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CROSS LICENSING OF COMPLEMENTARY TECHNOLOGIES

by

Chaim Fershtman*

Morton I. Kamien**

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*Department of Managerial Economics, J.L. Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois 60208 and Department of Economics, Tel Aviv University, Tel Aviv, Israel 69978.

**Department of Managerial Economics, J.L. Kellogg Graduate School of Management, Northwestern University, Evanston, Illinois 60208.

1. Introduction

It often happens that the introduction of a new product requires the development of several distinct complementary technologies. These new technologies may be in the form of component parts or production methods, or both. It also sometimes happens that these complementary technologies are independently developed by different firms. In these circumstances each firm possesses a technology that only has value when combined with those of the other firms'. Each firm may in these circumstances seek to develop the complementary technologies on its own or choose to provide access to its technology to others in exchange for access to their technologies. This exchange of complementary technologies typically takes the form of cross licensing. To the firm, cross licensing presents two advantages and one disadvantage. The first is avoidance of the costs of developing the complementary technology. The second is earlier introduction of the final product and realization of profits. The single disadvantage is creation of a rival provider of the final product. Obviously, each firm will balance the advantages against the disadvantage in deciding whether to engage in cross licensing or continuing to go it alone in developing the final product. However, the very potential for cross licensing may impact on a firm's choice of how rapidly to attempt to develop the new technologies while at the same time the potential continuation of the race may affect the cross licensing arrangement. It is this interdependence that is the main focus of our paper.

We address the role of the potential for cross licensing on the pace of the race in terms of a game between two firms, that involves both

cooperative and noncooperative elements. It is supposed that introduction of the final product requires development of two complementary technologies. At the outset of the race each firm may invest in the development of each technology. These decisions, along with how much to invest in developing the two technologies are made noncooperatively by the two firms. We assume that the development process for each of the technologies is stochastic but subject to influence through the rate of investment of resources. In practice this means that hiring of scientists and engineers does not guarantee succesful development of a new technology but that the more of them around, the better the chances. We use the commonly employed assumption that successful development of a technology at a point in time is independent of past development expenditures. Put technically, the hazard rate, the instantaneous conditional probability of success slightly after time t, given no success up to t, is independent of time. However, the hazard rate does depend on the rate of expenditure at time t. In analyzing this race we employ the subgame perfect Nash equilibrium (SPNE) as our solution concept.

The cooperative aspect of the game arises if one of the firms succeeds in developing one of the technologies while the other develops the complementary technology. Cooperation can take the form of cross licensing of the complementary technologies. We suppose that the outcome of the negotiations regarding the cross licensing terms can be described by the Nash bargaining solution in which continuation of the race acts as a threat point.

The study of patent races has not dealt with the cross licensing of complementary technologies as a recent survey by Reinganum (1985) discloses, nor has the study of patent licensing (see Kamien, 1989). Our analysis is closest to that of Grossman and Shapiro's (1987) paper in which the consequences of a two-stage race were studied. In their analysis, the firm that has successfully completed the first stage has the opportunity to license its results to the other firm. They follow Lee and Wilde's (1980) patent race formulation in that both firm's research and development costs cease as soon as the invention is successfully achieved by either, as do we. Gallini and Winter (1985) also addressed the role of licensing on the patent race but their focus differed from ours.

An interesting account of cross licensing of complementary technologies relating to the development of the radio telephone, which has evolved into the present cellular telephone, between GE and AT&T appears in Fagen (1975). Reference to cross licensing practices can be found in Taylor and Silbertson (1973) and Scherer (1980).

The paper is organized as follows. In Section 2 we outline our basic model and in section 3 analyze the complete specialization case in which each firm develops only one of the technologies. Strictly speaking, this is not a race, as neither firm can benefit from development of its technology until the other has developed the complementary technology. Indeed, while in a conventional patent race each firm increases its research expenditure in response to an increase by the other, here the opposite is true. Neither firm has an incentive to be first as it will have to wait for the other, before the final product can be introduced. Moreover, by prolonging development of its own technology it economizes on its overall development cost. However, once one of the firms has succeeded in developing its technology the other accelerates the development of the complementary technology. As might be expected, this independent development of the two technologies, with its "after you, Alphonse" characteristic, leads to a slower development pace than if they were developed by a single firm because each firm disregards the positive externality that its investment has on the other in terms of shortening the overall development period. This is demonstrated in Section 4. Section 5 is devoted to the cross licensing game itself and the effect of the threat of continuation of the race on the cross licensing agreement. We find that cross licensing does not always occur. In particular, there will be no cross licensing if one firm's advantage in developing the technology not yet developed exceeds the others' by so much as to make it unprofitable for it to license its technology. This absence of cross licensing can be overcome if the firms are allowed to collude in the production stage. In Section 6 we demonstrate how cross licensing can facilitate collusive outcomes in the duopoly game. Specifically, after licensing occurs firms play noncooperatively but the cross licensing agreement is designed so that the equilibrium outcome of this game is identical to the collusive outcome. Section 7 is devoted to the general case and the different possible effects of cross licensing on an innovation race. We show the effect of the possibility of cross licensing on the firms' research intensities and their evaluation of the race. A direct summary follows in the final section.

2. The Model: Innovation Race with Complementary Technologies

Consider a duopolistic market in which two risk-neutral firms must employ complementary technologies A and B to produce the same product. Possessing one of the technologies has no intrinsic value; only with the two of them together is production feasible. Once a firm has both technologies it can patent the product and commence production. We let π^M denote the monopoly profits. If at the production stage the two firms were to produce the market would be duopolistic and each would earn $\pi^D < \pi^M$.

The firms are posited to be engaged in a race to develop these technologies and we suppose that each firm's rate of investment and progress is observable by the other. We let $S_i = (A, B, \Phi, AB)$ denote the set of all possible states that firm i can be in, i.e., it may have developed neither technology, it may have developed either technology A or B, or may have developed both. We denote by $S = S_1 \times S_2$, an element of which describes the technologies that each firm has already succeeded in developing.

If intermediate innovations are patentable then at s = (A,B) each firm can block the others' production of the good. In such a situation cross licensing is a means of enabling both firms to provide the product. (Merger or a research joint venture are two other means of dealing with this situation). However, even if intermediate technologies are not patentable, firms might still consider cross licensing attractive, for by cross licensing they can avoid the cost of continuing the race and start to realize profits earlier. If cross licensing occurs each firm realizes the duopoly profit $\pi^D < \pi^M$. The attractiveness of cross licensing depends on the asymmetry between the firms regarding their research capabilities, their degree of time preference, the difference between π^M and π^D and the cost of continuing the innovation race. It should be noted that π^D is not necessarily independent of the terms of the cross licensing agreement. In Section 5 we discuss the relationship between the cross licensing game and the duopolistic profits π^D.

