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# EXISTENCE OF EQUILIBRIA IN ECONOMIES WITH SUBSISTENCE REQUIREMENTS AND INFINITELY MANY COMMODITIES

by

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## ABSTRACT

This paper proves the existence of equilibria in economies with $\mathscr{L}_\infty$ commodity spaces when consumption sets may not include the origin. The results apply to deterministic economies with a finite numbr of infinitely-lived agents and to models of commodity differentiation. An example is also given of an intertemporal economy with uncertainty in which no equilibrium exists.

## 1. Introduction

This paper is concerned with the existence of equilibria in the model of an economy with infinitely many commodities formulated by Bewley [2]. The main result in [2] requires the consumption sets to include the origin, a significant departure from the usual treatment of an economy with a finite number of commodities. It will be shown here that this assumption can be relaxed to some extent; in particular a fairly satisfactory result is obtained for the deterministic economy with a finite number of infinitely-lived agents. Existence is also proven under an assumption which seems reasonable for models of commodity differentiation.

We begin with an example of a dynamic stochastic economy in which there is no equilibrium. There is no private information in the economy. Existence fails simply because there is a continuum of possible states and consumers require at least a certain level of consumption in each state. The technology is such that if this level is provided with probability one, then there will be zero probability of the constraint binding. Thus even if the bare minimum alone is provided, the essential good may have a low expected marginal value.

If the expected utility hypothesis is taken as applicable to all gambles, even to those involving a risk of death (as suggested, e.g., by Arrow [1, p. 23]), then there should be no subsistence requirements in stochastic economies. Hence the example of nonexistence is not troublesome. However we hope it illustrates the general problem. Several details of the example are supplied in the appendix.

Let there be one consumer, one firm, two time periods ($i = 1,2$) and two goods ($E$ = energy and $F$ = food). The consumer has the von Neumann-Morgenstern utility function

$$u(E_1,F_1,E_2,F_2) = (E_1 - 1)^{1/2}(F_1 - 1)^{1/2} + 1/3(E_2 - 1)^{1/2}(F_2 - 1)^{1/2}$$

over the set of outcomes $(E_1,F_1,E_2,F_2)$ satisfying the subsistence requirements: $E_1 \geq 1$, $F_1 \geq 1$, $E_2 \geq 1$, $F_2 \geq 1$. He is endowed with 4 units of food in each period but no energy. The firm chooses a net output plan $(\hat{E}_1,\hat{F}_1,\hat{E}_2(\cdot),\hat{F}_2(\cdot))$ subject to

$$\hat{E}_1 + \hat{F}_1 \leq 0 \text{ and } \hat{E}_1 + \hat{F}_1 + \hat{E}_2(\theta) + \theta\hat{F}_2(\theta) \leq 0 \quad \forall\theta$$

where $\theta$ is uniformly distributed on $[0,1]$. The interpretation is that the firm can store energy costlessly and can also trade food and energy on the world market within each period, the current food price of energy being 1 and the future price being $1/\theta$.

Assuming complete markets, the firm maximizes its expected profit:

$$p_1\hat{E}_1 + q_1\hat{F}_1 + \int_0^1 p_2(\theta)\hat{E}_2(\theta)d\theta + \int_0^1 q_2(\theta)\hat{F}_2(\theta)d\theta$$

The consumer maximizes his expected utility subject to satisfying the subsistence requirements (a.s.) and the budget constraint:

$$p_1E_1 + q_1(F_1 - 4) + \int_0^1 p_2(\theta)E_2(\theta)d\theta + \int_0^1 q_2(\theta)(F_2(\theta) - 4)d\theta \leq 0$$

There is no price system $(p_1,q_1,p_2(\cdot),q_2(\cdot))$ in this economy at which markets would clear. It is essential that at least one unit of energy be stored; yet competitive markets cannot induce the firm to store it. In fact there is a unique social optimum (which involves storing exactly one unit) and it cannot be realized as a competitive equilibrium.

