Discussion Paper No. 677

THE KINKED DEMAND CURVE, FACILITATING PRACTICES, AND OLIGOPOLISTIC COMPETITION

by

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February 1986

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1. Introduction

Chamberlin in his treatment of oligopoly thought it self-evident that if "sellers have regard to their total influence upon price, the price will be the monopoly one" (1939, p. 54). His argument is persuasive at first, but it stumbles over its disregard for the fact that the monopoly price is not generally a Nash equilibrium for an oligopoly. Oligopolists have strong incentives to cheat on each other and drive price below the monopoly (or collusive) price. Chamberlin's discussion of how an oligopoly overcomes these incentives is vague; it is little more than the observation that self-interested agents (oligopolists, in this case) tend cooperatively to seek outcomes that are Pareto optimal from their perspective.

Somewhat later Sweezy (1939) observed that a rational oligopolist in altering his price should explicitly take into

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account the expected reaction of its fellow oligopolists.[1] In particular, he analysed the oligopolist's pricing decision for the case where it beleives that its competitors will match any price cuts, but not go along with any price increases. The resulting "kinked demand curve" leads to the conclusion that an oligopolist will cut price only if its price is greater than the monopoly price. Therefore equilibrium in the industry is the monopoly price provided the initial industry price is above the monopoly price. If, however, initial price is below the monopoly price, then price tends to stick at the initial price. Thus the virtue of Sweezy's model is that it shows how a collusive price can be much more stable for an oligopoly than the Nash equilibrium analysis predicts.

Nevertheless the kinked demand curve argument as it stands has serious shortcomings.[2] It predicts that if an oligopoly has succeeded in setting its price at a collusive level, then its members will revise that price downward immediately if costs fall or demand slackens, but will not revise its price upward if costs rise or demand strengthens. This is implausible and does not square with the empirical evidence as, for example, assembled by Stigler (1947) and Primeaux and Bomball (1974). In addition, the kinked demand curve theory does not adequately explain how price initially gets set in an oligopoly. It only explains why price becomes quite stable over a range of possibilities whenever oligopolists beleive that instantaneous matching of the lowest quoted price will always

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1. Hall and Hitch (1939) made essentially the same observation simultaneously.

2. Stigler (1947) is the best known criticism of Sweezy's theory. Scherer (1980, pp. 166-68) summarizes the various arguments against it.

occur.

In Sections 2, 3, and 4 we use a kinked demand curve type model to derive Chamberlin's observation that among sensible oligopolists who "have regard for their total influence upon price" the equilibrium price is the collusive price. Our argument is not vague; it is a well defined, strictly noncooperative, game theoretic argument based on firms not playing dominated strategies. It, since it predicts the collusive price and not a sticky price, does not admit the same objections that Sweezy's model admits and is not falsified by empirical studies that find oligopolistic prices are not rigid.

Statement of our result is straightforward. Consider an oligopolistic industry where all firms have identical profit functions. Suppose all firms set price simultaneously with each holding the belief that if its price is not the lowest prices report, then before any sales occur it will lower its price to match the lowest priced firm's price. This belief creates for all firms a kinked demand curve. We show that the industry, if each firm eliminates from consideration all prices that other prices dominate, immediately jumps to a collusive equilibrium at the monopoly price. This is Chamberlin's result made precise.

The logic of the result is easily explained for the simple case where the profit functions of the firms are extremely well behaved. Suppose the firms have identical cost and demand structure and all believe that each will instantly match the lowest price any firm sets. This means all sales will take place at a common price p that is the minimum of the prices that the firms initially report. Therefore each firms' profits is just a function of that market price p. Let $f_i(p)$ be this common profit function, let c be the price that maximizes it, and let f_i be unimodal around the collusive price c.

Suppose a firm i considers playing a price $p_i < c$. Let p_{min} be the lowest price set by any firm other than i. There are two possibilities. First, if $p_i \geq p_{min}$, then firm i's profits are $f_i(p_{min})$ because its price is not the lowest price reported (at least not uniquely so) and therefore does not determine the market price at which sales actually take place. Consequently if firm i had played the collusive price c rather than p_i, then its profits would have still been $f_i(p_{min})$. That is, given that $p_i > p_{min}$, firm i does not harm itself by playing the collusive price c rather than $p_i < c$.

The second possibility is that $p_i < p_{min}$. In this case firm i's profits are $f_i(p_i)$ because its price is the lowest reported price and therefore determines the market price at which sales take place. Because f_i is unimodal and $p_i < c$, firm i can increase its profits by setting its price not at p_i, but at a level higher than p_i. In fact, as is easily seen, if firm i sets its price at the collusive level c, then it is guaranteed profits higher than $f_i(p_i)$. Therefore, in the second case, firm i can increase its profits by playing c rather than p_i.

Thus, in either of the two possible cases, firm i does at least as well playing the collusive price c rather than playing the price $p_i < c$. In other words, the collusive price dominates all prices less than it. A similar argument shows that the collusive price dominates all prices greater than it. Therefore for this simple unimodal case playing the collusive price dominates all other prices and is the natural equilibrium.

One way to think about this paper's results is in relation to super game models of oligopoly. Anderson (1984) and Kalai and Stanford (1985) are examples of this literature that are particularly relevant to this paper. The collusive equilibria those models obtain are supported by the punishment of any firm that deviates from the collusive price. Our model is not dynamic, but its driving force is the same. The assumption that firms match the lowest quoted price instantly—before any trade takes place—is instant punishment, with no delay until the next period, for any firm that deviates from the collusive price. Thus our results can be thought of as a particular limiting case of the super game models. For example, consider Kalai and Stanford's model of a repeated duopoly game where the two firms play the tit-for-tat strategy against each other. If the length of each time period is shrunk toward zero, which means the delay between a firm's deviation and subsequent punishment is also shrunk toward zero, then the equilibrium price approaches the collusive price, which is the same result we obtain in this paper.

A second way to think about this paper's results is in terms of facilitating practices. A facilitating practice or device, as defined by Salop (1982), is an established custom governing the manner in which firms compete against each other having the effect of making coordination of their prices easier. In this paper the facilitating practice that drives equilibrium towards the collusive price is of the "meeting competition" type: all firms believe every firm will match the lowest priced competitor. Thus this paper is a precise theory of how a particular facilitating practice can lead to a monopoly outcome. In Section 5 we use the gasoline additive industry as an example where facilitating practices of the type modeled in this paper have apparently led to collusive prices.

