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A MARKOV MODEL OF EMPLOYMENT, UNEMPLOYMENT AND LABOR FORCE PARTICIPATION: ESTIMATES FROM THE DIME DATA

by

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Introduction

New developments in human capital, search and turnover theory and the availability of new evidence drawn from longitudinal panel data have stimulated considerable interest in labor supply and labor force participation dynamics. We now know that many workers experience frequent spells of employment separated by intervening periods out of the labor force or unemployed. Differences in unemployment and participation rates across demographic groups are due in large measure to differences in the durations of spells of employment, unemployment and non-participation. However, with the notable exception of the recent literature on unemployment duration and job separation there are few studies of the relationship between spell durations in the various participation states and earning power.[1] This is the principal topic of this paper.

The bulk of the existing literature on labor force participation is based on a static conceptualization of the income-leisure choice problem, a natural framework for the purpose of analyzing cross section data. Of course, neither the theory nor the empirical results has anything to say about durations in a particular participation state even though observed unemployment and participation rates are functions of duration in general. Consequently, it is not possible to reconcile existing studies of labor supply with observed unemployment and employment behavior. To remedy this situation, this paper presents a Markov decision model wherein individuals choose among the labor force states of employment, unemployment, and non-participation at each moment in time. This model is a natural extension of the static model, and includes recent developments in search and turnover theory as special cases. The model also provides a conceptual framework for interpreting parameter estimates obtained from data on individual laboar market histories. This framework is used to interpret results obtained on data from the Denver Income Maintenance Experiment (the DIME data).

An abstract model of qualitative choice over time is formulated in Section I. In Section II the model is interpreted as a characterization of the labor force decision process faced by the typical worker. Theoretical results concerning the state to state transition rates and workers' expected earnings are derived using the model. Section III treats the problem of estimating transition rate formulations from state duration data, Section IV introduces the DIME data and Section V reports parameter estimates and discusses their implications. As a check on the adequacy of the model, we report the results of simple diagnostic tests in this section also. Finally, in Section VI the estimated transition rates are used to construct the steady-state supply curve analogous to a static optimization model.

I. A Dynamic Qualitative Choice Model

We begin by sketching an abstract model of choice over time from a discrete set of alternatives. Formally, the model is both a special case of a general class of Markov decisions processes and a natural extension of the static random utility qualitative choice model to a dynamic context. Later the model is interpreted as a characterization of the problem of labor force participation state choice.

Associated with an individual agent is a vector $x \in X$ of stationary characteristics. Given this vector, at any date t, the agent is characterized by a pair $(e(t), n(t))$, where $e(t) \in E$ is a vector of stochastic time varying characteristics, which we will call disturbances, and $n(t) \in N$ is a particular state. The sets of stationary and stochastic characteristics, X and E, are both real vector spaces and the set of states, N, is a finite index set. The worker's preferences over states are represented by a stationary characteristic-contingent utility indicator $V_n(x,e)$, $n \varepsilon N$. It is the value function associated with a dynamic programming problem to be formulated subsequently.

At every date t the agent may choose to occupy the preferred state contingent on the current value of the disturbance. Given this assumption, changes in state occur only as a consequence of changes in the stochastic characteristic. The probability that the disturbances will take on a different value in the future depends on the state currently occupied and the agent's stationary characteristics but is independent of its own current value. These assumptions together with freedom to choose state occupancy imply that the process $\{n(t)\}$ is Markov.

Let

$$(1) \quad \mu_n(A;\ x,\ h) = \Pr(e(t+h) \varepsilon A |\ n(t = n \text{ and } x)$$

denote a probability measure defined on the Borel sets of E. In other words, $\mu_m(A;\ x,\ h)$ is the probability that the stochastic characteristic is contained in the subset A at future date $t + h$ given the worker's stationary characteristic and participation state. Assume that $\mu_n(\ )/h$ has a limit as h tends to zero. Without loss of generality

$$(2) \quad \lim_{h \to 0} \frac{1}{h} \mu_n(A;\ x,\ h) = \eta_n(x) \int_A F_n(d\tilde{e};\ x)$$

where $F_n(\tilde{e};\ )$ is a cumulative distribution function defined on E. Equation (2) has the following interpretation: Any change in the disturbance requires time, the mathematical expectation of the time required for the first change in e is $1/\eta_n(x)$ and $F_n(\tilde{e};\ \cdot)$ is the probability distribution over the new values given a change in e.

At any moment, the preferred state depends on the agent's characteristic. Given the stationary characteristics, the subset of stochastic characteristics such that a particular state m is preferred is defined by

$$(3) \quad A_m(x) = \{e \in E \mid V_m(x,e) = \max_{n \in N} V_n(x,e)\},\ m \in N$$

The probability of a transition from a given participation state n to another $m \neq n$ during any time interval of length h, given that $n(t) = n$, is

$$\begin{aligned} P_{nm}(x,\ h) &= \Pr\{n(t+h) = m \mid n(t) = n \text{ and } x\} \\ &= \Pr\{e(t+h) \in A_m(x) \mid n(t) = n \text{ and } x\} \\ &= \mu_m(A_m;\ x,\ h),\ m \neq n \end{aligned}$$

The associated state to state transition rates are

$$(4) \quad \begin{aligned} \lambda_{nm}(x) &= \lim_{h \to 0} P_{nm}(x,\ h)/h \\ &= \eta_n(x) \int_{A_m(x)} \tilde{F}_n(de;\ x) \qquad m \neq n, \text{ and } n \in N \end{aligned}$$

In other words, the probability of a transition from state n to another $m(\neq n)$ during any sufficiently short time interval of length h is equal to the product of the probability of a change in the stochastic characteristic,

$h\eta_m(x)$ and the probability that the new value, $\tilde{e}$, is such that state m is preferred.

In the general case, the agent's state history is a Markov process. Specifically, the hazard rates associated with any state is

$$(5) \quad \lambda_n(x) = \sum_{m \neq n} \lambda_{nm}(x), \; n \in N ,$$

the escape rate from the state. Consequently, duration of a spell in state n, t_n, is a random variable described by the c.d.f.

$$(6) \quad \Pr\{t < T \mid x\} = 1 - \exp(-T\lambda_n(x))$$

The unconditional probability distribution over states at date t is the solution to the differential equation system

$$(7) \quad \frac{dP_m(x)}{dt} = \sum_{n \neq m} \lambda_{nm}(x)P_n(x) - \lambda_m(x)P_m(x), \quad m \in N .$$

As t becomes large, the distribution converges uniformly to a unique steady state solution if $\lambda_{nm}(x) > 0$ for all $n \in N$ and $m \neq n$. Of course, this distribution represents the fractions of an infinite life time spent in the states as well as the distribution of a large sample of identical agents among the states.

In a stochastic dynamic decision context, the value funciton $V_n(x)$ is a derived rather than a primative construct. Its derivation requires the specification of the agent's preferences over future sequences of events and expectations regarding the future event sequence possibilities. In the remainder of this section the techniques of dynamic programming are applied for the purpose of establishing conditions under which the value function exists. Three assumptions are sufficient. First, the agent's preferences are representable by a stationary intertemporarily separable von Neumann-Morgenstern utility function defined on future participation-state stochastic-characteristic sequences. Second, the agent is assumed to know the process generating the stochastic characteristic, that defined in (1). Third, workers are endowed with infinitely long lives.

In keeping with the static random utility model, we assume that the instantaneous utility funciton takes the form $u_n(x) + e_n$, where $e = (e_1, e_2, \ldots e_m, \ldots)$ is a vector of utility disturbances. Let ρ denote the agent's discount rate. Finally, interpret $V_n(x,e)$ as the expected present value of the agent's future utility flow given that the state choices made in the future are optimal and that (n, e) is the agent's current state-disturbance. The value of the optimal choice given (x, e) is

$$(8) \quad \psi(x, e) = \max_{n \in N} V_n(x, e)$$

Hence, Bellman's principle of dynamic optimality implies that

$$V_n(x, e) = h[u_n(x) + e_n] + \frac{1}{1+h\rho} E\, \psi(x, e_{t+h})$$

holds as an approximation for all sufficiently small h. In other words, $V_n(x, e)$ is equal to the utility flow realization during the "current instant" plus the present value of the maximal future utility stream expected at the end of the instant. By substitution, (2) implies

$$V_n(x, e) = h[u_n(x)+e_n] + \frac{1}{1+h\rho} [h\eta_n(x) \int \psi(x, \tilde{e})\, F_n(d\tilde{e}; x) + (1-h\eta_n(x))\, V_n(x, e)] + O(h)$$

where $O(h)/h \to 0$ as $h \to 0$. By rearranging terms and taking the limits as $h \to 0$, one obtains

$$(9) \quad V_n(x, e) = \frac{1}{\rho + \eta_n(x)} [u_n(x) + e_n + \eta_n(x) \int \psi(x,\tilde{e}) F_n(d\tilde{e};x)]$$

$$(n,X,e) \in N \times X \times E .$$

To complete the derivation of the value function we need only establish the existence of the maximal value function $\psi(x,e)$. Obviously, (8) and (9) imply

$$(10) \quad \psi(x,e) = \max_{n \in N} \left\{ \frac{1}{\rho + \eta_n(x)} [u_n(x) + e_n + \eta_n(x) \int \psi(x,\tilde{e}) F_n(d\tilde{e}; x)] \right\}$$

Note that $\psi(\cdot)$ is a fixed point of a map, represented by the right side of (10), from the set of all functions defined on the vector space $X \times E$ to itself. If $\rho > 0$, for all $x \in X$, i.e., the discount rates for all workers are bounded away from zero, then the map is a contraction by virtue of Blackwell's (1962) sufficient conditions. Hence, a unique fixed point exists; we have an internally consistent theory.

