Intertemporal Bundling and Tacit Collusion

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Abstract

We analyze a symmetric, infinitely-repeated oligopoly price game and show that the level of profits that can be sustained in a subgame perfect Nash equilibrium (SPNE) is higher when firms are able to intertemporally bundle their output. In the absence of intertemporal bundling, it is well known that a SPNE exists that sustains any price and profit, up to the monopoly profit, as long as the number of firms does not exceed $\frac{1}{1-\delta}$, or equivalently, the discount factor is greater than $\frac{n-1}{n}$, and that marginal cost pricing is the unique SPNE otherwise. We show that when firms are free to offer intertemporal bundles, equilibria exist in which firms earn strictly positive profits for any number of firms and for any strictly positive discount factor. Firms can earn higher profits by temporally segmenting the market with staggered long-term contracts and by exploiting the fact that consumers anticipate future responses to current deviations. In some cases, firms can earn even higher profits by simultaneously offering intermittent discounts and multi-period contracts.

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1 Introduction

Intertemporal bundling is a common practice. Products, such as newspapers and magazines, and services, such as cellular telephone and DSL internet access, are often sold using multi-period service agreements enabling firms to easily bundle their goods and services across time. This practice, which we call intertemporal bundling, occurs when firms' contract offers require customers to commit to multi-period service agreements.[1] We argue that such service agreements may facilitate tacit collusion. That is, firms competing with one another may be able to sustain higher profits if they are free to offer multi-period service agreements. We show this is true as long as consumers are sufficiently small and sufficiently long lived.

In one respect our finding is quite counterintuitive. Since intertemporal bundling seems to increase the time between offers, one might think that it effectively lowers the discount factor and makes tacit collusion more difficult to sustain. But this intuition is wrong. Intertemporal bundling does not restrict the time between offers, only the time between consumers' purchase decisions. In particular, punishments are just as swift and consumers realize this when making their purchase decisions.

To understand the role of intertemporal bundling in our model, it helps to first imagine that firms are restricted to offer long-term contracts. For example, what happens if firms are required to offer annual contracts instead of monthly contracts? In fact, the annual contract game has equilibria in which the firms each offer the same price and earn the same profits as in the monthly contract game. To see this, suppose that firms' on-the-equilibrium-path prices are stationary, and that firms' off-the-equilibrium-path punishments consist of immediately pricing at marginal cost (the optimal punishment in a Bertrand game). Clearly, on the equilibrium path consumers all sign contracts at the start of each year, so firms are only tempted to deviate at that time. However, since consumers anticipate the rivals' subsequent pricing behavior, at the start of each year, the largest surplus a deviator can capture is the entire market surplus associated with the time between offers *not the length of the contracts*. If the deviator offers less surplus to consumers, they will delay their purchase (by one month) until the rivals offer marginal cost.

[1] Not all multi-period service agreements are examples of intertemporal bundling. In particular, some multi-period service agreements leave the consumer free to change service providers at anytime, but instead restrict the firm, either by limiting its ability to increase price or terminate service. However, this is not what we mean by intertemporal bundling.

However when annual contracts are feasible, equilibria exist with even higher profits as well. This is because firms can temporally segment the market. When the market is segmented, the ability to capture the entire market through a price deviation is removed. To see this, imagine that on the equilibrium path all firms offer stationary annual contracts, but that for some exogenous reason these contracts are staggered. Then each month only a twelfth of the consumers are available to sign a new contract. Each firm can increase its short run profits by lowering its price, but it can only capture one twelfth of the market. Just as when the market is not segmented, the deviator will loose all the future profits from this one-twelfth of the market. However, the punishment extends to the other eleven segments as well. The deviator will lose the profits it would have otherwise earned from its existing customers' contract renewals in the future. Therefore, in this environment, tacit collusion is easier to sustain.

In our paper, we show that intertemporal bundling facilitates higher profits even when consumers are not exogenously staggered and when the duration of firm's contracts is a decision made by the firm when the contract is offered. Because we assume all consumers are present at the start, when firms offer staggered long-term contracts in equilibrium, they must initially offer consumers incentives to induce them to stagger their purchases thereafter. Despite the short-run costs of these incentives, they can lead to higher profits than can be earned in any equilibrium when firms are restricted to one-period, or spot-market, contracts

Notice also that the incentive constraint is relaxed as the contract length increases. Since the contracts are staggered, when the length of contracts increases, there will be fewer available customers in each period, and the available customers will become less significant compared to the currently locked-in customers. As a result, firms' incentives to deviate diminish as the contract length increases.

We also consider a second mechanism through which firms may tacit collude. As part of the mechanism, firms offer a discount on the first month's service in every year. The same discount is offered through both an annual contract as well as the spot contract that each firm simultaneously offers. Since these two contracts are equally attractive, the consumers are willing to sign only the annual contracts. Because consumers all sign the annual contract at the beginning of each year, competition in the remaining eleven months is avoided. The existence of rival firm's spot contracts limits any firms' incentive to deviate because consumers always have the option to respond to a deviation by signing a rival's spot contract and then waiting for the price war to begin. In fact, a deep enough discount on the first month's service can eliminate any incentive to deviate. The sustainable profit increases in the contract length of the multi-period contract as this reduces the frequency of the discounts.

Our argument that increasing contract length does not lower profits relies on the forward looking behavior of consumers. That is, consumers anticipate retaliatory price wars will occur. The effect of forward-looking consumers was also seen in Gul (1987).[2] In his model of durable goods pricing, Gul showed that the Coase conjecture did not hold with more than one firm because firms find it easier to tacitly collude as the time between offers shrinks. Consumers who rationally anticipate a price war upon seeing a deviation will have an incentive to wait for the price war to begin, so a deviator will only be able to capture the time value of the consumers' surplus. He showed that as the time between offers shrinks, this value shrinks, making tacit collusion easier to sustain.

Durable goods can be thought of as a sequence of non-durable goods that have been intertemporally bundled. In this sense, Gul considered a model in which consumers are purchasing a service that was exogenously intertemporally bundled. In contrast, we allow firms to chose whether or not, and how, to intertemporally bundle their product. Firms have the flexibility to choose the length of the contract, and more importantly, they have the flexibility to offer a menu of contracts with different lengths. So in our analysis, firms can induce consumers to purchase in staggered contracts, and hence sustain tacit collusion more effectively than in Gul's model. We also differ from Gul in that we consider homogeneous consumers, so the commitment problem does not even arise.

Literature

One common justification for intertemporal bundling is that there are fixed costs of establishing service. Undoubtedly, these costs exist in many environments, but at the same time it isn't immediately apparent why this issue must be addressed with service agreements rather than installation fees.

