Information in Auctions: Disclosure

Ian Jewitt[*,†]
Nuffield College

This version September 29, 2004
PRELIMINARY AND INCOMPLETE

Abstract

`Working Version`

"As far as I can see, all a priori statements ... have their origin in symmetry" - Hermann Weyl..

1 Introduction

Should a seller release public information before, or indeed during, an auction? The posing and answering of this question by Milgrom and Weber (1982a) (MWa) provides one of the cornerstones of present day auction theory and the result that disclosure is good for revenue has even become part of folk wisdom in some policy areas. In context of their general symmetric affiliated model. Milgrom and Weber showed both how to characterise equilibria and that revealing information publicly is always expected revenue enhancing. Furthermore, they showed that under these assumptions the sort of information generated by common auction processes is affiliated in the requisite way. So, in particular, ascending button auctions yield higher revenues than second price sealed bid auctions.

Milgrom and Weber's results should *not* lead to a general belief that the public disclosure of information is usually, and certainly not always, expected revenue enhancing. It depends very much on the statistical context of the situation at hand and there are many natural cases where information disclosure decreases expected revenue. This is true even in symmetric situations once the

[*]I am grateful to seminar and conference participants at ESEM (Santiago), Tilburg, Bologna, Milan, Helsinki, Oxford, ESSET (Gerzensee), Toulouse, Pittsburgh, Exeter, Essex, St. Andrews, Washington, St. Louis, Southampton, LSE and UCL.

[†]It will be immediately aparent that this is a working draft and still in very rough form, please treat accordingly. Updated version to follow asap.

global affiliation assumption is relaxed and it holds with even more force once the symmetry assumption is abandoned[1].

The paper makes a number of methodological points. It relaxes, to a degree, two[2] of the key assumptions adopted by Milgrom and Weber, specifically the global *affiliation* and *symmetry ones*.

These assumptions both play a dual role in the literature. On the one hand, they are useful technical assumptions in that they make it much easier to characterise equilibria. On the other hand, they are also very substantive economic assumptions which have strong implications on the sign of some important effects. Symmetry makes the equilibrium allocation an invariant with respect to both auction form and to information disclosure and consequently it opens the door for revenue equivalence and revenue ranking results. Affiliation implies that information disclosure substitutes for bidders private information and thereby erodes their informational rents and enhances seller revenue (when the allocation is invariant). Much of our strategy is this paper is to relax these assumptions in a way which maintains their "technical" role but liberates the economic discussion somewhat.

We also make some reasonably practical suggestions about what sort of information should be disclosed by sellers, given the opportunity for choice in the matter and assuming the necessary data exists. As a rule of thumb we suggest calculating two statistics. One represents how much *relevant* information sellers have about the object in aggregate and the other how much information bidders have about each others signals. These two quantities impact on revenue in opposite directions and the disclosure decision is informed by how the conditional statistics compare with their marginal equivalents. What do we mean by relevant information? Decisions are made on the basis of calculations at the margin and what will be important is information at the relevant margin. Consider a second price auction, in the equilibrium each bidder will 'stay in' the auction up to the point at which they are indifferent whether their last competitor exits immediately or not. With independent private values, this calculation just requires exiting at ones value, bit more generally other bidders' behaviour will reflect their own signals which are pertinent to this calculation. If one's competitors do not react to their own signals, then their behaviour carries no information about one's own value, no matter how informative their signals might be. On the other hand, if competitors react sharply to their signals at the relevant point, then their behaviour will be informative. There is a *gearing effect.* How sharply a bidder's expected value of the object responds to her own signal is a measure of the informativeness of that signal for the value.[3]

[1] These assertions are not especially controversial and some examples which fit in with the scheme of this paper have already appeared in the literature - not least by Milgrom (2004) and Milfrom and Weber (1982b).

[2] The third is that bidder private information is scalar valued. This tends to make information disclosure efficiency enhancing in natural specifications (i.e. those satisfying Maskin's criterion). Cionsequently, revenue and bidder surpluses sum to a constant. Not necessarily so with multivariate signals.

[3] This is not invarian to how signals are measured, but the statement makes sense if we first scale signals to have a common marginal distribution.

By, analogy, how sharply the bidder's quit-evaluation of the object varies with her signal is a measure of how informative her signal is about her value in the equilibrium. We need to form a composition of all the signal informativeness and gearing effects and aggregate accross bidders.

It is perhaps worth stressing that we propose to identify three conceptually different items - the informativeness of bidder signals about values, the gearing effects on that information and the information bidder signals provide about each other. If the second price auction is a guide, the first of these two items will impact on revenue only through their composition. The third is distinct. Since bidder signals can be measured arbitrarily, then it must be captured by the shape of the copula of the joint distribution of bidder signals.

This is a different way of slicing the pie than in much of the existing literature on information disclosure. The revenue in an auction will be a function of the bidder signals and the statistic $T(X)$ of other information which is disclosed, we are currently comparing $T = X$ with $T' = \varnothing$, call this $R(S,T)$. If all auctions under all disclosure policies T have the same allocation, in terms of bidder signals then we can decompose this into $E[R(S,T)] = \sum_i E[R(S,T) - R(S,T')|\text{bidder } i \text{ wins}]$,that is the difference between bidder surpluses conditional on a statistic of S. Hence, revenue is determined by the $E[B_i(S,T) - B_i(S,T')|S]$. When the identity of the winning bidder depends on X, this is not possible even for common value auctions.

1.0.1 Symmetry

The symmetry assumption, as remarked above, ensures that the equilibria are invariant to the form of auction[4]. This invariance is convenient of course but also central to a good deal of the beautiful classical results of auction theory through the implications

$$\begin{array}{lclcl} \text{symmetry} & & \text{invariance of allocation} & & \text{revenue ranking} \\ \text{assumptions} & \underset{a}{\Rightarrow} & \text{to auction form} & \underset{b}{\Rightarrow} & \text{results.} \end{array}$$

Milgrom and Weber's symmetry assumptions achieve a but are stronger than is required. Symmetry makes it easy to guess what equilibrium allocations will be and effectively reduces the dimensionality of the system of differential equations which one uses to characterise bidding strategies and, of course, once we have found one we have found them all[5]. The revenue equivalence and revenue ranking results as is well understood, derive more or less directly from the auctions being compared having the same allocation. The above schema oversimplifies the situation particularly with regard to the role of monotonicity

[4] At least within an interesting class of mechanisms which include most familiar standard auctions. If a game is symmetric among players, then the set of equilibria must also be symmetric. This does not necessarily mean that any particular equilibrium must be symmetric.

[5] Vickrey (1961) was the first author to study asymmetric auctions. He solved for a particuar private value first price auction to show that the allocation need not be efficient. This exercise was important in order to illustrate his general point, that what we now call Vickrey auctions are more efficient in such environments.

in implications a and b but it remains true that if we could abandon or relax the symmetry assumption while retaining the invariance property, then one could hope to bring standard methods to bear on the implication b.

Symmetry and invariance are sufficiently closely related concepts that we cannot entirely dispense with one without the other[6]. However, one would like to identify precisely the symmetry we need for the invariance we want, for example

$$\begin{array}{lcl} \text{symmetry} & & \text{invariances of allocation} \\ \text{assumptions} & \underset{a'}{\Longleftrightarrow} & \text{to auction form.} \end{array}$$

Milgrom and Weber's symmetry assumptions also achieve another two invariances which are important in deriving comparative statics results:

$$\begin{array}{lclcl} \text{symmetry} & & \text{invariance of allocation} & & \text{revenue ranking} \\ \text{assumptions} & \underset{c}{\Rightarrow} & \text{to information disclosure policy } TX = x & \underset{d}{\Rightarrow} & \text{results.} \end{array}$$

Specifically, there exist a set of bidder signals S which determine the allocation of the object independently of what other information $T(X)$ is disclosed, the allocation is invariant to x. This information *disclosure invariance* is different from the auction *mechanism invariance,* one can have mechanism invariance without disclosure invariance and vice versa but it plays a similar role in the derivation of qualitative revenue effects.

A possible line of attack for extending Milgrom and Weber (1982a) therefore is to relax the symmetry assumptions in a way which maintains tractability. There are already important papers which effectively do this, specifically Milgrom and Weber (1982b) (MWb) study a two bidder common value model in which one of the bidders has private information pertinent to the value of the object but the other bidder is completely uninformed. In the equilibrium of a first price auction the uninformed bidder plays a mixed strategy which turns out to have the same distribution as the informed bidder's equilibrium bid distribution. Evidently, there is a symmetry in the joint distribution of bids even though the models is not fully symmetric. It must be because the model is *quasi* symmetric in some way.

Together, MWa and MWb Milgrom and Weber abstract some useful concepts with which to understand information disclosure.