A third way to think about this paper is in relation to the literature on price leadership. The kinked demand game is a reasonable formalization of what Markham (1951) called "price leadership in lieu of of an overt agreement" (p. 901-903). Price leadership in this model comes from whatever firm sets the lowest price. In our model if the structure is collusive, then the collusive price results. This is exactly as Markham argued would be the case whenever an industry's product is a nondifferentiable commodity for which the major firms are few in number, recognize their interdependence, and have similar profit functions. In the second half of Section 5 we suggest that the U.S. steel industry's behavior until recently was consistant with price leadership of this type.

Our model is quite distinct from Markham's other two categories of price leadership: dominant firm leadership and barometric firm leadership. The dominant firm model posits a fundamental asymmetry in power and size among the firms. The dominant firm sets price and the fringe firms take that price as given. Our model assumes that any firm will match the lower price of any other firm, even if that low priced firm is relatively small. The barometric firm model, as Markham points out, is a price leadership model in name only. In that model the price leader does little more than make official the price that actually exists already in the market. Markham comments that the barometric leader might better be called the "first 'price follower'" (p. 891).

2. Model

Let $N = \{1, 2, \ldots, n\}$ be the set of firms in the industry where $n \geq 2$ is the number of firms. Each firm i simultaneously and noncooperatively sets a nonnegative price $p_i \in S_i$ where $S_i = [0, \infty) \equiv R^+$ is firm i's set of admissible prices. Let $S = S_1 \times S_2 \times \ldots \times S_n$. A strategy tuple $s = (s_1, s_2, \ldots, s_n) \in S$ is a vector of n admissible prices, one from each firm. After the firms all set their prices, but before any sales are consumated, those firms whose prices are higher than the lowest quoted price all reduce their price to match that lowest price. This is the facilitating practice that drives our results and, harking back to Sweezy (1939), gives each firm a kinked demand curve.

Let $f = (f_1, f_2, \ldots, f_n)$ be a vector of functions, $f_i: S_i \to R$. Its interpretation is that, for every market price $p \in R^+$, $f(p) = (f_1(p), f_2(p), \ldots, (f_n(p))$ is the vector of profits the n firms earn when they behave optimally in response to the price p. Given that all firms immediately match the lowest price any firm sets, the <u>induced price</u> (or realized price) for a strategy tuple $s \in S$ is $p = \min_{j \in N}(s_j)$. Firm i's <u>realized profit</u> depends only on this induced price; it is $u_i(s) = f_i(\min_{j \in N}(s_j))$.

We assume that every one of the functions f_i is <u>multipeaked</u>. This means that, for each firm i, there is a finite sequence of real numbers $0 = r_1 < r_2 < r_3 < \ldots < r_k$ with f_i being weakly monotonic on every interval $[r_j, r_{j+1}]$ for $j = 1, 2, \ldots, k-1$ and nonincreasing on $[r_k, \infty)$. The <u>kinked demand game</u> is therefore this. The players are the set of firms N, the strategy space is S, and the payoff functions are the n-tuple of realized profit functions $u(s) = [u_1(s),...,u_n(s)]$ where each underlying f_i is multipeaked.

A price $p \in R^+$ is <u>self-enforcing</u> if for every price p', $0 \le p' < p$, $f(p') \le f(p)$. The notation $f(p') \le f(p)$ means that, for all $i \in N$, $f_i(p') \le f_i(p)$. It is <u>strictly self-enforcing</u> if for every p', $0 \le p' < p$, $f(p') < f(p)$ (< in every coordinate). Let SE denote the <u>set of self-enforcing prices</u> and stSE be the <u>set of strictly self-enforcing prices</u>. By the definitions $0 \in stSE \subset SE$ so that these sets are not empty. A self-enforcing price has the property that if it is lowered none of the firms gain. If a strictly self-enforcing price is lowered, then every firm loses.

Clearly payoff vectors are monotonic in self-enforcing prices--that is, if p' is a self-enforcing price and p is a self-enforcing price with $p > p'$ then $f(p) \ge f(p')$ ($f(p) > f(p')$ if p is strictly self-enforcing). It is also easy to check that, because the f_i's are multipeaked, the observations in the following proposition hold.

<u>Proposition 1</u>. For the kinked demand game the following properties are true:

1. SE is bounded above.
2. A maximal strictly self-enforcing price p* exists. That is, $p^* \in stSE$ exists such that $p^* \ge p$ for every $p \in stSE$.
3. The maximal strictly self-enforcing price is not necessarily maximal among all the self-enforcing prices; neither are its induced profits. That is, a self-enforcing $p > p^*$ may exist with $f(p) > f(p^*)$.
4. Generically (allowing for small perturbations in the f_i's)

p^* is maximal among the self-enforcing prices.

Figure 1 illustrates these concepts. Self-enforcing prices indicate joint self-interest of firms. As we suggested above, if a price p is self-enforcing, then all the firms—both individually and jointly—share the desire to prevent it from going down.

When the interests of the firms with respect to price are completely compatible, then we call the situation collusive. Specifically, the kinked demand game has a collusive structure if a price m exists that is the unique global maximum for each profit function f_i. The price m is then the collusive price. It is important to note that if m is the collusive price for f, then m is the maximal (strictly or not) self-enforcing price relative to f. Nevertheless, as stated in Proposition 1, even when a collusive price does not exist, the maximal strictly self-enforcing price does exist.

Certain aspects of the model should be emphasized. First, firms' realized profits depend only on the smallest price set by any firm. We incorporate this "meeting competition" behavior directly into the definition of u(s) above because we assume from the beginning that the firms always match. We therefore do not make each firm's profits depend on the full vector of stated prices, which we would have to do if we were permitted firms to make sales at a price different than their rivals' price.

