The obvious way to play repeated n-player Battle-of-the-Sexes games: Theory and evidence

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Object of Study

- Game: infinitely repeated n-player Battle-of-the Sexes with discounting
- Perfect (2-players) and limited ($n \geq 2$ players) feedback (monitoring)
- Setting: one-shot (explain!) and unlabeled (explain!)

What we do NOT study: suggest a place for a date

		Woman: FOOTBALL	Woman: OPERA
Man	FOOTBALL	1,0	0,0
	OPERA	0,0	0,1

What we DO study

		P_2: A_2	P_2: B_2
P_1	A_1	1,0	0,0
	B_1	0,0	0,1

Not typically considered a symmetric game

P_2

		A_2	B_2
P_1	A_1	1,0	0,0
	B_1	0,0	0,1

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, B = B^t = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, B^t \neq A$$

But, unlabelled, it is symmetric

P_2

		H	L
P_1	H	0,0	1,0
	L	0,1	0,0

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, B^t = A$$

Objective of this Paper

- Study implications of symmetry in this game
- Provide theory to identify the obvious way to play this game
- Test this theory using experiments

Solution Concept

- Focal point a la Schelling (1960)
- as interpreted by Alos-Ferrer and Kuzmics (2008)
- as attainable equilibria (compare Crawford and Haller, 1990)
- based on player symmetry
- and based on the meta-norm (Alos-Ferrer and Kuzmics, 2008) of Pareto-efficiency, and secondarily (lexicographically) simplicity

Meta-Objective of this Paper

- While, it is difficult to justify general (Nash, SP, sequential) equilibrium in repeated games,
- this paper provides a possible justification for some equilibria, and a somewhat general approach to do so in other (symmetric) games
- an example of (uninformed, naive, pure, constrained) theory first, then evidence
- highlights a possibly more general interplay between ex-ante efficiency, ex-ante symmetry, and ex-post symmetry

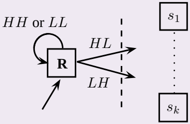
Roadmap and Results

- essentially almost any payoff-pair can be sustained by an attainable strategy profile
- we do NOT get a folk theorem for attainable equilibria
- the set of attainable equilibrium payoff-pairs is an interesting subset of the set of feasible individually rational payoff-pairs
- there is a unique ex-ante efficient attainable equilibrium outcome
- ex-ante efficiency implies ex-post symmetry
- many attainable strategy profiles implement it (especially as $\delta \to 1$)
- there is a (surprising) unique one, which uniformly dominates "all" others as $\delta \to 1$
- which we don't believe to be the obvious one, though
- there is a unique simplest attainable one (in theory only for $\delta = 1$)

Roadmap

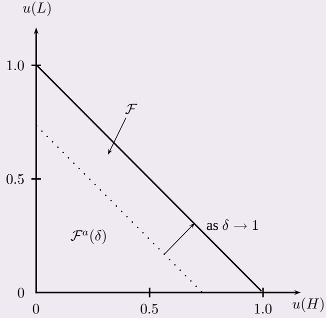
Attainability

Here attainability simply implies that both (all) players use the same repeated game strategy (or same automaton). E.g.



"Randomize until symmetry is broken, then play some usual repeated game strategy between the H-guy and the L-guy"

Set of feasible attainable payoffs



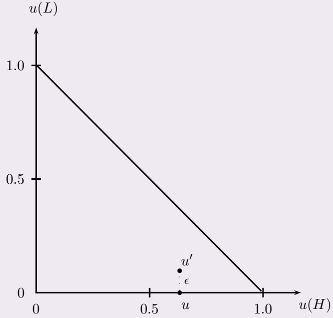
where $u(H)$ is the discounted, normalized payoff of the H-guy and $u(L)$ that of the L-guy

- $\mathcal{F}(\delta) = \mathcal{F}$ for all $\delta \geq \frac{1}{2}$ (Sorin, 1986; Mailath and Samuelson, 2006, Lemma 3.7.1)
- $\lim_{\delta \to 1} \mathcal{F}^a(\delta) = \mathcal{F}$

Roadmap

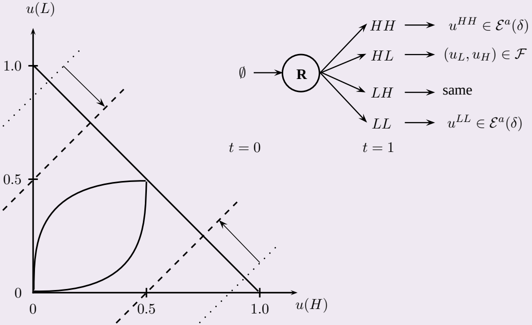
1. Attainable Payoffs
2. Attainable equilibria
3. Obvious way to play

NOT a folk theorem!



Payoff-vectors u and u' cannot be sustained in an attainable equilibrium.

Bounds on the attainable equilibrium payoff-set



- Lower bound is based on stationary (Markov) equilibria
- Upper bound (conjecture) is based on fixed point arguments of an appropriate mapping
- Upper bound (conjecture): $u \in \mathcal{F}$ with $|u(H) - u(L)| > \frac{1}{2}$ cannot be sustained in an attainable equilibrium

Roadmap

Ex-ante efficiency implies ex-post symmetry

- there are many ex-ante efficient attainable payoff-pairs (P-frontier)
- only one is part of an attainable equilibrium
- to maximize joint payoffs players need to initially randomize $\frac{1}{2}, \frac{1}{2}$ until symmetry is broken
- players only have an incentive to do so if continuation payoffs are symmetric

Implementing ex-post symmetric payoffs, Notation

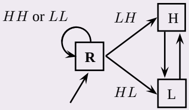
- Consider H-guy plays some sequence $\{y_t\}_{t=1}^{\infty}$, where $y_t \in \{L, H\}$ or $y_t \in \{0, 1\}$
- Example: $H, H, H, L, L, H, L, \ldots$ $(1, 1, 1, 0, 0, 1, 0, \ldots)$
- If H-guy plays H (or L), L-guy plays opposite L (or H)
- Define $u^H(y, \delta) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t y_t$
- Define $u^L(y, \delta)$
- Define $\Delta(y, \delta) = u^H(y, \delta) - u^L(y, \delta)$ (payoff-difference between H and L-guy)

Implementing ex-post symmetric payoffs, Result

- Recall $\Delta(y,\delta)$ is payoff-difference between H and L-guy
- Ex-post symmetric if $\Delta(y,\delta)=0$
- There is a special sequence y^s, such that
- for every periodic sequence, y, there is a $\bar{\delta}<1$ such that
- for every $\delta>\bar{\delta}$: $|\Delta(y^s,\delta)|<|\Delta(y,\delta)|$
- y^s "beats" all periodic sequences (is most symmetric)
- Special sequence is 1|0|01|0110|01101001|0110100110010110|...
- Explain special sequence and its properties!
- Dispense with "periodic"? Don't know.
- Restrict attention to balanced sequences: $\lim_{\delta\to 1}\Delta(y,\delta)=0$

Simplicity

- Is the special sequence the obvious way to play?
- It is uniquely most efficient as $\delta \to 1$
- But somewhat complicated?
- There is a unique simplest attainable ex-post symmetric strategy ($\delta \to 1$)
- in terms of state-complexity of automaton
- tit for tat:



- More interesting for $n \geq 3$ players