II. Wage Effects on Participation State Transitions

The typical worker can be viewed as having to choose among three labor force participation states over time: employment, unemployment, and out of the labor force. The purpose of this section is to study the qualitative relationships between the six transition rates and the worker's expected wage implied by this choice problem. For this purpose, the worker's expected market wage, denoted by the scalar x, is the only stationary non-stochastic characteristic of interest. Two assumptions are maintained. First, the process generating the worker's stochastic characteristic is independent of the wage. Second, the frequency of change in the stochastic characteristic and the distribution of any new value given change are independent of its current value. The first assumption implies that the qualitative relationship between any transition rate and the wage is a reflection solely of the worker's preference over participation states. Although the results presented in the section are special as a consequence, they represent the dynamic anologues of those obtained from the traditional static formulation of the participation choice problem. As we have already seen, the second assumption is that needed to model labor force participation histories as a semi-Markov process.

Denote the three participation states, employment, unemployment, and out of the labor force, as $n = 1, 2, 3,$ respectively. The two principal assumptions imply

(11.a) $\quad \eta_n(x) = \eta_n, \; n = 1, 2, \text{ and } 3$

and

(11.b) $\quad F_n(\tilde{e}; x) = F_n(\tilde{e}), \; n = 1, 2, \text{ and } 3$

where x is a scalar denoting the worker's wage. By virtue of (8)-(11), the value functions solve

(12)

$$(\rho+\eta_n)\, V_n(x, e) = u_n(x) + e_n + \eta_n \int \max_k V_k(x, \tilde{e})\, F_n(d\tilde{e}), \quad n = 1, 2, \text{ and } 3$$

Clearly, each is of the additively separable form

$$(13.a) \quad V_n(x, e) = V_n(x) + \frac{e_n}{\rho + \eta_n}$$

where

$$(13.b) \quad V_n(x) \equiv V_n(x, 0), \; n = 1, 2, \text{ and } 3.$$

As each disturbance can be assigned a zero mean without loss of generality, $V(x)$ can be regarded as the mathematical expectation of the value of occupying state n. As a consequence of (13), the model is a simple dynamic extension of the static random utility model of qualitative choice. The state to state transition rates are

$$(14) \quad \lambda_{nm}(x) = \eta_n \alpha_{nm}(x) \quad m \neq n, \; n = 1, 2, 3$$

where

$$(14.a) \quad A_m(x) = \left\{ e \in E \;\middle|\; V_m(x) + \frac{e_m}{\rho + \eta_m} = \max_k \left[V_k(x) + \frac{e_k}{\rho + \eta_k} \right] \right\}$$

and

$$(14.b) \quad \alpha_{nm}(x) = \int_{A_m(x)} \tilde{F}_n(de)$$

In other words, $A_m(x)$ is the set of random disturbances for which state m is preferred and $\alpha_{nm}(x)$ is the probability that a realization of the disturbance will be an element of $A_m(x)$ given that state n is currently occupied. Since $\sum_k \alpha_{nk}(x) = 1$ for all n, the hazard rates are

$$(15) \quad \lambda_n(x) \equiv \sum_{m \neq n} \lambda(x) = \eta_n(1-\alpha_{nn}(x)), \; n = 1, 2, 3$$

By virtue of (5), $1/\lambda_n(x)$ is the expected duration of a spell in state n.

The fact that the worker's instantaneous utility flow depends on the wage only when employed, is the principal distinction between employment and the other two states. For the purpose of interpretation it is convenient to assume risk neutrality; i.e.,

$$(16) \quad u_1(x) = x, \; u_2(x) = a, \; u_3(x) = b$$

where a and b are constants representing the mean value of leisure enjoyed in the two non-employment states. Given (16), $x + e_1$ can be regarded as the actual random wage earned while employed. Analogously, e_2 and e_3 are deviations from the expected value of leisure in the non-employment states.

Classical participation theory presumes that the worker can freely choose between employment at the "going wage" and non-employment at every instant. Consequently, there is no theoretical distinction between unemployment and out of the labor force. Search theory recognizes that employment opportunities are heterogeneous and that they arrive sequentially at rates which are at least partialy controlled by the worker. In the unemployment state the worker is making some effort to find an acceptable job. Consequently, the worker enjoys less leisure, $a < b$, and the worker becomes employed more quickly; i.e.,

(17) $\lambda_{21}(x) > \lambda_{31}(x)$

We are prepared to ask the following question: How do the transition rates of two otherwise identical workers compare if one can expect higher earnings when employed? An inspection of (14) reveals that the answer depends on the relative slopes of the participation-state contingent values. Expressions for these can be obtained by differentiating (12).[2] By virtue of (13)-(16), we obtain the system

(18) $$\Lambda(x)\,\frac{\partial V(x)}{\partial x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

where $\frac{\partial V(x)}{\partial x}$ is the gradient of the vector function

$$V(x) = \begin{bmatrix} V_1(x) \\ V_2(x) \\ V_3(x) \end{bmatrix}$$ and

$$\Lambda(x) = \begin{bmatrix} \rho+\lambda_{12}(x) + \lambda_{13}(x) & -\lambda_{12}(x) & -\lambda_{13}(x) \\ -\lambda_{21}(x) & \rho+\lambda_{21}(x) + \lambda_{23}(x) & -\lambda_{23}(x) \\ -\lambda_{31}(x) & -\lambda_{32}(x) & \rho+\lambda_{31}(x) + \lambda_{32}(x) \end{bmatrix}.$$

As $\lambda_{nm}(x) \geq 0$, the matrix $\Lambda(x)$ has positive main diagonal elements and is dominant diagonal. Therefore, the determinant $D(x) = |\Lambda(x)|$ is strictly positive if $\rho > 0$. This fact and (18) imply

$$(19.a) \quad \frac{\partial V_1}{\partial x} - \frac{\partial V_2}{\partial x} = \frac{\rho}{D(x)} \left[\rho + \lambda_{31}(x) + \lambda_{32}(x) + \lambda_{23}(x) \right] > 0$$

$$(19.b) \quad \frac{\partial V_1}{\partial x} - \frac{\partial V_3}{\partial x} = \frac{\rho}{D(x)} \left[\rho + \lambda_{21}(x) + \lambda_{23}(x) + \lambda_{32}(x) \right] > 0$$

In other words, the worker with greater expected earning power is more likely to prefer employment to either non-employment state. Finally, (17) and (19) imply

$$(20) \quad \frac{\partial V_2}{\partial x} - \frac{\partial V_3}{\partial x} = \frac{\rho}{D(x)} \left[\lambda_{21}(x) - \lambda_{31}(x) \right] > 0$$

Participation is also more likely to be preferred by the worker who commands a larger expected wage.

Formally, (19) implies that the set of disturbances such that employment is preferred, $A_1(x)$ as defined in (14), is increasing in the expected wage in the set inclusion sense; i.e., $A_1(x_2) \supseteq A_1(x_1)$ strictly if and only if $x_2 > x_1$. Consequently, (14) and (15) imply the following.

<u>Proposition 1</u>: The transition rates from both non-employment states to employment increase with the expected wage while the hazard rate associated with employment decreases with the wage.

Analogously, (19) and (20) imply that $A_3(x)$ is monotone decreasing in the expected wage. Therefore, (14) and (15) also yield the following.

<u>Proposition 2</u>: The transition rates from both participation states to out of the labor force decrease with the expected wage and the hazard rate associated with out of the labor force increases with the wage.

Because the qualitative effect of an increase in the wage on the set of disturbances for which unemployment is preferred is ambiguous, the derivatives of the remaining transition rates, those for which unemployment is the destination state, and of the hazard rate associated with unemployment, cannot be signed.

III. Estimation

The model developed above provides a conceptually simple framework for examining individual labor market histories. Because the focus of the model is on the dynamics of labor market activity, we are necessarily interested in both the length of time spent in a particular state n, and in which of the $m \neq n$ transitions occur. A description of an event (t_n, m)--a stay of length t in state n followed by a transition to state m--requires specification of the probability of both elements. Using the continuous time notation of section II, the instantaneous hazard rate from state n is defined as:

$$(21) \qquad \lambda_n(x) = \sum_{m \neq n} \lambda_{nm}(x)$$

Using the fundamental relationship between transition rates and distribution functions, and the constancy of the transition rates yields the distribution of exit times, t_n, from state n as:

$$(22) \qquad F_n(T \mid x) = 1 - \exp(-T \lambda_n(x)), \text{ with density,}$$

$$(23) \qquad f_n(T \mid x) = \lambda (x) \ (1 - F_n(T \mid x)$$ [3]

The conditional probability that $m \neq n$ will be the new state at the end of a completed spell is λ_{nm}/λ_n. Hence, the probability density of a complete spell characterized by origin state n, destination state m and duration T is

$$(24) \quad f_{nm}(T;x) = \frac{\lambda_{nm}(x)}{\lambda_n(x)} f_n(T;x)$$

$$= \lambda_{nm}(x)\ [1 - F_n(T;x)].$$

Therefore, the log likelihood for a sample of $\ell = 1,\dots, L$ spells can be written as

$$(25) \quad \ln \mathcal{L} = \sum_{\ell=1}^{L} [d_\ell \ln\lambda_{n_\ell m_\ell}(x_\ell) - T_\ell \lambda_{n_\ell}(x_\ell)]$$

where d_ℓ equals unity if the spell is complete and equals zero otherwise.

