Policy Dynamics and Legislative Bargaining

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1 Introduction

Recent theoretical and empirical studies on comparative constitutions have deepened our understanding of how political institutions shape economic policies. Models by Persson and Tabellini (1999), Lizzeri and Persico (2001), and Milesi-Ferretti et al. (2002), for example, compared how different electoral rules lead to different fiscal policies such as the size of general public goods, targeted transfers, local public goods, and corruption. Pagano and Volpin (2006) investigated how electoral rules shape government regulations on corporate governance. There also have been a few studies investigating the economic effects of legislative institutions. Persson and Tabellini (2000) compared the consequences of presidential versus parliamentary constitutions on fiscal policy. More recently, Battagilini and Coate (2005, 2006) modeled the relationship between legislative bargaining, public investment and debt. Finally, Fong (2006) and Baron, Diermeier, and Fong (2007) showed how coalition formation and voting under proportional representation can lead to policy inefficiency. This last approach combined both legislative and electoral institutions into a single, integrated model.

These theoretical advances have been accompanied by related empirical investigatios. In some cases the purpose was to test some of the theoretical predictions of the models, in others to establish new relationships. Persson and Tabellini (2001, 2003, 2004), for example, created a comprehensive data set on political institutions and then use the data to empirically investigate how constitutional arrangements shape fiscal policies.

Most of the existing studies, however, are based on static models or focus on static policy issues like the sizes of total government spending, welfare expenditures, or the level of waste and corruption. This is in marked contrast to the earlier generations of political economy models with their emphasis dynamic phenomena such as political business cycles (Rogoff 1990, Alesina, Roubini and Cohen 1997), public debt (Alesina and Drazen 1991, Alesina and Tabellini 1990, Persson and Svensson 1989, and Aghion and Bolton 1990), dynamics of welfare programs. (Hassler et al 2003, 2005 and

2006), public investment (Azzimonti 2006), and models of growth (Alesina and Rodrik 1994, Persson and Tabellini 1994 and Krusell 1996). However, these earlier models relied on very simplified models of political decision-making, such as the median voter theorem, that were unable to capture constitutional differences. To model constitutional difference an institutionalist approach is necessary.

This state of affairs leaves an important gap in our understanding of the relationship between political institutions and economic policy. It seems that we can either focus on institutional accounts of economic policy making or on dynamic policy models, but not both.[1] This state of affairs is particularly lamentable as recent work by e.g. Persson and Tabellini (2001, 2003, 2004) provided some empirical evidence of the constitutional effects on political business cycles, fiscal deficits as well as the responsiveness of government to business cycles. The main difficulty is the absence of suitable political economy frameworks, i.e. institutionally rich models with changing economic state variables. Existing legislative decision-making approaches run into technical difficulties once we enrich the choice space to include dynamic economic policy. Continuing policies in multiperiod models usually generate discontinuity and non-concavity of equilibrium value functions and policy rules that make the characterization of an equilibrium a challenging task (Baron and Herron 2003, Kalandrakis 2003, Baron, Diermeier and Fong 2007, Duggan and Kalandrakis 2007).

In this paper, we propose a new legislative decision-making framework to address these short-comings. The core legislative bargaining model is characterized by two key features: (1) A policy, once enacted, is in effect until a new law is made. (2) Any legislator with agenda-setting power is allowed to make a new policy proposal at any time and as frequently as possible. The first feature is reminiscent of Bernheim et al's (2006) concept of an evolving default policy. The idea is that during the a legislative period (i.e. before a new election must be held) policies can always be reconsidered.[2] The second feature distinguishes our model from all others in the literature. While most dynamic legislative bargaining models are extremely difficult to solve, our model

[1] A recent exceptions is the work by Battaglini and Coate on public debt (2006, 2007).

[2] The relationship with Bernheim et al. (2006) is discussed iin more detail below.

is tractable and exhibits continuous value functions, a rarity in models of collective choice. The model is extremely tractable and at the same time it yields predictions that not only seem plausible but appear to be consistent with recent empirical studies on the value of proposal power (Knight 2005).

These properties allow us to apply the model to an environment of dynamic policy choice. We first analyze a model of distributive politics in the tradition of Baron and Herron (2003) and Kalandrakis (2003). The model consists of a sequence of legislative periods. In each period legislators have to agree on a new distribution of benefits. The equilibrium choices of each legislator in each period, however, may depend on a *status quo* policy which is in turn determined by the policy choice in the previous period.

In this paper we focus on a particular structure of agenda-setting that is typical of parliamentary democracies. Comparative scholars have long observed that compared to presidential systems the constitutional features of parliamentary systems lead to high levels of agenda control for the executive, i.e. the cabinet (Doering 1995). In many cases, that power is concentrated within the prime minister. We capture this feature formally by considering a single, persistent agenda-setter during a given legislative period. Surprisingly, this framework does not necessarly lead to extreme proposal power, but constrains the agenda setter.

Specifically, we show that in the context of distributive policies, legislators have indirect preferences over distributions of benefits. That is, each legislator cares not only about his own allocation of benefits but also about the allocation to other legislators. This hold not because of altruistic preferences, but because current distributions affect each legislator's bargaining power in the future. As a consequence, in equilibrium, the legislators not included in the winning coalition are not fully expropriated, and the value of agenda-setting can be significantly smaller than what is predicted in other proposer models such as Baron and Ferejohn (1989) or Bernheim et al. (2006). This result of constrained proposal power is consistent with some recent empirical findings (e.g. Knight 2005).

We then apply our modeling modeling framework to a fully dynamic setting to capture richer political economy environments including the source of policy inertia, especially in the context of entitlement programs and the so-called "ratchet effect" of government spending, i.e. the observation that in some countries government spening increasese during recessions, but does not decrease during booms, leading to a stepwise increase in overall public spending. Persson and Tabellini (2004) show that the ratchet effect is only observed in parliamentary democracies with proportional representation, yet absent in other political systems.

Our point of departure is the Baron and Ferejohn model (1989). They analyzed how legislators bargain over a pie with majority rule and find a unique stationary equilibrium where only a bare majority of legislators receive positive shares of the pie, while the agenda setter captures a disproportionate share. The seminal paper was recently tested by Knight (2005) using US data on the distribution of transportation projects. The evidence supports the key qualitative prediction that proposal power is valuable, but more constrained that predicted by the model. In our model, we show that the possibility to reconsider a policy issue substantially weakens the proposal power for an agenda setter, even if he has the sole authority to make policy proposals throughout the whole legislative session. In existing legislative bargaining models a single proposer would always be able to capture the entire pie. However, this is not the case in our model, as legislators, out of fear that the agenda setter will use his agenda setting power to exploit legislators with low reservation values in the future, do not approve any policy that substantially lower the reservation values of others.

The paper belongs to the literature of dynamic legislative bargaining with a moving status quo where intertemporal tradeoff between current legislative and future status quo may lead to complex patterns of policy dynamics. With one-dimensional policy space and single-peaked preferences, Baron (1996) showed that, in the long run, the policy will converge to the alternative preferred by the median voter. Baron and Herron (2003) and Fong (2004) study the game in a multidimensional policy space. In models of a parliamentary democracy with proportional representation, Fong (2006) shows that an incumbent coalition government strategically manipulates to lower the bargaining position of the outside parties in order to create cheap coalition partners in the future. The incentive leads to more non-central policy outcomes and inefficiency. Baron et al (2007) show that with strategic voters the problem of inefficiency is worsened, since a more extreme status quo favors the incumbent parties in future elections.

Kalandrakis (2004) analyzes an infinitely repeated Baron-Ferejohn legislative bargaining where three players with linear utility divide a dollar in each period. The Markov perfect equilibrium in his model has the charateristc that irrespective of the discount factor or the initial division of the dollar, the proposer eventually extracts the whole dollar in all periods. In contrast, in the dynamic version of our model, full expropriation by the agenda setter rarely occurs. The distribution is more egalitarian.

Bernheim et al (2006) examines legislative policy making in institutions with real-time agenda setting and evolving default. Assuming finite rounds of proposal-making and voting within a pork barrel model of redistributive politics, the last proposer is able to pass his favorite policy under relatively weak conditions. As a consequence, the final policy outcome is highly unequal, and the last proposer is able to obtain his ideal policy. As the authors point out in the concluding section, it is natural to wonder whether particular procedures effectively promote a more egalitarian distribution of political power. Our model maintains the idea of an evolving default policy, but assumes an agenda setter with persistent power throughout the legislative session and no *ex ante* known last round of negotiation.

This paper is also linked to a recently emerging literature on the role of lack of commitment in policy making. While it is commonly accepted that lack of commitment by the policy maker is a source of inefficiency, our model shows that lack of commitment by the agenda-setter who holds power for a certain amount of time in fact leads to more egalitarian divisions. If we were to introduce some concavity into the legislators' payoff functions, this would imply more efficient policy outcomes.

2 The Model

2.1 The Setup

The political system is characterized by a legislature with three members, indexed by $l \in \{1, 2, 3\}$. The legislature must collectively decide on how to divide a total benefit of $G \in \mathbb{N}$ units for each period $t = 1, 2, ..., T$. We consider the case where T is either potentially large but finite and where T is (countably) infinite. A feasible policy in period t is therefore a triple $\mathbf{x}_t = (x_{1,t}, x_{2,t}, x_{3,t})$ with $x_{l,t} \in \mathbb{Z}_+$ for all $l \in \{1, 2, 3\}$ and $\sum_{l=1}^{3} x_{l,t} \leq G$. Denote the (time-invariant) policy space by $\Delta(G)$. The assumption of a discrete policy is made for technical convenience. The units can be as small as necessary, e.g. one cent.

Every legislator derives utility from the benefit he receives according to the policy. Given a sequence of policy, $\{\mathbf{x}_t\}_{t=1}^{T}$, the expected and discounted sum of utility of legislator l is given by

$$\sum_{t=1}^{T} \beta^{t-1} (1 - \beta) x_{l,t},$$

where β is a common discount factor. The multiplication by $(1 - \beta)$ is a normalization that yields simpler algebra in the infinite horizon model. The per period utility function is assumed to be linear since we do want to consider dynamic risk sharing in this paper.

In every period $t \in \{1, ..., T\}$, all legislators meet in a legislative session and one of them $a_t \in \{1, 2, 3\}$ is randomly selected to be the sole agenda setter. There is only one session in a period so we use the terms "period t" and "session t" interchangably. The agenda setter is conferred the solde power to make proposals from the policy space at any time throughout the legislative session. The political process in a session is modeled as the dynamic bargaining institution of Diermeier and Fong (2007) and its detailed description is relegated to the next paragraph. If at the end of the session a new policy $\mathbf{x}'$ has been adopted, it is implemented and $\mathbf{x}_t = \mathbf{x}'$. If the agenda setter waives his proposal power or no agreement on a new policy is reached before the session ends, the status quo policy remains in place and $\mathbf{x}_t = \mathbf{x}_{t-1}$. We assume an arbitrary initial status quo $\mathbf{x}_0 \in \Delta(G)$, which is the prevailing policy prior to period

1, the beginning of the model.