The traditional multi-product bundling literature, by analogy, suggests potential rationales for intertemporal bundling. For example, if consumers' valuations are negatively correlated across time, firms with market power can reduce the deadweight

[2] Ausubel and Deneckere (1987) and Dutta, Matros, and Weibull (2003) also explored how forward looking behavior of consumers helps firms sustain tacit collusion.

loss by intertemporal bundling (see Adams and Yellen, 1976). Also, under some conditions, intertemporal bundling might be a mechanism for leveraging market power into the future (see Whinston, 1990, etc.).[3]

Several other papers have looked at intertemporal bundling. Loewenstein, Donoghue, and Rabin (2003) considered a model of projection bias. Consumers possessing "projection bias" tend to rely too much on their current tastes to estimate their future preferences. They conjectured that a firm may use intertemporal bundling to take advantage of consumers with projection bias who currently place a high value on its product. DellaVigna and Malmendier (2004) analyze firms' optimal contract design when customers are time inconsistent and partially naive about it. The authors show that when a product brings consumers immediate benefit and postponed cost, firms tend to specify a high per usage fee in a subscription contract and when a product brings consumers immediate cost and postponed benefit, firms tend to set a per usage fee below the marginal cost. In our analysis, consumers are fully rational and their rational expectations play a crucial role in helping firm achieve higher profit through tacit collusion.

Considering a competitive setting, Cabral and Villas-Boas (2005) show that bundling of unrelated products can lead to more homogeneous tastes and thus more intense competition among firms. Based on this result, the authors suggest that intertemporal bundling may intensify competition. Since the current paper focuses on a different economic force of intertemporal bundling, our conclusion that intertemporal bundling can facilitate tacit collusion stands in sharp contract with theirs.

DeGraba and Mohammed (1999) studied how a capacity-constrained firm can induce a buying frenzy by first offering a bundle of multiple products and only sell products individually later. The authors term such tactic as "intertemporal mixed bundling" because the firm offers different bundles of products at different points of time but unlike in our analysis, the seller does not intertemporally bundle consumption of different points of time in their analysis.

Our paper is also closely related to the literature on multimarket contact of firms. When consumers sign multi-period contracts which are endogenously staggered, different cohorts of customers can be viewed as different "markets". One way to interpret our finding is that each firm's incentive to deviate is weakened when it

[3] As in Whinston (1990), bundling in our model is valuable because of its impact on the behavior of rivals. However, the strategic reason for bundling in Whinston's paper is to deter entry while in ours it is to soften price competition among existing firms.

knows that if it steal other firms' business in one market it will be punished in all markets. Although the reasoning sounds very intuitive, in the multimarket context the validity of such argument is not obvious. Generally speaking, when firms compete in multiple markets, these markets do not open in a staggered manner. Because of that, as pointed out by Bernheim and Whinston (1990), "once a firm knows that it will be punished in every market, if it decides to cheat, it will do so in every market." Therefore, multimarket contact facilitates tacit collusion in our analysis only because firms skillfully stagger these markets to open at different points of time.

The paper is organized as follows. We begin by describing the model and reviewing the set of SPNE when firms either cannot intertemporally bundle or can only offer life-long subscription contracts. We then analyze two specific classes of SPNE with intertemporal bundling. Equilibria in both classes sustain higher profits. While the second class of equilibria are capable of sustaining the same profits as the first, and for some discount factors sustain even higher profits, the first class of equilibria is of independent applied interest since it more closely resembles the observed behavior of firms. The first class of equilibrium are also more robust to relaxation of our modeling assumptions. Finally we prove that the all equilibrium profits that are sustainable in a SPNE can be sustained with a SPNE in our second class. That it, there are no symmetric equilibria which yield higher profits.

2 The Model

We consider a standard infinitely repeated oligopoly price game. As in the standard model, consumers are small and because they are small they do not act strategically. That is, they do not expect their actions to affect firms' future behavior. Unlike the standard model though, we explicitly assume that consumers are infinitely-lived.

Assumptions:

A1 There is a unit mass of infinitely lived and infinitely small consumers. Consumers have valuations V for the consumption of one unit of the good in each period of their lives. Consumers have valuation 0 for their outside option. Disposal of the good at any time is costless.

A2 There are n infinitely lived firms selling a homongeneous product. Each firm has zero unit costs. Firms, simultaneously, make one offer, or menu of offers, each period.

A3 Firms' one-period contract offers are denoted simply by their price, p_1. Firms' multi-period contracts are denoted by a price-duration pairs $\{P, k\}$ where P is the present discounted value of the stream of per-period payments specified in the contract and k is the contract length.

A4 Firms and consumers have a common strictly positive discount factor, $\delta \in (0, 1)$.

Since contracts are binding and both firms and consumers share a common discount factor, any two contracts whose streams of payments have the same present discounted value are equivalent. However, referring to the stream of payments sometimes helps the exposition. In those situations, we will write the stream of payments as $\{p_s\}_{s=1}^{k}$ and express the present discount value as

$$P = \sum_{s=1}^{k} \delta^{s-1} p_s.$$

Timing without intertemporal bundling

Each period, firms simultaneously announce their prices for a single period of service, and then consumers simultaneously choose among all of the firms' offers and their outside option.

Timing with intertemporal bundling

Each period, first each firm simultaneously announces a menu of prices and contract lengths and then consumers who do not have a prior contract choose either a contract from among the firms' menus of offers, their outside option (until the next period). Simultaneously, consumers who have a prior contract choose either a new contract from among the firms' menus of offers or keep consuming from their existing contract (which they are free to dispose of at any time).

We assume firms do not offer forward contracts. Later we argue in Section 6 this is without loss of generality, as long as consumers are forward looking.

Benchmark 1: SPNE without intertemporal bundling

An important benchmark is the case in which intertemporal bundling is not feasible. If the firms are restricted to make one-period offers, the SPNE of the game are well-known (see, for example, Tirole, 1988):

Claim 1 *When intertemporal bundling is not feasible, then i) if $n \leq \frac{1}{1-\delta}$ then any level of profit between zero and the monopoly profit is sustainable in a symmetric SPNE, and ii) if $n > \frac{1}{1-\delta}$, the unique sub-game perfect Nash equilibrium outcome is zero profits (marginal cost pricing).*

Benchmark 2: SPNE with "durable" goods

Another important benchmark is the case in which infinitely long, intertemporal bundled contracts are the only feasible contracts.

