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Human Capital, Product Quality, and Growth

by

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Abstract

A growth model is developed in which finite-lived individuals invest in human capital, and investments have a positive external effect on the human capital of later cohorts. Heterogeneous labor is the only factor of production, and higher-quality labor produces higher-quality goods. Stationary growth paths, along which human capital and the quality of consumption goods grow at a common, constant rate, are studied. It is also shown that if a small economy is very advanced or very backward relative to the rest of the world, then its rate of investment in human capital is lower under free trade than under autarky.

In many of the most successful of the newly industrialized economies, countries like Japan, Taiwan, Korea, and Hong Kong, rapid growth in per capita income has been accompanied by rapid expansion in the volume of exports, rapid growth in education, and rapid changes in the composition of output. The purpose of this paper is to develop a theoretical model that is useful for studying this phenomenon.

In the model developed here, heterogeneous labor, differentiated by level of human capital, determines a country's comparative advantage.[1] Empirical work supports this idea. Cross-country differences in human capital are large and are systematically related to patterns of production and trade. Leamer (1984), for example, finds that separating labor into three categories, defined in terms of human capital, is important in explaining world trade patterns for manufactured goods.[2]

In much of the existing literature on long-run growth, labor of different skill levels is assumed to be perfectly substitutable in production. That is, one hour of labor with human capital K is taken to be perfectly substitutable for K hours of labor with human capital of unity. Under this assumption, relative wage rates for labor of different types is determined entirely by the production technology. This simplifying device is useful for many purposes, but it puts severe limitations on the role international trade can play in determining incentives to invest in human capital. In the model developed here, wage rates are affected by the supplies of labor of various types, as well as the demands. The technology for human capital accumulation is important for determining the former; preferences and the technology for goods production for the latter.

The technology for human capital accumulation used here is one that distinguishes between the private human capital of individuals and the stock of knowledge of society as a whole. An individual accumulates human capital by investing--going to school--when young. His level of human capital upon leaving school and entering the workforce depends on the length of this investment period, which he chooses, and on the effectiveness of the time spent, which is determined by the social stock of knowledge available. His level of human capital upon entering the workforce determines his wage rate over the rest of his life, which he spends working. Thus, his choice about the length of the investment period is made by balancing the opportunity cost of later entry into the workforce against higher wage rate apid to more skilled labor. Private investment in schooling also has an external effect: it causes growth in the social stock of knowledge, which increases the effectiveness of time spent in school by later cohorts. Since individuals are finite lived, the external effect is the only source of steady-state growth.[3]

Imperfect substitutability among different types of labor is modelled here by allowing higher-quality labor to perform more highly valued services. Specifically, there is a continuum of goods, differentiated in terms of quality, where quality is defined in terms of Lancasterian (1966) characteristics. Labor is the only factor of production, and only higher-skill labor can produce higher-quality goods. In this setting, as aggregate human capital grows, output growth consists of dropping lower-quality goods from production and adding higher-quality goods. Household preferences over characteristics, together with the production technology, determine a derived demand for labor services of various skill levels.[4]

The model below is developed first for a closed economy. Existence is proved for a stationary growth path, a competitive equilibrium in which human capital and the quality of consumption goods grow at a common, constant rate. It is also shown that if the external effect of investment is sufficiently small, then the equilibrium is unique. In this case, changes in the discount rate, the intertemporal elasticity of substitution, and the productivity of the technology for human capital accumulation affect the equilibrium growth rate in sensible ways.

The effect of free trade is then examined for a small economy, under the assumption that the rest of the world is following a stationary growth path. It is shown that if the small economy is initially much less developed or much more developed than the rest of the world, then a shift from autarky to free trade slows its rate of human capital accumulation.

The rest of the paper is organized as follows. In section 1 the environment is described, and in section 2 competitive equilibria are defined. Stationary growth paths are described in section 3, and the existence and uniqueness of such paths is established in section 4. The small open economy is examined in section 5. Section 6 contains concluding comments. The proofs of all theorems are gathered in the Appendix.

1. The Environment

In this section the economic environment is described. Continuity restrictions and other technical issues are ignored, since they do not arise in the analysis of stationary growth paths.

The model is formulated in continuous time, beginning at date $t = 1$. The economy is composed of many, identical, infinitely-lived households, each composed of an infinite stream of continuously-overlapping generations. Each generation is the same size, and each individual lives for one unit of time. Hence the size and demographic composition of the population are constant over time. The size of each cohort is normalized to unity, so the size of the population at each date is also unity.

Consumption and time allocation decisions are made by the household. Its preferences over infinite consumption streams are stationary and additively separable over time, with a constant rate of pure time preference. There is no utility of leisure, so the time of every individual in every generation is allocated in a way that maximizes the individual's contribution to household income.

At the beginning of his life an individual can spend time investing in human capital. The effectiveness of this investment depends upon the stock of knowledge in society while the investment is undertaken. Let $G(t)$, $t \geq 0$, denote the stock of knowledge at date t, and let $\beta(t) \in [0,1]$ denote the amount of time invested by members of cohort t. Then $G(t)\phi[\beta(t)]$ is the human capital of an individual who is born at date t and spends $\beta(t)$ units of time investing.

<u>Assumption 1</u> The function $\phi: [0,1] \to \mathbf{R}_+$ is strictly increasing, strictly concave, and twice continuously differentiable, with $\phi(0) = 1$.

There is no acquisition of human capital on-the-job,[5] so the individual's human capital is constant over his working lifetime, the interval of time $[t + \beta(t),\ t + 1]$.

The stock of knowledge $G(t)$ grows over time at a rate that depends upon previous cohorts' decisions about investment in human capital. As noted above, this external effect provides the only "engine of growth." For simplicity, it is assumed that the rate of growth of the initial endowment at date t depends only on the investment decision of members of cohort t-1:

(1) $$G'(t)/G(t) = g[\beta(t-1)], \quad t \geq 1.$$

<u>Assumption 2</u> The function $g: [0,1] \rightarrow R_+$ is continuous and strictly increasing, with $g(0) = 0$.

The size and composition of the workforce at each date is described by a function $L(z,t)$, $z \geq 0$, $t \geq 1$, where $L(z,t)$ is the number (mass) of individuals in the workforce at date t who have human capital of at least z. That is, $L(\cdot,t)$ is a <u>right</u> cumulative distribution function for skills in the workforce at date t. Hence for each $t \geq 1$, $L(\cdot,t)$ is a nonincreasing, left-continuous function. Moreover, given the stock of knowledge at date 0, it follows from (1) that the stock at date t is bounded. Hence for each $t \geq 1$, the support of $L(\cdot,t)$ is bounded. Over intervals where $L(\cdot,t)$ is differentiable, $-\partial L(z,t)/\partial z$ is a density function for skills in the workforce. Each discontinuity in $L(\cdot,t)$ corresponds to a mass of workers with the same level of skill. Figure 1 depicts a typical cumulative distribution function for skills and its derivative. There is a continuous distribution of workers in each of the intervals $[z_1,z_2]$ and $[z_4,z_5]$, and there is a mass of workers with skill level z_3.

To compute L from G and β, note that $L(z,t)$ is the number of individuals who are in cohorts $\tau \in [t-1, t]$ (so they are alive at date t), for whom $\tau + \beta(\tau) \leq t$ (so they have finished investing and begun working by date t), and for whom $G(\tau)\phi[\beta(\tau)] \geq z$ (so they have human capital of at least z). Hence,

(2) $$L(z,t) = \int_{t-1}^{t} \chi_{\tau+\beta(\tau)\leq t}\, \chi_{G(\tau)\phi[\beta(\tau)]\geq z}\, d\tau, \quad z \geq 0, \quad t \geq 1,$$