To implement the model, a functional form for λ_{nm} must be chosen. A simple and, in the present context, natural specification is $\lambda_{nm}(x) = \exp(x\beta_{nm})$, a specification that has been widely used. Previous economic applications include Lancaster (1979), Tuma and Robbins (1980), and Lundberg (1981), all of which study movements among labor market states. The log likelihood function becomes

$$(26) \quad \ln(\mathcal{L}) = \sum_{\ell=1}^{L} [d_\ell x_\ell \beta_{n_\ell m_\ell} - T_\ell (\sum_{k \neq n_\ell} \exp(x_\ell \beta_{n_\ell k}))]$$

The additive separability and global concavity of the log likelihood function makes this specification particularly attractive. The results that follow were obtained by maximizing (26) using Newton's method with analytic first and second derivations within SAS's MATRIX procedure, a method that was fast even using starting values of zero. Asymptotic standard errors were obtained in the usual way by inverting the negative of the Hessian of the log likelihood function.

IV. The DIME data

The Denver Income Maintenance Experiment was one of the largest programs designed to measure the effects of negative income tax on labor supply. Families chosen for participation in the experiment did not constitute a random sample from the whole population; rather they were chosen to reflect

the characteristics of the population of families who were considered likely to participate in a full-scale negative income tax program. Excluded were:

(1) Families with heads over 58 years old or under 18 years old, or families with disabled heads unable to work;

(2) Families with pre-experiment earnings above $9,000 for a family of four with one head working or $11,000 with two heads working, and

(3) Individuals who did not belong to a family.

The sample was stratified by race and family type.

The Public Use Files are organized in a monthly format with data covering 48 months. They contain sufficient information to reconstruct histories of labor market status--employment, unemployment, and nonparticipation--using Current Population Survey definitions. About 60 percent of the families in the sample were assigned to negative income tax "treatments." These families are excluded from the sample in the analysis reported in this paper.

A potential hindrance to representability of the sample arises because of the truncation by pre-experiment income levels. As noted above, the cut-off level for eligibility was $11,000 in 1971 income for a two earner family, a level that is 102% of Denver median family income in 1970. This pre-experiment truncation overstates the potential bias, however, because of transitory fluctuations in income of some otherwise higher-income families who were eligible for the program. Ashenfelter ((1980), Table 1) finds that in the first year of the experiment 22.6% of the control families had income in excess of $11,204. Ashenfelter also finds that in the absence of truncation by income one would have expected to find 34% of the control families with incomes in excess of $11,000. This result suggest that the DIME controls are not completely representative of Denver families at the higher income levels. In consequence, since high incomes are well-known to be correlated with stable employment patterns, these data are likely to overstate labor market transitions.

Although the data contain information of sufficient detail to construct histories of labor market status, there are two points where CPS labor market data will differ from DIME labor market data. The CPS sample, being a point in time observation, is subject to length bias, that is, short spells of unemployment are less likely to be detected than long spells. The DIME sample, being a longitudinal data set, does not have this problem and consequently will contain more short spells of unemployment. If unemployment spells are distributed uniformly across a month, the unemployment rate calculated from a point-in-time sample should not differ from that calculated from a longitudinal basis, so no bias should arise. With regard to length of spells, however, it is obvious that a point in time sample will overstate the length of an average spell of unemployment.

Attached to each spell was the length of time of the completed spell and demographic variables relating to each individual as of the start of the spell. The means of these variables, by age group, are given in Table 1. Most of these variables are self-explanatory, the exceptions being the assets and wage variables. Assets were contructed from information on the value of

TABLE 1

Means by Age/Sex Groups: Dime Data-All Transitions

	Young Males	Adult Males	Young Females	Adult Females
Education	10.6	11.1	11.0	11.7
% Black	34.5	29.5	31.6	37.0
% Hispanic	31.7	36.2	28.3	28.5
ln Wage	4.6	5.5	4.3	4.8
Age	18.2	31.7	18.5	31.8
Assets/1000	0.4	0.2	0.3	0.2
# of kids	0.3	1.0	0.5	0.9
Black *ln Wage	1.5	1.63	1.3	1.8
Hispanic *ln Wage	1.4	2.0	1.2	1.3
N = number of separate-transitions	1,119	1,571	1,138	2,524
P = number of persons in each group in 1973	260	660	326	946

stocks, cash in checking accounts, equity value in house or cars, etc., which were available at each periodic interview. Linear interpolation was used to produce asset values for the intervening months between interviews.

The wage variable used is derived from the reported hourly wage on the longest job worked in a given month. Since the period of the experiment encompasses a wide range of annual inflation rates, all nominal wage rates were deflated by the Denver area CPI for each of the years 1971-74. Wage rates, however, are defined only for employed individuals. For individuals who are not employed, it is expected wages (more precisely, all the parameters of the wage distribution) that are important. To obtain expected wages, one could either obtain information from other data sources, or equivalently, impute values based on individual characteristics. We have chosen the latter option. The natural log of wages was regressed against age, age squared, education, and race/ethnic status for each of the age/sex groups. This procedure was done by quarters, resulting in a maximum of sixteen predicted wage rates for each individual. Each labor market spell was matched to the predicted wage appropriate to the quarter in which the spell began. In the empirical work presented below, when transitions out of the employment state are being considered, ln Wage refers to logarithm of the actual real wage; for all other transitions, ln Wage refers to the logarithm of the predicted real wage.

V. Estimates.

Here we consider the states employment, unemployment, and nonparticipation. Definitions conform to those used by the Current Population Survey. As regressors we consider education, measured by years of schooling; a quadratic in age; the DIME measure of assets; the number of children in the

Table 2
Transition Function Estimates:
Males 21 and Under

	E→U	E→N	U→E	U→N	N→E	N→U
Constant	-15.4691	10.9978	-15.9207	-8.9033	-16.6377	-17.7792
	(1.49)	(1.17)	(1.66)	(0.56)	(2.47)	(1.81)
Education	-0.3583	0.0444	-0.0010	0.1058	0.1074	-0.0987
	(5.09)	(0.51)	(0.01)	(0.69)	(1.62)	(0.95)
Age	1.9148	-1.1447	1.1493	1.0758	1.1582	1.3800
	(1.71)	(1.10)	(1.11)	(0.64)	(1.56)	(1.26)
$Age^2/100$	-4.9368	2.7047	-2.9876	-2.4367	-2.3284	-3.7750
	(1.63)	(0.96)	(1.09)	(0.53)	(1.13)	(1.20)
Assets/1000	-0.0066	0.0821	-0.0562	0.0102	-0.0143	0.0064
	(0.10)	(1.95)	(1.52)	(0.19)	(0.51)	(0.16)
Kids	0.0572	-0.1529	0.0999	0.2191	0.0851	0.0868
	(0.44)	(0.97)	(0.87)	(1.11)	(0.78)	(0.52)
Race:						
Black	1.1915	2.9494	1.4850	1.9201	0.5389	2.2017
	(1.23)	(3.62)	(0.78)	(0.61)	(0.30)	(0.82)
Hispanic	2.0258	2.6893	3.2149	-0.4751	2.1706	-3.0469
	(2.02)	(2.77)	(1.75)	(0.13)	(1.25)	(1.03)
ln Wage	-0.4096	-0.5442	0.7625	-1.4597	0.0377	0.6200
	(2.82)	(4.07)	(2.07)	(1.72)	(0.09)	(0.87)
B ln Wage	-0.2124	-0.6396	-0.4048	-0.3297	-0.2508	-0.3714
	(1.07)	(3.53)	(1.01)	(0.47)	(0.64)	(0.61)
H ln Wage	-0.3695	-0.5463	-0.7143	0.1743	-0.5970	0.6910
	(1.78)	(2.61)	(1.86)	(0.23)	(1.61)	(1.07)
ln (L)	-851.43	-739.03	-606.77	-289.80	-819.31	-515.19