Claim 2 *When the firms can sell only using infinitely long contracts, then i) if $n \leq \frac{1}{1-\delta}$, any level of profit between nV and the monopoly profit, $\frac{V}{1-\delta}$ is sustainable in a symmetric SPNE, and ii) if $n > \frac{1}{1-\delta}$, the unique sub-game perfect Nash equilibrium outcome is zero profits (marginal cost pricing).*

Proof:

> (i) Suppose $n \leq 1/(1-\delta)$. First, marginal cost pricing is obviously a SPNE outcome.
>
> The following are also symmetric SPNE: In every period firms each charge $p/(1-\delta)$ for an infinitely long contract, where $p \in (n(1-\delta)V, V]$. In period one, $1/n$ of the consumers purchase from each firm. If any consumer deviates by not purchasing at any time, all firms continue to charge

$p/(1-\delta)$. In this continuation subgame, the deviating consumer purchases from each firm with a probability $1/n$ in the next period. In any continuation subgame, if any firm deviates from the price $p/(1-\delta)$, in the following period all firms revert to marginal cost pricing.

Consumers have no incentive to deviate because the price never changes and consumers discount the future. Firms have no incentive to deviate either. Each firm's equilibrium profit is $p/n(1-\delta)$. A deviating firm can earn at most V, so the deviation profit is $\min\{p/(1-\delta), V\}$. However we have stipulated that $p \geq n(1-\delta)V$, which implies $p/(1-\delta) \geq V$, so no firm has an incentive to deviate as long as

$$\frac{1}{n}\frac{p}{(1-\delta)} \geq V$$

or $p \geq nV(1-\delta)$. Therefore, if $n \leq 1/(1-\delta)$, then the industry can support any profit in the range

$$\left[nV, \frac{V}{(1-\delta)}\right].$$

In the continuation subgame following a consumer's deviation, each firm faces exactly the same tradeoff when deciding whether to deviate, except that $1/n$ will be each firm's probability of earning the deviating consumer's business instead of its market share.

(ii) Suppose $n > 1/(1-\delta)$. Marginal cost pricing obviously remains as a SPNE outcome. We now show that no SPNE exists in which any firm earn positive profits. Let P_{it} denote firm i's equilibrium price in period t, $i \in \{1, 2, ..., n\}$ and $t \in \mathbb{N}$. Suppose a positive measure of transactions occur in period τ. The transactions must take place at price $\underline{P} \equiv \min_i \{P_{i\tau}\} \leq V/(1-\delta)$. Let Δ denote the measure of consumers that have not yet purchased in period τ, and note that this is strictly positive.

Equilibrium industry profits at time τ are bounded by $\Delta\underline{P}$. If profits were higher, consumers surplus could collectively be increased by every consumer purchasing at time τ at price $\underline{P}$. In any continuation game, total surplus is weakly lower, so if profit are strictly higher, consumer surplus must be lower. It follows that at least some consumers are not maximizing consumer surplus.

It follows that at time τ the least profitable firm earns no more than $\Delta \underline{P}/n$. The deviation profit is $\Delta \min\{V, \underline{P}\}$. Since $\underline{P} > \underline{P}/n$, $\min\{V, \underline{P}\} \leq \underline{P}/n$ holds only if $V \leq \underline{P}/n$ which contradicts $n > 1/(1-\delta)$ and $\underline{P} \leq V/(1-\delta)$.

Whether the product is sold period by period or intertemporally bundled into a durable good, the sufficient conditions for the monopoly price equilibrium to exist are the same. In the following two sections, we show that intertemporal bundling makes it easier for firms to tacit collude more effectively, but firms need to be able to offer contracts with multiple durations. Each of the two sections describes a different class of equilibrium strategies. Section 5 characterizes the equilibrium outcomes for all SPNE of the intertemporal bundling game.

3 *Staggered Contract Equilibria*: SPNE with Intertemporal Bundling I

In this section, and the next, we consider equilibria of the game in which firms are free to use intertemporal bundling in any way they choose. First, we consider a class of equilibria which we call *Staggered Contract Equilibria*. In this class of equilibria firms induce consumers to accept staggered contracts so that only a small fraction of consumers are at the end of the contractual obligation in any given period.

The set of *Staggered Contract Equilibria* are all SPNE in which strategies are of the following form:

Firms' equilibrium path actions: In Period 1, the firms offer a one-period contract at a price p_1, a two-period contract $(p_1 + \delta V, 2)$, and an n-period contract $\left(p_1 + \sum_{s=2}^{n} \delta^{s-1} V, n\right)$ for all n less than or equal to k. In Period 2 and beyond the firms' offer only the k-period contracts $\left(\sum_{s=1}^{k} \delta^{s-1} V, k\right)$.

Consumers' equilibrium path actions: In Period 1, an equal number of consumers accept each of the nk different contracts offers (k offers from each firm). In Period 2 and beyond an equal number of consumers (who aren't under contractual obligation) accept each of the firms' k-period contract offers.

Off-the-equilibrium-path actions: If any firm(s) deviates from these equilibrium strategies, every firm offers one-period, marginal-cost contracts forever. If any consumer(s) deviates by signing a contract different from what he is would sign in equilibrium in any period, firms continue to offer the same set of contracts the equilibrium prescribes that they offer in all the periods that follow.

Next, we find necessary and sufficient conditions under which a Staggered Contract Equilibrium exists (that is, under which the above strategies constitute a subgame-perfect Nash equilibrium).

Consumers are clearly optimizing since their surplus is the same whichever offer they accept. We need to show now that the firms' strategies are also deviation proof.

Period 1

We first consider deviations by firms in period 1. The firms' profit on the equilibrium path is

$$\frac{1}{n}\left[p_1 + \delta\frac{V}{1-\delta}\right] \tag{1}$$

The profit a firm can earn from undercutting with a one-period contract in Period 1, if the firm can grab the whole market, is equal to p_1. Consumers clearly prefer the deviator's offer to the rivals' one-period offers, and, because consumers expect to benefit from the price war as soon as their contract expires, they prefer to buy from the deviator as opposed to the rivals' k-period contracts because

$$V - p_1 + \delta\frac{V}{1-\delta} \geq V - p_1 + \delta^k\frac{V}{1-\delta} \tag{2}$$

clearly holds. So no one-period-contract deviation is profitable in Period 1 if

$$\frac{1}{n}\left[p_1 + \delta\frac{V}{1-\delta}\right] \geq p_1. \tag{3}$$

Assuming the deviating firm offers a one-period contract is without loss of generality. The fact that customers can always sign a one-period contract offered by other firms at the price p_1 and they anticipate a price war with a zero price starting from the second period suggest that any deviating contract that attracts customers must at least offer them a consumer surplus of $(V - p_1) + \delta\frac{V}{1-\delta}$. We can verify that this is the consumer surplus that the one-period contract at the price p_1 offers. Any deviation with a multi-period contract must provide at least the same consumer surplus and thus the present discounted value of consumers' payments throughout the contract must not exceed p_1. Therefore, the deviator may not make a higher profit by offering a multi-period contract.