Second, the model is not equipped to ask in a rigorous manner if adopting the matching competition is sensible for the firms involved. This is because, for a firm to decide if it should deviate from the matching competition practice, it must compare profits from following the practice against profits from deviating

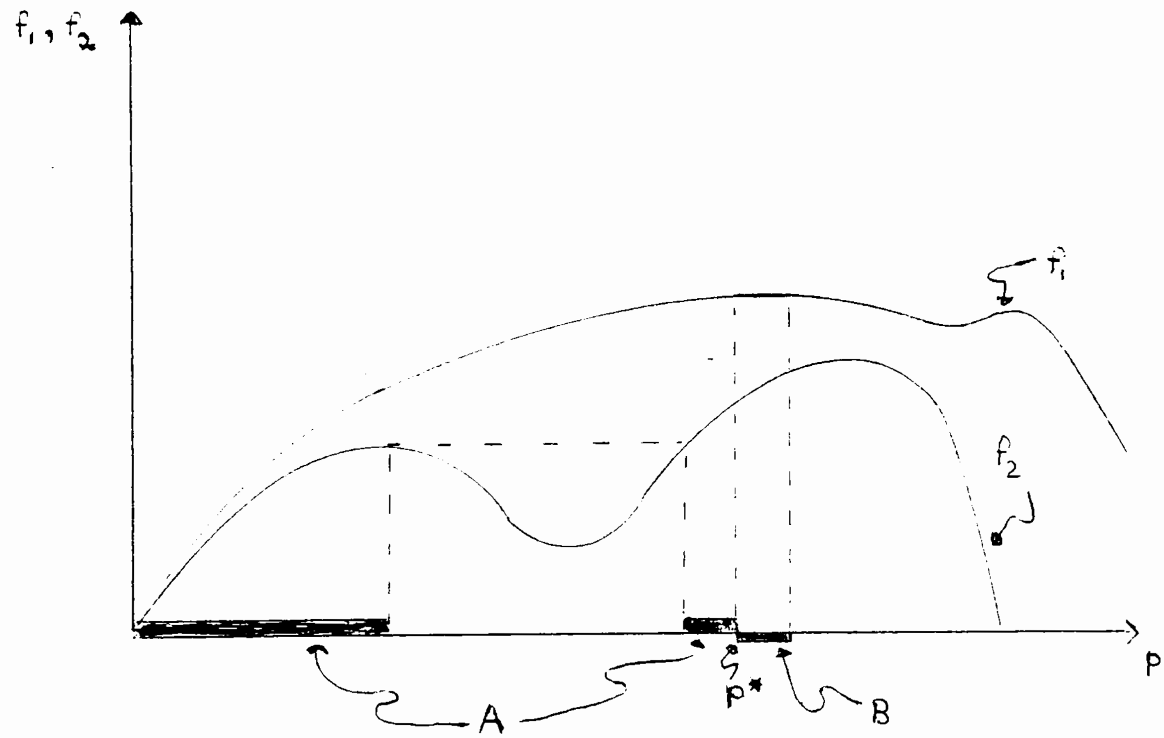

Figure 1. The price p* is the maximal stSE price, A is the set of stSE prices, and B is the set of SE prices at least as great as p*.

from it. We can not do this here because we have made $f_i(\cdot)$ depend only on the common price at which all firms settle after matching occurs. Nevertheless, except in situations where either (i) a preemptive deviation from the practice is likely to permanently give the preempting firm an advantage over the remaining firms or (ii) serious asymmetries in the firms' interests with respect to pricing exist, this is not a significant limitation.

The reason that this is not a limitation when reasonable commonality of interests exists among the firms with respect to pricing is most easily explained for the extreme case where a collusive structure does exist. The analysis of Section 4 shows that, for collusive structures, the meeting competition practice leads to the collusive price being the equilibrium price. Suppose firm i considers deviating from the meeting competition practice by charging a price p higher than the collusive price m.[3] For collusive structures $f_i(m) > f_i(p)$ for all $p > m$, i.e., firm i would not increase its profits even if the other n-1 firms followed the increase. Clearly if the other n-1 firms do not follow its price up to p, then firm i's realized profits are less than $f_i(p)$, which itself is less than $f_i(m)$. Therefore, if the kinked demand game has a collusive structure, it is in each firm's interest to follow the practice of matching the lowest price.

The third aspect of the model that merits emphasis is that the price matching behavior that we postulate each firm follows is strong, aggressive behavior. Its strength contasts with the

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3. It can not deviate by setting a price lower than the collusive price. If it did, then the other firms would match that price and deviation from the practice would be equivalent to following the practice.

weakness of the dominant firm's behavior in the dominant firm model of price leadership. In that model the sequence of events is implicitly that the dominant firm sets a price, each fringe firm selects that quantity that maximizes its profits at the given price, and the dominant firm selects its quantity to maintain the price it initially selected. Thus, if a fringe firm selects a quantity that is higher than the dominant firm expected, the dominant firm cuts back on its quantity in order to maintain the price. The dominant firm does not punish the fringe firm in any way for its temerity. It does not protect its market share. Ono (1982) has shown that unless the dominant firm is substantially more efficient than the other firms, acting the part of the dominant firm is against the dominant firm's interests in the sense that it would prefer one of the other firms to take the part.

This contrasts to the story that is implicit to our model. For a fringe firm j and any given induced market price p, $f_j(p)$ represents the profit it earns when it prices the same as the other firms (including the dominant firm if one exists). Implicit in $f_j(p)$ is some sales quantity q_j that is the number of units firm j sells if the price is p. If firm j decides to produce and sell more units than q_j, then in order to sell the extra units it must reduce its price below p.[4] But if it reduces its price, then the other firms match and make the price reduction necessary to sell the additional units much larger than would otherwise be the case. No firm cuts back its production in order to maintain the price and

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4. Presumably firm j in order to sell its units at the common market price p had to pursue optimally all available avenues of nonprice competition. Therefore, if it wants to increase its unit sales, it must reduce its price.

make room for firm j's extra production. Quite the contrary, every other firm in order to keep its market share boosts its production and thereby magnifies the price drop that firm j must induce in order to sell a given quantity of additional production.

3. Inadequacy of the Nash Equilibrium Concept

For the kinked demand curve the Nash equilibrium concept leads to the set of equilibria that Sweezy (1939) informally identified in his paper. This set includes equilibria where the the industry is stuck at a price lower than the collusive price.