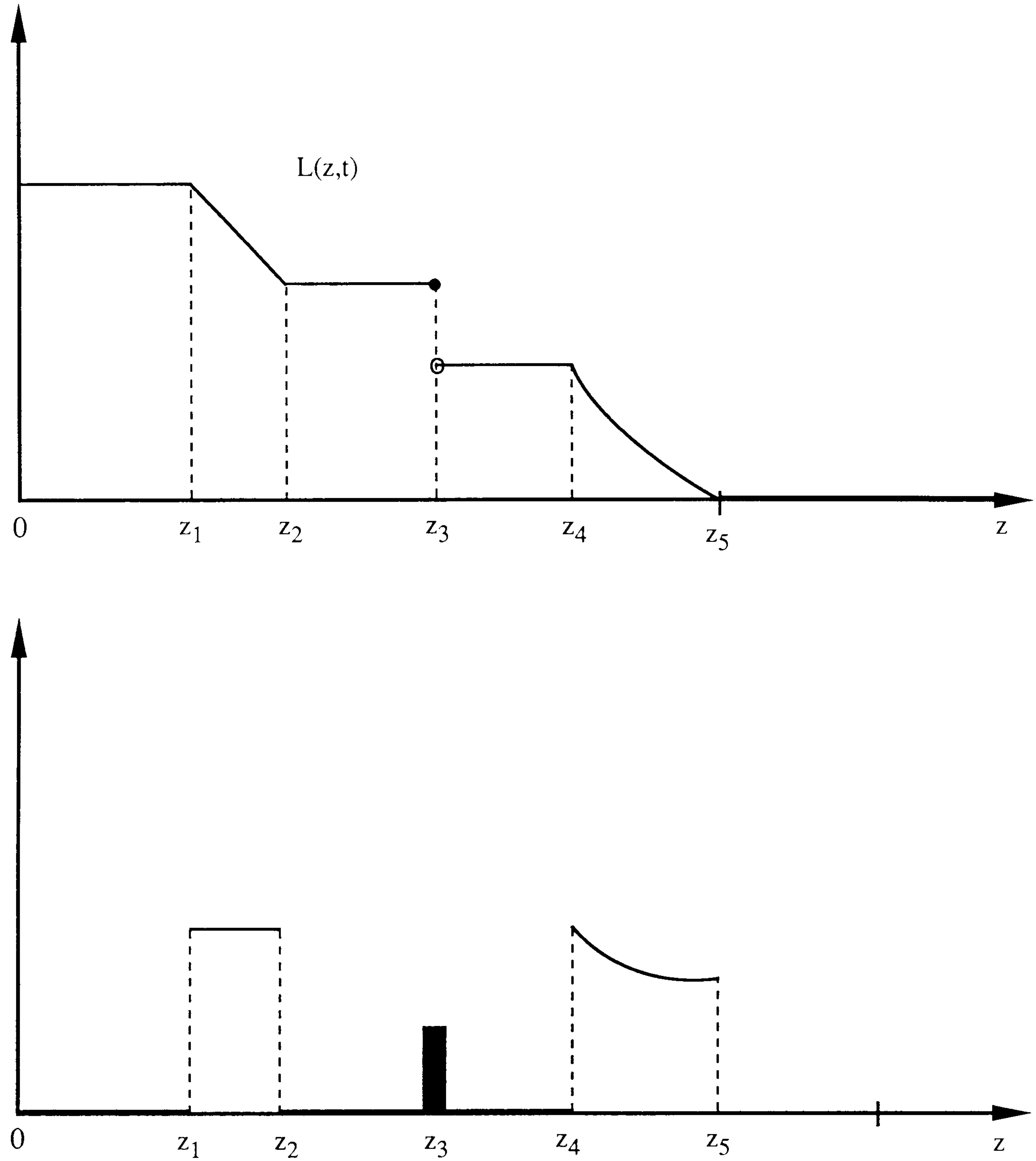


Figure 1

where χ_A denotes the indicator function for the set A.

Goods are valued for the characteristics they contain. At each date there is a continuum of goods and a continuum of characteristics, both indexed on R_+. A unit of the good of quality z provides one unit of each of the characteristics $\xi \in [0,z]$, so higher-index goods are better in the sense that they provide more characteristics. The allocation at every date is described by a function $Q(z,t)$, $z \geq 0$, $t \geq 1$, where $Q(z,t)$ is the quantity of goods consumed at date t that have quality of at least z. Therefore, like $L(\cdot,t)$, the function $Q(\cdot,t)$ is a right cumulative distribution function, so it is nonincreasing and left-continuous. For reasons that will become apparent below, the bound on skill at each date will also be a bound on the quality of goods available at that date.

Since each unit of each good of quality z and above contains one unit of characteristic z, $Q(z,t)$ is quantity of characteristic z contained in the allocation at date t. Over intervals where $Q(\cdot,t)$ is differentiable, $-\partial Q(z,t)/\partial z$ is a density function for the quality levels of goods in the allocation. Each discontinuity in $Q(\cdot,t)$ corresponds to a mass point of consumption goods of the same quality level. The two panels of Figure 1 can, without change, be interpreted as depicting a typical allocation of characteristics (the cumulative function) and the corresponding allocation of goods (its derivative).[6]

The technology is unchanging over time and displays constant returns to scale at each date. Labor of various skill levels is the only input into production. An individual with human capital z can produce (a flow of) one unit of any good of quality less than or equal to z. Hence the feasiblity constraint is

(3) $$Q(z,t) \leq L(z,t), \quad \text{all } z \geq 0,\ t \geq 1.$$

In equilibrium, since higher-quality products will command higher prices, each individual will produce the highest-quality he is capable of producing, and (3) will hold with equality.

The utility function of the representative household is additively separable over time, with a constant discount rate $\rho > 0$ and a constant elasticity of intertemporal substitution $1/\sigma > 0$. In addition, its preferences over characteristics at each date are stationary over time, additively separable, and symmetric. Hence, the intertemporal utility function has the form

(4) $$\int_1^\infty e^{-\rho t} \frac{1}{1-\sigma} \{U[Q(\cdot,t)]\}^{1-\sigma} dt,$$

where $\sigma > 0$, and where

(5) $$U[Q(\cdot,t)] = \int_0^\infty u[Q(z,t)]dz.$$

For $\sigma = 1$, (4) is interpreted as $\int e^{-\rho t} \ln\{U[Q(\cdot,t)]\} dt$.

<u>Assumption 3</u> The function u is strictly increasing, (weakly) concave, and twice continuously differentiable, with $u(0) = 0$ and $u'(0) < \infty$.

It is important that $u'(0)$ be finite, so that zero consumption of some characteristics, and hence of some goods, is possible.

In the limiting case where u is linear, all characteristics are perfect substitutes. In this case, let $c(t) = U[Q(\cdot,t)] = \int Q(z,t)dz$ denote the total quantity of characteristics consumed at date t. The intertemporal utility function in (4) then has the standard form

$$\int_1^\infty e^{-\rho t} \frac{1}{1 - \sigma} [c(t)]^{1-\sigma} dt.$$

Definition A feasible allocation, given the initial conditions $[G(t), \beta(t), 0 \leq t < 1]$, consists of functions $[G(t), \beta(t), L(z,t), Q(z,t), z \geq 0, t \geq 1]$ such that (1)-(3) hold and the integrals in (4) and (5) are well defined.

2. Competitive Equilibria

At each date $t \geq 1$, there are perfectly competitive spot markets for goods of every quality level and labor of every skill level. Let $P(z,t)$, and $W(z,t)$, $z \geq 0$, $t \geq 1$, denote goods prices and wage rates, and let $R(t)$, $t \geq 1$, denote interest rates.

Firms, taking prices and wage rates as given, hire labor of various skill levels and use it to produce goods of various quality levels. Since higher-quality goods always command strictly higher prices, a worker with human capital z always produces the good of quality z. Hence, in equilibrium (3) holds with equality:

$$Q(z,t) = L(z,t), \quad \text{all } z \geq 0, \quad \text{all } t \geq 1. \tag{3'}$$

Since perfect competition implies that labor is paid its marginal product, the wage function satisfies[7]

(6) $$W(z,t) = P(z,t), \quad \text{all } z \geq 0, \quad \text{all } t \geq 1.$$

Firms earn no profits.

Households, taking as given wages, prices, interest rates, and the stock of knowledge, make decisions about investments in human capital, labor supply, and goods purchases. Households can borrow or lend at the market rate of interest, and they have rational expectations (perfect foresight). The household's objective is to maximize its total utility, as given by (4) and (5), subject to an intertemporal budget constraint.

First, consider the household's investment decisions. Since leisure is not valued, each household member divides his time between human capital accumulation and work with the objective of maximizing the present discounted value of his lifetime earnings--his contribution to family income. If an individual born at date t, when the stock of knowledge is $G(t)$, invests for b units of time, then his human capital is $G(t)\phi(b)$, and he works over the time interval $[t + b, t + 1]$. Therefore, given the paths $R(\cdot)$ and $W(\cdot,\cdot)$ for interest rates and wage rates, his investment problem is

(7) $$\max_{b\in[0,1]} \int_b^1 \exp\left[- \int_t^{t+s} R(v)dv\right] W[G(t)\phi(b), t + s]ds.$$

Notice that in solving the investment problem, the household ignores the external effect of its investment decision on the stock of knowledge. Since the external effect is a function of the economy-wide average rate of investment, and since each household is negligably small relative to the whole economy, each household correctly perceives that its own investment plans have no effect on the aggregate.

Given the initial conditions $G(t)$, $\beta(t)$, $0 \le t < 1$, at date 1 and the function $G(t)$, $t \ge 1$, describing the stock of knowledge at all later dates, and with investment decisions $\beta(t)$, $t \ge 1$, determined by (7), the household's labor supply function $L(h,t)$, $h \ge 0$, $t \ge 1$, can be calculated from (2). The household's total, discounted income can then be calculated by summing over family members, to find the flow of income at each date, and then summing over time. The household's income at any date $t \ge 1$ is computed by integrating the distribution function $L(\cdot,t)$ against the wage function $W(\cdot,t)$. Hence family income at date t is $-\int_0^\infty W(h,t)L(dh,t)$, and total, discounted, family income at date 1 is[8]

$$(8) \qquad Y = \int_1^\infty \exp\left[-\int_1^t R(v)dv\right]\left[-\int_0^\infty W(h,t)L(dh,t)\right] dt.$$

Next, consider the household's expenditures. Given market prices $P(\cdot,t)$, at all dates $t \ge 1$, the cost of any allocation function $Q(\cdot,t)$, $t \ge 1$, can be calculated by summing expenditures on various goods at each date and then summing over time. The cost of the allocation at any date $t \ge 1$ is computed by integrating the cumulative distribution function $Q(\cdot,t)$ for goods consumed at that date against the price function $P(\cdot,t)$. Hence total expenditure at date t is $-\int_0^\infty P(z,t)Q(dz,t)$, and the lifetime budget constraint for a household with total discounted income $Y > 0$ at date $t = 1$ is

$$(9) \qquad \int_1^\infty \exp\left[-\int_1^t R(v)dv\right]\left[-\int_0^\infty P(z,t)Q(dz,t)\right] dt - Y \le 0.$$

The household chooses a consumption allocation $Q(z,t)$, $z \geq 0$, $t \geq 1$, to maximize lifetime utility, as given by (4) and (5), subject to the budget constraint (9).

Definition A competitive equilibrium, given the initial conditions $[G(t), \beta(t), 0 \leq t < 1]$ at date 1, consists of functions $[G(t), \beta(t), L(z,t), Q(z,t), P(z,t), W(z,t), R(t), z \geq 0, t \geq 1]$, such that (1), (2), (3′), and (6) hold; $\beta(t)$ solves (7), for all $t \geq 1$; and $Q(\cdot,\cdot)$ maximizes (4)-(5) subject to (8)-(9).