Solving for p_1, (3) holds if

$$p_1 \leq \frac{1}{n-1}\delta\frac{V}{1-\delta}. \tag{4}$$

Period 2 and beyond

Each firm's profit on the equilibrium path from Period 2 on is

$$\frac{1}{n}\frac{V}{1-\delta}. \tag{5}$$

The profit a firm can earn by deviating to one-period contract at time 2 is

$$\frac{1}{n}\sum_{j=1}^{k-1}\sum_{i=1}^{j}\frac{1}{k}\delta^{i-1}V + \frac{1}{k}V, \tag{6}$$

where the first term is the profit earned from existing contractual obligations, and the second term is the profit earned by capturing one period of consumer surplus of all of the uncommitted consumers. So no deviation is profitable in Period 2 if

$$\frac{1}{n}\frac{V}{1-\delta} \geq \frac{1}{n}\sum_{j=1}^{k-1}\sum_{i=1}^{j}\frac{1}{k}\delta^{i-1}V + \frac{1}{k}V. \tag{7}$$

Because firm profits, (5), can be written as

$$\frac{1}{n}\frac{V}{1-\delta} = \frac{1}{n}\frac{1}{k}\left[\sum_{j=1}^{k-1}\sum_{i=1}^{j}\delta^{i-1}V + \sum_{j=1}^{k-1}\sum_{i=j+1}^{\infty}\delta^{i-1}V + V + \sum_{i=2}^{\infty}\delta^{i-1}V\right], \tag{8}$$

equation (7) can be written

$$\frac{1}{n}\frac{1}{k}\left[\sum_{j=1}^{k-1}\sum_{i=j+1}^{\infty}\delta^{i-1}\right] + \frac{1}{n}\frac{1}{k}\sum_{i=2}^{\infty}\delta^{i-1} \geq (1-\frac{1}{n})\frac{1}{k}, \tag{9}$$

or

$$\left[\sum_{j=1}^{k-1} \frac{\delta^j}{1-\delta}\right] + \frac{\delta}{1-\delta} \geq n-1, \tag{10}$$

and the larger is k, the easier this constraint is to satisfy. For $k = 2$, this condition becomes

$$2\frac{\delta}{1-\delta} \geq n-1, \tag{11}$$

or $\delta \geq 1 - \frac{2}{n+1}$. Letting k go to infinity, it is clear that there exists a k such that if

$$\frac{\delta}{(1-\delta)^2} + \frac{\delta}{1-\delta} \geq n-1, \tag{12}$$

then the incentive constraint is satisfied. This is equivalent to

$$\delta \geq 1 - \frac{1}{\sqrt{n}} \tag{13}$$

or

$$n \leq \frac{1}{(1-\delta)^2}. \tag{14}$$

Note that for $n = 2$, the incentive constraint is $\delta \geq 1 - 1/\sqrt{2} \approx .29$. In other words, intertemporal bundling using staggered contracts helps a duopoly sustain profitable tacit collusion when the discount factor lies between .29 and .5.

So, if (4) and (14) are satisfied, there exists a k such that a staggered contract equilibrium exists.

Now we ask what the most profitable staggered contract equilibrium is. The firm's profit in equilibrium is equal to

$$\frac{1}{n}\left[p_1 + \frac{\delta}{1-\delta}V\right]. \tag{15}$$

The equilibrium with the highest profit is the one with the highest possible p_1, and since p_1 must satisfy (4), or

$$p_1 \leq \frac{1}{n-1}\frac{\delta}{1-\delta}V \tag{16}$$

and $p_1 \leq V$, the firm's maximal equilibrium profit is

$$\frac{1}{n-1}\frac{\delta}{1-\delta}V \tag{17}$$

or

$$\frac{1}{n}\frac{1}{1-\delta}V, \tag{18}$$

whichever is smaller. Note that k affects whether or not the equilibrium is sustainable but not the maximal profit level when it is.

This clearly implies the following result.

Proposition 1 *When firms can offer subscription contracts of any lengths, a staggered contract equilibrium exists that sustains any industry profit level between zero and*

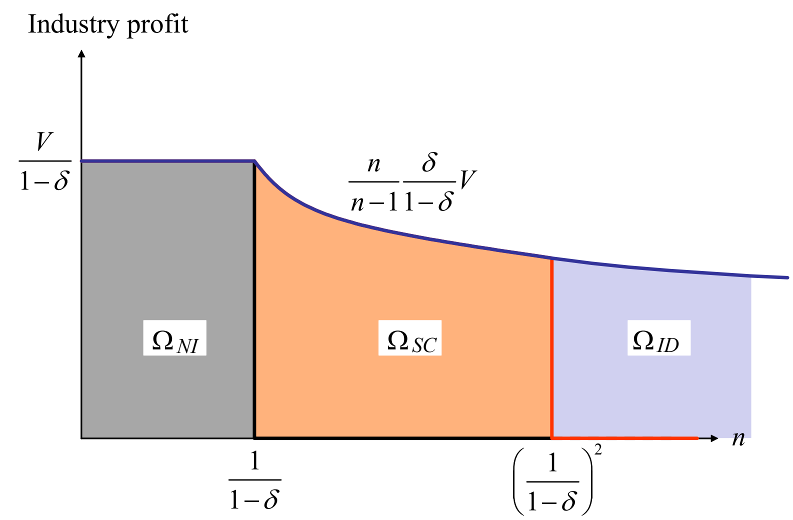
$$\min\{\frac{V}{1-\delta}, \frac{n}{n-1}\frac{\delta}{1-\delta}V\} \tag{19}$$

if and only if $n \leq \frac{1}{(1-\delta)^2}$.

Comparing Claim 1 and Proposition 1 we can easily see that intertemporal bundling using staggered contracts helps the industry sustain tacit collusion when the number of firms is between $\frac{1}{1-\delta}$ and $\frac{1}{(1-\delta)^2}$. To get an idea of the power of staggering contracts, suppose, for example, that $\delta = 0.9$. In this case, when intertemporal bundling is not feasible, tacit collusion can be sustained in an industry with up to ten firms. But when firms can offer staggered subscription contracts, tacit collusion can be sustained among a hundred firms.