The nonexistence can be credited to our requirement that the values of contracts be expected values. If we allow the energy supply to be valued as $\int_0^1 E_2(\theta)d\pi$, where $\pi$ is any finitely additive measure on [0,1], then the economy will have an equilibrium; in fact the existence of an equilibrium of this general type is shown by Bewley [2, Theorem 1] (which also implies that there would be an equilibrium if the state space were finite). The use of such a functional essentially allows us to assign a very high value to guaranteeing to provide energy during a crisis, i.e., when $\theta$ is small (the value of a contract would still be independent of the amount supplied when $\theta$ = 0, since that has zero probability of occurrence). If one were truly modeling a futures market there would be little reason to exclude such functionals; however the interpretation as a sequence economy with securities is possible only when they are disallowed.

The nonexistence could also be attributed to the subsistence requirements. Indeed all of Bewley's assumptions in [2, Theorem 3] (Mackey continuity of preferences, the Exclusion Assumption,etc.) are satisfied except that concerning the structure of the consumption sets. Thus, were it not for the subsistence requirements, the existence of an equilibrium of the type described in the preceding paragraph would imply the existence of an equilibrium of the type desired. Subsistence requirements can be problematic in economies with only finitely many commodities, but here the difficulty is more fundamental. It is not just Arrow's exceptional case which must be dealt with; the failure in the example is in the central part of the second welfare theorem — the optimum cannot be supported as a compensated equilibrium.

Finally, the nonexistence could be credited to the nature of the information constraints on the agents' decisions -- specifically to the fact that the storage decision must be made in the first period whereas demands are not determined until the second. Similar disparities in agents' information occur in team theory, and indeed examples resembling ours have already been given in that context; cf. Welch [11, Example 4.1]. Note that the existence problem, while inherent in team theory, does not arise in dynamic stochastic economies unless subsistence requirements are present (see Bewley [3, Section 8] for an analysis of the dynamic stochastic economy).

## 2. Results

We refer to Bewley [2] for a full statement of the model. The commodity space is $\mathscr{L}_\infty(M,\mathscr{M},\mu)$ and the economy is a list

$$\mathscr{E} = (Y_j, X_i, \succsim_i, \omega_i, \theta_{ij}, i = 1,\ldots,I, j = 1,\ldots,J).$$

Bewley gives conditions [2, Theorem 1] under which the economy will have an equilibrium $(\mathbf{x},\mathbf{y},\pi)$ with $\pi \in \underline{ba}$ and $\pi > 0$. The consumption sets are assumed to be convex, Mackey closed, contained in $\mathscr{L}_\infty^+$ and to satisfy a monotonicity assumption. As an intermediate step in the existence proof, Bewley demostrates [2, p. 523] that no consumer is at a minimum wealth point at $(\mathbf{x},\mathbf{y},\pi)$, i.e., that $\pi\cdot\mathbf{x}_i > \inf\{\pi\cdot x | x \in X_i\}$, $\forall i$.

The absence of subsistence requirements is important only for deducing the existence of an equilibrium $(\mathbf{x},\mathbf{y},p)$ with $p \in \mathscr{L}_1$, from the existence of the equilibrium $(\mathbf{x},\mathbf{y},\pi)$. Accordingly we will simply assume:

(i) $\mathscr{E}$ has an equilibrium $(\mathbf{x},\mathbf{y},\pi)$ with $\pi \in \underline{ba}$, $\pi > 0$, and $\pi\cdot\mathbf{x}_1 > \inf\{\pi\cdot x | x \in X_i\}$, $\forall i$.

We also adopt the following as general assumptions:

(ii) $\forall i$, $X_i$ is convex and contained in $\mathscr{L}_\infty^+$.

(iii) $\forall i$, and $\forall x \in X_i$, $\{z \in X_i | z \succ x\}$ is relatively Mackey open in $X_i$.

