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TOBIN'S MARGINAL q AND AVERAGE a : A NEOCLASSICAL INTERPRETATION

by

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ABSTRACT

It is increasingly recognized that Tobin's conjecture that investment is a function of marginal q is equivalent to the firm's optimal capital accumulation problem with adjustment costs. This paper formalizes this idea in a very general fashion and derives the optimal rate of investment as a function of marginal q adjusted for tax parameters. An exact relationship between marginal q and average q is also derived. Marginal q adjusted for tax parameters is then calculated from data on average q assuming the actual U.S. tax system concerning corporate tax rate and depreciation allowances.

1. Introduction

In the last decade and a half, the literature on investment has been dominated by two theories of investment -- the neoclassical theory originated by Jorgenson and the "q" theory suggested by Tobin. The neoclassical theory of investment starts from a firm's optimization behavior. The objective of the firm is to maximize the present discounted value of net cash flows subject to the technological constraints summarized by the production function. It seems useful to divide the neoclassical theory into two stages. The earlier version of the neoclassical approach developed by Jorgenson [11] derives the optimal capital stock under constant returns to scale and exogenously given output. To make the rate of investment determinate, the model is completed by a distributed lag function for net investment. This earlier version of the neoclassical investment theory has a couple of drawbacks. The assumption of exogenously given output (which makes the optimal capital stock determinate) is inconsistent with perfect competition. The theory itself cannot determine the rate of investment; rather, it relies on an *ad hoc* stock adjustment mechanism. Some sort of adjustment costs are introduced implicitly through the distributed lag function for investment.

This point was recognized by Lucas [13], Gould [9], Uzawa [17], and Treadway [16]. Their solution was to introduce the cost of installing new investment goods in the firm's optimization problem. In this formulation, capital stock is given to the firm at each moment of time because of the adjustment costs in changing capital stock.

What the firm can control at each moment of time is the rate of investment, not the stock of capital. This modification of the earlier version of neoclassical theory was in fact recommended by Jorgenson [12] who wrote:

> A derivation of this model incorporating installation costs explicitly with constant returns to scale in both production and installation is obviously much more satisfactory than the original derivation. [Jorgenson [12], pp.223-224]

The alternative theory, suggested by Tobin [15], is that the rate of investment is a function of q, the ratio of the market value of new additional investment goods to their replacement cost. Here again, some sort of adjustment costs lies behind the theory. If a firm can freely change its capital stock, then it will continue to increase or decrease its capital stock until q is equal to unity. Also, the role of the production function is never clear in Tobin's [15] exposition. One may wonder if the "q" theory can be derived from the firm's optimization.

It is increasingly recognized that the modified neoclassical investment theory with installment costs and the "q" theory are equivalent. Lucas and Prescott [14] were the first to recognize this, although they never indicated the connection to the "q" theory. Later Abel [1] has shown that the optimal rate of investment is the rate for which q-1 is equal to the marginal cost of installmemt. However, his discussion is focused primarily on the Cobb-Douglas technology. Yoshikawa [18] arrived at the same conclusion as Abel did, but his model

is characterized by static expectations. Section 2 of the present paper integrates the two theories of investment in a very general model of the firm's present value maximization and derives the optimal rate of investment as a function of q. It turns out that the form of investment function is independent of both the production function and the demand curve for the firm's output. All this comes from a simple application of Pontryagin's maximum principle.

The "q" theory (or, equivalently, the modified neoclassical theory) is not operational as long as q is not observable. Remember that q, which we call marginal q, is the ratio of the market value of an additional unit of capital to its replacement cost. What we can observe is average q, namely the ratio of the market value of existing capital to its replacement cost. Empirical work based on the "q" theory has utilized average q as a proxy for marginal q (see, e.g., von Furstenberg [8]). Section 3 of the present paper derives an exact relationship between marginal q and average q. If the firm is a price-taker with constant returns to scale in both production and installation, then marginal q is equal to average q. If the firm is a price-maker, then average q is higher than marginal q by what is legitimately called as the monopoly rent. Section 3 also indicates how the relationship should be modified if we take account of taxes and depreciation allowances. The (marginal) q that is relevant to the firm's investment decision should reflect tax rules concerning corporate tax rate, investment tax credits, and depreciation formulas. We will call this q the modified q. In section 4 we calculate modified q from data on

average q taking into account the actual U.S. tax system and estimate a simple linear investment function.

