

Discussion Paper No. 601  
COMPETITION AND TWO-PART TARIFFS

by

Beth Hayes

March 1984

J. L. Kellogg Graduate School of Management  
Northwestern University  
Evanston, Illinois 60201

I am grateful to V.V. Chari, Mike Riordan, Tom Ross, Joel Sobel, the participants of the EPO-Dept. of Justice Seminar, and the Sloan workshop at Northwestern University for their help.

## I. Introduction

Monopoly power--of some sort--is thought to be a necessary condition for a firm to have the power to price discriminate. For example, a firm prefers to use a two-part tariff (See Oi, 1971) rather than charging a single price for output. However, a competing firm could always charge a single price that consumers would prefer and earn nonnegative profits. Thus, it seems that the existence of competition in a market would result in the elimination of any two-part tariff. However, the existence of two-part tariffs in many markets that are (or appear to be) competitive belies this standard assumption. This paper explains why monopoly power is not required for the existence of a two part tariff. I show circumstances where price discrimination using a two-part tariff is preferred by consumers to a single price, and therefore why this pricing method will be found in competitive markets. Such pricing occurs in environments with uncertainty because two part tariffs act as a form of insurance. Ex ante consumers plan consumption; however, there is a random component to their consumption decisions. Before they choose to consume, the uncertainty is resolved, but the choice of pricing is made before the random component is known. In a decentralized world where individuals make their consumption decisions after the uncertainty is resolved, the two-part tariff is optimal.

Two very common examples of competitive markets with two-part tariff are health clubs and bars. Frequently, health clubs (country clubs, tennis clubs, etc.) charge a yearly fee and a per-use price. Bars often have a cover charge as well as a price per drink. And, in the same market area, health clubs and bars that charge only a single price can be found. Other examples of competitive firms that use two-part tariffs include long-distance phone

services, rent-a-record stores, video-tape rental stores, and all you can eat buffets. To gain some intuition consider the example of a health club. Suppose that the environment is competitive and that the club is charging a single price equal to marginal cost so as to just cover costs. Customers then make consumption decisions based on this price and a variety of other factors. Their consumption decision may also depend on the outcome of a random variable. The specific reason for the ex-ante uncertainty may be from many sources. It is possible that income may be random. Or perhaps the consumption value of health club services is variable. For example, if the consumer wants to go to the club early each morning, some mornings it will be more difficult to rise and therefore result in lower utility. Or the opportunity cost of a health club visit may be variable because of work requirements or other opportunities for pleasure. For whatever reason, if such uncertainty exists, the individual's marginal rate of substitution between health club visits and other goods will be random. And because of the random component in the utility function, the individual's consumption of health club visits (and of other goods) will be random.

Now, suppose the health club offers a two-part tariff instead of the single price, so that the consumer pays a fixed fee at the beginning of the year plus a price per visit. Assume that the market for health clubs is competitive, so that the firm's profits will be driven to zero. Thus, if the fixed fee is positive, the per visit charge will be lower than the marginal cost. The consumer's income (effectively) declines by the amount of the fixed fee and consumption of health club services in each state will rise relative to other goods because the marginal cost has fallen. This paper shows that the consumer's utility decreases in low states and increases in high states because of the income and price changes, with a net positive effect on

utility. The result is that two part tariff is a form of insurance. Because of incomplete markets for insurance consumers cannot accomplish this insurance except through the two-part tariff. I will also discuss the alternative solutions for insuring this type of uncertainty. However, the first best insurance contract is not incentive compatible and the incentive constrained first best insurance contract cannot be achieved with decentralized markets. The two-part tariff is the best solution that satisfies incentive compatibility and allows for decentralization.

Other authors (see Oren et al. [1983], Phillips and Battalio [1983]) have explained the use of optimal price discrimination in markets as a method of separating various consumer types. However, these results do not carry through to competitive markets. The explanation for two-part tariffs in this paper does not depend on differences in consumers, but rather exogenous uncertainty about the future. Berglas (1976) discusses why two-part tariff pricing is not necessary (and why marginal pricing is optimal) in the theory of clubs. This paper shows why two-part tariffs would dominate marginal cost pricing in the club model.

## II. Model

To understand why consumers will prefer a two-part tariff to a constant per-unit price, consider the following model. There are two goods in this world,  $x$  and  $y$ . Let  $x$  be one particular product (i.e., health club services) and let  $y$  be all other goods.