Figure 1 graphically illustrates how staggered contracts help expand the set of equilibrium profit:[4]

Figure 1 Comparison of sets of equilibrium profits



In Figure 1, Ω_{NI} denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when intertemporal bundling is not feasible; and $\Omega_{NI} \cup \Omega_{SC}$ denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when firms tacit collude using staggered subscription contracts. Finally, $\Omega_{NI} \cup \Omega_{SC} \cup \Omega_{ID}$ denotes the set of equilibrium profits that are sustainable, as a function of the number of firms, when firms tacit collude using any contract. We show this, and explore more general contracts, in the next two section of paper.

Given the expectation that the subscription price will stay at V forever starting from period two in equilibrium, customers feel indifferent between the k contracts of different lengths offered in period one. That is why firms can induce $1/k$ customers to choose each of the k contracts of different lengths and initiate the staggering of all the future contracts. The staggering of contracts plays an important role in facilitating tacit collusion. Since customers correctly anticipate the price to fall to zero if any firm deviates to attract their businesses, the deviation profit from each customer will be the market price the customer is paying in the current period, i.e., p_1 in the first period and V in all remaining periods. The corresponding deviation profit is p_1/k in period one and V/k in any other period. By triggering the price

[4]Note that $n \in \mathbb{N}$, although for easy of illustration it is treated as belonging to $\mathbb{R}_+$ in the figure.

war, the deviating firm will loss the future profits from renewing its contracts with its existing customers once the contracts with these customers end. Among the existing customers, $1/nk$ of them will have their contract ended in each of the following $k-1$ periods. In other words, for every one customer the deviating firm steals, it will loose the renewal contracts of $1/n$ customers in every of the following k periods. As the contract length k increases, each cohort of customers will constitute a smaller fraction of the population of customers. The benefit of deviation will become less significant both in absolute term and relative to the cost of deviation. That explains how intertemporal bundling through staggered subscription contracts weakens firms' incentives to deviate from a collusive outcome.

In the next section, we consider an alternative tactic firms may adopt to intertemporally bundle consumption. As it turns out, with such tactic, for any discount factor, profitable tacit collusion may be sustainable among arbitrarily large number of firms.

4 *Intermittent Discount Equilibria:* SPNE with Intertemporal Bundling II

In this section, we consider a class of equilibria which we call *Intermittent Discount Equilibria*. In this class of equilibria, firms offer both long term contracts, and in the period in which the long term contracts are signed, a one-period discount contract. Though consumers don't ever choose the discount contract on the equilibrium path, its presence reduces the rival firms' incentives to try to steal market share and hence makes it easier to tacit collude.

The set of *Intermittent Discount Equilibria* are all the SPNE in which strategies are of the following form:

Firms' equilibrium path actions: In Period 1, the firms offer a one-period contract at a price p_1, and a k-period contract (p_1, V, k). In Periods 2 through k the firms offer only one-period contracts at a price V. These offers repeat every k periods, so, for example, in periods $k+1$, $2k+1$, $3k+1$, etc. the firms offer the same contracts as in Period 1.

Consumers' equilibrium path actions: In periods 1, $k+1$, $2k+1$, $3k+1$, etc. all consumers purchase k-period contracts, $1/n$ from each firm. In every other period, consumers (if there are any who aren't under contractual obligation) purchase the one-period contracts, $1/n$ from each firm.

Off-the-equilibrium path actions: If any firm deviates from these equilibrium path strategies, then all firms revert to one-period, marginal-cost contracts forever. If consumers deviate from their equilibrium path strategies, firms continue to adopt their equilibrium path strategies as long as the mass of deviating consumers is sufficiently small. Otherwise, firms revert to one-period, marginal-cost contracts forever.

First, we show that this is a subgame-perfect Nash equilibrium for all δ and n. Consumers' strategies are clearly optimal since their surplus is the same whichever offer they accept, and given firms' strategies, they are indifferent between signing the k-period and one-period contracts. To show that firms' strategies are optimal we look separately at their behavior in the first period of the contract and the remaining periods of the contract.

Period 1, $k+1$, $2k+1$, $3k+1$, etc.

If either firm deviates, consumers have the option to purchase in Period 1 at the price p_1 and in periods 2 through ∞ at the price 0. So the profit a deviating firm can earn is only p_1. This means the firm prefers its equilibrium strategy as long as its equilibrium profit in period 1, which we denote by π_1, is greater than p_1, or

$$\pi_1 = \frac{1}{n}\sum_{i=1}^{\infty}\delta^{i-1}V - \frac{1}{n}\sum_{i=1}^{\infty}\delta^{(i-1)k}(V - p_1) \geq p_1. \tag{20}$$

This is clearly satisfied for $p_1 = 0$ (and for some $p_1 > 0$ sufficiently close to zero). Also, note that π_1 is increasing in k and as k approaches infinity this expression becomes

$$\frac{1}{n}\sum_{i=2}^{\infty}\delta^{i-1}V + \frac{1}{n}p_1 \geq p_1 \tag{21}$$

or

$$\delta \frac{V}{1-\delta} \geq (n-1)p_1, \tag{22}$$

so the highest sustainable first period price is

$$p_1 = \frac{1}{n-1} \delta \frac{V}{1-\delta}. \tag{23}$$

All other periods

If a consumer deviates and begins purchasing one-period contracts, the firms are better off offering the equilibrium one-period price V than offering a lower price as long as

$$\frac{1}{n} \sum_{i=1}^{k-t} \phi \delta^{i-1} V + \frac{1}{n} \delta^{k-t} \pi_1 > \phi V \tag{24}$$

where $t = T - \text{Int}(T/k)k$ is the time since the last k-period contract offer, T is the current period, and ϕ is the size of the consumer who do not sign the k-period contract the last time it was offered.

Since ϕ is the size of the customer (we do not consider collective actions by consumers), under our assumption that consumers are arbitrarily small, $\phi = 0$ and no deviation is profitable.

Equilibrium Profits

The equilibrium profits are

$$\pi_1 = \frac{1}{n} \sum_{i=1}^{\infty} \delta^{i-1} V - \frac{1}{n} \sum_{i=1}^{\infty} \delta^{(i-1)k} (V - p_1), \tag{25}$$

which are increasing in k. As k approaches ∞ these profits become

$$\frac{1}{n} p_1 + \frac{1}{n} \delta \frac{V}{1-\delta}, \tag{26}$$

Equilibrium profit is the highest when $p_1 = \frac{1}{n-1}\delta\frac{V}{1-\delta}$. In this case, each firm's profit is $\frac{1}{n-1}\delta\frac{V}{1-\delta}$ and the industry profit is $\frac{n}{n-1}\delta\frac{V}{1-\delta}$.

