Expanding “Choice” in School Choice

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October 15, Northwestern Seminar

Introduction

- Traditionally, students are assigned to public schools according to where they live.
  - Limited and unequal freedom of choices.
- Starting with Minnesota in 1987, several school districts adopted school choice programs. (New York City, Boston, Cambridge, Charlotte, Columbus, Denver, Seattle and St. Petersburg-Tampa)
- *General idea gaining political support, but the exact method still debated. The "strategy-proofness" has become a focal issue in the redesign of Boston and NY programs.*

Before Redesign: “Boston” mechanism

- Students submit ordinal rankings. Schools assign seats to those who listed them as first choices, according to their priorities. Not enough seats, then use a lottery. Enough seats, then assign the remaining seats to those (turned down by their top choices) who list them as second choices .... The process ends when no students are rejected.

  - How a student lists a given school in her ranking matters for her chance of assignment at that school.

  - But this choice induces “gaming” the system: Not strategy proof.

  - Eliminating “gaming” is important from a practical as well as from a fairness standpoint.

Redesign: Gale-Shapley deferred acceptance algorithm (DA)

- Students rank schools. Schools rank students (based on priorities, and lottery). Schools assign seats to those who listed them as first choices, according to their priorities, **but only tentatively.** Move to the next round in which all those previously held and the new applicants are considered on the equal footing, and again seats are assigned tentatively..... The process ends when no students are rejected, at which point the tentative assignment become final.

$\Rightarrow$ Strategyproof: Listing the best school as a top choice doesn't sacrifice her shot at less preferred schools.

- But strategy-proofness involves limiting students' choice.

- *But this cannot be the only rationale for DA....*

Welfare Rationale for DA

- When both sides have strict preferences, DA selects the "student optimal" stable matching (SOSM); whereas the Boston may select any stable matching (Ergin and Sonmez).

- But in practice, schools' priorities are very coarse (e.g., "siblings" and "walk zone"), and are indifferent to a large group of students. How do we break a tie in DA?

- Two tie-breaking procedures

  - STB: Single (common) tie-breaking for all the schools

  - MTB: Multiple (separate) tie-breaking for each school

Problems with DA When There are Ties

- **Ex Post Welfare Issue (recognized by others):** No strategyproof mechanism implements SOSM.

- **Ex Ante Cardinal Welfare issue (our focus):** Ex post welfare (incl. SOSM) less relevant than ex ante welfare. The lack of parent "choice" with DA has real welfare consequences, well grounded on the parents' sentiment expressed in BPS hearings:

> I'm troubled that you're considering a system that takes away the little power that parents have to prioritize... what you call this strategizing as if strategizing is a dirty word...

> ... if I understand the impact of Gale Shapley, ... I thought I understood that in fact the random number in fact [has] preference over your choices...

Example: Ex ante welfare

- 3 students, $I = \{1, 2, 3\}$, and 3 schools, $S = \{A, B, C\}$, each with one seat to fill.

- The schools are indifferent to all three students. All students have the same rankings: $A \succ B \succ C$, but with different vNM values:

	v_j^1	v_j^2	v_j^3
$j = A$	4	4	3
$j = B$	1	1	2
$j = C$	0	0	0

- *Every assignment is SOSM and ex post Pareto efficient, so no difference between DA and Boston based on these criteria.*

- Under DA, all three submit true (ordinal) preferences, and they will be assigned to the schools with equal probabilities.

  $\rightarrow EU_1 = EU_2 = EU_3 = \frac{5}{3}$

- Pareto-dominated by the following assignment: *Assign student 3 to B, and students 1 and 2 randomly between A and C.*

  $\rightarrow EU_1' = EU_2' = EU_3' = 2 > \frac{5}{3}$

- Boston mechanism implements this latter matching:

  - It allows one to affect tie-breaking, but this extra "choice" is what led to the failure of strategy-proofness.

- *How do we optimally balance the tradeoff between strategyproofness and ex ante welfare?*

Our Proposal: Choice-Augmented Deferred Acceptance (CADA)

- (i) All students submit ordinal preferences, plus an "auxiliary message" the name of one's "target" school.

- (ii) Two random priority list of students are generated, labeled **T** and **R**. A school's priority list is determined so that (1) its inherent priorities are respected; that (2) among those at tie, the students who named the school as a target are listed first, according to **T**, and then those who didn't are listed according to **R**.

- *Run the DA based on the ordinal preferences from (i) and the priority lists from (ii).*

$\Rightarrow$ Strategy-proof with respect to *ordinal preferences*; the gaming aspect is limited to tie-breaking.

- If the schools have strict priorities, the CADA coincides with DA, implementing SOSM.

- If not, CADA does better than DA (in the sense made precise later).... In the above example, 1 and 2 will name A for their target, and 3 will name B for her target.

$\longrightarrow$ *Best of both worlds...*

Model

- There are $n \geq 2$ schools, $S = \{1, ..., n\}$, each with a unit mass of seats to fill.