The development process is assumed to be stochastic. Let $\lambda_i^j dt$ denote the conditional probability that firm i will develop technology j in the short time interval dt given that it has not yet been developed, i.e., the hazard rate, where $\lambda_i^j \geq 0$ is a measure of the i'th firm research intensity in developing technology j. In order to succeed with probability λ_i^j the firm has to spend $h_i^j(\lambda_i^j)$ where $\partial h_i^j/\partial \lambda_i^j > 0$, $\partial^2 h_i^j/\partial \lambda_i^{j2} > 0$, $h_i^j(0) = 0$ for $j \in \{A,B\}$, $i = 1,2$. The convexity of h_i^j reflects diminishing returns to development spending. We further assume that $\partial h_i^j(0)/\partial \lambda_i^j = \beta_i^j > 0$ for j = A, B, and i = 1,2. Note that λ_i^j is assumed to be independent of time. Thus, given λ_1^j and λ_2^j the time of discovery of technology j is distributed as a negative exponential with expectation of $[\lambda_1^j + \lambda_2^j]^{-1}$.

We let $\Psi_i: S \rightarrow R_+^2$ be the i'th firm's strategy. Ψ_i specifies the i'th firm research intensities $(\lambda_i^A, \lambda_i^B)$ devoted to the development of each technology given the state of the race $s = (s_1, s_2)$. The above strategy space is a set of feedback decision rules that describe the players' research effort depending on his observation of the state of the race.

Let $V_i(s_i, s_j)$ be the value of the game for player i when the game starts at state (s_i, s_j). When one of the firms succeeds in developing both technologies it has complete monopoly power. Therefore

$$V_1(AB,\Phi) = V_1(AB,A) = V_1(AB,B) = \pi^M$$

$$V_2(AB,\Phi) = V_2(AB,A) = V_2(AB,B) = 0$$

If s = (AB,AB) both firms develop the technologies required for production simultaneously, the race is a draw and each firm realizes $\pi_i(AB,AB) = \pi^D$.

We solve for SPNE in the standard way--that is, for every $s \in S$ we find the firms equilibrium strategies. Thus, following Grossman and Shapiro (1987), for every $s = (s_1, s_2)$ such that $s_i \neq AB$ for $i = 1,2$, if firm 2 chooses research intensities λ_2^A, λ_2^B and firm 1 chooses λ_1^A, λ_1^B in the development of the two technologies, then firm 1's expected payoff is given implicitly by:

$$
\begin{aligned}
(1) \qquad rV_1(s_1,s_2) = {} & \lambda_1^A(s)(V_1(s_1 \cup A,\ s_2) - V_1(s_1,s_2)) \\
& + \lambda_1^B(s)(V_1(s_1 \cup B,\ s_2) - V_1(s_1,s_2)) \\
& + \lambda_2^A(s)(V_1(s_1,\ s_2 \cup A) - V_1(s_1,s_2)) \\
& + \lambda_2^B(s)(V_1(s_1\ s_2 \cup B) - V_1(s_1,s_2)) - h_1^A(\lambda_1^A(s)) - h_1^B(\lambda_1^B(s))
\end{aligned}
$$

where r is a common discount rate and $s_i \cup A$ indicates the technologies specified in s_i and technology A. Equation (1) states that the rate of return on $V_1(s_1,s_2)$ is equal to the expected capital gains, the first four terms, and minus the expenditure flow, the last two terms. A similar relationship applies for firm 2. Rearranging (1) yields that

$$(2) \qquad V_1(s_1,s_2) = \text{Max}\{(0,\ R_1(s_1,s_2)\},$$

as the firm's value cannot be driven below zero, and where

$$
\begin{aligned}
(3) \qquad R(s_1,s_2) = {} & [\lambda_1^A(s)V_1(s_1 \cup A,\ s_2) + \lambda_1^B(s)V_1(s_1 \cup B,\ s_2) \\
& + \lambda_2^A(s)V_1(s_1,\ s_2 \cup A) + \lambda_2^B(s)V_1(s_1,\ s_2 \cup B) - h_1^A(\lambda_1^A(s)) \\
& - h_1^B(\lambda_1^B(s))](r + \lambda_1^A(s) + \lambda_1^B(s) + \lambda_2^A(s) + \lambda_2^B(s))^{-1}.
\end{aligned}
$$

The equilibrium strategies $(\tilde{\lambda}_1(s), \tilde{\lambda}_2(s))$ satisfy the best response property; that is, given $\tilde{\lambda}_2(s)$, firm 1 chooses research intensities $\tilde{\lambda}_1(s)$ that maximize $V_1(s_1, s_2)$.

If $V_1(s_1 \cup A, s_2) - V_1(s_1, s_2) \geq \beta_1^A$, then differentiating (3) implies the equilibrium strategy satisfy

$$\text{(4a)} \qquad \partial h_1^A(\tilde{\lambda}_1^A(s_1, s_2))/\partial \lambda_1^A = V_1(s_1 \cup A, s_2) - V_1(s_1, s_2).$$

When $V_1(s_1 \cup A, s_2) - V_1(s_1, s_2) < \beta_1^A$, maximization of $V_1(s_1, s_2)$ implies that $\tilde{\lambda}_1^A$ should be zero since in this case $\partial V_1(s_1, s_2)/\partial \lambda_1^A < 0$ for every value of λ_1^A.

In a similar way if $V_1(s_1 \cup B, s_2) - V_1(s_1, s_2) \geq \beta_1^B$ then

$$\text{(4b)} \qquad \partial h_1^B(\tilde{\lambda}_1^B(s_1, s_2))/\partial \lambda_1^B - V_1(s_1 \cup B, s_2) - V_1(s_1, s_2).$$

Otherwise $\tilde{\lambda}_1^B(s_1, s_2) = 0$.

Similar conditions hold for firm 2. Note that conditions (4a,b) imply that if firm i has already succeeded in developing technology A, then $\tilde{\lambda}_i^A = 0$ since $s_1 \cup A = s_1$ implies that $V_1(s_1 \cup A, s_2) = V_1(s_1, s_2)$.

In Appendix 1 we provide an example and a short proof for the case $s = (A,B)$ in which each firm has already succeeded in developing one technology and the firms continue the race to develop the complementary technology.

3. Complete Specialization: Cross Licensing

The effect of cross licensing on innovation may be best illustrated in the complete specialization scenario. In this case we assume that the two firms have completely different innovation capabilities. Specifically we let $\beta_1^B > \pi^M$ and $\beta_2^A > \pi^M$ which implies that for firm 1 developing technology B is too costly regardless of the state achieved. Similarly firm 2 has a disadvantage in developing technology A. Consider now the state s = (A,B) and assume that cross licensing is not feasible. As both firms cannot benefit from developing only one intermediate technology the equilibrium of the innovation race is that for every $s \in S$, $\lambda_i^j(s) = 0$ for i = 1,2, J = A,B, i.e., neither firm will undertake development of either technology.

