A Welfare Analysis of Arbitration*

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Abstract

The paper compares conventional and final-offer arbitration from the welfare perspective. By some existing contractual arrangement, one party is supposed to make a payment to the other party, depending on the state of the world. Under asymmetric information, one party has a better signal about the state of the world and the other party is aware about its informational disadvantage. Then the ranking of the two arbitration procedures depends on the informational assumptions about the arbitrator. If the arbitrator is no better informed than the less-informed party, final-offer arbitration dominates conventional arbitration in the sense that the probability of arbitration award is no higher in any pair of equilibria, and it is lower for some pairs. If the arbitrator is no worse informed than the more-informed party, the opposite conclusion can be drawn. It is slightly more ambiguous under which of the two schemes the existing contractual arrangements are better approximated. Under non-common prior, parties believe that their opponents have wrong signals about the state of the world. Then the conventional arbitration better approximates the existing contractual arrangement, and (if parties are risk-averse) it also results in a lower probability of arbitration award.

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1 Introduction

Arbitration by third-party neutrals has become an often-used method of conflict resolution. It is frequently prescribed to resolve labor-management disputes when the labor unions are legally prohibited from striking, for example in public services such as police and fire protection. The use of arbitration in the settlement of disputes under existing contracts includes: buyers and sellers in commercial contracts, baseball players and club owners, and divorce settlements.[1]

Various compulsory-arbitration schemes are, or have been, in use in many states, yet other schemes have been proposed. Two main schemes are: *conventional arbitration*, where the arbitrator is free to impose any settlement that she of he wishes, and *final-offer arbitration*, where the arbitrator is constrained to choose one of the final (or last) offers of the disputing parties without any possibility of compromise.[2]

Conventional arbitration was implemented first, but a number of observers of early experience with this scheme (see Stevens (1966), Feuille (1975) and Feigenbaum (1975)) suggested that the arbitrators have a tendency of splitting the difference between the positions of the parties. This results in a *"chilling" effect* in bargaining, and implies excessive reliance on arbitration. Stevens (1966) proposed final-offer arbitration as a remedy designed to counteract this tendency. This form of arbitration was then implemented in some states and in some industries. For example, final-offer arbitration is used in the Major League Baseball, while conventional arbitration prevails in bargaining under some commercial contracts; Lester (1984) discusses in detail the use of both procedures in public-service collective bargaining in different

[1] Lester (1984), and Najita and Stern (2001) summarize the actual experience with binding arbitration of collective bargaining in public services, and Dworkin (1986) provides a review of highlights and controversies surrounding baseball's salary arbitration.

[2] Other schemes include: tri-offer arbitration, where the arbitrator is constrained to choose one of the final offers or the recommendation of a neutral fact-finder; an arbitration where each party makes two offers, the arbitrator announces the party that made better offers, and the other party chooses one of the offers made by the announced party; finally, if the conflict concerns several issues, parties make a final offer, specifying their position of each issue, and the arbitrator may either select one of the offers, or she or he may make a separate decision on each issue.

states. The subsequent theoretical and empirical literature often questions, or contains evidence against, the hypothesis that final-offer arbitration indeed counteracts the chilling effect (see, for example, Crawford (1979), Farber and Katz (1979), Farber (?), Chatterjee (1981), Brams and Merrill III (1983), Lester (1984), Ashenfelter et al. (1992)).

The reliance on arbitration, or the percentage of conflicts that end up with arbitration award, is the welfare criterion emphasized in the literature most. It reflects the view that a quality system makes parties reach a settlement without using the system. In particular, parties typicality incur a costs of arbitration, and it is believed that an agreement for which both parties are responsible is likely to be better for their future relations that one imposed by binding arbitration. Other welfare criteria include fairness (or appropriateness) of arbitrator's awards or the settlements reached by parties themselves, their freedom from biases, etc.; see again Lester (1984) for a discussion of the aims that were to be achieved by state laws providing for arbitration of negotiating impasses.

Assuming no uncertainty about the arbitrator's decision, Crawford (1979) proves that both conventional and final-offer arbitration lead (under reasonable assumptions) to equivalent outcomes. However, most of the literature, following Farber (1980), assumes that the optimal decision from arbitrator's perspective is unbiased in expectation but uncertain. The assumption that it is uncertain has firm support from the empirical literature (see Ashenfelter and Bloom (1984), Farber and Bazerman (1986), or Ashenfelter (1987)).[3]

With the uncertainty about the arbitrator's decision, the two arbitration procedures are no longer outcome-equivalent, but the existing literature provides little insight about the welfare comparison of the two procedures. First, it focuses on questioning the hypothesis that final-offer arbitration counteracts the chilling effect.

[3] The assumption that it is unbiased is usually motivated by the rules for arbitrators selection. Typically, an arbitrator is selected at least in part by a mutual agreement of the parties to a dispute; since parties can view the arbitrator's record in related arbitration cases, biased arbitrators are unlikely to be selected (see Bloom and Cavanagh (1986) for an analysis of the selection of arbitrators).

Second, the industrial relations literature typically measures the probability of reaching a settlement without a binding arbitrator's decision by the size of *contract zones* (see Farber and Katz (1979) and Farber (1980)). A contract zone is defined as the interval between the two settlements (one for each party) such that each party is indifferent between the settlement (for that party) and the expected arbitrator's award. Although some insight may be obtained in this way, the view that the probability of impasse is directly determined by the size of the contract zone is, quoting Crawford (1981), a serious oversimplification, one which may be misleading.[4][5]

This paper attempts to address the welfare questions within a game-theoretic model, where, in consistence with the existing literature (see Farber (?)), differences in parties' information are assumed to be the major explanation for the failure of collective bargaining. The main contribution seems to be pointing out that the conclusions depend on the nature of these differences, or more precisely, whether the bargaining failure is driven by assymetric information or non-common prior.

I study a two-stage model. In a bargaining stage (period 1), a settlement payment is offered by one party. If the other party rejects this offer, bargaining breaks down, and parties go for a binding arbitration (period 2). To capture differences in parties' information, I assume that there are two states of the world. According to some existing contractual arrangements, one party is supposed to make a higher, or lower, payment to the other party, depending on the state of the world. I study two scenarios. In the asymmetric information case, one party has a better signal about the state of the world and the other party is aware about its informational disadvantage. In the non-common prior case, parties believe that their opponents have wrong signals about the state of the world, and each party believes that the arbitrator will find out that its opponent's signal is wrong.

[4]It should be emphasized here that any welfare analysis of arbitration encounters the problem that bargaining must be inefficient, while the existing bargaining theories lead to, or assume, efficiency. Crawford (1981) elaborates on this issue.

[5]Another method of measuring the probability of reaching a settlement has been used by Chatterjee (1981). Namely, parties make final offers (in a one-shot game), and the settlement is reached if the payment demanded by one party does not exceed the payment offered by the other party.

Under asymmetric information, it is assumed that with a probability η the arbitrator can recognize the signal of the better-informed party. If η tends to 0, which seems to be a more reasonable assumption, the probability of rejecting the settlement payment is no higher in any equilibrium outcome of the final-offer arbitration game, and it is lower for some equilibrium outcomes, than that in a unique equilibrium outcome of the conventional arbitration game. The rough intuition is (see Section 4.3 for details) that final offers can be used as a signalling device. When η is small, all equilibria are close to pooling. It is believed in some equilibria that the better-informed party to prove that its information supporting its case is "strong" makes final offers that would not be optimal if the arbitrator made no inference from the final offers. This makes the better-informed party willing to accept lower settlement payments. It turns out that the probability of accepting the settlement payment by the better-informed party with "weak" information supporting its case may increase compared to that with no signalling device (conventional arbitration).

If η tends to 1, the probability of rejecting the settlement payment is no lower in any equilibrium outcome of the final-offer arbitration game, and it is higher for some equilibrium outcomes, than that in a unique equilibrium outcome of the conventional arbitration game. The rough intuition is (see Section 4.4 for details) that when η is large, all equilibria are close to separating. Then the only effect of replacing conventional with final-offer arbitration is that the better-informed party can extract informational rents, as its opponent have to make the final offer not knowing the signal of the better-informed party. Thus, the better-informed party demands higher settlement payments. It turns out that the probability of accepting the settlement payment by the party with "weak" information supporting its case may decrease compared to that with no informational rents (conventional arbitration).

Under non-common prior, some sort of dutch-book argument applies. Parties make "exaggerated" final offers trying to take advantage of their opponents being wrong. Assuming that each party is wrong half of times[6], the "exaggerated" final offers expose the parties to more uncertainty, compared to conventional arbitration,

[6] This assumption is rather inessential for the argument.

and move the arbitration outcome away from the existing contractual arrangement.

I follow Farber (1980) in the modelling of arbitration procedures. That is, I assume that the optimal decision from arbitrator's perspective is unbiased in expectation but uncertain. Under conventional arbitration, the arbitrator just imposes the optimal decision from his or her perspective, and under final-offer arbitration, the final offer that is closer to the optimal decision is picked.

Some other ingredients of the present model are also known from the literature. In Chatterjee (1981), agents have private information about the cost of arbitration, and in Samuelson (1991), the private information is introduced in a similar manner to the present model. Gibbons (1988) emphasizes the role of learning by the arbitrator from the parties' offers about the state of the employment relationship, which is known to the parties but not to the arbitrator.