First, there is the distinction between information substitutes and complements. MWa make the important point that, their symmetry and affiliation assumptions define an environment of information substitutes because revenue is increased bysuch disclosure. It is left somewhat implicit how to construct information complements in minimal departures from their general symmetric model without violating the assumptions needed to characterise the equilibria in the first place. Constructing such departures is not hard to do given the machinery already provided in MWa and we give some examples below. We propose to maintain the information complements and substitutes terminology but will use it slightly differenty. First, we distinguish the uses to which the

[6] [symmetry is] ... "invariance of a configuration of elements under a group of automorphic transformations" (Weyl, 1952)

information is being put. In an auction, there are essentially two uses for information, to learn about ones value and to learn the private information of ones opponents. Bidders use their private signals to do so and make informational rents as a result. The quality of these two types of information might impact on revenues in different ways and information disclosed might be substitute for one sort but be a complement to the other. Second, MWb and Milgrom (2004) introduce an interesting distinction between the "weighting" and "publicity" effects. Roughly, the weighting effect measures the impact on the revenue (or bidder surplus) of changing the sensitivity of bidders value estimates to their signals. It depends, therefore, on some sort of measure of how informative bidders signals are about the value of the object. The publicity effect is the residual. The decomposition of the overall revenue effect into weighting and publicity effects is possible in MWb because given only one bidder is informed it is natural to measure the sensitivity of the expected value of the object to bidder signals simply as the sensitivity to the informed bidder's signal, but this is clearly very special limiting case. In MWa, if $\frac{\partial}{\partial x}E[U_i|S=s,X=x]=0$, then $\frac{\partial}{\partial s_i}E[U_i|S=s,X=x]$ is independent of x, for each bidder i, then one might say that there is no weighting effect. This condition also implies that there are no revenue effects of disclosure in a second price auction and consequently, the publicity effect is defined to be zero (Milgrom (2004)). On the other hand, if the information disclosure does not change the quality of information bidders have about each others' signals, then one can also show that there are no revenue effects in a second price auction, even if $\frac{\partial}{\partial x}E[U_i|S=s,X=x]>0$. We give an exampe below (oil prices) to illustrate. In our second job market example below, disclosure of information makes one bidder's value estimate more sensitive to her signal and the other's less sensitive. How should we aggregate? It would therefore be very desirable to rehearse these concepts in a general class of models which included MWa and MWb as special cases. This is hard. However, it will still be interesting to do so in a class of models less polarised than MWa and MWb and we attempt this in the sequel.

It will be helpful to discuss the distinction in light of our proposed interpretation of MNWb as a quasisymmetric model. The uninformed bidder's strategy can, of course, be interpreted as a map from a privately observed random signal distributed independently from the other random variables in the model conditional on public information. Since signals can be measured arbitrarily this random signal can moreover be taken without loss of generality to have the same distribution as the informed bidder's signal. Specifically, if the informed bidder has signal S_1 with conditional cdf $F_{S_1|X}(s|x)$, then we construct another conditionally independent random variable S_2 with the same cdf $F_{S_2|X}(s|x)=F_{S_1|X}(s|x)$. Given conditional independence, the joint distribution of bidder signals is then symmetric both with and without disclosure of information. Assuming common values layers more symmetry on top of this and the assumptions together make the model tractable. This construct allows us to interpret MWb's results in a somewhat different way, more akin to what follows and in line with the discussion of gearing above.

Given our construct for the signal of the uninformed bidder, the allocation is invariant to which signal $X = x$ is disclosed, as in MWa. This implies that the gearing effect is constant accross different realisations of X. On the other hand, if there is no disclosure, there will be a different gearing effect. It seems strange to assert that there can be such gearing effects in such a model - one bidder is completely uninformed in a first price auction so it seems there is nothing to gear up or down. The model is clearly a limiting case however. Suppose an ascending auction is conducted instead. As the price increases, the uninformed bidder will revise her expected value of the object and how quickly depends on how sensitive the informed bidder's value is to his signal. So, gearing makes sense for ascending auctions. Moreover, under the assumptions, a version of the revenue equivalence theorem holds. The first price auction is revenue equivalent to an ascending auction which attains the same allocation.[7]

To date there has been much more discussion of monotonicity in auctions and other games (Athey (2001?), McAdam (2004), Reny (2004) than symmetry. Systematically investiating the symmetries required to achieve the invariances that we can utilise in economic models would also seem to be an important project for further study of auctions and othe Bayesian games.

1.1 Monotonicity

Recall, MWa assume that there is a vector of affiliated random variables $(S, X) = (S_1, ..., S_n, X_1, ..., X_m)$. S_i is the private signal of bidder i, $i = 1, ..., n$ and the X variables are other pieces of information which may impact on bidder valuations of the objects. Bidder utilities $(U_1, ..., U_n)$ are assumed to be nondecreasing functions of (S, X). These assumptions imply, if information $X = x$ is disclosed before the auction, first that each bidder i's conditional expected utility $E[U_i|S = s, X = x]$ is nondecreasing in s, and second, the conditional joint distribution of bidder signals, $[S|X = x]$, is affiliated in S for each possible realisation x of X. We call this conditional affiliation. Similarly, if the X variables are not disclosed prior to the auction, then each bidder i's conditional expected utility $E[U_i|S = s]$ is nondecreasing in s, and the marginal joint distribution of bidder signals, S, is affiliated. We call this marginal affiliation. In other words, global affiliation implies both conditional and marginal affiliation but is stronger than these two assumptions taken separately. Conditional and marginal affiliation are what is required in order to characterise equilibria with and without disclosure of X. The main difficulty in extending MWa in this way is to establish consistency of the conditions. One can see the nature of the

[7] A cash on the nail auction achieves this.

problem by inspecting the following identity[8]

$$\frac{\partial^2 \ln f_S(s)}{\partial s_i \partial s_j} - E[\frac{\partial^2 \ln f_{S|X}(S|X)}{\partial s_i \partial s_j}|S=s]$$
$$= Cov[\frac{\partial \ln f_{S|X}(S|X)}{\partial s_i}, \frac{\partial \ln f_{S|X}(S|X)}{\partial s_j}|S=s]$$
$$= Cov[\frac{\partial \ln f_{X|S}(X|s)}{\partial s_i}, \frac{\partial \ln f_{S|X}(X|s)}{\partial s_j}|S=s]$$

In words, the cross partial log derivative(s) of the marginal density $f_S(s)$ is equal to the sum of the conditional expectation of the corresponding cross partials of the conditional density and the conditional covariance of likelihood ratios. One can think very informally of the left hand side of this equation as measuring the difference between how informative signals are about each other without disclosure and with disclosure of $X = x$ (If $\frac{\partial^2 \ln f_S(s)}{\partial s_i \partial s_j} = 0$, then bidder signals are independent, if $\frac{\partial^2 \ln f_{S|X}(S|X)}{\partial s_i \partial s_j} = 0$ then they are conditionally independent). Global affiliation implies that this quantity is always positive. Our discussion suggests that it will be interesting to explore models where it is negative but this creates a tension with ensuring S is affiliated. Another identity illustrates an important decomposition of the sensitivity of conditional expectations, $\frac{\partial E[U|S=s]}{\partial s}$ measures the sensitivity of the expectation of U conditional on $S = s$ to s, $\frac{\partial E[U|S=s,X=x]}{\partial s}$ measures the sensitivity when the expectation is also conditioned on $X = x$. The decomposition which follows directly from the law of iterated expectations is

$$\frac{\partial E[U|S=s]}{\partial s_i} - E[\frac{\partial E[U|S,X]}{\partial s_i}|S=s] = Cov(U, \frac{\partial \ln f_{X|S}(X|s)}{\partial s_i}|S=s). \quad (1)$$

The left hand side of this equation is evidently a measure of the differential sensitivity of the the expected value to bidder signals without and with disclosure. If $E[U|S = s, X = x]$ is increasing in x and X and S are affiliated then the quantity is positive. Suppose that $S = (S_1, S_2)$ the signals of two bidders in a second price auction and U is the utility of the object to bidder i, and that MWa symmetry assumption hold, then if the covariance is always positive the expected revenue in such an auction is increased by disclosure of X[9]. We briefly summarise. For regularity we want $\frac{\partial^2 \ln f_{S|X}(S|X)}{\partial s_i \partial s_j} > 0$ and $\frac{\partial E[U|S,X]}{\partial s} > 0$ so that auctions following disclosure of X satisfy the required regularity conditions and we want $\frac{\partial^2 \ln f_S(S)}{\partial s_i \partial s_j} > 0$ and $\frac{\partial E[U|S]}{\partial s} > 0$ so that they hold without disclosure and, for revenue to be decreased we want $Cov(U, \frac{\partial \ln f_{X|S}(X|s)}{\partial s_i}|S = s) < 0$. We look for interesting specifications satisfying these inequalities.

[8] Consider the formula heuristic, dealing with affiliation more generally, and precisely, without the use of the differentiability assumptions implicit in this formula are now familiar.

[9] Again the discussion is heuristic, one can easily generalise to more than two bidders at the cost of more notation. The revenue result is from MWa although the covariance representation may be new.

The following sections gives three different classes of examples. The first class consists of cases within the general symmetric environment of Milgrom and Weber but in which the global affiliation assumption does not hold but for which we can establish that both the conditional and marginal affiliation conditions do hold. This means that we can characterise equilibria a la Milgrom and Weber and we can carry out revenue comparisons. The examples show that there are tractable and natural cases in which seller-disclosed is complementary to bidder information and which (therefore) enhances bidder information rents and decreases expected revenue. This is not the whole story however, indeed it is probably less than half the whole story. The next class abandons the general symmetry condition but is designed to satisfy the weaker assumption of quasisymmetry once we make special assumptions on the component densities. As we pointed out above, this assumption is designed to fix the allocation and thereby facilitate characterisation of equilibria. The remaining class of examples contains a case where we do not impose quasisymmetry. This example has some counterintuitive features: for instance in a first price auction, the "less well informed" bidder bids more aggressively than the better informed bidder in the sense of first order stochastic dominance but both win the auction with equal probability, whereas in a cash on the nail second price auction both bidders have the same distribution of bids but Section 1.2 of this introduction presents another example in which the symmetry assumption is dropped (or rather, as we shall argue later, relaxed). In this example there is a well informed bidder and a poorly informed bidder

2 Examples

2.1 Symmetric Environments

The first example is a standard one which fits within the Milgrom Weber' (1982a) framework.