- There are mass n of students who are indexed by vNM values $\mathbf{v} = (v_1, ..., v_n) \in \mathcal{V} = [0, 1]^n$, with a measure μ that admits strictly positive density in the interior of $\mathcal{V}$.

- An *assignment* is a student's probability distribution over S.

- An *allocation* is a function $\phi := (\phi_1, ..., \phi_n) : \mathcal{V} \mapsto \Delta$ s.t. $\int \phi_i(\mathbf{v}) d\mu(\mathbf{v}) = 1$ for each $i \in S$.

Ex Ante Welfare Standards

- Allocation $\phi \in \mathcal{X}$ is **Pareto efficient (PE)** if there is no other allocation in $\mathcal{X}$ that Pareto-dominates ϕ.

- Allocation $\phi \in \mathcal{X}$ is **Ordinally efficient (OE)** if there is no other allocation in $\mathcal{X}$ that ordinally-dominates ϕ. (NB: ordinal domination means stochastically dominating for all agent types and strictly for some positive measure.)

Fix any allocation ϕ and fix $K \subset S$. Call an allocation $\tilde{\phi}$ a *within* K *reallocation of* ϕ if $\tilde{\phi}$ if it differs from ϕ in the probability shares of K. (K measures the scope of markets.)

- For any $K \subset S$, an allocation $\phi \in \mathcal{X}$ is **PE (OE) within** K if there is no within K reallocation of ϕ that Pareto (ordinally) dominates ϕ. Say **pairwise PE** if PE within every pair of schools.

Characterization of Welfare Notions

- Define a binary relation $\rhd^{\phi}$ on S:

$$i \rhd^{\phi} j \Leftrightarrow \exists A \subset \mathcal{V}, \mu(A) > 0, \text{ s.t. } v_i > v_j \text{ and } \phi_j(\mathbf{v}) > 0, \forall \mathbf{v} \in A.$$

- Allocation ϕ *admits a trading cycle within* K if there exist $i_1, i_2, ...i_l \in K$ such that $i_1 \rhd^{\phi} i_2, ..., i_{l-1} \rhd^{\phi} i_l$, and $i_l \rhd^{\phi} i_1$.

Lemma 1: (Bogomolnaia-Moulin) An allocation ϕ is OE within $K \subset S$ if and only if ϕ does not admit a trading cycle within K.

Lemma 2: (i) If an allocation is PE (resp. OE) within K', then it is is PE (resp. OE) within $K \subset K'$;
(ii) An allocation is OE within $K \subset S$ if it is PE within K.
(iii) If an allocation is OE, then it is pairwise PE.

Welfare Properties of DA-STB and DA-MTB

Theorem 2: (i) (Che and Kojima (2008)) The DA-STB allocation is OE, and is thus pairwise PE.

(ii) Generically, there exists no $K \subset S$ with $|K| > 2$ such that the DA-STB allocation is PE within K.

Theorem 3: (i) The DA-MTB allocation is not OE within every pair of schools. (In fact, it is not even ex post PE within a pair of schools.)

(ii) There exists no $K \subset S$ with $|K| > 2$ such that the DA-MTB allocation is OE within K.

$\boxed{\text{Intuition: Example}}$ Unit mass students of each type.

	v_j^1	v_j^2	v_j^3
$j=1$	5	4	1
$j=2$	1	2	5
$j=3$	0	0	0

- DA-STB gives $\phi^S(\mathbf{v}^1) = \phi^S(\mathbf{v}^2) = (\frac{1}{2}, \frac{1}{6}, \frac{1}{3})$ and $\phi^S(\mathbf{v}^3) = (0, \frac{2}{3}, \frac{1}{3})$. **PE within** $\{S1, S2\}$.

- DA-MTB has $\phi^M(\mathbf{v}^1) = \phi^M(\mathbf{v}^2) \approx (0.392, 0.274, 0.333)$ and $\phi^M(\mathbf{v}^3) \approx (0.215, 0.451, 0.333)$. **Not OE within** $\{S1, S2\}$. Type 1 and 2 may get bad draw at S1 but good draw and S2, and the other way around for type 3.

- DA-STB is NOT PE: Type 1 can exchange shares at 1 and 3 in exchange for a share at 2, with type 2.

Welfare Properties of CADA

Theorem 4: (i) The equilibrium allocation is OE, and thus pairwise PE.

(ii) The allocation is PE within set K of "oversubscribed" schools (those whose capacity does not exceed the measure of all who name them as targets).

(iii) If all but one ($n-1$) schools are oversubscribed, then the equilibrium allocation is PE.

Proof idea: Fix an equilibrium ϕ^* (whose existence is shown). In equilibrium, each student with $\mathbf{v}$ can be seen to choose a *within* K *reassignment* $\mathbf{x} = (x_1, ..., x_n)$ of ϕ^* to

$$\max \sum_{i \in S} v_i x_i$$

s.t.

$$\sum_{i \in S} p_i x_i \leq \sum_{i \in S} p_i \phi_i^*(\mathbf{v}),$$

where p_i is the mass of people naming i as the target, i.e., "shadow price" of buying school i's share.

