Context Dependent Forward Induction Reasoning

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Abstract

This paper studies the case where a game is played in a particular context. The context influences what beliefs players hold. As such, it may affect forward induction (FI) reasoning: If players rule out specific beliefs, they may not be able to rationalize observed behavior. The effects are not obvious. Context-laden FI may allow different outcomes than context-free FI. At the formal level, contextual reasoning is defined within an epistemic structure. In particular, we represent contextual FI reasoning as "rationality and common strong belief of rationality" (RCSBR) within an arbitrary type structure. (The concept of RCSBR is due to Battigalli-Siniscalchi [2002].) What strategies are consistent with RCSBR (defined on an arbitrary type structure)? We show that the RCSBR is characterized by a new solution concept we call Extensive Form Best Response Sets (EFBRS's). We go on to study the EFBRS concept in games of interest. In particular, we establish a relationship between EFBRS's and Nash outcomes, in perfect-information games satisfying a 'no ties' condition. We also show how to compute EFBRS's in certain cases of interest.

1 Dynamic interactive epistemology:

Analysis of players' beliefs and their beliefs about opponents' beliefs in the context of an extensive form game

At each state of the world ω a player is characterized by probabilistic beliefs conditional on each information set

Cognitive unity of the self across information sets given by perfect recall and application of the rule of conditionalization

We consider assumptions on interactive beliefs involving the following ingredients:

Rationality=best response to conditional beliefs at each relevant information set

Strong Belief (of E)=event E is believed (w/ prob. 1) at each info set h consistent with E

Correct Common Belief (CCB): *there is CCB of* E at state ω if $\omega \in E$ and (every i believes at every $h \in H^i)^n(E)$ for all n; loosely, E is *"transparent"*

Self-evident event: E^* is *self-evident* if for every state $\omega \in E^*$ every i believes E^* at every information set $h \in H^i$

Remark:
E^* *is self-evident iff* $E^* = \{\omega : \text{there is CCB of } E \text{ at } \omega\}$ *for some* E.

Focus:
self-evident events E^* that obtain from *"transparency of epistemic events"* E

(an event is "epistemic" if it concerns players' beliefs)

2 Conceptual Motivation

Forward induction reasoning

Backward induction (BI)=each player believes that the *future* behavior of the opponent is "strategically rational" (sophisticated in some sense) and best responds

Forward Induction (FI)=each player believes, if possible, that the *past* behavior of the opponent is "strategically rational", and best responds.

In other words, according to FI reasoning a player "rationalizes" (if possible) the observed behavior of the opponent.

We argue that "rationalization" depends on what is transparent to the players, which in turn depends on the context behind the game.

Example: BoS w/ outside option, given "ladies' choice" convention

Bob chooses between an outside option (Out) that yields 2 and playing a coordination game with Ann (In) [**Bob**'s payoffs in **bold**]

(*,**2**) $\longleftarrow$ Out **Bob** $\longrightarrow$ In

Ann**Bob**	A	B
A	1,**3**	0,**0**
B	0,**0**	3,**1**

Let E^*=[It is transparent that Bob believes Ann would play A after In] ("ladies' choice" convention).
E^* is a self-evident event.

If we assume E^*, every other assumption about players' rationality and beliefs must be made in conjunction with E^* (otherwise E^* would not be self-evident!).

Let R^i=i is rational

$\forall E$, SB$^i(E)$=i strongly believes E (believes E whenever possible)

Claim: the following assumptions (representable as events in a canonical, universal state space)

R^a and R^b and E^*

SB$^a(R^b \cap E^*)$

SB$^b(R^a \cap E^* \cap$ SB$^a(R^b \cap E^*))$

are *mutually consistent* and *imply* [Out]

Intuition: $R^b \cap E^*$ implies [Out], thus SB$^a(R^b \cap E^*)$ has no implication for Ann's beliefs about Bob after In $\Rightarrow$ consistency. The "usual" forward induction argument does not work!

"Context free" FI in the BoS w/ outside option:
On the other hand, suppose now that no event is self-evident (except the sure event), implying that there is no "ladies' choice" convention. Then the "usual" forward induction argument works: the assumptions

R^a and R^b

$\mathsf{SB}^a(R^b)$

$\mathsf{SB}^b(R^a \cap \mathsf{SB}^a(R^b))$

are *mutually consistent* and *imply* [In]

Intuition: $\mathsf{SB}^a(R^b) \Rightarrow$ after In, Ann would believe that Bob plays In.B (In.A is dominated by Out) and would reply with B. Thus,
$R^b \cap \mathsf{SB}^b(R^a \cap \mathsf{SB}^a(R^b)) \Rightarrow$ Bob expects B by Ann and plays In and then B.