Milgrom Weber

$$\begin{aligned} S_i &= V + \varepsilon_i \\ X &= V + \eta \end{aligned}$$

This next example is somewhat extreme but it makes the point that information disclosure can be bad for revenue in a particularly clear way. In this example the private signals of the bidders are ancillary to the value of the object - they provide no information by themselves, but do provide information when used in conjunction with information which may be disclosed by the seller.

The Job Market A student, is going on the job market and competing economics departments will make cash salary offers for her services. Her advisor, who may choose to exaggerate the student's qualities, is known personally to the institutions to which the student is applying. Will the circulation of identical letters of reference written by the adviser improve the students expected salary offer? Here is one representation of the situation, the various institutions have private information stemming from their experience of the student's advisor, ψ represents how much the advisor exaggerates the qualities of his students and S_i are the impressions that the various institutions have received about this quantity measured so that higher signals are good news for student quality. We suppose for simplicity that there is no private-information content to the students job market papers and transcript - everyone knows how to read these the same way. The private information of the bidders for the student's services is represented as a noisy measure of how *little* the advisor exaggerates, $S_i = -\psi + \varepsilon_i$, $i = 1, ..., n$. Suppose, if written, the letters' message X can be measured by the sum of the student's quality V plus the advisor's habitual degree of exaggeration, ψ, so $X = V + \psi$. Suppose that no letters are sent, then the competing departments have no useful private information - all they can do is estimate how much the advisor would have exaggerated if she had written a letter. Therefore, with no letters, the departments are effectively in a Bertrand bidding war for a student of average, i.e. expected, quality. The departments dissipate any surplus through competition, get no informational rents and bequeath all the expected surplus to the student. On the other hand, if the letters are sent, then the departments do have private information and will receive informational rents at the expense of the student. In terms of the Milgrom Weber assumptions, note that $E[V|S = s, X = x] = E[V|V + \varepsilon - x = s - x]$. Each S_i and X are negatively correlated and therefore not affiliated. However conditional and unconditionally on X, S is affiliated.

$$\begin{aligned} S_i &= -\psi + \varepsilon_i \\ X &= V + \psi. \end{aligned}$$

The Vineyard Sale A vineyard is of quality V, which we take to be its common value to the bidders. V depends on a combination of the soil quality and prevailing weather patterns. The vineyard produces a harvest in the current year and bidders each measure the quality of the current crop by testing the sugar content of the grapes with error. The farmer collected statistics on rainfall and sunshine over the year of the harvest, should he release these to the bidders?[10] Bidder i's signal S_i equals the quality of the present harvest, made up of the vineyard quality V plus contribution of the current year's weather ψ measured with error ε_i , $i = 1, ..., n$. The farmer's weather measurements are summarised as $X = \psi$.

$$\begin{aligned} S_i &= V + \psi + \varepsilon_i \\ X &= \psi. \end{aligned}$$

[10] Alternatively, the bidders each privately measure the sugar content of their individual grape samples using an instrument borrowed from the seller. The instrument may be biased, should the seller have it recalibrated publicly?.This is essentially a restatement of the previous paragraph in different language $S_i = V + \varepsilon_i$, $\varepsilon_i \sim N(\psi, \sigma)$, $i = 1, ..., n$, $X = \psi + \eta$.

The global affiliation assumption does not hold in this example, to see this consider the conditional expectation $E[V|S=s,X=x]=E[V|S-X=s-x]$ which is evidently decreasing in x if it is increasing in s. What about the other regularity assumptions? Are $E[V|S=s]$ and $E[V|S=s,X=x]$ increasing in s and are S and $[S|X=x]$ affiliated? If the noise terms ε_i all have logconcave densities, then $[S,V|X=x]$ is affiliated and also S is affiliated The first of these implies that $[S|X=x]$ is affiliated and $E[V|S=s,X=x]$ is increasing in s and therefore decreasing in x. However, S and X need not be jointly affiliated, but this will follow if V in addition to the ε_i's has a logconcave density. Referring to (1) one sees the potential problem, under the assumptions, so far $\frac{\partial}{\partial s}E[V|S=s]$ is shown to be the sum of a positive and negative term so we need some condition to imply that the sum is positive. The appropriate condition is easily furnished however, supposing that X in addition to the ε_i's has a logconcave density then S and V are jointly affiliated and hence $E[V|S=s]$ is increasing. To summarise, if the ε_i's are iid and together with X have logconcave densities, then all Milgrom and Weber's regularity conditions hold both conditionally on X being disclosed and if X is not disclosed. If, in addition V has a logconcave density, then the proofs runs along standard lines to show that disclosure of X is bad for revenue. We have elaborated at some length on the requisite assumptions in this example because there is a methodological point to be made. It was crucial to show that $f_{V,S}(v,s)=\int f_{V,S,X}(v,s,x)dx$ is affiliated when $f_{V,S,X}(v,s,x)$ is not affiliated. We pursue this in more detail in a later section but it may be helpful to set out the simple proof for this case. We have

$$f_{V,S}(v,s)=\int\prod_i f_{\varepsilon_i}(s_i-x-v)f_V(v)f_X(x)dx$$

and the "problem" is that the $f_{\varepsilon_i}(s_i-x-v)$ are not affiliated in s_i,x,v whatever assumptions we make on f_{ε_i}. After a change of variables $z=x+v$ we have

$$f_{V,S}(v,s)=\int\prod_i f_{\varepsilon_i}(s_i-z)f_V(v)f_X(z-v)dz$$

under the assumptions $\prod_i f_{\varepsilon_i}(s_i-z)f_V(v)f_X(z-v)$ is affiliated in (s,v,z) and the conclusion follows from the standard theorem of affiliation being preserved under marginalisation.

Oil Prices This is an example in which $E[V|S=s,X=x]$ is increasing in x but in which there is no effect on revenue in a second price auction. As oilmen, bidders for an OCS tract, pride themselves in being able to estimate the productivity of an oilfield, but they do not pretend to be able to predict the future path of oil prices. However, they nevertheless consider the future price of oil to be very relevant to the value of the tract. We might represent this as follows, the private information arises from a noisy measure of the amount of oil in the tract. The value of the tract will depend on the amount of oil, its (random) price P and the unit cost of extraction c which we assume known. X is a public pronouncement on the price of oil. Disclosure of this information s not effect the expected revenue in second price auction but will do so in a first price auction. The reason is that disclosure of this information does not impact on the quality of information that bidders have about each others signals and therefore does not affect information rents or expected revenue.

$$\begin{aligned} S_i &= Q + \varepsilon_i,\ i = 1, ..., n \\ V &= PQ - cQ,\ P > c \\ X &= P + \eta. \end{aligned}$$

We can show, under standard regularity conditions, that the impact of disclosure of X in each of the above examples impacts on revenues in different ways. The first example is the only one in which disclosure is good for the seller. This example fits in with Milgrom and Weber's framework providing we make the obvious appropriate assumption on the random variables.[11]

2.2 Quasisymmetric Environments

Wilson's Drainage Tract Model We discussed this class of models above. V is the value of the drainage tract, bidder 1 is the neighbour who observes the informative signal S_1 and bidder 2 observes noise, under the usual assumptions, revenue will be increased by disclosure

$$\begin{aligned} S_1 &= V + \varepsilon \\ S_2 &= \eta \\ X &= V + \xi. \end{aligned}$$

Espionage I In this example, bidder 1 is better informed than bidder 2. Bidder 1's signal is a noisy observation of the value of the object, bidder 2 observes a noisy version of bidder 1's signal, one can think of this as an espionage model where an uninformed bidder is known to have stolen some market information from the informed bidder

$$\begin{aligned} S_1 &= V + \varepsilon \\ S_2 &= S_1 + \eta \\ X &= \eta + \xi. \end{aligned}$$

Suppose ε, η and ξ all have logconcave densities, then S is affiliated both marginally and conditionally on $X = x$. (S_1, V, X) is affiliated but *minus* S_2 and X are affiliated. X and S_2 might be thought of as informational complementsfor bidder 2 in the sense that disclosure enables this bidder to interpret her own signal better. In the job market example disclosure of complementary information was bad for revenue. In this example however, there is clearly another more important force at work. It is tempting to think in the following terms: without

[11] Note in this case how informative signals are about the value depends on X.

disclosure of X, the 'informational playing field' is not level and disclosure of X levels the field. When X is disclosed, then both bidders know more about each others private information and in the limit as $\sigma_{\xi}^2 \to 0$ neither has private information. Bidding game effectively becomes Bertrand competition in which neither achieves any rents. Note that $E[V|S_1, S_2, X] = E[V|S_1, X] = E[V|S_1]$ is increasing in S_1 but independent of X, in MWa this condition eliminates any revenue effects. In MWb, i.e. if $S_2 = \eta$ again there will similarly be no revenue effects. The example is not covered by the analysis in MWb however for two reasons. First, the bidder signals are not independent and second, the allocation will not be invariant to information disclosure.