To understand the set of Nash equilibria we need to introduce some additional notation and a definition. For a strategy tuple $s \in S$ and a strategy $t_i \in S_i$ we use the symbols s_{-i} to denote the vector $(s_1, s_2, \ldots, s_{i-1}, s_{s+1}, \ldots, s_n) \in \times_{j \in N \setminus i} S_j$, and $(s_{-i} : t_i)$ to denote the vector $(s_1, s_2, \ldots, s_{i-1}, t_i, s_{i+1}, \ldots, s_n)$. As usual we define a strategy tuple $s^* \in N$ to be a <u>Nash equilibrium</u> if $u_i(s^*_{-i} : t_i) \le u_i(s^*)$ for every $i \in N$ and every $t_i \in S_i$. We say that a price $p \in R^+$ is a <u>Nash equilibrium price</u> if a Nash equilibrium strategy s^* induces it. with $p = \min_{i \in N}(s^*_i)$. The following theorem, which is identical to Macgregor's (1983) Theorem 1, characterizes all Nash equilibria for the kinked demand game.[5]

<u>Theorem 1</u>. For the kinked demand game a price p is a Nash equilibrium price if and only if it is self-enforcing.

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5. Also see her Ph.D. dissertation at the University of California, Berkeley.

An informal, alternative way of stating the result is this. A strategy tuple is a Nash equilibrium if and only if every firm reporting the lowest price chooses its price to maximize its payoff in the range of prices below or equal to the price of its lowest priced competitor.

Proof. It is obvious that if p is not self-enforcing then some firm can improve its own utility by lowering the minimum price of any strategy inducing p. Conversely, if p is self-enforcing, let $s^* = (p,p,\ldots,p)$. It is obvious now (recall that $n \geq 2$) that s^* is a Nash equilibrium inducing p.‖

Theorem 1 makes apparent the great multiplicity of Nash equilibria that exist for the kinked demand game. On Figure 1 the set $A \cup B$ is the set of Nash equilibria for the two profit functions (f_1,f_2) illustrated. Many of these equilibria appear on their face to be unstable because they involve firms playing dominated strategies. For example, suppose the industry consists of two firms and both of them report the same price p and that price is less than the collusive price c. According to Theorem 1 that strategy $s = (p, p)$ is a Nash equilibrium. But, as we showed in Section 1, to play the price $p < c$ is to play a dominated strategy.

4. Sequentially Dominant Strategies[6]

A reasonable equilibrium concept should exclude dominated strategies from the equilibrium set while still remaining within the noncooperative framework. It is with this purpose in mind that we define the concept of a sequentially dominant strategy. Such a strategy is a Nash equilibrium, but a Nash equilibrium is generally not a maximal sequentially dominant strategy. Use of this equilibrium concept excludes all Nash equilibria that depend on firms playing dominated strategies.

A reduction of the kinked demand curve game is described by a set $R = R_1 \times R_2 \times \ldots \times R_n$ where each component R_i is a subset (not necessarily proper) of firm i's original admissible set of prices S_i. Thus, for all $i \in N$, $R_i \subset S_i$ and $R \subset S$. In the reduced game described by R, each firm's profits are still $U_i(s)$, but the strategy tuple s is restricted to be an element of R. In particular, for each firm i the requirement is $s_i \in R_i$, which means that some $s_i \in S_i$ may exist that are no longer admissible.

Given a reduction $R \subset S$ of the original game, a firm $i \in N$, and two strategies r_i, $t_i \in R_i$, we say that r_i (weakly) <u>dominates</u> t_i relative to R if, for every $s_{-i} \in R_{-i}$, $U_i(s_{-i}:r_i) \geq U_i(s_{-i}:t_i)$. We then write $r_i \mathrm{dom}_R t_i$. Thus if r_i dominates the strategy t_i, then, no matter what admissible prices the other firms report, firm i does at least as well reporting r_i as it would do if it reported t_i. Discarding t_i from further consideration as a possible price to

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6. The idea of sequential domination has been discussed by many authors. For example, see Luce and Raiffa (1957) and Moulin (1979).

report therefore makes eminant good sense. For two strategy tuples s, $t \in S$ we say that s <u>dominates</u> t <u>relative to</u> R if, for every $i \in N$, $s_i \mathrm{dom}_R t_i$. We denote this with $s \mathrm{ dom}_R\ t$.

Given two reductions R and T of the game where $R \subset T$, R is a <u>dominant reduction</u> of T if, for every $t \in T$, an $r \in R$ exists such that $r \mathrm{ dom}_T\ t$. In words, a reduction R of T is a dominant reduction if, for each firm i, any strategy t_i that is discarded from the feasible set of strategies is dominated by some strategy r_i that is not discarded. Observe that dominant reductions always exist because a reduction T is always a dominant reduction of itself.

A reduction R of the original space of admissible strategies S is a <u>sequentially dominant reduction</u> of S if a sequence of reductions $T^0, T^1, \ldots, T^\ell$ exist for some finite, positive ℓ such that $T^0 = S$, $T^\ell = R$, and T^j is a dominant reduction T^{j-1} for all $j \in \{1, 2, \ldots, \ell\}$. Therefore, in words, R is a sequentially dominant reduction of S if R is obtained from S through a sequence of dominant reductions. A reduction R is a <u>maximal sequentially dominant reduction</u> if the only dominant reduction of R is R itself.

A strategy $s \in S$ is a <u>sequentially dominant strategy</u> if the singleton set $\{s\}$ is a maximal sequentially dominant reduction. Obviously every sequentially dominant strategy (or the singleton consisting of it) is a maximal sequentially dominant reduction. Provided that a sequentially dominant strategy exists, then it is a very plausible noncooperative equilibrium point for the game, particularly if it is unique. This is so for two reasons. First, it is a Nash equilibrium, though generally the converse is not true. Second, each player arrives at his equilibrium strategy through a well defined, noncooperative process of eliminating dominated strategies from consideration. The only assumption that each firm makes about the other firms is that they also eliminate from consideration dominated strategies.[7]

In general maximal sequentially dominant reductions may not result in singletons or, even, may not exist. The following theorm shows that for the kinked demand curve game the situation is simple. Maximal sequentially dominant reductions always exist, they always yield sequentially dominant strategies, and, whenever a collusive price exists, a stratety is sequentially dominant strategy if and only if it induces the collusive price.

Theorem 2. Every maximal sequentially dominant reduction of the kinked demand curve game is a sequentially dominant strategy s whose induced price is self-enforcing and greater than or equal to p*, the maximal strictly self-enforcing price. Conversely, every self-enforcing price that is greater than or equal to p* is induced by some sequentially dominant strategy s.

In terms of Figure 1, sequentially dominant strategies induce precisely the prices within the set B. Proof of the theorem is in the Appendix.