The example shows that the context affects FI reasoning. Specifically, it shows that *"context-laden" FI reasoning may allow outcomes excluded by "context-free" FI-reasoning*.

[Technically, this is due to the non-monotonicity of strong belief]

Therefore, the context affects the outcome of FI reasoning in a non-obvious way.

Now, let's be more specific about context dependent FI reasoning.

Ingredients of a situation of **strategic interaction**:

1. *rules of the game:* who moves when, with what constraints and information (→possible strategies), material consequences of play

2. *payoffs:* "utilities" of consequences

3. *interactive beliefs:* what the players might conceivably think about each other, conditional on their information

[We informally assume, just for simplicity, that there is "common knowledge" of 1 (rules)+2 (utility-payoffs), that is, we assume complete information.]

1 (rules)="game" in the natural language

1+2 (rules and utility payoffs)="game" in the language of game theory

What about 3? The *actual* beliefs that players hold should be consistent with strategic reasoning, but strategic reasoning is affected by pre-conceived ideas that determine the set of *possible* (or, conceivable) beliefs that players may have.

Context and beliefs

We argue that the **context** (previous history, social conventions, or preplay communication) may make an epistemic event E transparent, thus giving rise to a self-evident event E^*, e.g.:

- Players live in a society where it is transparent there is a lady's choice convention—i.e., if She gets to move in an asymmetric coordination game She will try to obtain "her best outcome."

- Players of some game Γ live in a society with a "keep right" convention. It is transparent to them that while playing Γ they strongly believe that "keep right" holds.

FI reasoning and RCSBR

Battigalli & Siniscalchi (2002) provide a rigorous epistemic formalization of FI reasoning by means of a set of assumptions called *Rationality and Common Strong Belief in Rationality* (RCSBR).

These assumptions are represented as events in a state space Ω whereby each $\omega \in \Omega$ specifies what players would do and what players would think at each history/node of the game.

If "no beliefs are ruled out by the context", RCSBR yields the Extensive Form Rationalizable strategies (Pearce, 1984), such as (In.B,B) in the BoS w/ an Outside Option.

How can we capture the context?

Implicit representation: Ω itself captures the context by allowing some beliefs and not others, Ω *is* the event made self-evident by the context.

Explicit representation: look at a canonical, universal state space Ω^u, fix the context-dependent, self-evident event E^*, recognize that in the assumptions stated above "rationality of i" is contextual and formally corresponds to event $R^i \cap E^*$ in Ω^u:

Each representation has its own advantages and they are, in a precise sense, equivalent. Whatever representation we choose, RCSBR is context-dependent in non obvious ways. To emphasize this, let's write the RCSBR event as

$$RCSBR_{\text{context}}$$

MAIN QUESTION: behavioral implications of contextual FI reasoning

The analyst need not know what is transparent to the players.

What are the behavioral consequences of assuming $RCSBR_{\textsf{context}}$?

Formally, let $Q \subset S$ the be product set of strategy profiles consistent with $RCSBR_{\textsf{context}}$, that is

$$Q = \textsf{proj}_S RCSBR_{\textsf{context}} \qquad (*)$$

Can we state properties of Q (without using interactive beliefs and state spaces) that are necessary and sufficient for the existence of a context that yields Q as in (*)? This would help answer the following question:

Can we (analysts) identify the observable implications of $RCSBR_{\textsf{context}}$ without knowing what is self-evident?

We build on related work by Brandenburger & Friedenberg (2004) on Self Admissible Sets to obtain:

a solution concept, the Extensive Form Best Response Set (EFBRS), providing necessary and sufficient conditions for Q so that $Q = \textbf{proj}_S RCSBR_{\text{context}}$ for some context.

In some interesting special cases, this characterization yields observable implications that are independent of the context.

Transparency of restrictions on first-order beliefs

First-order beliefs= beliefs about the opponent's strategy.