2. Optimal Capital Accumulation

Consider a firm acting to maximize the present value of future after-tax net receipts:

$$(1) \quad V(0) = \int_0^\infty R(t) \exp[-\int_0^t r(s)ds]dt ,$$

where r(s) is the nominal discount rate. In (1) net receipts R(t) are written as profits after tax plus depreciation tax deductions minus purchases of investment goods plus investment tax credits:

$$(2) \quad R(t) = [1-u(t)]\ \pi(t) + u(t) \int_0^\infty D(x,t-x)p_I(t-x)I(t-x)dx$$

$$- [1-k(t)]p_I(t)I(t),$$

where $\pi(t)$ is profits before tax at time t, u(t) corporate tax rate, D(x,t-x) depreciation allowance per dollar of investment for tax purposes on an asset of age x according to the tax code that was in effect at time t-x,[2] $p_I(t)$ the price of investment goods, I(t) investment, and k(t) is the rate of investment tax credit. In the earlier version of the neoclassical investment theory, profits are written as

$$(3a) \quad \pi(t) = p(t)F(K(t),N(t),t) - w(t)N(t)$$

where p(t) is the price of the firm's output at time t, F the production function, K(t) capital stock, N(t) the vector of variable factor inputs (e.g., labor), and w(t) is the associated vector of input prices. If the

firm is a price maker, the output price p(t) depends on output F(t). (1) is maximized subject to the capital accumulation equation:

(3b) $\dot{K} = I - \delta K,$

where δ is the rate of physical depreciation. As is well known, the rate of optimal investment is indeterminate in this model, while the optimal level of capital stock can be defined under the assumption that output is exogenously given and the production function is linearly homogenous.[3] If the existing level of capital stock K(0) is lower (higher) than the optimal level K*, investment will be infinitely positive (negative).

The modification introduced by Lucas [13], Gould [9] and Treadway [16] is to introduce installation costs in (3a):[4]

(4a) $\pi = p[F(K,N;t) - G(I,K;t)] - wN.$

The installation function G depends on K as well as on I because the cost of installing I units of investment goods is likely to depend on the size of I relative to K. G will be an increasing and convex function of I: $G_I > 0$, $G_{II} > 0$, reflecting the presumption that the cost of installment per unit of investment will be greater, the greater the rate of investment for any given K.

An alternative way to introduce adjustment costs associated with investment was introduced by Uzawa [17]. He leaves (3a) intact but modifies (3b) as follows:

(4b) $\dot{K} = \psi(I,K;t) - \delta K.$

In this formulation, I units of gross investment do not necessarily

turn into capital; only ψ x 100% of investment does. The graph of ψ is drawn in Figure 1. ψ is increasing and concave in I, reflecting the same presumption stated above. In Figure 1, ψ drops sharply as I changes from 0 to negative, reflecting the irreversibility of investment. We shall call ψ the installation function.

Since the two formulations of adjustment costs give similar results concerning the optimal investment rule, we henceforth focus on Uzawa's formulation. Under (4a), the reader can easily derive formulas corresponding to the ones we will derive below under (4b).[5] Thus the firm is assumed to maximize (1) with π defined by (3a) subject to the capital accumulation constraint (4b). After some manipulations, (1) reduces to

$$(5)\quad V(0) = \int_0^\infty [(1-u)\pi - (1-k-z)p_I I]\exp(-\int_0^t r ds)dt$$

$$+ \int_0^\infty \{u(t)[\int_{-\infty}^0 D(t-v,v)p_I(v)I(v)dv]\exp(-\int_0^t r ds)\}dt,$$

where:

$$(6)\quad z(t) = \int_0^\infty u(t+x)D(x,t)\exp[-\int_0^x r(t+s)ds]dx.$$

The second term in (5), often neglected in the literature, represents the present value of current and future tax deductions attributable to past investments. This important term will be referred to as A(0). Note that z(t) corresponds to uz in the notation in Hall and Jorgenson [10] which assumes static expectations about future tax rate u(t). Since A(0) is independent of current and future decisions by the firm,

the optimization problem is equivalent to maximizing the first term in (5) with respect to I and N subject to (4b).