Espionage II Suppose now that instead of the above, after the espionage has taken place, the seller also releases a garbled version of the informed bidders signal, we substitute $X = S_1 + \zeta$.

$$\begin{aligned} S_1 &= V + \varepsilon \\ S_2 &= S_1 + \eta \\ X &= S_1 + \zeta. \end{aligned}$$

S_1 is sufficient for the value so, as before, there is no weighting effect as defined by MWb. In MWb, the publicity effect is always positive, but this is a different model so it isn't clear that the effect here will be positive.We argue as follows, since S_1 and X are independent there is no gearing effect either[12],[13]. This implies that the allocation is invariant to $X = x$ and that bidder surpluses are invariant to $X = x$. We propose therefore to consider the amount of information the bidders' signals convey about each other. This suggests a different conclusion than we would arrive at by falsly exprapolating from MWb that there is only a positive publicity effect at work. Suppose for instance that we measure S_1 to be unit normal and that the η and ζ are also normal noise terms. The joint distribution of signals is joint normal and the copula is determined by the correlation coefficient. The correlation between S_1 and S_2 is $2/\sqrt{3}$ whereas conditional on X the correlation is $2/\sqrt{5}$- bidders have less information about each other after disclosure and therefore more privacy of information. This effect increases informational rents and reduces revenue. It is interesting to ask how publication of X affects the distribution of surplus between the bidders. A natural intuition is that publishing X diminishes bidder 1's informational advantage and so shifts the surplus towards bidder 2, in the limit as X becomes more accurate about bidder 2's signal the information advantage is completely eroded. This is the wrong intuition. S_1 is more informative for x than S_2 and the three variables are affiliated under the usual logconcavity assumptions.

[12] Consider the conditional variance $var(E[V|S_1, X = x] \ |X = x)$, by independence this equals $var(E[V|S_1] \ )$.

[13] This can be generalised as follows. The proof is an application of Basu's theorem. Suppose $T(S)$ is a boundedly complete sufficient statistic for estimating the value. If X is ancillary to the value, then there are no weighting or gearing effects.

These conditions together with quasisymmetry of the auction imply that the allocation shifts towards bidder 2 as x increases[14]. That is, we can show that if bidder 2 wins at the realisation (s_1, s_2, x), they also win at the allocation (s_1, s_2, x). Since $E[V|X = x]$ is increasing one establihes a positive covariance $EE[[V \Pr[2 \text{ wins}]|X]$ which is the result [15]. Now to test the previously offered intuition, consider the case

$$\begin{aligned} S_1 &= V + \varepsilon \\ S_2 &= S_1 + \eta + \psi \\ X &= S_1 + \zeta + \psi, \end{aligned}$$

before and after disclosure the situation is as before, just different noise terms. But the transition is different if $\psi$is a large source of the noise for bidder 2. This will make S_2 more informative for X than is S_1. Hence the result is reversed and disclosure shifts the surplus towards bidder 1.

Job market II Here is a different version of the job market example. There are two hiring departments, in department 1 the chair of the hiring committee is an experienced professor who knows the candidate's adviser well. However, he is a busy man, works in a different field and does not have time to read the candidates papers. In department 2, the chair of the recruiting committee is less experienced, she knows nothing about the candidate's advisor but does work in the candidate's field and has more time to read the papers. Should the letters be sent? Here is a representation of the situation, suppose for neatness that $\varepsilon_1, \varepsilon_2$ are iid noise variables

$$\begin{aligned} S_1 &= -\psi + \varepsilon_1 \\ S_2 &= V + \varepsilon_2 \\ X &= V + \psi. \end{aligned}$$

Without disclosure of the letter S_1 has no relevant information whatsoever whereas bidder 2 clearly does. The informational playing field is therefore distinctly uneven. It is interesting to break the impact of disclosing the letter into 2 parts. Call the above scenario without disclosure of X, A, and with disclosure of X, C. The intermediate scenario, C gives the bidders signals

$$\begin{aligned} S_1' &= V + \varepsilon_1 \\ S_2 &= V + \varepsilon_2 \end{aligned}$$

(where $S_1' = S_1 - X$) but without disclosure of X. One moves to C from either A or B by disclosing X. The shift from A to B evidently moves from an unlevel informational playing field to a level one. The shift from B to C, supposing the densities are logconcave, is obtained by disclosure of an informational substitute

[14] This is Lehmann (1988)'s informativeness criterion.
[15] `State as a proposition?`

in a MWa world of symmetry and global affiliation and therefore increases expected revenue. Evidently, we are hinting at an intuition that this disclosure will be good for revenue. This would be an incorrect intuition however, the "levelling the playing field" terminology is largely a red herring. The shift from A to B is one from independent signals to correlated signals and this part of the equation is bad for revenue. Specifically, assuming independence and unit variances for all the component random variables we can show that disclosure leads to lower revenue in, for instance, first price and cash on the nail auctions[16],[17],[18]. Note that both with disclosure and without disclosure, we are in quasisymmetric environments. Specifically, the allocation with disclosure is independent of $X = x$ implying that the sum of bidder surplusses is independent of x. Hence, expected revenue conditional on X is equal to $A(X) = E[V|X]$ minus a constant.

2.2.1 A Classification of the Models.

Here we give our proposed regression based approach to a classification of the models. Suppose that we can sampe the random variables and run regressions, the following figure illustrates how we propose to present the data.

`-NB the figure is wrong - espionage II is wrong, maybe others.-`

2.3 Examples: Non-Symmetric Environments II

This section presents some more examples in which the Milgrom Weber symmetry assumption is relaxed, this time in a more profound way. First, we discuss a quasisymmetric example of the same sort of disclosure.

Suppose there are two statistics, T_1 is informative about the value of the object, specifically, let $V = T_1$. T_2 is not informative about the value of the object. The signals are independent and have the same distribution. Bidder 1 observes the signal T_1 if event $X = 1$ occurs and signal T_2 if event $X = 0$ occurs, similarly bidder 2 observes the signal T_2 if $X = 1$ and T_1 if $X = 0$. X takes the values 0 and 1 each with probability half. The situation is quasisymmetric with and without disclosure and MW regularity obtains since the joint marginal distribution of signals is independent and $E[U_1|S_1, S_2] = 0.5S_1 + 0.5S_2$. There are no revenue effects of disclosure in this case.

We will consider a similar situation in which the two signals have the same supports but not necessarily the same marginal distributions and are no longer independent. This case will be one in which the allocation of the object generally varies both with the disclosure policy and with the format of auction. However,

[16] Cash on the nail auctions are Vickrey auctions in which there is a small cost attached to having the funds available for immediate payment, e.g. the overnight interest rate. A cash on the nail auction is therefore simply a refinement of the Vickrey auction which is known to have multiple equilibria (see Milgrom (1981)) in two bidder common value environments.

[17] In this example revenue equivalence holds pre disclosure, post disclosure cash on the nail auctions generate more revenue than first price. Direct calculation shows that disclosure is bad for cash on the nail auctions, therefore it is bad for first price auctions.

[18] But I didn't keep the calculations - need to check this.

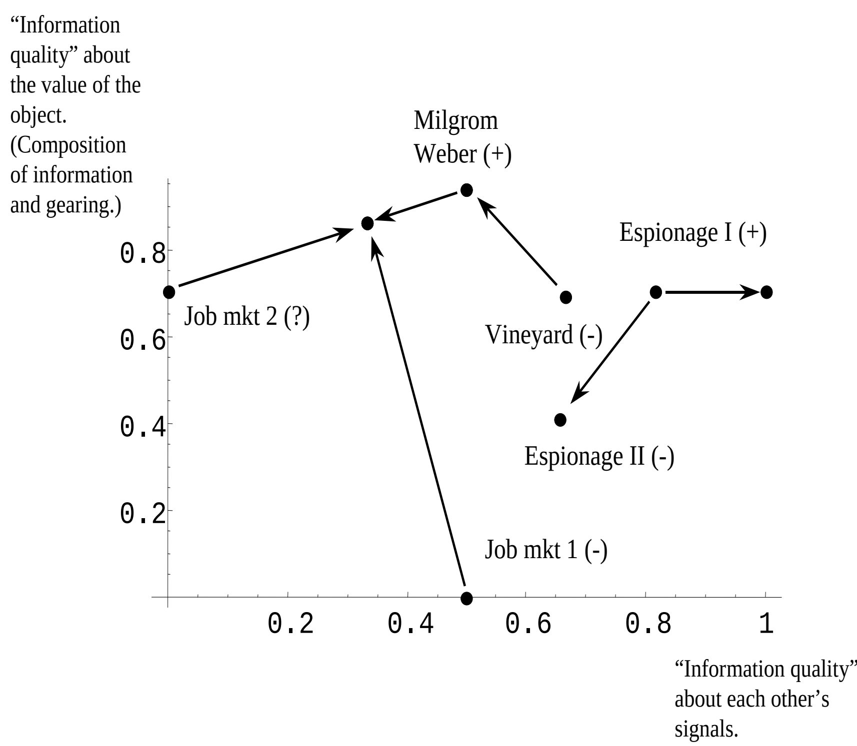

Figure 1: β, ρ plots of the impact of information disclosure.

there are still some symmetries left and we can use these to make the model tractable.