Two important corollaries follow immediately from Theorems 1 and 2 and their proofs.

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7. The fully noncooperative nature of our equilibrium concept contrasts with Macgregor (1983). She asserts that the maximal strictly self-enforcing price is the natural noncooperative equilibrium point. What argument she presents justifying this notion, however, cooperative. She appears to argue that the firms will cooperatively select from the set of Nash equilibria that equilibrium which, relative to the set of Nash equilibria, is Pareto optimal for them.

Corollary 1. If the kinked demand game has a collusive structure with collusive price m, then a strategy $s \in S$ is a sequentially dominant strategy if and only if it induces the collusive price. In particular, the strategy tuple $s = (m, \ldots, m)$ is a sequentially dominant strategy.

Corollary 2. Let p be the maximal self-enforcing price. Generically, for the kinked demand game, a strategy s is a sequentially dominant strategy if and only if it induces the price p.

Corollary 1 says that if the firms have a unity of interests with respect to price, then all equilibria (of the sequentially dominant variety) are identical in the important sense that they all induce the collusive price. Corollary 2 says that generically the price induced by equilibrium strategies is uniquely the maximal self-enforcing price. It and the fact that all sequentially dominant strategies are Nash equilibria combine to make a very strong statement: generically the concept of sequential domination noncooperatively picks a unique induced price as the equilibrium price from among a continuum of Nash equilibrium induced prices.

It is important to note that Theorem 2 and its corollaries imply that if one firm has lower costs than the other firms and therefore prefers that the market price be set below the price the other firms prefer, then the low cost firm gets its way in equilibrium. Specifically, suppose p' is the price the low cost firm prefers and p'' ($p'' > p'$) is the price that the other firms prefer. Generically all sequentially dominant strategies for the industry induce price p'. Thus our model implies that the low cost firm has price setting power in an industry where price matching is the norm. If p' is enough below p'', then the other firms might find it advantageous to either not match the low cost firm or to exit from the industry. Both those decisions, however, lie outside the scope of our model and analysis.

Anderson (1984) constructed a repeated game model of oligopoly and obtained results that are are similar and complementary to our results. In his model firms are not restricted to follow a "meeting the competition" strategy. Firms, however, are assumed to incur adjustment costs whenever they change prices. He looks for perfect quick-response equilibria. These are subgame perfect, ε-Nash equilibria that exist when the time period between replays of the stage game becomes short enough, i.e., when firms' responses to other firms becomes quick enough. His Theorem 3.10 illustrates the types of results that he has obtained: Conditional on some technical restrictions on firm's profit functions, he identifies a particular strategy tuple as being a strongly perfect quick-response equilibrium. The strategy tuple that he identifies as an equilibrium is a generalization of the tit-for-tat strategy; it thus endogenously derives the "meeting competition" property that is central to our formulation. In order to obtain the result he restricts himself to looking only at <u>strongly</u> perfect quick-response equilibria. This restriction eliminates from consideration all perfect quick-response equilibria that are not Pareto optimal for the firms.

Our results and Anderson's results are complementary because, while similar, they do it with models and assumptions that differ significantly. Three examples illustrate these differences. Anderson's model is a repeated game where endogenously determined responses are allowed to become arbitrarily quick. Our model is a one shot game where responses are instantaneous and exogenously prescribed. Anderson's model uses ε-Nash equilibria. Our model uses sequential domination. Anderson in his Theorem 3.10, by looking for strong equilibria, guarantees the collusiveness of whatever equilibria he can identify. Our result shows that sequential domination leads uniquely to the collusive price without any assumption that the equilibrium must be Pareto optimal, but only individually optimal.

5. Two Applications

In this section we discuss two industries and their pricing behavior: the producers of lead based antiknock gasoline additives and the producers of steel in the United States. Our purpose is to point out that the model and analysis presented above fits reasonably well the behaviors of these industries as we understand them. Our analysis here should be regarded as exploratory and suggestive; we are not experts in either industry. We hope our discussion will stimulate fresh empirical research.

Consider first the case of the lead antiknock gasoline additive industry, which is now dying, but which until the advent of the environmental movement was the gasoline additive of choice to prevent engine knock. This industry has received substantial legal and economic attention because the Federal Trade Commission (FTC) during the early 1980s charged the participants with unfair competitive practices.[8] These additives, which purchasers consider to be commodities, were manufactured during the 1974-79 period by Ethyl Corporation, Du Pont, Nalco, and PPG Industries. The FTC in their complaint, which ended up being aimed primarily against Ethyl and Du Pont, focused on three competitive practices that existed within the industry during the 1974-79 period: (i) all four companies quoted prices in terms of a delivered price rather than a fob price, (ii) Ethyl and Du Pont often gave customers more than the 30 day notice that they were contractually required to give regarding price increases, and (iii) Ethyl and Du Pont (and occasionally PPG) included a "most favored nation clause" into supply contracts whereby each customer was guaranteed the same price as every other custumer at each moment of time. A fourth competitive practice, which the FTC did not attack, but which all four companies offered in their supply contracts and which is important for our analysis, was inclusion into supply contracts of "meet or release" meeting competition clauses.[9] These stated that if a buyer received an offer from another supplier at a lower price, then the current supplier had to either meet the lower price or to release the buyer from its current contract at the higher price. The full Commission decided in 1983 that the first three practices constituted unfair competition. Their decision, however, was

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8. Unless otherwise noted the facts that we use may be found in the decision of the Second Circuit of the U.S. Court of Appeals in the case of E.I. Du Pont de Nemours & Co. v. Federal Trade Commission, 729 F.2d 128 (1984).

9. See Salop (1982, Section V, footnote 36).

overturned on appeal in 1984 and consequently the practices have been allowed to continue.[10]

The effect of these four competitive practices, we argue, is to make the pricing problem for Ethyl and Du Pont in competing against each other that of the kinked demand game. The requirement of the kinked demand game is that firms believe all trade will take place at the lowest reported price. Of the four practices, two in particular—giving "more than 30 day notice" of price increases and incorporating "meet or release" clauses in the supply contracts—made it almost unavoidable that Du Pont and Ethyl would hold this belief about each other. The meet or release clause meant that each would immediately learn if the other was reducing price and, because of the release part of the clause, provided a powerful incentive to match immediately. Neither firm consequently could reasonably consider price cutting a way of stealing market share from the other. The greater than 30 day notification for price increases provided the firms with a coordinated way of increasing prices. If one firm thought prices should be increased due to changed demand or cost conditions, then it could announce the price increase for a date in the future and see if other firms went along. If other firms did go along, then the increase became effective. If they did not go along, then the increase was canceled before it went into effect and cost market share.