Table 3

Transition Function Estimates:
Males 22 and Over

	E→U	E→N	U→E	U→N	N→E	N→U
Constant	1.6264 (1.60)	4.2304 (3.53)	-6.9535 (2.20)	-7.7707 (0.88)	-18.0220 (4.21)	-15.1749 (1.84)
Education	0.0186 (0.79)	-0.0001 (0.00)	0.0066 (0.26)	0.0596 (1.00)	-0.0952 (2.72)	-0.0819 (1.32)
Age	-0.1346 (3.11)	-0.2331 (4.46)	-0.1866 (3.33)	-0.1543 (1.50)	-0.2312 (3.64)	-0.1149 (1.10)
$Age^2/100$	0.1442 (2.40)	0.2933 (4.19)	0.2126 (2.81)	0.2466 (1.93)	0.2472 (3.04)	0.1083 (0.82)
Assets/1000	-0.9922 (5.02)	-0.3346 (2.07)	0.3923 (4.32)	-0.3209 (0.56)	0.1963 (1.87)	-0.3998 (1.05)
Kids	0.0693 (1.27)	-0.0342 (0.44)	0.1188 (2.14)	0.1002 (0.69)	0.0006 (0.01)	-0.1879 (1.35)
Race:						
Black	2.4180 (2.75)	0.1911 (0.17)	-5.0105 (1.14)	4.0144 (0.36)	-7.3915 (1.17)	-8.4390 (0.73)
Hispanic	0.7259 (0.73)	2.8305 (2.89)	-10.5040 (2.26)	-1.1940 (0.11)	-2.1147 (0.37)	1.9822 (0.20)
ln Wage	-0.5539 (4.58)	-0.8277 (6.47)	1.6299 (2.70)	1.081 (0.66)	3.8086 (4.67)	2.7149 (1.75)
B ln Wage	-0.4322 (2.67)	-0.0431 (0.20)	0.8110 (1.04)	-0.7539 (0.39)	1.3109 (1.18)	1.5087 (0.75)
H ln Wage	-0.0902 (0.50)	-0.4941 (2.69)	1.8027 (2.21)	0.2288 (0.12)	0.3953 (0.40)	-0.2642 (0.15)
ln (L)	-2087.77	-1255.98	-1.038.09	-323.23	-783.38	-357.22

Table 4

Transition Function Estimates
Model: Females Under 21

	E→U	E→N	U→E	U→N	N→E	N→U
Constant	-20.2854 (2.22)	-25.2351 (3.99)	-1.0345 (0.09)	-11.9666 (0.69)	-5.8334 (0.68)	17.6153 (1.50)
Education	-0.3731 (4.15)	-0.1972 (3.18)	-0.0078 (0.09)	-0.2321 (1.75)	0.0269 (0.43)	-0.1674 (1.87)
Age	2.5252 (2.54)	2.9890 (4.40)	-0.1918 (0.14)	1.6489 (0.82)	-0.0323 (0.03)	-2.5754 (1.82)
$Age^2/100$	-0.0665 (2.50)	-0.0777 (4.32)	0.6363 (0.18)	-4.3313 (0.83)	-0.3274 (0.12)	6.2507 (1.70)
Assets/1000	-0.0454 (0.89)	-0.0640 (1.36)	0.1064 (2.36)	0.0778 (0.68)	0.0301 (0.93)	-0.1020 (0.85)
Kids	-0.6992 (3.82)	0.0009 (0.01)	-0.3123 (2.12)	-0.1781 (1.08)	-0.2151 (1.99)	-0.2272 (1.46)
Race:						
Black	-0.3313 (0.36)	-1.6977 (2.82)	-6.8361 (2.97)	-0.2206 (0.06)	-1.2658 (0.97)	-0.3549 (0.24)
Hispanic	-1.8682 (1.65)	-1.3201 (2.09)	-1.4145 (0.59)	-2.2939 (0.61)	-0.8858 (0.67)	-3.2725 (1.68)
ln Wage	-0.5485 (3.35)	-0.8049 (7.89)	0.2861 (0.69)	-0.8527 (1.01)	1.0261 (2.42)	1.4518 (2.34)
B ln Wage	0.1381 (0.68)	0.3284 (2.47)	1.3809 (2.71)	0.1282 (0.15)	0.3024 (1.00)	0.3191 (0.91)
H ln Wage	0.4703 (1.90)	0.2443 (1.67)	0.2165 (0.41)	0.5832 (0.67)	0.1781 (0.57)	0.8300 (1.79)
ln (L)	-582.11	-918.98	-456.16	-303.58	-1275.23	-704.51

Table 5

Transition Function Estimates
Model: Females 22 and Over

	E→U	E→N	U→E	U→N	N→E	N→U
Constant	3.0943	2.3540	-2.3364	-7.8167	-10.3372	-17.1868
	(3.09)	(3.71)	(1.08)	(2.56)	(6.46)	(5.79)
Education	-0.0458	-0.0687	-0.1353	-0.2117	-0.1667	-0.2658
	(1.41)	(3.38)	(2.48)	(3.12)	(4.24)	(4.35)
Age	-0.2049	-0.0846	-0.3555	-0.1651	-0.3338	-0.4084
	(4.87)	(2.97)	(4.60)	(1.68)	(6.47)	(5.18)
$Age^2/100$	0.2090	0.0789	0.4191	0.1789	0.3227	0.3913
	(3.86)	(2.14)	(4.53)	(1.51)	(5.57)	(4.35)
Assets/1000	-1.0329	-0.0757	0.1038	0.0289	-0.0214	-0.0572
	(4.12)	(1.27)	(1.14)	(0.22)	(0.37)	(0.53)
Kids	-0.3371	0.1280	-0.1805	0.1407	-0.2348	-0.2489
	(3.74)	(2.49)	(2.67)	(1.78)	(4.91)	(3.10)
Race:						
Black	1.7380	-0.1358	-4.3931	-2.4317	0.2265	8.1160
	(2.57)	(0.39)	(1.76)	(0.73)	(0.12)	(2.61)
Hispanic	0.9845	0.1574	-3.2420	1.6755	-5.3018	5.2322
	(1.31)	(0.43)	(1.09)	(0.46)	(2.88)	(1.52)
ln Wage	-0.4547	-0.6316	1.8694	2.2820	3.3638	5.0282
	(4.16)	(12.06)	(2.64)	(2.49)	(6.14)	(5.69)
B ln Wage	-0.3286	-0.0679	0.7237	0.3936	-0.1364	-1.5789
	(2.35)	(0.92)	(1.44)	(0.59)	(0.37)	(2.51)
H ln Wage	-0.2161	-0.0150	0.6309	-0.3685	1.0724	-1.0638
	(1.35)	(0.19)	(1.03)	(0.49)	(2.83)	(1.49)
ln (L)	-1371.15	-2740.44	-946.48	-722.92	-3139.44	-1345.19

household; and the wage and race variables. A linear term (with intercept) in the log of wages is included separately for hispanics and for blacks. Results are reported in Tables 2-5.

The variables of primary interest are the wage variables. Although the theoretical model is very highly structured, the empirical results are in remarkable agreement with the model's predictions. Proposition 1 implies that the wage coefficients in the U-E and N-E transitions should be positive. This prediction is borne out for all groups except young males, for whom the sign is negative (for blacks and hispanics) in the N-E transition. Note the small t-statistic for these effects, however. Proposition 1 also states that the total effect of wages on the flow out of employment is negative; this holds in our estimates since the wage coefficients are negative for both the E-U and the E-N transitions. Proposition 2 implies that the wage effects in the E-N and U-N transitions are negative. This prediction also holds except in the case of the U-N transition for adult women for whom the effect is positive and in the case of the same transition for adult men, for whom the effect, though positive, is insignificant. Finally, proposition 2 implies that the total effect of wages on the flow into the labor force from nonparticipation is positive. This prediction is upheld by our estimates for all groups.[4]

For young males and females education reduces the incidence of unemployment but does not markedly affect duration directly; this is in accordance with the results of Ashenfelter and Ham (1979) and Kiefer and Neumann (1979). For adult males, education does not appear to be an important variable in explaining transitions although it is important to recall that we are holding constant a predicted wage, which itself is affected by education. Education appears to decrease the rate of all transitions of adult females. For young males, the age variables are not separately significant; it appears however that higher ages are associated with higher transition rates from employment to unemployment, lower rates from employment to nonparticipation, and higher transition rates out of unemployment and nonparticipation. For young females higher ages are associated with higher rates of leaving employment for both unemployment and nonparticipation. Higher ages also are associated with a lower rate of leaving nonparticipation for unemployment, but age seems not to affect the other flows. Age ranges for these groups between 16 and 21, and thus the range of variation of age and age-squared is small. For adult males and females, the age effect is significantly nonlinear.

Assets are associated with higher transition rates of employment to nonparticipation and lower rates from unemployment to employment for young males, and seem to have no other effect for that group. For young females, higher assets are associated with a higher transition rate for unemployment to employment and appears to have little other effect. For adult males, higher assets are associated with smaller flows out of employment, and larger flows from unemployment to employment and perhaps from nonparticipation into employment. For adult females, higher assets are associated with smaller flows out of employment in to unemployment, but with little else. The simplest search models suggest that assets should increase unemployment duration, since workers with higher assets can be more choosy about accepting a job. Our results suggest that this may be too simple.

The presence of children seems to have no effect on the transition rates for young males, possibly because only a small fraction of the sample had children. For young females, children are associated with smaller transition rates from employment to unemployment and nonparticipation to employment.

Children are associated with a higher transition rate from unemployment to employment for adult males. The results for adult females imply that children reduce the flows from employment to unemployment, from unemployment to employment, and both flows out of nonparticipation. Children are associated with increased flows from employment to nonparticipation and perhaps from unemployment into participation.

The race variables are significant for some transitions. It should be noted that whenever the dummy variable is significant the corresponding interaction with the wage is as well, so some care in interpretation is required. In particular, when difference by race in transitions occur, typically wage effects on transitions will also differ by race.