The assumption of a moving status quo captures the idea that, once enacted, a policy is in effect until it is reformed through the political process. Intuitively, this means that our model applies to continuing policies as in Baron (1996). Exampes are entitlement programs that stay in place until they are changed by the legislature. Examples include social security and welfare programs, but also subsidies for certain industries. In many cases,e.g. the U.S. Social Security Act of 1935, beneficiaries can sue the government if benefits are with-held. The U.S. Congressional Budget Office reported that in 1996 more than 55% of all Federal expenditures (excluding interest payments) were dedicated to entitlement programs which amounted to 10.3% of U.S. GDP (Baron 1996; p. 317). Other examples are various forms of regulation if they have distributive consequences. That is, regulatory policies usually impose (net) benefits on some economic actors and (net) costs on others. Examples include regulatory policies in the areas of trade, work-place safety, zoning, or price controls.

Legislative bargaining in a session proceeds in potentially multiple rounds of proposal making and voting. The number of rounds depends on both exogenous factors that may randomly terminate the session and the decision by the agenda setter. Initially, in each session t there is a default policy $\mathbf{d}_{t,0} \in \Delta(G)$. A "default" is the policy that will be implemented if no new policy proposal is passed subsequently in the same session. To make it consistent with the assumption of a moving status quo we assume that in each session the initial default is the policy implemented in the previous session, i.e. $\mathbf{d}_{t,0} = \mathbf{x}_{t-1}$; i.e., Denote the prevailing default in round r by $\mathbf{d}_{t,r-1}$. In each round r, the agenda setter can choose to make a new policy proposal $\mathbf{x}'_{t,r}$ or to pass. To simplify the mathematical formulation, a "pass" is modeled as a proposal identical to the prevailing default; i.e., $\mathbf{x}'_{t,r} = \mathbf{d}_{t,r-1}$. Once a proposal (different from the default) is made, it is voted on against the default. Voting is by simple majority rule. If a new proposal passes it becomes the default in the next round, i.e., $\mathbf{d}_{t,r} = \mathbf{x}'_{t,r}$. Otherwise the original default remains, i.e., $\mathbf{d}_{t,r} = \mathbf{d}_{t,r-1}$. Collective decision-making then continues in the same fashion conditional on the continuation of the session. The default evolves as legislation in a session progresses. The default

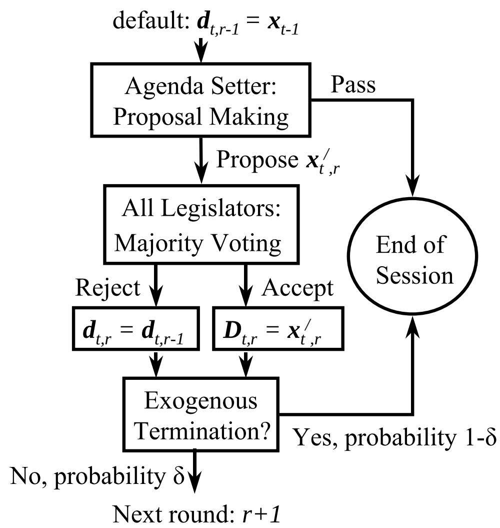

Figure 1: Flow chart: the political process in round r of session t.

policy that survives till the end of the session is the policy outcome in that period and the status quo for the next session

There are two ways to terminate a legislative session. First, at the end of each round of negotiation, the session may end *exogenously* with probability $(1-\delta)$, where $\delta \in [0,1]$. In other words, conditional on any round of negotiation, with probability δ a session continues and the agenda setter gets a chance to revisit the same policy issue and make a new proposal to replace the bill that has been passed. Throughout the paper, we assume that a session may exogenously end only with a negligible probability and only characterize an equilibrium with δ sufficiently close to 1. Second, the session ends *endogenously* if the ongoing default is such that the agenda setter no longer wants to propose any new policy to defeat it.

The political process in round r of session t is summarized in Figure 1.

Note that in the existing dynamic bargaining literature *status quo* and *default policy* (e.g. Baron 1996, Kalandrakis 2004, Fong 2006, Baron, Diermeier and Fong 2007, and Duggan and Kalandrakis 2007). However, this is not the case in our model. The *status quo* for a session is the initial policy at the beginning of that session. In the first session it is given exogenously. But if a new policy is passed in any session that policy becomes the next session's status quo. In contrast, we refer to a *default* as the policy to be implemented at the end of the session if no new bill is passed on the same policy issue in the remainder of that same session.

The assumption of an evolving default is similar to the approach proposed by Bernheim et al (2006). That is, the passage of a bill does not prevent the legislature from revisiting the issue at a later date; rather, it changes the default for subsequent deliberations. Bernheim et al. assumed an exogenously fixed, commonly known number of bargaining rounds. In our model, however, there is not a well-defined last round. Rather, the number of actual bargaining rounds is endogenously determined in equilibrium.

Our set-up is closely related to Kalandrakis (2004) but with only one critical difference. Kalandrakis (2004) assumes that an agenda setter is restricted to make a policy proposal only once in every legislative session, whereas here we assume that an agenda setter could potentially make proposals more than once in one session. Conceptually, the passage of a bill does not prevent the legislature from revisiting the same policy issue before the bill becomes law. Like Kalandrakis, we characterize a stationary equilibrium and a comparison of equilibrium policy outcomes shows how the possibility of reconsideration substantially changes the nature of enacted policies as well as the dynamics of policy choice. Specifically, Kalandrakis (2004) constructs an equilibrium in which, in the long run, the agenda setter takes all the benefit, whereas such policy dynamics do not occur in our model for almost all initial status quo policies.

2.2 Equilibrium Definition

Consider an arbitrary period $t \in \{1, ..., T\}$. No matter in which round of negotiation, the only payoff relevant state variable is the prevailing default $\mathbf{d}_{t,r-1}$. If no new bill is passed in this session, the default $\mathbf{d}_t$ is the policy outcome in period t and becomes the initial default in session $t+1$. Since the probability that a session exogenously ends is the same, the legislators face an identical dynamic choice problem in legislative bargaining rounds r and $r' \neq r$ if the default policies in the two rounds are the same, i.e., $\mathbf{d}_{t,r-1} = \mathbf{d}_{t,r'-1}$. Therefore, we restrict analysis to cases in which the legislators condition their strategies only on the prevailing default. In other words, we assume stationarity within a legislative session.

For any legislator $l \in \{1, 2, 3\}$, let $U_l(\mathbf{x}_t, a_t, t)$ be the legislator's expected and discounted sum of utility evaluated in period t with agenda setter a_t, if a policy $\mathbf{x}_t$ is passed in the current round of negotiation. We refer to

$$U_l : \Delta(G) \times \{1, 2, 3\} \times \{1, ..., T\} \to \mathbb{R}$$

as a dynamic payoff function. Given this definition, with prevailing default $\mathbf{d}_t$, $U_l(\mathbf{d}_t, a_t, t)$ is the reservation value of legislator l, which is his expected and discounted sum of utility if policy $\mathbf{d}_t$ remains to be default in the subsequent round.

We make two behavioral assumptions regarding proposal making and voting. First, an agenda setter proposes a new policy (different from the prevailing default) if doing so makes him strictly better off. Otherwise, he passes or, equivalently, he proposes the default policy. This assumption can be justified by an infinitesimal cost of proposal making. Given stationarity within a legislative session, once an agenda setter passes a round in some session t, he would pass all potential subsequent rounds and therefore this session ends. Second, a legislator votes against a policy proposal if and only if passage of the bill makes him strictly worse off. This is equivalent to a case in which a legislator has to overcome an infinitesimal cost in order to vote against the agenda setter. Since a legislator as voter simply compares a proposal and the default policy, it is not necessary to specify voting strategies explicitly.

For any agenda setter $a \in \{1, 2, 3\}$ in an arbitrary round of negotiation in period t with default $\mathbf{d}_t$, let $\mathbf{g}(\mathbf{d}_t, a_t, t) \in \Delta(G)$ be his optimal proposal. This is the policy that maximizes the his dynamic payoff $U_a(\cdot, a_t, t)$ subject to the constraint that at least one other legislator is weakly better off if this policy becomes a new default. In other words, $\mathbf{g}(\mathbf{d}_t, a_t, t)$ solves

$$\begin{aligned} \max_{\mathbf{x}' \in \Delta(G)} \quad & U_a(\mathbf{x}', a_t, t) \\ s.t. \quad & U_i(\mathbf{x}', a_t, t) \geq U_i(\mathbf{d}_t, a_t, t) \text{ for some } i \neq a. \end{aligned} \tag{1}$$

By the two behavioral assumptions in the previous paragraph, majority voting is modeled by an incentive compatibility constraint in the maximization problem. An agenda setter would never make any proposal that is destined to be rejected by majority voting. Therefore, any proposal, if made, would satisfy at least one of the other legislators at his reservation value. Note that the prevailing default $\mathbf{d}_t$ always satisfies the incentive compatibility constraint. If the default policy is such that an agenda setter cannot pass any proposal that leaves him a strictly higher dynamic payoff than his reservation value, he proposes, and of course, passes the default policy. In this case, the default policy solves the constrained maximization problem.

In the rest of the paper we call the function

$$\mathbf{g} : \Delta(G) \times \{1, 2, 3\} \times \{1, ..., T\} \to \Delta(G)$$

a (pure-strategy) policy rule. Diermeier and Fong (2007) show that in one-session dynamic legislative bargaining with a discrete distributive policy space, only pure strategies are played in the unique stationary Markov perfect equilibrium. Therefore, in this paper we restrict attention to pure-strategy equilibria.

For any legislator $l \in \{1, 2, 3\}$, let $V_l(\mathbf{x}_{t-1}, t)$ be his expected and discounted sum of utility evaluated in the beginning of period $t+1$ (or at the end of period t) with a status quo $\mathbf{x}_{t-1}$ before an agenda setter is randomly selected. We call

$$V_l : \Delta(G) \times \{1, ..., T\} \to \mathbb{R}$$

a continuation value function. The dynamic payoff function and the continuation value function can be jointly and recursively calculated once the policy rule is known. In particular, for any $\mathbf{x}_{t-1} \in \Delta(G)$,

$$V_l(\mathbf{x}_{t-1}, t) = \begin{cases} \sum\limits_{a_t \in \{1,2,3\}} \frac{1}{3} U_l(g_l(\mathbf{x}_{t-1}, a_t, t), a_t, t+1), & \text{if } t \in \{1, ..., T-1\}, \\ 0, & \text{if } t = T. \end{cases} \tag{2}$$

The continuation value at the end of period T is obviously zero. For any $t \in \{1, ..., T\}$, any $a_t \in \{1, 2, 3\}$ and any $\mathbf{x}_t \in \Delta(G)$,

$$U_l(\mathbf{x}_t, a_t, t) = (1-\delta)\left[(1-\beta) x_{t,l} + \beta V_l(\mathbf{x}_t, t)\right] + \delta U_l(\mathbf{g}(\mathbf{x}_t, a_t, t), a_t, t). \tag{3}$$

We are ready to summarize the equilibrium definition.

Definition 1 *A subgame perfect equilibrium (stationary in each session) of this political system with a finite horizon is a set of dynamic payoff functions* (U_1, U_2, U_3), *a set of continuation value functions* (V_1, V_2, V_3), *and a policy rule* $\mathbf{g}$, *such that:*

1. *Given* (U_1, U_2, U_3), $\mathbf{g}$ *solves maximization problem (1).*

2. *Given* $\mathbf{g}$ *and* V_l, *for all* $l \in \{1, 2, 3\}$, U_l *solves the functional equation (3).*

3. *Given* $\mathbf{g}$ *and* (U_1, U_2, U_3), *for all* $l \in \{1, 2, 3\}$, V_l *is defined by equation (2).*

For the purpose of this paper, we only characterize an equilibrium for δ sufficiently close to 1.