The other two practices played a supporting role in making Ethyl and Du Pont's pricing problem into the kinked demand game. Quoting delivered prices rather than fob prices meant that each

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10. See E.I. Du Pont de Nemours & Co. v. Federal Trade Commission, 729 F.2d 128 (1984).

firm's price was uniform across buyers and would not inadvertently be undercut by another seller due to locational differences. The most favored nation clauses reinforced this. It made credible sellers' protestations to buyers that they could not make special deals because that would cause a general price cut. Therefore, to summarize the first part of our argument, the competitive practices that Du Pont and Ethyl each followed gave each other very good reasons to see their competitive interaction as being identical to that in the kinked demand game.

The presence and behavior of Nalco and PPG, the other two firms in the industry during the 1974-79 period, causes trouble for the neat argument just presented. During this period Nalco and PPG gave price discounts on a majority of their sales. This price discounting became increasingly prevalent during the latter parts of the period as the market for lead antiknock compounds shrunk in response to the legally mandated increased useage of unleaded gasoline. Ethyl and Du Pont sought to meet this price competition with increased customer service. These facts on their face seem inconsistent with the kinked demand game being a defensible description of the industry.

A reconciliation, however, is possible. Ethyl had monopolized the market until 1948 when Du Pont had entered. Nalco entered in 1964. In 1974, when PPG entered the industry, Ethyl had 34% market share and Du Pont had 38% market share, leaving only 28% for Nalco and PPG together. Thus Ethyl and Du Pont were traditionally the dominant firms in the industry and, arguably, this continued to be so throughout the 1974-79 period. A reasonable way to conceptualize the industry during the 1974-79 period is as a dominant firm (Ethyl and Du Pont together) competing with two fringe firms that are trying to gain market share over time through discounting from the dominant firms' price.[11] The profit functions, f_1 for Du Pont and f_2 for Ethyl, of the two dominant firms are then the discounted value of future profits taking full account of how their pricing decisions affect the fringe firms behavior. In particular, f_1 and f_2 incorporate effects of the following type: if Du Pont and Ethyl set p at a high level, than the fringe firms are likely to discount more aggresively and grow in market share more rapidly than if Du Pont and Ethyl set p close to marginal cost.[12] Given f_1 and f_2, our analysis suggests that the equilibrium market price is the maximal strictly self-enforcing price. Provided f_1 and f_2, as perceived by Du Pont and Ethyl respectively, have a common global maximum, this price is also the collusive price.

The second industry we consider is the steel industry in the United States from the turn of the century to the present as described by Scherer (1980, pp. 178-80). The facts in this case are simple. From about 1900 to 1958 U.S. Steel Corporation set prices and the other domestic producers followed with identical prices. From 1958 to 1968 price leadership rotated among different members of the industry, primarily because U.S Steel was no longer willing to take the political heat from Washington, D.C. of being the overt

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11. See, as noted above, Markham (1951) and Ono (1982) for discussions of dominant firm price leadership.

12. To calculate the dominant firms' profit functions explicitly, taking full account of the dynamics of the fringe firms' growth, is difficult. Judd and Peterson (1984) have constructed and solved a model in this spirit for the case of a dominant firm facing a competitive fringe that must finance their growth through retained earnings. In their model the dominant firm controls the rate at which the fringe firms can grow because the dominant firm's price determines the fringe firms' profits.

price leader. Beginning in 1968 and continuing into the late seventies Bethlehem Steel attempted to keep some coordination in prices by periodically punishing defectors from the established price structure. But, as Scherer (1980) summarized, Bethlehem's "efforts were at best only partly successful. Sub rosa chiseling was widespread in times of excess capacity and sharp import competition" (p. 180). Nothing is left of the past coordination today in 1986. For instance, beginning in 1982 General Motors has put its requirements for steel up for competitive bid.[13]

Four questions must be asked to determine if the kinked demand game as we have analyzed it applies to the steel industry through 1968. On the whole we answer these questions affirmatively; therefore we think the theory does have application to the steel industry's history. The first question is this. Up until 1968 when discipline first began to disintegrate, did the steel firms beleive that all sales would take place at the minimum of the reported prices? The answer to this is unquestionably yes. U.S. Steel certainly expected other firms to mimic its prices. Each smaller firm certainly believed that if it quoted lower prices than U.S. Steel, then U.S. Steel would either match the lower prices or quote a revision that the smaller firm would then match.

The second question is: why was U.S. Steel instead of another company the price leader until 1958? One possible reason exists within the context of our theory. If U.S. Steel, which was the biggest firm, had scale economies that gave it a cost advantage, then Theorem 2 suggests that its maximizing price would be the

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13. See T. F. O'Boyle, Big Steel is Hurting in Buyer Market, Wall Street Journal, 28 May 1985, pp. 16.

equilibrium price actually observed in the market. If, on the other hand, U.S. Steel had cost parity with other firms, then our analysis does not suggest a special role for U.S. Steel. Explanation of why U.S. Steel did play a special role may then have to depend on noneconomic factors such as tradition. A possible origin of U.S. Steel's special role may have been Judge Elbert Gary's dinners, held by him early in the century for executives from competing companies while he was chairman of U.S. Steel, where "respect and affectionate regard" for each other and for each other's interests was cultivated.[14]

The third question is: why did coordination break down beginning in the late sixties? The root cause appears to have been the rising flow of imported steel. By 1968 imports accounted for just under 20% of the U.S market, almost as much as U.S..Steel's 21% market share. Our theory suggests that this increasing presence was disruptive for the following reasons. Imported producers had different cost, demand, and profit functions than the American producers. This meant the foreign companies had little reason to join the domestic producers' kinked demand game. The domestic producers did not adopt a policy of matching the prices of foreign competitors; with this ommission domestic producers implicitly acquiesced to foreign producers' interest in staying outside of the game.[15] Instead the U.S. producers appear to have collectively

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14. Quoted from Scherer (1980, p. 170); his source was Machlup (1952, p. 87) who quoted it from a government antitrust brief.