The first-order conditions for optimality are:

(7) $\pi_N = 0,$

(8) $(1-k-z)p_I = \lambda\psi_I,$

(9) $\dot{\lambda} = (r+\delta-\psi_K)\lambda - (1-u)\pi_K,$

and the transversality condition is:

(10) $$\lim_{t\to\infty} \lambda(t)K(t)\exp(-\int_0^t rds) = 0,$$

where λ is the shadow price for constraint (4b). (7) is the familiar marginal productivity condition. (9) states that λ is the present discounted value of additional future (after-tax) profits that are due to one additional unit of current investment.[6] To interprete (8) in economic terms, write (8) as

(8') $$(1-k)p_I + (1-\psi_I)\lambda = \lambda + zp_I.$$

The first term in (8') is the acquisition price of new investment goods from the viewpoint of the firm. Because of the investment tax credits, it is less than the market price by kp_I where k is the rate of investment tax credits. The second term in (8') represents (implicit) adjustment costs associated with investment. If there were no adjustment costs so that $\psi(I,K;t) = 1$, then the market value of the firm would increase by λ for one additional unit of investment. But the capital stock actually increases by only ψ_I. Thus $(1-\psi_I)\lambda$ represents the market

value foregone due to the concave installation function. The second term on the right side of (8') is the present value of tax deductions due to one unit of current investment. Therefore (8') states that the marginal benefit of installing one unit of new investment goods is equal to the marginal cost of doing so.

Now we can rigorously define Tobin's <u>marginal</u> q as

(11) $$q = \lambda/p_I,$$

and <u>average</u> q as

(12) $$h = V/(p_I K).$$

In terms of q just defined, (8) and (9) can be written as:

(8") $$\frac{q}{1-k-z} = \frac{1}{\psi_I},$$

(9') $$\dot{q} = (r+\delta-\hat{p}_I-\psi_K)q - (1-u)\frac{\pi_K}{p_I},$$

where $\hat{p}_I = \dot{p}_I/p_I$. We can solve (8") for I to obtain the optimal investment rule:

(13) $$I = \alpha(\tilde{q},K;t),$$

where $\tilde{q}$, to be called the modified q, is defined as $q/(1-k-z)$. The remarkable information content of q should be noted. Once q is known (along with 1-k-z and K), the firm can decide the optimal rate of investment through the knowledge of the installation function ψ alone. All the information about the demand curve for the firm's output and the production function that are relevant to investment decision is summarized by q. Expectations about future course of the rate of

investment tax credits k are also incorporated in q and do not affect the form of the investment function (13).

In passing, we note that (13) reduces to the form:

$$(13') \qquad I/K = \beta(\tilde{q};t),$$

if and only if the installation function is linear homogeneous in I and K. The linear homogeneity of ψ will pay an important role in the discussion of marginal q and average q, to which we now turn.

3. Marginal q and Average q

If we knew marginal q, then econometric implementation of the "q" theory would be quite straightforward. Unfortunately, however, marginal q is not directly observable. What we can (in principle) observe is average q. There have been increasing efforts to measure average q for the U.S. corporations (Ciccolo [5], von Furstenberg [8]), and people are busying themselves regressing investment on average q. The researchers should feel uneasy about doing this, unless they are sure that average q and marginal q are practically the same thing. The following proposition states that marginal q and average q are essentially the same in the special yet important case where the firm is a price-taker and the production function and the installation function are homogeneous.

PROPOSITION L: <u>Let</u> A(0) <u>be the present discounted value of current and future tax deductions attributable to past investments (the last term in</u> (5)). <u>Suppose the firm is a price-taker in its output market and suppose the transversality condition</u> (10) <u>holds. Then</u>:

(14) $$q(0) = h(0) - \frac{A(0)}{p_I(0)K(0)}$$

<u>if and only if the installation function</u> $\psi(I,K;t)$ <u>is linearly homogeneous in</u> I <u>and</u> K <u>and the production function</u> $F(K,N;t)$ <u>is linearly homogeneous in</u> K <u>and</u> N.

PROOF: First suppose F and ψ are linearly homogeneous. Since the firm is a price-taker, we have, from (7),

(15) $$F_N = w/p.$$

Since F is homogeneous, (15) implies

(16) $$\pi/K = \pi_K.$$

Since ψ is homogeneous, we have

(17) $$\psi_I I + \psi_K K = \psi.$$

Now consider

(18) $$\frac{d}{dt}\left[\lambda(t)K(t)\exp(-\int_0^t rds)\right] = [\dot{\lambda}K + \lambda\dot{K} - r\lambda K]\exp(-\int_0^t rds)$$

along an optimal path. Using (16),(17),(8),(9),(4b), we can easily establish

(19) $$\frac{d}{dt}\left[\lambda K\exp(-\int_0^t rds)\right] = -[(1-u)\pi - (1-k-z)p_I I]\exp(-\int_0^t rds).$$

Integrating (19) from 0 to infinity and using the transversality condition (10), we obtain

(20) $$\lambda(0)K(0) = \int_0^\infty [(1-\mu)\pi - (1-k-z)p_I I]\exp(-\int_0^t rds)dt,$$

which immediately implies (14). The converse is now obvious. Q.E.D.