In a certain world, a consumer of good  $x$  has a utility function that depends on  $x$  and all other goods,  $y$ .

$$(2.1) \quad U = U(x,y)$$

Assume that utility is increasing in both arguments so that  $U_x > 0$ ,  $U_y > 0$ , and that utility is strictly concave in both components and that  $U_{xy} > 0$ . Let  $p$  be the price unit of good  $x$ , and normalize the price  $y$  so that  $p_y = 1$ . If a two-part tariff is used for pricing  $x$ , the lump-sum (or entry fee) portion is  $R^1$ . Let  $I$  be income for the individual consumer. The consumer's budget constraint in a certain world is

$$(2.2) \quad px + y + R < I.$$

As argued in the introduction, assume that the consumer's utility function depends on a random variable.<sup>2</sup> Thus, in state  $\theta$ , the utility function is

$$(2.1') \quad U = U(x,y,\theta)$$

Let  $\theta \in [-\infty, \infty]$  be a random variable where the distribution of  $\theta$  is known by both the consumers and the firm. Let  $f(\theta)$  be the probability density function on  $\theta$  with the distribution function  $F(\theta)$ . We will assume that the consumer can choose  $x$  and  $y$  after  $\theta$  is determined.

Before the realization of  $\theta$ , the firm must determine the price  $p$  and the entry fee  $R$  and the consumer must decide whether or not to pay  $R$ . We will assume that firms are risk neutral. If the consumer pays  $R$ , after  $\theta$  is realized he can choose to purchase any amount of  $x$  at price  $p$ . If he elects

---

<sup>1</sup> See Oi (1971) for a discussion of the two-part tariff.

<sup>2</sup> Again, the uncertainty could be due to variable income; we will use randomness in the utility function.

not to pay  $R$ , his consumption of good  $x$  must be zero. Assume that firms which provide the good are risk neutral and have a constant marginal cost,  $c$ , per unit of output.<sup>3</sup> Also assume that the industry is competitive so that no individual firm has market power. In a competitive equilibrium each firm will earn zero profits. As we will see, firms need not be "price-takers" in the usual sense because many  $(p,R)$  combinations may be possible each yielding zero-profits.

If the firm charges a two part tariff the zero profit condition in a certain world implies that

$$R = (c - p) \cdot x$$

where  $x$  is the quantity demanded at price  $p$  by consumers who have paid  $R$ , and therefore have net income  $I - R$ . In an uncertain world, the risk neutral firm will set

$$R = (c - p) E(x)$$

where  $E(x)$  is the expected value of  $x$ , at prices  $p$ . The expectation is with respect to the distribution  $f(\theta)$ . Notice that if the entry fee  $R$  is positive the price  $p$  must be below marginal cost  $c$ . Therefore, the assumption of a competitive market implies that with a two-part tariff, per-unit prices are below marginal cost.

Consumers, then, maximize utility (2.1') subject to (2.2) after the state is realized. Let the partial derivative of utility with respect to the

---

<sup>3</sup>This cost assumption can be generalized.

$j^{\text{th}}$  argument be  $U_j$ . If the consumer pays  $R$ , in state  $i$  he chooses  $(x_i, y_i)$  so that

$$(2.3) \quad \frac{U_x(x_i, y_i, \theta_i)}{U_y(x_i, y_i, \theta_i)} = p$$

and so that the budget constraint is satisfied with equality. Throughout the paper I will assume that the demand for good  $x$  is increasing in  $\theta$ , so  $x_\theta > 0$ . Demand for  $x$  could be decreasing in  $\theta$  or increasing and decreasing. This assumption is made for convenience.

With this model we have:

Proposition 1. If the market for good  $x$  is competitive and if an individual's consumption of  $x$  and  $y$  are state dependent, then the following is true. Before the uncertainty is resolved, consumers strictly prefer a two-part tariff with  $R > 0$  and  $p < c$  to a single price  $p = c$  if the covariance of the marginal utility of income and  $x$  consumption is positive. If there is no uncertainty, or if consumption decisions are state independent, consumers strictly prefer price equal to marginal cost.