Apart from addressing an example of this substantive question, and more to the point we will construct an example designed to illustrate the general symmetry properties of the joint distribution of bids in such auctions which will form the basis of much of the subsequent analysis and which better explains the principles underlying many of the other asymmetric examples.

The example has a number of very striking features some of which appear nonsensical without a little reflection. For instance, in the first price auction, one of the bidder's can have a sufficient statistic for estimating the value of the object but bid less aggressively than the other bidder by first order stochastic dominance. The more aggressive bidder's signal is ancillary[19]. The example therefore provides a fairly stark rebuttal to a common argument that information disclosure enhances revenue because it reduces the fear of the winners' curse and encourages bidders to bid less conservatively. Also surprising is that the aggressive and less aggressive bidders both win the auction with equal probability. In a cash on the nail auction[20] or an all pay auction, the two bidders have identical bid distributions in equilibrium, nevertheless one of the bidders wins the object with a strictly higher probability. Again the less aggressive bidder, this time in terms of win probability, can be the one with the sufficient statistic.

We begin with a joint distribution on the unit square which is constructed to have the following symmetry: the conditional random variable $[S_1|S_1 \geq S_2]$ has the same distribution as the conditional random variable $[S_2|S_2 \geq S_1]$,

$$\Pr(S_2 \leq s|S_2 \geq S_1) = \Pr(S_2 \leq s|S_2 \geq S_1) = s^2.$$

The joint density is

$$f_{S_1,S_2}(s_1,s_2) = \begin{matrix} 1 \\ 2 \\ 0 \end{matrix} \text{ on } \begin{matrix} 0 \leq s_1 \leq s_2 \\ s_2 \leq s_1 \leq 2s_2 \\ 2s_2 \leq s_1 \leq 1 \end{matrix}$$

One can verify the marginal cdf's are as follows

$$\Pr(S_1 \leq s_1) = s_1$$

$$\Pr(S_2 \leq s) = \begin{matrix} \frac{3}{2}s^2 \\ 2s - \frac{s^2}{2} - \frac{1}{2} \end{matrix} \text{ on } \begin{matrix} 0 \leq s \leq 0.5 \\ 0.5 \leq s \leq 1 \end{matrix}$$

S_1 is uniform on $[0,1]$, S_2 has a skewed kite shaped density. S_2 first order stochastically dominates S_1. Note that the mean of S_1 is $\frac{1}{2}$ and S_2 is $\frac{7}{12}$.

Now we consider a first price auction in which bidders observe s as the signals and the common value is given by $v = aS_1 + (2-a)S_2$ for $0 \leq a \leq 2$. If $a = 0$,we

[19] We mean that, the conditional distribution of the bidder's signal $F_{S_i|V}(s|v)$ is independent of v. In other words the random variables are independent. The term ancillary is normally reserved for families of distributions determined, e.g. by some parameter. Similar comments apply to our use of "sufficient statistic" but we will find the abuse of language useful.

[20] A common expression for having to pay immediately on completion of the sale. "Nails" were plinths on which samples of wheat were inspected prior to auction.

say bidder 1's information is ancillary and bidder 2's is sufficient, and vice versa when $a = 2$.

One can verify that an equilibrium for this game is that bidders simply bid their own signals independently of the values of a, b. For bidder 1, conditional on winning, the marginal distribution of signals is a triangular density with distribution equal to the maximum of two iid $U[0, 1]$ distributions. If their information is ancillary, then onditional on winning the distribution of S_1 and S_2 is identical, the expected win value will just be twice the expected bid. Exactly the same calculations work for bidder 2. Their expected bids and surpluses are identical and independent of a, but one needs to divide by $\frac{14-a}{12}$ to get the shares of the total which are evidently increasing in a. Here, revenue share is decreasing in a. Evidently, the revenue share is increasing in a, bidder 1's share is increasing and bidder 2's share is decreasing. Bidder 2 gets no informational rents when $a = 2$ when their information is ancillary. However bidder 1

Share of Surplus With Disclosure

Bidder 1 Ancillary

	CoN	FP
Seller	$\frac{23}{28}$	$\frac{18}{28}$
Buyer 1	0	$\frac{2}{28}$
Buyer 2	$\frac{5}{28}$	$\frac{4}{28}$

Bidder 2 Ancillary

	CoN	FP
Seller	$\frac{9}{24}$	$\frac{20}{24}$
Buyer 1	$\frac{15}{24}$	$\frac{4}{24}$
Buyer 2	$\frac{5}{24}$	$\frac{8}{24}$

Share of Surplus Without Disclosure

Bidder 1 Ancillary

	CoN	FP
Seller	$\frac{16}{28}$	$\frac{12}{28}$
Buyer 1	$\frac{2}{28}$	$\frac{4}{28}$
Buyer 2	$\frac{2}{28}$	$\frac{4}{28}$

Bidder 2 Ancillary

	CoN	FP
Seller	$\frac{20}{24}$	$\frac{16}{24}$
Buyer 1	$\frac{2}{24}$	$\frac{4}{24}$
Buyer 2	$\frac{2}{24}$	$\frac{4}{24}$

Probability of winning, Mean Bid

Bidder 1 Ancillary

	CoN		FP	
Buyer 1	$\frac{7}{12}$,	$\frac{7}{3}$	$\frac{1}{2}$,	$\frac{7}{12}$
Buyer 2	$\frac{5}{12}$,	$\frac{7}{3}$	$\frac{1}{2}$,	$\frac{1}{2}$

Bidder 2 Ancillary

	CoN		FP	
Buyer 1	$\frac{7}{12}$,	2	$\frac{1}{2}$,	$\frac{7}{12}$
Buyer 2	$\frac{5}{12}$,	2	$\frac{1}{2}$,	$\frac{1}{2}$

In this example, disclosure is always good for revenue for cash on the nail auctions, but not for first price auctions. Presumably, this reflects a differential sensitivity of the cash on the nail auction to asymmetries. ...

3 Symmetry without Global Affiliation

3.1 The Model

We will express the model somewhat differently from Milgrom Weber but it is essentially the same setup. The reason for the departure is that it is crucial to separate out bidders' inference problem from the specification of their preferences. Milgrom and Weber make assumptions which mean that inferences and tastes always pull in the same direction but the purpose of the present exercise is to relax this feature.

Definition 1 (Regularity) *We will say that the triple (U, S, X) is MWa regular if the following three conditions hold.*

Monotonicity $E[U_i|S = s, X = x]$ is nondecreasing in s for each $x \in \mathcal{X}$.

Conditional Affiliation $[S|X = x]$ is affiliated for each $x \in \mathcal{X}$

Marginal Affiliation S is affiliated.

Proposition 2 (MW) *If (i) (S, X) is affiliated (global affiliation), (ii) $U_i = u_i(S, X)$, for some nondecreasing functions u_i, $i = 1, ..., n$, then (U, S, X) satisfies MW regularity.*

Proof. See Milgrom and Weber (1982) ■

3.2 An Affiliation Lemma

4 Asymmetric Common Value Auctions

We assume that the environment is one of pure common values, secondly that there are only two bidders.

4.1 Standard Linear Auctions

A general class of auction forms, which we shall utilise for the analysis, are those in which the object is allocated to the highest bidder and the payments made by the winning and losing bidders are linear functions of both bids. Define therefore, γ_{wl} is the fraction of the losers bid that the winner pays, γ_{ww} is the fraction of the winner's bid that the winner pays, γ_{lw} is the fraction of the winner's bid that the loser pays and γ_{ll} is the fraction of the loser's bid that the loser pays. Similarly for bidder 2. We assume for the time being that the bidders are treated anonymously by the auction form[21].

[21] See Bulow and Klemperer (1999) for an application which dispenses with this assumption.

Second price auction The second-price auction allocates the object to the highest bidder with probability one. Payments are zero for the underbidder and the lower bid for the highest bidder. In this framework, the auction is equivalent to an ascending bid (English) open outcry auction.

$$\begin{bmatrix} \gamma_{ww} & \gamma_{wl} \\ \gamma_{lw} & \gamma_{ll} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Milgrom (1981) pointed out that in the common value environment, second price auctions have multiple equilibria. We shall refer to such auction forms (with $(\gamma_{ww}, \gamma_{ll}) = (0, 0)$) as *singular.*

Cash on the nail auction A cash on the nail[22] auction is an English one in which cash must be paid immediately. Bidders must therefore, we suppose, withdraw the amount of their intended bid from their interest bearing deposit account in order to participate. The few days interest lost is ε times the sum bid

$$\begin{bmatrix} \gamma_{ww} & \gamma_{wl} \\ \gamma_{lw} & \gamma_{ll} \end{bmatrix} = \lim_{\varepsilon \to 0} \begin{bmatrix} \varepsilon & 1 \\ 0 & \varepsilon \end{bmatrix}.$$

5 Analysis

6 Symmetry and Invariance

6.1 Symmetry properties of bid distributions

Proposition 3 (general symmetry) *(Symmetry of bid distribution). For non-singular auctions the equilibrium bid distribution has the following symmetry property. For all measurable sets $\mathcal{B}$,*

$$a_{ww} \Pr[B_1^A \in \mathcal{B}, B_1^A \geq B_2^A] + a_{ll} \Pr[B_1^A \in \mathcal{B}, B_1^A < B_2^A] = \\ a_{ww} \Pr[B_2^A \in \mathcal{B}, B_2^A \geq B_1^A] + a_{ll} \Pr[B_2^A \in \mathcal{B}, B_2^A < B_1^A]$$

Proof. Suppose for a given auction A, bidder equilibrium strategies are $B_i^A = b_i^A(S_i)$, $i = 1, 2$. Bidder 1's bid $\widehat{b}_1 = b_1^A(s_1)$ therefore maximises

$$\int_{\{b_2 \leq \widehat{b}_1\}} \left(E[V|B_2^A = b_2, S_1 = s_1] - \gamma_{ww}\widehat{b}_1 - \gamma_{wl} b_2 \right) f_{B_2^A|S_1}(b_2|s_1) db_2 \\ - \int_{\{b_2 \geq \widehat{b}_1\}} (\gamma_{ll} b_1 + \gamma_{lw} b_2) f_{B_2^A|S_1} db_2.$$

[22] A "nail" was a sort of plinth on which purchasers could spread a sample of corn for inspection before purchase, hence the expression. It means one needs to supply the cash immediately on purchase.