Suppose the analyst knows that some restrictions F on *first-order* beliefs are transparent (e.g. "ladies choice" convention). Let

$$F^* = [\text{restrictions } F \text{ hold and are "transparent"}]$$

be the corresponding self-evident event and consider FI reasoning in this F^*-context: $RCSBR_{F*}$

Then $Q_F = \text{proj}_S RCSBR_{F*}$ is a particular EFBRS. Is there an algorithm (based only on extensive form game and F) to compute Q_F?

We build on related work by Battigalli & Siniscalchi (2002, 2003) and show:

The particular EFBRS $Q_F = \textbf{proj}_S RCSBR_{F^*}$ can be computed with a *modified Extensive Form Rationalizability* procedure where first-order beliefs are restricted to F.

3 Setup

Two person game between a (Ann) and b (Bob)

1. *Rules of the game* $\longrightarrow$ information partitions H^a, H^b and strategy sets S^a, S^b for a (Ann) and b (Bob) (technical: we allow for simultaneous moves and specify a player's information even when she is not active); everything is **finite.**

Information about strategies:

$$S^a(h) = \{s^a \in S^a : s^a \text{ allows } h\},\ h \in H^a \cup H^b$$

2. *Payoffs* $\longrightarrow$ strategic form payoff functions $\pi^a, \pi^b : S^a \times S^b \to \mathbb{R}$

3. *Beliefs (implicit representation):*
T^i=abstract set of epistemic *types* of player i (technically, a Polish space).

Events about Bob: $E^b \subset S^b \times T^b$

Info. of Ann about Bob: $\forall h \in H^a$, write $[h]^b = S^b(h) \times T^b$ for brevity

A *belief* of Ann about Bob is a *conditional probability system (CPS):*

$$\mu^a(\cdot|\cdot) \;=\; (\mu^a(\cdot|[h]^b))_{h\in H^a} \in \prod_{h\in H^a} \Delta([h]^b) \text{ such that}$$
$$[h]^b \;\subset\; [g]^b \Rightarrow \mu^a(E^b|[h]^b)\mu^a([h]^b|[g]^b) = \mu^a(E^b|[g]^b)$$

$\mathcal{C}^a(S^b \times T^b)$=set of CPSs of Ann about Bob.

The belief of type t^a of Ann is $\beta^a(t^a)$, where $\beta^a : T^a \longrightarrow \mathcal{C}^a(S^b \times T^b)$ (Likewise for Bob).

Type structure (based on the given extensive form game Γ)*:*

$$\mathcal{T} = \left\langle S^a, S^b; T^a, T^b; \beta^a, \beta^b \right\rangle$$

States of the world:
$\Omega^i = S^i \times T^i$, $\Omega = (S^a \times T^a) \times (S^b \times T^b)$, $\omega = (\omega^a, \omega^b) = (s^a, t^a, s^b, t^b)$

Ω, hence $T^a \times T^b$, captures the epistemic implications of the context.

[We represent an event about Bob as $E^b \subset \Omega^b$, rather than $\Omega^a \times E^b \subset \Omega$]

First-order beliefs: given $\mu^a \in \mathcal{C}^a(S^b \times T^b)$,

$$\text{marg}_{S^b}\mu^a = (\text{marg}\mu^a(\cdot|[h]^b))_{h\in H^a}$$

is a CPS on S^b. $\text{marg}_{S^b}\beta^a(t^a)$ is the first-order belief of type t^a about Bob's strategies

Sequential best response: s^a is a seq. best response to $\nu^a \in \mathcal{C}^a(S^b)$, if

$$s^a \in \arg\max_{r^a\in S^a(h)} \sum_{s^b} \nu(s^b|S^b(h))\pi^a(r^a, s^b)$$

whenever s^a allows $h \in H^a$ (that is, $s^a \in S^a(h)$)

$$\rho^a(\nu^a) = \{s^a : s^a \text{ is seq. best resp. to } \nu^a\}$$

Rationality: (s^a, t^a) is *rational* if $s^a \in \rho^a(\beta^a(t^a))$
$R^a = \{(s^a, t^a) : (s^a, t^a) \text{ is rational}\}, R^b = ..., R = R^a \times R^b$

Strong Belief: μ^a *strongly believes* $E^b \subset \Omega^b$ ($E^b \neq \emptyset$) if

$$\forall h \in H^a, E^b \cap [h]^b \neq \emptyset \Rightarrow \mu^a(E^b|[h]^b) = 1.$$