Proof. The consumer's expected utility maximization problem is

$$\text{Max}_{p, R} \int_{-\infty}^{\infty} f(\theta) U(x(p, I-R, \theta), I - R - px(p, I-R, \theta), \theta) d\theta$$

(2.4)

$$\text{subject to} \quad R = (c - p) \int_{-\infty}^{\infty} f(\theta)x(p, I-R, \theta)d\theta$$

To simplify notation, let  $x(\theta)$  refer to the utility maximizing  $x$  consumption in state  $\theta$ . Because  $x(\theta)$  is determined from first order conditions for the consumer maximization problem, (2.3) must be satisfied. This is because the consumer cannot (does not) commit himself to consuming  $x(\theta)$  ex ante, but can choose  $(x(\theta), y(\theta))$  after  $\theta$  is realized.

Thus, the maximizing choice of  $P$  solves

$$\begin{aligned}
 \frac{\partial L}{\partial p} &= \int_{-\infty}^{\infty} f(\theta) [U_x(\theta) - pU_y(\theta)] \frac{\partial x(\theta)}{\partial p} d\theta \\
 (2.5) \quad &- \int_{-\infty}^{\infty} f(\theta) U_y(\theta) x(\theta) d\theta \\
 &- \lambda \left( \int_{-\infty}^{\infty} f(\theta)x(\theta)d\theta - (c-p)\int_{-\infty}^{\infty}f(\theta) x_p(\theta) d\theta \right)
 \end{aligned}$$

with

$$-\lambda = \frac{\int_{-\infty}^{\infty} f(\theta)U_y(\theta) d\theta}{(c-p)\int_{-\infty}^{\infty}f(\theta) x_I(\theta) + 1}$$

(from  $\partial R/\partial P$ ),

where  $U_I = \partial U/\partial i$ , and  $x_I(\theta) = \partial x(\theta)/\partial I$  and  $x_p(\theta) = \partial x(\theta)/\partial p$ .

The first term in (2.5) is always equal to zero from (2.3). Consider the solution when price equals marginal cost. With  $p = c$ , the last term in (2.5) is zero, and

$$-\lambda = \int_{-\infty}^{\infty} f(\theta) U_y(\theta) d\theta.$$

Then, at  $p = c$ ,  $\partial L/\partial P \stackrel{>}{<} 0$  as



$$(2.6) \quad \frac{\partial L}{\partial P} = \int_{-\infty}^{\infty} f(\theta) U_y(\theta) [ \int_{-\infty}^{\infty} f(z)x(z)dz - x(\theta) ] d\theta \stackrel{>}{<} 0$$

If  $\partial L/\partial p < 0$ , then expected utility will rise when the two-part tariff has  $R > 0$  and  $p < c$ .

Rewrite (2.6) as

$$(2.7) \quad \begin{aligned} \frac{\partial L}{\partial P} &= E_{\theta} U_y(\theta) E_{\theta} x(\theta) - E_{\theta} U_y(\theta) x(\theta) \\ &= - \text{cov}(U_y(\theta), x(\theta)) \end{aligned}$$

Then, from first order conditions for the individual's utility maximization,  $U_y(\theta) = \lambda(\theta)$ , where  $\lambda(\theta)$  is the marginal utility of income in state  $\theta$ . Thus the necessary and sufficient condition for  $\partial L/\partial P < 0$  is that the covariance between consumption of  $x$  and marginal utility of income be positive.

Note that

$$\frac{dU_y(\theta)}{d\theta} = U_{yx} x_{\theta} + U_{yy} y_{\theta} + U_{y\theta} .$$

If we are considering an example where the consumer is uncertain about his preferences, then  $x$  and  $y$  consumption must vary inversely with the state. So

$$x_{\theta} \stackrel{>}{<} 0 \quad \text{as} \quad y_{\theta} \stackrel{<}{>} 0 .$$

(If income is constant, more  $x$  must mean less  $y$ .)

Therefore,  $U_{xy} x_{\theta} + U_{yy} y_{\theta} \begin{matrix} \geq \\ < \end{matrix} 0$  as  $x_{\theta} \begin{matrix} \geq \\ < \end{matrix} 0$ .