Multiplying by the nonnegative factor $f_{S_1}(s_1)$ maximisation of the above expression implies maximisation, expressed in unconditional probabilities, of

$$\int_{\{b_2 \leq \widehat{b}_1\}} \left(E[V|B_2^A = b_2, S_1 = s_1] - \gamma_{ww}\widehat{b}_1 - \gamma_{wl}b_2\right) f_{B_2^A,S_1}(b_2, s_1)db_2 - \int_{\{b_2 \geq \widehat{b}_1\}} \left(\gamma_{ll}\widehat{b}_1 + \gamma_{lw}b_2\right) f_{B_2^A,S_1}(b_2, s_1)db_2.$$

or upon expanding the conditional expectation, we maximise

$$\int \int_{\{b_2 \leq \widehat{b}_1\}} v f_{V,B_2,S_1}(v, b_1, s_1)dvdb_2 - \int_{\{b_2 \leq \widehat{b}_1\}} (\gamma_{ww}\widehat{b}_1 + \gamma_{wl}b_2) f_{B_2,S_1}(b_2, s_1)db_2 - \int_{\{b_2 \geq \widehat{b}_1\}} \left(\gamma_{ll}\widehat{b}_1 + \gamma_{lw}b_2\right) f_{B_2,B_1,S_1}(b_2, s_1)db_2.$$

The following first order condition holds at $b = b_1(s_1)$

$$\int v f_{V,B_2,S_1}(v, b, s_1)dv - (\gamma_{ww} + \gamma_{wl} - \gamma_{ll} - \gamma_{lw}) b f_{B_2,S_1}(b, s_1) = \gamma_{ww} \int_{\{b_2 \leq b\}} f_{B_2,B_1,S_1}(b_2, s_1)db_2 + \gamma_{ll} \int_{\{b_2 \geq b\}} f_{B_2,S_1}(b_2, s_1)db_2.$$

Integrating over the event $b_1(S_1) = B_1 = b$ now yields

$$\int v f_{V,B_2,B_1}(v, b, b)dv - (\gamma_{ww} + \gamma_{wl} - \gamma_{ll} - \gamma_{lw}) b_1 f_{B_2,B_1}(b, b) = \gamma_{ww} \int_{\{b_2 \leq b_1\}} f_{B_2,B_1}(b_2, b)db_2 + \gamma_{ll} \int_{\{b_2 \geq b_1\}} f_{B_2,B_1}(b_2, b)db_2.$$

For bidder 2, one obtains a similar equation. Observing that the left hand sides of these equations are the same for both bidders, the right hand sides must also be identical, this proves the proposition. ■

Remark 4 *Note that the proof does not rely on the bidders signals being scalar.*

6.2 The three types of auction

The general symmetry proposition allows us to classify linear auctions into three types according to whether $\gamma_{ww} = \gamma_{ll} \neq 0$, $\gamma_{ww} \neq \gamma_{ll}$ or $\gamma_{ww} = \gamma_{ll} = 0$. The following propositions are corollaries of the general symmetry proposition which highlight some economically relevant aspects of the symmetries of the equilibrium bid distribution. There are a number of important consequences of the assumed symmetry in the Milgrom Weber model. Firstly, all bidders win with equal probability, they contribute an identical amount to the expected revenue of the seller and the allocation of the object is invariant to the design of the auction, at least within the class of "standard auctions". These consequences of the symmetry assumption are important ones in the analysis of auctions, for instance the standard proof of the revenue equivalence theorem and revenue ranking results rely crucially on the fact that the allocation of the object is invariant to the auction design among the class being compared. The following propositions explores what remains of these invariance properties in the standard linear auction set up with two bidders and common values.

Type I: Equal win probability If $a_{ww} \neq a_{ll}$ as in first price auctions and wars of attrition, (3) implies on setting $B = \mathbb{R}$ that

$$\Pr[B_1 \geq B_2] = \Pr[B_2 \geq B_1],$$

each bidder wins with equal probability.

Proposition 5 *For auctions within the type I class, bidder equilibrium win probabilities are equal* $\Pr[B_1^A = b_1^A(S_1) \geq B_2^A = b_2^A(S_2)] = \frac{1}{2}$.

The allocation of the object is not generally invariant with respect to the auction design.

If $\det \begin{bmatrix} a_{ww} & a_{wl} \\ a_{ll} & a_{lw} \end{bmatrix} = 0$, *as in a first price auction but not war of attrition, then both bidders always contribute the same expected revenue.*

Proof. The first sentence of the proposition has already been established. The second sentence will be established in the example which follows below in which the allocation of a first price auction is calculated, the allocation in this equilibrium is shown to be different from that of, for instance an all pay auction. Auctions of the class $A = \begin{bmatrix} 1-\delta & 0 \\ 0 & \delta \end{bmatrix}$ include first price auctions ($\delta = 0$) and all pay auctions ($\delta = 0.5$) so the allocation does not remain constant as δ traverses the interval $[0, 0.5]$. This proves the statement if there is no discontinuity at 0.5. Proof of this is omitted. To establish the equal revenue contribution condition, note that the expected revenue contribution of bidder 1 is

$$E[(a_{ww}B_1 + a_{wl}B_2)1\{B_1 \geq B_2\}] + E[(a_{lw}B_2 + a_{ll}B_1)1\{B_1 < B_2\}],$$

similarly for bidder 2.

$$E[(a_{ww}B_2 + a_{wl}B_1)1\{B_2 \geq B_1\}] + E[(a_{lw}B_1 + a_{ll}B_2)1\{B_2 < B_1\}].$$

The general symmetry condition evidently implies

$$a_{ww}E[B_1 1\{B_1 \geq B_2\}] + a_{ll}E[B_1 1\{B_1 < B_2\}] = \\ a_{ww}E[B_2 1\{B_2 \geq B_1\}] + a_{ll}E[B_2 1\{B_2 < B_1\}]$$

hence, the difference between the expected revenue contributions of the two bidders can be written

$$a_{wl}E[B_2 1\{B_1 \geq B_2]] + a_{lw}E[B_2 1\{B_1 < B_2\}] -$$
$$a_{wl}E[B_1 1\{B_2 \geq B_1]] + a_{lw}E[B_1 1\{B_2 < B_1\}].$$

In matrix terms, we have shown that the revenue difference can be written as $k' - k''$ where for some k we have

$$\begin{bmatrix} E[B_1 1\{B_1 \geq B_2\}] & E[B_1 1\{B_1 < B_2\}] \\ E[B_2 1\{B_2 \geq B_1\}] & E[B_2 1\{B_2 < B_1\}] \end{bmatrix} \begin{bmatrix} a_{ww} & a_{wl} \\ a_{ll} & a_{lw} \end{bmatrix} = \begin{bmatrix} k & k' \\ k & k'' \end{bmatrix}.$$

Evidently, assuming $k > 0$, the revenue difference vanishes if and only if the "K" matrix is singular, hence at least one of the matrices on the left hand side must also be singular. Note if $a_{wl} = a_{lw} = a$ then the difference in revenue contributions becomes

$$ER_1 - ER_2 = a(EB_2 - EB_1)$$

when $a > 0$ this means that the bidder with the smaller expected bid contributes more to revenue than the one with the larger expected bid, ■

Corollary 6 *In a*

Type II: Equal bid distribution If $a_{ww} = a_{ll} \neq 0$, as in all pay and cash on the nail auctions then (3) implies

$$\Pr[b_1 \in B, b_1 \geq b_2] + \Pr[b_1 \in B, b_1 < b_2] =$$
$$\Pr[b_2 \in B, b_2 \geq b_1] + \Pr[b_2 \in B, b_2 < b_1]$$

i.e.

$$\Pr[b_1 \in B] = \Pr[b_2 \in B],$$

both bidders have the same distribution of bids. Assuming the bidding functions are invertible, we can write this condition as

$$F_1(b_1^{-1}(b)) = F_2(b_2^{-1}(b)),$$

hence

$$b_1^{-1}(b_2(s)) = F_1^{-1}(F_2(s)).$$

The upshot of this is that it fixes the allocation of the object in terms of the marginal distributions of the bidders' signals independently on how those signals are correlated with the value of the object and independently of the auction forms satisfying $\gamma_{ww} = \gamma_{ll} \neq 0$. Such auctions are rendered relatively tractable.