Proposition 2 *When firms can offer subscription contracts of any lengths, for all δ and n, a SPNE exist in the class of Intermittent Discount Equilibria that sustains any industry profit level between zero and*

$$\min\{\frac{V}{1-\delta}, \frac{n}{n-1}\frac{\delta}{1-\delta}V\}. \tag{27}$$

and no SPNE in this class can sustain higher profit.

When a Staggered Contract Equilibrium is unable to facilitate tacit collusion, an Intermittent Discount Equilibrium may still facilitate tacit collusion. Strikingly, tacit collusion is sustainable among any number of firms with any discount factor and the industry profit is uniformly bounded away from zero. In Figure 1, the set of equilibrium payoffs supported by Intermittent Discount Equilibria for different number of firms is denoted by $\Omega_{NI} \cup \Omega_{SC} \cup \Omega_{ID}$.

In an Intermittent Discount Equilibrium, firms do not offer multi-period contracts every period. When these contracts are offered, firms induce all consumers to purchase long-term contracts at the end of which the firms offer multi-period contracts again. So there is no incentive to deviate in the intervening periods. Whenever multi-period contracts are offered, firms simultaneously offer one-period contracts with the same discount. Although the one-period discounted contracts are never accepted, they serve an important function by setting a bound on the deviation profit. A deviating firm can only capture one period of demand because consumers always have the option of signing any other firm's one-period discounted contract and then waiting for the price war to begin. When the discount on the one-period contract is set arbitrarily low, the deviation profit becomes arbitrarily small. That explains why tacit collusion can be sustained among arbitrary number of firms with any discount factors.

The price of the multi-period contract makes consumers indifferent between accepting the one-period contract and accepting the multi-period contract, and the incentive constraint is relaxed by lowering the price of the one period contract. This means that the collusive profit can be increased by increasing the length of the multi-period contract and reducing the frequency with which one-period discounts are offered.

However, while increasing contract length increases the collusive profit, sellers cannot benefit from offering an infinite contract length. The Intermittent Discount Equilibrium relies on the arbitrarily small size of consumers, and as the contract length increases, the importance of this assumption increases. The equilibrium relies on firms maintaining a one-period price of V if any consumer deviates and accepts a one-period offer. This is valid as long as the loss from a future price war is greater than the gain from serving the deviating consumer. But the cost of future price wars goes to zero as k goes to infinity, so the size of consumers is more and more limited as k goes to infinity.

To see this, suppose that sellers offer only an infinitely long contract, i.e., $k = \infty$, and a one-period contract. Then under some conditions consumers are able to trigger a price war in the second period by signing the one-period contract in the first period and reentering the market in the second. Let the size of a customer be ϕ. The deviating customer will be the only source of business starting from the second period. If any firm undercuts the others, it will be able to earn an immediate profit of ϕV which is the customer's one-period valuation. A firm will be tempted to cut its price and steal the customer's business if

$$\begin{aligned} & \phi V > \tfrac{1}{n}\tfrac{\phi V}{1-\delta} \\ \Leftrightarrow \quad & n > \tfrac{1}{1-\delta}, \end{aligned}$$

where $1/n$ is the probability of each firm obtaining the consumer's full consumer surplus. In other words, although the intermittent discounts can be arbitrarily far apart, they have to be infinitely repeated. Without repeatedly offering these discounts, firms cannot do better than offering only one-period contracts.

5 General Results

The following Proposition establishes that the above class of SPNE achieves the highest feasible profit sustainable in any equilibrium.

Proposition 3 *The highest industry profit that can be supported in a symmetric*

SPNE is $\min\{\frac{V}{1-\delta}, \frac{n}{n-1}\delta\frac{V}{1-\delta}\}$*, or more precisely* $\frac{n}{n-1}\delta\frac{V}{1-\delta}$ *if* $n > \frac{1}{1-\delta}$ *and* $\frac{V}{1-\delta}$ *if* $n \leq \frac{1}{1-\delta}$.

Proof:

First, note that by Proposition 2, a profit level of $\min\{\frac{V}{1-\delta}, \frac{n}{n-1}\delta\frac{V}{1-\delta}\}$ can be supported. Next, consider any symmetric equilibrium that sustains the highest possible industry profit in the set of all SPNE. Let π_1 denote the present value of each firm's equilibrium profit stream in this equilibrium beginning from period 1 onwards. Let π_2 denote the present value, in period 1, of each firm's equilibrium profit stream in this highest profit equilibrium from period 2 onwards.

In period 1, each firm can either earn its equilibrium profit, π_1, or it can deviate. And, there always exists a deviation that yields a profit of $n(\pi_1 - \pi_2)$. To see this, first note that regardless of the deviator's offer and consumers' expectations about the future, the best the consumers can ever hope to do if they buy from the other firm is capture $\frac{V}{1-\delta} - n(\pi_1 + \pi_2)$. This is the surplus they would earn if subsequent pricing dissipated all of the firms' future profits. So a deviator can capture the entire market by offering consumers surplus less than or equal to $\frac{V}{1-\delta} - n(\pi_1 + \pi_2)$, which means the deviator can capture a profit of $n(\pi_1 - \pi_2)$.

This implies that the highest profits equilibrium must satisfy $\pi_1 \geq n(\pi_1 - \pi_2)$. If not, firm have an incentive to deviate. This in turn implies $\pi_1 \leq \frac{n}{n-1}\pi_2$. Since any symmetric equilibrium necessarily satisfies $\pi_2 \leq \delta\frac{1}{n}\frac{V}{1-\delta}$, it follows that $\pi_1 \leq \frac{1}{n-1}\delta\frac{V}{1-\delta}$. So total industry profit cannot exceed $\frac{n}{n-1}\delta\frac{V}{1-\delta}$. $\square$

6 Extensions

Naïve Consumers

In this subsection, we study the case when consumers are naïve in the sense that they do not understand that a price war will follow when any firm offer a price cut. Consumer naïveté does hamper firms' ability to tacit collude because when consumers are naïve, a deviating firm can offer all available consumers infinitely long contracts to capture all their future demand. In fact, there does not exist an equilibrium in which tacit collusion is sustained by one period contracts or infinitely long contracts. Neither do Intermittent Discount Equilibria exist. However, we also show that Staggered Contract Equilibria exist, although only among smaller number of firms. Such comparison points out that temporally partitioning the market into multiple segments is particularly important when consumers are naïve because the other mechanisms that we know of all fail.