Allowing for cross licensing implies that once s = (A,B) is reached, the firms can cross license their technologies. The firms can anticipate such cross licensing at the commencement of development of the technologies, $s = (\Phi,\Phi)$, and therefore undertake their development. The firms develop the two different technologies independently but the innovation effort of one firm affects the optimal effort of the other. Unlike a race each firm would like the other to succeed in its development. A rival's success implies that when the firm succeeds in developing its technology, they can immediately cross license and start to realize profits. Specifically we claim the following:

<u>Claim 1</u>: In the complete specialization case with cross licensing, undertaking develement is worthwhile:

$$V_1(\Phi,B) > V_1(\Phi,\Phi), \text{ and } V_2(A,\Phi) > V_2(\Phi,\Phi).$$

<u>Proof</u>: Using (3) yields that

(5) $$V_1(\Phi,B) = \frac{\lambda_1^A(\Phi,B)\pi^D - h_1^A(\lambda_1^A(\Phi,B))}{r + \lambda_1^A(\Phi,B)}$$

Since $\lambda_1(\Phi,B)$ is the optimal research intensity that maximizes the value of the game at (Φ,B) it is evident that

(6) $$V_1(\Phi,B) \geq \frac{\lambda_1^A(\Phi,\Phi)\pi^D - h_1^A(\lambda_1^A(\Phi,\Phi))}{r + \lambda_1^A(\Phi,\Phi)}.$$

Multiplying both sides of (6) by $r + \lambda_1^A(\Phi,\Phi)$ then adding to both sides $\lambda_2^B(\Phi,\Phi)V_1(\Phi,B)$ and rearranging yields that

(7) $$V_1(\Phi,B) \geq \frac{\lambda_1^A(\Phi,\Phi)\pi^D + \lambda_2^B(\Phi,\Phi)V_1(\Phi,B) - h_1^A(\lambda_1^A(\Phi,\Phi))}{r + \lambda_1^A(\Phi,\Phi) + \lambda_2^B(\Phi,\Phi)}$$

Bearing in mind that

(8) $$V_1(\Phi,\Phi) = \frac{\lambda_1^A(\Phi,\Phi)V_1(A,\Phi) + \lambda_2^B(\Phi,\Phi)V_1(\Phi,B) - h_1^A(\lambda_1^A(\Phi,\Phi))}{r + \lambda_1^A(\Phi,\Phi) + \lambda_2^B(\Phi,\Phi)},$$

and $\pi^D > V_1(A,\Phi)$ it is evident that

$$V_1(\Phi,B) > V_1(\Phi,\Phi). \qquad []$$

In the typical patent race model, a firm's optimal research intensity is an increasing function of its rival's research intensity. In other words the reaction functions are upward sloping. In the complete specialization case we get the opposite characterization:

Claim 2: At the initial stage, i.e., $s = (\Phi,\Phi)$, of the complete specialization game, the firms' reaction functions are downward sloping, i.e., firm i's optimal research intensity is a decreasing function of firm j's research intensity.

Proof: Using (8) to differentiate $V_1(\Phi,\Phi)$ with respect to $\lambda_2(\Phi,\Phi)$ yields that

$$(9) \qquad \frac{\partial V_1(\Phi,\Phi)}{\partial \lambda_2^B} = \frac{V_1(\Phi,B) - V_1(\Phi,\Phi)}{r + \lambda_1^A(\Phi,\Phi) + \lambda_2^B(\Phi,\Phi)} > 0$$

which is positive due to claim 1. Since the optimal $\lambda_1(\Phi,\Phi)$ is derived from the f.o.c.

$$(10) \qquad \partial h^A(\lambda_1^A(\Phi,\Phi))/\partial \lambda_1^A = V_1(A,\Phi) - V_1(\Phi,\Phi)$$

it is evident from the convexity of $h^A(\bullet)$ that an increase in $\lambda_2(\Phi,\Phi)$ results in a higher $V_1(\Phi,\Phi)$ which implies a lower optimal $\lambda_1^A(\Phi,\Phi)$. []

Note that, since in an ordinary patent race $\partial V_1(\Phi,\Phi)/\partial \lambda_2^B < 0$, the firm's reaction functions are upward sloping.

Claim 3: An innovation success by one firm induces higher research intensity by the other firm, i.e.,

(11a) $\lambda_1^A(\phi,B) > \lambda_1^A(\phi,\phi)$

(11b) $\lambda_2^B(A,\phi) > \lambda_2^B(\phi,\phi)$.

Proof: Assume to the contrary that $\lambda_1^A(\phi,\phi) > \lambda_1^A(\phi,B)$. Using the f.o.c. (10) and the convexity of the cost function $h_1^A(\lambda_1^A)$ our assumption yields that

(12) $V_1(A,B) - V_1(\phi,B) < V_1(A,\phi) - V_1(\phi,\phi)$.

From (1) and since $\lambda_1^A(\phi,\phi)$ is the research intensity that maximizes $V_1(\phi,\phi)$ we obtain the following:

(13)
$$\begin{aligned} rV_1(\phi,\phi) &= \lambda_1^A(\phi,\phi)[V_1(A,\phi) - V_1(\phi,\phi)] \\ &+ \lambda_2^B(\phi,\phi)[V_1(\phi,B) - V_1(\phi,\phi)] - h_1^A(\lambda_1^A(\phi,\phi)) \\ &\geq \lambda_1^A(\phi,B)[V_1(A,\phi) - V_1(\phi,\phi)] - h_1^A(\lambda_1^A(\phi,B)). \end{aligned}$$

Substituting (12) into (13) yields

(14) $$rV_1(\phi,\phi) \geq \lambda_1^A(\phi,B)[V_1(A,B) - V_1(\phi,B)] - h_1^A(\lambda_1^A(\phi,B)) = rV_1(\phi,B)$$

which contradicts Claim 1. []

The above analysis is better illustrated by the following simulation.

Consider a complete specialization race in which firm 1 specialized in innovating technology A while firm 2 specialized in technology B. Assume that both firms have quadratic cost functions such that $h_1^A(\lambda_1^A) = .5(\lambda_1^A)^2$ and $h_2^B(\lambda_2^B) = .5b(\lambda_2^B)^2$. The discount rate is $r = .10$ and the duopolistic profits are $\pi^D = 20$. We now let the parameter b change from 1 to 100 and calculate the equilibrium of the game. Our calculations are presented in Table 1.