3 Analysis

We solve the model by backward induction. However, deriving the dynamic payoff functions is a nontrivial task; it involves solving a fixed-point problem, which results from stationarity within a session.

3.1 The Last Period

The last period is isomorphic to the one-session model of Diermeier and Fong (2007). The proposition below restates their results relevant for this paper, followed by a discussion of the main implications.

Proposition 1 *For any* δ *sufficiently close to* 1, *there exists a unique legislative equilibrium, in which for any* $\mathbf{x} \in \Delta(G)$ *and any* $a, l \in \{1, 2, 3\}$,

$$g_l(\mathbf{x}, a, T) = \begin{cases} \min\limits_{i \neq a} x_i, & \text{if } l \neq a, \\ G - 2 \min\limits_{i \neq a} x_i, & \text{if } l = a. \end{cases}$$

In equilibrium and for any initial default policy $\mathbf{x}_{T-1}$ *in period* T *such that* $\mathbf{g}(\mathbf{x}_{T-1}, a) \neq \mathbf{x}_{T-1}$, *the agenda setter makes a policy proposal* $\mathbf{g}(\mathbf{x}_{T-1}, a)$ *only in the first round and this proposal is passed.*

The result has some immediate implications.

First, although the agenda setter is allowed to make a policy proposal at any time and as frequently as possible during the legislative session, in equilibrium, there is only one round of proposal making and voting. In an environment without uncertainty, the collective decision is made once and for all without any further modifications.

Second, although it looks as if the legislators played a one-shot legislative bargaining game with closed rule, the possibility of reconsideration changes the nature of the game and makes the equilibrium policy outcome substantially different from what the equilibrium outcome of a static game. Consider the following numerical example.

EXAMPLE 3.1. Assume that $G = 60$, the initial default policy $\mathbf{x}_{T-1} = (30, 20, 10)$, and $a_T = 1$ is the agenda setter. In a static model with closed rule the policy outcome would be $(50, 0, 10)$. Legislator 3 is most disadvantaged by the default policy, and therefore becomes the cheapest coalition partner for the agenda setter. Excluded from the coalition, legislator 2 is fully expropriated since her vote is not needed at all to pass the proposal. The agenda setter leaves legislator 3 just enough benefit to break even and takes the rest of the pie.

In our setup, however, the agenda setter could never pass the policy $(50, 0, 10)$. To see why, consider counter-factually, what would happen if legislator 3 approved the proposal. With probability $1 - \delta$ the legislators would not have be able to revisit the policy issue and therefore $(50, 0, 10)$ would be the final policy outcome. With probability δ, however, the agenda setter would be able to propose a new policy $(60, 0, 0)$, which would be accepted by the fully expropriated legislator 2. This implies that by accepting the policy $(50, 0, 10)$, legislator 3 becomes vulnerable. Foreseeing such an adverse consequence, legislator 3 will always vote against the proposal of $(50, 0, 10)$ even though according to this proposal he does not lose any benefit right away. By similar arguments, we can conclude that legislator 3 will not accept any new policy where legislator 2 receives strictly less benefit than legislator 3. Therefore, legislator 1 as agenda setter can guarantee himself at most 40 and pass the policy of $(40, 10, 10)$. Notice that the possibility to reconsider policies limits agenda control *even in the case where there is a sole agenda setter.* This in marked contrast not only to the Baron-Ferejohn-type models, but also to agenda control models with sincere voting (McKelvey 1976), where an agenda could achieve any point in the policy space, or sophisticated voting (Banks 1980, Shepsle and Weingast 1980) where the set of attainable policies is only limited to the Banks set or the Uncovered Set, respectively.

Note also that our result is very different from the result obtained by Bernheim et al. where the last proposer can capture all or almost all of the benefits. The "power of the last word" disappears once we allow for possible ongoing consideration of policy proposals.

Third, the legislators have indirect preferences over the *distribution* of benefits, although the legislators derive utilities only from the benefits they receive. In Example 3.1, legislator 3 strictly prefers $(40, 10, 10)$ to $(50, 0, 10)$ even though both policies leave him 10 units of benefit. Distribution of benefits matters, because through the evolving default, it affects distribution of bargaining power in the rest of the legislative session.

Fourth, except for the agenda setter, all legislators have preferences towards more egalitarian distribution of benefits. In particular, a legislator without the agenda setting power does not want other legislators to be fully expropriated by the agenda setter. However, this demand for "fairer allocations" result from self-interested legislators who want to improve their long-term bargaining position. It does not depend on primitive preference for fair allocations. In a model of decision-making over legislative procedures this insight may have implications for the existence of minority rights and benefits in legislatures.

Five, as a consequence, the agenda setter has limited ability to expropriate the legislator excluded from his winning coalition. Specifically, the value of proposal power in our model is in general smaller that what is implied by a one-shot legislative bargaining game with closed rule.

Six, depending on the initial default policy and the identity of the agenda setter, the policy outcome can be either full equality, full expropriation by the agenda setter, partial expropriation, or policy inertia. That is, the amount of agenda setting power depends on the session's status quo policy. This is illustrated in the next set of examples.

EXAMPLE 3.2. Suppose that $a_T = 1$. (A) Full equality. Suppose that $\mathbf{x}_{T-1} = \left(w, \frac{1}{3}G, \frac{2}{3}G - w\right)$, where $w \leq \frac{1}{3}G$, then the equilibrium policy outcome is an egalitarian distribution of benefits, $\left(\frac{1}{3}G, \frac{1}{3}G, \frac{1}{3}G\right)$. (B) Full expropriation. Suppose that $\mathbf{x}_{T-1} = (w, 0, G - w)$, then in equilibrium the policy outcome is $(G, 0, 0)$ and the agenda setter captures the entire benefit. (C) Partial expropriation. Suppose that $G = 60$ and $\mathbf{x}_{T-1} = (5, 30, 25)$, then the equilibrium policy is $(10, 25, 25)$. (D) Policy Inertia. Suppose that $\mathbf{x}_{T-1} = (G - 2w, w, w)$ for some $w \in \left(\frac{1}{3}G, \frac{1}{2}G\right]$. Then the agenda setter is not able to change the policy at all.

Seven, in contrast to implications derived from legislative bargaining models in the tradition of Baron and Ferejohn (1989), the agenda setter may *not* be one who receives the most benefit in equilibrium. Indeed, the agenda setter may be the legislator who gets the least amount, as shown in Example 3.2 (C). This, however, happens only if the agenda setter is sufficiently disadvantaged by that session's status quo. This is consistent with episodes in which parties with insufficent representation take control of the government. Possible cases include minority and especially caretaker governments.

Finally, the possibility to reconsider policy may create incentives for agenda-setters to rationally make future proposals that will make him worse off than the current proposal. In this case agenda setters would like to commit to making a proposal only once. The possibility to reconsider policy at any time, however, rules out such commitment. Note that, intuitively, commitment would amount to the credible belief that certain policy areas will never be revisited. It is difficult to see how this can be accomplished in a constitutional fashion in a democracy.

3.2 The Penultimate Period

Since the status strongly impact the equilibrium in each session agents may have an incentive to deviate from a strategy that would be optimal in order to strategically position the status quo for the next and subsequent rounds. This holds, for example, in the model of Kalandrakis (2003). Surprisingly, this is not the case in our model .So see why consider the penultimate period. Now the problem is that a policy choice in period $T-1$ affects not only a legislator's instantaneous utility in that period but also continuation value in period T through the moving status quo. However, we claim that an agenda setter does not strategically manipulate the status quo. Instead, he makes a policy proposal as if this was the last period.

The following captures the intuition for the formal argument.

Consider the case where the legislators bargain over the default policy for period T before a new agenda setter is randomly selected. Suppose that legislator 1 is the period-$T-1$ agenda setter and he has to the power to make a proposal of a default policy with the proposal being put to a vote against the current default, i.e. the policy chosen in the previous period. A simple backward induction calculation shows that in equilibrium the chosen initial default for period T gives the legislator 1 a benefit of $\frac{1}{2}G$, some other legislator (say, legislator 2) also $\frac{1}{2}G$, and the third legislator 0. This default policy $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ leaves both legislators 1 and 2 a unconstrained maximal continuation value of $\frac{1}{2}G$, and therefore once proposed, it would be supported by 1 and 2 in the majority voting.

Suppose then that in period $T-1$ the initial default is $\left(\frac{1}{6}G, \frac{1}{3}G, \frac{1}{2}G\right)$ and keep in mind that legislator 1 is the agenda setter. Could he propose and pass a policy of $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$? Notice that this proposal gives legislator 1 his maximal continuation value in the last period and higher current utility that what the default gives him. However, $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ will never be a policy outcome in period $T-1$. This holds because once a bill of $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ is passed, it also becomes the default for the rest of the legislative session in $T-1$. With probability δ, which is assumed to be close to 1, the agenda setter gets a chance to revisit the policy and then would have a strong incentive change the default policy. That is, with $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ as the default, legislator 1 as agenda setter would now propose a policy of $(G, 0, 0)$, which would be supported by legislator 3, and thus pass.Therefore the proposal of $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ would not be approved by voter 2.

More formally, if $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ is the policy outcome in period $T-1$, legislator 1 has a continuation value of $\frac{1}{2}G$ in the final period. Therefore, in the last two periods legislator 1 receives a total utility of

$$\left(\tfrac{1}{2}G\right) + \beta\left(\tfrac{1}{2}G\right) = (1+\beta)\left(\tfrac{1}{2}G\right).$$

If $(G, 0, 0)$ is the policy outcome in period $T-1$, legislator 1 has a continuation value of $\frac{1}{3}G$ in the final period. Therefore, in the last two periods legislator 1 receives a total utility of

$$(1-\beta)\left[G + \beta\left(\tfrac{1}{3}G\right)\right] = (1-\beta)\left(1+\tfrac{1}{3}\beta\right)G.$$

For any $\beta \in [0, 1)$, the policy $(G, 0, 0)$ gives legislator 1 a larger total utility in the last two periods. This implies that $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$ cannot be the final policy outome in period $T-1$. Foreseeing that legislator 1 as agenda setter will not finalize the policy choice at $\left(\frac{1}{2}G, \frac{1}{2}G, 0\right)$, legislator 2 will not accept this proposal. This eliminates strategic manipulation of the status quo by any agenda setter for T. More generally, it is straightforward to show that in period $T-1$ an agenda setter makes the same policy proposal as in a one-sesseion game. Repeating this argument by backward inductions yields the next proposition which is formally proved in the appendix.

Proposition 2 *Suppose that T is finite. For any δ sufficiently close to* 1, *there exists a unique symmetric subgame perfect equilibrium, in which for any $t \in \{1, ..., T\}$, any $g^{t-1} \in \Delta(G)$, and any $a, l \in \{1, 2, 3\}$,*

$$f_l(g, a, t) = \begin{cases} \min\limits_{i \neq a} g_i, & \textit{if } l \neq a, \\ 1 - 2\min\limits_{i \neq a} g_i, & \textit{if } l = a. \end{cases}$$

3.3 The Infinite-Horizon Equilibrium

Again, we restrict our analysis to cases in which the agenda setter conditions his proposal strategy only on the prevailing default. Consider an arbitrary round of negotiation in an arbitrary session with agenda setter a and default g. Let $U_l^*(\mathbf{x}, a)$ be legislator l's dynamic payoff function with policy $\mathbf{x}$. This corresponds to the expected utility of legislator l if policy $\mathbf{x}$ is passed in this round of negotiation in the current session. Let $V_l^*(\mathbf{x})$ be legislator l's continuation value with a status quo policy $\mathbf{x}$ at the beginning of the period before an agenda setter is randomly selected. Again we model voting by an incentive compatibility constraint and only explicity formulate the proposal strategy of an agenda setter. Let $g^* : \Delta(G) \times \{1,2,3\} \to \Delta(G)$ be the policy rule. Given stationarity, we can drop the time variable from the functions.