For all events $E^b \neq \emptyset$ about Bob let

$$\text{SB}^a(E^b) = \{(s^a, t^a) : \beta^a(t^a) \text{ strongly believes } E^b\}$$

Correct mutual Strong Belief (and iterations): for all $E = E^a \times E^b$

$$\begin{aligned} \text{SB}(E) &= \text{SB}^a(E^b) \times \text{SB}^b(E^a) \\ \text{CSB}(E) &= E \cap \text{SB}(E) \\ \text{CSB}^0(E) &= E \\ \text{CSB}^m(E) &= \text{CSB}(\text{CSB}^{m-1}(E)),\ m = 1, 2, ... \end{aligned}$$

Rationality and Common Strong Belief in Rationality

The auxiliary CSB(·) operator yields a compact representation of RCSBR:

$$\begin{aligned}
\mathsf{CSB}^0(R) &= R \\
\mathsf{CSB}^1(R) &= R \cap \mathsf{SB}(R) \\
\mathsf{CSB}^2(R) &= R \cap \mathsf{SB}(R) \cap \mathsf{SB}(R \cap \mathsf{SB}(R)) \\
&\dots \\
\mathsf{CSB}^\infty(R) &= \bigcap_m \mathsf{CSB}^m(R) = RCSBR
\end{aligned}$$

To emphasize that RCSBR is defined within a context-determined type structure $\mathcal{T}$, write

$$RCSBR_{\mathcal{T}}$$

4 Results

Extensive Form Best Response Sets (EFBRS) and RCSBR

Recall: $\rho^a(\cdot)$ is the sequential best response correspondence.

DEF. $Q^a \times Q^b$ is an EFBRS if $\forall s^a \in Q^a$, $\exists \mu^a \in \mathcal{C}(S^b)$ (a 1st-order cps) such that
(1) $s^a \in \rho^a(\mu^a)$,
(2) μ^a strongly believes Q^b,
(3) $\rho^a(\mu^a) \subset Q^a$.
Likewise for Bob.

Comments to EFBRS

(1)-(2): kind of "internal stability" property

(3): kind of "maximality" or "external stability" property

Crucial features: (2) requires *strong* belief, mere initial belief is not enough. (3) requires *maximality* w.r.t. best responses to "allowed" beliefs; intuition: context may exclude beliefs, not strategies (which are freely chosen), a player who best responds to allowed beliefs *must* be deemed rational.

Formally: fix $\mathcal{T}$ and suppose Bob observes actions by Ann consistent with some $s^a \in \rho^a(\mu^a)$, where $\mu^a =$marg$_{S^s}\beta^a(t^a)$ for some t^a; then strong belief in rationality implies that Bob must believe that Ann is rational. A similar intuition holds for higher levels of "strategic sophistication". Thus s^a must be in the set Q^a strongly believed by Bob.

Example on the role of maximality:

$(\mathbf{1},*) \xleftarrow{\text{Out}} \mathbf{A} \xrightarrow{\text{In}}$

A\B	L	C	R
U	**2**,0	**0**,0	**3**,1
M	**0**,0	**2**,0	**3**,1
B	**0**,1	**0**,1	**0**,0

Only the belief $\hat{\mu}^a$ s.t. $\hat{\mu}^a(\mathsf{L}) = \hat{\mu}^a(\mathsf{C}) = \frac{1}{2}$ justifies Out;
In.B is strictly dominated.
$\{\text{Out}\} \times \{\text{L}, \text{C}\}$ satisfies (1)-(2), but not (3): $\rho^a(\hat{\mu}^a) = \{\text{Out}, \text{In.U}, \text{In.B}\}$

If $Q \neq \emptyset$ satisfies (1)-(2), then **either** $Q = \{\text{Out}\} \times \{\text{L}, \text{C}\}$ (FI does not bite),
or $Q = \{\text{In.U}, \text{In.M}\} \times \{\text{R}\}$ (FI bites)

$\Rightarrow$ by (3), the only non-empty EFBRS is $Q = \{\text{In.U}, \text{In.M}\} \times \{\text{R}\}$
(the extensive form rationalizable strategies)

THEOREM 1 $Q = \mathsf{proj}_S RCSBR_{\mathcal{T}}$ for some type structure $\mathcal{T}$ *if and only if* Q is an EFBRS.

Very rough sketch of proof

If $\exists\mathcal{T}$ s.t. $Q = \mathsf{proj}_S RCSBR_{\mathcal{T}}$, (1)-(2)-(3) can be verified by inspection.