Diagnostic checks

The empirical results presented above provide substantial evidence that variations in personal characteristics, particularly wages, induce variations in labor market behavior. In the following section we employ these estimates to construct the steady state distributions of time spent in each labor market state, the dynamic analogues of labor supply functions. In order for these steady state calculations to be valid it is necessary that the underlying transition process be Markov, an assumption that we have exploited in the estimation. Recent work (Clark and Summers (1979), Kiefer and Neumann (1980)) suggests that this assumption may be overly strong, and hence, it should be tested. One way to see the effects of divergences from an assumed distribution is to examine the cumulative probability plots of time spent in a spell. Specifically, from (21) we have the cumulative distribution of a spell of, say, unemployment is:

$$(27) \quad Z_i = F_2(t_i^*; X) = 1 - \exp(-t^*(\exp(X_i B_{21}) + \exp(X_i B_{23})))$$

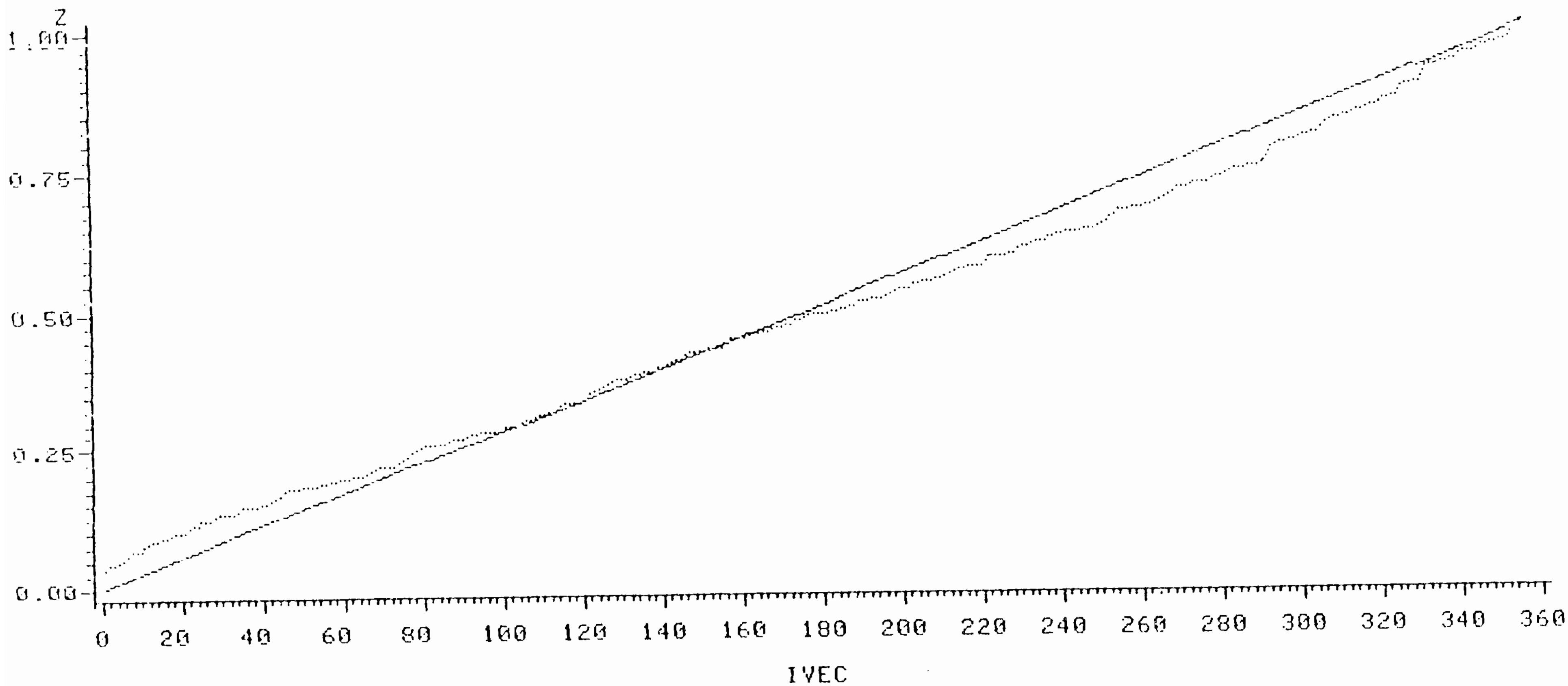

Figure 1

Cumulative Distribution of Unemployment Duration
By Age and Sex Groups: Young Males

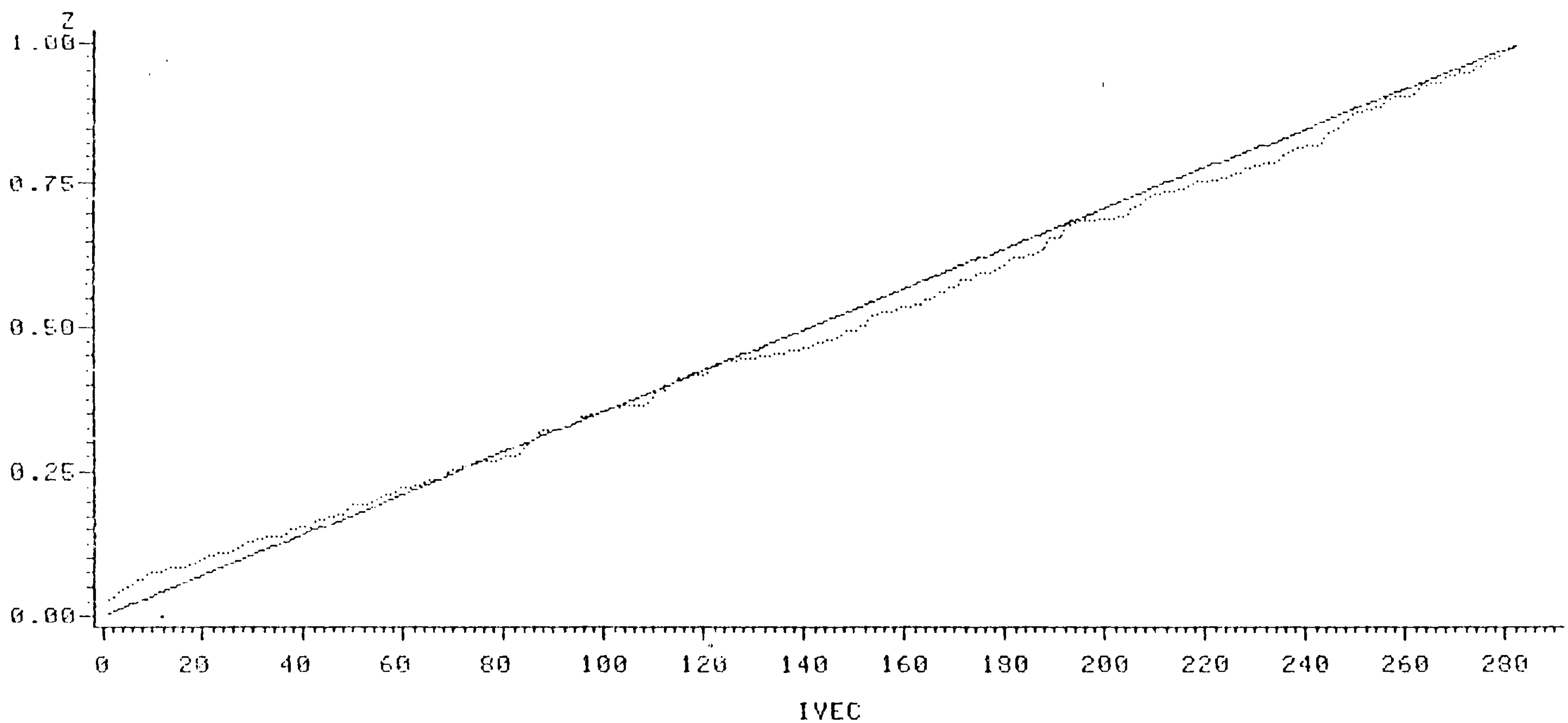

Figure 2

Cumulative Distribution of Unemployment Duration
By Age and Sex Groups: Young Females