3. Stationary Growth Paths

None of the existing theorems on existence of a competitive equilibrium appear to apply to this system. The analysis below considers the more limited issue of the existence of a stationary growth path, a competitive equilibrium in which all cohorts invest in human capital at a constant rate a, and the stock of knowledge grows at the constant rate $g(a)$ due to the external effect.

Definition A stationary growth path is a competitive equilibrium in which, for some $a \in [0,1]$,

(10a) $\qquad \beta(t) = a, \quad \text{all } t \geq 0;$ and

(10b) $\qquad G(t) = G(0)e^{g(a)}, \quad \text{all } t \geq 0.$

The main idea behind the proof of existence of a stationary growth path is as follows. Fix any constant rate of investment $\beta(t) = a \in [0,1]$, and, without loss of generality, let $G(0) = 1$. Then the path $G(\cdot)$ for the stock of knowledge is given by (10b), and $L(\cdot,1)$, the distribution function for skills in the workforce at date 1, can be computed from (2). Let $q(\cdot) = L(\cdot,1)$. It then follows immediately from (2) that the distribution functions for skill at all later dates satisfy

$$\text{(10c)} \qquad L[e^{g(a)(t-1)}z,t] = q(z), \quad \text{all } z \geq 0, \text{ all } t \geq 1.$$

The upper panel of Figure 2 depicts $L(\cdot;\cdot)$ at date 1 and at a date t when the stock of knowledge has doubled. The doubling in human capital shifts the distribution function to the right by a factor of two. The lower panel of Figure 2 depicts the corresponding density functions. Since each individual spends <u>a</u> units of time investing in human capital and the size of the population is normalized to unity, the size of the workforce is constant at $1 - a$. Hence $1 - a$ is the height of each distribution function and the area under each density function.

Paths for wages and interest rates can then be constructed from the marginal utilities of a household that consumes the allocation $Q(\cdot,\cdot) = L(\cdot,\cdot)$ given by (10c). As will be shown below, the wage profile also shifts at the constant rate $g(a)$, and the interest rate is constant. That is

$$\text{(10d)} \qquad W[e^{g(a)(t-1)}z,t] = p(z), \quad \text{all } z \geq 0, \quad \text{all } t \geq 1, \quad \text{and}$$

$$\text{(10e)} \qquad R(t) = r, \quad \text{all } t \geq 1,$$

where $p(\cdot) = W(\cdot,1)$ is the wage profile at date 1.[9] Note that labor quality available at a fixed wage rate increases at the rate $g(a)$ over time.

Figure 3 displays the wage profiles corresponding to the quantities in Figure 2. The upper panel of Figure 3 shows the marginal utilities associated with the quantities at date 1, the function $\partial W(z,1)/\partial z = p'(z) = u'[q(z)]$, all $z \geq 0$. The lower panel shows the corresponding integral, the wage profile $W(\cdot,1)$ at date 1. It also shows how wages change over time. Since the skill distribution shifts to the right by a factor of two by date t, the wage profile also shifts to the right by a factor of two.

It can be shown that (10d) and (10e) imply that the investment problem (7) has a stationary form. Therefore, the only equilibrium condition that must be checked is the solution to this single maximization problem. If the solution is $\underline{a}$, the constant investment rate fixed at the beginning of the exercise, then there is a stationary growth path with investment rate $\underline{a}$. Thus, establishing the existence and uniqueness of a stationary growth path involves establishing that a certain mapping from investment rates into investment rates has one and only one fixed point. This mapping is developed formally in the current section and analyzed in the next.

Let $G(0) = 1$, fix an investment rate $a \in [0,1]$, and let $q(\cdot;a)$ denote the distribution function for human capital at date 1. There are two case to consider, $a > 0$ and $a = 0$.

If $a > 0$, then $g(a) > 0$ and the stock of knowledge is growing over time. At date 1, cohorts $s \in [0, 1 - a]$ are in the workforce, and cohort s has human capital $e^{g(a)s}\phi(a)$. Hence, for any $s \in [0, 1 - a]$, all workers in cohorts $s' \in [s, 1 - a]$ have human capital of at least $e^{g(a)s}\phi(a)$. This is a group of workers of size $1 - a - s$, so

$$(11a) \qquad q[e^{g(a)s}\phi(a);a] = \begin{cases} 1 - a, & s \in (-\infty, 0], \\ 1 - a - s, & s \in (0, 1 - a], \\ 0, & s \in (1 - a, +\infty). \end{cases}$$

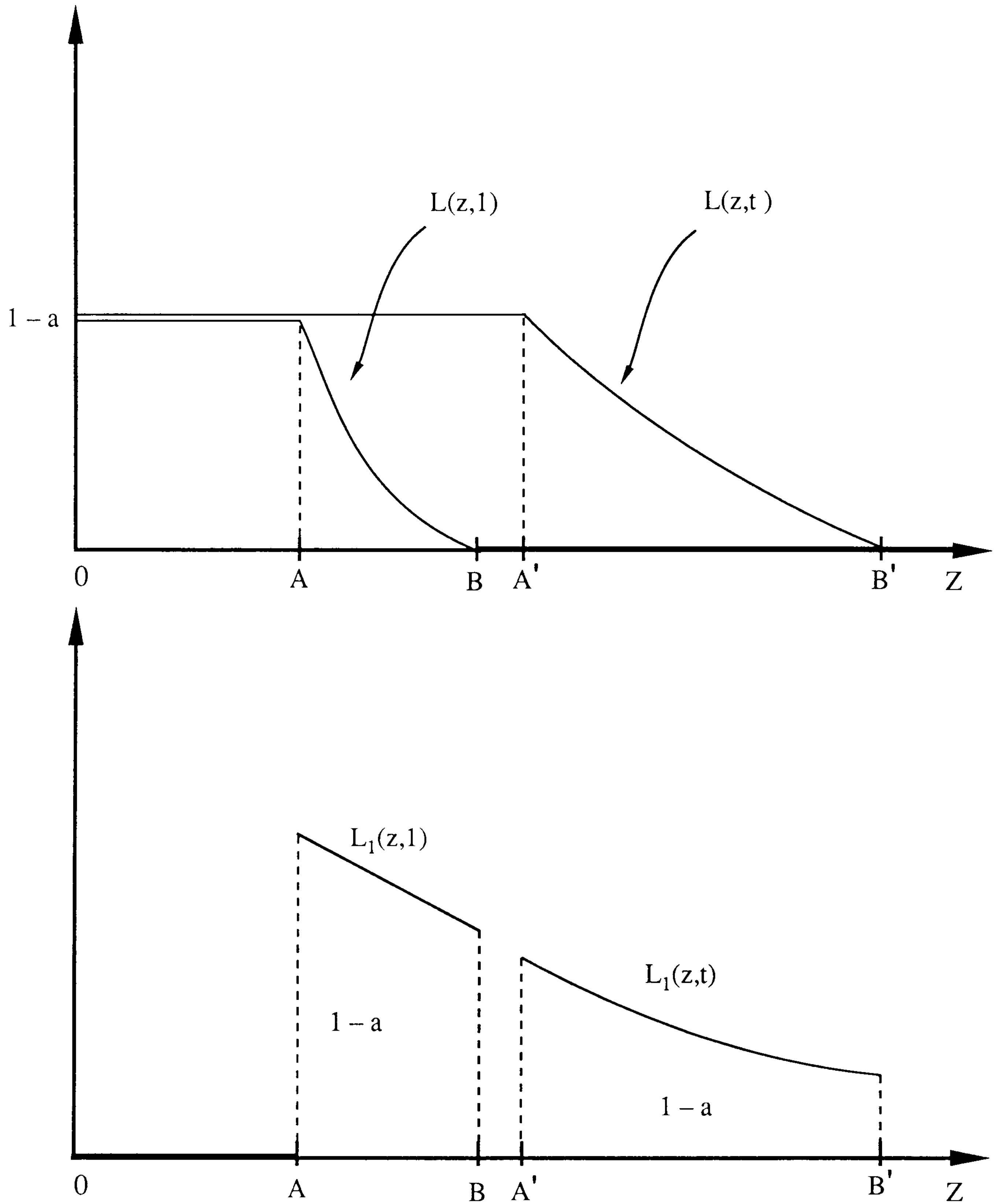
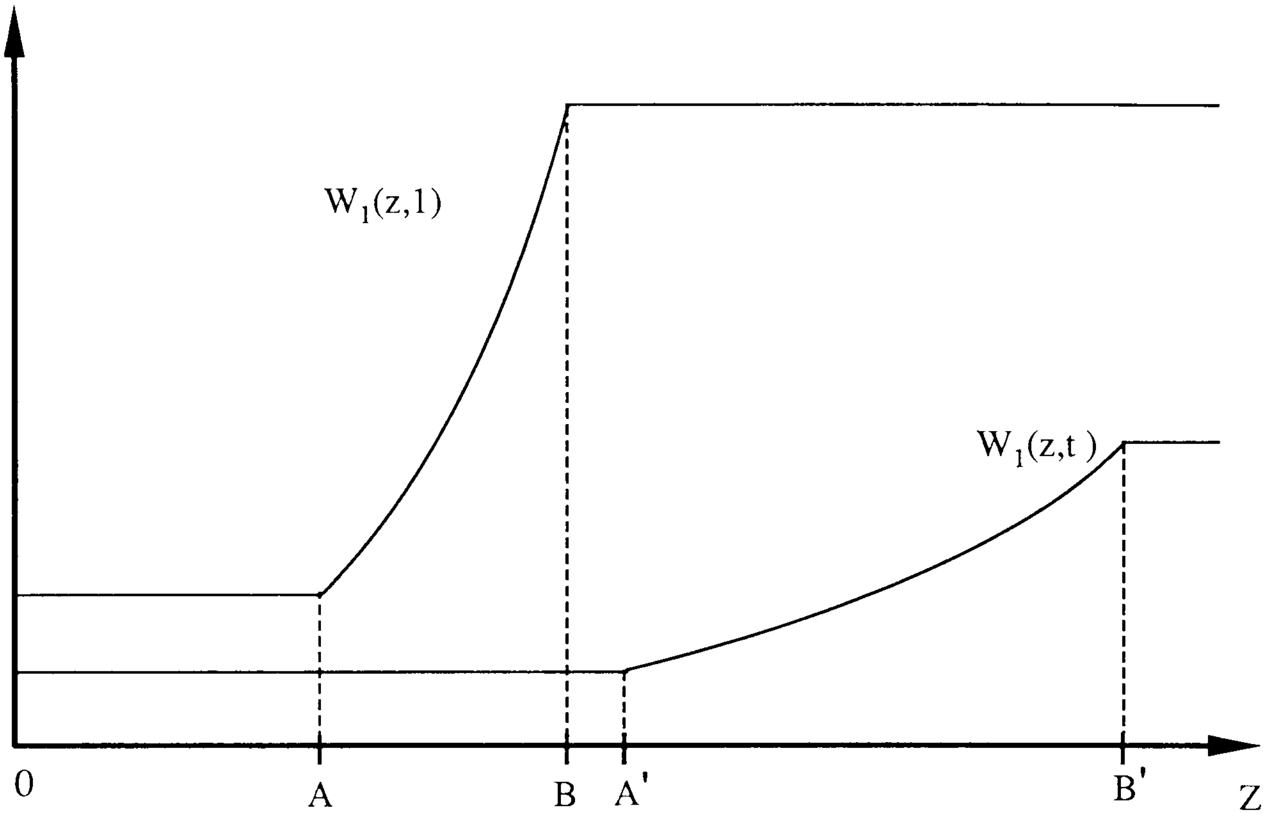