(iv) $\forall i$, and $\forall x \in X_i$, $x \in$ w-*cl$\{z \in X_i | z \succ_i x\}$.

(v) $\quad \forall i,\ \omega_i \in \mathscr{L}^+_\infty$.

The lower-semicontinuity assumption (iii) is discussed by Bewley [2] (see also Brown and Lewis [4]). The local nonsatiation assumption (iv) is very weak; in particular it is implied by Bewley's monotonicity condition.

These assumptions are satisfied by the example in the introduction. To ensure the existence of an equilibrium with a price system in $\mathscr{L}_1$, we will impose additional conditions on the structure of the consumption and production sets. The conditions on these two types of sets will be symmetric. Imposing these has the effect of ruling out the simultaneous occurrence of subsistence requirements and an accumulation of information during the making of production decisions. In nonstochastic models, however, these allow for fairly general subsistence requirements, as we discuss below.

Let Z be a given subset of $\mathscr{L}_\infty$. We consider respectively a relaxed version of Bewley's Exclusion Assumption [2] and a similar version of Majumdar's mixture property [8].

PROPERTY E. For each $\pi \in$ <u>ba</u> and $z \in Z$ there exist sequences $F_n \in \mathscr{M}$ and $z_n \in \mathscr{L}_\infty$ such that $\lim_n \pi_c(F_n) = 0$, $\pi_p(M\backslash F_n) = 0$, $\forall n$, $\lim_n \pi_p \cdot z_n = 0$, $\lim_n z_n = 0$ in $\tau(\mathscr{L}_\infty, \mathscr{L}_1)$, and $z\chi_{M\backslash F_n} + z_n \in Z$, $\forall n$.

THEOREM 1. Suppose each $X_i$ and each $Y_j$ has property E. Then $\mathscr{E}$ has an equilibrium $(\mathbf{x},\mathbf{y},p)$ with $p \in \mathscr{L}_1$.

In a model of differentiated commodities, condition E requires that, if one were to consider a sufficiently small subset $F_n$ of the commodity list, consumers could survive using only small amounts $z_n(m)$ of those goods by substituting small amounts $z_n(m)$ of goods of other types. The interpretation for producers is similar.

PROPERTY M. For each $\pi \in \underline{ba}$ and $z, z' \in Z$, there exist sequences $F_n \in \mathscr{M}$ and $z_n \in \mathscr{L}'_\infty$ such that $\lim_n \pi_c(F_n) = 0$, $\pi_p(M \backslash F_n) = 0$, $\forall n$, $\lim_n \pi_p \cdot z_n = 0$, $\lim_n z_n = 0$ in $\tau(\mathscr{L}_\infty, \mathscr{L}_1)$, and $z\chi_{M\backslash F_n} + z'\chi_{F_n} + z_n \in Z$, $\forall n$.

THEOREM 2. Suppose each $X_i$ and each $Y_j$ has property M. Assume further that $0 \in Y_j$, $\forall j$, and that $(\{\omega_i\} + A(\sum_j Y_j)) \cap X_i \neq 0$, $\forall i$. Then $\mathscr{E}$ has an equilibrium $(\mathbf{x}, \mathbf{y}, p)$ with $p \in \mathscr{L}_1$.

In the last part of the hypothesis of Theorem 2, the symbol $A(\sum_j Y_j)$ denotes the largest convex cone at 0 contained in $\sum_j Y_j$. The assumption is a weak form of the Adequacy Assumption used by Bewley to obtain the equilibrium $(\mathbf{x}, \mathbf{y}, \pi)$ [2, Theorem 1].