Proposition 7 *For auctions within the type II class, the equilibrium bids* $B_1^A = b_1^A(S_1)$ *and* $B_2^A = b_2^A(S_2)$ *have the same distribution.*

The allocation of the object is an invariant with respect to the auction design.

If $a_{wl} = a_{lw}$*, as in an all pay auction but not cash on the nail auction then both bidders always contribute the same expected revenue.*

Proof. The first two sentences are established in the text preceding this proposition. The proof of the third follows the proof of proposition (?? previous).

■

One might imagine that in the case $\gamma_{ww} = \gamma_{ll} \neq 0$ where bid distributions are equal that bidders will also have an equal probability of winning, that this is not true is displayed in the example in section ??.

The equal contribution to revenue conditions can be generalised as follows.

Type III: Singular The remaining auction type with $\gamma_{ww} = \gamma_{ll} = 0$, the general symmetry condition does not have any content, the equilibrium allocation is not determined and the auction game has multiple equilibria. We call these singular auctions.

6.3 Special Distributions Quasi-Symmetry

We have seen in the last section that equilibria are naturally endowed with a high degree of symmetry and invariance of allocation with respect to auction design. It is not generally true however that the allocation is invariant to auction design when we move, say, from a first price auction to an all pay one or to a cash on the nail second price one. In this section we identify a class of distributions for which the invariance of allocation property is extended.

The general symmetry proposition can be viewed as a condition determining the distribution of the object: suppose the equilibrium bid of bidder 1 $b_1(s_1)$ in an increasing invertible function of the signal and define $\varphi(s_2) = b_1^{-1}(b_2(s_2))$. Evidently, the function φ determines the allocation, if $s_1 > \varphi(s_2)$ then bidder 1 wins the object and if $s_1 < \varphi(s_2)$ bidder 2 wins. The general symmetry condition can be restated as follows: for any measurable $\mathcal{T}$ $(= b_1^{-1}(\mathcal{B}))$

$$a_{ww} \Pr[S_1 \in \mathcal{T}, S_1 \geq \varphi^A(S_2)] + a_{ll} \Pr[S_1 \in \mathcal{T}, S_1 < \varphi^A(S_2)] = \\ a_{ww} \Pr[\varphi^A(S_2) \in \mathcal{T}, S_1 \leq \varphi^A(S_2)] + a_{ll} \Pr[\varphi^A(S_2) \in \mathcal{T}, S_1 > \varphi^A(S_2)].$$

Our task is to characterise conditions on the joint distribution of signals such that the φ^A so determined is independent of the design parameters of the auction a_{ww}, a_{ll}. We already know what the allocation is when $a_{ww} = a_{ll} \neq 0$, $\varphi = F_1^{-1}F_2$. Hence, we need for all a_{ww}, a_{ll} that

$$a_{ww} \Pr[S_1 \in \mathcal{T}, S_1 \geq F_1^{-1}F_2(S_2)] + a_{ll} \Pr[S_1 \in \mathcal{T}, S_1 < F_1^{-1}F_2(S_2)] = \\ a_{ww} \Pr[S_2 \in F_1^{-1}F_2(\mathcal{T}), S_1 \geq F_1^{-1}F_2(S_2)] + a_{ll} \Pr[S_1 \in F_1^{-1}F_2(\mathcal{T}), S_1 < F_1^{-1}F_2(S_2)].$$

or

$$a_{ww}\Pr[F_1(S_1)\in\mathcal{T},F_1(S_1)\geq F_2(S_2)]+a_{ll}\Pr[S_1\in\mathcal{T},F_1(S_1)<F_2(S_2)]=$$
$$a_{ww}\Pr[F_2(S_2)\in\mathcal{T},F_1(S_1)\geq F_2(S_2)]+a_{ll}\Pr[F_2(S_2)\in\mathcal{T},F_1(S_1)<F_2(S_2)].$$

$$a_{ww}\Pr[F_1(S_1)\in\mathcal{T},F_1(S_1)\geq F_2(S_2)]+a_{ll}\Pr[S_1\in\mathcal{T},F_1(S_1)<F_2(S_2)]=$$
$$a_{ww}\Pr[F_2(S_2)\in\mathcal{T},F_1(S_1)\geq F_2(S_2)]+a_{ll}\Pr[F_2(S_2)\in\mathcal{T},F_1(S_1)<F_2(S_2)].$$

which since, $\Pr[F_2(S_2)\in\mathcal{T},F_1(S_1)<F_2(S_2)]+\Pr[F_1(S_1)\in\mathcal{T},F_1(S_1)\geq F_2(S_2)]=\Pr[F_1(S_1)\in\mathcal{T}]$ and similarly for bidder 2, and moreover since $F_1(S_1)$ and $F_2(S_2)$ both have identical (uniform) distributions, this is equivalent to requiring

$$\Pr[F_1(S_1)\in\mathcal{T},F_1(S_1)\geq F_2(S_2)]=\Pr[F_2(S_2)\in\mathcal{T},F_2(S_2)\geq F_1(S_1)]. \quad (2)$$

(#) is therefore our required symmetry condition. The signals bidders receive carry the same information after any invertible transformation, so the economically relevant aspects of the joint distribution of signals is fully described by the copula

$$C(p_1,p_2)=\Pr\{F_1(S_1)\leq p_1,F_2(S_2)\leq p_2\}$$

we have for $p_1'=p_1+dp_1>p_1$

$$\begin{aligned}C(p_1',p_2)-C(p_1,p_2)&=\Pr\{p_1<F_1(S_1)\leq p_1',F_2(S_2)\leq p_2\}\\&\approx\frac{\partial}{\partial p_1}C(p_1,p_2)dp_1=C_1(p_1,p_2)dp_1\end{aligned}$$

Definition: we will say a joint distribution is quasisymmetric if it has a symmetric copula

Definition: we will say a joint distribution is locally quasisymmetric if it has a copula which is asymmetric in a neighborhood of the ray $(p_1,p_2)=(p,p)$, $0\leq p\leq 1$.

Proposition 8 *If the allocation is invariant to auction form, the joint distribution of signals possesses a locally quasisymmetric copula.*

Proof. This is the content of equation (#). ■

Proposition 9 *1.*

2. Any pair of independently distributed signals is quasisymmetric.

3. If (T_1,T_2) are joint normally distributed, then $(S_1,S_2)=(f_1(T_1),f_2(T_2))$ are quasisymmetric.

4. `To be added ...`

6.4 Invariance Properties of Equilibria

6.4.1 Invariance of the allocation to the stochastic environment

Equation **??** means that the allocation of the object is determined entirely by the marginal joint distribution of signals (s_1, s_2) the bidders receive and is invariant to changes in the value of the object or even in how the signals relate to the value.

Proposition 10 *The equilibrium allocation of the object depends only on the copula of the marginal joint distribution of signals. That is, if we measure bidder signals so that the marginal distribution function of bidder signals is the copula, then auction environments with the same copula will have the same equilibrium allocation of the object in terms of the signals so measured.*

6.5 Quasi-symmetric environments

Proposition 11 *In any quasi-symmetric environment, all non singular auctions have the same allocation bidders have identically distributed bids.*

Remark 12 *Auctions in which the joint marginal distribution of signals can be measured as joint normally distributed are quasi symmetric.*

Proposition 13 *In a quasisymmetric environment, the equilibrium bidding functions can be written in terms of standard normal signals as*

$$b_1(x) = b_2(x) = \frac{\int_{-\infty}^{x} \varphi(t,t) H(t) \mu(t) dt}{H(x)}$$

where

$$\begin{aligned} \mu(t) &= \frac{f_{s_2|s_1}(t|t)}{\gamma_{ww} F_{s_2|s_1}(t|t) + \gamma_{ll}(1 - F_{s_2|s_1}(t|t))} \\ H(x) &= \exp\left[(\gamma_{ww} + \gamma_{wl} - \gamma_{lw} - \gamma_{ll}) \int_{-\infty}^{x} \rho(t) dt \right]. \end{aligned}$$

This is essentially an adaptation of the formula for equilibrium first price equilibrium bidding strategies in Milgrom and Weber (1982a).

Note that we could

Proof. Standard calculation. ■

Some cases simplify relatively conveniently. For the cash on the nail auction, H and $\rho(t)$ are not defined, so one needs to take limits to obtain what appears to be the "standard" bidding strategy in normalised signals, (alternatively use a more direct argument)

$$b_1^{CON}(x) = b_2^{CON}(x) = \varphi(t,t).$$

For the all pay auction,

$$b_1^{AP}(x) = b_2^{AP}(x) = \int \varphi(t,t) f_{s_2|s_1}(t|t)dt.$$

The other formulae are greatly simplified if bidder signals can be measured so that they have a joint normal distribution, for instance any independent signal model. Given the variances of the signals have been normalised, the only remaining parameter in the joint distribution is the correlation coefficient of the signals ρ. In this case, for e.g. the first price auction,

$$\begin{aligned} b_1(x) &= b_2(x) = \frac{\int_{-\infty}^{x} \varphi(t,t)dH(t)}{H(x)} \\ &= \int \varphi(t,t)dH(t|x), \end{aligned}$$

where H is the distribution function

$$H(t) = F\left(t\sqrt{\frac{1-\rho}{1+\rho}}\right)^{\frac{1}{1-\rho}}$$

and $H(t|x)$ is the conditional distribution function given the "event" $t \leq x$.