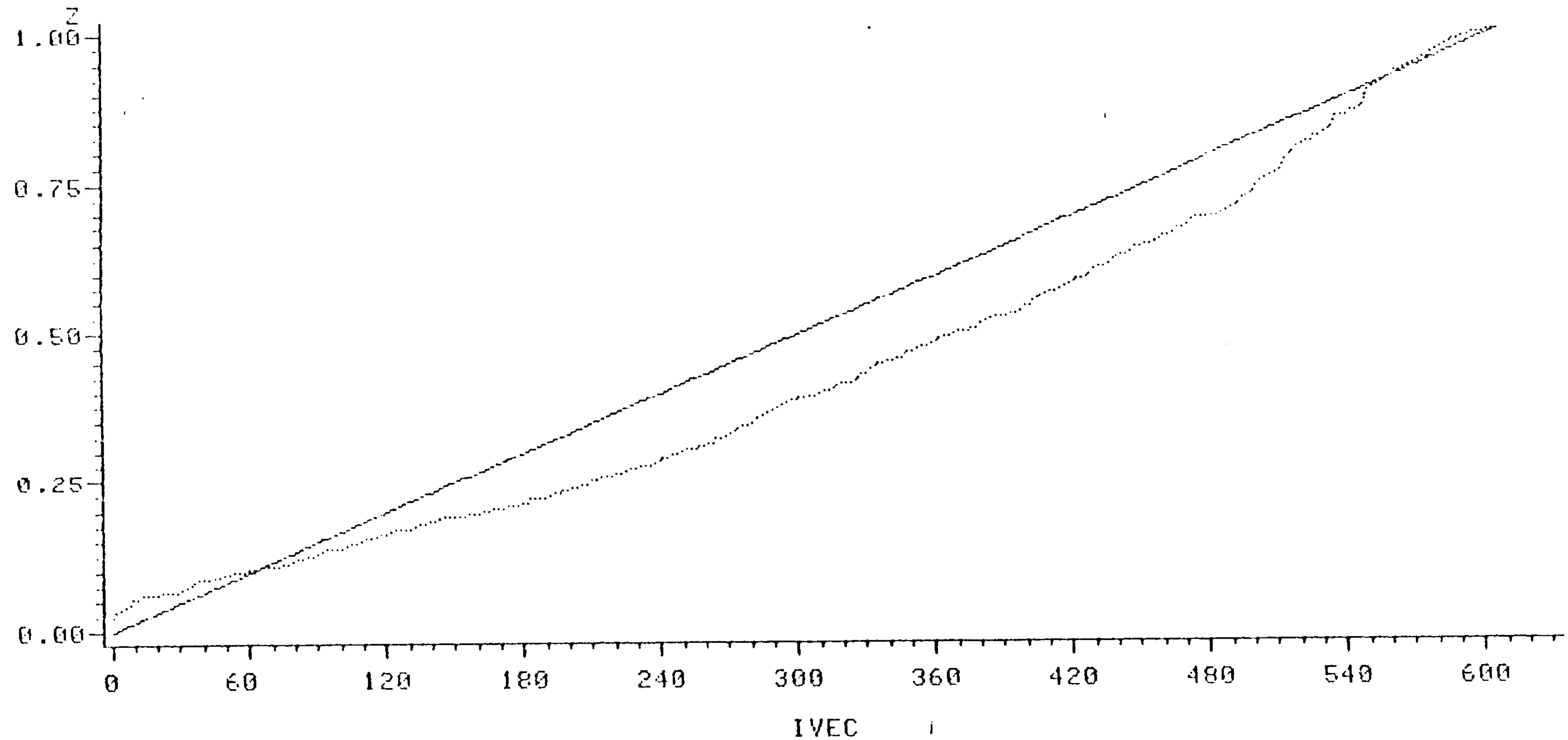

Figure 3

Cumulative Distribution of Unemployment Duration
By Age and Sex Groups: Adult Males

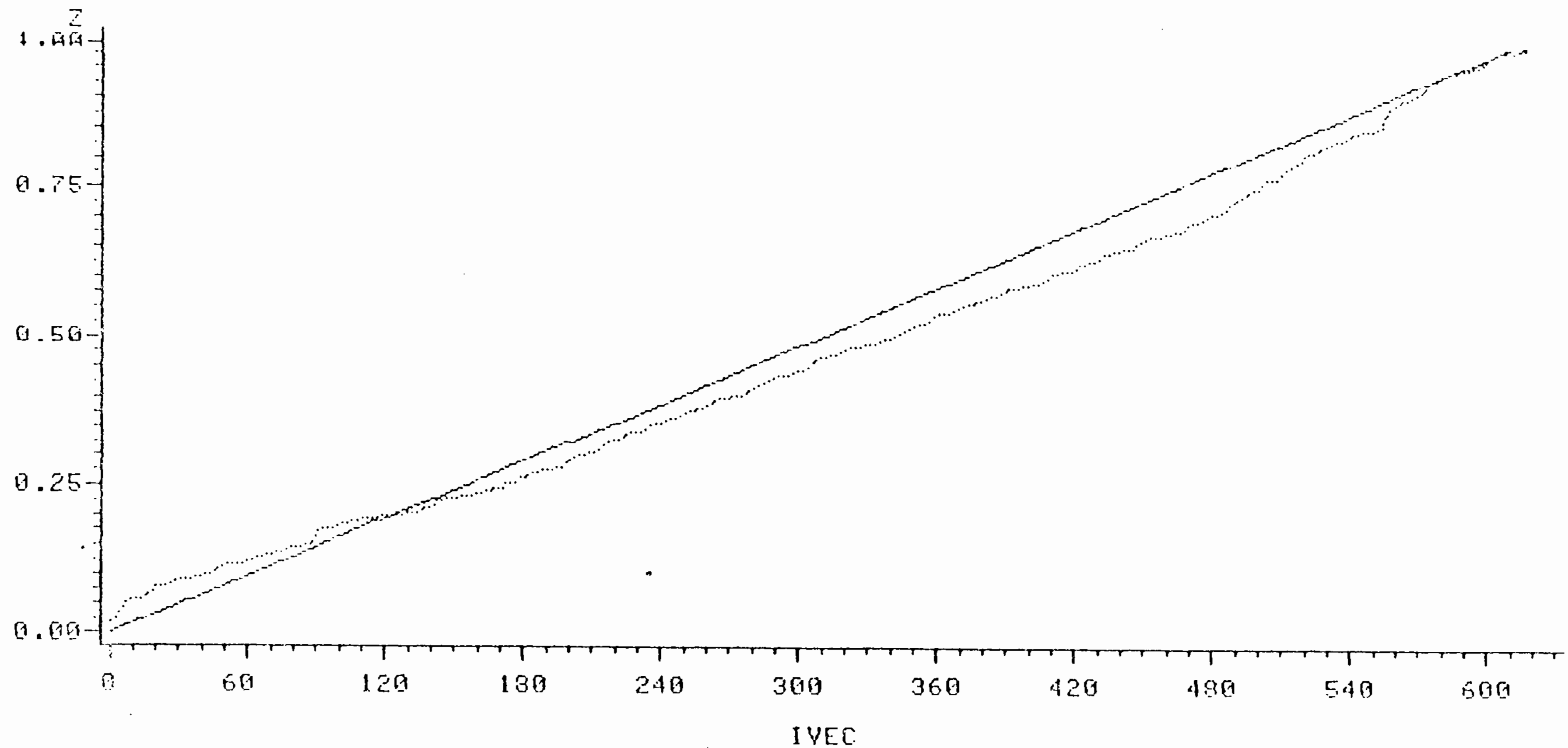

Figure 4

Cumulative Distribution of Unemployment Duration
By Age and Sex Groups: Adult Females

It is well known (Kendall and Stuart (1973)) that the distribution of Z is uniform on the unit interval. Thus, the Z's can be sorted and plotted against a 45° line (the cumulative function of the uniform distribution) and a visual diagnostic check can be made of the adequacy of the model. Alternatively, a formal distributional test such as the Kolmogorov-Smirnov test can be applied to check the goodness of fit.[5] One could describe the distribution of spell lengths in this manner for each of the states, but as their is little prior evidence to compare our results for employment and not in the labor force, we focus our attention here on unemployment spells, whose cumulative residual functions are displayed in figures 1-4 for each of the age/sex groups.[6]

For young males and for females of both age groups, the predicted and actual distributions line up very nicely. For these three groups, we calculate K-S statistics of: older females -1.34; young females -0.72; and young males -1.30. The 5% critical value is obtained from the asymptotic distribution of the K-S statistic is 1.36, and thus we cannot reject the hypothesis that the predicted and observed distributions are the same. Thus, conditional on observed characteristics, transitions out of unemployment appears to be Markov for these groups.[7] For adult males, however, the predicted distribution of unemployment spell lengths remains significantly different from the observed, regardless of the number of regressors added, and thus the Markov assumption does not seem to hold here. Notice that in figure 4 the source of the disparity between actual and predicted spell length is that we underpredict long spells of unemployment. This finding for adult males is consistent with the previous findings of Kiefer and Neumann (1979, 1981) and with Clark and Summers (1979), and it suggests that

Table 6

Kolmogorov-Smirnov Statistics Corresponding to Alternative Specifications of Transition Functions

Age/Sex Group	Regressors Included[a] Constant Only (1)	All Except Wages (2)	All (3)
Adult Males (22-59)	3.93	3.00	2.68
Young Males (16-21)	1.42	1.31	1.30
Adult Females (22-59)	2.32	1.95	1.34
Young Females (16-21)	1.53	0.66	0.73

[a] All regressors includes: Education, Assets, Age, Age^2, Number of Children less than six, Race dummies, Wages and Race-Wage interactions.

TABLE 7

Steady State Monthly Transition Rates by Age, Sex and Race, In Percent.

	E-U (1)	E-N (2)	U-E (3)	U-N (4)	N-E (5)	N-U (6)
Young Males: (16-21)	6.5	7.6	22.1	7.6	8.0	4.0
Black	6.9	10.1	16.6	9.5	6.3	4.8
Hispanic	7.4	7.7	22.1	6.6	6.4	3.7
White	5.4	5.9	28.6	5.6	11.5	2.9
Adult Males: (22-59)	1.7	1.4	28.2	4.7	13.7	4.3
Black	1.9	1.1	24.4	3.8	13.7	3.9
Hispanic	1.9	2.0	25.6	5.1	13.1	5.1
White	1.5	1.2	35.5	5.4	14.3	3.6
Young Females: (16-21)	4.5	8.2	20.4	9.1	4.9	3.0
Black	5.2	7.1	14.9	10.3	5.1	4.9
Hispanic	4.7	7.9	17.6	8.8	4.1	2.5
White	3.9	9.1	28.4	7.2	5.4	1.9
Adult Females: (22-59)	1.8	4.7	16.6	10.6	1.0	1.4
Black	1.9	3.2	13.6	11.0	5.1	2.5
Hispanic	1.8	5.8	14.3	7.7	0.3	0.6
White	1.6	5.1	24.8	14.0	4.6	1.3

Source: Calculated from the estimates in Tables 2-5 and the mean values of the exogenous variables reported in Table 1.

either true duration dependence or unmeasured heterogeneity is of importance for these individuals.