The rest of the proof parallels the 1st Welfare Theorem: If there were any within K reallocation of ϕ^* that Pareto dominates it, then it is not feasible. Q.E.D.

- **Remark:** The parallel with the 1st Welfare Theorem is suggestive of the economic benefit associated with the CADA: Students' signaling of their targets activates competitive markets within oversubscribed schools, in which "congestion" serves as a price.

- **When is a school oversubscribed?**

Say a school is **popular** if the measure of students who prefer it the most is as large as its capacity.

Proposition 3: *Every popular school is oversubscribed.*

Corollary: *If all but one school are popular, then the CADA allocation is PE.*

Overall three-way rankings:

DA-MTB: *Not ex post* PE; PE within some but not every pair of schools.

DA-STB: OE, and pairwise PE.

CADA: OE, and PE within each pair *and* within oversubscribed schools. If all but one school is popular, full PE is achieved.

Recall the Example

	v_j^1	v_j^2	v_j^3
$j = 1$	5	4	1
$j = 2$	1	2	5
$j = 3$	0	0	0

- CADA has $\phi^*(\mathbf{v}^1) = \phi^*(\mathbf{v}^2) = (\frac{1}{2}, 0, \frac{1}{2})$ and $\phi^*(\mathbf{v}^3) = (0, 1, 0)$. Schools 1 and 2 are popular, so ϕ^* is **PE**.

Special Case: Uniform Preferences

Theorem 5: Suppose all students have the same ordinal preferences. For a DA with any random tie-breaking rule, there exists a CADA which makes all students weakly better off than they are from the DA.

$\boxed{\text{Intuition:}}$ DA produces random assignment, which a student can replicate in CADA by randomizing over the "target" schools with probabilities equal to the shares of the population choosing alternative schools for their targets.

Extension: Enriching the Auxiliary Message

- Our CADA is practical because of its simplicity. But the auxiliary message can be expanded to include more than one school, perhaps at the expense of becoming less practical.

- In general, the auxiliary message can include a rank order of schools up to $k \leq n$, with the tie broken in the lexicographic fashion according to this rank order.

- We call the associated CADA a *CADA of degree* k.

$\boxed{\text{Example: “More is better”}}$ (4 schools, and two types of students) $\mathcal{V} = \{\mathbf{v}^1, \mathbf{v}^2\}$, each with $\mu(\mathbf{v}^1) = 3$ and $\mu(\mathbf{v}^2) = 1$.

	v_j^1	v_j^2
$j = 1$	20	20
$j = 2$	4	3
$j = 3$	1	2
$j = 4$	0	0

- **CADA of degree 1:** All students name S1, so $\phi^*(\mathbf{v}^j) = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$, $j = 1, 2$.

- **CADA of degree 2:** All name S1 as Target 1; but type 1 students name S2 and type 2 students name S3 respectively as Target 2. Hence, $\phi^{**}(\mathbf{v}^1) = (\frac{1}{4}, \frac{1}{3}, \frac{1}{12}, \frac{1}{3})$ and $\phi^{**}(\mathbf{v}^2) = (\frac{1}{4}, 0, \frac{3}{4}, 0)$. ϕ^{**} Pareto dominates ϕ^*.

$\boxed{\text{Example: "More is worse"}}$ $\mu(\mathbf{v}^1) = 3$ and $\mu(\mathbf{v}^2) = 1$.

	v_j^1	v_j^2
$j = 1$	12	8
$j = 2$	2	4
$j = 3$	1	3
$j = 4$	0	0

- **CADA of degree 1:** *Type 1 students name S1 as target, and all type 2 students name S2 as their target.* Hence, $\phi^*(\mathbf{v}^1) = (\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3})$ and $\phi^*(\mathbf{v}^2) = (0, 1, 0, 0)$, which is **PE**.

- **CADA of degree 2:** *Type 1 students choose school 1 and 2 as their first and second targets, respectively. Type 2 students choose school 1 (instead of school 2!) as their first target and school 3 as their second target.* Hence, $\phi^{**}(\mathbf{v}^1) = (\frac{1}{4}, \frac{1}{3}, \frac{1}{12}, \frac{1}{3})$ and $\phi^{**}(\mathbf{v}^2) = (\frac{1}{4}, 0, \frac{3}{4}, 0)$. **Not PE**!

Simulation

- 5 schools each with a capacity of 20, and 100 students.

- Student i's vNM value for school j, v_{ij}:

$$v_{ij} = \alpha u_j + (1 - \alpha) u_{ij}$$

where u_j is a common shock, u_{ij} is an idiosyncratic shock, and $\alpha \in [0, 1]$ is a "commonality" parameter. Averaged over 100 drawings of the preference shocks, and 2000 draws for tie-breaking lottery.