Figure 2



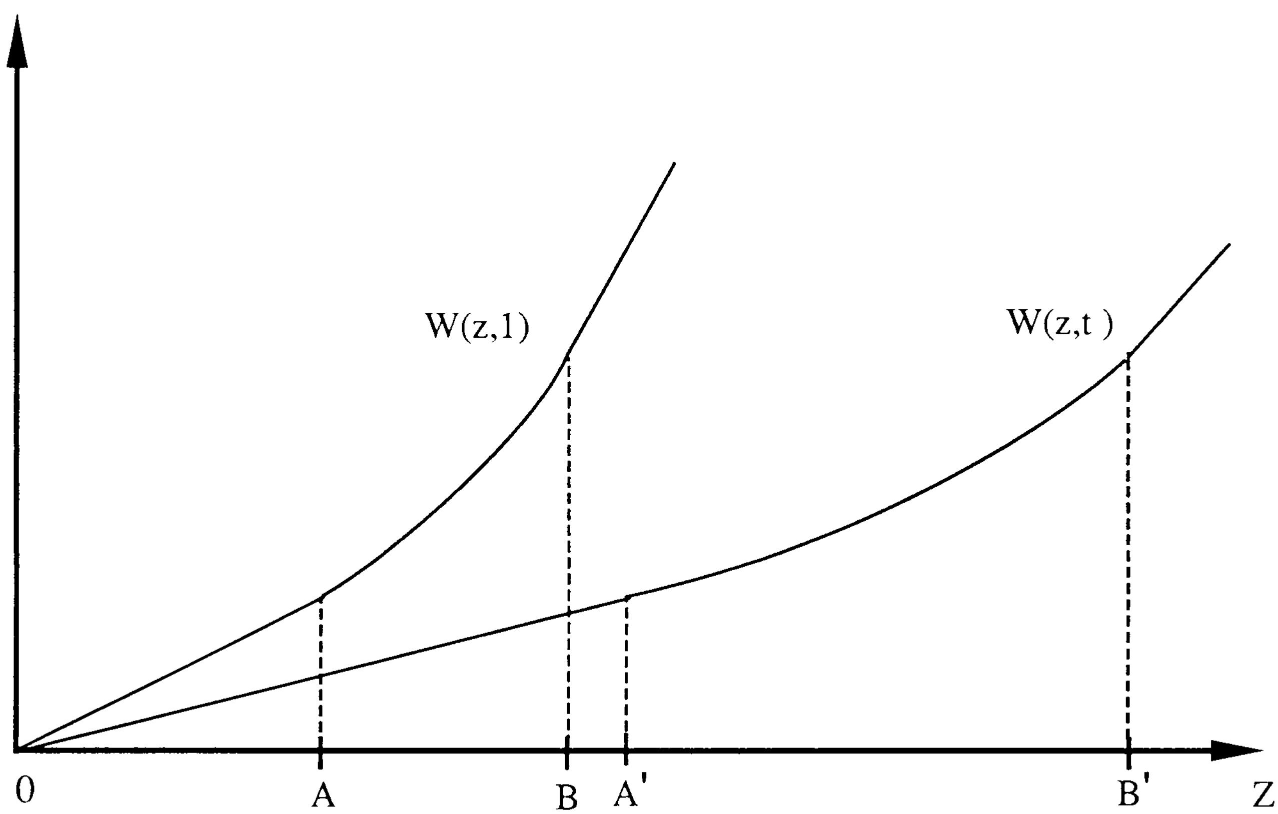


Figure 3

In this case the distribution function for skill is continuous, with a strictly decreasing region connecting two flat regions. Let $p(\cdot;a)$ be the price function given by the marginal utilities associated with $q(\cdot;a)$:

$$p(0;a) = 0, \tag{12}$$

$$p_1[e^{sg(a)}\phi(a);a] = \begin{cases} u'(1-a), & s \in (-\infty, 0], \\ u'(1-a-s), & s \in (0, 1-a], \\ u'(0), & s \in (1-a, +\infty). \end{cases} \tag{13a}$$

If u is strictly concave, then the price function has a strictly convex region between two linear regions. If u is linear, then the price function is linear on all of $\mathbf{R}_+$.[10]

If $a = 0$, then Assumption 2 implies that $g(a) = 0$, so the stock of knowledge is constant over time. At date 1, cohorts $t \in [0,1]$ are in the workforce, and Assumptions 1 and 2 imply that each of them has human capital $G(0)e^{g(0)\tau}\phi(0) = 1$. Since the size of the workforce is unity and all workers have human capital level $z = 1$,

$$q(z;0) = \begin{cases} 1, & z \in [0, 1], \\ 0, & z \in (1, +\infty). \end{cases} \tag{11b}$$

In this case the distribution function for skill has a discontinuity at $z = 1$. The associated prices are given by (12) and

$$p_1(z;0) = \begin{cases} u'(1), & z \in [0, 1], \\ u'(0), & z \in (1, +\infty). \end{cases} \tag{13b}$$

If u is strictly concave, then the price function is composed of two linear regions, with a kink at $z = 1$. If u is linear, then the price function is also linear.

The following assumption ensures that utility is bounded along any stationary growth path. This restriction is needed to ensure that the equilibrium interest rate is positive.

<u>Assumption 4</u> $\rho > (1 - \sigma)g(1)$.

Theorem 1 establishes necessary conditions for a stationary growth path with investment rate $\underline{a}$.

<u>Theorem 1</u> Let Assumptions 1 - 4 hold. If there is a stationary growth path with investment rate $a \in [0,1]$, then the allocation $L(\cdot,\cdot)$ satisfies (10c), where $q(\cdot)$ is given by (11). Supporting wage rates and interest rates $W(\cdot,\cdot)$ and $R(\cdot)$ satisfy (10d) and (10e), where $p(\cdot)$ is given by (12) and (13) and r by[11]

(14) $$r(a) = \rho - (1 - \sigma)g(a).$$

The final equilibrium condition involves the investment problem for a typical family member. In an economy that is following a stationary growth path with investment rate $\underline{a}$, the stock of knowledge is given by (10b), the wage profile by (10d), and the interest rate by (14). Hence the lifetime income of an individual born at date t who invests for b units of time, as given by (7), is

$$\psi(b;a) = \int_b^1 e^{-r(a)s} p[e^{g(a)(1-s)}\phi(b);a]ds, \tag{15}$$

which is independent of t. The following result is then immediate.

Theorem 2 Let Assumptions 1-4 hold. Then there is a stationary growth path with investment rate $a^* \in [0,1]$ if and only if $a^* = \text{argmax}_{b\in[0,1]} \psi(b;a^*)$.

4. Existence of a Stationary Growth Path

To establish the existence of a stationary growth path, (15) will be used to define a continuous mapping from economy-wide investment rates $\underline{a}$ to optimal individual investment rates b(a). Since $p(\cdot;a)$ is convex, however, the problem in (15) is not concave. Therefore, the following assumption is needed to establish that the optimal response b(a) is unique and varies continuously with $\underline{a}$.

Assumption 5 For some $\epsilon > 0$,

$$(1 - b)\phi'(b)/\phi(b) \geq e^{r(a)+g(a)}u'(0)/u'(1), \tag{16}$$

$$\text{all } b \in [0,\epsilon], \quad \text{all } a \in [0,1],$$

$$r(a) + \phi''(b)/\phi'(b) < 0, \quad \text{all } a,b \in [\epsilon,1], \quad \text{and} \tag{17}$$

$$\left[1 - \frac{g(a)\phi(b)}{\phi'(b)}\right]^2 \leq \frac{u'(1 - a)}{u'(0)}, \quad \text{all } a,b \in [\epsilon,1]. \tag{18}$$

The ratio $\phi'(b)/\phi(b)$ is the percentage rate of growth in human capital for additional time invested, for an individual who has already invested b. The restriction in (16) ensures that for sufficiently low rates of investment, this rate of growth is large. The restriction in (17) holds if the technology for human capital accumulation shows strongly diminishing returns: if $\phi''(b)/\phi'(b)$ is large in absolute value. The restriction in (18) holds if the utility function for characteristics shows only mildly diminishing returns: if $u'(1)/u'(0)$ is close to unity. Under Assumption 5, a stationary growth path exists.[12]

<u>Theorem 3</u> Let Assumptions 1 - 5 hold. Then there exists at least one stationary growth path, and all stationary growth paths have investment rates a^* that lie in the interval $(\epsilon,1)$.