It is tempting, but unfortunately it turns out rather inconclusive, to relate the present results to the data. Dworkin (1986) reports that in the Major League Baseball, where final-offer arbitration is in use, most cases are settled short of arbitration, and those that are not are often characterized by widely divergent salary positions between the parties (with a similar number of winners on each side). The case of the Major League Baseball is particularly interesting for the present paper, because there seem to be little room for asymmetric information between baseball players and club owners.[7] Under non-common prior, my model indeed predicts widely divergent final offers[8], it makes no prediction regarding the frequency of cases that are settled short of arbitration, and it suggests that conventional arbitration is better from the welfare perspective.

There is also substantial evidence from public-service sector, which unfortunately is rather ambiguous. For example, Lester (1984) reports that (conventional) arbitration awards averaged around 29 percent of total negotiations in 1960s 1970s in Philadelphia, while in the New York State, where conventional arbitration was also

[7] I would like to thank Jean-Pierre Benoit for communicating this point to me.

[8] One should, however, be cautious about this consistency, because there is an evidence that even players who lose their cases in arbitration gain substantial salary increases, which suggests that, in practice, arbitration in baseball departs from the final-offer scheme.

in use, awards declined from 15.9 percent of total negotiations to 9.4 percent between 1975-1976 and 1982-1983. For comparison, the average of (final-offer) arbitration awards to total negotiations in Michigan was 16.4 percent in the period 1973-1974 through 1976-1977, and 6.7 percent for the six-year period 1977-1978 through 1982-1983 with the figure for 1982-1983 rising to 12 percent.[9] Additionally, it is rather difficult, based on the available data, to identify the sources of collective-bargaining failure in the public services. Not only I cannot say which of the two: asymmetric information and non-common prior is more important, but there could be also other sources; for example, Farber (?) points out that another important reason of bargaining failure is that at least one party may want to place the responsibility for an unfavorable outcome on the shoulders of a third party (the arbitrator).

2 Model

There are two eqi-probable states of the world: a and b, and two risk-neutral interacting agents. In the existing literature on arbitration, risk-aversion plays an important role. Here, I assume risk-neutrality, because all my arguments, but one, do not require assuming of risk-aversion, and risk-aversion makes the analysis substantially more complicated. The place that requires the assumption of risk-aversion is the welfare comparison of arbitration procedures under non-common prior (see Section 4 for details). Agent A is supposed to make the payment of \$1 to agent B contingent on state b; no payment is due in state a. Agent B can search for evidence that the state is b, and if it is so she finds the evidence with certainty.[10] She does not find any evidence if the state is a. Then agent B can present the evidence to agent A. Agent A recognizes the evidence with probability ξ. With probability $1 - \xi$, agent A does not recognize the presented evidence, which means that by itself the evidence does

[9]My impression, based on the data from the public services, is that factors other than the choice of conventional versus final-offer arbitration may contribute more to the success or failure of arbitration laws.

[10]Results would not change qualitatively if player B found the evidence with a positive probability only.

not affect agent A's prior about the state of the world.

I analyze two scenarios. Under *asymmetric information (AI)*, if agent B finds no evidence, and therefore knows that the state of the world is a, she can, with probability $1-\xi$, forge some "evidence" and present to agent A. Agent A certainly does not recognize the forged "evidence", but she does not recognize that the "evidence" has been forged either, i.e. the evidence by itself does not affect agent A's prior about the state of the world.[11] With probability ξ, agent B cannot forge the "evidence", or (which leads to the same outcome) the other agent and any third party recognize with certainty that the "evidence" has been forged. The possibility of forging evidence creates asymmetric information, i.e. agent B learns the state of the world while agent A does not know if she was unable to recognize the evidence or the presented "evidence" has been forged by agent B. It would not change qualitatively the results, if agent A learned with a positive probability that the "evidence" has been forged; it would increase the number of cases to be considered.

Under *non-common prior (NP)*, there is no possibility of forging evidence, or agent A learns with certainty when the presented "evidence" has been forged. It would again not change the results qualitatively, if agent A learned only with a positive probability that the "evidence" has been forged, but it would increase the number of cases to be considered. If agent A is unable to recognize the evidence, she assumes that the state of the world is a, and she believes that any third party will assume that the state of the world is a as well. For symmetry, I assume that when the state of the world is a it happens with probability $1-\xi$ that agent B believes that she found "evidence" (and any third party will recognize this "evidence"), but neither agent A nor any third party are able to recognize this "evidence".

Both under AI and NP, the timing of the following events is the same: In Period 1, agent A offers a (settlement) payment to agent B, which is assumed to be non-negative. Agent B can either accept or reject the payment; in the former case the game ends in period 1. In the latter case, agent B asks for arbitration. Then an

[11] The probability of forging evidence is assumed to be $1-\xi$ to ensure that player A who does not recognize the evidence presented by player B finds both states of the world eqi-probable.

arbitrator decides about the amount of payment that agent A makes to agent B. The arbitrator's decision is enforceable. The arbitration happens in period 2 and imposes a legal cost c on each player. This cost includes explicit costs: arbitrators' fees, stenographic expenses, renting a room for hearing, etc., but also implicit costs (e.g. time) of being involved in an arbitration procedure. It in practice need not be the same for both parties[12], and this assumption is inessential but convenient. It is sensible to assume that

$$c < \frac{1}{2}.$$

In the present paper, I wish to compare two forms of arbitration: In *Conventional Arbitration* game, an arbitrator listens to the evidence presented by agent B; if the evidence has been recognized by agent A, the arbitrator recognizes it as well. Moreover, under AI, the arbitrator recognizes the evidence (learns that the state is b) with a probability $\eta > 0$ when agent A does not, and the arbitrator also learns with probability η that the "evidence" is forged when agent A does not. Under NP, if agent B does not recognize the evidence, the arbitrator does not recognize it either. Then the arbitrator assumes that both states of the world are eqi-probable.

Of course, the arbitrator makes also statistical inference from equilibrium strategies; for example if agent B asks (under AI) for arbitration only if she knows that the state of the world is b, then the arbitrator knows as well that the state is b (whenever she is asked for arbitration) even if she is unable to recognize agent B's evidence. Following Farber (1980), I assume that the arbitrator is statistically objective but she makes systematic mistakes. That is, under conventional arbitration, agent A has to make the payment of

$$\pi = p_b(1+c) + (1-p_b)(-c) + \varepsilon, \tag{1}$$

where p_b denotes the arbitrator's probability that the state of the world is b, and ε is a noise term.[13] For convenience, define p_a as $1 - p_b$.

[12]The explicit cost are shared equally in most states, but not always (e.g. Pennsylvania); see Lester (1984).

[13]It is assumed here that if an agent loses the case, she has to reimburse the other agent for the

I assume that ε is distributed symmetrically, uni-modally around 0, which implies that $E\varepsilon = 0$; I also assume that ε has a density $f(\pi_\varepsilon)$ satisfying the monotone hazard-rate condition

$$\frac{d}{d\pi_\varepsilon}\left(\frac{f(\pi_\varepsilon)}{1 - F(\pi_\varepsilon)}\right) > 0, \tag{2}$$

where F stands for the cumulative distribution function. Given the assumption that f is symmetric around 0, the monotone hazard-rate condition is equivalent to the log-concavity of F, i.e. to

$$\frac{d}{d\pi_\varepsilon}\left(\frac{f(\pi_\varepsilon)}{F(\pi_\varepsilon)}\right) < 0. \tag{3}$$

The monotone hazard-rate rate assumption is satisfied for many probability distributions of interest, e.g. uniform, normal, logistic, chi-squared, exponential, Laplace, and (under some restrictions on the parameters) Weibull, gamma and beta. The role of the monotone hazard rate assumption will be to guarantee that there always exists equilibria in the final-offer arbitration game.

In *Final-Offer Arbitration* game, an arbitrator listens to the evidence presented by agent B. Then each agent suggests a payment (makes a final offer), π_A and π_B, and the arbitrator selects the final offer that is closer to π given by (1), i.e. agent A has to pay π_A if

$$\mid \pi_A - \pi \mid < \mid \pi_B - \pi \mid$$

and she has to pay π_B if the opposite inequality holds. Assume that the arbitrator tosses a fair coin in the case of equality. The arbitrator can elicit final offers in conventional arbitration; moreover, this usually happens in practice. The difference between the two procedures is that under conventional arbitration, the arbitrator is not committed to choose of the final offers.[14] Note that the arbitrator has no

legal cost. It is typically not the case in practice (see, for example, Lester (1984)). The assumption is inessential, but convenient in this particular model.

[14]In first years of arbitration in New Jersey, a number of award cases were prepared and mediated under final-offer arbitration. When the parties has moved their position close enough together, the case may have been converted to conventional arbitration to facilitate the drafting of the agreement and to avoid the arbitrator having to select a winner.

information about the state of the world on top of what players A and B know, which could help in making the decision.

To conduct a welfare analysis the two forms of arbitration, I apply two criteria. First, arbitration is costly as players must pay the legal cost. One would like therefore to keep the probability of asking for arbitration as low as possible. More precisely, I compare

$$2cq,$$

where q denotes the probability of asking for arbitration.[15] Call this criterion the comparison of *deadweight-losses*; I will often say that the deadweight-loss of one form of arbitration exceeds the deadweight loss of another.

Denote by w_A, respectively w_B the total payoff of agent A, respectively agent B. Note that $w_B = -w_A = \pi$ is the payment made by agent A in period 1 if the game ends in period 1, and $w_B = \pi - c$ and $w_A = -\pi - c$ if the game ends in period 2 and agent A makes payment π to agent B.