<Insert Table 1 about here>

As Table 1 indicates $V_1(\Phi,B) > V_1(\Phi,\Phi)$ and $V_2(A,\Phi) > V_2(\Phi,\Phi)$. Moreover an innovation success by a rival induces higher research intensities, i.e., $\lambda_1^A(\Phi,B) > \lambda_1^A(\Phi,\Phi)$ and $\lambda_2^B(A,\Phi) > \lambda_2^B(\Phi,\Phi)$. Note also that our simulation indicates that $\lambda_1^A(\Phi,\Phi)$ is a decreasing function of b. This is due to the specific nature of the game. Each firm wishes for its rival to succeed so it can realize profits earlier.

4. Complete Specialization: Merger vs. Cross Licensing

Beside cross licensing the two firms have the option to merge into one firm. The outcome of such a merger is a monopolistic firm that needs to develop the two complementary technologies prior to production. Letting $V^m(s)$ be the monopolist's value function, the optimal innovation effort has to satisfy the following conditions:

$$V^M(B) = (\lambda^A(B)\pi^M - h^A(\lambda^A(B))/(r + \lambda^A(B)) \tag{15a}$$

$$\partial h^A(\lambda^A(B))/\partial\lambda^A = \pi^M - V^M(B) \tag{15b}$$

$$(15c) \qquad V^M(A) = (\lambda^B(A)\pi^M - h^B(\lambda^B(A))/(r + \lambda^B(A))$$

$$(15d) \qquad \partial h^B(\lambda^B(A))/\partial\lambda^B = \pi^M - V^M(A)$$

$$(15e) \qquad V^M(\Phi) = \frac{\lambda^A(\Phi)V^M(A) + \lambda^B(\Phi)V^M(B) - h^A(\lambda^A(\Phi)) - h^B(\lambda^B(\Phi))}{r + \lambda^A(\Phi) + \lambda^B(\Phi)}$$

$$(15f) \qquad \partial h^A(\lambda^A(\Phi))/\partial\lambda^A = V^M(A) - V^M(\Phi)$$

$$(15g) \qquad \partial h^B(\lambda^B(\Phi))/\partial\lambda^B = V^M(B) - V^M(\Phi).$$

We can now use the above conditions to compare the research intensities of a single producer and two specialized firms that engage in cross licensing.

Claim 4: In the complete specialization case a monopolist's research intensity is higher than that of the duopolistic firms.

Before demonstrating the above claim, we note that this result is the opposite of what occurs in an ordinary patent race in which there is an excessive innovation effort by the duopolistic firms (see, for example, Reinganum (1985)).

Proof: Assume that $\lambda_1^A(\Phi,B) > \lambda^A(B)$. Using (10) and (15b) and the convexity of $h^A(\bullet)$ implies that $\lambda_1^A(\Phi,B) > \lambda^A(B)$ iff

(16) $\pi^M - V^M(B) \le \pi^D - V_1(\Phi,B)$.

Now observe that since $\lambda_1^A(\Phi,B)$ maximizes $V_1(\Phi,B)$ and $\pi^M > \pi^D$.

(17) $$\pi^D - V_1(\Phi,B) \le \pi^D - [\lambda^A(B)\pi^D - h^A(\lambda^A(B))]/(r + \lambda^A(B))$$
$$\le \pi^M - [\lambda^A(B)\pi^M - h^A(\lambda^A(B))]/(r + \lambda^A(B) = \pi^M - V^M(B)$$

which yields a contradiction.

In a similar way we can prove that $\lambda^B(A) > \lambda_2^B(A,\Phi)$. []

Table 2: Simulation of Duopoly vs. Single Innovator

b	10		20		30		50		75	
	D	M	D	M	D	M	D	M	D	M
$\lambda^A(\Phi,\Phi)$	.38	.52	.34	.46	.31	.43	.28	.38	.25	.34
$\lambda^B(\Phi,\Phi)$	.38	.52	.26	.37	.21	.30	.15	.23	.12	.18
$\lambda^A(\Phi,B)$	.54	.80	.54	.80	.54	.80	.54	.80	.54	.80
$\lambda^B(A,\Phi)$	.54	.80	.36	.54	.27	.42	.20	.31	.15	.24

The difference between duopoly that cross licenses and the innovation effort of a single producer is illustrated in Table 2. In this simulation we assume that the monopolist has the same research technologies as the two firms in the duopoly case. We assume that $\pi^M = 40$ and $\pi^D = 20$. Letting might indicate collusive behavior.

If intermediate technologies are patentable the firms cannot move to the production stage without cross licensing, and they cannot gain from continuing the race. Using the Nash bargaining solution implies that such cross licensing will be carried out without any additional payments, i.e.,

$$(18) \qquad F^* = 0 = \text{Argmax}_F((\pi^D + F)(\pi^D - F).$$

If patents for intermediate technologies are not granted, potential continuation of the race plays an important role in determining the cross licensing terms. Using Figure 1 we distinguish among three possible cases:

<Insert Figure 1 about here>

Case a (Figure 1a): If $\pi^D \geq \text{Max}\{V_1(A,B), V_2(A,B)\}$ then cooperative cross licensing is feasible even if side payments are excluded. Maximizing $[\pi^D + F - V_1(A,B)][\pi^D - F - V_2(A,B))]$ yields that

$$(19) \qquad F^* = (1/2)[V_1(A,B) - V_2(A,B)]$$

which implies that the two firms split the gains from cross licensing between them. Thus, if $\pi^D - V_2(A,B) > \pi^D - V_1(A,B)$, cross licensing is accompanied by the second firm's payment to the first. The firms' overall profits after cross licensing and side payments are

$$(20) \qquad \pi_i^{CL} = \pi^D + (1/2)(V_i(A,B) - V_j(A,B)), \; j \neq i.$$

$\pi^D < .5\pi^M$ will only reinforce our main conclusions. Comparing the research intensities of a duopoly and a monopolist, in the above table indicates that the research intensities of the monopolist are always higher than those of the duopolistic firms.

5. The Cross Licensing Game

We model cross licensing as a cooperative game in which the two firms have to agree whether or not to cross license their technologies, and on what terms. Our main concern is with the interdependence between the cross licensing game and the innovation race.

We denote the firms' profits from cross licensing as $(\pi_1^{CL}(s), \pi_2^{CL}(s))$. The cross licensing payoffs consist of the duopolistic payoffs realized by the firms in the production stage and the side payment which might accompany it. The cross licensing payoffs depend on s, since the development cost asymmetry might lead to a difference between cross licensing terms when s = (A,B) and when s = (B,A). We regard the cross licensing game as a bargaining problem and adopt the Nash bargaining solution.