- Four procedures: DA-STB, DA-MTB, CADA, CADA-Naive (Simply picks one's top choice as target).

- Without and with priorities

- Priorities: 50 have priority in the top choice, 30 have priority in second choice, 20 have in third choice.

Figure 1: Welfare as Percentage of the First Best Welfare (mean adjusted)

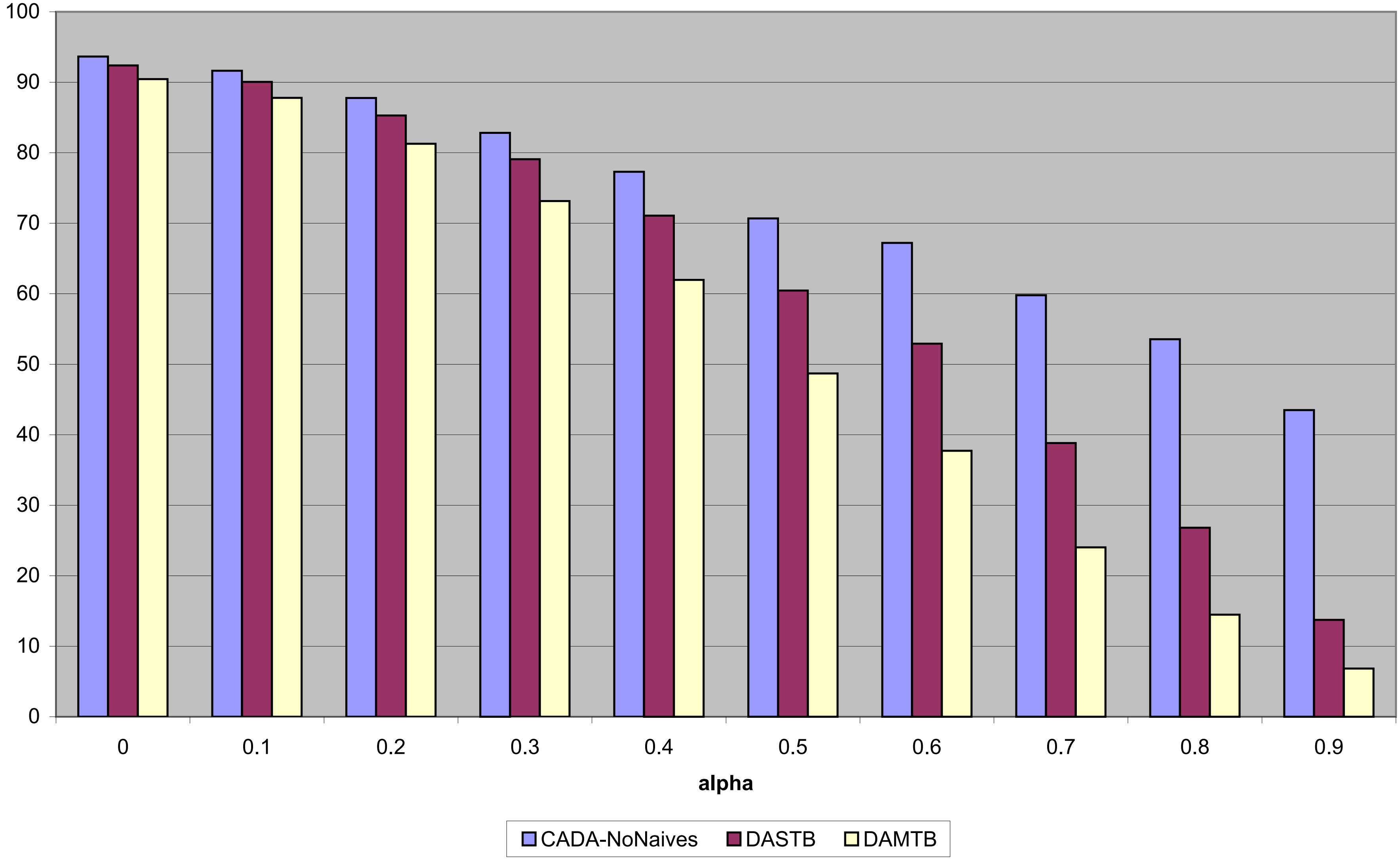


Figure 2: Percentage of Students Getting Their First Choice

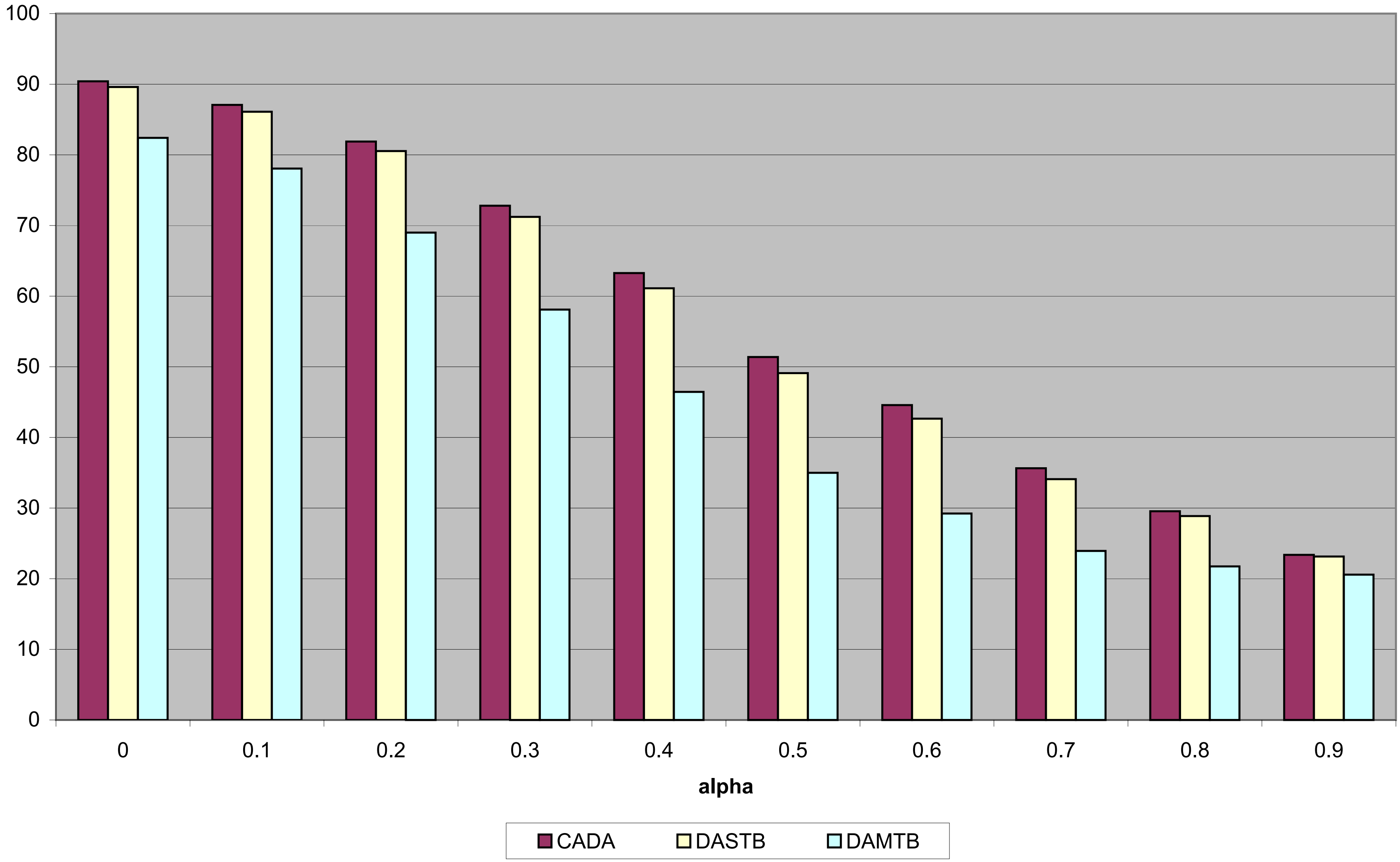


Figure 3: Average Utility of Receivers of kth Choice, CADA vs DASTB