Theorem 2 applies to an economy with a finite number of infinitely-lived agents and a deterministic environment. The commodity space would be identifiable with $\ell^k_\infty$, the bounded sequences in $\mathbb{R}^k$, and a production set $Y_j$ would have the form

$$Y_j = \{b - a \mid a, b \in \ell^k_\infty, b_1 = 0, (a_t, b_{t+1}) \in P_{jt}, \forall t\}$$

for given sets $P_{jt} \subset \mathbb{R}^{2k}$. Let $\pi \in \underline{ba}\ (\mathbb{N} \times \{1, \ldots, k\})$ and $y, y' \in Y_j$. Set $F_n = \{n, n+1, \ldots\} \times \{1, \ldots, k\}$. We have $\lim_n \pi_c(F_n) = 0$ and $\pi_p(M \backslash F_n) = 0$, $\forall n$. Define $y_n \in \ell^k_\infty$ as $y_{nt} = 0$, $\forall t \neq n$, and $y_{nn} = b_n - b'_n$. Then $\pi_p \cdot y_n = 0$, $\forall n$, and $y\chi_{M\backslash F_n} + y'\chi_{F_n} + y_n \in Y_j$, $\forall n$. Finally $\lim_n y_n = 0$ in $\tau(\mathscr{L}_\infty, \mathscr{L}_1)$ since this topology coincides with the product topology on norm bounded subsets of $\ell^k_\infty$ (Kelley-Namioka [7, p. 173], Dunford-Schwartz [6, p. 294]).

This use of the mixture property is similar to that of Radner [10, Theorem 4.1]. A similar , but simpler, argument shows that consumption sets would also have property M in this model, regardless of subsistence requirements.

Theorem 2 would also apply to a model in which there were markets for contingent claims, with no real activity occuring before the resolution of uncertainty. If producers had to choose inputs before the uncertainty was resolved, as in the example in the introduction, then the production sets would not have property M. The production sets would satisfy property E in either case, but property E would not hold for the consumption sets unless there were no subsistence requirements. If there are no subsistence requirements, then Bewley's result applies with the same generality as Theorem 1 above.

In both proofs we first show that $(\mathbf{x},\mathbf{y},\pi_c)$ is a compensated equilibrium, $(\mathbf{x},\mathbf{y},\pi)$ being the equilibrium assumed to exist and $\pi_c$ being the countably additive measure in the Yosida-Hewitt decomposition of $\pi$. For Theorem 1, this argument is essentially the same as that of Prescott-Lucas [9, Theorem 1]. The argument for Theorem 2 follows Majumdar [8, Theorem 4].

PROOF OF THEOREM 1. Choose, for any i, any bundle x such that $x \succ_i \mathbf{x}_i$. Set $x'_n = x\chi_{M\setminus F_n} + x_n$ where $F_n$ and $x_n$ are as given by property E. Since $\lim_n x'_n = x$ in $\tau(\mathscr{L}_\infty, \mathscr{L}_1)$ -- see Bewley [2, p. 534] -- it must be that $x'_n \succ_i \mathbf{x}_i$ for n sufficiently large. Thus $\pi\cdot x'_n > \pi\cdot\mathbf{x}_i$. But $\pi\cdot x'_n = \pi_c\cdot x\chi_{M\setminus F_n} + \pi\cdot x_n \to \pi_c\cdot x$. Hence $\pi_c\cdot x \geq \pi\cdot\mathbf{x}_i$, which in view of the local nonsatiation assumption, implies in fact that

$$\inf\{\pi_c\cdot x \mid x \succsim_i \mathbf{x}_i\} \geq \pi\cdot\mathbf{x}_i \quad (1)$$

Summing over i and noting that $\pi \geq \pi_c$, we may write

$$\pi_c \cdot \sum_i (\mathbf{x}_i - \omega_i) \geq \sum_i \inf\{\pi_c \cdot x | x \succsim_i \mathbf{x}_i\} - \pi_c \cdot \sum_i \omega_i \geq \pi \cdot \sum_i (\mathbf{x}_i - \omega_i) \tag{2}$$

Using property E for the sets $Y_j$ in the same way, we deduce that

$$\pi_c \cdot \sum \mathbf{y}_j \leq \sum \sup\{\pi_c \cdot y | y \in Y_j\} \leq \pi \cdot \sum \mathbf{y}_j \tag{3}$$

Furthermore the right-hand sides of (2) and (3) must be equal and the left-hand sides as well, since $(\mathbf{x},\mathbf{y})$ is an attainable allocation. Thus each $\mathbf{y}_j$ is profit maximizing and each $\mathbf{x}_i$ is expenditure minimizing on the consumer's upper-contour set.