The main idea of the proof is as follows. First (16) is used to show that, for any rate of investment in the rest of the economy, the optimal rate of investment for an individual exceeds ϵ. Hence there can be no stationary growth path with a rate of investment less than ϵ. Then (17) and (18) are used to show that, for economy-wide investment rates exceeding ϵ, the individual's best response--the solution to (15)--is unique. That is, together (17) and (18) ensure that ϕ is "concave enough" to offset the convexity of $p(\cdot;a)$, so that (15) has only one local maximum. That solution is a continuous function of $\underline{a}$, so (15) defines a continuous mapping from the interval $[\epsilon,1]$ into itself. Fixed points of that mapping--and there must be at least one--correspond to stationary growth paths.

Along a stationary growth path, the rate of growth of output, as conventionally measured, is constant over time. To see this, choose any two dates t and t + h, and evaluate the labor supplied at date t + h at the wages prevailing at date t. It follows from (10c) and (10d) that

$$- \int W(\xi,t)L_1(\xi,t + h)d\xi$$

$$= \; - \int W[e^{g(t+h-1)}z,t]L_1[e^{g(t+h-1)}z,t+h]e^{g(t+h-1)}dz$$

$$= \; - \int p(e^{gh}z)q'(z)dz \; = \; \Gamma(h).$$

The measured rate of output growth between t and t + h is $\Gamma(h)/\Gamma(0) - 1$. This expression depends on h, the length of time between observations, but not on the date t. Hence the rate of growth, if measured at regular intervals, is constant over time.

The presence of an external effect in this model means that competitive equilibria are inefficient and that there may be multiple equilibria. Theorem 4 establishes that if the external effect is sufficiently small, then the equilibrium is unique. In this case, the effect on the investment rate of changes in the rate of time preference ρ and in the elasticity of intertemporal substitution $1/\sigma$ can be determined, as well as the effect of a change in productivity of the technology for human capital accumulation, for the case $\phi(b) = (1 + b)^{\mu}$, $0 < \mu < 1$. Theorem 5 summarizes these results.

<u>Theorem 4</u> Let Assumptions 1 - 5 hold. If g' is sufficiently small, then the stationary growth path is unique.

Theorem 5 Let Assumptions 1 - 5 hold, and suppose that the stationary growth path is unique. Then a higher rate of time preference ρ leads to a lower rate of investment along the stationary path, as does a lower elasticity of intertemporal substitution $1/\sigma$. If the production function for human capital has the form $\phi(b) = (1 + b)^{\mu}$, $0 < \mu < 1$, then a higher value for μ leads to a higher rate of investment along the stationary path.

5. Investment in a Small Open Economy

In this section, the consequences of a free-trade policy are examined for a small economy. Throughout the section, the stationary investment rate a^* is taken to be unique. The small economy and the rest of the world have identical preferences and technologies, and initially each is following a stationary growth path of the type described above. The two have different initial stocks of knowledge, however, and knowledge does not spill over across international boundaries.

Without loss of generality, let the stock of knowledge in the rest of the world at date 0 be unity, $G(0) = 1$. Let the stock in the small economy be $\hat{G}(0) = \theta > 0$. As long as autarky prevails, both regions invest at the rate a^*, both stocks grow at the rate $g(a^*)$, and the ratio $\hat{G}(t)/G(t)$ is constant. If $\theta \neq 1$, however, then relative prices differ in the two countries, and there are potential (static) gains from trade.[13]

In the rest of the world, a shift from autarky to free trade leaves the paths for prices, wages, and the interest rate are unchanged. Hence the rate of investment a^* and the rate of growth of the stock of knowledge $g(a^*)$ there are also unchanged. In the small economy, the shift does alter the paths for prices and wage rates, and therefore does alter incentives to invest in human capital. The question, then, is whether a shift to free trade strengthens or weakens the incentives for human capital accumulation in the small economy. That is, do individuals in the small economy, under free trade, choose to invest more or less than a*?

Recall that the investment problem for an individual born at date τ in the rest of the world is given by (15). Suppose that the small economy makes a permanent shift to free trade at date τ. Then the investment problem for an individual born at that date in the small country is similar, except that the human capital term, the first argument of p in (15), must be multiplied by θ. Hence, the modified version of (15) takes the form

$$\Psi(b,\theta;a^*) = \int_b^1 e^{-rs} p[e^{g(1-s)}\theta\phi(b);a^*]ds. \tag{19}$$

If Assumptions 1-5 hold, then for each $\theta > 0$ the problem $\max_{b\in[0,1]}\Psi(b,\theta;a^*)$ has a unique solution (see Lemma 4 in the Appendix). This solution, call it $b^*(\theta)$, lies on the interval $(\epsilon,1)$ and is characterized by the first-order condition $\Psi_1[b^*(\theta),\theta;a^*] = 0$. The function b^* describes the optimal rate of investment for an individual in the small economy as a function of the relative size of the stock of knowledge there. By definition, $b^*(1) = a^*$.

First, note that if the utility function u over characteristics is linear, then (12) and (13) imply that p is linear. Hence the parameter θ simply multiplies the expression on the right side of (19), so the optimal investment rate is independent of θ. That is, $b^*(\theta) = a^*$, all $\theta > 0$. Therefore, the stock of knowledge in the small economy grows at the rate $g(a^*)$, and its relative position does not change. In this case, free trade has no effect on the investment rate or growth rate of the small economy, or on its relative position over time.

The intuition behind this result is very simple. If u is linear, then all characteristics are perfect substitutes. In effect, there is only one characteristic, and higher-quality labor produces proportionately more of it. Therefore, labor inputs of all quality levels are perfectly substitutable, and there is no incentive for dissimilar countries to trade. Hence free trade does not affect investment or growth rates.

If u is strictly concave, then (12) and (13) imply that p has a strictly convex region between two linear regions. In this case, the following additional restriction is needed.

<u>Assumption 6</u> $\phi'(b)/\phi(b) \geq g(a^*)$, all $b \in [0,1]$.

Assumption 6 states that the percentage rate of increase in an individual's human capital for incremental time investments always exceeds the percentage rate of increase in the stock of (social) knowledge due to the external effect as time passes. This restriction, like those made previously, holds if the external effect is not too strong.

Theorems 6 - 8 describe the effects of free trade on the incentives to invest in human capital in the case where u is strictly concave. Note that $\theta\phi[b^*(\theta)]$ describes the human capital upon entry into the labor force, under free trade, for an individual in the small economy.

<u>Theorem 6</u> If Assumptions 1 - 6 hold, then $\theta\phi[b^*(\theta)]$ is strictly increasing in θ.

<u>Theorem 7</u> Let Assumptions 1 - 6 hold, and assume that u is strictly concave. Then there exists $\underline{\theta} < 1$ and $\underline{b} < a^*$ such that $b^*(\theta) = \underline{b}$ for

$\theta \leq \underline{\theta}$; and there exists $\overline{\theta} > 1$ such that $b^*(\overline{\theta}) < a^*$ and b^* is strictly decreasing for $\theta > \overline{\theta}$, with $\lim_{\theta \to \infty} b^*(\theta) = \underline{b}$.

Theorem 8 Let Assumptions 1 - 6 hold, and assume that u is strictly concave and that $-u''(q)/u(q)$ is nonincreasing on $[0, 1 - a^*]$. Then $b^*(\theta)$ is strictly increasing at $\theta = 1$.

Theorem 6 states that under free trade, the optimal final level of human capital for an individual in the small economy is a strictly increasing function of the stock of knowledge there. It is reassuring to find that the model delivers this sensible, if unsurprising, conclusion. Theorems 7 and 8 imply that $b^*(\theta)$ is as sketched in Figure 4. Since the relative position of the small economy improves or deteriorates as $b^*(\theta)$ exceeds or falls short of a^*, this figure can be used to study the short-run and long-run dynamics of the system.

The first part of Theorem 7 states that if the small economy is sufficiently backward relative to the rest of the world, then the optimal investment rate for an individual there is less than the steady-state rate and is independent of the degree of relative backwardness. The intuition behind this result is very simple. Since high-skill labor is relatively abundant in the rest of the world, the effect of free trade in the small economy is to lower the relative price of the goods produced by high-skill labor. Hence the incentives to acquire skill are reduced. The long-run dynamics are then clear: the small economy falls ever farther behind the rest of the world in terms of human capital. It does not follow, however, that the small economy is made worse off by free trade. The gains from trade may outweigh the loss from slower growth in human capital.