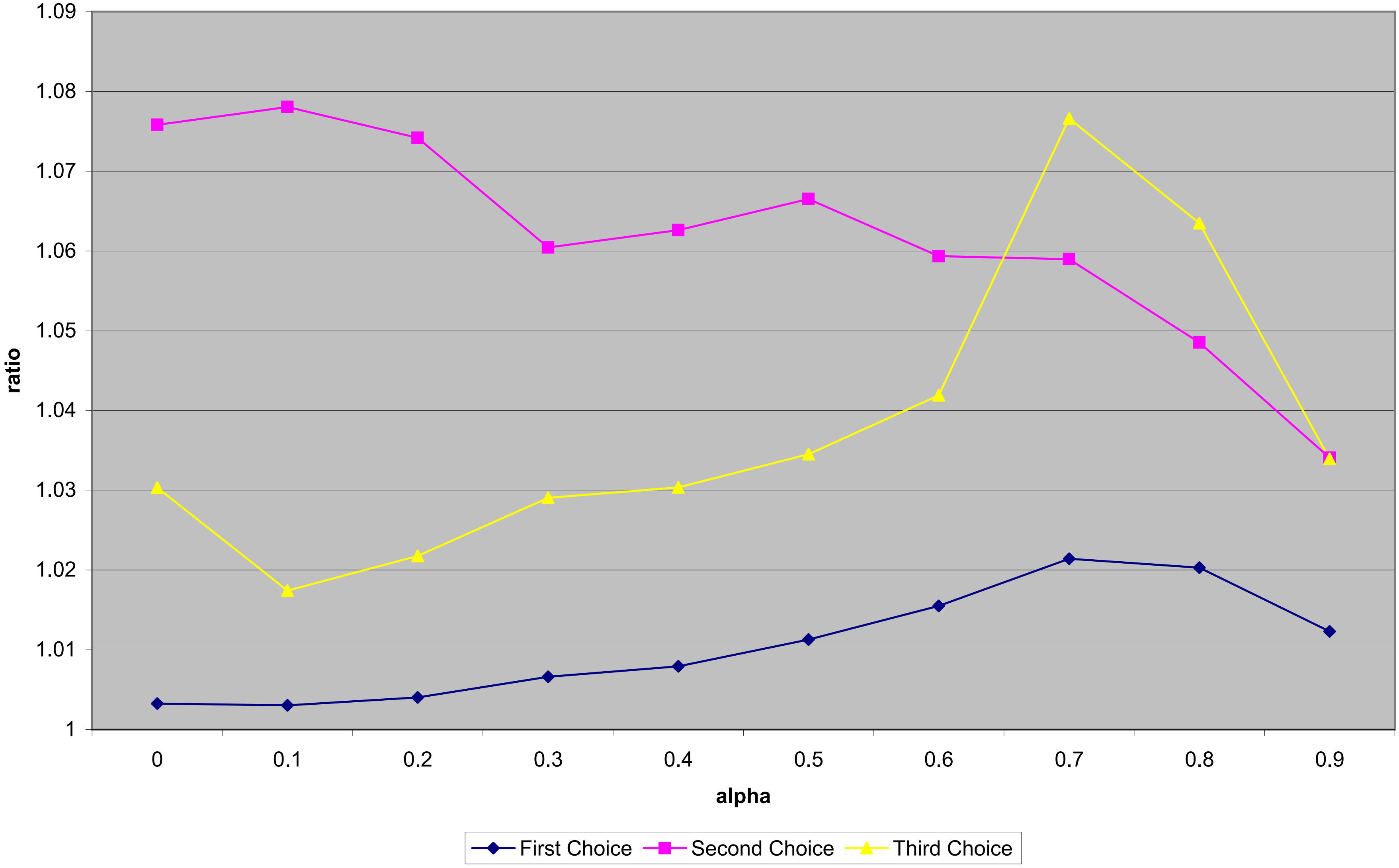


Figure 4: Average Number of Popular Schools and Oversubsribed Schools

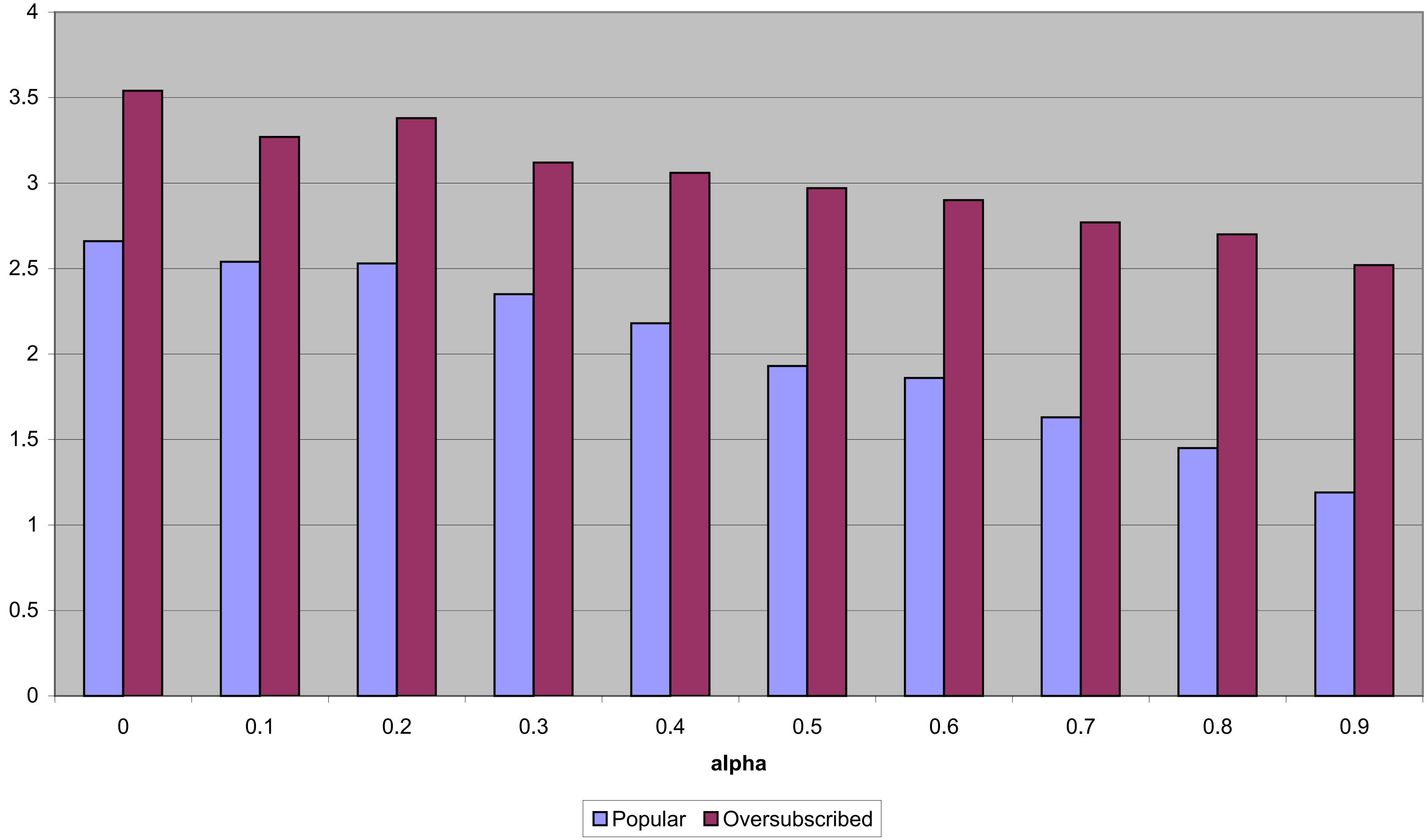


Figure 6: Average Number of Students Selecting kth Choice as Target in CADA Equilibrium