We are now ready to define the equilibrium in the infinite-horizon model.

Definition 2 *A subgame perfect equilibrium of this political system with a finite horizon is a set of dynamic payoff functions (U_1^*, U_2^*, U_3^*), a set of continuation value functions (V_1^*, V_2^*, V_3^*), and a policy rule $\mathbf{g}^*$, such that:*

1. *Given (U_1^*, U_2^*, U_3^*), $\mathbf{g}^*$ solves the following maximization problem:*

$$\begin{aligned} \max_{\mathbf{x}' \in \Delta(G)} \quad & U_a^*(\mathbf{x}', a) \\ s.t. \quad & U_i^*(\mathbf{x}', a) \geq U_i^*(\mathbf{d}, a) \text{ for some } i \neq a. \end{aligned}$$

2. *Given $\mathbf{g}^*$ and V_l^*, for all $l \in \{1,2,3\}$, U_l^* solves the following functional equation:*

$$U_l^*(\mathbf{x}, a) = (1-\delta)\left[(1-\beta)x_l + \beta V_l^*(\mathbf{x})\right] + \delta U_l^*(\mathbf{g}^*(\mathbf{x}, a), a).$$

3. *Given $\mathbf{g}^*$ and (U_1^*, U_2^*, U_3^*), for all $l \in \{1,2,3\}$, V_l^* is defined by equation*

$$V_l^*(\mathbf{x}) = \sum_{a_t \in \{1,2,3\}} \tfrac{1}{3} U_l^*(g_l^*(\mathbf{x}, a), a).$$

Condition 1 says that any agenda setter proposes a policy to maximize his own dynamic payoff subject to the constraint that this proposal can be approved by at least one of the other legislators. Condition 2 defines dynamic payoff functions. Due to stationarity within a session, which here means that an agenda setter behaves in the same way when facing identical defaults, this condition lays out a fixed point problem for the dynamic payoff functions. The last condition states that continuation values are expected dynamic payoffs in every period before an agenda setter is randomly selected.

Similar to the finite-horizon game, the existence of a legislative equilibrium can be easily established for any δ. However, for the purpose of the paper, we only characterize the equilibrium for δ sufficiently close to 1. The next proposition presents an infinite-horizon equilibrium that is the limit of finite-horizon subgame perfect equilibria as time horizon T goes to infinity. In this equilibrium, an agenda setter does not strategic manipulate the status quo and his strategy does not depend on how the legislators negotiate over the policy in the past.

Proposition 3 *Consider the infinite horizon model. For any δ sufficiently close to* 1, *there exists a stationary Markov perfect equilibrium, in which for any $\mathbf{x} \in \Delta(G)$, and any $a, l \in \{1, 2, 3\}$,*

$$g_l(\mathbf{x}, a) = \begin{cases} \min\limits_{i \neq a} x_i, & \text{if } l \neq a, \\ G - 2\min\limits_{i \neq a} x_i, & \text{if } l = a. \end{cases}$$

Along this equilibrium path, policy dynamics in the long run depends on the initial status quo policy $\mathbf{x}_0$ and identity of the first agenda setter a_1. If $x_{a_1,0} \geq \min_{i \neq a_1} x_{i,0}$ or $x_{a_1,0} < \min_{i \neq a_1} x_{i,0} \leq \frac{1}{3}$, in any period, the agenda setter receives $G - 2\min_{i \neq a_1} x_{i,0}$, and each of the other legislators receives $\min_{i \neq a_1} x_{i,0}$. If $\max\left\{x_{a_1,0}, \frac{1}{3}\right\} < \min_{i \neq a_1} x_{i,0}$, then until any $i \neq a_1$ obtains power for the first time, in every period a_1 as agenda setter gets $G - 2\min_{i \neq a_1} x_{i,0}$, which is strictly less than $\min_{i \neq a_1} x_{i,0}$, what each of the other legislators obtains; after the first time the initial agenda setter loses power, in every period the agenda setter $4\min_{i \neq a_1} x_{i,0} - G$, which is strictly greather than $G - 2\min_{i \neq a_1} x_{i,0}$, what each of the other legislators receives.

3.4 Discussion

First, as in the one-shot game in equilbrium each period agenda setters immediately propose a new policy, even though the game form would allow him to propose new policies as long as the period lasts. Without any uncertainty other than stochastic turnover of power, the policy does not change as long as the agenda setter does not change. With frequent possibilities of reconsideration, the equilibrium policy outcome is substantially different from a setup in which every agenda setter are *constrained* to make a new policy only once as in, for example, the model of Kalandrakis (2004).

Second, if an agenda setter holds power for consecutive periods, the policy is stable over the consecutive periods; an agenda setter does not change the policy in his second term. This is because both legislators without proposal power are given the same size of benefit and there is no room for the agenda setter to further expropriate any of them in the second term. This result is in contrast to policy dynamics in some dynamic legislative bargaining models, for example, Fong (2006) and Baron et al (2007). In Kalandrakis (2004), during transitional periods when an agenda setter is not able to expropriate all benefits, he proposed different policies and switched winning coalitions from one period to the other when obtaining proposal power for two consecutive periods. In other words, the respective models have competing implications on the dynamics of policy choice that can be empirically investigated.

Third, note that the policy an agenda setter chooses in a multiperiod setup is equal to what he would choose in a one-session setup. There is no manipulation of the status quo by any agenda setter.

Fourth, the model implies very rich policy dynamics. Depending on the initial status quo policy and the identity of the first agenda setter, in the long run there are three different patterns of policy dynamics.

1. Full Expropriation by Any Agenda Setter

After finite periods of transition, whoever is the agenda setter takes all and leaves nothing to the others. Illustrated by Example 4.1, such policy dynamics is reminiscent of Kalandrakis (2004). However, in our model, full expropriation happens only if the initial status quo is so unequal that some legislator originally gets nothing.

Example 4.1. $G = 60$, $g^0 = (30, 30, 0)$

time period	first	second	third	fourth	...
agenda setter	3	3	1	2	...
policy outcome	$(30, 30, 0)$	$(30, 30, 0)$	$(60, 0, 0)$	$(0, 60, 0)$	...

2. Inequality-Inclined Allocation of Benefit

After finite periods of transition, in every period the agenda setter receives more benefits than the others, although he does not capture all. An agenda setter therefore has limited ability to expropriate the legislator excluded from his winning coalition. The legislators without proposal power receive equal shares. This is illustrated by Examples 4.2 and 4.3.

Example 4.2. $G = 60$, $g^0 = (25, 25, 10)$					
time period	first	second	third	fourth	...
agenda setter	3	3	1	2	...
policy outcome	$(25, 25, 10)$	$(25, 25, 10)$	$(40, 10, 10)$	$(10, 40, 10)$	...

Example 4.3. $G = 60$, $g^0 = (30, 20, 10)$					
time period	first	second	third	fourth	...
agenda setter	2	3	1	2	...
policy outcome	$(10, 40, 10)$	$(10, 10, 40)$	$(40, 10, 10)$	$(10, 40, 10)$	...

3. Full Egalitarian Distribution of Benefits

The policy converges to a fully egalitarian distribution of benefits right away, and thereafter no agenda setter is able to change it anymore. All agenda setters, except for the first one, do not benefit from being proposers. This is illustrated by Example 4.4.

Example 4.4. $G = 60$, $g^0 = (30, 20, 10)$					
time period	first	second	third	fourth	...
agenda setter	3	2	1	2	...
policy outcome	$(20, 20, 20)$	$(20, 20, 20)$	$(20, 20, 20)$	$(20, 20, 20)$	...

Finally, as shown in Examples 4.3 and 4.4 with the same initial status quo, policy dynamics in the long run is path-dependent. In particular, it not only depends on the initial status qup, but also who obtains power in the first period. This model thus suggests how political players in an early stage of democracy and the initial distribution of benefits may affect the long-run pattern of fiscal policy.

4 Application: The Ratchet Effect of Government Spending

Next we investigate the model's implications in the context of economic fluctuations. As we show below, the model's equilibrium provides a possible explanation for the "ratchet effect" of public spending in parliamentary democracies (Persson and Tabellini 2004).

4.1 Stylized Facts and General Ideas

Recent empirical studies by Persson and Tabellini (2001, 2003, and 2004) showed that, in parliamentary countries with proportional representation, government spending as a fraction of GDP increases during cyclical downturns but does not come down during cyclical upturns, whereas this "ratchet effect" is not apparent in countries with other constitutional arrangements. This section presents a simple political economy model based on the analytical framework developed in Section 2 and shows how the legislative institutions typical of parliamentary democracies may lead to the asymmetric movements of government spending.

Specifically, Persson and Tabellini divided democratic countries into four constitutional groups, and empirically investigate how different constitutional arrangements shape fiscal policies. They show that proportional-parliamentary democracies differ from all other groups – majoritarian-presidential, majoritarian-parliamentary, and proportional-presidential – in terms of fiscal policy dynamics.

First, government expenditure, fiscal deficit and welfare spending are more persistent in this group than in the others. Second, spending as a percentage of GDP increases in cyclical down-turns but does not decrease in booms. In other words, downturns lead to a lasting expansion of outlays and welfare spending in proportion to GDP that is not reversed during upturns. Third, the difference in the size of government between this group and the others grew particularly large in the period up to the early 1980s (or the early 1990s in the case of welfare spending).

What could account for the special status of proportional-parliamentary democracies? Proportional representation leads to minority parliaments; i.e. no party obtains a majority of seats in parliament. This is true even if voters can vote strategically (Baron and Diermeier 2001) and if governments can strategically manipulate future status quos (Fong 2006, Baron, Diermeier, and Fong 2007). Therefore government policy needs to be conceptualized as bargaining among multiple parties, either among *all* parties represented in the parliament or among the parties represented in the governing coalition (Diermeier and Feddersen 1998). Note that this feature is absent in all other constitutional groups. For example, parliamentary democracies with plurality rule (e.g. the UK) usually lead to two major political parties. Except for the rare case of a hung parliament, the party who controls a majority of seats usually has full control over policy. One the other hand, presidential democracies (whether multi-party or two-party) lack the constitutional feature of effective agenda control by the executive. So, our model combines the features typical of parliamentary democracies (the government's agenda control) with multi-party bargaining typical of proportional representation. In the context of a simple model we show below how the friction resulting from multilateral bargaining is the key to explaining the ratchet effect.