Claim 3 *If consumers are naïve about the consequence of a price cut, then in all equilibria in which firms offer only one-period contracts on the equilibrium path, the industry profit is necessarily zero. Similarly, in all equilibria in which firms offer only infinite-period contracts on the equilibrium path, the industry profit is necessarily zero*

Proof. First, consider the case that firms offer only one-period contracts. Note that marginal cost pricing using one-period contracts is an equilibrium, so equilibria in this set do in fact exist. Let Π_t be the present discounted value of the industry profit at time $t \in \mathbb{N}$. At time t, the profit of the least profitable firm is no larger than Π_t/n. Since consumers are naïve about the consequence of a price cut, the least profitable firm can deviate by offering consumers a life-time contract providing them a consumer surplus arbitrarily slightly higher than they would receive in equilibrium. Therefore, the deviation profit is arbitrarily close to Π_t which is obviously larger than Π_t/n, for $n \geq 2$. Next, if firms can offer only infinitely long contracts and consumers are naïve, then firms essentially are playing an one-shot game and clearly the unique equilibrium outcome is marginal cost pricing. □

With a proof based on logic similar to that used in the proof for Claim 3, we will immediately get this result:

Claim 4 *If consumers are naïve about the consequence of a price cut, then Intermittent Discount Equilibria do not exist.*

Now, suppose firms offer contracts of different lengths, p_1, $\left(p_1 + \sum_{s=2}^{\tau} \delta^{s-1}V, \tau\right)$, $\tau = 2, ..., k$, in the first period to create the staggering of submarkets and starting from period two offer only k-period contracts $\left(\sum_{s=1}^{k} \delta^{s-1}V, k\right)$. Take any period

$t \geq 2$. Since consumers are naïve, any deviating offer as good as $\left(\sum_{s=1}^{\infty} \delta^{s-1} V, \infty\right)$ will be accepted. Therefore, the deviating profit is

$$\frac{1}{n} \sum_{j=1}^{k-1} \sum_{i=1}^{j} \frac{1}{k} \delta^{i-1} V + \frac{1}{k} \frac{V}{1-\delta}.$$

So no deviation is profitable in Period 2 if

$$\frac{1}{n} \frac{V}{1-\delta} \geq \frac{1}{n} \sum_{j=1}^{k-1} \sum_{i=1}^{j} \frac{1}{k} \delta^{i-1} V + \frac{1}{k} \frac{V}{1-\delta}.$$

Since

$$\frac{1}{n} \frac{V}{1-\delta} = \frac{1}{n} \frac{1}{k} \left[\sum_{j=1}^{k-1} \sum_{i=1}^{j} \delta^{i-1} V + \sum_{j=1}^{k-1} \sum_{i=j+1}^{\infty} \delta^{i-1} V + V + \sum_{i=2}^{\infty} \delta^{i-1} V \right],$$

the inequality can be rewritten into

$$\frac{1}{nk} \left[\sum_{j=1}^{k-1} \sum_{i=j+1}^{\infty} \delta^{i-1} \right] + \frac{1}{nk} \sum_{i=2}^{\infty} \delta^{i-1} \geq \frac{1}{k} \left(\frac{1}{1-\delta} - \frac{1}{n} \right).$$

$$\left[\sum_{j=1}^{k-1} \frac{\delta^j}{1-\delta} \right] + \frac{\delta}{1-\delta} \geq \frac{n}{1-\delta} - 1.$$

When k goes infinity, we have

$$\frac{\delta}{(1-\delta)^2} + \frac{\delta}{1-\delta} \geq \frac{n}{1-\delta} - 1.$$

$$\begin{aligned} \frac{\delta}{(1-\delta)^2} + \frac{\delta}{1-\delta} &= \frac{n}{1-\delta} - 1. \\ n &\leq \frac{1}{1-\delta}. \end{aligned}$$

As long as tacit collusion starting from period 2 can be maintained, tacit collusion in period one can also be maintained with a low enough p_1. From our previous analysis, we know that the highest sustainable first-period price is $p_1 = \frac{1}{n-1} \delta \frac{V}{1-\delta}$. This leads to the following:

Claim 5 *If consumers are naïve about the consequence of a price cut, then tacit collusion is sustainable by firms offering staggered contracts if and only if* $n \leq \frac{1}{1-\delta}$.

Claim 4 suggests that staggering contracts helps firms to restore the potential to tacit collude that consumer naïveté takes away. Recall that when consumers are forward looking and firms do not intertemporally bundle, tacit collusion is sustainable if and only if $n \leq 1/(1-\delta)$. When consumers are naïve, however, without intertemporal bundling tacit collusion is never sustainable.

Forward Contracts

When firms are allowed to offer forward contracts, they may attempt to simultaneously steal other firms' businesses in all market segments which otherwise open at different points of time and this may threaten the stability of tacit collusion. Whether such threat constitutes a real impact depends crucially on whether consumers are forward looking or naïve. Suppose consumers are naïve and fail to infer that the deviating firm's offering of discounted contracts will trigger a price war in the future. In this case, a deviating firm can offer infinitely long contracts starting in different periods to consumers who become available at different points of time to successfully attract all consumers. Moreover, there is no punishment enforceable on the deviating firm because its deviation allows it to capture all consumers for infinitely long. In other words, tacit collusion can never be sustained.

When consumers are forward looking, allowing forward contracts does not affect firms' abilities to tacit collude. This is because consumers will not let the deviating firm lock them in beyond the current period unless the deviating firm offers a free forward contract. As a result, the deviating firm cannot increase deviation profit by offering forward contracts.

Finitely-lived Consumers

Here we discuss two natural ways to model finitely-lived consumers. The first is a model in which a new cohort of consumers arrive in each period in overlapping generations and these consumers each live l periods and then exit the market forever.

The second is a model in which consumers arrive simultaneously every l periods and each live l periods at which time a new cohort of consumers arrive.

Model One

In this extension, consumers are exogenously partitioned into l submarkets each of which open at different points of time. For simplicity, suppose that no consumers are in the market at time 0. That is, the market grows each period from period 1 to period l, at which time the market reaches steady state.