Second, one would wish to depart as little as possible from the outcome, $w_B = 1$ and $w_A = -1$ in the state b and $w_A = w_B = 0$ in the state a. This is so for a number of reasons, e.g. hold-up problems or facilitating future contracting, etc. In the present paper, I do not model any specific reason. I simply compare the loss caused by departures from the first-best outcome

$$E[l(|w_A + 1|, |w_B - 1|) \mid b] + E[l(|w_A|, |w_B|) \mid a],$$

where l stands for an increasing in both argument (loss) function, which takes value 0 at the point 0. Call this criterion the comparison of *outcome-accuracy*. I will say that one form of arbitration is more outcome-accurate than another.[16]

[15] See Section 5 for a generalization of this criterion to the case of risk-averse agents.

[16] It may be more sensible to disregard the cost of arbitration in the variables of l. More precisely, one may wish to replace w_A and w_B with variables $w_A^{/}$ and $w_B^{/}$ such that the legal costs get reimbursed at the very end of the game (assuming, of course, that this reimbursement does not affect the equilibrium strategies). However, it will not the results of this paper, making some reasonings slightly more complicated.

3 Asymmetric Information

3.1 Conventional Arbitration

Begin the analysis with the characterization of equilibrium under conventional arbitration. The assumptions on information structure imply that there are tree possibilities: (1) Both agents know the state of the world; (2) Agent B knows that the state is b, but agent A does not recognize agent B's evidence; (3) Agent B knows that the state is a, but she is able to forge "evidence". The analysis of possibility (1) is straightforward. Since players have the same expectation about the arbitrator's decision, agent A offers the settlement payment that makes agent B exactly indifferent between accepting and rejecting the offer (that is, 0 and 1 in the state a and b, respectively). Agent B accepts this offer and the game ends in period 1.

The possibilities (2) and (3) have to be analyzed simultaneously. Recall that agent B knows the state of the world, and agent A does not and she believes that both possibilities are equally likely. Notice that agent B who knows that the state of the world is a (call him type a) is willing to accept a strictly lower settlement payment than agent B who knows that the state of the world is b (call him type b); this is so, because there is a positive probability η that the arbitrator recognizes the evidence even when agent B does not as well as the arbitrator learns with the probability η that the "evidence" has been forged although B does not.

There are therefore five possible responses to the equilibrium offer of settlement payment: (a) both types reject the offer; (b) type a randomizes and type b rejects; (c) type a accepts and type b rejects; (d) type a accepts and type b randomizes; (e) both types accept the offer. Observe that (d) cannot happen on equilibrium path. Indeed, this would mean that the arbitrator knows in period 2 that agent B's type is b. Therefore the settlement payment in period 1 would have to be equal to 1 to make agent B indifferent between accepting and rejecting the settlement payment. Suppose now that agent A proposes in period 1 a settlement payment higher by a little ε. Both types of agent B accept this higher payment. This is so, because otherwise they face the arbitrator's decision in period 2 and the arbitrator at best assumes that agent B

is of type b; however, the payoff to this decision is equal to 1 for type b, and it is even lower for type a, whose forged "evidence" can be detected by the arbitrator[17]. Since agent A does not have to pay the legal cost by offering the slightly higher payment, and she pays even less in expectation, as she does not have to pay agent B's legal cost, she strictly prefers the higher payoff provided ε is small enough.

Observe further that if (c) happens on equilibrium path, then type a must be indifferent between accepting and rejecting the settlement payment or the settlement payment must be 0. Indeed, if she strictly preferred accepting and the settlement payment were not 0, then agent A would strictly prefer offering a slightly lower settlement payment. Such a payment would still result in a higher payoff for type a than facing the arbitrator's decision in period 2 contingent on the information agent B is of type b, and paying the legal cost. It would be therefore accepted by type a, which would obviously make the payoff of agent A strictly higher if type b rejected this offer. It can happen that the alternative settlement payment gets accepted by type b (if the arbitrator beliefs that agent B is of type b only with probability 1/2 contingent on rejecting the alternative settlement payment), but this makes the payoff of agent A even higher.

Consider first (potential) equilibria with the settlement payment s that makes type a indifferent between accepting and rejecting the settlement payment. Let β denote the probability that agent B is of type b assigned by the arbitrator contingent on rejecting the settlement payment. Note that the arbitrator assigns exactly the same probability that agent B is of type b when she neither recognizes the evidence nor that the "evidence" has been forged. This is so, because the arbitrator recognizes the evidence and that the "evidence" has been forged with the same probability η. Since type a is indifferent between accepting and rejecting, type a has to be indifferent between s and the lottery

$$-c+\varepsilon-c \text{ with probability } \eta \text{ and } \beta(1+c)-(1-\beta)c+\varepsilon-c \text{ with probability } 1-\eta, \quad (4)$$

[17] Also, the arbitrator can recognize type b's eveidence.

i.e this offer s has to be equal to

$$s_\beta := (1-\eta)\beta - 2c\,[\eta + (1-\eta)(1-\beta)] \tag{5}$$

for some $\beta \in [1/2, 1]$.

Observe that if η is not too large, $s_\beta \geq 0$ for every $\beta \in [1/2, 1]$, assuming that $c < 1/2$. For large η, it may happen that $s_\beta < 0$. Then, since I assumed that the settlement payment must be non-negative, it cannot happen that β is the probability that agent B is of type b assigned by the arbitrator contingent on rejecting the settlement payment. Then I have to consider the interval of $[\beta^*, 1]$, where $s_{\beta^*} = 0$[18] instead of the interval $[1/2, 1]$. In the extreme case when $s_\beta < 0$ even for $\beta = 1$, the only settlement payment to be considered is 0, which is accepted with probability 1 by type a.

I shall now show that, in equilibrium, the settlement payments from the interval $[s_{1/2}, s_1]$ (from the interval $[0, s_1]$, respectively) that can be offer by agent A has to maximize her payoff over this interval, assuming that rejecting s_β results in the belief β that agent B is of type b.

Indeed, suppose that she offers s_β but s_γ yields a higher payoff. Then she may be willing to offer s_β only when the off equilibrium offer s_γ does not result in the belief γ that agent B is of type b contingent on rejecting it. If it resulted, however, in a higher (than γ) belief, then type a would have to reject the settlement payment of s_γ, but then rejecting it cannot result in the higher belief. If it resulted in a lower belief, then type a would have to accept the settlement payment of s_γ, and the lower belief would be possible only if type b accepted it as well; this, however, makes the payoff of agent A even higher compared to the case when s_γ is rejected by type b and accepted by type a with some probability.

Moreover, the settlement payments s_β that maximize agent A's payoff over the interval $[s_{1/2}, s_1]$ (from the interval $[0, s_1]$) can appear in equilibrium only if they yield to agent A a (weakly) higher payoff than $s = 1$ that is accepted with probability 1 by both types of agent B, because making any offer higher than 1 guarantees that both

[18] Notice s_β increases with β.

types of agent B accept.

On the other hand, it readily checks that if the constraint from the previous paragraph is satisfied, then for every $s_\beta \in [s_{1/2}, s_1]$ (from the interval $[0, s_1]$) that maximizes agent A's payoff over this interval (assuming that rejecting s_β results in the belief β that agent B is of type b), there exists an equilibrium where she offers the settlement payment s_β. In this equilibrium, the belief that agent B is of type b contingent on rejecting the settlement payment $s < s_{1/2}$ is $1/2$ (no payment lower than $s_{\beta^*} = 0$ is allowed, respectively) and no type accepts any payment $s < e_{1/2}$; it is γ for every $s_\gamma \in [s_{1/2}, s_1]$, and only type a accepts s_γ with the probability q determined by the equation

$$\frac{1}{2-q} = \gamma \tag{6}$$

type b rejects this settlement payment; finally, it is 1 for $s > s_1$, type a accepts any such offer, and type b accepts only offers $s \geq 1$. Note that if $s_1 < 0$, then it is always an equilibrium (assuming that $c < 1/2$) to offer no settlement payment that is accepted by type a and rejected by type b.

To find out s_β that maximizes agent A's payoff I shall estimate the marginal cost and benefit of increasing the β. Increasing the β implies that type a of agent B accepts the settlement payment with a higher probability, and agent A saves then on the legal cost as well as she extracts the premium that agent B is willing to pay in order to avoid the legal cost. This means that the marginal benefit of increasing β is

$$(2c)\frac{1}{2}\frac{dq}{d\beta} = \frac{c}{\beta^2}, \tag{7}$$

where q has been calculated from (6) for $\gamma = \beta$.

Increasing the β implies also making a higher payment. This higher payment has to be made both when agent B accepts the settlement payment and, because of an increase in the arbitrator's belief of the state of the world being b, also when she rejects the settlement payment. The marginal cost of increasing β is

$$\frac{ds_\beta}{d\beta} = (1-\eta)(1+2c), \tag{8}$$

where $ds_\beta/d\beta$ has been derived from (5).

The marginal cost is therefore constant, and the marginal benefit is decreasing in β, and so the optimal β is when the two are equal, i.e.

$$\beta = \sqrt[2]{\frac{c}{(1-\eta)(1+2c)}}, \tag{9}$$

unless the right-hand side exceeds 1, and then $\beta = 1$ in equilibrium, or the right-hand side falls below 1/2 (respectively, β^*), and then $\beta = 1/2$ (respectively, $\beta = \beta^*$) in equilibrium.