If firms decide not to cross license or they fail to reach an agreement, they continue the innovation race. Thus, the threat point in the cross licensing bargaining game is their expected payoffs in such an innovation race, i.e., $V_1(A,B)$ and $V_2(A,B)$. Denote the net money transfer from firm 2 to firm 1 as F (F can also be negative), the firms' net payoffs after the cross licensing agreement are $(\pi_1^{CL}(A,B), \pi_2^{CL}(A,B)) = (\pi^D + F, \pi^D - F)$.[1] Clearly such side payments are not always feasible as it

[1] At this stage F is just a fixed fee. We will later distinguish between royalties and fee per unit and discuss their different implications on the cross licensing games.

Note also that if side payments are excluded, the firms will still wish to cooperate since for both of them $\pi^D > V_i(A,B)$.

Case b (Figure 1b): $\pi^D < V_1(A,B)$, $\pi^D > V_2(A,B)$ and $\pi^D > (V_1(A,B) + V_2(A,B))/2$.

Since $\pi^D < V_1(A,B)$, the first firm will not agree to cross license if it does not receive any compensation from the second firm and if its duopolistic profit plus the compensation does not exceed $V_1(A,B)$. Since $2\pi^D > V_1(A,B) + V_2(A,B)$ there is, however, a possibility for benefit from cross licensing. Thus, if side payments are allowed the firms will cross license. The side payment F* is as defined in (19) and the firms' payoffs, including the side payment, are as defined in (20).

Case c (Figure 1c): $\pi^D < V_1(A,B)$, $\pi^D > V_2(A,B)$ and $\pi^D < (V_1(A,B) + V_2(A,B))/2$.

Unlike case b, in this case the use of side payments does not facilitate cross licensing. The first firm has a huge advantage in the race, not in terms of the technology it already developed, but by its advantage in developing its complementary technology. Individual rationality implies that the first firm will engage in cross licensing only if it is compensated for the loss of the advantage in the race. But, since $2\pi^D < V_1(A,B) + V_2(A,B)$ such compensation implies that the second firm has to pay more than its own gain from cross licensing. Thus, we can conclude that under the circumstances of case c there is no cross licensing.

The situation described in case c raises an important policy issue.

Consider the case in which firm one has technology A and firm two has technology B. From society's point of view it is desirable that the firms share their information and start production. Society does not benefit from the continuation of the race since resources will be expended on research for technologies already known and its outcome will be the presence of a single monopolistic firm. A possible solution to this dilemma is to let the firms collude in the production stage. Such collusion leads to higher profits in the production stage. Let $\tilde{\pi}^D$ be the profit each firm gets when the collude. If collusion raises their profit sufficiently so that $2\tilde{\pi}^D >$ $V_1(A,B) + V_2(A,B)$, as described in Figure 1d, then there is a possibility of cross licensing and we return to a situation similar to case b. Since $\pi^M > V_1(A,B)$ it is always possible to guarantee that cross licensing will take place by letting the firms charge the monopolistic price, dividing among them the monopolistic profits and letting the second firm pay the first a side payment according to (18), where $\pi^M/2$ replaces π^D. We can conclude that if side payments are allowed and collusion is possible, so case c is excluded, then $\pi_i^{CL} > V_i(A,B)$ and firms always prefer to cross license, to continuing the race.

6. Cross Licensing as a Scheme that Facilitates Collusive Outcomes in the Duopoly Game

In describing the cross licensing problem we posit a two-stage game, the first stage of which involves a cooperative cross licensing game and the second stage a noncooperative duopolistic game.[2] Thus, although side

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[2]In Section 5 we discussed one exception in which allowing collusion is necessary in order to facilitate cross licensing.

payments might be paid, the two firms combined profits in the productive stage is $2\pi^D$. In discussing the cooperative cross licensing game we assumed that firms had to agree whether to cross license or not and on the side payments that accompany such cross licensing. Let us now modify this formulation and consider the use of royalties as part of the cross licensing agreement. In negotiating the terms of cross licensing the firms realize the effects of their agreement on the duopolistic market. Explicit collusion in the duopolistic game is prohibited by law. Thus the problem the firms face is how to implement a cross licensing agreement such that the resultant noncooperative duopolistic game yields equilibrium profits identical to the cooperative outcome. Clearly if they agree only on fixed side payments, they will be treated as a fixed cost or gain in the duopolistic stage and therefore will have no effect on the game's equilibrium.

To illustrate the possibility of achieving the collusive outcome through cross licensing by means of a royalty consider a duopolistic market with a linear demand function $p = a - b(q_1 + q_2)$ and linear production cost function $TC_i(q_i) = cq_i$. Let μ_i be the royalties that firm i gets. Firm i's profits are

(21) $$\pi_i(q_i, q_j, \mu_i, \mu_j) = [a - b(q_1 + q_j) - c - \mu_j]q_i + \mu_i q_j$$

which is its profits from selling its output, paying the royalties $\mu_j q_i$ and getting $\mu_i q_j$ as revenues from royalties.

<u>Claim 5</u>: Letting $\mu_1^* = \mu_2^* = (a - c)/4$ guarantees the collusive outcome is

the above duopoly game.

Proof: Given (μ_1, μ_2), which are agreed upon at the cross licensing stage, the Cournot-type equilibrium at the production stage is:

(22) $$q^*(\mu_1, \mu_2) = (a - c - 2\mu_j + \mu_i)/3b, \quad j \neq i.$$

Given (μ_1^*, μ_2^*) the equilibrium outputs are $q_1^*(\mu_1^*, \mu_2^*) = q_2^*(\mu_1^*, \mu_2^*) = (a - c)/4$ which yields the monopolistic profits. []

The division of the monopolistic profits are of course subject to negotiation. Using, as before, the Nash bargaining solution yields that such cross licensing will be accompanied by the following side payments:

(23) $$F^* = \text{argmax}[\pi^M/2 + F - V_1(A,B)][\pi^M/2 - F - V_2(A,B)].$$

Note that although firms realize monopolistic profits they do not collude at the production stage in which there is a standard noncooperative Cournot-type game.

7. The Effect of Cross Licensing on the Innovation Race

The possibility of cross licensing may affect the firms' strategies in the innovation race as firms' research intensities depend on the prize they get upon success while this prize depend on the possibility of cross licensing. In order to discuss this effect we now distinguish between $V_i(s)$, the value function when cross licensing is not feasible, and $\tilde{V}_i(s)$, the value function when firms may cross license. Our analysis will be carried out mainly by comparing the two.