REMARK 1: The proposition holds at any point in time along the optimal path.

REMARK 2: Since the installation function is concave in I, the optimal path is unique if it exists.

REMARK 3: The relationship (20) has already been noted in a different context by Blinder and Weiss [3].

The economic intuition behind this result is the following. For simplicity, let us ignore for a moment taxes and depreciation. Let $I(t)$ and $N(t)$ be the optimal policy for the firm with capital stock K_0 at time 0. Now consider another firm with identical production function and installation function but with a different level of capital stock K_0'. It is clear that this second firm's optimal policy is $I(t)K_0'/K_0$ and $N(t)K_0'/K_0$ if the production and installation functions are characterized by constant returns to scale. Hence the expected future profits of the second firm are equal to K_0'/K_0 times those of the first firm, implying that the first firm's average q is equal to the second firm's average q. In other words, average q is independent of the initial capital stock if the production and installation functions are linearly homogeneous and if the firm is a price-taker. Now consider a firm undertaking I units of investment which will be turned into $\Delta K = \psi(I,K;t)$ units of additional capital stock. What is the market value of these additional units of capital? It is ΔK times average q because the average market value of the firm with K is equal to that of the firm with $K + \Delta K$.

As we have seen at the end of section 2, (13') is a necessary and

sufficient condition for the installation function to be linearly homogeneous. A corollary to Proposition 1 is therefore the following. If (i) the optimal investment rule is (13'), (ii) the production function is linearly homogeneous, and (iii) the firm is a price-taker, then (13') is written as:

$$(13'') \qquad I/K = \beta(\frac{h-a}{1-k-z};t),$$

where $a = a/(p_I K)$.[7]

The economic intuition behind Proposition 1 does not carry over to a price-making firm, since if the firm expands its output, the output price will fall. The market value of additional units of capital is therefore less than the average market value of the existing capital stock. However, a fairly simple relationship between marginal q and average q still exists. The following proposition is a generalization of Proposition 1.

PROPOSITION 2: Suppose the firm is a price-maker in the output market. If the production and installation functions are linearly homogeneous and if the transversality condition holds, then:

$$(21) \qquad q(0) = h(0) - a(0) - \frac{1}{p_I(0)K(0)} \int_0^\infty [\eta(t)(1-u)pF \exp(-\int_0^t rds)]dt,$$

where $a(0) = A(0)/(p_I(0)K(0))$ and $\eta(t) = -\frac{F}{p}\frac{dp(t)}{dF}$, the inverse of the elasticity of demand for the firm's output.

PROOF: The first-order condition (7) becomes

$$(22) \qquad (1-\eta)pF_N = w.$$

This and the homogeneity of F imply

(23) $$\pi = pF_K K + \eta pF_N N.$$

We also have

(24) $$\pi_K = (1-\eta)pF_K$$

from (3a). (23) and (24) imply

(25) $$\pi/K = \pi_K + \eta pF/K.$$

Using (25),(17),(8),(9),(4b), it is easy to show that

(26) $$\frac{d}{dt}[\lambda \exp(-\int_0^t rds)] = -[(1-u)\pi - (1-k-z)p_I I + \eta pF(1-u)]\exp(-\int_0^t rds).$$

Integrating (26) from 0 to infinity and using (10), we obtain (21). Q.E.D.

The last term in (21) has the clear interpretation of monopoly rent. Suppose the price-making firm chooses I and N as optimum. We note that a price-taking firm with the same production and installation functions and with the same capital stock will choose the same I and N if the output price is $(1-\eta)p$. Hence the difference between the price-maker's profits and the price-taker's profits is ηpF.

4. Some Empirical Evidence

We have seen in eq.(13) that the optimal investment is a function of modified q. We have also seen that modified q can be written as[8]