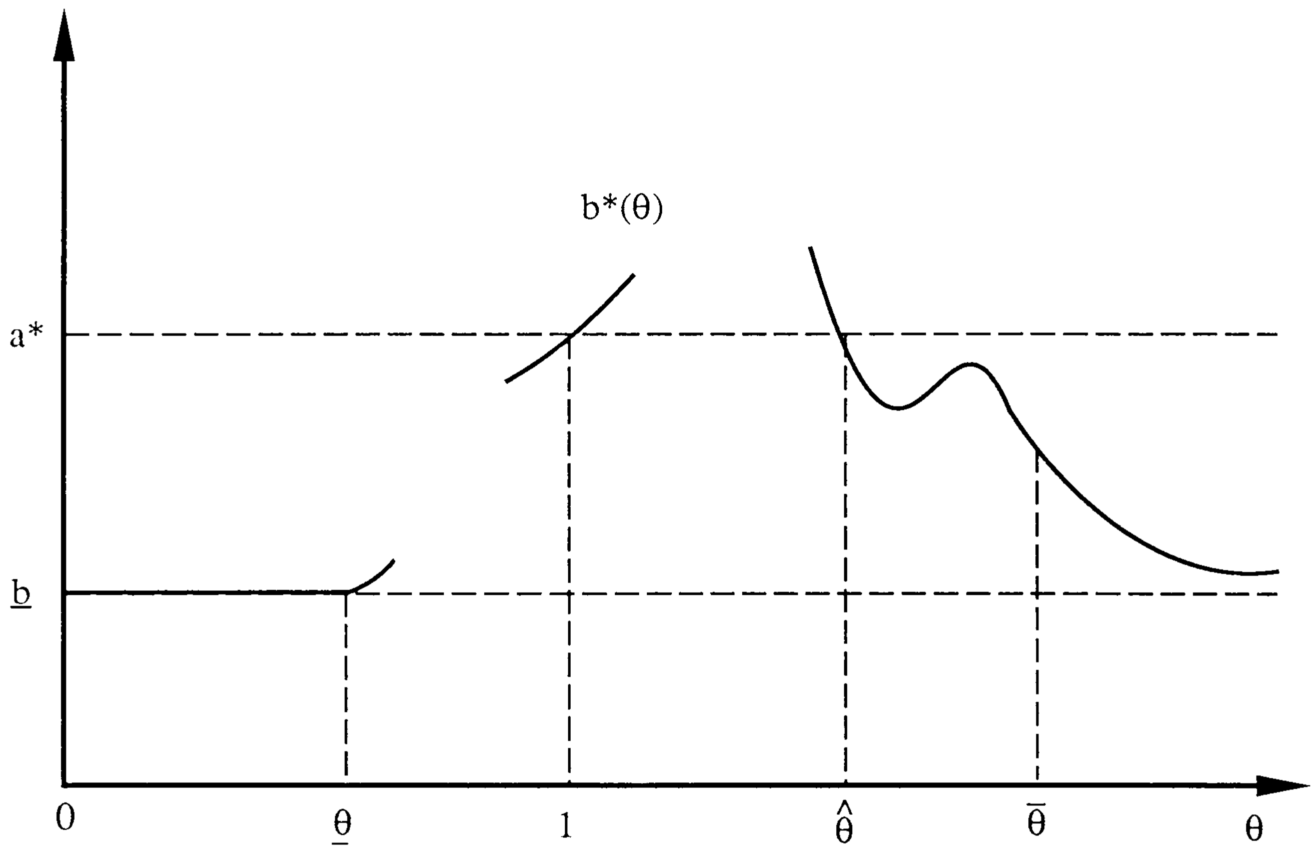


Figure 4

The second part of Theorem 7 states that if the small economy is sufficiently highly developed relative to the rest of the world, then the optimal investment for an individual there is less than a* and decreases with the relative level of development.[14] The intuition behind this result is that even with modest levels of investment, individuals in the small economy are highly skilled relative to labor in the rest of the world. Hence their opportunity cost of investment is high and their optimal investment rate is low. In the short-run, then, the stock of knowledge in the small economy grows at a rate less than g(a*), and its relative advantage shrinks. Theorem 8 provides information about the long-run dynamics. Since b* is increasing at $\theta = 1$ and $b^*(\overline{\theta}) < a^*$, it follows that $b^*(\hat{\theta}) = a^*$ for some $1 < \hat{\theta} < \overline{\theta}$, as shown in Figure 4. After its relative position has fallen to $\hat{\theta}$, the small economy invests at the rate a*, its stock grows at the rate g(a*), and its relative position is unchanged. As before, the welfare effects of free trade are ambiguous.

Figure 4 also provides information about the effect of free trade on investment in a small open economy that begins with a stock of knowledge just slightly larger (smaller) than the stock in the rest of the world. At least in the short run, the small economy invests at a higher (lower) rate than a*, so its stock of knowledge diverges even farther from the stock in the rest of the world. The long-run behavior of the system is also clear from Figure 4: in general, there are an odd number of steady states, including (at least) the points $\theta = 0$, 1, and $\hat{\theta}$, with stable and unstable points alternating.

6. Conclusions

The model analyzed here has emphasized the role of decisions about human capital accumulation in determining the rate of growth. Within this context, international trade affects growth by affecting the incentives for schooling or other investments in human capital. This view of growth and of the relationship between trade and growth raises a number of questions.

Distinguishing between individual human capital and the social stock of knowledge, as has been done here, allows a clearer discussion of the incentives and mechanisms governing the growth of each. The individual investment problem can be treated in a standard, decision-theoretic way, as it has been here. The growth in the stock of knowledge is more problematic, however. Here it has been modeled simply as an external effect.

An interesting extension of the present work would be to introduce a separate research activity, like new product development, that augments the stock of knowledge. If both new blueprints and better-trained workers are needed to produce higher-quality goods, then investments in R&D and in conventional human capital are complementary, and the incentives governing them are linked. The models of growth based on R&D in Aghion and Howett (1989) and Grossman and Helpman (1989), for example, provide frameworks within which conventional human capital might be incorporated.

The location of the external effect, here at a level that can be called national, is also important. The presence of effects that are external to the family immediately implies that the competitive equilibria of the model are inefficient. Too little investment is undertaken, so at the margin, subsidies to education, child labor laws, and other policies that encourage investment will raise welfare. Similarly, as shown in section 5, the presence of effects that are internal to the nation implies that free trade may adversely affect investment and growth. To the extent that the externalities operate at a lower level, within the family, or at a higher one, internationally, these conclusions will be changed.

The analysis above has stressed increases in the quality of schooling rather than the quantity (years) as the source of long-run growth. Conventional methods of measurement pick up only the latter, however, and it is far from obvious how the former can be measured. In the model above, quality improvements can be determined from the shape of the age-earnings profile. But if on-the-job learning is present as well, then the age-earnings profile confounds the two. An interesting empirical issue is how increases in human capital due to improvements in the quality of schooling might be measured.

The conclusion that trade may impede growth for a small, backward economy also follows from a variety of other models in which static comparative advantage determines patterns of long-run growth and trade. Recent papers by Boldrin and Scheinkman (1988), Krugman (1987), Lucas (1988), Stokey (1989), and Young (1989) have explored models in which learning by doing is the only source of productivity gains. If the industries in which the less developed country has a static comparative advantage are industries in which there are limited opportunities for learning, then the effect of free trade is to speed up learning in the more developed country and to slow it down in the less developed one. The model here shows that similar reasoning applies when the external effect operates at arm's length from the production process. An interesting question is whether selective trade restrictions might be useful in allowing a country to protect the incentives to invest in human capital accumulation, while at the same time allowing it to capture a substantial portion of the static gains from trade.

Finally, it is clear that the production technology, which includes no complementarities between labor of different skill levels, is important in arriving at many of the conclusions. If such complementarities were present, and if trade in intermediate goods allowed them to be exploited across international boundaries, then free trade might have very different effects on the incentives for human capital accumulation.

APPENDIX

The proof of Theorem 1 draws on the following two lemmas.

Lemma 1 Fix $a \in [0,1]$ and $y \geq 0$, define $p(\cdot;a)$ by (12)-(13), and suppose that $q^*(\cdot;a,y)$ is the solution to the problem

$$\text{(A.1)} \qquad \max_{\hat{q}} \int_0^\infty u[\hat{q}(z)]dz$$
$$\text{s.t.} \quad -\int_0^\infty p(z;a)d\hat{q}(z) - y \leq 0.$$

Define

$$P[e^{g(a)(t-1)}z,t;a] = p(z;a), \quad \text{all } z \geq 0, \quad \text{all } t \geq 1, \quad \text{and}$$

$$Q^*[e^{g(a)(t-1)}z,t;a,y] = q^*(z;a,y), \quad \text{all } z \geq 0, \quad \text{all } t \geq 1.$$

Then for each $t \geq 1$, the function $Q^*(\cdot,t;a,y)$ solves (A.1) for the prices $P(\cdot,t;a)$ and income y.

Proof Write (A.1) with $P(\cdot,t;a)$ in place of $p(\cdot;a)$, make the change of variable $z = e^{g(a)t}\xi$, and use the definitions of $Q^*(\cdot,t;a)$ and $P(\cdot,t;a)$. □

Lemma 2 Let Assumptions 3 and 4 hold; fix $a \in [0,1]$; define $Q^*(\cdot,\cdot;a,y)$, all $y \geq 0$ as above; and define

$$V(t;a,y) = U[Q^*(\cdot,t;a,y)], \quad \text{all } t \geq 1, \quad \text{all } y \geq 0.$$

Then the solution to the problem

$$\text{(A.2)} \qquad \max_{\{\hat{y}(t)\}} \int_1^\infty e^{-\rho t} \frac{1}{1-\sigma} \{V[t;a,\hat{y}(t)]\}^{1-\sigma} dt$$

$$\text{s.t.} \quad \int_1^\infty \exp\left[-\int_1^t R(s)ds\right] \hat{y}(t)dt \ - \ Y \ \le \ 0,$$

is a constant path, $\hat{y}(t) = \bar{y}$, if and only if $R(t) = \rho - (1-\sigma)g(a)$, all t.