Then a sufficient condition for  $dx/d\theta$  and  $dU_y/d\theta$  to have the same sign is

$$U_{y\theta} = 0 \quad \text{or} \quad U_{y\theta} \begin{matrix} \geq \\ < \end{matrix} 0 \quad \text{as} \quad x_{\theta} \begin{matrix} \geq \\ < \end{matrix} 0.$$

If the utility function is separable in good  $x$  and other goods, and only good  $x$  is directly affected by the random variable, this condition is satisfied.

This means that equation (2.7) is negative and thus

$$\frac{\partial L}{\partial p} < 0 \quad \text{at } p = c \text{ whenever } \text{cov}(U_y(\theta), x(\theta)) > 0.$$

When (2.7) is negative, consumers will ex ante prefer a price lower than marginal cost with a strictly positive entry fee,  $R$  to a single price equal marginal cost. This result holds when the allocation,  $x$ , and the marginal utility for other goods,  $U_y$ , both depend on the state  $\theta$ .

If the covariance of the marginal utility of income and  $x$  is negative, from (2.7),  $\partial L/\partial p > 0$ . Then consumers will strictly prefer a negative entry fee (a "bonus") and a price above marginal cost.<sup>4</sup> Thus, when uncertainty about consumption is due to income uncertainty, if  $x$  is a normal good, consumers strictly prefer a two-part tariff with a negative entry fee and a price greater than marginal cost. If  $x$  is an inferior good, consumers will prefer a standard two-part tariff. Q.E.D.

---

<sup>4</sup>Two examples are book and record clubs that give you free items to join, then charge prices that are above those found in many retail outlets for each additional unit purchased.

Thus the expected gains from the lower price in each state outweigh the losses from a lower net income when the above conditions are satisfied.

Consider a simple example to see this. Let the individual's utility function be

$$U = \theta x^{1/2} + y^{1/2}$$

with budget constraint

$$px + y + R = I.$$

Let there be two possible states, each occurring with probability 1/2, where  $\theta_1 = 1/2$  and  $\theta_2 = 2$ . The individual sets

$$y_1 = \frac{p}{\theta_1} x_1$$

$$x_1 = \frac{I - R}{p + \frac{p}{\theta_1}}$$

where  $x_1$  and  $y_1$  refer to the maximizing choices for state 1. Let marginal cost be  $c = 1$ , so the firm sets

$$R = (c - p) E(x(\theta)) = (1 - p)E(x(\theta))$$

Setting Income  $I = 1$  and solving this problem with  $p = c = 1$  and  $R = 0$  results in an allocation  $x_1 = .2$ ,  $y_1 = .8$ , and utility  $U_1 = 1.118$  in state 1. Then the allocation in state 2 is  $x_2 = .8$ ,  $y_2 = .2$  with utility  $U_2 = 2.236$ . This yields expected utility  $EU_1 = 1.677$ . If price is decreased to  $p = .761$  and an entry fee is set at  $R = .146$  the allocations are  $x_1 = .278$   $y_1 = .643$  in state 1 with  $U_1 = 1.065$ , and  $x_2 = .943$ ,  $y_2 = .137$  with  $U_2 = 2.312$  in state 2. This yields expected utility of  $EU_2 = 1.688 > EU_1$ . See figure 1.