Equality in (2) and the relations $\pi_c \cdot \mathbf{x}_i \geq \pi \cdot \mathbf{x}_i$ (from (1)) and $\pi_c \cdot \omega_i \leq \pi \cdot \omega_i$ (from $\pi_c \leq \pi$) imply that $\pi_c \cdot \mathbf{x}_i = \pi \cdot \mathbf{x}_i > \inf\{\pi \cdot x | x \in X_i\}$, $\forall i$. Since $X_i \subset \mathscr{L}_\infty^+$ and $\pi > 0$ it follows that the $\mathbf{x}_i$ are not minimum wealth points at the price system $\pi_c$. Since we have lower-semicontinuous preferences on convex consumption sets, we can deduce in the usual way (Debreu [5, p. 591]) that the $\mathbf{x}_i$ are utility maximizing on the sets $\{x \in X_i | \pi_c \cdot x \leq \pi_c \cdot \mathbf{x}_i\}$. It is now straightforward to derive the relations $\pi_c \cdot \mathbf{x}_i = \pi_c \cdot (\omega_i + \sum \theta_{ij} \mathbf{y}_j)$, $\forall i$, from the various equalities already established, and this completes the argument.

PROOF OF THEOREM 2. Choose, for any i, any $x \in X_i$ satisfying $x \succ_i \mathbf{x}_i$. Set $x'_n = x\chi_{M\setminus F_n} + \mathbf{x}_i \chi_{F_n} + x_n$, where $F_n$ and $x_n$ are as given by property M. For sufficiently large n we must have $x'_n \succ \mathbf{x}_i$, since $\lim_n x'_n = x$ in $\tau(\mathscr{L}_\infty, \mathscr{L}_1)$. Hence $\pi \cdot x'_n > \pi \cdot \mathbf{x}_i$. But $\pi \cdot x'_n = \pi_c \cdot (x\chi_{M\setminus F_n} + \mathbf{x}_i \chi_{F_n}) + \pi_p \cdot \mathbf{x}_i + \pi \cdot x_n$, so $\lim_n \pi \cdot x'_n = \pi_c \cdot x + \pi_p \cdot \mathbf{x}_i$. Therefore $\pi_c \cdot x \geq \pi_c \cdot \mathbf{x}_i$. Using the local nonsatiation assumption we obtain

$$\pi_c \cdot \mathbf{x}_i = \inf\{\pi_c \cdot x \mid x \succsim_i \mathbf{x}_i\}. \tag{4}$$

Now choose an arbitrary $x \in X_i$ and $\varepsilon > 0$. Let $x' \in X_i$ be such that $x' \succ_i \mathbf{x}_i$ and $\pi_c \cdot x' \leq \varepsilon + \pi_c \cdot \mathbf{x}_i$. Set $x'_n = x'\chi_{M\setminus F_n} + x\chi_{F_n} + x_n$ where $F_n$ and $x_n$ are as given by property M. Since $\lim_n x'_n = x'$ in $\tau(\mathscr{L}_\infty, \mathscr{L}_1)$, we have $x'_n \succ_i \mathbf{x}_i$ for n sufficiently large. Therefore $\pi \cdot x'_n > \pi \cdot \mathbf{x}_i$ for such n. But $\pi \cdot x'_n = \pi_c \cdot (x'\chi_{M\setminus F_n} + x\chi_{F_n}) + \pi_p \cdot x + \pi \cdot \mathbf{x}_n$, which converges to $\pi_c \cdot x' + \pi_p \cdot x$. Hence it must be that $\pi_c \cdot x' + \pi_p \cdot x \geq \pi \cdot \mathbf{x}_i$. Since $\varepsilon$ and x were chosen arbitrarily this implies that