For example, for $\eta = 2/5$ and $c = 1/4$,

$$\beta = \sqrt[2]{\frac{5}{18}} \in \left(\frac{1}{2}, 1\right).$$

It is easy to check that, for these parameters, $s_{1/2} > 0$ and even $s_{1/2}$ yields to agent A a (weakly) higher payoff than $s = 1$ that is accepted with probability 1 by both types of agent B.

Consider now the two other sorts of (potential) equilibria. Notice that if (a) can happen on equilibrium path, then the settlement offer cannot be higher than $s_{1/2}$, because type a cannot reject (with probability 1) any offer higher than $s_{1/2}$. It is easy to see that the same equilibrium outcome obtains with the settlement offer equal to $s_{1/2}$, which is rejected by both types of agent B. Finally, an analogous argument to that from the case where type a is indifferent between accepting and rejecting the settlement payment, yields that $s_{1/2}$ has to maximize agent A's payoff over all offers $s_\beta \in [s_{1/2}, s_1]$, assuming that rejecting s_β results in the belief β that agent B is of type b. That is, if an equilibrium where both type of agent B reject the settlement payment exists, the same equilibrium outcome obtains in an equilibrium described in previous paragraphs.

If (e) happens on equilibrium path, the settlement payment s has to (weakly) exceed $r_{1/2}$, the expected value of

$$1 + \varepsilon \text{ with probability } \eta \text{ and } (1+c)/2 - c/2 + \varepsilon - c \text{ with probability } 1 - \eta,$$

as type b would not accept it otherwise; moreover, agent A has to (weakly) prefer making such an offer s, assuming that it will be accepted by both types, to making any offer $s_\beta \in [s_{1/2}, \min(s_1, s)]$ (or, respectively, $s_\beta \in [0, \min(s_1, s)]$), assuming that rejecting s_β results in the belief β that agent B is of type b; again it follows from an analogous argument to that from the case where type a is indifferent between accepting and rejecting the settlement payment. In the extreme case when $s_\beta < 0$ even for $\beta = 1$, agent A has to (weakly) prefer making such an offer s to making no settlement payment, assuming that it will be accepted by type a and rejected by type b.

On the other hand, it readily checks that there exists an equilibrium where agent A offers a settlement payment s satisfying the two constraints. In this equilibrium, the belief that agent B is of type b contingent on rejecting the settlement payment $t < s_{1/2}$ is $1/2$, and no type of agent B accepts such a settlement payment; it is β for $s_\beta \in [s_{1/2}, \min\{s_1, s\})$ (respectively, $[0, \min\{s_1, s\})$), and it is accepted by type a with the probability q determined by (6) and rejected by type b; it is 1 for $t \in [s_1, s)$ (for $t \in [0, s)$ in the extreme case when $s_1 < 0$) and it is accepted only by type a; it is again $1/2$ for $t \geq s$ and it is accepted by both types of agent B.

Thus, for some range of the parameters of the model there exist equilibrium outcomes, in which the settlement payment is accepted by type a with some probability and it is rejected by type b; for another range of parameters there exist equilibrium outcomes, in which the settlement payment is accepted by both types of agent B; there is also a range of parameters of the model with equilibrium outcomes of each of the two sorts.

Assuming that rejecting s_β results in the belief β that agent B is of type b, let U^A_β denote the expected payoff of agent A from making the offer s_β.

Proposition 1 *The conventional arbitration game may have two sorts of equilibrium outcomes.*

*(**a**) In both sorts, if player B finds evidence and player A recognizes the evidence of player B, then agent A offers the settlement payment of* \$1*, and this payment gets accepted by agent B. If player B does not find the evidence and she cannot forge it, then agent A offers no payment, and this gets accepted by agent B.*

*(**b**) In one of the two sorts of equilibrium outcomes, if player A does not recognize*

the evidence of player B, then she offers the settlement payment from the interval $[s_{1/2}, s_1]$ (respectively, $[0, s_1]$) that maximizes her payoff over this interval, assuming that rejecting s_β results in the belief β that agent B is of type b. This settlement payment gets accepted by type a of agent B with probability q determined by (6), and it gets rejected by type b of agent B.

Such an equilibrium outcome exists if and only if

$$\max_{\beta\in[1/2,1]} U_\beta^A \geq -1 \text{ (respectively, } \max_{\beta\in[\beta^*,1]} U_\beta^A \geq -1\text{).}$$

In the extreme case when $s_\beta < 0$ even for $\beta = 1$, the equilibrium of this sort always exists; the settlement payment is equal to 0, it is accepted with probability 1 by type a and it is rejected with probability 1 by type b.

(c) *In the other sort of equilibrium outcomes, if player A does not recognize the evidence of player B, then she offers the settlement payment $s \geq r_{1/2}$, and this settlement payment gets accepted by both types of agent B.*

This equilibrium outcome exists if and only if

$$\max_{\beta\in[1/2,\gamma]} U_\beta^A \leq -s \text{ (respectively, } \max_{\beta\in[\beta^*,\gamma]} U_\beta^A \leq -s\text{),}$$

where $\gamma = 1$ if $s \geq s_1$ and $s = s_\gamma$ otherwise. In the extreme case when $s_1 < 0$, this existence condition becomes

$$-\frac{1}{2}(1+2c) \leq -s.$$

3.2 Final-Offer Arbitration

As it is usually the case with signalling games, the subgame beginning in period 2 has multiple equilibria, including separating, pooling, and a number of sorts of hybrid equilibria. It will be of no use to characterize all of them. This multiplicity typically precludes general conclusions. I will therefore analyze only the case of η close to 0 and η close to 1. The case of η close to 0 seems more consistent with the spirit of the literature, as it assumes that the arbitrator does not have informational advantage over any disputing party. I decided to include the case of η close to 1, because it exhibits an important effect, which appears for any η and which becomes dominant, and therefore easier to describe, for η close to 1.

I wish to point out only two observations that apply to any η. First, in Appendix I prove the following existence result:

Proposition 2 *Under the additional assumption that final offers have to belong to an interval $[-C, 1+C]$, where C can be arbitrarily large but exogenously given, there exists an equilibrium of the two-stage, final-offer arbitration game.*

I do not know if the additional assumption is essential. A general problem with the present analysis of final-offer arbitration under asymmetric information is that the *single-crossing property* is not guaranteed by the assumptions (2) and (3). Thus, equilibria can be of the form, rather different than for signalling games studied in the literature, and to prove the existence result I refer to a fixed-point argument. The single-crossing property can be obtained by some (rather mild) additional assumptions on the noise term ε, but it is not necessary for the objectives of the present paper. Similarly, the requirement that final offers belong to an interval $[-C, 1+C]$ is dispensable under some additional assumptions on ε.

Second, the form of equilibria of the two-stage, final-offer arbitration game is similar to that under conventional arbitration. More specifically, either the settlement payment is accepted by type a with a probability $q \in [0, 1]$ and it is rejected by type b with probability 1 or both types accept the settlement payment with probability 1. It follows from the same arguments as under conventional arbitration.[19]

[19] The difference is that under-final offer arbitration there exist equilibria such that type a accept with probability 1, type b rejects with probability 1, yet type a is not indifferent between accepting and rejecting. The argument that agent A could offer then a slightly lower payment that would be accepted by type a does not work here, as agent B may anticipate a "better" equilibrium in period 2 contingent on the rejection of that slightly lower payment, compared to that contingent on the rejection of the equilibrium payment.

3.3 The case of small η

Under conventional arbitration, it follows from (9) that for $c < 1/2$ there exists $\overline{\eta} > 0$ such that for every $\eta < \overline{\eta}$ agent A offers in equilibrium a settlement payment that is either rejected with probability 1 by both types of agent B or accepted with probability 1 by both types of agent B. If agent A offers a settlement payment that is rejected, then her expected payoff is

$$U^A_{1/2} = -\eta\frac{1}{2}(1+2c) - (1-\eta)(\frac{1}{2}+c),$$

which is preferred by agent A to making the payment of 1 that is accepted by both types of agent B (maintaining the assumption that $c < 1/2$). Thus this equilibrium always exists.

There is a continuum of equilibria, in which agent A offers a settlement payment that is accepted. The settlement payment in this sort of equilibria varies from

$$\eta + (1-\eta)(\frac{1}{2} - c)$$

to $-U^A_{1/2}$ (see Proposition 1).

Under final-offer arbitration, the game has multiple equilibria. First, observe that every outcome of the conventional arbitration game can be "approximately replicated". More precisely, consider the pooling equilibrium of the subgame beginning in period 2, where agent B is of type b with probability β, in which each agent plays the best-response to her opponent's offer. In this equilibrium, player A chooses π_A to minimize and player B chooses π_B to maximizes

$$F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\pi_A + \left[1 - F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\right]\pi_B,$$

where F^β is the *cdf* of $\beta(1+c) + (1-\beta)(-c) + \varepsilon$. The offers are, thus, jointly determined by two first-order conditions:

$$\frac{\pi_B - \pi_A}{2} = \frac{F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)}{f^\beta(\frac{\pi_A+\pi_B}{2})},$$

$$\frac{\pi_B - \pi_A}{2} = \frac{1 - F^{\beta}\left(\dfrac{\pi_A + \pi_B}{2}\right)}{f^{\beta}(\dfrac{\pi_A + \pi_B}{2})},$$

If follows immediately from the first-order conditions that $\pi_A < \pi_B$ and $\beta(1+c) + (1-\beta)(-c)$ is the middle of the segment $[\pi_A, \pi_B]$; with probability $1-\eta$, each of the two final offers is chosen by the arbitrator with probability 1/2, and with probability η the offer of agent A is chosen in the state a and the offer of agent B is chosen in the state b. When η is small, this yields agent B the total payoff of approximately $\beta(1+c)+(1-\beta)(-c)$, and the payoff of agent A is approximately equal to the negative of this number, where "approximately" can be replaced with "exactly" for $\eta = 0$.[20]

It is easy to see now that every equilibrium outcome of the two-stage, conventional arbitration game can be approximated in an equilibrium of the two-stage, final-offer arbitration game, in which agents anticipate such pooling equilibria in period 2 contingent on rejecting the corresponding settlement payments in period 1.