The intuition is as follows. As discussed above a sizable fraction of total government expenditure is related to continuing entitlement programs. In those programs benefits are distributed and once enacted, they are in effect until they are reformed in subsequent legislative sessions. When an economy is hit by a temporary negative income shock, the party that controls agenda setting faces a strong resistance on expenditure cuts. This is because a more stringent entitlement program on any socioeconomic group implies a worse status quo in the future and therefore a permanently lower bargaining power of that group or party. Fiscal adjustment in response to a temporary shock has a permanent effect. This makes it extremely difficult for an agenda setter, whose power is persistent and may last for a certain amount of time, to cut down expenditures on the other groups. On the other hand, with a temporary positive income shock, the leading party can easily satisfy its coalition partners by their reservation values and pass a more generous entitlement program to benefit the socioeconomic group it represents. An asymmetric, upward, movement of public spending thus results.

4.2 A Simple Model: Impulse Response Analysis

In this section we present a dynamic model similar to that in Section 4 with one important difference. Now the total size of benefits, or the total government expenditure, is endogenous. Imagine that every period the three legislators divide a pie but the size of the pie is not fixed. They have to jointly produce a total benefit of $G \in \mathbb{N}$ and then divide it. A policy is therefore a vector (x_1, x_2, x_3, G), where $G = \sum_{l=1}^{3} x_l$. Public production is costly and parties equally share the cost.[3] The cost function depends on the state of the economy denoted by s. For simplicity, we assume that the cost function is piecewise linear:

$$C(G, s) = \begin{cases} 0, & \text{if } G \leq \overline{G}_s, \\ (1+c)\left(G - \overline{G}_s\right), & \text{if } G > \overline{G}_s, \end{cases}$$

for some $c > 0$. Assume that $s \in \{H, N, L\}$, where H stands for high, N for normal, and L for low. In good states, the marginal cost is smaller; In bad states, the marginal cost is larger. Therefore we assume that $\overline{G}_H > \overline{G}_N > \overline{G}_L$. We do not explicitly model income shocks. However, we conjecture that in this simple framework the effect of a negative public production shock should be similar to that of a negative income shock in a fully specified public finance model.

The preference of legislator l is represented by

$$E\left[\sum_{t=0}^{\infty} \beta^t \left(x_{l,t} - \tfrac{1}{3} C(G_t, s_t)\right)\right],$$

where $\beta \in [0, 1)$ is a common discount factor as before.

As a normative benchmark, in the first best solution, every period the total size of government expenditure is equal to $\overline{G}_s$; public spending is fully responsive to state of the economy. In our model, the policy is chosen by the political process of legislative bargaining. We want to show how the equilibrium policy deviates from the first best solution and identify a possible source of bargaining frictions.

[3] The model does not consider endogeous taxation or borrowing.

Instead of fully characterizing this model with a general stochastic process that governs transition of states, we conduct an impulse response experiment. We assume that for every period, the state is stable and normal, but that in period 1, the economy is hit by an unexpected temporary shock; $s_1 \in \{H, L\}$. The exact interpretation of this shock is not critical. What really matters is that the economy temporarily deviates from its long-run trend; we now want to investigate how this fluctuation results in fluctuation of government spending. Moreover, we assume that power is persistent and legislator 1 is the agenda setter for every period. To obtain our result we only require some degree of power persistence. That is, the party who will control agenda setting in the next quarter is very likely to be the party currently in power. In what follows, we divide the discussion into cases with a positive shock, $s_1 = H$, and a negative shock, $s_1 = H$. We assume that the initial status quo policy is such that the total spending $G_0 = \overline{G}_N$ and $g_{2,0} \geq g_{3,0}$.

Case I: A Temporary Positive Shock.

In period 1 with $s_1 = H$, it is as if the agenda setter chooses a policy $(x_{1,1}, x_{2,1}, x_{3,1}, G^1) \in \mathbb{Z}_+^4$ in order to maximize

$$(1-\beta)\left(x_{1,1} - \frac{1}{3}(1+c)\max\left\{0, G_1 - \overline{G}_H\right\}\right) + \beta\left(\overline{G}_N - 2x_{3,1}\right),$$

subject to

$$x_{1,1} = G^1 - x_{2,1} - x_{3,1},$$

$$(1-\beta)\left(x_{3,1} - \tfrac{1}{3}(1+c)\max\left\{0, G^1 - \overline{G}_H\right\}\right) + \beta x_{3,1} \geq (1-\beta)\, x_{3,0} + \beta x_{3,0},$$

and

$$\begin{aligned} & (1-\beta)\left(x_{3,1} - \frac{1}{3}(1+c)\max\left\{0, G_1 - \overline{G}_H\right\}\right) + \beta x_{3,1} \\ = \; & (1-\beta)\left(x_{2,1} - \frac{1}{3}(1+c)\max\left\{0, G_1 - \overline{G}_H\right\}\right) + \beta x_{2,1}. \end{aligned}$$

This equivalent maximization problem needs some explanation. Given any period-one policy choice $x_1 = (x_{1,1}, x_{2,1}, x_{3,1})$, the continuation values of all three legislators from the second period are $\overline{G}_N - 2x_{3,1},\ x_{3,1},\ x_{3,1}$ respectively. The first constraint is the resource constraint (or balanced budget constraint). According to what we learned from Sections 2 to 4, the agenda setter makes a policy proposal that satisfies one of the other legislators with the lowest reservation value. In our example, this is legislator 3 and this explains the second constraint. Finally, in order for legislator 3 to accept the proposal, it has to be the case that legislator 2 does not receive a lower utility level than him. Otherwise, in the rest of the session in period 1, the agenda setter would revisit the policy and propose another policy that seeks support from legislator 2. The last constraint summarizes this equal-utility property, which is equivalent to the constraint that $x_{2,1} = x_{3,1}$.

In equilibrium, (A) $x_{2,t} = x_{3,t} = \min\{x_{2,0}, x_{3,0}\}$ for all t, (B) $G_1 = \overline{G}_H$ and $G_t = \overline{G}_N$ for all $t > 1$, and (C) $x_{1,1} = \overline{G}_H - 2\min\{x_{2,0}, x_{3,0}\}$ and $g_{1,t} = \overline{G}_N - 2\min\{x_{2,0}, x_{3,0}\}$.

With a temporary positive shock, total expenditure expands accordingly, and all extra spending goes to the party that controls the agenda. The legislators without power do not benefit from the positive shock. After the shock, total spending is back to its normal level.

Case II: A Temporary Negative Shock.

In period 1 with $s_1 = L$, it is as if the agenda setter chooses a policy $(x_{1,1}, x_{2,1}, x_{3,1}, G_1) \in \mathbb{Z}_+^4$ in order to maximize

$$(1-\beta)\left(g_{1,1} - \tfrac{1}{3}(1+c)\max\left\{0, G_1 - \overline{G}_L\right\}\right) + \beta\left(\overline{G}_N - 2x_{3,1}\right),$$

subject to

$$x_{1,1} = G_1 - x_{2,1} - x_{3,1},$$

$$(1-\beta)\left[x_{3,1} - \tfrac{1}{3}(1+c)\max\left\{0, G_1 - \overline{G}_L\right\}\right] + \beta x_{3,1} \geq (1-\beta)\left[x_{3,0} - \tfrac{1}{3}(1+c)\left(\overline{G}_N - \overline{G}_L\right)\right] + \beta x_{3,0},$$

and

$$\begin{aligned} & (1-\beta)\left(x_{3,1} - \tfrac{1}{3}(1+c)\max\left\{0, G_1 - \overline{G}_L\right\}\right) + \beta x_{3,1} \\ = \; & (1-\beta)\left(x_{2,1} - \tfrac{1}{3}(1+c)\max\left\{0, G_1 - \overline{G}_L\right\}\right) + \beta x_{2,1}. \end{aligned}$$

The second constraint guarantees that legislator 3, the "cheaper" possible coalition partner, is indifferent to the new period-1 policy. The last constraint requires that legislator 2, the one excluded from the winning coalition, is offered the same total utility as legislator 3. This is equivalent to the constraint that $x_{2,1} = x_{3,1}$. Combining the last two conditions we have

$$x_{2,1} = x_{3,1} = \widehat{x} \equiv x_{3,0} + \tfrac{1}{3}(1-\beta)(1+c)\max\left\{0, G_1 - \overline{G}_N\right\}.$$

Substituting the binding constraints, the agenda setter's objective function is simplified to

$$\begin{aligned}&(1-\beta)\left\{G_1 - 2\left[x_{3,0} - \tfrac{1}{3}(1-\beta)(1+c)\max\left\{0, G_1 - \overline{G}_N\right\}\right] - \tfrac{1}{3}(1+c)\max\left\{0, G_1 - \overline{G}_L\right\}\right\} \\ &+\beta\left\{\overline{G}_N - 2\left[x_{3,0} - \tfrac{1}{3}(1-\beta)(1+c)\max\left\{0, G_1 - \overline{G}_N\right\}\right]\right\}.\end{aligned}$$

There is only one unknown G_1 in the objective function. It can be verified that every $G_1 > G_N$ makes the agenda setter worse off than $G_1 = G_N$, so the equilibrium period-1 total spending must be no greater than $\overline{G}_N$. Moreover, $G_1 \geq \overline{G}_L$. Given these, the objective function is simplified to

$$-(1-\beta)\,c\,G_1 + \textit{constant}.$$

Therefore, the agenda setter chooses G_1 as small as possible given that that $G_1 \geq \overline{G}_L$ and the incentive constraints for legislator 3 to accept the policy.

In equilibrium, if $2\widehat{g} \leq \overline{G}_L$, i.e., $\overline{G}_L$ is sufficiently large, we have $G_1 = \overline{G}_L$. In this case, $x_{2,1} = x_{3,1} = \widehat{x}$ and $x_{1,1} = \overline{G}_L - 2\widehat{g}$. If the negative shock is not too bad, total spending is fully adjusted downward to its socially optimal level. The spending on the legislators without power are only slightly cut down. With reasonable parameter values, $x_{2,1}$ and $x_{3,1}$ are very close to $\min\{x_{2,0}, x_{3,0}\}$. Most of the fiscal adjustment is done by cutting the spending that benefits the agenda setter.

In equilibrium, if $2\widehat{g} > \overline{G}_L$, i.e., $\overline{G}_L$ is sufficiently small, we have $G_1 = 2\widehat{g} > \overline{G}_L$. In this case, $x_{2,1} = x_{3,1} = \widehat{x}$ and $x_{1,1} = 0$. Now, if the negative shock is very severe, total spending cannot be fully adjusted downward to its socially optimal level. There is overspending in the bad state. The spending on the legislators without power are only slightly cut down, whereas the spending on the agenda setter is totally cut down to zero. The fact that the agenda setter is receiving zero benefit in period

1 should not be interpreted literally. If the agenda setter's marginal utility of his benefit becomes sufficiently large as spending on him is cut, in equilibrium, $x_{1,1}$ may be strictly positive even if the bad shock is very severe.

This simple model with an impulse analysis yields various testable empirical implications. Importantly it generates a version of the ratchet effect of total spending. As the economy is hit by an unexpected temporary positive shock, the total spending expands and all extra spending benefits the agenda setter. As the economy is hit by an unexpected temporary negative shock, the total spending may not be fully downward adjusted. Whether it does depends on the size of the negative shock. The intuition is that the agenda setter has difficulty cutting spending on the other legislators; he can mainly cut down his own benefit. If the negative shock is small, the agenda setter is able to do so and adjust total spending to its new socially optimal value. However, if the negative shock is sufficiently large, the agenda setter leaves zero benefit to himself and at this corner solution, he is not able to further reduce public spending. As a consequence, there is overspending compared to the first best solution.