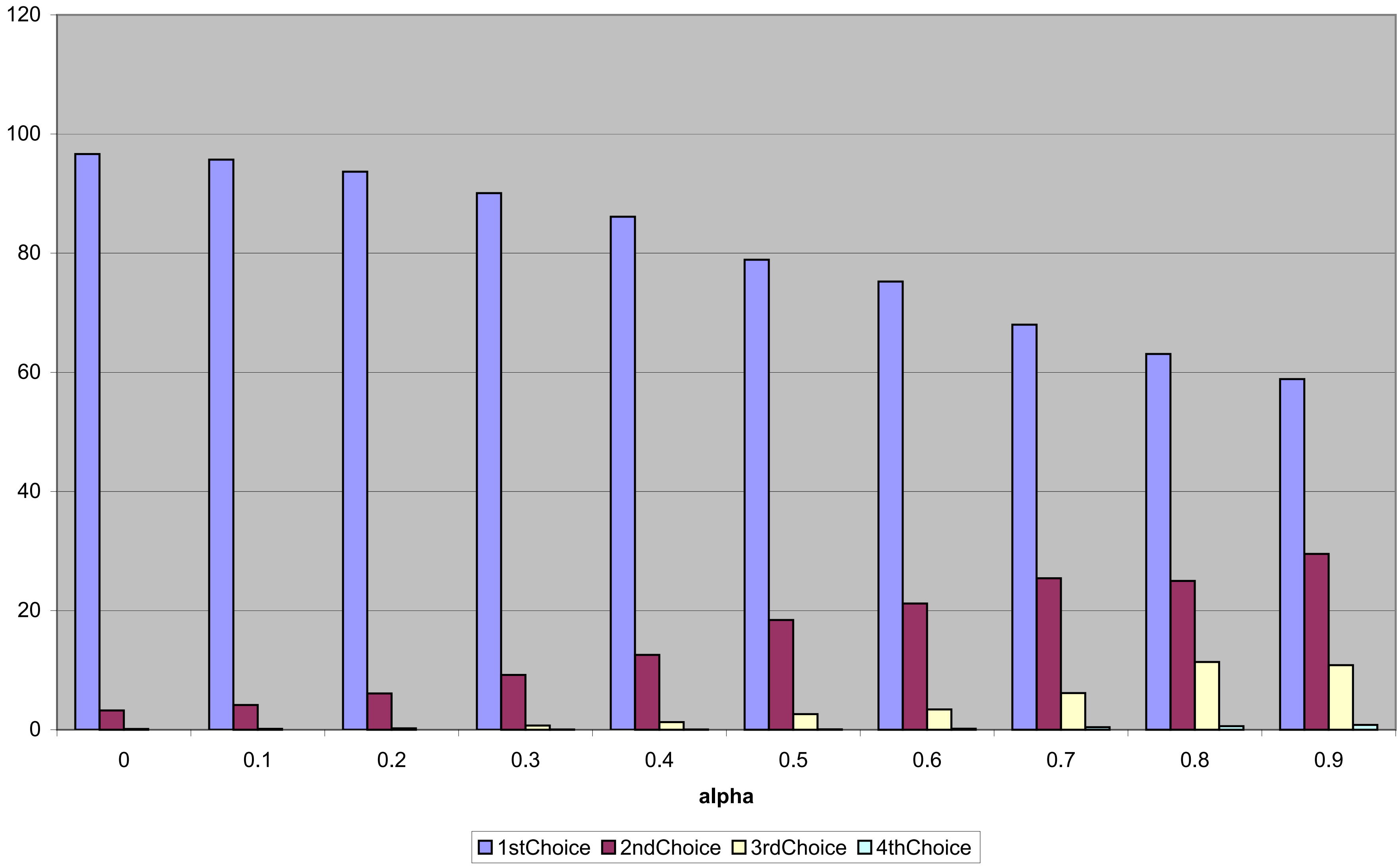
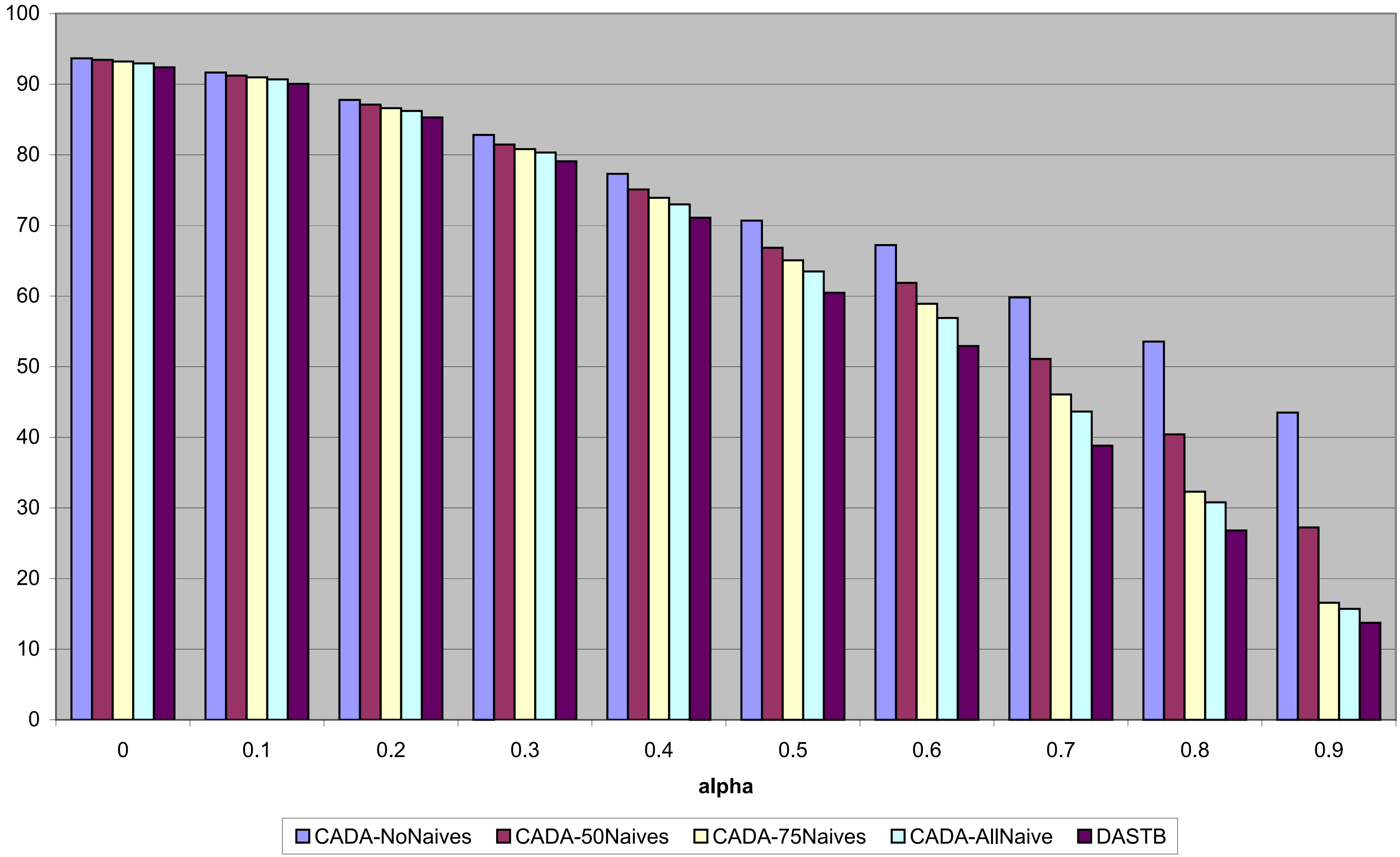


Figure 7: Welfare with Naive Players (mean adjusted)



Practical Issues

- Naive students: the same welfare benefits

- Extra protection: "Opt out" choice included in the auxiliary message; can make CADA Pareto dominate DA.

- Dynamic implementation:

Conclusion

- We propose a new deferred acceptance procedure in which students are allowed, via signaling of their preferences, to influence how they are treated in a tie for a school.

  - CADA combines desirable features of DA and the Boston mechanisms.

  - CADA strikes a better balance between incentives and choice, implementing a more efficient allocation than the DA without sacrificing the strategyproofness of ordinal preferences.

- The idea of "congestion" acting as a competitive market seems novel and general applicable beyond school choice.

- Related to the idea of "linking" decisions (Jackson-Sonnenschein; Casella-Palfrey).