$$\inf\{\pi_p \cdot x \mid x \in X_i\} = \pi_p \cdot \mathbf{x}_i. \tag{5}$$

We also obtain from Property M that

$$\pi_c \cdot \mathbf{y}_j \geq \pi_c \cdot y, \ \forall y \in Y_j \text{ and} \tag{6}$$

$$\pi_p \cdot \mathbf{y}_j \geq \pi_p \cdot y, \ \forall y \in Y_j \tag{7}$$

by passing to the limit in the assumed inequality $\pi \cdot \mathbf{y}_j \geq \pi \cdot y'_n$, setting $y'_n = y\chi_{M\setminus F_n} + \mathbf{y}_j\chi_{F_n} + y_n$ in the case of (6) and $y'_n = \mathbf{y}_j\chi_{M\setminus F_n} + y\chi_{M\setminus F_n} + y_n$ in the case of (7).

From (4) and (6) we see that $(\mathbf{x}, \mathbf{y}, \pi_c)$ is a compensated equilibrium. Furthermore from (i) and (5) we obtain that $\pi_c \cdot \mathbf{x}_i > \inf\{\pi_c \cdot x \mid x \in X_i\}$, $\forall i$. In view of the convexity of the $X_i$ and the lower-semicontinuity of preferences, this implies (Debreu [5]) that the $\mathbf{x}_i$ are $\succsim_i$-maximal in $\{x \in X_i \mid \pi_c \cdot x \leq \pi_c \cdot \mathbf{x}_i\}$.

It remains only to show that $\pi_c \cdot \mathbf{x}_i = \pi_c \cdot (\omega_i + \sum_j \theta_{ij} \mathbf{y}_j)$ for each i. Since

$(\mathbf{x},\mathbf{y})$ is an attainable allocation this will follow from showing that $\pi_c \cdot \mathbf{x}_i \geq \pi_c \cdot (\omega_i + \sum_j \theta_{ij} \mathbf{y}_j)$, $\forall i$, and since budgets are balanced at $(\mathbf{x},\mathbf{y},\pi)$ this in turn will follow upon showing that $\pi_p \cdot \mathbf{x}_i \leq \pi_p \cdot (\omega_i + \sum_j \theta_{ij} \mathbf{y}_j)$, $\forall i$. From (7) we have that $\pi_p \cdot y \leq 0$, $\forall y \in \mathbf{A}(\sum Y_j)$. Combined with the adequacy assumption and (5) this yields $\pi_p \cdot \mathbf{x}_i \leq \pi_p \cdot \omega_i$. Since it also follows from (7) that $\pi_p \cdot \mathbf{y}_j \geq 0$, $\forall j$, this completes the proof.

# APPENDIX

We will show that in the example in the introduction there is no equilibrium with a price system in $\mathscr{L}_1$ but there is an equilibrium when we admit valuation functionals in ba. Taking the commodity space to be $\mathbb{R}^2 \times \mathscr{L}_\infty([0,1] \times \{1,2\})$ with dual $\mathbb{R}^2 \times \mathscr{L}_1([0,1] \times \{1,2\})$, the assumptions of Debreu [5, Theorem 1] are satisfied, so any equilibrium must be Pareto optimal. We will show that there is a unique Pareto optimum, and that it cannot be supported by a price system in $\mathscr{L}_1$.

Let S denote the amount of energy stored, $(4 - E_1 - F_1)$. For feasibility, we must have $1 \leq S \leq 2$. Given such an S, it is optimal to choose $E_1 = \frac{1}{2}(4-S)$, $F_1 = \frac{1}{2}(4-S)$, $E_2 = \frac{1}{2}(S + 1 + 3\theta)$ and $F_2 = \frac{1}{2\theta}(S - 1 + 5\theta)$. This gives a total expected utility of $5/3 - S/6$ which is maximized, subject to the feasibility constraint, at $S = 1$. The optimal consumption levels are therefore $E_1 = 3/2$, $F_1 = 3/2$, $E_2 = 1 + \frac{3\theta}{2}$, $F_2 = 5/2$.