Our model also raises some new empirical questions. First, in the empirical studies Persson and Tabellini identify the ratchet effect in parliamentary democracies with proportional representation. Our intuition suggests that what matters is the form of governance that makes fiscal policy decisions, here whether or not the fiscal policy is a bargaining outcome by multiple parties with a sole agenda setter in each legislative session. To test this tuition, we could possibly look at fiscal policy dynamics in those countries during different regimes: regimes with a majority party, and regimes with a minority parliament and coalition governments. We conjecture that the ratchet effect is more prevalent when fiscal policy is determined by a coalition government. Second, ratchet effects are particularly pronounced for large negative shocks. Third, it is worth analyzing the composition of government spending. In particular, over the business cycles, how does spending on different items fluctuate? Our model implies that in a bad state, the agenda setter's spending is cut down the most. Whether these predictions are observed in the data is an open question.

5 Concluding Remarks

This paper proposes a new analytical framework of legislative bargaining. The setup, we believe, captures important features of legislative decision-making such as endogenous defaulty, the possibility of reconsideration without a well-defined last round of decision-making and persistent agenda setting power. The model is tractable and can be easily applied to dynamic models where it can a positive analysis of fiscal policy.

An immediate next step of this research agenda is to extend the core model to include an arbitrary number of legislators with any decision rule and fully characterize all legislative equilibria. We think the same intuitions carry over to a more general context, including the key insight that an agenda setter has limited ability to expropriate legislators excluded from the winning coalition if he holds the power to reconsider the policy issue at some later dates. It also would be interesting to investigate how the value of agenda setting and distribution of benefits are affected by the size of the legislature and voting rules.

Another important extension of the core model is to replace the agenda setter by a gatekeeper. We define a gatekeeper as the legislator who is conferred the veto right to block any policy proposal made by some others and at the same time able to propose a new policy in some situations. The sequence of events in the game can be modified as follows: There is an initial default and one legislator is randomly assigned to be a gatekeeper. The legislators then are able to make policy proposals in turn. A legislator can choose to pass his turn if proposing a policy does not make himself better off. Once a proposal is made, it has to be approved by the gatekeeper and then voted on against the default by majority rule. A passed proposal becomes the new default in future rounds of negotiation. Legislative interaction ceases after all legislators pass. The final default policy is implemented. In a model like this, we would be able to compare the respective value of proposal and veto power.

Finally, this analytical framework could be incorporated into fuller developed models of public finance and macroeconomic policy choice. As recent empirical studies on political economy and comparative constitutions have established more stylized facts and raise new questions about how political institutions shape dynamics of policy, we expect fruitful insights from such an approach.

Appendix

Proof of Proposition 2. We prove the proposition by induction on T. By Diermeier and Fong (2007), this proposition is true for the last period. Suppose that the proposition is true for all of the last n periods, where $n \in \mathbb{N}$ and $1 \leq n \leq T-1$. We want to show that the proposition is also true for period $T-n$.

We prove the proposition for period $T-n$ by a standard technique of "conjecture and verification", and break the argument into three steps. We first construct the continuation value functions by the proved equilibrium policy rules in all subsequent periods and conjecture the equilibrium policy rule in the current period. Given this conjecture, we then construct dynamic payoff functions that satisfy the functional equations. Finally, we show that the conjectured policy rule solves the maximization problem with all the constructed dynamic payoff functions. We use a hat to denote any function that is either conjectured or constructed based on conjecture.

Step 1. Calculating continuation value functions.

Without loss of generality, take any $\mathbf{x} \in \Delta(G)$ such that $x_1 \geq x_2 \geq x_3$. The patterns of continuation value functions depend on the relative size of x_2. Consider the two cases below.

CASE 1. $x_2 \leq \frac{1}{3}G$.

In period $T-n+1$, legislator 1 as agenda setter would propose $(G-2x_3, x_3, x_3)$, legislator 2 as agenda setter would propose $(x_3, G-2x_3, x_3)$, and legislator 3 as agenda setter would propose $(x_2, x_2, G-2x_2)$. The expected per period utility of legislator 1 or 2 in period $T-n+1$ is therefore,

$$\tfrac{1}{3}\left[(G-2x_3)+x_3+x_2\right] = \tfrac{1}{3}(1-\beta)(G+x_2-x_3),$$

and that of legislator 3 is

$$\tfrac{1}{3}\left[x_3+x_3+(G-2x_2)\right] = \tfrac{1}{3}(1-\beta)\left[G-2(x_2-x_3)\right].$$

In period $T-n+2$, the status quo could be a permutation of $(G-2x_3, x_3, x_3)$ or $(x_2, x_2, G-2x_2)$. Since $x_3 \leq x_2 \leq \frac{1}{3}G$, we have $x_2 \leq G-2x_2$ and $x_3 \leq G-2x_3$. If any permutation of $(G-2x_3, x_3, x_3)$, according to the new equilibrium policy, any legislator as agenda setter gets $G-2x_3$, and each of the other legislators gets x_3. The expected per period utility of any legislator is therefore $\frac{1}{3}G$. Similarly, If any permutation of $(x_2, x_2, G-2x_2)$, according to the new equilibrium policy, any legislator as agenda setter gets $G-2x_2$, and each of the other legislators gets x_2. The expected per period utility of any legislator is there $\frac{1}{3}G$. It is then straightforward to establish that in all subsequent periods, if any, the expected per period utility of any legislator equals $\frac{1}{3}G$. Given this, we calculate the continuation value of any legislator in the beginning of period $T-n+1$ (or at the end of period $T-n$) as follows:

$$\begin{aligned}
\widehat{V}_1(\mathbf{x}, T-n) &= \widehat{V}_2(\mathbf{x}, T-n) \\
&= \tfrac{1}{3}(1-\beta)(G+x_2-x_3) + \sum_{\tau=1}^{n-1}\beta^\tau(1-\beta)\left(\tfrac{1}{3}G\right) \\
&= (1-\beta^n)\left(\tfrac{1}{3}G\right) + \tfrac{1}{3}(1-\beta)(x_2-x_3),
\end{aligned}$$

and

$$\begin{aligned}
\widehat{V}_3(\mathbf{x}, T-n) &= \tfrac{1}{3}(1-\beta)[G-2(x_2-x_3)] + \sum_{\tau=1}^{n-1}\beta^\tau(1-\beta)\left(\tfrac{1}{3}G\right) \\
&= (1-\beta^n)\left(\tfrac{1}{3}G\right) - \tfrac{2}{3}(1-\beta)(x_2-x_3)
\end{aligned}$$

CASE 2. $x_2 > \frac{1}{3}G$.

Again, the expected per period utility of legislator 1 or 2 in period $T-n+1$ is $\frac{1}{3}(G+x_2-x_3)$, and that of legislator 3 is $\frac{1}{3}[G-2(x_2-x_3)]$. The expected per period utilities in all subsequent periods, however, depend on the realized sequence of agenda setters. In period $T-n+2$, the status quo could be a permutation of $(G-2x_3, x_3, x_3)$ or $(x_2, x_2, G-2x_2)$. Since $x_3 = \min_{i\in\{1,2,3\}} x_i$, we have $x_3 \leq G-2x_3$. However, since $x_2 > \frac{1}{2}G$, we have $x_2 < G-2x_2$. If either legislator 1 or 2 is the agenda setter in period $T-n+1$ so that the period $T-n+2$ status quo is a permutation of $(G-2x_3, x_3, x_3)$, according to the new equilibrium policy, any legislator as agenda setter gets $G-2x_3$, and each of the other legislators gets x_3. The expected per period utility of any legislator is therefore $\frac{1}{3}G$. However, if legislator 3 is the agenda setter in period $T-n+1$ so that the period $T-n+2$ status quo is $(x_2, x_2, G-2x_2)$, legislator 1 or 2 as agenda setter would get $G-2(G-2x_2) = 4x_2-G$ and leave $G-2x_2$ to each of the others, whereas legislator 3 as agenda setter would not be able to change the policy. Therefore, the expected per period utility of legislator 1 or 2 is there $x_2 > \frac{1}{3}G$ and that of legislator 3 is $G - 2x_2 < \frac{1}{3}G$. It is then straightforward to establish the fact that in all periods after period $T - n + 1$, as long as legislator 3 keeps serving as an agenda setter, the expected per period utility of legislator 1 or 2 is x_2 and that of legislator 3 is $G - 2x_2$. Otherwise, the expected per period utility of any legislator is $\frac{1}{3}G$. Given this, we calculate the continuation value of any legislator in the beginning of period $T - n + 1$ (or at the end of period $T - n$) as follows:

$$\begin{aligned} V_1(\mathbf{x}, T-n) &= V_2(\mathbf{x}, T-n) \\ &= \tfrac{1}{3}(1-\beta)(G + x_2 - x_3) + \sum_{\tau=1}^{n-1} \beta^\tau (1-\beta) \left[\left(\tfrac{1}{3}\right)^\tau x_2 + \left(1 - \left(\tfrac{1}{3}\right)^\tau\right)\left(\tfrac{1}{3}G\right)\right] \\ &= (1-\beta^n)\left(\tfrac{1}{3}G\right) + \tfrac{1}{3}(1-\beta)(x_2 - x_3) + \tfrac{1}{3}\beta\left(1-\beta^{n-2}\right)\left(x_2 - \tfrac{1}{3}G\right), \end{aligned}$$

and

$$\begin{aligned} V_3(\mathbf{x}, T-n) &= \tfrac{1}{3}(1-\beta)\left[G - 2(x_2 - x_3)\right] \\ &\quad + \sum_{\tau=1}^{n-1} \beta^\tau (1-\beta)\left[\left(\tfrac{1}{3}\right)^\tau (G - 2x_2) + \left(1 - \left(\tfrac{1}{3}\right)^\tau\right)\left(\tfrac{1}{3}G\right)\right] \\ &= (1-\beta^n)\left(\tfrac{1}{3}G\right) - \tfrac{2}{3}\left((1-\beta)x_2 - x_3\right) - \tfrac{2}{3}\beta\left(1-\beta^{n-2}\right)\left(x_2 - \tfrac{1}{3}G\right). \end{aligned}$$

Step 2. Conjecturing a policy rule.

We guess that for any $\mathbf{x} \in \Delta(G)$ and any $a \in \{1, 2, 3\}$,

$$\widehat{g}_l(\mathbf{x}, a, T-n) = \begin{cases} \min\limits_{i \neq a} x_i, & \text{if } l \neq a, \\ G - 2\min\limits_{i \neq a} x_i, & \text{if } l = a. \end{cases}$$

Observe that by this conjecture, for any $\mathbf{x} \in \Delta(G)$ and any $x \in \{1, 2, 3\}$,

$$\widehat{\mathbf{g}}\left(\widehat{\mathbf{g}}(\mathbf{x}, a, T-n), a, T-n\right) = \widehat{\mathbf{g}}(\mathbf{x}, a, T-n). \tag{4}$$

This implies that if this is an equilibrium policy rule, in equilibrium, there is at most one round of negotiation in any period. Any incentive compatible policy proposal is stable in a way that the agenda setter would not want to change it anymore should there be chances of reconsideration in the rest of the session.

Step 3. Constructing dynamic payoff functions over equilibrium policy outcomes.