The influence of the different regressors is revealed somewhat by considering the adequacy of fit of the model under various specifications of the relevant list of included variables. Table 6 contains the results of fitting transition rates out of unemployment that depend on: (column 1) a constant, (column 2) all variables included in Table 5 except wages, and (column 3) all variables in Table 5.[8]

For all groups, the "fit" of the model improves with the inclusion of individual specific regressors (compare columns (1) and (2)) although for young males the improvement is small. Note that for both young males and females the inclusion of all regressors except wages results in a K-S statistic that would not exceed the critical value, and thus when, in column (3), wages are included little change is noted. On the other hand, for adults, the inclusion of wages does make a significant difference in the fit of the model, and, in the case of females, results in a statistic less than the critical value.

These results can be interpreted in several ways, but the message is essentially the same. Higher wages, or the prospect of same, have a significant effect on the time spent in unemployment for adults, but appreciably less so for teenagers, and in the case of young females, almost no effect.

VI. Steady-State Labor Supply

The transition function parameters given in Tables 2-5 combined with the sample means yield estimates of the steady-state transition rates, which are presented in Table 7 for each of the age and sex groups and within each group by race.[9] Following Marston (1976), the steady state unemployment and labor force participation rates can be shown to be:

$$(28) \qquad = \frac{U}{E+U} = A/(A+B)$$

$$(29) \qquad = (E+U)/(E+U+N) = (1+A/B)/(1+A/B+C/D)$$

where

$$A = \lambda_{12} + (\lambda_{13} \cdot \lambda_{32})/(\lambda_{31} + \lambda_{32})$$
$$B = \lambda_{21} + (\lambda_{23} \cdot \lambda_{31})/(\lambda_{31} + \lambda_{32})$$
$$C = \lambda_{31} + (\lambda_{21} \cdot \lambda_{32})/(\lambda_{21} + \lambda_{23})$$
$$D = \lambda_{13} + (\lambda_{23} \cdot \lambda_{12})/(\lambda_{21} + \lambda_{23})$$

Estimates of these steady-state rates are presented in Table 8.

A rather familiar picture is evident from these data. Unemployment rates vary substantially across the population, being lowest for prime age white males and highest for black teenage males. This conforms well with the findings of previous studies, e.g., Marston (1976), and suggests that DIME controls mirror forces that exist in the national labor market.

For comparison with results obtained from static supply models, it is convenient to express the labor force participation rates in a different form. Figures 5-8 present graphs of the labor force participation rates for each age-sex group. Within each graph the labor supply function of each of the race groups are super imposed. All labor force participation rates were evaluated over the range $w+50\%$ to $w-40\%$, where w is the average wage of

TABLE 8

Steady State Labor Force Participation Rates by Age, Sex and Race

	Labor Force Participation Rate (2)
Young Males: (16-21)	61.2
Black	52.9
Hispanic	57.7
White	71.1
Adult Males: (22-59)	91.8
Black	93.2
Hispanic	89.0
White	92.9
Young Females: (16-21)	48.6
Black	55.2
Hispanic	44.7
White	45.3
Adult Females: (22-59)	30.2
Black	64.6
Hispanic	9.4
White	50.9

Source: Computed from Table 7.

TABLE 9

Elasticity of Labor Force Participation Rates by Age, Sex and Race

DIME Controls

Young Males (16-21)	
Black	0.69
Hispanic	0.44
White	0.24
Young Females (16-21)	
Black	0.71
Hispanic	0.95
White	0.97
Adult Males (22-59)	
Black	0.33
Hispanic	0.51
White	0.27
Adult Females (22-59)	
Black	1.03
Hispanic	0.78
White	0.68

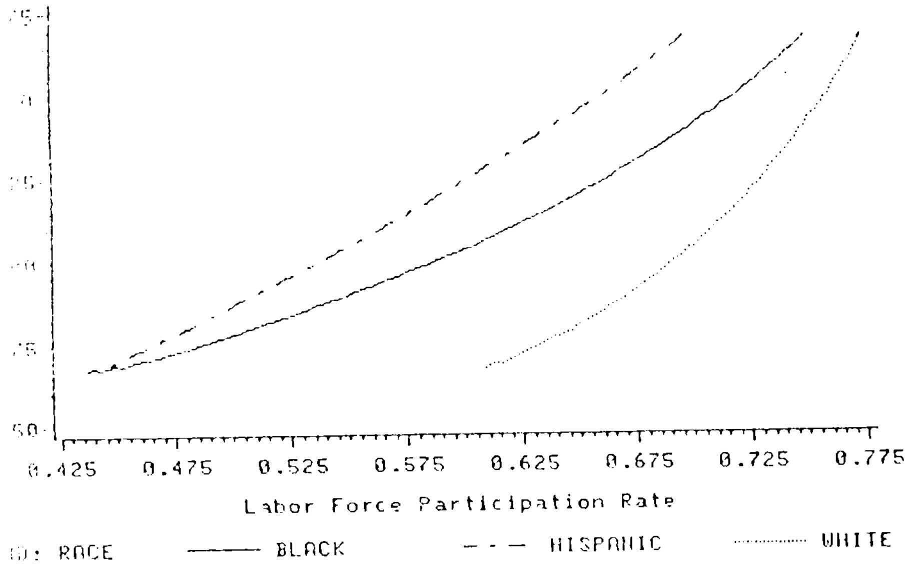

Figure 5

Young Male Labor Force Participation Rates
(By Race: White, Black, and Hispanic)

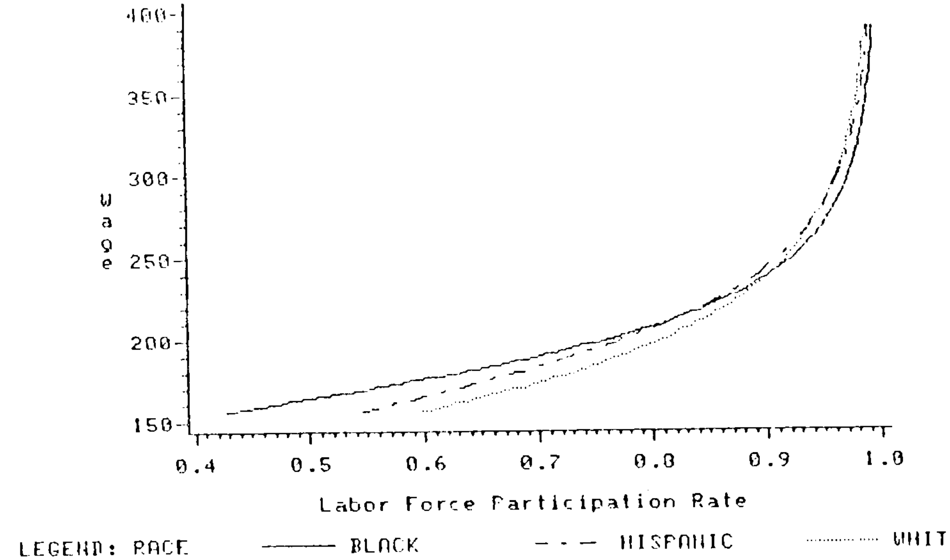

Figure 6

Adult Male Labor Force Participation Rates
(By Race: White, Black, and Hispanic)

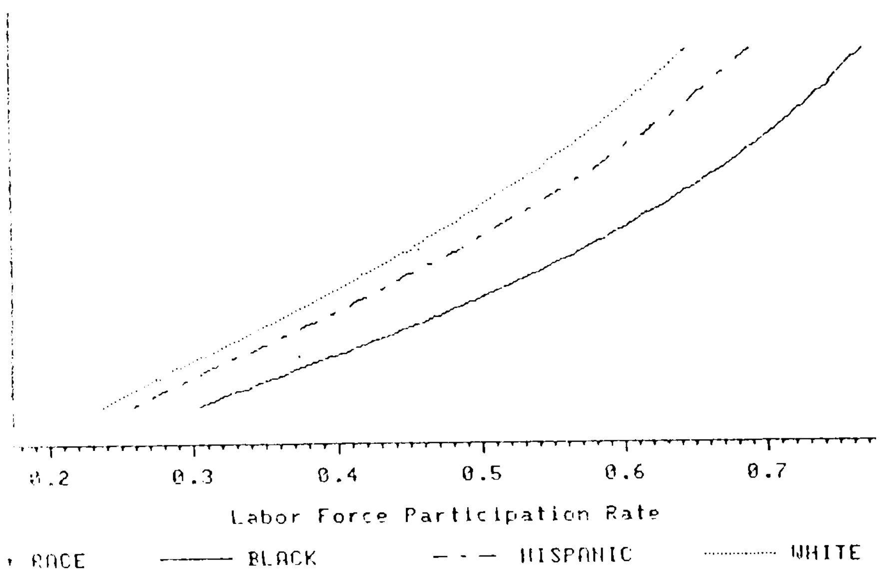

Figure 7

Young Female Labor Force Participation Rates
(By Race: White, Black, and Hispanic)