Previous discussion indicates that $V_i(AB,A) = \tilde{V}_i(AB,A) = \pi^M$; $V_i(A,A) = \tilde{V}_i(A,A)$; $V_i(B,B) = \tilde{V}_i(B,B)$ and $\tilde{V}_i(A,B) = \pi_i^{CL}(A,B) > V_i(A,B)$. Thus our main focus is to investigate how the possibility of cross licensing affects the early stages of the race. In particular consider the case in which one of the firms has an advantage in the race, for example when $s = (A,\Phi)$.[3] The innovation race that starts at $s = (A,\Phi)$ depends on the existing patent law. If patents for intermediate technologies are granted the two firms are now engaged in a race to obtain technology B. Clearly such a race will take place only when cross licensing is feasible. Otherwise, the first firm that has either technology has the power to terminate the race. If patents for intermediate technologies are not granted it is possible that firm 2, besides developing technology B, also tries to develop technology A. This type of behavior occurs only if

$$(24) \qquad \partial h_2^A(0)/\partial\lambda_2^B = \beta_2^A < \tilde{V}_2(A,A) - \tilde{V}_2(A,\Phi).$$

Clearly, from society's point of view, this behavior is not optimal as firm 2 expends resources on developing a technology that already exists.

Claim 6: Given the state (A,Φ): (i) firm 2 is better off when cross licensing is feasible, i.e., $\tilde{V}_2(A,\Phi) > V_2(A,\Phi)$; (ii) $\lambda_2^A(A,\Phi) > \tilde{\lambda}_2^A(A,\Phi)$, i.e., the second firm's research intensity in technology A is higher when

[3] In a similar way we can analyze the cases where $s = (\Phi,A)$, (B,Φ), or (Φ,B).

cross licensing is <u>not</u> feasible.

<u>Proof</u>: From the f.o.c. and since $V_2(A,A) = \tilde{V}_2(A,A)$ note that $\lambda_2^A(A,\Phi) > \tilde{\lambda}_2^A(A,\Phi)$ occurs iff $\tilde{V}_2(A,\Phi) > V_2(A,\Phi)$.

Assume to the contrary that $\tilde{\lambda}_2^A(A,\Phi) > \lambda_2^A(A,\Phi)$. This assumption implies that $V_2(A,\Phi) > \tilde{V}_2(A,\Phi)$. Since $(\tilde{\lambda}_2^A(A,\Phi), \tilde{\lambda}_2^B(A,\Phi))$ maximizes $\tilde{V}_2(A,\Phi)$ we obtain the following

$$(25) \qquad \tilde{V}_2(A,\Phi) = \frac{\tilde{\lambda}_2^B(A,\Phi)\pi_2^{CL} + \tilde{\lambda}_2^A(A,\Phi)\tilde{V}_2(A,A) - h_2^A(\tilde{\lambda}_2^A(A,\Phi)) - h_2^B(\tilde{\lambda}_2^B(A,\Phi))}{r + \tilde{\lambda}_2^A(A,\Phi) + \tilde{\lambda}_2^B(A,\Phi) + \tilde{\lambda}_1^B(A,\Phi)}$$

$$> \frac{\lambda_2^B(A,\Phi)\pi_2^{CL} + \lambda_2^A(A,\Phi)\tilde{V}_2(A,A) - h_2^A(\lambda_2^A(A,\Phi)) - h_2^B(\lambda_2^B(A,\Phi))}{r + \lambda_2^A(A,\Phi) + \lambda_2^B(A,\Phi) + \tilde{\lambda}_1^B(A,\Phi)}$$

Since $\pi_2^{CL} > V_2(A,B)$ and $\tilde{V}_2(A,A) = V_2(A,A)$ we obtain that $\tilde{\lambda}_1^B(A,\Phi) > \lambda_1^B(A,\Phi)$ is a necessary condition for our assumption that $V_2(A,\Phi) > \tilde{V}_2(A,\Phi)$.

Given our assumption $\tilde{V}_2(A,B) - \tilde{V}_2(A,\Phi) > V_2(A,B) - V_2(A,\Phi)$ which implies, using the f.o.c., that $\tilde{\lambda}_2^B(A,\Phi) > \lambda_2^B(A,\Phi)$. We can now use the above inequalities to demonstrate the following contradiction:

$$\begin{aligned}(26) \qquad r\tilde{V}_2(A,\Phi) &= \tilde{\lambda}_1^B(A,\Phi)(0 - \tilde{V}_2(A,\Phi)) + \tilde{\lambda}_2^A(A,\Phi)(\tilde{V}_2(A,A) - \tilde{V}_2(A,\Phi)) \\
&+ \tilde{\lambda}_2^B(A,\Phi)(\tilde{V}_2(A,B) - \tilde{V}_2(A,\Phi)) - h_2^B(\tilde{\lambda}_2^B(A,\Phi)) - h_2^A(\tilde{\lambda}_2^A(A,\Phi)) \\
&> \tilde{\lambda}_1^B(A,\Phi)(0 - \tilde{V}_2(A,\Phi)) + \lambda_2^A(A,\Phi)(\tilde{V}_2(A,A) - \tilde{V}_2(A,\Phi)) \\
&+ \lambda_2^B(A,\Phi)(\tilde{V}_2(A,B) - \tilde{V}_2(A,\Phi)) - h_2^B(\lambda_2^B(A,\Phi)) - h_2^A(\lambda_2^A(A,\Phi)) \\
&> \lambda_1^B(A,\Phi)(0 - V_2(A,\Phi)) + \lambda_2^A(A,\Phi)(V_2(A,A) - V_2(A,\Phi)) \\
&+ \lambda_2^B(A,\Phi)(V_2(A,B) - V_2(A,\Phi)) - h_2^B(\lambda_2^B(A,\Phi)) - h_2^A(\lambda_2^A(A,\Phi))\end{aligned}$$

$= rV_2(A,\Phi)$

which contradicts our assumption. []

Claim 7: When $\pi_1^{CL}(A,B) > \tilde{V}_1(A,\Phi)$:

(i) $\tilde{V}_1(A,\Phi) > V_1(A,\Phi)$;

(ii) $\lambda_1^B(A,\Phi) > \tilde{\lambda}_1^B(A,\Phi)$.

Proof: As before note that the f.o.c. imply that (i) implies (ii).

