## A Subjective Foundation of Objective Probability<sup>\*</sup>

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## Abstract

De Finetti's concept of exchangeability provides a way to formalize the intuitive idea of similarity and its role as guide in decision making. His classic representation theorem states that exchangeable expected utility preferences can be expressed in terms of a subjective beliefs on parameters. De Finetti's representation is inextricably linked to expected utility as it simultaneously identifies the parameters and Bayesian beliefs about them. This paper studies the implications of exchangeability assuming that preferences are monotone, transitive and continuous, but otherwise incomplete and/or fail probabilistic sophistication. The central tool in our analysis is a new *subjective ergodic* theorem which takes as primitive preferences, rather than probabilities (as in standard ergodic theory). Using this theorem, we identify the i.i.d. parametrization as sufficient for all preferences in our class. A special case of the result is de Finetti's classic representation. We also prove: (1) a novel derivation of subjective probabilities based on frequencies; (2) a subjective sufficient statistic theorem; and that (3)differences between various decision making paradigms reduce to how they deal with uncertainty about a common set of parameters.

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