Now, consider the pooling equilibrium of the subgame beginning in period 2, where agent B is of type b with probability β, in which agent A plays the best-response to her opponent's offer assuming that F^{β} is the *cdf* of the arbitrator's peak points and agent B plays the best-response to her opponent's offer assuming that $F^{1/2}$ is the *cdf* of the arbitrator's peak points. Notice that this is indeed an equilibrium, letting for any the out of equilibrium offer of agent B agent A and the arbitrator believe that agent B is of type b with probability 1/2.

The offers in this equilibrium of are jointly determined by two first-order conditions:

$$\frac{\pi_B - \pi_A}{2} = \frac{F^{\beta}\left(\dfrac{\pi_A + \pi_B}{2}\right)}{f^{\beta}(\dfrac{\pi_A + \pi_B}{2})}, \qquad (10)$$

[20]The result that outcomes of the conventional arbitration game can be "approximately replicated" under final-offer arbitration typically does not hold under risk-aversion, because parties are exposed to different type of risk in period 2 under the two procedures. In early literature on arbitration (see Farber (?) for a summary of this literature), this different type of risk was perceived as the main source of different welfare consequences of the two procedures.

$$\frac{\pi_B - \pi_A}{2} = \frac{1 - F^{1/2}\left(\frac{\pi_A + \pi_B}{2}\right)}{f^{1/2}(\frac{\pi_A + \pi_B}{2})}. \tag{11}$$

I shall describe an equilibrium of the two-stage game, in which players anticipate pooling equilibria described in the pervious paragraph in period 2, and the settlement payment is accepted with a probability by type a and it is rejected with probability 1 by type b. This settlement payment s must make type a indifferent between accepting and rejecting, an so s has to be equal to

$$t_\beta := -2c\eta + (1-\eta)\left\{\pi_A F^\beta\left(\frac{\pi_A + \pi_B}{2}\right) + \pi_B\left[1 - F^\beta\left(\frac{\pi_A + \pi_B}{2}\right)\right]\right\}, \tag{12}$$

where π_A and π_B are jointly determined by (10) and (11). Further, the equilibrium settlement payment t_β must maximize the payoff of agent A over this interval $[d_{1/2}, d_1]$, assuming that rejecting t_β results in the belief β that agent B is of type b.

The equilibrium β can be therefore determined by comparing marginal benefit and marginal cost of increasing β. The marginal benefit is given by (7), and the marginal cost is given by

$$\frac{dt_\beta}{d\beta} = \frac{d\left\{(1-\eta)\pi_A F^\beta\left(\frac{\pi_A + \pi_B}{2}\right) + (1-\eta)\pi_B\left[1 - F^\beta\left(\frac{\pi_A + \pi_B}{2}\right)\right]\right\}}{d\beta}.$$

Lemma 3

$$\frac{dt_\beta}{d\beta} < \frac{ds_\beta}{d\beta}$$

Recall that s_β denotes the settlement payment in the conventional arbitration game that makes type a indifferent between accepting and rejecting, assuming that rejecting s_β results in the belief β that agent B is of type b.

Proof.

$$\begin{aligned}
&\frac{d\left\{(1-\eta)\pi_A F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)+(1-\eta)\pi_B\left[1-F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\right]\right\}}{d\beta}\\
=\;&\frac{\partial\left\{(1-\eta)\pi_A F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)+(1-\eta)\pi_B\left[1-F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\right]\right\}}{\partial\pi_A}\cdot\frac{d\pi_A}{d\beta}+\\
&\frac{\partial\left\{(1-\eta)\pi_A F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)+(1-\eta)\pi_B\left[1-F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\right]\right\}}{\partial\pi_B}\cdot\frac{d\pi_B}{d\beta}+\\
&\frac{\partial\left\{(1-\eta)\pi_A F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)+(1-\eta)\pi_B\left[1-F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\right]\right\}}{\partial\beta}
\end{aligned}$$

The first term on the right-hand side equals 0 due to the first-order condition (10). The first term equals to

$$\begin{aligned}
&(1-\eta)\left\{\frac{\pi_A-\pi_B}{2}f^\beta(\frac{\pi_A+\pi_B}{2})+\left[1-F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\right]\right\}\frac{d\pi_B}{d\beta}\\
=\;&(1-\eta)\left[1-2F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\right]\frac{d\pi_B}{d\beta}
\end{aligned}$$

due to the first-order condition (10), and it can be computed directly that the third term equals to

$$(1-\eta)(1+2c)\left(\pi_B-\pi_A\right)f^\beta(\frac{\pi_A+\pi_B}{2})=(1-\eta)(1+2c)2F^\beta\left(\frac{\pi_A+\pi_B}{2}\right),$$

again due to the first-order condition (10). It follows from the first-order conditions (10) and (11) that

$$\frac{d\pi_B}{d\beta}<(1+2c)$$

(see Figure 1(b)). Finally,

$$F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)<\frac{1}{2};$$

indeed, in Figure 1(a) the curve $\frac{\pi_B-\pi_A}{2}=1-F^{1/2}\left(\frac{\pi_A+\pi_B}{2}\right)/f^{1/2}(\frac{\pi_A+\pi_B}{2})$ obtains by moving the curve $\frac{\pi_B-\pi_A}{2}=1-F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)/f^\beta(\frac{\pi_A+\pi_B}{2})$ to the left.

Thus,

$$\begin{aligned}
\frac{dt_\beta}{d\beta} &< (1-\eta)(1+2c)\left[1-2F^\beta\left(\frac{\pi_A+\pi_B}{2}\right)\right]+(1-\eta)(1+2c)2F^\beta\left(\frac{\pi_A+\pi_B}{2}\right) \\
&= (1-\eta)(1+2c)=\frac{ds_\beta}{d\beta}
\end{aligned}$$

by (8), which completes the proof . ■

The observations about the marginal benefit and cost of increasing β imply that the equilibrium β in the final-offer arbitration game can be higher than that in the conventional arbitration game (see Figure 2(a)); in other words, the deadweight-loss in final-offer arbitration can be lower than that in conventional arbitration. Not much can be said in terms of outcome-accuracy. Comparing the two equilibria, final-offer arbitration can, but again need not, benefit agent A at the expense of agent B, but this happens independently of the state of the world.

The two-stage, final-offer arbitration game has also equilibria in which the settlement payment is accepted by both types of agent B with probability 1. The interval of such settlement payments typically contains that under conventional arbitration in the interior. The reason is that agents may anticipate the pooling equilibrium, in which each agent plays the best-response to her opponent's offer when type b rejects the settlement payment, and they may anticipate the pooling equilibrium, in which agent A plays the best-response and agent B does not (in order not to be perceived as type a) when type b accepts the settlement payment, as well as the other way round, they may anticipate the former pooling equilibrium when type b accepts and the latter pooling equilibrium when type b rejects. This observation shows that final-offer arbitration can also benefit agent B at the expense of agent A, again independently of the state of the world.

Claim 1 *Both games have two sorts of equilibria.*

(a) In one of them, the settlement payment is rejected with probability 1 *by both types of agent B under conventional arbitration, but it may be accepted with a positive probability by type a under final-offer arbitration (while it is still rejected with probability* 1 *by type b).*

If this happens, the deadweight-loss of final-offer arbitration falls below that of conventional arbitration; the comparison in term of outcome-accuracy is ambiguous.

(b) In the other sort of equilibria, the settlement payment is accepted with probability 1 *by both types of agent* B*, both under conventional arbitration and under final-offer arbitration.*

The intuition behind part (a) of this proposition is that final offers contain signals about the quality of evidence. This signalling may be costly for a privately informed party; moreover, the cost (in comparison to no signalling under conventional arbitration) can increase with the probability that the evidence is "strong". This encourages the other party to making settlement payments that are accepted with higher probability.

One way of interpreting part (a) is that when the percentage of arbitrator's award in total negotiations is very high under conventional arbitration, replacing it by final-offer arbitration can indeed be helpful in encouraging collective bargaining. This seems especially possible when the arbitrator expects privately informed parties to prove that they have a "strong" evidence supporting their case by making final offers that would not be optimal when they had no private information.

Note finally that the payoff of agent B in some pooling equilibria of final-offer arbitration is lower than that of conventional arbitration, which makes possible that $t_{1/2} < 0$ although $s_{1/2} > 0$ (assuming that $c < 1/2$ and η is not too large). If this happens, the settlement payment cannot be rejected with probability 1 by both types of agent B, which provides yet another argument that the deadweight-loss of final-offer arbitration may fall below that of conventional arbitration.