Take any $\mathbf{x} \in \Delta(G)$ and any $a \in \{1,2,3\}$. By equations (2) and (4),

$$\begin{aligned}
\widehat{U}_l\left(\widehat{\mathbf{g}}\left(\mathbf{x},a,T-n\right),a,T-n\right) &= (1-\delta)\left[(1-\beta)\,\widehat{g}_l\left(\mathbf{x},a,T-n\right)+\beta\widehat{V}_l\left(\mathbf{x},T-n\right)\right] \\
&\quad +\delta\widehat{U}_l\left(\widehat{\mathbf{g}}\left(\widehat{\mathbf{g}}\left(\mathbf{x},a,T-n\right),a,T-n\right),a,t\right) \\
&= (1-\delta)\left[(1-\beta)\,\widehat{g}_l\left(\mathbf{x},a,T-n\right)+\beta\widehat{V}_l\left(\mathbf{x},T-n\right)\right] \\
&\quad +\delta\widehat{U}_l\left(\widehat{\mathbf{g}}\left(\mathbf{x},a,T-n\right),a,t\right),
\end{aligned}$$

which implies that

$$\widehat{U}_l\left(\widehat{\mathbf{g}}\left(\mathbf{x},a,T-n\right),a,T-n\right)=\begin{cases}(1-\beta)\min\limits_{i\neq a}x_i+\beta\widehat{V}_l\left(\mathbf{x},T-n\right), & \text{if } l\neq a,\\ (1-\beta)\left(G-2\min\limits_{i\neq a}x_i\right)+\beta\widehat{V}_l\left(\mathbf{x},T-n\right), & \text{if } l=a.\end{cases}$$

Step 4. Constructing dynamic payoff functions over all policy alternatives.

Without loss of generality, take any $\mathbf{x} \in \Delta(G)$ such that $x_1 \geq x_2 \geq x_3$. The patterns of dynamic payoff functions depend on the relative size of x_2 as the continuation value functions do in Step 1. We discuss the two cases below.

CASE 1. $x_2 \leq \frac{1}{3}G$.

For any $l \in \{1,2\}$, if $a \neq l$, then

$$\begin{aligned}
\widehat{U}_l\left(\mathbf{x},a,T-n\right) &= (1-\delta)\left[(1-\beta)\,x_l+\beta\widehat{V}_l\left(\mathbf{x},T-n\right)\right]+\delta\widehat{U}_l\left(\widehat{\mathbf{g}}\left(\mathbf{x},a,T-n\right),a,T-n\right) \\
&= (1-\delta)\left[(1-\beta)\,x_l+\beta\widehat{V}_l\left(\mathbf{x},T-n\right)\right] \\
&\quad +\delta\left[(1-\beta)\min_{i\neq a}x_i+\beta\widehat{V}_l\left(\widehat{\mathbf{g}}\left(\mathbf{x},a,T-n\right),T-n\right)\right] \\
&= (1-\delta)\left\{(1-\beta)\,x_l+\beta\left[(1-\beta^n)\left(\tfrac{1}{3}G\right)+\tfrac{1}{3}(1-\beta)(x_2-x_3)\right]\right\} \\
&\quad +\delta\left[(1-\beta)\min_{i\neq a}x_i+\beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right];
\end{aligned}$$

if $a = l$, with a similar calculation as above, then

$$\begin{aligned}
\widehat{U}_l\left(\mathbf{x},l,T-n\right) &= (1-\delta)\left\{(1-\beta)\,x_l+\beta\left[(1-\beta^n)\left(\tfrac{1}{3}G\right)+\tfrac{1}{3}(1-\beta)(x_2-x_3)\right]\right\} \\
&\quad +\delta\left[(1-\beta)\left(G-2\min_{i\neq a}x_i\right)+\beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right].
\end{aligned}$$

Consider the dynamic payoff function for legislator 3. If $a \neq 3$, then

$$\begin{aligned}
\widehat{U}_3\left(\mathbf{x},a,T-n\right) &= (1-\delta)\left\{(1-\beta)\,x_l+\beta\left[(1-\beta^n)\left(\tfrac{1}{3}G\right)-\tfrac{2}{3}(1-\beta)(x_2-x_3)\right]\right\} \\
&\quad +\delta\left[(1-\beta)\min_{i\neq a}x_i+\beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right];
\end{aligned}$$

if $a = 3$, then

$$\begin{aligned}\widehat{U}_3(\mathbf{x},3,T-n) &= (1-\delta)\left\{(1-\beta)x_l + \beta\left[(1-\beta^n)\left(\tfrac{1}{3}G\right) - \tfrac{2}{3}(1-\beta)(x_2-x_3)\right]\right\} \\ &\quad +\delta\left[(1-\beta)\left(G - 2\min_{i\neq a} x_i\right) + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right].\end{aligned}$$

CASE 2. $x_2 > \frac{1}{3}G$.

For any $l \in \{1,2\}$, if $a \neq l$, then

$$\begin{aligned}\widehat{U}_l(\mathbf{x},a,T-n) &= (1-\delta)(1-\beta)\left\{x_l + \beta\left[\begin{array}{c}\left(\frac{1-\left(\frac{1}{3}\beta\right)^n}{1-\frac{1}{3}\beta}\right)\left(\frac{1}{3}G\right) + \frac{1}{3}(x_2-x_3) \\ +\frac{1}{3}\beta\left(\frac{1-\left(\frac{1}{3}\beta\right)^{n-2}}{1-\frac{1}{3}\beta}\right)\left(x_2-\frac{1}{3}G\right)\end{array}\right]\right\} \\ &\quad +\delta\left[(1-\beta)\min_{i\neq a} x_i + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right];\end{aligned}$$

if $a = l$, then

$$\begin{aligned}\widehat{U}_l(\mathbf{x},l,T-n) &= (1-\delta)(1-\beta)\left\{x_l + \beta\left[\begin{array}{c}\left(\frac{1-\left(\frac{1}{3}\beta\right)^n}{1-\frac{1}{3}\beta}\right)\left(\frac{1}{3}G\right) + \frac{1}{3}(x_2-x_3) \\ +\frac{1}{3}\beta\left(\frac{1-\left(\frac{1}{3}\beta\right)^{n-2}}{1-\frac{1}{3}\beta}\right)\left(x_2-\frac{1}{3}G\right)\end{array}\right]\right\} \\ &\quad +\delta\left[(1-\beta)\left(G - 2\min_{i\neq a} x_i\right) + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right].\end{aligned}$$

Finally consider the dynamic payoff function for legislator 3. If $a \neq 3$, then

$$\begin{aligned}\widehat{U}_3(\mathbf{x},a,T-n) &= (1-\delta)(1-\beta)\left\{x_l + \beta\left[\begin{array}{c}\left(\frac{1-\left(\frac{1}{3}\beta\right)^n}{1-\frac{1}{3}\beta}\right)\left(\frac{1}{3}G\right) - \frac{2}{3}(x_2-x_3) \\ -\frac{2}{3}\beta\left(\frac{1-\left(\frac{1}{3}\beta\right)^{n-2}}{1-\frac{1}{3}\beta}\right)\left(x_2-\frac{1}{3}G\right)\end{array}\right]\right\} \\ &\quad +\delta\left[(1-\beta)\min_{i\neq a} x_i + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right];\end{aligned}$$

if $a = 3$, then

$$\begin{aligned}\widehat{U}_3(\mathbf{x},3,T-n) &= (1-\delta)(1-\beta)\left\{x_l + \beta\left[\begin{array}{c}\left(\frac{1-\left(\frac{1}{3}\beta\right)^n}{1-\frac{1}{3}\beta}\right)\left(\frac{1}{3}G\right) - \frac{2}{3}(x_2-x_3) \\ -\frac{2}{3}\beta\left(\frac{1-\left(\frac{1}{3}\beta\right)^{n-2}}{1-\frac{1}{3}\beta}\right)\left(x_2-\frac{1}{3}G\right)\end{array}\right]\right\} \\ &\quad +\delta\left[(1-\beta)\left(G - 2\min_{i\neq a} x_i\right) + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right].\end{aligned}$$

Step 5. Verifying optimality of the conjectured policy rule.

To complete the fixed-point problem for period $T-n$, we have to verify that given the constructed dynamic payoff functions, the policy rule maximizes an agenda setter's dynamic payoff, subject to the constraint that it would be approved by at least one of the other legislators. Here, the approval constraint imposed by the legislators without proposal power is crucial in shaping the policy outcome. Without loss of generality let $a=1$ and take any $\mathbf{x} \in \Delta(G)$ such that $x_2 \geq x_3$. Let $\mathbf{g}^* \in \Delta(G)$ be a policy that solves

$$\begin{aligned} \max_{\mathbf{x}' \in \Delta(G)} \quad & \widehat{U}_a(\mathbf{x}', a, T-n) \\ s.t. \quad & \widehat{U}_i(\mathbf{x}', a, T-n) \geq \widehat{U}_i(\mathbf{x}, a, T-n) \text{ for some } i \neq a. \end{aligned}$$

We want to show that $\mathbf{g}^* = \widehat{\mathbf{g}}(\mathbf{x}, a, T-n)$. We prove this by a series of claims. In the claims especially note where and how the assumptions of a sufficiently large δ plays a role. Some claims below without proofs are obvious ones.

CLAIM 1. For any δ sufficiently close to 1,

$$\min_{i \neq 1} g_i^*(\mathbf{x}, a, T-n) \geq \min_{i \neq 1} x_i.$$

PROOF OF CLAIM 1. Suppose, to the contrary, that $g_j^*(\mathbf{x}, 1, T-n) < \min_{i \neq 1} x_i$ for some $j \neq 1$. With probability δ, after this current round of negotiation the session continues and the agenda setter gets another chance to make a policy proposal. The total discounted and expected utility of any legislator $i \neq 1$ derived from this subgame is

$$(1-\beta) \min_{i \neq 1} g_i^*(\mathbf{x}, 1, T-n) + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right),$$

which is strictly smaller than the total discounted and expected utility derived from the subgame in which the current default $\mathbf{x}$ remains, that is

$$(1-\beta) \min_{i \neq 1} x_i + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right).$$

For all δ sufficiently close to 1, this implies that

$$\widehat{U}_i(\mathbf{g}^*(\mathbf{x}, 1, T-n), 1, T-n) < \widehat{U}_i(\mathbf{x}, 1, T-n)$$

for any $i \neq 1$, which is a contradiction.

CLAIM 2. For distinct $j, k \neq 1$, if $\widehat{U}_j(\mathbf{g}(\mathbf{x}, 1, T-n), 1, T-n) \geq \widehat{U}_j(\mathbf{x}, 1, T-n)$, then $g_k^*(\mathbf{x}, 1, T-n) \leq g_j^*(\mathbf{x}, 1, T-n)$.