Consider an equilibrium in which firms simply offer l-period contracts $\left(\sum_{s=1}^{l} \delta^{s-1}V, l\right)$ every period. The incentive constraint every period is

$$\frac{1}{n}\sum_{i=1}^{\infty}\sum_{s=1}^{l}\delta^{s-1}V \geq \frac{1}{l}V \tag{28}$$

which, by analogy to (7) through (10), holds as long as

$$\left[\sum_{j=1}^{k-1}\frac{\delta^{j}}{1-\delta}\right]+\frac{\delta}{1-\delta} \geq n-1. \tag{29}$$

Model Two

Next, we demonstrate how firms can tacit collude more easily in the second extended model. Here we propose a class of equilibria that we term as *Modified Staggered Contract Equilibria.*

The set of *Modified Staggered Contract Equilibria* are all SPNE in which strategies are of the following form:

Firms' equilibrium path actions: In the first period of the life time of each generation of consumers, each firm simultaneously offers l different subscription contracts of different lengths: $p_1 \leq V$ and $\left\{\left(p_1 + \sum_{s=2}^{\tau}\delta^{s-1}V, \tau\right)\right\}_{\tau=2}^{l}$. In period $\tau \in \{2, 3, ..., l\}$ within each generation, each firm offers a $(l-\tau)$-period contract $\left(\sum_{s=1}^{l-\tau}\delta^{s-1}V, l-\tau\right)$.

Consumers' equilibrium path actions: In the first period of the lifetime of each generation of consumers, an equal number of consumers accept each of the nl different contracts offers (l offers from each firm). In period $\tau \in \{2, 3, ..., l\}$ within each generation, an equal number of consumers (who aren't under contractual obligation) accept each of the firms' $(k - \tau)$-period contract offers.

Off-the-equilibrium-path actions: If any firm(s) deviates from these equilibrium strategies, every firm offers one-period, marginal-cost contracts forever. If any consumer(s) deviates by signing a contract different from what he is would sign in equilibrium in any period, firms continue to offer the same set of contracts the equilibrium prescribes that they offer in all the periods that follow.

The incentive compatibility constraint for firms staying collusive in periods 1, $l+1$,..., $al+1$,... is analogous to (20), with k replaced by l:

$$\pi_1 = \frac{1}{n}\sum_{i=1}^{\infty}\delta^{i-1}V - \frac{1}{n}\sum_{i=1}^{\infty}\delta^{(i-1)l}(V - p_1) \geq p_1. \tag{30}$$

The incentive compatibility constraint in the tth period of the current generation of consumers' lives is

$$\sum_{j=0}^{l-t}\sum_{i=j}^{l-t}\delta^i\frac{V}{nl} + \frac{1}{n}\delta^{l-t+1}\pi_1 \geq \frac{V}{l},\ t \in \{2, 3, .., l\}. \tag{31}$$

We now show that tacit collusion can be supported for some (δ, n) pairs which do not satisfy $n \leq 1/(1-\delta)$.

Claim 6 *Suppose a measure one of consumers arrive in periods* 1, $l+1$,..., $al+1$,... *and each live* l *periods, where* $l \geq 2$. *Modified Staggered Contract Equilibria exist for all* (δ, n) *satisfying* $n \leq 1/(1-\delta)$ *and also for some* (δ, n) *pairs such that* $n > 1/(1-\delta)$.

Proof. It is obvious that if $n \leq 1/(1-\delta)$, by setting $p_1 = V$ so that $\pi_1 = V/(1-\delta)$, both (30) and (31) are satisfied. However, (31) is also satisfied for some

$\hat{\delta}$ such that $n > 1/(1-\hat{\delta})$, i.e., $\hat{\delta} < (n-1)/n$, for all $t \in \{2, 3, ..., l\}$. Focus on $n > 1/(1-\delta)$. Since

$$\begin{aligned} \frac{d\pi_1}{dp_1} &= \frac{1}{n}\sum_{i=1}^{\infty}\delta^{(i-1)l} \\ &= \frac{1}{n\left(1-\delta^l\right)} < \frac{1}{n\left(1-\delta\right)} < 1, \end{aligned}$$

(30) is satisfied more easily for $p_1 < V$, there exists $p_1 < V$ such that (30) holds for some $\tilde{\delta} = (n-1)/n - \varepsilon$. By continuity, for ε sufficiently small, (31) still holds for some $\delta' \in (\hat{\delta}, (n-1)/n)$. Therefore, both (30) and (31) hold for $\delta'' = \max\{\tilde{\delta}, \delta'\} < (n-1)/n$. □

7 Summary and Discussion

In this paper, we have demonstrated how intertemporal bundling may help soften competition by facilitating tacit collusion. In one class of equilibria, by offering a menu of multi-period contracts of different lengths, firms break up the market into multiple segments each open at different points of time. Tacit collusion is easier to sustain because a deviating firm can steal business only in one market segment at one point of time but will be punished in every market segments.

In another class of equilibria, multi-period contracts are always offered with a discount on the first period's consumption/service. Moreover, when these multi-period contracts are offered, each firm also offers a discounted one-period contracts. In equilibrium consumers only sign up for multi-period contracts so firms compete infrequently. Firms have little incentive to deviate because consumers will respond to a deviation by accepting other firms' discounted one-period contract and then waiting for the price war to follow.

Our analysis crucially relies on the assumption that consumers are forward looking, being able to foresee a price war upon observing a deviation by any firm. When consumers are naïve about the relationship between deviation and price war, a deviating firm can capture large profit by offering life long contracts with a small discount, and this makes sustaining tacit collusion harder. In this case, among all the mechanisms we have studied in this paper, only by staggering multi-period contracts firms may be able to sustain tacit collusion.

Another important assumption we have made is that consumers are of zero measures and a positive measure of consumers cannot coordinate to collectively deviate. If a positive measure of consumers can collectively deviate, firms' ability to tacit collude will be affected. Suppose firms tacit collude on a Staggered Contract Equilibrium. If a significant measure of available consumers collectively refuse to purchase for one period, the total number of available consumers in the following period will increase substantially and this may cause the incentive compatibility constraint to break down. Knowing that, consumers will coordinate the collective deviation to trigger a price war. Similarly, if firms tacit collude on an Intermittent Discount Equilibrium, the deviation by a significant measure of consumers may also trigger a price war, as discussed in the end of section 4. In general, the larger is the measure of consumers who can collectively deviate, the shorter will be the multi-period contract firms can offer and the lesser will be the power of multi-period contracts in helping firms sustain tacit collusion.

The paper has some important empirical implications. First, firms margins are potentially sensitive to the expected lifetime of consumers and more generally to the feasibility of long-term contracts. If consumers are not long-lived, firms are unable to use long-term contracts. Similarly, firms margins are potentially sensitive to the stability of consumers demands over time. Absent predicable, stable demand, long-term contracts would be inefficient and unable to facilitate tacit collusion. However, as with any model of tacit collusion, there are multiple equilibria which presents a serious limitation to empirical analysis. While we show that the highest sustainable profit level is sensitive to consumer lifetime, it does not follow that the profit level must be higher when consumers lifetimes grow. As importantly, intertemporal bundling could alter the equilibrium payoffs through equilibrium selection even if the highest profit sustainable is unchanged.

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