Remark 1 *It is legitimate to suspect that the conclusion that the deadweight-loss of final-offer arbitration never exceeds, but may fall below, that of conventional arbitration in equilibria from Claim 1, (a) only because of the particular assumption that both states of the world are eqi-probable; indeed, if the state* a *is more likely, the conventional-arbitration curves in Figure 2(a) may intersect, and then the settlement payment is accepted with positive probability by type* a*.*

However, it can be easily shown that if the conventional-arbitration curves in Figure 2(a) intersect, then $s_\beta < 0$ for every β to the left of the intersection, and consequently, $t_\beta < 0$ for every such β and every settlement payment t_β that makes type a indifferent between accepting and rejecting (no matter what equilibrium is anticipated in period 2). Thus, the conclusion of Claim 1, (a) holds for any probability distribution over the two states of the world.

3.4 The case of large η

Under conventional arbitration, there exists $\overline{\eta} < 1$ such that for $\eta > \overline{\eta}$, this is the extreme case where $e_1 < 0$. The game has an equilibrium, in which the settlement payment equals to 0, type a accepts this settlement payment with probability 1 and type b rejects it with probability 1. Further, since $r_{1/2} > (1 + 2c)/2$, there is no equilibrium in which the settlement payment is accepted by both types, i.e. the conventional arbitration game has a unique equilibrium.

Under final-offer arbitration, the subgame beginning in period 2 where agent B is of type b with probability β has a separating equilibrium in which type a of agent B picks the best-response to agent A's final offer, assuming that F^0 is the *cdf* of the arbitrator's peak points, type b of agent B picks the best-response to agent A's final offer, assuming that F^1 is the *cdf* of the arbitrator's peak points, and type agent A picks the best-response to agent B's final offers, assuming that F^β is the *cdf* of the arbitrator's peak points. Moreover, since this is a unique equilibrium in the limit (for $\eta = 1$), the final offers in any other equilibrium are very close when η is large enough.[21]

That is, every equilibrium π_A, $\pi_{B,a}$ and $\pi_{B,b}$ of the subgame in period 2 can be

[21]For $\eta < 1$, there typically exist other equilibria. For example, think of a situation where the arbitrator who does not recognize the presented evidence believes that agent B is of type a unless her final offer slightly differs to her best-response to agent A's offer. Then agent B of type b may indeed be willing to pick that offer, losing some payoff contingent on the arbitrator being able to recognize the presented evidence in exchange of a higher payoff contingent on the arbitrator not being able to recognize the presented evidence.

approximately determined by the following set of first-order conditions:

$$\frac{\pi_{B,b}-\pi_A}{2} = \frac{1-F^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right)}{f^1(\frac{\pi_A+\pi_{B,b}}{2})}, \tag{13}$$

$$\frac{\pi_{B,a}-\pi_A}{2} = \frac{1-F^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)}{f^0(\frac{\pi_A+\pi_{B,a}}{2})}, \tag{14}$$

$$\begin{aligned} 0 \;\; = \;\; & (1-\beta)\left\{F^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)-\frac{\pi_{B,a}-\pi_A}{2}f^0(\frac{\pi_A+\pi_{B,a}}{2})\right\} \\ & +\beta\left\{F^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right)-\frac{\pi_{B,b}-\pi_A}{2}f^1(\frac{\pi_A+\pi_{B,b}}{2})\right\}. \end{aligned} \tag{15}$$

Notice that for "sensible" distributions of the noise term $\pi_A > 0$ in this equilibrium for $\beta = 1$, and so the payoff of type a of player B exceeds 0.

In the two-stage game, there is an equilibrium in which the settlement payment is accepted with a probability by type a and is rejected (with probability 1) by type b, so that a $\beta \geq 1/2$ is the probability that agent B is of type b contingent on rejection. This equilibrium β can be determined by comparing marginal benefit and marginal cost of increasing β. The marginal benefit is given by (7), and the marginal cost is given by

$$\begin{aligned} & \frac{1}{2}\frac{d\left\{\pi_A F^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)+\pi_{B,a}\left[1-F^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)\right]\right\}}{d\beta} \\ & +\frac{1}{2}\frac{d\left\{\pi_A F^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right)+\pi_{B,b}\left[1-F^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right)\right]\right\}}{d\beta}. \end{aligned}$$

Lemma 4 *The marginal cost is strictly greater than 0 for $\beta > 1/2$ (it equals 0 for $\beta = 1/2$).*

Proof. The marginal cost can be expressed as

$$\frac{1}{2}\left\{f^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)\left(\frac{\pi_A-\pi_{B,a}}{2}\right)+F^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)\right\}$$
$$+\frac{1}{2}\left\{f^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right)\left(\frac{\pi_A-\pi_{B,b}}{2}\right)+F^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right)\right\},$$

and further, by the first-order conditions (13) and (14), as

$$F^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)+F^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right)-1. \tag{16}$$

Combining (13)-(15),

$$2(1-\beta)F^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)+2\beta F^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right)-1=0,$$

and so (16) equals 0 for $\beta = 1/2$. Since it follows from (13) and (14) that

$$F^0\left(\frac{\pi_A+\pi_{B,a}}{2}\right)>F^1\left(\frac{\pi_A+\pi_{B,b}}{2}\right),$$

(16) must be greater than 0 for $\beta > 1/2$. ■

Notice that the marginal cost is independent of η, and so, by (8), it is greater (except the neighborhood of 1/2) than the marginal cost under conventional arbitration for η close enough to 1. This yields that the β that makes the marginal cost and marginal benefit equal is lower under final-offer arbitration compared to conventional arbitration (see Figure 2(b)).

There is no other equilibrium in the two-stage game. Indeed, the settlement payment that can be accepted by both types of agent B must be at least as high at the payoff from the subgame beginning in period 2 where agent B is of type b with probability 1, which is higher than the settlement payment that makes type a indifferent between accepting and rejecting, assuming that agent B is of type b with probability 1 contingent on rejection.

Summarizing,

Claim 2 *Each game has a unique equilibrium.*

(a) Under conventional arbitration, no settlement payment is offered in period 1*. This payment is accepted with probability* 1 *by type* a *and it is rejected with probability* 1 *by type* b*.*

(b) Under the final-offer arbitration, it may happen that the settlement payment is positive; it may also happen that it is accepted by type a *only with a probability* $q < 1$ *(while it is rejected with probability* 1 *by type* b*), and then the equilibrium payoff of type* b *exceeds* 1*.*

That is, the deadweight-loss of final-offer arbitration never falls below, but it may exceed, that of conventional arbitration, as well as final-offer arbitration is never more, but it may be less, outcome-accurate than conventional arbitration.

The intuition behind the claim is that final-offer arbitration (unlike conventional arbitration) allows a privately informed party to extract a rent. The rent of the party that lacks "strong" evidence supporting its case increases with the strength of the other party's belief that the evidence is "strong". This reduces the other party's willingness to offering settlement payments that are accepted with positive probability as such settlement payments raise the strength of its belief that the evidence of the privately informed party is "strong" (contingent on rejection) and, consequently, they raise the rent of the party that lacks "strong" evidence.

However, the rent of the party with "strong" evidence supporting its case decreases with the strength of the other party's belief that the evidence is "strong". Thus, settlement payments that are accepted with positive probability reduce the rent of the party with "strong" evidence. The total rent cost is therefore ambiguous, but the rent cost of offering settlement payments that are accepted with positive probability is certainly positive, because the uninformed party makes final offers with the belief that the evidence of the privately informed party is "strong" that is stronger than the actual probability that the evidence is "strong".

4 Non-Common Priors

The analysis of this scenario is relatively straightforward. If agent B does not find any evidence, or she finds evidence and agent A recognizes it, then the arbitrator also learns that the state of the world is a or b, respectively. In both cases, both players have the same expectation about the arbitrator's decision. If agent B finds evidence and agent A does not recognize it, then they differ in their expectation about the arbitrator's decision; agent A believes that the arbitrator knows that the state is a, and agent B believes that the arbitrator knows that the state is b, while the arbitrator believes that the states are eqi-probable. Of course, the arbitrator makes systematic mistakes, which is common knowledge.

Consider first conventional arbitration. If players have the same expectation about the arbitrator's decision, then agent A offers the settlement payment that makes agent B exactly indifferent between accepting and rejecting the offer (if the offer that makes agent B exactly indifferent is non-negative) of the random payment $\pi - c$; if the offer that makes agent B exactly indifferent is negative, agent A offers no settlement payment. Under conventional arbitration, $\pi = -c + \varepsilon$ when agent B does not find any evidence or $\pi = 1 + c + \varepsilon$ when she finds evidence and agent A recognizes it.

If players have different expectations about the arbitrator's decision, the settlement payment that agent A is willing to offer is 0, and agent B is not willing to accept less than 1. Thus, agent B rejects agent A's settlement payment.

Summarizing, I obtain the following proposition:

Proposition 5 *The conventional arbitration game has a unique equilibrium.*

(a) *In the equilibrium, if player B finds no evidence, the game ends in period* 1*, and no payment is made.*

(b) *If player B finds evidence and player A recognizes the evidence of player B, the game also ends in period* 1*. The payment then made by player A equals to* 1*.*

(c) *If player B finds evidence and player A does not recognize it, the game ends in period* 2*. The random payment then made by player A equals to*

$$1/2 + \varepsilon.$$

Under final-offer arbitration, if the play reaches period 2, players choose their final offers π_A and π_B to maximize

$$F^A\left(\frac{\pi_A+\pi_B}{2}\right)(-\pi_A)+\left[1-F^A\left(\frac{\pi_A+\pi_B}{2}\right)\right](-\pi_B), \tag{17}$$

and, respectively,

$$F^B\left(\frac{\pi_A+\pi_B}{2}\right)\pi_A+\left[1-F^B\left(\frac{\pi_A+\pi_B}{2}\right)\right]\pi_B. \tag{18}$$

given the final offer of their opponents. If players have the same expectation about the decision, $F^A = F^B$ is the *cdf* of $p_b(1+c)+(1-p_b)(-c)+\varepsilon$; recall that p_b denotes the arbitrator's probability that the state of the world is b, and ε is a noise term with a density function f; note that $p_b = 0$ if agent B does not find evidence and $p_b = 1$ if agent B finds evidence that is recognized by player A. If players have different expectations about the arbitrator's decision, F^A is the *cdf* of $-c+\varepsilon$ and F^B is the *cdf* of $(1+c)+\varepsilon$; this happens in state b when player B finds evidence that is not recognized by player A and in state a when player B believes that she found evidence but the evidence is not recognized by player A.