PROOF OF CLAIM 2. We prove it by contradiction. Suppose that $g_k^*(\mathbf{x}, 1, T-n) > g_j^*(\mathbf{x}, 1, T-n)$ and consider $\widetilde{\mathbf{g}} \in \Delta(G)$ such that $\widetilde{g}_1 = g_1(\mathbf{x}, 1, T-n) + 1$ and $\widetilde{g}_k = g_k^*(\mathbf{x}, 1, T-n) - 1$. Following the dynamic payoff functions constructed in Step 4, we can show that

$$\widehat{U}_j(\widetilde{\mathbf{g}}, 1, T-n) = \widehat{U}_j(\mathbf{g}^*(\mathbf{x}, 1, T-n), 1, T-n) \geq \widehat{U}_j(\mathbf{x}, 1, T-n),$$

and

$$\widehat{U}_1(\widetilde{\mathbf{g}}, 1, T-n) > \widehat{U}_1(\mathbf{g}^*(\mathbf{x}, 1, T-n), 1, T-n),$$

which is a contradiction. To do the above comparison, we have to discuss the following six cases: Case 1: $g_1^*(\mathbf{x}, 1, T-n) \leq g_j^*(\mathbf{x}, 1, T-n) \leq \frac{1}{3}G$. Case 2: $g_1^*(\mathbf{x}, 1, T-n) \leq g_j^*(\mathbf{x}, 1, T-n)$ and $g_j^*(\mathbf{x}, 1, T-n) > \frac{1}{3}G$. Case 3: $g_j^*(\mathbf{x}, 1, T-n) < g_1^*(\mathbf{x}, 1, T-n) \leq g_k^*(\mathbf{x}, 1, T-n)$ and $g_1^*(\mathbf{x}, 1, T-n) \leq \frac{1}{3}G$. Case 4: $g_j^*(\mathbf{x}, 1, T-n) < g_1^*(\mathbf{x}, 1, T-n) \leq g_k^*(\mathbf{x}, 1, T-n)$ and $g_1^*(\mathbf{x}, 1, T-n) > \frac{1}{3}G$. Case 5: $g_1^*(\mathbf{x}, 1, T-n) > g_k^*(\mathbf{x}, 1, T-n)$ and $g_k^*(\mathbf{x}, 1, T-n) \leq \frac{1}{3}G$. Case 6: and $g_1^*(\mathbf{x}, 1, T-n) > g_k^*(\mathbf{x}, 1, T-n) > \frac{1}{3}G$. The calculation is trivial.

CLAIM 3. For distinct $j, k \neq 1$, if $\widehat{U}_j(\mathbf{g}(\mathbf{x}, 1, T-n), 1, T-n) \geq \widehat{U}_j(\mathbf{x}, 1, T-n)$, then $g_k^*(\mathbf{x}, 1, T-n) = g_j^*(\mathbf{x}, 1, T-n)$.

PROOF OF CLAIM 3. We prove it by contradiction. Suppose, to the contrary, that $g_k^*(\mathbf{x}, 1, T-n) < g_j^*(\mathbf{x}, 1, T-n)$. Note that, by the construction in Step 3, $\widehat{U}_l(\mathbf{x}, 1, T-n)$ can be expressed by the summation of

$$(1-\delta)\,\Phi(\mathbf{x}, T-n)$$

and

$$\delta\left[(1-\beta)\, g_k^*(\mathbf{x}, 1, T-n) + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right)\right],$$

where $\Phi(\mathbf{x}, T-n)$ is shorthand of the complete expression of function. If

$$\Phi(\mathbf{x}, T-n) > (1-\beta)\, g_k^*(\mathbf{x}, 1, T-n) + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right),$$

then it must be the case that $g_k^*(\mathbf{x}, 1, T-n) \geq \min_{i \neq 1} x_i + 1$, and if

$$\Phi(\mathbf{x}, T-n) \leq (1-\beta) g_k^*(\mathbf{x}, 1, T-n) + \beta(1-\beta^n)\left(\tfrac{1}{3}G\right),$$

then it must be the case that $g_k^*(\mathbf{x}, 1, T-n) \geq \min_{i \neq 1} x_i$, so that $\widehat{U}_j(\mathbf{g}^*(\mathbf{x}, 1, T-n), a, T-n) \geq \widehat{U}_j(\mathbf{x}, 1, T-n)$ as δ is sufficiently close to 1. Consider $\widetilde{\mathbf{g}} \in \Delta(G)$ such that $\widetilde{g}_1 = g_1^*(\mathbf{x}, 1, T-n) + 1$ and $\widetilde{g}_j = g_j(\mathbf{x}, 1, T-n) - 1$. It is straightforward to verify that

$$\widehat{U}_j(\widetilde{\mathbf{g}}, 1, T-n) \geq \widehat{U}_j(\mathbf{x}, 1, T-n),$$

for all δ sufficiently close to 1 and

$$\widehat{U}_1(\widetilde{\mathbf{g}}, 1, T-n) > \widehat{U}_1(\mathbf{g}^*(\mathbf{x}, 1, T-n), 1, T-n),$$

which is a contradiction.

CLAIM 4. Consider any $\overline{\mathbf{g}} \in \Delta(G)$ such that $\overline{g}_j = \overline{g}_k = \overline{g}$ for any $j, k \neq 1$ and for some $\overline{g} \in \mathbb{N}$. Then $\widehat{U}_1(\overline{\mathbf{g}}, 1, T-n)$ is decreasing in $\overline{g}$.

By Claims 1 to 4, we have shown that a policy $\widehat{\mathbf{g}} \in \Delta(G)$, such that $\widehat{g}_1 = G - \min_{i \neq 1} x_i$ and $\widehat{g}_j = \min_{i \neq 1} x_i$ for any $j \neq 1$, is the policy that maximizes $\widehat{U}_1(\cdot, 1, T-n)$ subject to the constraint that it is approved by at least one of the other legislators.

Thus far we have proved the optimality of the conjectured policy rule $\widehat{\mathbf{g}}(\mathbf{x}, a, T-n)$ and therefore completed the fixed-point problem in period $T-n$. ■

Proof of Proposition 3. Initially we conjecture that the policy rule is as given in the proposition. Then following Steps 1, 3, and 4 in the proof of Proposition 2, we can construct the following continuation value functions and dynamic payoff functions: For any $\mathbf{x} \in \mathbf{\Delta}(G)$ such that $x_1 \geq x_2 \geq x_3$,

$$\begin{aligned} V_1^*(\mathbf{x}) &= V_2^*(\mathbf{x}) = \tfrac{1}{3}G + \tfrac{1}{3}(1-\beta)(x_2 - x_3) + \tfrac{1}{3}\beta\left(x_2 - \tfrac{1}{3}G\right), \\ V_3^*(\mathbf{x}) &= \tfrac{1}{3}G - \tfrac{2}{3}(1-\beta)(x_2 - x_3) - \tfrac{2}{3}\beta\left(x_2 - \tfrac{1}{3}G\right); \end{aligned}$$

for any $l \in \{1, 2\}$, if $x_2 \leq \frac{1}{3}G$ and $a \neq l$, then

$$\begin{aligned} \widehat{U}_l(\mathbf{x}, a) &= (1-\delta)\left\{(1-\beta)x_l + \beta\left[\tfrac{1}{3}G + \tfrac{1}{3}(1-\beta)(x_2 - x_3)\right]\right\} \\ &\quad + \delta\left[(1-\beta)\min_{i \neq a} x_i + \beta\left(\tfrac{1}{3}G\right)\right]; \end{aligned}$$

if $x_2 \leq \frac{1}{3}G$ and $a = l$, then

$$\begin{aligned}\widehat{U}_l(\mathbf{x}, l) &= (1-\delta)\left\{(1-\beta)x_l + \beta\left[\tfrac{1}{3}G + \tfrac{1}{3}(1-\beta)(x_2 - x_3)\right]\right\} \\ &\quad + \delta\left[(1-\beta)\left(G - 2\min_{i \neq a} x_i\right) + \beta\left(\tfrac{1}{3}G\right)\right];\end{aligned}$$

if $x_2 \leq \frac{1}{3}G$ and $a \neq 3$, then

$$\begin{aligned}\widehat{U}_3(\mathbf{x}, a) &= (1-\delta)\left\{(1-\beta)x_l + \beta\left[\tfrac{1}{3}G - \tfrac{2}{3}(1-\beta)(x_2 - x_3)\right]\right\} \\ &\quad + \delta\left[(1-\beta)\min_{i \neq a} x_i + \beta\left(\tfrac{1}{3}G\right)\right];\end{aligned}$$

if $x_2 \leq \frac{1}{3}G$ and $a = 3$, then

$$\begin{aligned}\widehat{U}_3(\mathbf{x}, 3) &= (1-\delta)\left\{(1-\beta)x_l + \beta\left[\left(\tfrac{1}{3}G\right) - \tfrac{2}{3}(1-\beta)(x_2 - x_3)\right]\right\} \\ &\quad + \delta\left[(1-\beta)\left(G - 2\min_{i \neq a} x_i\right) + \beta\left(\tfrac{1}{3}G\right)\right];\end{aligned}$$

if $x_2 > \frac{1}{3}G$, $l \in \{1, 2\}$ and $a \neq l$, then

$$\begin{aligned}\widehat{U}_l(\mathbf{x}, a) &= (1-\delta)(1-\beta)\left\{x_l + \beta\left[\left(\tfrac{3}{3-\beta}\right)\left(\tfrac{1}{3}G\right) + \tfrac{1}{3}(x_2 - x_3) + \tfrac{1}{3}\beta\left(\tfrac{3}{3-\beta}\right)\left(x_2 - \tfrac{1}{3}G\right)\right]\right\} \\ &\quad + \delta\left[(1-\beta)\min_{i \neq a} x_i + \beta\left(\tfrac{1}{3}G\right)\right];\end{aligned}$$

if $x_2 > \frac{1}{3}G$, $l \in \{1, 2\}$ and $a = l$, then

$$\begin{aligned}\widehat{U}_l(\mathbf{x}, l) &= (1-\delta)(1-\beta)\left\{x_l + \beta\left[\left(\tfrac{3}{3-\beta}\right)\left(\tfrac{1}{3}G\right) + \tfrac{1}{3}(x_2 - x_3) + \tfrac{1}{3}\beta\left(\tfrac{3}{3-\beta}\right)\left(x_2 - \tfrac{1}{3}G\right)\right]\right\} \\ &\quad + \delta\left[(1-\beta)\left(G - 2\min_{i \neq a} x_i\right) + \beta\left(\tfrac{1}{3}G\right)\right];\end{aligned}$$

if $x_2 > \frac{1}{3}G$, $l = 3$ and $a \neq l$, then

$$\begin{aligned}\widehat{U}_3(\mathbf{x}, a,) &= (1-\delta)(1-\beta)\left\{x_l + \beta\left[\left(\tfrac{3}{3-\beta}\right)\left(\tfrac{1}{3}G\right) - \tfrac{2}{3}(x_2 - x_3) - \tfrac{2}{3}\beta\left(\tfrac{3}{3-\beta}\right)\left(x_2 - \tfrac{1}{3}G\right)\right]\right\} \\ &\quad + \delta\left[(1-\beta)\min_{i \neq a} x_i + \beta\left(\tfrac{1}{3}G\right)\right];\end{aligned}$$

if $x_2 > \frac{1}{3}G$, $l = a = 3$, then

$$\begin{aligned}\widehat{U}_3(\mathbf{x}, 3) &= (1-\delta)(1-\beta)\left\{x_l + \beta\left[\left(\tfrac{3}{3-\beta}\right)\left(\tfrac{1}{3}G\right) - \tfrac{2}{3}(x_2 - x_3) - \tfrac{2}{3}\beta\left(\tfrac{3}{3-\beta}\right)\left(x_2 - \tfrac{1}{3}G\right)\right]\right\} \\ &\quad + \delta\left[(1-\beta)\left(G - 2\min_{i \neq a} x_i\right) + \beta\left(\tfrac{1}{3}G\right)\right].\end{aligned}$$

Following the procedure in Step 5 in proof of Proposition 2, we can show the optimality of the policy rule. ■

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