If (1)-(2)-(3) hold for Q, construct $\mathcal{T}$ s.t. $Q = \mathsf{proj}_S RCSBR_{\mathcal{T}}$:
- define $T^a = Q^a$, $T^b = Q^b$
- for each $s^a \in Q^a = T^a$, let $\mu^a(s^a)$ be the belief justifying s^a as per (1)-(2)
- and construct $\beta^a(s^a) \in \mathcal{C}^a(S^b \times T^b)$ so that $\mu^a(s^a)$ =marg$_{S^b}\beta^a(s^a)$ and $\forall h \in H^a$ with $S^b(h) \cap Q^b \neq \emptyset$

$$\beta^a(s^a)((s^b, t^b)|[h]^b) > 0 \text{ only if } s^b = t^b.$$

EFBRS, self-evident events and RCSBR

Each $\mathcal{T}$ (satifying a weak "technical" condition) corresponds to a self-evident epistemic event $E^*(\mathcal{T})$ within the canonical, universal type structure. Therefore $RCSBR_{\mathcal{T}}$ is equivalent to $\mathsf{CSB}^{\infty}(R \cap E^*(\mathcal{T}))$.

COROLLARY In the canonical, universal type structure

$$Q = \mathsf{proj}_S \mathsf{CSB}^{\infty}(R \cap E^*)$$

for some self-evident epistemic event E^**if and only if* Q is an EFBRS.

Transparent restrictions on first-order beliefs

Fix *closed* sets $F_h^a \subset \mathbf{\Delta}(S^b(h))$ $(h \in H^a)$ and let
$F^a = \{\mu^a \in \mathcal{C}^a(S^b) : \forall h \in H^a, \mu(\cdot|S^b(h)) \in F_h^a\}$ (likewise for F^b)

In the canonical, universal type structure $\mathcal{T}^u$ let

$$\begin{aligned} F &= \{(s^a, t^a, s^b, t^b) : \mathsf{marg}_{S^b}\beta^a(t^a) \in F^a, \mathsf{marg}_{S^a}\beta^b(t^b) \in F^b\} \\ F^* &= \{\omega : \text{there is CCB of } F \text{ at } \omega\} \text{ (self-evident)} \end{aligned}$$

How to compute $Q_F = \mathsf{proj}_S \mathsf{CSB}^\infty(R \cap F^*)$ (an EFBRS)?

F-rationalizability (Battigalli & Siniscalchi 2003)

$S^a_{F,0} = S^a$, $S^b_{F,0} = S^b$
$S^a_{F,m} = \{s^a : \exists \mu^a \in F^a$, μ^a strongly believes $S^b_{F,m-1}$ and $s^a \in \rho^a(\mu^a)\}$
likewise for $S^b_{F,m}$
$S^a_F = \bigcap_m S^a_{F,m}$, $S^b_F = \bigcap_m S^b_{F,m}$

By inspection, $S^a_F \times S^b_F$ is an EFBRS

RESULT $S^a_F \times S^b_F =$proj$_S$CSB$^\infty(R \cap F^*)$

Note: Battigalli & Siniscalchi proved a related, but different result,
$S^a_F \times S^b_F =$proj$_S$CSB$^\infty(R \cap F)$

Examples of observable implications

Using Theorem 1, we can obtain the observable implications of RCSBR, independently of the context, in interesting special cases:

RESULT: In the *Finitely Repeated Prisoners' Dilemma,* if $(s^a, t^a, s^b, t^b) \in RCSBR$ (for some $\mathcal{T}$) then (s^a, s^b) yields the BI path (sequence of defections).

RESULT: In the *Centipede* game, if $(s^a, t^a, s^b, t^b) \in RCSBR$ (for some $\mathcal{T}$) then (s^a, s^b) yields the BI path (immediate exit).

Observable implications of RCSBR in perfect information games

DEF. s^a is *sequentially justifiable* if $s^a \in \rho^a(\mu^a)$ for some $\mu^a \in C^a(S^b)$.

DEF. A perfect information game has *No Relevant Ties* if, for every pair of distinct terminal nodes z', z'', the player moving at the last common predecessor of z' and z'' is not indifferent between z' and z''.