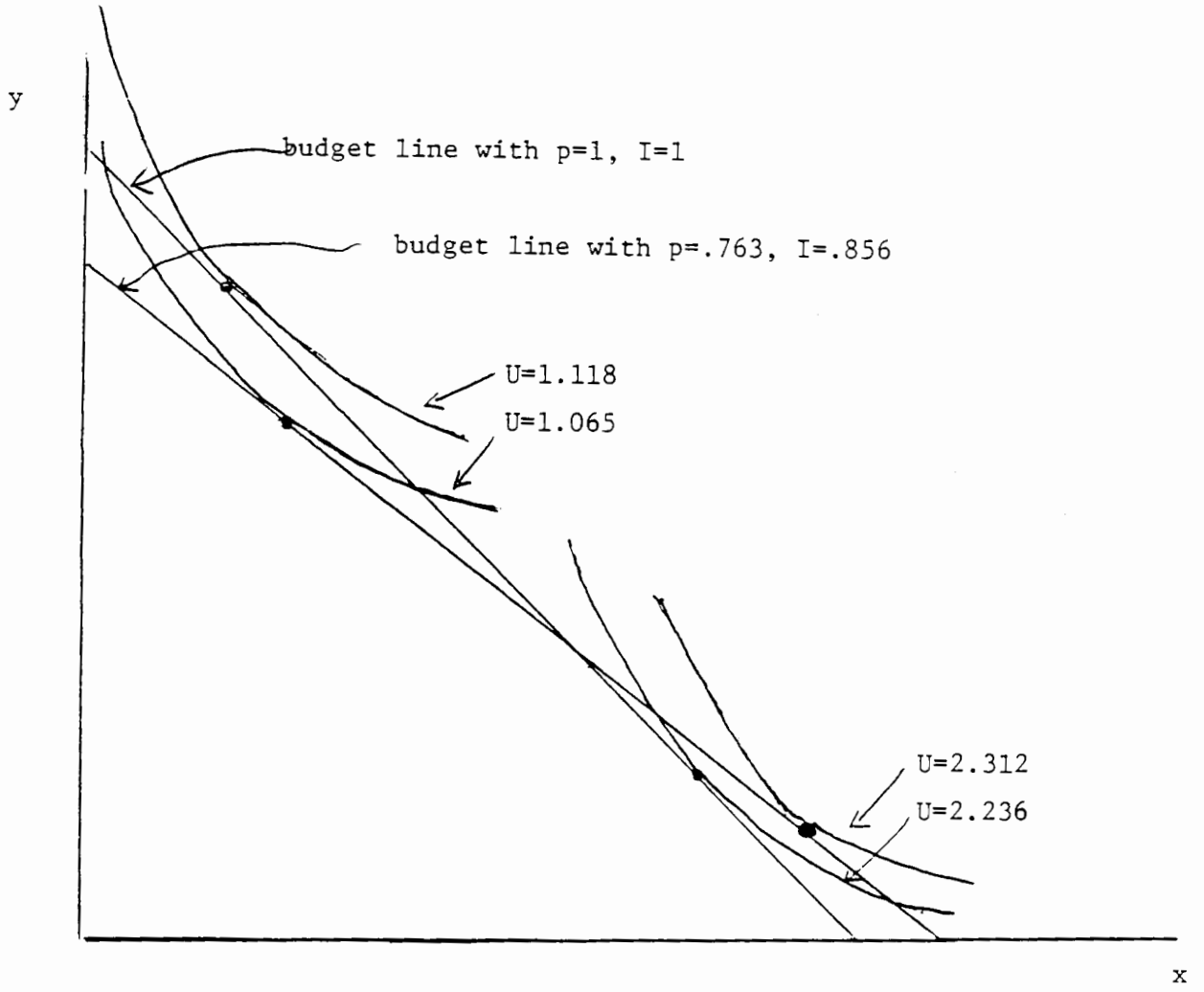


Figure 1

Because the price of good  $x$  decreases the relative price of  $y$  increases when a two-part tariff is used. Combined with the decrease in income, consumption of  $y$  will fall in all states relative to  $y$  consumption in a price equal to marginal cost world. Consumption of good  $x$  must rise in high states because the price of  $x$  is lower and because the two-part tariff budget set lies outside (northeast) of the single price budget set for large  $x$ . Consumption of good  $x$  in low states may rise or fall.

Changing from a single price at marginal cost to an optimal two-part tariff pricing scheme causes utility to fall in low  $\theta$  states and causes utility to rise in high  $\theta$  states. The utility loss in low states is outweighed by gains in high states. This two-part tariff provides insurance because the marginal utility of income is greater in states with higher  $\theta$ . The two-part tariff allows the consumer to transfer income from low to high states.

### III. Alternative Solutions

We have seen that using a two-part tariff can improve a consumer's ex-ante expected utility. Because the consumer "commits" himself before the state is known (by paying the entry fee), but can ex-post choose any allocation, there are not incentive problems with this solution. Also, there is not ex-post discrimination between different types of consumers: each faces the same prices for all good and can choose to consume any amount subject only to the budget constraint.