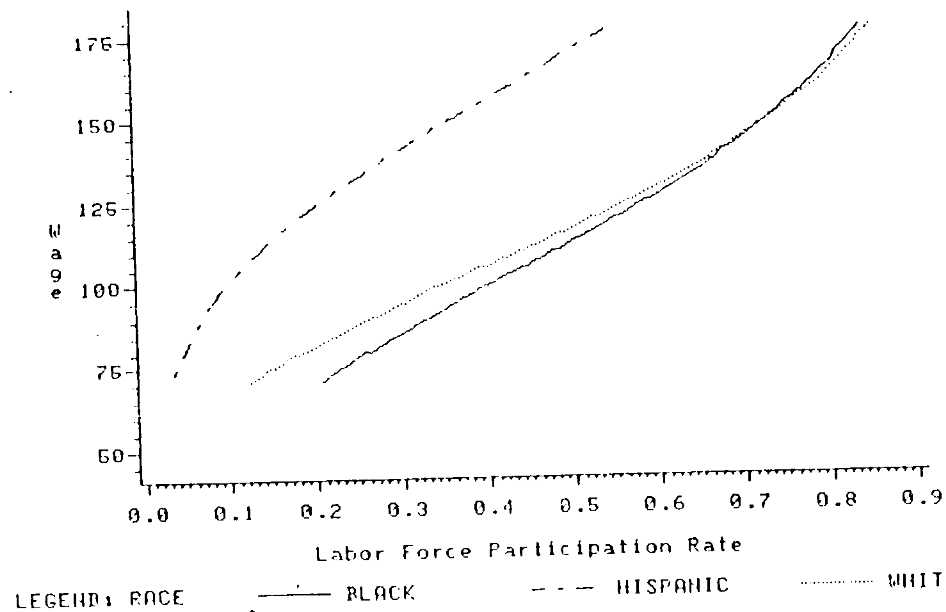

Figure 8

Adult Female Labor Force Participation Rates
(By Race: White, Black, and Hispanic)

whites in each group. Elasticities of labor supply evaluated at the means of all variables are given in table 9.

In conventional studies of labor force participation rates, the responsiveness of supply to wages is usually summarized in a table of wage elasticities such as are presented in Table 7. These estimates reproduce the by now well known facts about labor supply patterns: adult males have the least elastic labor participation schedules and adult females have the most elastic with teenagers falling in between according to sex. Within each age/sex group there are fairly large differences in elasticities which because of differences in the mean values of wages may obscure more fundamental similarities in labor market behavior. The graphs in Figures 5-8 tell an interesting story about these elasticities. For adult males, there is essentially no difference in labor force participation behavior, which implies that the observed differences must be due to the different average wage rates that each group faces. Observation of Figure 6 reveals the same story for black and white adult females, but not for Hispanics. The graphs for teenagers, on the other hand, give no indication of similar responses for the same wage which suggests that variations in non-market opportunities accounts for a significant share of the differential in labor supply among teenagers.

VII. Conclusions

The simple model developed in this paper conveys the idea of how wages function in allocating workers across different labor market states. Individuals are subject to random utility shocks, which may be quite varied in origin--i.e., sickness, unusually favorable education opportunities, wage changes, etc.--and they respond by optimally choosing their labor market status. The central results of the model, stated in propositions 1 and 2, are that higher wage earnings will induce individuals to remain employed longer, that is, any given utility shock is less likely to result in a quit, and if a job separation occurs, higher wage workers are more likely to remain in the labor market. In this sense, the model can be viewed as a dynamic analogue of the static labor market model wherein movements between labor market and non-labor market activity is specifically treated.

Applying the insights of this model to data on the labor market histories of controls in the Denver Income Maintenance Experiment yields several important findings. First, the model appears to fit the data quite well, even though stationarity in the transition rates is imposed. In contrast to the work by Clark and Summers (1979), we find no evidence of duration dependence for females or young males, although there is some evidence that the transition rates of adult males may not be stationary.

A major reason why we find little evidence of non-stationary transition rates is that the estimates are conditional upon individual characteristics, including wages. Consistent with the theory described in Section II, wages have a statistically and quantitatively important effect on transition rates, even for teenagers about whom it is generally believed that non-wage considerations are paramount.[10]

The implications of the effects of wages on the various transition rates are seen most clearly by looking at the steady-state labor force participation rates that can be derived from the underlying transition rates. Although our approach, both conceptually and empirically, differs from conventional studies of labor force participation, the results obtained are in broad agreement with prior studies: adult males have the smallest elasticities of participation with respect to wages and adult females have the largest, with teenagers falling in the middle along sex lines. In contrast to other studies, we find that for adults, with the exception of Hispanic females, the differences in participation rates implied by the wage elasticities is an artifact; when graphed out, the labor force participation functions are almost identical, implying that the observed differences in participation and in the elasticities is accounted for by differences in wages alone.

While the results obtained so far are encouraging in terms of modelling dynamic labor market processes, there is much that needs to be done yet. For example, black male teenagers leave (or lose) jobs at a monthly rate of 17%, while black adult males leave (or lose) jobs at a rate of 3% per month, a pattern which is well known is the major reason for high unemployment rates among black teenagers. (See Hall (1970) and Marston (1976)). How much of this difference is due to choice and how much can be attributed to chance? Do "bad" events, e.g., Sickness, inability to make it to work on time, etc., have a greater incidence among some groups or do existing patterns mirror the tastes of individuals for work versus non-work activity? Answers to these questions are critical for the evaluation of labor market policies, but they are as yet unavailable. We are hopeful that the model and results presented in this paper will be a step in this direction.

Appendix A: The Derivation of Equation (18)

Define

$$\Delta f(x) = f(x + \Delta x) - f(x)$$

for any function $f(x)$. By virtue of (12) and (13) we have

$$(\rho + \eta_n) \frac{\Delta V_n(x)}{\Delta x} = \frac{\Delta u_n(x)}{\Delta x}$$

(A1)

$$+ \eta_n \int \frac{\left[\max_k \left(V_k(x + \Delta x) + \frac{e_k}{\rho+\eta_k}\right) - \max_k \left(V_k(x) + \frac{e_k}{\rho+\eta_k}\right)\right]}{\Delta x} G'_n(e)\,de$$

If e is an interior point of some acceptance set, given x, say $A_j(x)$, the term in square brackets is simply $\Delta V_j(x)$ for all sufficiently small Δx. The same cannot be said if e lies on a boundary between two acceptance sets; equivalently, $\max \left[V_k(x) + \frac{e_k}{\rho+\eta_k} \right]$ is not differentiable in any x on boundaries even if $V_k(x)$ were differentiable everywhere.

However, because $dG(e)$ is a density, the probability that e will fall on a boundary is zero. Hence

$$(\rho + \eta_n) \frac{\Delta V_n(x)}{\Delta x} = \frac{\Delta u_n(x)}{\Delta x}$$

(A2)

$$+ \eta_n \cdot \sum_{m \neq n} \int_{\dot{A}_m(x)} \frac{\Delta V_m(x)}{\Delta x} dG_n(e)$$

for all small Δx, where $\dot{A}_j(x)$ is the interior of $A_j(x)$. By taking limits one obtains:

$$\text{(A3)} \quad (\rho + \eta_n) \frac{dV_n}{dx} = \frac{du_n(x)}{dx} + \sum_{m \neq n} [\eta_n \int_{A_m(x)} G'(e)de] \frac{dV_m}{dx}$$

for all n . Finally, these relations together with (13)-(16) imply (18) by appropriate substitutions.

FOOTNOTES

[1]Mincer and Jovanovic (1981), Bartel (1975), and Kiefer and Neumann (1979, 1981).

[2]See Appendix A for the derivation of equation (18).

[3]See Kalbfleisch and Prentice (1980), p. 6

[4]It should be noted that the theory developed above has even stronger predictions when coupled with the assumption that hazard rates are exponential. Equation (14) implies that the individual transition rates, λ_{nm} , can be factored into an arrival rate, η_n , which is independent of x , and a choice probability, $\alpha_{nm}(x)$. If the λ_{nm} are exponential, the implied choice probabilities are logistic, and in principle then, one can sort out "choice" from "chance," that is, whether some transitions are due to bad luck--a low η_n-- or to preferences. We intend to report on this decomposition in the near future.

[5]It should be noted that neither the visual test nor the K-S test can separate out true duration dependence from unmeasured population heterogeneity (heckman (1980)). The residual analysis described above gets at only the net influence of both factors.

[6]Plots of the remaining states can be obtained from the authors.

[7]This finding is at odds with the results of Clark and Summers (1979). However (see Table 6), when no allowance is made for individual characteristics the predicted and observed distributions differ markedly. Thus the Clark and Summers' results appear to be due to their inability to control for inter-individual variation in factors such as wage rates, and not to duration dependence.

[8]The calculated K-S statistics are conditional on the estimated coefficients, and because estimates may differ from true population values,

the test statistics may understate the true confidence levels. Moreover, the use of estimated coefficients leads to some correlation in the values of the distribution function of duration times due to the common error in the used to calculate each probability. Thus the K-S tests should be viewed heuristically. It should be noted that observation of the residual plots tells the same story. We thank Jim Heckman for bringing this point to our attention.

[9]We have calculated overall transition rates for each age-sex group using the overall means for the entire group. Since the transition funcitons are non-linear, these overall rates are not equal to a weighted averaged of the corresponding race groups.

[10]See Anderson and Sawhill (1980) for a discussion of the factors influencing teenager labor market behavior.

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