text{def}}{=} \frac{K}{N},$$

which is the fraction of the Virtual-Walrasian quantity. Further, it is easy to show that this mechanism is incentive compatible and gives zero utility to the most inefficient agents. Therefore, by Lemma 1 it maximizes the intermediary's profit. □

Proposition 10: Fee setting and price posting mechanism for one seller and infinite number of buyers.

Proof of Proposition 10. As N_B converges to infinity, the highest bid almost surely converges to $\bar{v}$. Hence we are back to the one sided incomplete information problem. By Myerson (1981) the optimal mechanism is $p^B = \bar{v}$ and $p^S = \Gamma^{-1}(\bar{v})$ when considering price posting mechanisms. This can also be represented as a fee setting mechanism with $\omega(p) = p[1 - \Gamma^{-1}(\bar{v})/\bar{v}]$, which induces the seller to set $P(c) = \bar{v}$.[62]

□

Proposition 11: Integrated Seller.

Proof of Proposition 11. The value function for an integrated seller is

$V(\tilde{c}) = \max_p \{(1 - F(p))(p - \tilde{c}) + F(p)\delta V(\tilde{c})\}$, where the first (second) term on the right hand side is the payoff in case of a sale (no sale). Rearranging yields $V(\tilde{c}) = \max_p \left\{\frac{1-F(p)}{1-\delta F(p)}(p - \tilde{c})\right\}$. Observe that for $\delta = 0$ this reduces, of course, to the static

[62] Any other ω that induces the seller to set $P(c) = \bar{v}$ would do, e.g. $\omega(p) = \bar{v} - \Gamma^{-1}(\bar{v})$ for $p = \bar{v}$ and $\omega(p) = \infty$ else.

problem. For F uniform, we get $V(\tilde{c}) = \max_p \left\{ \frac{2(1-p)}{1+\delta-2\delta p}(p-\tilde{c}) \right\}$, which is maximized at $P_I(\tilde{c}) = \frac{1+\delta-\sqrt{(1-\delta)(1+\delta-2\tilde{c}\delta)}}{2\delta}$. Therefore, the equilibrium price difference between an independent and an integrated seller who both have the same true cost $\tilde{c}$ is

$$P(\tilde{c}) - P_I(\tilde{c}) = \frac{\sqrt{(1-\delta)(1+\delta-2\tilde{c}\delta)} - \sqrt{(1-\delta)(1+\delta-4\tilde{c}\delta)}}{2\delta}, \tag{46}$$

which is negative for $\tilde{c} < 0$.[63] This means that an intermediary-seller may charge a higher price than an independent seller, even if their true costs $\tilde{c}$ are equal. □

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[63] The true cost $\tilde{c}$ is negative for some sellers. This can be seen from the fact that for continuation cost $c = 0$ the seller needs to have a negative true cost $\tilde{c}$, since c is the sum of $\tilde{c}$ and the net present value of future trade.

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