Using this form of price discrimination is not a first best optimum. The first best solution is the solution to

$$\text{Max}_{x(\theta), y(\theta)} \int_{-\infty}^{\infty} f(\theta) U(x(\theta), y(\theta), \theta) d\theta$$

(3.1) subject to 
$$\int_{-\infty}^{\infty} f(\theta) [c x(\theta) + y(\theta)] d\theta = I$$

Here the only constraint is that, in expected terms, we can afford the allocations. The solution to this problem sets the marginal rate of substitution in each state equal to the ratio of marginal cost. Also, the solution equates the marginal utility of income in each state. Thus, the consumer still bears risk because the utility will be different in each state, but he is able to transfer consumption to more "valuable" (in utility terms) states. One good feature of this solution is that the allocation could be decentralized. If types were ex-post observable, to achieve this solution only income needs to be transferred between types. So, insurance could be provided independently of goods  $x$  and  $y$ , and all trade between  $x$  and  $y$  could occur at marginal cost.

The problem with this allocation is that it is not incentive compatible. This means that if individuals' types are not ex-post observable, all will claim to be the type with the highest  $\theta$ , which is not affordable.

Because of the incentive compatibility problem, the first best solution cannot be implemented. Consider the incentive constrained solution to (3.1). The additional constraints are:

$$(3.2) \quad U(x(\theta), y(\theta), \theta) > U(x(\theta'), y(\theta'), \theta) \quad \text{for all } \theta, \theta'$$

These guarantee that individuals will prefer the  $(x(\theta), y(\theta))$  allocation in state  $\theta$  over all other possibilities.

The solution to this problem will be incentive compatible by design. However, at the optimal allocation, the marginal rate of substitution between  $x(\theta)$  and  $y(\theta)$  will depend on  $\theta$ . This result means that the allocation must be completely controlled by the firm providing insurance. Ex-post, each individual can choose his utility maximizing consumption, but the consumption must be either provided as a bundle or a single price may be charged with the quantities of  $x$  constrained, or different lump-sum entry fees with different marginal prices.

In the example provided, this means providing only two goods,  $x$  and  $y$ . In a world with utility over many goods, providing consumption, it would involve providing consumption bundles composed of many commodities which is impractical if not impossible. As an alternative method for implementing this incentive constrained solution, a firm could charge a single price for  $x$  with quantity constraints. However, this may not be feasible, or it may be very costly to accomplish in a market situation. Quantity constraints require that the firm keep records of the number of visits. This may be difficult because of long time horizons (health club yearly membership), multi-branch stores (i.e. video tape stores) or because general monitoring is costly (keeping track of number of trips to the all-you-can-eat buffet). The alternative of different entry fees and different ex-post prices would not be feasible if goods are fungible ex-post, so that trade between two consumers is possible.

The solution to (3.1) that is incentive compatible as well as decentralizable through pricing is the optimal two-part tariff described in section II. Decentralization means that insurance is provided through one market, and that each individual maximizes ex-post utility subject to the same budget constraint. This allows for some insurance, while allowing the market to clear ex-post as in any competitive equilibrium. No consumer has an

to clear ex-post as in any competitive equilibrium. No consumer has an incentive to choose an alternative firm at price equal to marginal cost because the ex-post price is below marginal cost. There is no incentive compatibility problem because all consumers face the same ex-post prices. And, the problems associated with the incentive constrained solution above (i.e. quantity constraints) are not a problem.

Individuals' ex-ante expected utility is greatest with the first best solution; expected utility declines when incentive compatibility constraints are imposed and it declines further with the optimal two-part tariff. However, as shown in section II, ex-ante expected utility is greater with a two-part tariff than with marginal cost pricing. Table 1 shows the solutions to the example in section II under first best, incentive constrained, two-part tariff and marginal cost pricing solutions.

#### IV. Optimal Two-Part Tariffs with Many Types of Individuals

Consumers in an uncertain world will prefer two-part tariffs. If all consumers were identical with the same distribution over states, in a competitive market for good  $x$ , all firms would offer the two-part tariff  $(R^*, p^*)$  that solves (2.3).

The next question to consider is what set of two-part tariffs would result in an equilibrium with non-identical consumers. Consumers will differ from each other with respect to uncertainty. Suppose that there are many types of consumers, each with the same utility function, but with varying degrees of uncertainty about the stochastic element of their consumption decision. The difference will mean that each consumer has a different probability density function over the random variable  $\theta$ , where more uncertainty will mean a mean-demand preserving spread of the distribution.