It is easy to see that in any Nash equilibrium (of the subgame beginning in period 2) $\pi_A \leq \pi_B$. Moreover, by (2) and (3), π_A and π_B are determined by the first-order conditions:

$$\frac{\pi_B-\pi_A}{2}=\frac{F^A\left(\frac{\pi_A+\pi_B}{2}\right)}{f^A(\frac{\pi_A+\pi_B}{2})} \tag{19}$$

$$\frac{\pi_B-\pi_A}{2}=\frac{1-F^B\left(\frac{\pi_A+\pi_B}{2}\right)}{f^B(\frac{\pi_A+\pi_B}{2})}. \tag{20}$$

Note that by (3) the right-hand side of (19) is non-decreasing in π_A, and the left-hand side of (19) is decreasing in π_A; since the left-hand side is equal to 0 for $\pi_A = \pi_B$, and the right-hand side is positive, the equilibrium π_A satisfies (19). Similarly, by (2) the right-hand side of (20) is non-increasing in π_B, and the left-hand side of (20) is increasing in π_B; since the left-hand side is equal to 0 for $\pi_B = \pi_A$, and the right-hand side is positive, the equilibrium π_B satisfies (20).

Summarizing, I obtain the following proposition:

Proposition 6 *The final-offer arbitration game has a unique equilibrium.*

*(**a**) In the equilibrium, if player B finds no evidence, the game ends in period 1, and no payment is made.*

*(**b**) If player B finds evidence and player A recognizes the evidence of player B, the game also ends in period 1. The payment then made by player A equals to 1.*

*(**c**) If player B finds evidence and player A does not recognize it, the game ends in period 2. The equilibrium final offers are jointly determined by (19) and (20).*

Having characterized the equilibria of the two arbitration games, I can compare their welfare properties. The comparison reduces to the case when player B finds evidence and player A does not recognize it. The following proposition asserts that for "reasonable" density functions f^M, f^M is the density of $1/2 + \varepsilon$, and legal cost parameters c, the conventional arbitration is more outcome-accurate than the final-offer arbitration. I say that the density f is up to δ in an interval $[-z, z]$ when

$$\left| \int_{-\infty}^{\infty} l(|-x-c+1|, |x-c-1|) f^M(x)dx - \int_{1/2-z}^{1/2+z} l(|-x-c+1|, |x-c-1|) f^M(x)dx \right| \leq \delta$$

and

$$\left| \int_{-\infty}^{\infty} l(|-x-c|, |x-c|) f^M(x)dx - \int_{1/2-z}^{1/2+z} l(|-x-c|, |x-c|) f^M(x)dx \right| \leq \delta.$$

Note that, under conventional arbitration, when f^M is the density of $1/2 + \varepsilon$

$$E[l(|w_A + 1|, |w_B - 1|) \mid b] = \int_{-\infty}^{\infty} l(|-x-c+1|, |x-c-1|) f^M(x)dx$$

and

$$E[l(|w_A|, |w_B|) \mid a] = \int_{-\infty}^{\infty} l(|-x-c|, |x-c|) f^M(x)dx.$$

In words, a density is up to δ in an interval $[-z, z]$ when if tails (realizations to the left of $-z$ and to the right of z) can be disregarded if one is satisfied with computing outcome-accuracy approximately up to δ. It seems reasonable to assume that the tails will not matter much indeed for $z = 1/2$.

Proposition 7 *Suppose that $c \leq 1/2$. There exists $\overline{\delta} > 0$ such that if the density f of ε is up to a $\delta \leq \overline{\delta}$ in the inteval $[-1/2, 1/2]$, then conventional arbitration is more outcome-accurate than final-offer arbitration.*

Proof. Notice also that f^B and F^B are translations of f^A and F^A by $1 + 2c$. This, together with the assumption that ε is symmetric around 0 and (19) and (20), implies that

$$\frac{\pi_A + \pi_B}{2} = \frac{1}{2}.$$

Since f is symmetric and uni-modal around 0, f^B is symmetric and uni-modal around $1 + c$. This which implies that

$$f^B(\frac{1}{2}) \leq \frac{1}{1 + 2c}.$$

Moreover, since the area surrounded by the graph of f, the vertical lines at $1/2$ and $1 + c$, and the horizontal line at 0 exceeds

$$(\frac{1}{2} + c)f^B(\frac{1}{2}),$$

$$F^B\left(\frac{1}{2}\right) < \frac{1}{2} - (\frac{1}{2} + c)f^B(\frac{1}{2}).$$

This, by (20), implies that

$$\frac{\pi_B - \pi_A}{2} > 1 + 2c.$$

Thus,

$$\pi_A < -\frac{1}{2} - 2c \text{ and } \pi_B > \frac{3}{2} + 2c. \tag{21}$$

Now, I can conclude that

$$\int_0^1 l(|-x-c+1|, |x-c-1|)f^M(x)dx =$$

$$\int_0^{1/2} l(|-x-c+1|, |x-c-1|)f^M(x)dx + \int_{1/2}^1 l(|-x-c+1|, |x-c-1|)f^M(x)dx$$

$$< \frac{1}{2}l(\left|\frac{1}{2}+2c-c+1\right|, \left|-\frac{1}{2}-2c-c-1\right|) + \frac{1}{2}l(\left|-\frac{3}{2}-2c-c+1\right|, \left|\frac{3}{2}+2c-c-1\right|);$$

the inequalities follow from the assumption that l is increasing in both variables. Similarly,

$$\int_0^1 l(|-x-c|, |x-c|)f^M(x)dx$$

$$= \int_0^{1/2} l(|-x-c|, |x-c|)f^M(x)dx + \int_{1/2}^1 l(|-x-c|, |x-c|)f^M(x)dx <$$

$$< \frac{1}{2}l(\left|\frac{1}{2}+2c-c\right|, \left|-\frac{1}{2}-2c-c\right|) + \frac{1}{2}l(\left|-\frac{3}{2}-2c-c\right|, \left|\frac{3}{2}+2c-c\right|).$$

which completes the proof. ■

Here is the intuition behind this result: When players believe in their opponents being wrong, they are unlikely to reach any agreement on settlement payment. Each of them simply believes that the arbitrator will support his or her claim. Then the form of arbitration does not affect much their willingness of asking for arbitration. Final-offer arbitration (unlike conventional arbitration) allows players to, as they believe, take advantage of their opponents being wrong by making "strong" claims against them (see (21)). This, however, makes the arbitrator take more extreme decisions. In consequence, it reduces the accuracy of arbitration.

The logic behind the proposition is particularly well-seen in the limit case when there is no uncertainty about the arbitrator's decision. Then when players have different expectations about the arbitrator's decision, player A expects $-c$ to be the "peak" of the arbitrator's preferences, and player B expects the "peak" to be at $1+c$. Thus, if the play reaches period 2, A's final offer never exceeds $-c$, while B's final offer never falls below $1+c$. This in turn imply that A's final offer never exceeds

$$-c - [(1+c) - (-c)] = -1 - 3c,$$

while B's final offer never falls below

$$1 + c + [(1 + c) - (-c)] = 2 + 3c.$$

Continuing this reasoning, one concludes that agents' final offers will tend to $-\infty$ and $+\infty$, respectively.

The deadweight-loss is identical under both forms of arbitration as under both scenarios parties end up with arbitration if and only if player B finds evidence and player A does not recognize it. Note, however, that this result is driven by risk-neutrality. If parties were risk-averse, one would like not only to keep the probability of asking for arbitration as low as possible, but also, if arbitration happens, to keep the uncertainty of the arbitrator's decision as low as possible. Then a more proper measure of deadweight loss, given concave utility indices u_A and u_B, seems to be

$$d(q\,[c + Ew_A - e_A]\,, q\,[c + Ew_B - e_B]),$$

where where d stands for an increasing (in both arguments) function, q denotes the probability of asking for arbitration, e_A (resp. e_B) denotes the certainty equivalent of the equilibrium monetary payoff w_A (resp. w_B) of agent A (resp. B), i.e. e_A and e_B are defined by the equations

$$u_A(e_A) = Eu_A(w_A) \text{ and } u_B(e_B) = Eu_B(w_B).$$

Since the final-offer arbitration typically exposes the parties to more risk, its deadweight loss typically exceeds that of the conventional arbitration. Again, it is well-seen in the limit case when there is no uncertainty about the arbitrator's decision. Then there is no uncertainty in the outcome of the conventional arbitration game, and the arbitrator randomizes between the offers that tend to $-\infty$ and $+\infty$ in the final-offer arbitration game.

5 Comparative Statics, Extensions

5.1 Quality of Arbitration

A number of earlier papers discuss the role of the quality of arbitration. The quality of arbitration has been usually measured by comparing (in terms of mean-preserving spread) the distributions of the noise term ε. The main point is that arbitration of lower quality can (a bit paradoxically) result in better outcomes, i.e. encourage more collective bargaining of risk-averse parties.

