Influential Opinion Leaders

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We rely on expert advice in choices of:

- investments,
- technologies,
- political candidates.

Do experts influence mass opinions?
“Rational players cannot be fooled”:
- Cheap talk,
- Signalling.

But there is evidence:

Rational players can be coordinated:

Many experts:
- How do they coordinate?
Social Learning under Strategic Uncertainty

Global games:
- Strategic uncertainty has large equilibrium consequences.
- Strategic uncertainty only in rare contingencies.
- But that set is pivotal.

Social learning in global games:
- Voters interpret actions of experts.
- Voters neglect the contingencies with strategic uncertainty.
- When strategic uncertainty arises, the voters “get fooled”.
Overview

Setup:

- Many experts: endorse candidates.
- Voters:
  - observe endorsements,
  - elect a winner by majority rule.
- Coordination motive.

Experts’ strategic position is weak:

- No individual “market power”.
- Heterogenous preferences.
- Distribution of biases is common knowledge.

Yet, manipulation arises.
### Intuition

<table>
<thead>
<tr>
<th>Typical State</th>
<th>Pivotal State</th>
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<tbody>
<tr>
<td>- experts know the outcome</td>
<td>- uncertain election outcome</td>
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<tr>
<td>- no impact of biases</td>
<td>- impact of biases</td>
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- Bayesian updating: based on **typical** state.
- Equilibrium condition: based on **pivotal** state.
Conformism

- Desire for united party.
Election Without Experts
model
Voters

- Voters \( i \in [0, 1] \).
- \( a^i \in \{A, B\} \).
- Majority rule; \( w \in \{A, B\} \) — the winner.
- Partisan voters, \( 2/3 \).
- Swing voters, \( 1/3 \):

\[
u_v(a^i, w) = 1_{a^i = w}.
\]

- (0 probability of being pivotal.)
Uncertainty

- Uncertainty over distribution of partisans.
- State $\theta$:
  - if $\theta < 0$ then $A$ wins,
  - if $\theta > 1$ then $B$ wins,
  - if $\theta \in (0, 1)$ then
tie arises when share of swing votes for $A$ equals $\theta$.
- $\theta \sim$ strength of candidate $B$. 
Information

- Common prior: $\theta \sim U[\theta, \bar{\theta}]$.
- Voters’ signals $y^i = \theta + \epsilon^i$.
- Strategy maps $y^i$ to $a^i$. 
analysis
Pivotal Condition

Symmetric monotone Bayes Nash equilibrium:

 ► Outcome monotone in $\theta$.

 ► $\theta^*$ — pivotal state in which tie arises:

\[
\begin{aligned}
w &= A, \text{ if } \theta < \theta^*, \\
tie, \text{ if } \theta = \theta^*, \\
w &= B, \text{ if } \theta > \theta^*.
\end{aligned}
\]

 ► Pivotal condition:

\[
\theta^* = \Pr(s(y^i) = A \mid \theta^*).
\]
Behavior in the Pivotal State

Beliefs $\rightarrow$ actions:
  - Vote for the likely winner.

Distribution of beliefs:
  - $\pi(y^i)$ — posterior that $A$ wins.
  - $\pi(y^i) | \theta^* \sim U[0, 1]$.

Recall pivotal condition:

$$\theta^* = \Pr(\pi(y^i) > 1/2 | \theta^*) = 1/2.$$
Result

Summary

1. Unique monotone BNE.
2. \( \theta^* = \frac{1}{2} \).
Election With Experts
model
Experts

- Experts $j \in [0, 1]$.
- $a^j \in \{A, B\}$.
- $1/3$, partisans supporting $A$.
- $1/3$, partisans supporting $B$.
- $1/3$, swing experts:

  $$u_e(a^j, w) = b^j 1_{a^j=A} + 1_{a^j=w},$$

  $$-1 < b^j < 1.$$  

- Distribution of biases known.

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Information

Experts’ signals:
- \( x^j = \theta + \sigma \xi^j \),
- Support: \( \xi^j \in [-1/2, 1/2] \).

Social Learning:
- Each voter \( i \) privately observes a random sample of \( n \) endorsements.
- \( \lambda^i \in \{0, \ldots, n\} \) — # of endorsements for \( A \) in \( i \)'s sample.
- Unobserved preferences.

Strategy of
- expert maps \( x^j \) to \( a^j \),
- voter maps \((y^i, \lambda^i)\) to \( a^i \).
analysis
Pivotal Condition

- Monotone Weak Perfect Bayesian equilibrium.
- Pivotal Condition:

\[ \theta^* = \Pr(s(y^i, \lambda^i) = A \mid \theta^*) \].

We need to understand:
- experts’ behavior,
- voters’ interpretation of experts’ behavior,
- voters’ behavior.
Experts’ Behavior

\( e(\theta, \theta^*) \) — fraction of experts endorsing \( A \).

Depends on:

- realized state \( \theta \),
- pivotal state \( \theta^* \),
- bias and error distribution.

Simple cases:

\[
e(\theta, \theta^*) = \begin{cases} 
\frac{1}{3}, & \text{for } \theta > \theta^* + \sigma; \\
\frac{2}{3}, & \text{for } \theta < \theta^* - \sigma; \\
\frac{1}{2} + \frac{\bar{b}}{6}, & \text{for } \theta = \theta^*. 
\end{cases}
\]
Voters

Bayesian updating

- As $\sigma \to 0$ voters assign vanishing probability to atypical $\theta$.
- $p_v(y, \lambda)$ — voter’s posterior that $A$ wins.
  - monotone in $\lambda$,
  - independent of the distribution of biases.
Voters
behavior in the pivotal state

Distribution of signals at $\theta^*$:

- $\lambda | \theta^* \sim B \left( \frac{1}{2} + \frac{\bar{b}}{6}, n \right)$.

Updating rule:
- Does not correct for the bias.

Optimal behavior:
- Vote for the more likely winner.

Recall pivotal condition:

\[ \theta^* = \Pr \left( s(y^i, \lambda^i) = A \mid \theta^* \right). \]
Results

Summary

1. *Unique monotone weak perfect equilibrium.*
2. *Characterization of* $\theta^{**} = \lim_{\sigma \to 0} \theta^*(\sigma)$.
3. $\theta^{**}$ *increases with average bias* $\bar{b}$.
4. 
   \[
   \lim_{n \to \infty} \theta^{**}(n) = \begin{cases} 
   1, & \text{if } \bar{b} > 0, \\
   0, & \text{if } \bar{b} < 0.
   \end{cases}
   \]
Noise Independence

Consequences:

▶ Providing additional information to voters does not help.
▶ Experts’ influence can be large.
Summary
“Rational players cannot be fooled”…

Weak interpretation:

- Rational social learning:

\[ I_e | I_v. \]

Truism.

Strong interpretation:

- Correct beliefs in every state:

\[ (I_e | I_v) = (I_e | I_v, \theta). \]

Incorrect.
Voters get fooled at the pivotal state:

- Gap between $I_e|I_v$ and $I_e|(I_v, \theta^*)$.
- Monotone in experts’ biases.
- Equilibrium is monotone in experts’ biases.
- Voters get fooled on a small set of $\theta$.
- But equilibrium consequences are large.