<u>Proof</u> Define $q^*(\cdot;a,y)$ as above and $v(a,y) = U[q^*(\cdot;a,y)]$, all $y \ge 0$. It follows from the definitions of v, V, q^*, and Q^* that

$$\begin{aligned} V(t;a,y) &= \int_0^\infty u[Q^*(\xi,t;a,y)]d\xi \\ &= e^{g(a)(t-1)} \int_0^\infty u[q^*(z;a,y)]dz \\ &= e^{g(a)(t-1)} v(a,y), \quad \text{all } t \ge 1, \quad \text{all } y \ge 0. \end{aligned}$$

Write (A.2) in terms of $v(a;\cdot)$. Since $v(a,\cdot)$ is concave and Assumption 4 holds, the claim follows from a standard variational argument. □

<u>Proof of Theorem 1</u> Suppose there is a stationary growth path with investment rate <u>a</u>. By (2), the allocation $L(\cdot,\cdot)$ satisfies (10c), and by construction of $p(\cdot;a)$, $q(\cdot;a)$ solves (A.1) for the prices $p(\cdot;a)$ and expenditure $\bar{y}(a) = -\int p(z;a)q(dz;a)$. That is, $q(\cdot;a) = q^*[\cdot;a,\bar{y}(a)]$. Hence by Lemma 1, for each $t \ge 1$, $L(\cdot,t)$ solves (A.1) for the prices $W(\cdot,t)$ given by (10d) and expenditures $\bar{y}(a)$. Hence by Lemma 2, the interest rate must be constant at the rate $r(a)$ given by (14). □

The proof of Theorem 3 draws on the following two lemmas.

Lemma 3 Let Assumptions 1,3, and 5 hold, fix $a \in [0,1]$ and $\theta > 0$, and define

$$\text{(A.3)} \qquad \Psi(b,\theta;a) = \int_b^1 e^{-r(a)s} p[e^{g(a)(1-s)}\theta\phi(b);a]ds, \quad \text{all} \quad b \in [0,1].$$

Then $\Psi(\cdot,\theta;a)$ is differentiable, with $\Psi_1(b,\theta;a) > 0$, all $b \in [0,\epsilon]$.

Proof Differentiability follows from Assumptions 1 and 3, with

$$\text{(A.4)} \qquad \Psi_1(b,\theta;a) = - e^{-r(a)b} p[e^{g(a)(1-b)}\theta\phi(b);a]$$
$$+ \theta\phi'(b) \int_b^1 e^{-r(a)s} e^{g(a)(1-s)} p_1[e^{g(a)(1-s)}\theta\phi(b);a]ds.$$

Since ϕ is strictly concave; $u'(1) \le p_1(z;a) \le u'(0)$, all z; and (16) holds,

$$\theta\phi'(b) \int_b^1 e^{-r(a)s} e^{g(a)(1-s)} p_1[e^{g(a)(1-s)}\theta\phi(b);a]ds$$
$$> \theta\phi'(b)(1 - b)e^{-r(a)}u'(1)$$
$$\ge \theta\phi(b)e^{g(a)(1-b)}u'(0)$$
$$\ge e^{-r(a)b} p[e^{g(a)(1-b)}\theta\phi(b);a], \quad \text{all} \quad b \in [0,\epsilon]. \ \square$$

<u>Lemma 4</u> Let Assumptions 1, 3, and 5 hold, and fix $a \in [\epsilon,1]$ and $\theta > 0$. Then there exists exactly one value $b \in (\epsilon,1)$ such that $\Psi_1(b,\theta;a) = 0$, and this value is the unique solution to the problem: $\max_{b\in[0,1]} \Psi(b,\theta;a)$.

<u>Proof</u> It follows from Lemma 3 that $\Psi_1(\epsilon,\theta;a) > 0$, from (A.4) that $\Psi_1(1,\theta;a) < 0$, and from Assumptions 1 and 3 that $\Psi_1(\cdot,\theta;a)$ is continuous on $[\epsilon,1]$. Hence there exists at least one value b for which $\Psi_1(b,\theta;a) = 0$, and it suffices to prove that $\Psi_1(b,\theta;a) = 0$ implies that $\Psi_{11}(b,\theta;a) < 0$.

Differentiating (A.4), suppressing <u>a</u> as an argument of all functions, and substituting from (A.4), we find that $\Psi_1(b,\theta;a) = 0$ implies

(A.5)
$$\begin{aligned}\Psi_{11}(b,\theta;a) &= [r + \phi''(b)/\phi'(b)]\, e^{-rb}p[e^{g(1-b)}\theta\phi(b)] \\ &\quad - \theta[2\phi'(b) - g\phi(b)]\, e^{-rb}e^{g(1-b)}p_1[e^{g(1-b)}\theta\phi(b)] \\ &\quad + [\theta\phi'(b)]^2 \int_b^1 e^{-rs}e^{2g(1-s)}p_{11}[e^{g(1-s)}\theta\phi(b)]ds.\end{aligned}$$

By (17), the first term on the right is negative. Therefore, since $e^{-rb}e^{g(1-b)} \geq e^{-rs}e^{g(1-s)}1$, all $s \in [b,1]$, it suffices to show that

(A.6)
$$\begin{aligned}&[2\phi'(b) - g\phi(b)]p_1[e^{g(1-b)}\theta\phi(b)] \\ &\geq \theta[\phi'(b)]^2 \int_b^1 e^{g(1-s)}p_{11}[e^{g(1-s)}\theta\phi(b)]ds \\ &= \frac{[\phi'(b)]^2}{g\phi(b)} \{p_1[e^{g(1-b)}\theta\phi(b)] - p_1[\theta\phi(b)]\},\end{aligned}$$

or

$$\frac{p_1[\theta\phi(b)]}{p_1[e^{(1-b)g(a)}\theta\phi(b)]} \geq \left[1 - \frac{g(a)\phi(b)}{\phi'(b)} \right]^2.$$

Since $p_1(z;a)/p_1(z';a) \geq u'(1 - a)/u'(0)$, all z,z', (18) suffices. □

Proof of Theorem 3 Note that $\psi(b;a) = \Psi(b,1;a)$, all $a,b \in [0,1]$. By Theorem 2 and Lemma 4, there is a stationary growth path with investment rate $a^* \in [\epsilon,1]$ if and only if $\psi_1(a^*;a^*) = 0$. It follows from Lemma 3 that $\psi_1(\epsilon;\epsilon) > 0$, from (A.4) that $\psi_1(1;1) < 0$, and from Assumptions 1 and 3 that $\psi_1(a;a)$ is continuous on $[\epsilon,1]$. Hence there exists at least value a^* for which $\psi_1(a^*;a^*) = 0$. Together, Theorem 2 and Lemma 3 rule out stationary growth paths with investment rates on $[0,\epsilon]$. □

Proof of Theorem 4 It follows from (12) and (13a) that for $s \in [0, 1 - a]$,

$$p[e^{sg(a)}\phi(a);a] = \phi(a)u'(1 - a) + \int_0^s g(a)e^{vg(a)}\phi(a)p_1[e^{vg(a)}\phi(a)]dv.$$

Therefore, evaluating (A.4) at $b = a$ and $\theta = 1$, substituting from above for p, using (13a) to eliminate p_1, changing the two variables of integration, and dividing by $e^{-r(a)a}e^{g(a)(1-a)}\phi(a)$, we find that $\psi_1(a;a) = 0$ if and only if $H(a) = 0$, where

$$H(a) = - u'(1 - a)e^{-g(a)(1-a)} + \int_0^{1-a} e^{-g(a)s}u'(s)\left[\frac{\phi'(a)}{\phi(a)} e^{-r(a)s} - g(a)\right]ds. \tag{A.7}$$

Hence if H is monotone, then the stationary growth path is unique.

By (14), $g' \approx 0$ implies $r' \approx 0$, so in this case

$$\begin{aligned} H'(a) \approx {} & [u''(1 - a) - g(a)u'(1 - a)]e^{-g(a)(1-a)} \\ & - e^{-g(a)(1-a)}u'(1 - a) \left[\frac{\phi'(a)}{\phi(a)} e^{-r(a)(1-a)} - g(a)\right] \\ & + \left\{\frac{\phi''(a)}{\phi(a)} - \left[\frac{\phi'(a)}{\phi(a)}\right]^2\right\} \int_0^{1-a} e^{-[r(a)+g(a)]s} u'(s)ds. \end{aligned}$$

Cancelling the terms involving $u'(1 - a)$ and using the fact that u is weakly concave and ϕ is strictly concave, we find that $H'(a) < 0$. □

<u>Proof of Theorem 5</u> Let $r(a,\rho)$ and $H(a,\rho)$ denote the functions defined in (14) and (A.7), viewed as a function of ρ as well as <u>a</u>. Since $r_\rho(a,\rho) > 0$ and since $H(a,\rho)$ depends on ρ only through the interest rate, $H_\rho(a,\rho) < 0$. The claim then follows from the fact that $H_a(a,\rho) < 0$.

Define $r(a,\sigma)$ and $H(a,\sigma)$ as above. Since $r_\sigma(a,\sigma) > 0$ and since $H(a,\sigma)$ depends on σ only through the interest rate, the same argument applies.