TABLE 1

Example

$$U = \theta x^{1/2} + y^{1/2}$$

$$\theta_L = .5$$

$$\theta_H = 2$$

$$\pi_L = .5$$

$$\pi_H = .5$$

$$I = 1$$

$$c = 1$$

<u>Solution</u>	$x_1$	$y_1$	$U_1$	$x_2$	$y_2$	$U_2$	EU
First Best	.08	.32	.707	1.28	.32	2.828	1.767
Incentive Compat.	.14	.561	.936	1.136	.163	2.535	1.735
Two-Part Tariff	.278	.643	1.065	.943	.137	2.312	1.688
MC pricing	.2	.8	1.118	.8	.2	2.236	1.677

Using the Rothschild-Stiglitz definition of a mean preserving spread, let the probability density function be

$$f(\theta, r),$$

where  $r$  is a parameter. Then an increase in  $r$  represents a mean-demand preserving spread if

$$\int_{-\infty}^{\infty} F_r(\theta, r) x_{\theta}(\theta) d\theta = 0$$

and

$$\int_{-\infty}^K F_r(\theta, r) x_{\theta}(\theta) d\theta \rightarrow 0 \quad \forall K < \infty.$$

Since the demand for  $x$  also depends on the price  $p$ , the mean-demand preserving spread also depends on price. This mean-demand preserving spread of the distribution is taken at the optimal price for some  $r$ . An individual will be said to be in risk class  $r$  if he has the probability density function  $f(\theta, r)$ .

Now, consider the set of feasible two-part tariffs  $(R_r, p_r)$  that a competitive firm could offer to individuals in risk class  $r$ . Again, to earn zero profits, the firm must set

$$R_r = (c - p_r) \int_{-\infty}^{\infty} f(\theta, r) x(\theta) d\theta.$$

Then with a mean-demand preserving spread,

$$(4.1) \quad \frac{dR_r}{dr} = (c-p) \int_{-\infty}^{\infty} f_r(\theta, r) x(\theta) d\theta - (c-p) \int_{-\infty}^{\infty} f(\theta, r) x_I(\theta) \frac{dR_r}{dr} d\theta \\ - \left( \int_{-\infty}^{\infty} f(\theta, r) x(\theta) - (c-p) \int_{-\infty}^{\infty} f(\theta, r) x_p(\theta) \right) \frac{dp}{dR_r} \frac{dR_r}{dr} d\theta$$

Because this is a mean-demand preserving spread, the first term in (4.1) is zero, and therefore,  $\frac{dR_r}{dr} = 0$ .

So, the firm's set of feasible two-part tariffs does not vary with a mean-demand preserving spread. This means that a different two-part tariff can be offered for each risk-type of consumer without any type jeopardizing the choice of any other.

The next proposition explains how the optimal two-part tariff varies with the amount of uncertainty.

Proposition 2. Individuals who are more uncertain (in the mean-demand preserving sense) will prefer a more insurance, where more insurance means a higher entry fee and lower per-unit price if

$$(4.2) \quad U_y(\theta) \left( x(\theta) + \frac{dR}{dp} \right) \text{ is concave in } \theta.$$

If (4.2) is convex (linear) in  $\theta$ , more uncertainty yields a smaller (the same) entry fee and higher (the same) per-unit price.

Proof: Substitute (4.1) into (2.5) which yields

$$(4.3) \quad \frac{\partial L}{\partial p} = - \int_{-\infty}^{\infty} f(\theta) U_y(\theta) \left( x(\theta) + \frac{dR}{dp} \right) d\theta .$$

Let  $p^*$  solve  $\partial L / \partial p = 0$ , which is the utility maximizing price for some

distribution  $f(\theta, r^*)$ . If we take a mean-demand preserving spread of (4.3) at  $p^*$  and find that  $\frac{\partial^2 L}{\partial p \partial r} < 0$ , then more the optimal price is less than  $p^*$  with increasing uncertainty, and therefore, uncertainty results in more insurance.