`To be completed`

6.6 References

ATHEY, S. (1996): "Comparative Statics Under Uncertainty: Single Crossing Property and Log-Supermodularity," MIT Working Paper, July 1996.

ATHEY, S. (2001) "Single crossing Properties and the Existence of Pure strategy Equilibria in Games of Incomplete Information", Econometrica, 69, 861-889.

BULOW J., M. HUANG and P. KLEMPERER (1998): "Toeholds and Takeovers," *Journal of Political Economy.*

De FINETTI

ENGELBRECHT-WIGGANS, P. MILGROM and R. WEBER (198?): *Journal of Mathematical Economics*,

KLEMPERER, P. (1999): "Auctions with Almost Common Values: The Wallet Game and its Applications," *European Economic Review, 42, 757-769.*

LEHMANN, E.

MASKIN, E. and J. RILEY: "Asymmetric Auctions," *Review of Economic Studies*, 67, 413-454.

MILGROM, P.R. (1981): "Rational Expectations, Information Acquisition and Competitive Bidding," *Econometrica*, 49, 921-943.

MILGROM, P.R. and R. WEBER (1982a), "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089-1122.

MILGROM, P.R. and R. WEBER (1982b), "The Value of Information in a Sealed Bid Auction," *Journal of Mathematical Economics*, 10 (1), 105-114.

RENY, P.R. (2004) "On the Role of Single-Crossing for the Existence of Monotone Pure Strategy Equilibria in Bayesian Games", University of Chicago, July.

PERSICO, N. () "Information Acquisition in Auctions," *Econometrica*, ...

VICKREY, W. (1961) "Counterspeculation, Auctions and Competitive Sealed Tenders," *Journal of Finance*, XVI, 8-37.

A Appendix

KARLIN, S. (1956): "On Decision Theory for Polya Distributions, Case of two Actions, I", Proceedings of the Third Berkeley Symposium on Probability and Statistics, Volume 1, University of California Press, Berkeley, 115-129.tbc

B Quasisymmetry

The class of symmetric distributions which MW use is more restrictive than required for our purposes. Denote bidder i's value and signal as $R_i = (U_i, S_i)$, $(U, S) = ((U_1, S_1), ..., (U_1, S_n))$ and let X be the vector of public information which might or might not be disclosed.

Definition 14 $((U, S), X)$ *is a quasisymmetric environment if there exist conditional scales for bidder signals* $T = (T_1, ..., T_n) = (t_1(S_1, X), ..., t_n(S_n, X))$ *such that for each* x *in the support of* X, $[(U, Y)|X = x]$ *is finitely exchangeable. In other words, there exists a conditional joint distribution function which for any permutation* $(\cdot)$ *of bidder names (1,...,n) satisfies*

$$F((u_1, t_1), ..., (u_n, t_n)|x) = F((u_{(1)}, t_{(1)}), ..., (u_{(n)}, t_{(n)})|x).$$

Proposition 15 $((U, S), X)$ *is quasisymmetric iff there exists a representation of the form*

$$\int \prod_{i=1}^{n} F(\tau_i(s_i, x), u_i|x, \omega)g(\omega, x)d\omega.$$

Proof. This is essentially an extension of de Finetti's theorem to transformed finitely exchangeable random variables. ?? made the extension to finitely exchangeable random variables - the generalisation requiring the existence of "negative probabilities" in the kernel g. ■

B.1 Quasi Symmetric

There is an amount V of oil in tract for sale. There are two bidders, each with similar drainage tracts, one North and one South of the field for sale. At the time of the auction, the Northern bidder has taken 10 test drills the Southern bidder has taken only 1, so the Northern bidder is better informed. The seller herself has taken two tests, one close to the Northern border and one close to the Southern border. Should the seller disclose the results of her Northern test or her Southern one, or both, or neither? We might represent this in the obvious notation as

$$S_N = V + \varepsilon_N,\ S_S = V + \varepsilon_S$$
$$X_N = V + \eta_N,\ X_S = V + \eta_S$$
$$(\varepsilon_S, \varepsilon_N, \eta_S, \eta_N) \sim N(0, \Sigma).$$

Where the covariance matrix of the error variables reflects the text above, for instance we should have ε_N, η_N more highly correlated than ε_N, η_S and the variance of ε_S larger than the variance of ε_N. The example raises a number of issues, specifically whether having one bidder better informed than another is bad for the seller and whether sellers might wish to 'level playing fields' by publishing information which is highly correlated with the informationally favoured bidder? This is a very rich environment which we will return to in detail later.

Not $\underset{c}{\Rightarrow}$ This example is designed to highlight the information invariance assumption implicit in Milgrom and Weber, the example departs from (global) symmetry in a minimal way - mechanism invariance is preserved but information invariance does not hold. Bidder valuations each have private and common value elements and bidder signals each equal the sum of two noisy measurements of these respective elements. The information to be disclosed is a vector equal to the vector of the bidder noisy measures of the common value element:

$$\begin{aligned} S_i &= V + \eta_i + V_i + \varepsilon_i \\ U_i &= V + V_i \\ X_i &= V + \eta_i, \ X = (X_1, ..., X_n). \end{aligned}$$

Suppose all the component random variables $V, \eta_i, V_i, \varepsilon_i$ are independently distributed etc. If X is not disclosed then the situation is symmetric among the bidders in the sense that the random payoff signal pairs (U_i, S_i) are finitely exchangeable. Suppose now that X is disclosed, X is sufficient for V in the sense that conditional on X, S and V are independently distributed - therefore S carries no further information about V. Similarly, $S_i - X_i$ is sufficient for V_i. The situation effectively becomes

$$\begin{aligned} S_i' &= S_i - x_i = V_i + \varepsilon_i \\ U_i &= E[V|x] + V_i \end{aligned}$$

which is now symmetric among the bidders in the sense that the random payoff signal pairs (U_i, S_i') are finitely exchangeable. We are evidently in a symmetric Milgrom Weber world both before and after information disclosure but the transition between the two is not of the Milgrom Weber form. What can we say about the impact of information disclosure on revenue in this case? It is easy to see that - it depends. Suppose that the η_i have zero variance, then we are back in a MW scenario and information disclosure, given the usual supporting assumptions on distributions, is good for revenue. On the other hand, suppose instead that V has zero variance, then disclosure is complementary to bidder information as in the vineyard example.

Not Milgrom Weber II This example departs from (global) symmetry in a minimal way - mechanism invariance is preserved but information invariance does not hold. Bidder valuations each have private and common value elements and bidder signals each equal the sum of two noisy measurements of these respective elements. The information to be disclosed is a vector equal to the vector of the bidder noisy measures of the common value element:

$$\begin{aligned} S_i &= U_i + \varepsilon_i + \eta_i \\ X_i &= \eta_i, \ X = (X_1, ..., X_n). \end{aligned}$$

Suppose all the component random variables $U_i, \eta_i, \varepsilon_i$ are independently distributed etc. If X is not disclosed then the situation is symmetric among the bidders in the sense that the random payoff signal pairs (U_i, S_i) are finitely exchangeable. Suppose now that X is disclosed, the situation effectively becomes

$$\begin{aligned} S_i' &= S_i - x_i = V_i + \varepsilon_i \\ U_i &= E[V|x] + V_i \end{aligned}$$

which is now symmetric among the bidders in the sense that the random payoff signal pairs (U_i, S_i') are finitely exchangeable.

B.1.1 Two kinds of Information

Part of the thesis of this paper is that given some symmetry assumptions and sufficient affiliation, there are two channels through which information impacts on the revenue of an auction. There is an aggregated measure of how sensitive expected values are to signals, call this B.if there is no disclosure and B_x if $X = x$ is disclosed. It will be invariant to auction formats to the degree the allocation is invariant. There is also a measure of how much information bidder signals carry about each other. As remarked above, this is characterised by the copula of S, which does not depend on the auction format denote the unconditional and conditional versions as C and C_x respectively. If the joint distribution of signals can be written as joint normal the copula is determined by the correlation coefficient so in this case C_x is invariant to x but will generally be different from C[23].. Revenue given disclosure of $X = x$ can therefore be written $=$ $A(x) - \psi^{\alpha}[B_x, C_x]$. We need to explore the nature of the functional ψ^{α}.

The gist of what we attempt is captured by the following simple observation: if bidders equilibrium bids are determined in a second price auction by $a_x + bS_{xi}$ when $X = x$ is disclosed for some exchangeable[24] random variables $\{(S_{x1}, S_{x2}) :$ x is a realisation of $X\}$ and $a' + b'S_i'$ when there is no disclosure then the expected aggregate bidder surplus comparison will be based on $bE[\max\{S_{x1}, S_{x2}\}|X = x]$ versus $b'E[\max\{S_1', S_2'\}]$ for an arbitrary realisation of $X = x$. If all the signals can be measured to be joint normal, then the comparison will depend on the b coefficients and the correlation coefficients.

[23] More generally, if the copula satisfies some local symmetry property (characterised below) both conditionally and unconditionally and conditional and marginal affiliation hold, then the allocation will be invariant to auction form. If the conditional copula is the same for all realisations of $X = x$ and so is the sensitivity of valuations to signals, then bidder surplus will be independent of x.

[24] Assuming X is a continuous distribution, see the discussion of exchangeability below.