THEOREM 2: Fix a *perfect information* game with *NRT*.
1. If Q is an EFBRS, then there is a *Nash* equilibrium $\hat{s}$ that is *path*-equivalent to every $s \in Q$.
2. If $\hat{s}$ is a *Nash* equilibrium in sequentially justifiable strategies, then there is an EFBRS Q whereby every $s \in Q$ is *path*-equivalent to $\hat{s}$.

[We extend part 1 to PI games satisfying the Single Payoff Condition, such as zero-sum PI games.]

Example: sequential justifiability is needed in part 2

$$
\begin{array}{ccccc}
A & \longrightarrow & B & \longrightarrow & \begin{pmatrix} 2 \\ 2 \end{pmatrix} \\
\downarrow & & \downarrow & & \\
\begin{pmatrix} 1 \\ 1 \end{pmatrix} & & \begin{pmatrix} 0 \\ 0 \end{pmatrix} & &
\end{array}
$$

(Down if In) is not sequentially justifiable (it is conditionally dominated) for Bob, hence Nash outcome (1,1) is inconsistent with RCSBR.
Only Nash outcome (2,2) is consistent with RCSBR.

Example: $RCSBR_{\mathcal{T}}$ need not yield the BI path

$$
\begin{array}{ccccccc}
A & \longrightarrow & B & \longrightarrow & A & \longrightarrow & \begin{pmatrix} 3 \\ 3 \end{pmatrix} \\
\downarrow & & \downarrow & & \downarrow & & \\
\begin{pmatrix} 2 \\ 2 \end{pmatrix} & & \begin{pmatrix} 1 \\ 1 \end{pmatrix} & & \begin{pmatrix} 0 \\ 0 \end{pmatrix} & &
\end{array}
$$

Nash outcome $(2,2)$ is implied by $RCSBR_{\mathcal{T}}$ for some $\mathcal{T}$ whereby $\beta^a(t^a)(\text{across}^b) < \frac{1}{2}$ for all $t^a \in T^a$.

Only the BI path $(3,3)$ is consistent with Extensive Form Rationalizability, and hence with $RCSBR$ in the universal type structure (cf. Battigalli & Siniscalchi, 2002).

As a consequence of Theorem 2 (part 1) we obtain

COROLLARY In all perfect information games satisfying *NRT* and with a *unique Nash path*, *RCSBR yields the Backward Induction path*.

The Centipede is a case in point.

5 Conclusions

We represent FI reasoning with the assumption of Rationality and Common Strong Belief in Rationality, corresponding to an event $RCSBR_{\mathcal{T}}$ in a type structure $\mathcal{T}$. The type structure captures restrictions on players beliefs made transparent by the context.

Despite the fact that the context affects FI reasoning in non obvious ways, we can characterize $RCSBR_{\mathcal{T}}$ (for some $\mathcal{T}$) with a solution concept, the Extensive Form Best Response Set, which allows to derive the context independent observable implications of $RCSBR$ in interesting cases.

For the sake of simplicity, we restricted our analysis to two-person games of complete information without chance moves. These assumptions can be removed:

The main issue is how to model correlation/independence of beliefs concerning different opponents and chance. If correlation is allowed, the extension of our characterization is straightforward.

References

[1] Battigalli, P., and M. Siniscalchi, “Hierarchies of Conditional Beliefs and Interactive Epistemology in Dynamic Games,” *Journal of Economic Theory*, 88 (1999), 188-230.

[2] Battigalli, P., and M. Siniscalchi, “Strong Belief and Forward-Induction Reasoning,” *Journal of Economic Theory*, 106 (2002), 356-391.

[3] Battigalli, P., and M. Siniscalchi, “Rationalization and Incomplete Information,” *Advances in Theoretical Economics*, 3 (2003), Article 3. [http://www.bepress.com/bejte/advances/vol3/iss1/art3]

[4] Ben Porath, E., "Rationality, Nash Equilibrium, and Backward Induction in Perfect Information Games," *Review of Economic Studies*, 64 (1997), 23-46.

[5] Brandenburger, A., and A. Friedenberg, "Self-Admissible Sets," unpublished, 2004. Available at www.stern.nyu.edu/∼abranden.

[6] Brandenburger, A., A. Friedenberg, and H.J. Keisler, "Admissibility in Games," *Econometrica*, 76 (2008), 307-352.

[7] Pearce, D., "Rational Strategic Behavior and the Problem of Perfection," *Econometrica*, 52 (1984), 1029-1050.