Finally, define $H(a,\mu)$. Since $\phi'(b)/\phi(b) = \mu/(1 + b)$, clearly $H_\mu(a,\mu) > 0$. The claim then follows from the fact that $H_a(a,\rho) < 0$. □

<u>Proof of Theorem 6</u> Suppress θ as an argument of b^*. The claim holds if and only if $0 < \phi(b^*) + \theta\phi'(b^*)b^{*\prime}$, all $\theta > 0$. From Lemma 4 and its proof, $b^{*\prime} = -\Psi_{12}(b^*,\theta)/\Psi_{11}(b^*,\theta)$ and $\Psi_{11}(b^*,\theta) < 0$. Hence the claim holds if and only if

$$0 > \phi(b^*)\Psi_{11}(b^*,\theta) - \theta\phi'(b^*)\Psi_{12}(b^*,\theta), \quad \text{all } \theta > 0.$$

Differentiating (A.4) and using the fact that $\Psi_1(b^*,\theta;a^*) = 0$, we find that

(A.8) $$\Psi_{12}(b^*,\theta;a^*)$$

$$= e^{-rb^*}\frac{1}{\theta}\{p[e^{g(1-b^*)}\theta\phi] - e^{g(1-b^*)}\theta\phi p_1[e^{g(1-b^*)}\theta\phi]\}$$

$$+ \theta\phi'\phi\int_{b^*}^{1} e^{-rs}e^{2g(1-s)}p_{11}[e^{g(1-s)}\theta\phi]ds,$$

where ϕ is evaluated at b*. Substituting from (A.5) and (A.8), we find that the inequality above holds if and only if

$$0 > \phi\cdot[(r + \phi''/\phi')p + (g\phi - 2\phi')\theta e^{g(1-b^*)}p_1]$$

$$- \phi'\cdot[p - e^{g(1-b^*)}\theta\phi p_1], \quad \text{all } \theta > 0,$$

where p and its derivative are evaluated at $e^{g(1-b^*)}\theta\phi$. The stated assumptions ensure that this is so. □

<u>Proof of Theorem 7</u> It follows from (A.4), (12), and (13a) that

(A.9) $$\Psi_1(b,\theta;a^*) = \theta u'(1 - a^*)\Gamma(b), \quad \text{if } e^{g(a^*)(1-b)}\theta\phi(b) \le \phi(a^*),$$

where

$$\Gamma(b) = e^{-rb}e^{g(1-b)}\phi(b)\{-1 + \frac{\phi'(b)}{\phi(b)}\frac{1}{r + g}[1 - e^{-(r+g)(1-b)}]\}.$$

On the other hand, (A.7) implies that

(A.10) $$0 = - e^{-g(1-a^*)} + \int_0^{1-a^*} e^{-gs}\frac{u'(s)}{u'(1-a^*)}\left[\frac{\phi'(a^*)}{\phi(a^*)}e^{-rs} - g\right]ds.$$

Since the first term on the right of (A.10) is negative, the term in brackets must be positive for at least some values of <u>s</u>. Since that term is decreasing in $\underline{s}$, we can choose $\hat{s} \in (0, 1 - a]$ so that the integrand is postive for $s < \hat{s}$ and negative for $s > \hat{s}$. Then, since u′ is a decreasing function, it follows that

$$0 \;>\; - e^{-g(1-a^*)} + \frac{u'(\hat{s})}{u'(1-a^*)} \int_0^{1-a^*} e^{-gs} \left[\frac{\phi'(a^*)}{\phi(a^*)} e^{-rs} - g\right] ds .$$

Since $u'(\hat{s})/u'(1 - a^*) \geq 1$, it then follows that

$$0 \;>\; - e^{-g(1-a^*)} + \int_0^{1-a^*} e^{-gs} \left[\frac{\phi'(a^*)}{\phi(a^*)} e^{-rs} - g\right] ds$$

$$= \; - 1 + \frac{\phi'(a^*)}{\phi(a^*)} \frac{1}{r + g} [1 - e^{-(r+g)(1-a^*)}] .$$

Hence $\Gamma(a^*) < 0$. Define $\underline{\theta} = e^{-g(1-a^*)}$. Then by (A.9), $\Psi_1(a^*, \underline{\theta}; a^*) < 0$, so by Lemma 4, $b^*(\underline{\theta}) = \underline{b} < a^*$, where $\Gamma(\underline{b}) = 0$. Then (A.9) also implies that $b^*(\theta) = \underline{b}$, all $\theta < \underline{\theta}$.

Next, define θ^o by $\theta^o \phi[b^*(\theta^o)] = e^{g(a^*)(1-a^*)}\phi(a^*)$. By Theorem 6, θ^o is well defined and $\theta^o > 1$. It also follows from (13a) and (A.4) that

$$\text{(A.12)} \qquad \Psi_1(b, \theta; a^*) \;=\; C + \theta u'(0)\Gamma(b), \quad \text{if} \quad \theta\phi(b) \geq e^{g(a^*)(1-a^*)}\phi(a^*),$$

where

$$C = - e^{-ra^*}\{p[e^{g(1-a^*)}\phi(a^*)] - e^{g(1-a^*)}\phi(a^*)u'(0)\} \;>\; 0.$$

Since $\Gamma(a^*) < 0$, for some $\overline{\theta}$ sufficiently large, $\Psi_1(a^*, \theta; a^*) < 0$, all $\theta > \overline{\theta}$. Then by Lemma 4, $b^*(\theta) < a^*$, all $\theta > \overline{\theta}$.

Recall that $b^{*\prime}(\theta)$ has the sign of $\Psi_{12}[b^*(\theta), \theta]$. Using (12) and (13a) to evaluate (A.8), we see that for $\theta \geq \theta^o$, the term in braces is negative and the integrand is identically zero. Hence $b^{*\prime}(\theta) < 0$, all $\theta \geq \theta^{o}$. Finally, since $C > 0$, since $\Gamma(b) = 0$ only if $b = \underline{b}$, and since Γ is continuous at $\underline{b}$, it follows from (A.12) that $b^*(\theta) \to \underline{b}$ as $\theta \to \infty$. □

<u>Proof of Theorem 8</u> It suffices to show that $\Psi_{12}(a^*,1;a^*) > 0$. Note that for any function f that is differentiable on an interval $[A,B]$, $f(B) = f(A) + Bf'(B) - Af'(A) - \int_A^B vf''(v)dv$. Choosing $A = \phi(a^*)$ and $B = e^{g(1-a^*)}\phi(a^*)$, and noting that $p[\phi(a^*)] = \phi(a^*)p_1[\phi(a^*)]$, we find that

$$p[e^{g(1-a^*)}\phi] = e^{g(1-a^*)}\phi p_1[e^{g(1-a^*)}\phi] - \int_0^{1-a^*} e^{2gv}g\phi^2 p_{11}(e^{gv}\phi)dv.$$

Therefore, evaluating (A.8) at $(a^*,1)$ and using (13a), we find that

$$\Psi_{12}(a^*,1;a^*)$$

$$= e^{-ra^*}\int_0^{1-a^*} e^{2gv}g\phi^2 p_{11}(e^{gv}\phi)[\frac{\phi'}{\phi}\frac{1}{g}e^{-r(1-a^*-v)} - 1]dv$$

$$= e^{-ra^*}e^{g(1-a^*)}\int_0^{1-a^*} e^{-gs}u''(s)\ [1 - \frac{\phi'}{\phi}\frac{1}{g}e^{-rs}]ds. \tag{A.13}$$

But since $H(a^*) = 0$, (A.7) implies that

$$\int_0^{1-a^*} e^{-gs}u'(s)\ [1 - \frac{\phi'}{\phi}\frac{1}{g}e^{-rs}]ds < 0. \tag{A.14}$$

Since the term in brackets is strictly increasing in s, and $-u''(s)/u'(s)$ is nonincreasing, (A.14) implies that the expression in (A.13) is positive. □

Footnotes

[1] A static model in which human capital accumulation determines comparative advantage is developed by Findley and Kierzkowski (1983), who analyze a two-sector, two-country model of trade in which unskilled labor and "classrooms" are primary factors, and skilled labor is an intermediate product. They do not consider the issue of growth, however.

[2] Leamer's results confirm earlier work by Keesing (1966, 1971) analyzing the mix of labor skills in imports and exports of the industrialized countries, and by Baldwin (1971), Branson and Junz (1971), and Waehrer (1968) establishing that U.S. exports are intensive in the use of human capital.

[3] The model of threshold effects in Azariadis and Drazen (1990) uses a technology for human capital accumulation very similar to the one used here.

[4] A similar framework is used in Stokey (1988) in a model of learning by doing, and in Stokey (1989) in a static, two-country model of trade.

[5] The assumption that no skills are acquired on the job leads to an odd age-earnings profile: it is downward sloping over the individual's entire working lifetime. This could be remedied by incorporating some version of Rosen's (1976) model of human capital accumulation. This would permit human capital to grow after an individual begins working, giving a more reasonable age-earnings profile.

[6] See Stokey (1988) for a more detailed description of the relationship between allocations of characteristics and goods, and of prices for allocations of either sort.

[7] The wage rates for types of labor in zero supply are, within some range, indeterminate. Equation (6) imposes a particular pricing convention for these types of labor: they are priced at the lowest wage rate consistent with zero demand by firms.

[8] It is also possible to calculate total, discounted, family income by calculating the total discounted income of each family member and then aggregating across family members. Care must be taken to include the income earned after date 1 by family members in cohorts $\tau \in [0,1)$.

[9] Implicit in (10d) is a particular normalization for prices at each date: they are normalized so that current output evaluated at current prices is constant over time. The choice of normalization convention is, of course, purely a matter of convenience, but it does affect the interest rate.

[10] The prices of goods in zero supply are, within some range, indeterminate. Equations (12) and (13) impose a particular convention for them: they are priced at the lowest price consistent with zero demand by households.

[11] The supporting prices are unique, given the conventions for pricing commodities in zero supply (see footnotes 7 and 10) and for normalizing spot prices at each date (see footnote 9).

[12] A parametric family of examples that satisfy Assumptions 1 - 5 is the following: $\phi(b) = 1 + b^{1/2}$; $g(a) = \lambda a$, $0 < \lambda < 1/2$; $u(q) = (1 - e^{-\nu q})/\nu$, $\nu > 0$; $\sigma = 1$; $0 < \rho < 1/2$; $\rho + \lambda + \nu < \ell n(3/2)$; $\epsilon = 1/16$. It is immediate that under these parameter restrictions, Assumptions 1 - 4 and (16) and (17) hold. For $\nu > 0$ sufficiently small, (18) also holds.

[13] If some commodities, like services, are not tradeable, then labor heterogeneity also creates migration pressures. These are ignored in the analysis here.

[14] Under free trade with a small, more advanced economy, small quantities of previously unproduced goods become available in the rest of the world. Under autarky, the prices of these goods were, within some range, indeterminate. Under the pricing convention in equations (12) and (13), however (see footnote 10), the prices of these goods remain unchanged under free trade with the small economy.

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