In order for the import-export division of the firm to be maximizing profits we must have $p_1 = q_1$ and $p_2(\theta) = \frac{1}{\theta}q_2(\theta)$ a.s. Take $p_1 = q_1 = 1$ and let $\frac{1}{\theta}q_2(\theta) = p_2(\theta) \equiv p(\theta)$. In order to induce the consumer to purchase the optimal bundle we must have, since the subsistence requirements would be slack a.s., $p(\theta) = \frac{1}{3}\theta^{-1/2}$ a.s. But then $\int_0^1 p(\theta) = 2/3 < 1$, so the firm would not store the unit of energy.

According to Yosida-Hewitt [13, Theorem 4.1] there is a (purely) finitely additive measure $\pi_p \in \underline{ba}([0,1])$ satisfying $\pi_p([0,1]) = \pi_p([0,\varepsilon]) = 1/3$ for every $\varepsilon > 0$. If we assign the value

$$E_1 + F_1 + \frac{1}{3}\int E_2(\theta)\theta^{-1/2}d\theta + \int E_2(\theta)d\pi + \frac{1}{3}\int F_2(\theta)\theta^{1/2}d\theta$$

to any contract $(E_1,F_1,E_2(\cdot),F_2(\cdot))$ then markets will clear, the resulting allocation being the social optimum. To see that the consumer would purchase that bundle, note that

$$\int(1 + \frac{3\theta}{2})d\pi_p = 1/3 + \int\frac{3\theta}{2}d\pi_p = 1/3$$

which is the minimum cost attainable on the consumption set. The introduction of $\pi_p$ therefore could not overturn the supporting property established in the preceding paragraph. On the production side, notice that the firm could make no profit through $\pi_p$ by trading in the second period, so the decisions of the import-export division would be unaffected by its introduction. Finally under the above valuation functional the profit from storing energy would be zero.

REFERENCES

1. K.J. ARROW, Exposition of the theory of choice under uncertainty, in "Decision and Organization," C.B. McGuire and R. Radner, eds., North Holland, Amsterdam, 1972.

2. T. BEWLEY, Existence of equilibria in economies with infinitely many commodities, J. Econ. Theory 4 (1972), 514-540.

3. T. BEWLEY, Stationary equilibrium, J. Econ. Theory 24 (1981), 265-295.

4. D.J. BROWN and L.M. LEWIS, Myopic economic agents, Econometrica 49 (1981), 359-368.

5. G. DEBREU, Valuation equilibrium and Pareto optimum, Proc. Nat. Acad. Sci. U.S.A. 40 (1954), 588-592.

6. N. DUNFORD and J.T. SCHWARTZ, "Linear Operators" (Part I), Interscience, New York, 1957.

7. J.L. KELLEY and I. NAMIOKA, "Linear Topological Spaces," Springer-Verlag, New York, 1976.

8. M. MAJUMDAR, Some general theorems on efficiency prices with an infinite dimensional commodity space, J. Econ. Theory 5 (1972), 1-13.

9. E.C. PRESCOTT and R.E. LUCAS, JR., A note on price systems in infinite dimensional spaces, Int. Econ. Rev. 13 (1972), 416-422.

10. R. RADNER, Efficiency prices for infinite horizon production programs, Rev. Econ. Stud. 34 (1967), 51-66.

11. R.L. WELCH, The representation of shadow values in resource allocation teams, J. Econ. Theory 20 (1979), 23-40.

12. K. YOSIDA and E. HEWITT, Finitely additive measures, Trans. Amer. Math. Soc. 72 (1952), 46-66.