$$(4.4) \quad \frac{\partial^2 L}{\partial p \partial r} = - \int_{-\infty}^{\infty} f_r(\theta) U_y(\theta) \left( x(\theta) + \frac{dR_r}{dp} \right) d\theta \\ - \int_{-\infty}^{\infty} f(\theta, r) \frac{dU_y(\theta)}{dr} \left( x(\theta) + \frac{dR_r}{dp} \right) d\theta - \int_{-\infty}^{\infty} f(\theta, r) U_y(\theta) \frac{d(x(\theta) + dR/dp)}{dr}$$

$$\text{Then, } \frac{dx(\theta)}{dr} = \left( -x_I(\theta) + x_p(\theta) \frac{dp}{dR} \right) \frac{dR_r}{dr} = 0$$

and therefore,  $\frac{dU_y(\theta)}{dr} = 0$  also. Thus,

$$\frac{\partial^2 L}{\partial p \partial r} \left\{ \begin{array}{l} > \\ < \end{array} \right\} 0 \quad \text{as} \quad U_y(\theta) \left( x(\theta) + \frac{dR_r}{dp} \right) \text{ is } \left\{ \begin{array}{l} \text{concave} \\ \text{linear} \\ \text{convex} \end{array} \right\} \text{ in } \theta$$

So, expression (4.3) is negative when conditions (4.2) is satisfied. With this condition, more uncertain consumers prefer a lower per-unit price (and higher entry fee). QED

Table 2 shows the optimal two-part tariffs in the earlier example as uncertainty increases.

### V. Conclusion

We have shown that a two-part tariff price discrimination scheme can be beneficial to consumers because it provides a form of insurance. If individuals are uncertain about their demand for a good, but do not decide on consumption until the uncertainty is resolved, they always prefer a pricing

scheme with a positive up-front payment and a per-unit price less than marginal cost to a scheme with price equal to marginal cost. As a result, utility is higher with the two-part tariff in states where demand for the good is high, and utility is lower when demand is low, than in a price equal marginal cost system. This is insurance because the marginal utility of income is greater in states where demand is high, and the consumer effectively transfers income to those states through the lower price.

Because consumers strictly prefer this form of pricing, it will be used by firms even in competitive environments. It is a form of price discrimination because ex-post, consumers in different states pay a different average price per unit.

Consumers who are more uncertain (in the Rothschild-Stiglitz mean preserving distribution sense) about their future demand will prefer a different two-part tariff scheme from that preferred by more certain consumers. Those with no uncertainty strictly prefer a price equal to marginal cost, while a variety of entry fee-per unit prices can be offered to individuals with varying amounts of uncertainty. Exactly how the optimal tariffs vary with uncertainty depends on the utility function and probability density function.

Table 2

P	$\theta_H=2, \theta_L=.5$		$\theta_H=2.1, \theta_L=.4$		$\theta_H=2.2, \theta_L=.3$		$\theta_H=-1, \theta_L=3.5$	
	R	EU	R	EU	R	EU	R	EU
1	0	1.677	0	1.701	0	1.73	0	1.113
.9	.054	1.684	.052	1.711	.049	1.742	.075	1.139
.8	.118	1.688 **	.112	1.718	.107	1.753	.157	1.165
.7	.193	1.687	.184	1.721 **	.175	1.760	.247	1.19
.6	.279	1.68	.267	1.719	.254	1.764 **	.345	1.212
.5	.379	1.664	.363	1.709	.346	1.761	.449	1.212
.4	.492	1.635	.474	1.686	.454	1.747	.558	1.248
.3	.617	1.589	.6	1.645	.578	1.715	.671	1.259
.2	.751	1.517	.737	1.575	.718	1.654	.784	1.264 **
.1	.884	1.41	.878	1.457	.867	1.53	.895	1.261

References

- Berglas, Eitan, "On the Theory of Clubs," American Economic Review, (66), May 1976, pp. 116-121.
- Oi, Walter, " A Disneyland Dilemma: Two-Part Tariffs for a Mickey-Mouse Monopoly," Quarterly Journal of Economics, (85), February 1971, pp. 76-96.
- Oren, Shmuel S., Steven Smith and Robert Wilson, "Competitive Non-Linear Tariffs," Journal of Economic Theory, (29), 1983, pp. 49-71.
- Phillips, Owen R. and Raymond C. Battalio, "Two-Part Tariffs and Monopoly Profits When Visits are Variable," Bell Journal of Economics, (14), Autumn 1983, pp. 601-606.
- Rothschild, M. and J. Stiglitz, "Increasing Risk: I. A Definition," Journal of Economic Theory, (2), 1970, pp. 225-243.
- Spence, Michael, "Non-Linear Prices and Welfare," Journal of Public Economics, (81), 1977, p. 1-18.