15. We are not clear why the U.S. companies did not act aggressively to meet foreign price competition. Perhaps it was because foreign price cutting was always sub rosa and therefore difficult to monitor and expensive to match. Or perhaps it was the general passiveness of the industry towards change. Adams (1977, pp. 115-16) excoriates the steel firms for their passive pricing

acted as a dominant firm price leader against their foreign competitors, much as Du Pont and Ethyl acted as price leaders towards Nalco and PPG..

As we mentioned in Section 2, acting as the dominant firm price leader is a weak response to burgeoning competition. Ono (1982) showed that accepting the passive role of dominant firm price leader is disadvantageous. Thus U.S. steel producers, to the extent they did accept this role vis-à-vis foreign firms, abetted the market penetration of imports.

Eventually as imports rose opinions among the domestic steel producers as to the best long term strategy for coping with the challenge imports presented may have begun to diverge seriously. To the extent that they did diverge, this caused their perceived profit functions (f_1, f_2,..., f_n) to become dissimilar to the point where, for some domestic steel producers, the twin policies of always selling at the posted price and of always matching the lowest price any other domestic producer quoted no longer made sense. Once domestic firms began reaching that conclusion coordination became increasingly difficult in the industry.

The fourth (and hardest) question is: did domestic steel producers set the maximal strictly self-enforcing price as our Theorem 2 suggests they should have? We can not answer this question but we can cite some indirect evidence. Before World War II certain classes of steel products showed remarkable price rigidity. It is unreasonable to believe that the maximal strictly self-enforcing price was equally rigid. Nevertheless since World

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behavior in the face of rising imports.

War II prices have been less rigid.[16] In particular, during the 1960s in response to import competition the rate at which U.S. firms increased steel prices slowed sharply in comparison to the rate in the 1950s.[17] Such a slowing is what one would expect if, as we just argued, the U.S. producers (1) played the kinked demand game among themselves and (2) collectively acted as a dominant firm price leader vis-à-vis the foreign firms.[18]

6. Concluding Remarks

In this paper we have shown that if the member firms of an oligopoly have similar profit functions and if they believe that every firm will immediately match the lowest price quoted on the market so that that price becomes the market price at which all sales take place, then the resulting equilibrium market price is the maximal strictly self-enforcing price. If the firms have identical profit functions, then the equilibrium price is also the collusive

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16. See the discussion of Adams (1977, pp. 108-110) regarding price rigidity. A possible explanation for the observed rigidity is this. Monopolies tend towards price rigidity, presumeably because they can afford to lapse into slothful, nonmaximizing behavior. For example, see the nice study of Primeaux and Bomball (1974) comparing the pricing behavior of electric utilities that are monopolies with the the pricing behavior of electric utilities that are duoplies. The domestic steel industry, particularly before World War II, may have collectively regarded itself as a monopoly and, consequently, paid insufficient attention to keeping its prices at optimal levels.

17. See Adams (1977, p. 111).

18. Adams (1977) asserts that "since World War II . . . steel prices have . . . shown a remarkable insensititivity to market conditions" (p. 110). This slowing of price increases in response to import competition stands in direct contradiction to his generalization.

price. This equilibrium is a Nash equilibrium in which no firm quotes a price that is either directly or sequentially dominated by another price that it could have quoted.

This result may be regarded as a synthesis of (i) Chamberlin's argument that oligopolists, because they understand their interdependence and recognize their common interest in setting a collusive price, will immediately jump to the collusive price and (ii) Sweezy's observation that if every firm matches the lowest price quoted by any competing firm, then pricing within the oligopoly becomes quite stable. This marriage is nice because on one side it eliminates the objection to Chamberlin's argument that the collusive price is not a Nash equilibrium, i.e., each firm has a strong incentive to undercut the collusive price in an attempt to gain market share. On the other side it eliminates two prime objections to the kinked demand curve model as originally proposed. First, since the collusive price is the equilibrium price, no indeterminancy exists concerning what price will initially be set. Second, this theory does not predict that prices will be any more rigid in a kinked demand game oligopoly than in a monopoly. Thus this theory appears to retain the strengths of both Chamberlin and Sweezy, while shedding some of their more important weaknesses.

Our results also tie into the interest antitrust scholars have in facilitating practices. The driving forces in our model are that the firms in the oligopoly have common interests (i.e., similar profit functions) and believe that undercutting on price is impossible because any undercutting will be instantly matched. An important way that this belief that undercutting is impossible can come about and be maintained is through adoption of appropriate facilitating practices. For example, our analysis of the lead antiknock additive industry shows that meeting competition clauses coupled with prior announcement of price increases makes this belief almost mandatory for Ethyl Corporation and Du Pont. Facilitating practices, however, are not necessary for this belief. The long history of the steel industry's lock step following of U.S. Steel's pricing decisions illustrates that the driving force here is the belief that matching will occur, not the particular mechanism by which that belief came about or is maintained. Finally, as the example of the steel industry also illustrates, similarity of profit functions is important. The oligopolistic coordination that this paper identifies depends critically on the existence among the firms of a common interest with respect to price.

Appendix: Proof of Theorem 2

The proof depends on a sequence of lemmas.

Lemma 1. If $R = \times_{i \in N} R_i$ is a maximal sequentially dominant reduction of the kinked demand game, then every R_i is finite.

Proof. Suppose to the contrary that, for some i, R_i is infinite. Then one of the monotonicity intervals of f_i contains at least two distinct points r_i and r_i' of R_i. But then it is easy to see that either $r_i \; \mathrm{dom}_R \; r_i'$ or $r_i' \; \mathrm{dom}_R \; r_i$ (according to whether $f_i(r_i) \geq f_i(r_i')$ or conversely) and thus the reduction R is not maximal.‖

Lemma 2. Every maximal sequentially dominant reduction R of the kinked demand game is a singleton.

Proof. We already know that R is finite. Assume to the contrary that R is not a singleton. Define m to be the largest feasible price relative to R, that is,

$m = \max\{r: [r,\infty) \cap R_i \neq \emptyset \text{ for all } i \in N\}$.

This definition directly implies, first, $|[m,\infty) \cap R_i| \geq 1$ for all $i \in N$ and, second, $m \in R_j$ for some $j \in N$. Furtermore, it is easily shown by contradiction that the maximality of the reduction R implies that $|[m,\infty) \cap R_i| = 1$ for every $i \in N$.