Assume to the contrary that $V_1(A,\Phi) > \tilde{V}_1(A,\Phi)$ which implies that $\tilde{\lambda}_1^B(A,\Phi) > \lambda_1^B(A,\Phi)$. Given this assumption and using Claim 6 and that $\tilde{V}_1(A,B) > V_1(A,B)$; $\partial\tilde{V}_1(A,\Phi)/\partial\lambda_2^A < 0$, and that $\tilde{\lambda}_1^B(A,\Phi)$ maximizes the value function $\tilde{V}_1(A,\Phi)$ one can obtain the following inequality:

$$\text{(27)} \qquad \tilde{V}_1(A,\Phi) = \frac{\tilde{\lambda}_1^B(A,\Phi)\pi^M + \tilde{\lambda}_2^B(A,\Phi)\tilde{V}_1(A,B) + \tilde{\lambda}_2^A(A,\Phi)\tilde{V}_1(A,A) - h_1^B(\tilde{\lambda}_1^B(A,\Phi))}{r + \tilde{\lambda}_1^B(A,\Phi) + \tilde{\lambda}_2^A(A,\Phi) + \tilde{\lambda}_2^B(A,\Phi)}$$

$$> \frac{\lambda_1^B(A,\Phi)\pi^M + \tilde{\lambda}_2^B(A,\Phi)V_1(A,B) + \lambda_2^A V_1(A,A) - h_1^B(\lambda_1^B(A,\Phi))}{r + \lambda_1^B(A,\Phi) + \lambda_2^A(A,\Phi) + \tilde{\lambda}_2^B(A,\Phi)}$$

Now note that $\partial V_1(A,\Phi)/\partial\lambda_2^B < 0$. Thus, if $\lambda_2^B(A,\Phi) > \tilde{\lambda}_2^B(A,\Phi)$ inequality (27) implies that $\tilde{V}_1(A,\Phi) > V_1(A,\Phi)$ which contradicts our assumption. Therefore, a necessary condition for our assumption to hold is that $\tilde{\lambda}_2^B(A,\Phi) > \lambda_2^B(A,\Phi)$. Using the above inequality and Claim 6, our assumption yields that when $\tilde{V}_1(A,B) = \pi_1^{CL}(A,B) > \tilde{V}_1(A,\Phi)$ we have the following inequality which implies a contradiction and finishes the proof.

$$\begin{aligned}(28)\qquad r\tilde{V}_1(A,\Phi) &= \tilde{\lambda}_1^B(A,\Phi)(\pi^M - \tilde{V}_1(A,\Phi)) + \tilde{\lambda}_2^B(A,\Phi)(\tilde{V}_1(A,B) - \tilde{V}_1(A,\Phi)) \\ &+ \tilde{\lambda}_2^A(A,\Phi)(\tilde{V}_1(A,A) - \tilde{V}_1(A,\Phi)) - h_1^B(\tilde{\lambda}_1^B(A,\Phi)) \\ &> \lambda_1^B(A,\Phi)(\pi^M - V_1(A,\Phi)) + \lambda_2^B(A,\Phi)(V_1(A,B) - V_1(A,\Phi)) \\ &+ \lambda_2^A(A,\Phi)(V_1(A,A) - V_1(A,\Phi)) - h_1^B(\lambda_1^B(A,\Phi)) \\ &= rV_1(A,\Phi). \qquad []\end{aligned}$$

Concluding Remarks

The main focus of this paper is the interdependence between cross licensing and the race to innovate when firms have to use two complementary technologies to produce a final product. Circumstances under which the firms would cross license rather than continue the race were characterized. It was found that allowing the firms to collude in the provision of the final product may induce them to cross license in situations in which they would not otherwise. For society, this poses the choice between more competitive provision of the final product versus its earlier introduction, ableit at a lower price, and elimination of duplication of effort in the development of a known technology. It was also found that cross licensing by means of a royalty can lead to tacit collusion among the providers of the final product.

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Appendix 1

Equilibrium in the Innovation Race for s = A,B

Since $s_1 = A$ it is evident from (3a) that $\tilde{\lambda}_1^A = 0$. Similarly, $s_1 = B$ yields that $V_2(s_1, s_2 \cup B) = V_2(s_1, s_2)$ which implies that $\tilde{\lambda}_2^B = 0$. Since $s_2 \cup A = (A,B)$ we obtain in this case that $V_1(s_1, s_2 \cup A) = V_2(s_1 \cup B, s_2) = 0$ while $V_1(s_1 \cup B, s_2) = V_2(s_1, s_2 \cup A) = \pi^M$. Thus using (1)-(3) it is evident that the equilibrium research intensities $(\tilde{\lambda}_1^B, \tilde{\lambda}_2^A)$ and $V_1(A,B)$, $V_2(A,B)$ satisfy the following conditions

$$\text{(A1.1)} \qquad V_1(A,B) = [\tilde{\lambda}_1^B(A,B)\pi^M - h_1^B(\tilde{\lambda}_1^B(A,B))]/(r + \tilde{\lambda}_1^B(A,B) + \tilde{\lambda}_2^A(A,B))$$

$$\text{(A1.2)} \qquad \partial h_1^B(\tilde{\lambda}_1^B(A,B))/\partial\lambda_1^B = \pi^M - V_1(A,B)$$

and similar conditions hold for firm 2.

We now demonstrate how the above conditions are derived (for more details see Lee and Wilde (1980)). Similar proofs can, of course, be presented for all the possible subgames (s_1, s_2).

Given λ_1^B and λ_2^A and given that one of the firms succeeds in developing its complementary technology at time t, then the conditional probability that the discovery is made by firm 1 is $\lambda_1^B/(\lambda_1^B + \lambda_2^A)$. Thus, given that discovery occurred at time t the expected present profits of firm 1 is

$$e^{-rt}\pi^M\lambda_1^B/(\lambda_1^B + \lambda_2^A) - h_1^B(\lambda_1^B)(1 - e^{-rt})/r.$$

Let $\pi_j(\lambda_1^B, \lambda_2^A)$ denotes the i'th firm expected payoff from the innovation race as a function of the firms' research intensities $(\lambda_1^B, \lambda_1^A)$. Thus

$$\text{(A1.3)} \qquad \pi_1(\lambda_1^B, \lambda_1^A) = \int_0^\infty [e^{-rt}(\lambda_1^B/(\lambda_1^B + \lambda_2^A))\pi^M - h_1^B(\lambda_1^B)(1 - e^{-rt})/r]dt$$

Solving the above integral yields that $\pi_1(\lambda_1^B, \lambda_2^A) = (\lambda_1^B\pi^M - h_1^B(\lambda_1^B))/(r + \lambda_1^B + \lambda_2^A)$. Maximizing $\pi_1(\lambda_1^B, \lambda_2^A)$ with respect to λ_1^B yields (the convexity of h_1^B guarantees that the second-order conditions are satisfied)

$$\text{(A1.4)} \qquad \frac{\partial\pi_1}{\partial\lambda_1^B} = \frac{\pi^M - \partial h_1^B/\partial\lambda_1^B}{r + \lambda_1^B + \lambda_2^B} - \frac{\lambda_1^B\pi^M - h_1^B(\lambda_1^B)}{(r + \lambda_1^B + \lambda_2^B)^2} = 0$$

where $(\tilde{\lambda}_1^B, \tilde{\lambda}_2^A)$ is the equilibrium in the innovation race if $\pi_1(\tilde{\lambda}_1^B, \tilde{\lambda}_2^A) \geq \pi_1(\lambda_1^B, \tilde{\lambda}_2^A)$ for all λ_1^B and $\pi_2(\tilde{\lambda}_1^B, \tilde{\lambda}_2^A) \geq \pi_2(\tilde{\lambda}_1^B, \lambda_2^A)$ for all possible λ_2^A. From the definition of $V_1(A,B)$ we obtain that $V_1(A,B) = \pi_1(\tilde{\lambda}_1^B, \tilde{\lambda}_2^A)$. Thus, rewriting (A1.4) yields that

$$\text{(A1.5)} \qquad V_1(A,B) = \pi^M - \partial h_1^B(\tilde{\lambda}_1^B)/\partial\lambda_1^B.$$