I wish to make to points on this issue. First, lower quality (measured in the traditional manner) leads to better outcomes under non-common prior even for risk-neutral agents. For the conventional arbitration game, the result is somewhat ambiguous. For example, it depends on whether the loss function l is convex or concave whether the arbitrator who always awards 1/2 is better than the arbitrator who awards 1/4 and 3/4 with probability fifty-fifty. For final-offer arbitration, it is easy to see that, under mild assumptions on f, a mean-preserving spread moves both curves (19) and (20) downwards around $(\pi_A + \pi_B)/2 = 1/2$ (in terms of Figure 1(a)), which makes $(\pi_B - \pi_A)/2$ lower, i.e. it results in a better outcome.[22]

It is again somewhat in contrast with asymmetric information. Under conventional arbitration, quality of arbitration measured in terms of mean-preserving spread has no effect because of the assumption that agents are risk-neutral. However, the parameter η can be used as another measure of quality of arbitration. If one restricts attention to the equilibrium where type a randomizes and type b rejects, higher η implies higher β (see (9)), i.e. lower quality leads to a larger deadweight-loss. By construction, lower quality leads typically to a lower outcome-accuracy as well. On the other hand, $r_{1/2}$ increases and $\max_{\beta\in[1/2,\gamma]} U_\beta^A$ (respectively, $\max_{\beta\in[\beta^*,\gamma]} U_\beta^A$) decreases with η, so (by Proposition 1) the range of the parameter c such that there are equilibria where both types of agent B accept the settlement payment shrinks. Not much can be said under

[22] Again, this result is particularly well-seen in the limit case when there is no uncertainty about the arbitrator's decision. Then if the quality of arbitration becomes almost perfect, the final offers tend to $-\infty$ and $+\infty$.

final-offer arbitration because of the multiplicity of equilibria in period 2; however, similar conclusions can be made for some families of equilibria, parametrized by η.

5.2 Future Research

This paper compares only two (although most common) forms of arbitration. There is a number of other, welfare-related questions that can potentially be addressed in a modified version of the present setting. It happens in practice that other forms of arbitration have been (or are) in use. In some states, parties can choose the form of arbitration: the final-offer arbitration applies if (and only if) one of the parties refuses conventional arbitration; in Iowa, a arbitration procedure involves three tiers: mediation, fact-finding with recommendations, and final-offer arbitration with the arbitrator to chose one of three offers where a fact finder's recommendation is included as a separate offer. There have been also proposed schemes (see Crawford (1981)) where each party makes two offers, the arbitrator announces the party that made better offers, and the other party chooses one of the offers made by the party announced. Lester (1984) provides an excellent survey of arbitration experience. More generally, one may try to address the question of optimal mechanism design.[23]

One may also wish to explore the potential role of other signalling or screening procedures, combined with or alternative to final offers, such as "burning money" or the possibility of making another settlement payment offer after asking for arbitration but before the arbitrator making the award. This leads to a number of interesting questions regarding both the present method of modelling arbitration as well as the reality, but unfortunately, the final-offer version of present setting gets very complicated with such modifications.

Finally, it should be emphasized that the present model is "one-dimensional", i.e. the conflict concerns just one issue. In practice, collective bargaining is often "multi-dimensional" (or it concerns several issues). Two forms of final-offer arbitration have been developed for dealing with multi-dimensional situations. In the *package* arbi-

[23]Brams and Merrill III (1986) can be viewed as an attempt of addressing this question.

tration, parties make a final offer, specifying their position of each issue, and the arbitrator selects one of them. In *issue-by-issue* arbitration, the arbitrator makes a separate decision on each issue. See Crawford (1981), and Lester (1984) for a survey discussion, and Çelen (2003) for an attempt of formal analysis of the two forms of final-offer arbitration.

6 Appendix

I will now prove Proposition 2. Since the arguments are rather standard, I will only sketch them. Observe first show that the subgame from period 2 has an equilibrium under an additional exogenous constraint on final offers:

$$\exists_{n=1,2,\ldots}\exists_{k=0,1,\ldots,n} \quad \pi = -C + \frac{k}{n}(1+2C) \text{ where } \pi = \pi_A, \pi_B. \tag{22}$$

That is, final offers can be chosen only from a finite grid of the interval $[-C, 1+C]$. Indeed, the existence of equilibria follows then immediately from the Kakutani's fixed point theorem. One can simply take a fixed point of the correspondence defined on the actions of player A, i.e. π_A, the actions of each type of player B, denote them by $\pi_{B,a}$ and $\pi_{B,b}$, and the arbitrator's beliefs contingent on each offer from the grid, which assigns the best-responses in terms of actions and the beliefs determined by the Bayes rule if an offer is used with positive probability and the set of all possible beliefs if and offer is used with probability 0.

The next step is to show condition (22) is dispensable. This relies on the following "limit" argument. For a given n the equilibrium distributions π_A^n, $\pi_{B,a}^n$ and $\pi_{B,b}^n$ are regular probability measures on $[-C, 1+C]$. Therefore, since the space of such measures is metrizable and compact in the weak*-topology, one can assume, passing to a subsequence if necessary, that they converge (in the weak*-topology) as n goes to ∞ to some regular measures π_A, $\pi_{B,a}$ and $\pi_{B,b}$. For details, see Dudley (1989) and Rudin (1973); more specifically, see the Riesz Theorem (Dudley 1989, Theorem 7.4.1), the Banach-Alaoglu Theorem (Rudin 1973, Theorem 3.15) and Rudin 1973, Theorem 3.16.

By Dudley (1989), Theorem 10.2.2, $\pi_{B,a}$ and $\pi_{B,b}$ determine conditional probabilities $\mu(a \mid \pi)$ and $\mu(b \mid \pi)$ with $\mu(a \mid \pi) + \mu(b \mid \pi) = 1$, unique up to a set of of final offers π that are used with probability 0 by both types of agent B when they play $\pi_{B,a}$ and $\pi_{B,b}$, respectively. The convergence of $(\pi^n_{B,a}, \pi^n_{B,b})$ to $(\pi_{B,a}, \pi_{B,b})$ implies that for almost every π (i.e. except a set of final offers π that are used with probability 0 by agent B) and every $\varepsilon > 0$ there exists a $\delta > 0$ such that if n is large enough, $\left|\pi^/ - \pi\right| < \delta$ and $\pi^/$ is of the form (22), then $\mu^n(a \mid \pi^/)$ and $\mu^n(b \mid \pi^/)$ differ from $\mu(a \mid \pi)$ and $\mu(b \mid \pi)$, respectively, by at most ε. It can be easily checked now that π_A, $\pi_{B,a}$ and $\pi_{B,b}$ together with $\mu(a \mid \pi)$ and $\mu(b \mid \pi)$, where $\mu(a \mid \pi) := 1$ and $\mu(b \mid \pi) := 0$ for final offers π that are used with probability 0 by agent B, is an equilibrium of the subgame beginning in period 2.

Thus, for every β, probability that agent B is of type b, the subgame beginning in period 2 has an equilibrium. It is easy to check that the correspondence assigning to every β the set of equilibria in strategies π_A, $\pi_{B,a}$ and $\pi_{B,b}$ that are regular measures is upper hemi-continuous (of course, assuming that the space of regular measures is equipped with the weak*-topology), which in turn implies that the correspondence assigning to every β the set of equilibrium payoff vectors in the subgame beginning in period 2 is upper hemi-continuous.

This implies that there exist a $\beta \geq 1/2$, an equilibrium E_β of the subgame from period 2 (given this β), and a settlement payment t_β making type a of agent B indifferent between accepting t_β and asking for arbitration (given E_β) that maximize agent A's payoff (in the two-stage game) over all such triples β, E_β, and t_β. Denote by β^* the β at which the maximum is reached, and by u this maximum payoff of agent A. Now, take any equilibrium for $\beta = 1$ (say E_1), and consider the minimum settlement payment that makes type b of agent B indifferent between accepting the payment and asking for arbitration (given E_1). Denote by v this settlement payment.

If $u \geq -v$, then the two-stage game has an equilibrium in which agent A offers the settlement payment t_{β^*}. This payment is accepted by type a of agent B with probability q, given by (6) for $\gamma = \beta^*$, and it is rejected with probability 1 by type b of agent B. Agents anticipate the equilibrium E_{β^*} contingent on rejecting this payment. Off the equilibrium path, agents anticipate any E_β contingent on rejecting t_β for any other $\beta \neq \beta^*$, and such payments are accepted by type a of agent B with probability q, given by (6) for $\gamma = \beta$ and they are rejected with probability 1 by type b of agent B. Agents anticipate E_1 contingent on rejecting the settlement payments higher than all t_β; such a payment is accepted with probability 1 by type a, and it is accepted or rejected with probability 1 by type b, depending on whether this type prefers the settlement payment or the payoff in the two-stage game contingent on asking for arbitration and the equilibrium E_1being played in period 2. Finally, they anticipate $E_{1/2}$ contingent on rejecting the settlement payments lower than all t_β, and both types of agent B reject such payments with probability 1.

If $u \leq -v$, then the two-stage game has an equilibrium in which agent A offers the settlement payment v. This payment is accepted by both types of agent B with probability 1, given by (6) for $\gamma = \beta^*$, and it is rejected with probability 1 by type b of agent B. Off the equilibrium path, the play is defined as in the previous paragraph.

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