Let m^- be defined by

$$m^- = \max\{r: r < m \text{ and } r \in R_k \text{ for some } k \in N\}.$$

The price m^- must exist because otherwise R would be a singleton. Now, for a firm k such that $m^- \in R_k$, observe that for $r_k \in R_k \cap [m,\infty)$ either $m^- \operatorname{dom}_R r_k$ or vice versa depending upon whether $f_k(m^-) \geq f_k(m)$ or vice versa. In either case we get a contradiction to the fact that R is a maximal reduction.‖

Lemma 3. Sequentially dominant reductions have a no regret property: If R is a sequentially dominant reduction, if $r_{-i} \in R_{-i}$, and if $s_i \in S_i$, then an $r_i \in R_i$ exists such that $u_i(r_{-i}: r_i) \geq u_i(r_{-i}: s_i)$.

Lemma 3 is easily proven by induction on the steps that reduce S to R. It immediately implies:

Lemma 4. Every sequentially dominant strategy is a Nash equilibrium.

The next lemma completes the proof of the theorem's first sentence.

Lemma 5. Every sequentially dominant strategy s of the kinked demand game induces a price greater than or equal to any strictly self-enforcing price m.

Proof. Let $S = R^1, R^2, \ldots, R^j, \ldots R^k = \{s\}$ be the sequence of dominant reductions leading to s. Let $K = \{1,2,..,k\}$. We show by induction on j that, for every $j \in K$ and every $i \in N$, $R_i^j \cap [m,\infty) \neq \varnothing$. The statement of the lemma is then the case where $j = k$. For $j = 1$ the claim is obviously true because $m \in R_i^1 \equiv S_i$ for every i. Suppose, contrary to the induction step, that the claim holds for some $j \in K\backslash\{k\}$, but does not hold for j+1. Thus $R_i^j \cap [m,\infty) \neq \varnothing$ for every $i \in N$ and $R_\ell^{j+1} \cap [m,\infty) = \varnothing$ for some $\ell \in N$. Therefore a strategy tuple $r \in \times_{i\in N}(R_i^j \cap [m,\infty))$ exists.

The no regret property implies that a strategy tuple $r' \in R^j$ with $u_t(r') \geq f_t(m)$ exists. This means r' is also in $\times_{i\in N}(R_i^j \cap [m,\infty))$ because m is strictly self-enforcing and the game is the kinked demand game. The self-enforcing property of m further implies that no $w_\ell \in S_k \cap [0,m)$ exists such that $w_\ell \operatorname{dom}_T r'_\ell$ where $T = R^j$. This contradicts the assumption $R_\ell^{j+1} \cap [m,\infty) = \varnothing$.‖

Lemma 6. Let R be a sequentially dominant reduction of the kinked demand game satisfying $R_1 = R_2 = \ldots = R_n = \{a_1, a_2, \ldots, a_k\}$ where $a_1 < a_2 \ldots < a_k$. Then a sequentially dominant reduction T exists with $T_1 = T_2 = \ldots = T_n = \{a_q\}$ satisfying $q \in \{1,\ldots,k\}$ and, for $h \in \{1,\ldots,q-1\}$, $f(a_q) > f(a_h)$.

Proof. For any $\ell \in \{1,\ldots,k\}$ define W, W', and W": (1) $W = \{a_1, a_2, \ldots, a_\ell\}$; (2) $W' \subset W$ such that, for all $i \in N$, $W'_i = \{a_1, a_2, \ldots, a_{\ell-1}\}$; and (3) $W'' \subset W$ such that, for all $i \in N$, $W''_i = \{a_1, a_2, \ldots, a_{\ell-2}, a_\ell\}$.

Observe that W can be reduced to either W' or W". If, for some $j \in N$, $f_j(a_{\ell-1}) \geq f_j(a_\ell)$, then W may be reduced to W' in two stages. First, remove a_ℓ from W_j because $a_{\ell-1}$ dominates a_ℓ for firm j. Second, remove a_ℓ from the other n-1 components of W because, with a_ℓ removed from W_j, $a_{\ell-1}$ dominates a_ℓ for all firms $i \in N\backslash\{j\}$. This produces W'. Otherwise if, for all $i \in N$, $f(a_{\ell-1}) < f(a_\ell)$, then W can be reduced to W" immediately because, for every firm a_ℓ dominates $a_{\ell-1}$.

Application of the observation repeatedly allows reduction of R down to T.‖

Proof of Theorem 2. Lemma 5 completes the proof of the first part of Theorem 2. To prove the second part we assume that p is a self-enforcing price greater than or equal to any strictly self-enforcing price and proceed to show that $\times_{i \in N}\{p\}$ is a sequentially dominant reduction.

Each f_i is multipeaked. Therefore a finite sequence of prices $0 = r_1 < r_2 < \ldots < r_k$ exist such that: (1) for every $i \in N$ and every $j \in \{1,2,\ldots,k-1\}$, f_i is weakly monotonic on $[r_j, r_{j+1}]$; (2) for every $i \in N$, f_i is monotonically nonincreasing on $[r_k,\infty)$; and (3) for some $\ell \in \{1,2,\ldots,k\}$, $p = r_\ell$.

The first dominant reduction of S is R^1 where, for all $i \in N$, $R_i^1 = \{r_1,r_2,\ldots,r_k\}$. Next observe that, if $\ell > 1$, a dominant reduction of R^1 is R^2 where, for all $i \in N$, $R_i^2 = \{r_1,r_2,\ldots,r_{\ell-2}, r_\ell, r_{\ell+1},\ldots,r_k\}$. Continuing in this fashion produces the sequentially dominant reduction R^ℓ where, for all $i \in N$, $R_i^\ell = \{r_\ell,r_{\ell+1},\ldots,r_k\}$. Application of Lemma 6 to R^ℓ leads to two mutually exclusive cases:

1. $\times_{i\in N}\{r_\ell\}$ is a sequentially dominant strategy tuple as required in the statement of the theorem, or
2. an r_q with $q > \ell$ exists such that $f(r_q) > f(r_h)$ for all $h \in \{1,2,\ldots,\ell,\ldots,q-1\}$.

Notice that the second case implies that r_q is a strictly self-enforcing price with $r_q > r_\ell \equiv p$. This contradicts our assumption that p is a self-enforcing price greater than or equal to any strictly self-enforcing price. Therefore the second case is impossible and the first case must be true.‖

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