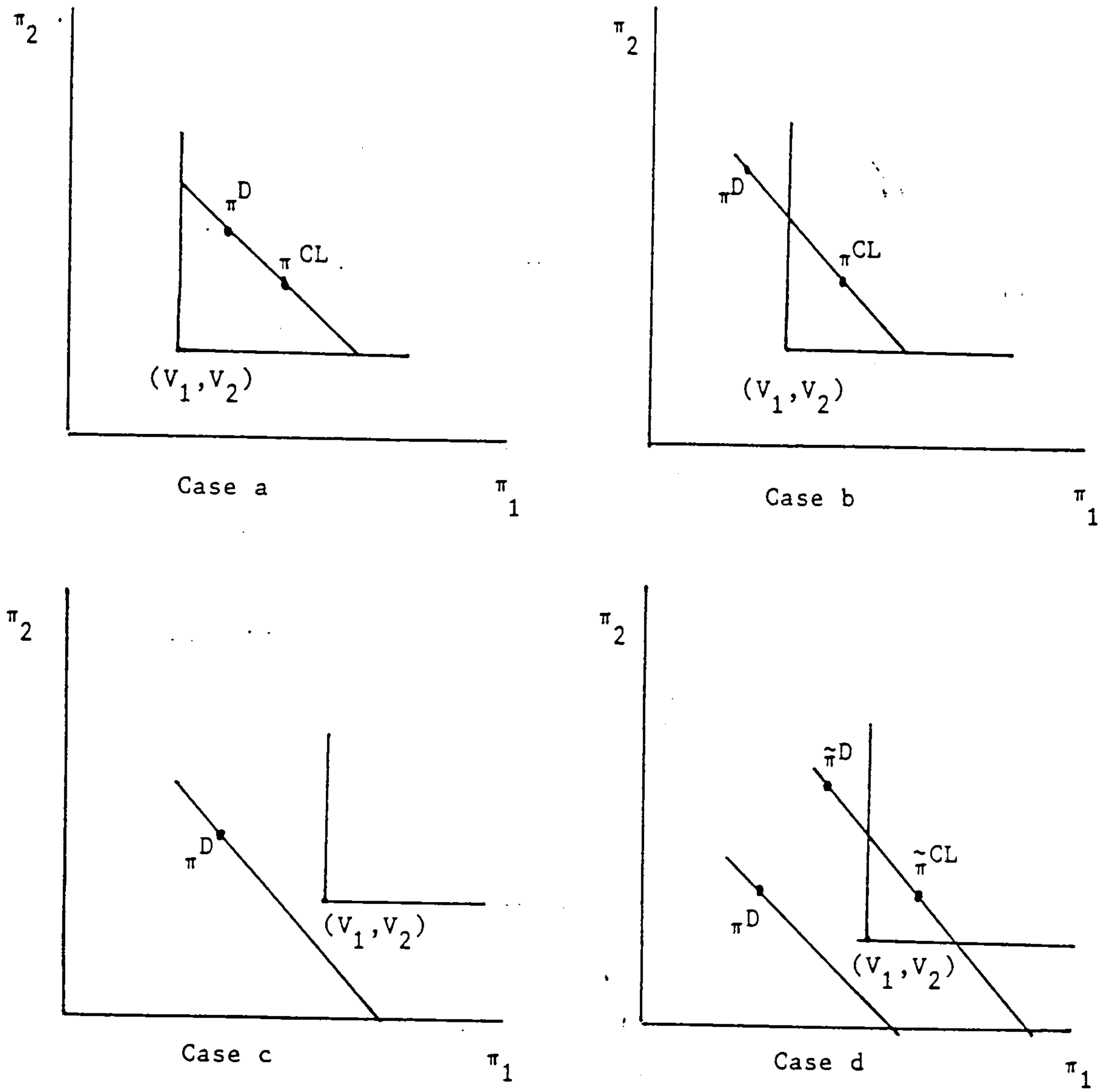
π2
πD
π CL
(V1,V2)
Case a
π1
π2
πD
πCL
(V1,V2)
Case b
π1
π2
πD
(V1,V2)
Case c
π1
π2
~πD
~πCL
πD
(V1,V2)
Case d
π1

Table 1: Numerical Simulation of a Complete Specialization Game

b	5	7	10	12	14	16	18	20	22	24	26	28	30	50	75	100	125	150	200
$\lambda_1^A(\phi,\phi)$	.41	.40	.38	.37	.36	.35	.34	.34	.33	.33	.32	.32	.31	.28	.25	.23	.22	.20	.18
$\lambda_1^A(\phi,B)$	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54	.54
$\lambda_2^B(\phi,\phi)$	.53	.45	.38	.35	.32	.30	.28	.26	.25	.24	.23	.22	.21	.15	.12	.10	.08	.07	.06
$\lambda_2^B(A,\phi)$	.80	.66	.54	.49	.44	.41	.38	.36	.34	.32	.30	.29	.27	.20	.15	.12	.10	.09	.07
$V_1(\phi,\phi)$	13.6	13.4	13.0	12.8	12.7	12.5	12.3	12.2	12.0	11.9	11.8	11.6	11.5	10.4	9.5	8.7	8.0	7.4	6.6
$V_1(\phi,B)$	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6	14.6
$V_1(A,\phi)$	17.8	17.4	16.8	16.6	16.3	16.0	15.8	15.6	15.4	15.2	15.0	14.8	14.7	13.3	12.0	11.0	10.2	9.5	8.4
$V_2(\phi,\phi)$	14.2	13.7	13.0	12.7	12.3	12.0	11.8	11.5	11.3	11.1	10.8	10.6	10.4	9.0	7.7	6.8	6.1	5.6	4.7
$V_2(A,\phi)$	16.0	15.3	14.6	14.1	13.8	13.4	13.1	12.8	12.5	12.3	12.0	11.8	11.6	10.0	8.6	7.6	6.8	6.2	5.3
$V_2(\phi,B)$	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8	16.8