(27) $$\tilde{q} = \frac{h-a}{1-k-z},$$

if the firm is a price-taker and if the production and installation functions are linear homogeneous. In this section we calculate a,z, and $\tilde{q}$ for the period 1952-78 for the U.S. corporate sector as a whole assuming the actual U.S. postwar tax system. To compute a and z, we

have to make some assumptions on expectations about future tax rate u(t) and the future nominal discount rate r(t). We assume static expectations about r(t). Since we will use the long rate -- the Baa corporate bond rate plus 4% -- for r(t), this assumption seems innocuous. We also assume that future corporate tax rate is expected to be constant at 48%. Under these assumptions, the discrete-time analogue of the expression for A is:

$$(28) \quad A_t = u \sum_{n=1}^{\infty} \sum_{i=1}^{\infty} D(n+i,t-n)(1+r_t)^{-i+1} p_I(t-n) I(t-n), \qquad (t=1952,\ldots,1978)$$

where: I(t-n) is investment in year t-n, T_t asset life for tax purposes, r_t Baa rate at time t plus 4%, D(n,t-n) depreciation formula, i.e., depreciation allowance on an asset of age n according to the tax code as of t-n, and u is corporate tax rate (48%). Similarly, the present discounted value of tax deductions on new investment, z, is calculated by the formula

$$(29) \quad z_t = u \sum_{n=1}^{T_t} D(n,t)(1+r_t)^{-n}. \qquad (t=1952,\ldots,1978)$$

It is assumed that the straight line depreciation formula was adopted by the corporations prior to 1954 and that the sum-of-years-digits formula was adopted after 1954.[9] The data on tax life (T_t) and corporate investment for each of the three types of assets -- producers durable equipments, nonresidential structures, and residential structures -- are taken from Christensen and Jorgenson [4]. A_t and z_t are calculated for each type of assets.[10] Column 1 of Table I reports A_t aggregated over the three types of assets. Column 2 reports the replacement cost of corporate capital (source: Christensen and Jorgenson, [4]).[11] Column 3 is the ratio of

column 1 and column 2. Golumn 4 gives the weighted average of z_t with weights being corporate investment in each of the three types of assets. Column 5 reports the rate of investment tax credits which is defined as the ratio of corporate investment tax credits claimed (source: Christensen and Jorgenson, ibid.) to corporate investment in the three types of assets. Column 6 gives average q taken from von Furstenberg [8].[12] Modified q as defined by (27) is given in column 7. Column 8 reports corporate investment in the three types of assets.

Several features stand out in Table 1. Both A_t and z_t increase during the period ending the mid-sixties. This is because the U.S. tax law has been allowing faster write-offs (shorter tax lives). From the late sixties the nominal interest rate began to increase noticeably. This explains the downward trend in z_t since 1967. This also partly explains the drastic downward trend in a_t in the late sixties and the seventies, but the main reason for the downward trend in a_t is that A_t is evaluated at acquisition prices. The denominator (i.e., the capital stock at replacement costs) immediately reflects changes in the price of investment goods, while the numerator A_t depends on the prices at which the existing assets were acquired in the past. Therefore, in a period of sustained inflation, the denominator grows faster than the numerator.[13] Comparing average q with modified q, we notice that the movement in modified q is less pronounced than that in average q. This is explained by the downward trend in a_t after the mid-sixties and by the upward trend in the rate of investment tax credits.

Finally, to get a rough idea about how much $\tilde{q}$ can explain aggregate investment, the linear form of (13") is estimated by the ordinary least

squares (OLS) on the data presented in Table 1. Our OLS estimate is:

$$(30) \qquad 1/K = \underset{(.00840)}{.0980} + \underset{(.00912)}{.0423}\,\tilde{q}, \qquad D.W. = .43,\ R^2 = .46,$$

sample period: 1953-1976, mean of the dependent variable = .136. The figures in the parentheses are standard errors. The Durbin-Watson statistic (D.W.) is very low and indicates a strong positive serial correlation in the error term.

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FOOTNOTES

[1]I am grateful to Olivier Blanchard, Dale Jorgenson, and Marty Sullivan for helpful discussions. An anonymous referee also provided useful comments, one of which lead to a simplification of the proof of Proposition 1.

[2]This formulation implicitly assumes that changes in the depreciation formula $D(x,t)$ do not apply retrospectively to past investments, which is the case in the postwar U.S. except for the 1962 change.

[3]An alternative way to make the optimal capital stock determinate is to assume diminishing returns to scale. See Arrow [2] for the derivation of the optimal capital stock under this assumption.

[4]This is anticipated by Eisner and Strotz [7].

[5]If (4a) instead of (4b) is assumed, then eq.(8) becomes: $(1-k-z)p_I + pG_I(1-u) = \lambda$. Also, the homogeneity of ψ is replaced by that of G in Propositions 1 and 2.

[6]There are two benefits from increasing capital stock in our formulation. The first is the resulting increase in the productive capacity of the firm. This is represented by π_K. The second is that the installation function ψ as a function of I shifts downward as K increases. This is represented by ψ_K in (9).

[7]If taxes and subsidies are all ignored, then (13") reduces to the investment function derived by Lucas and Prescott [14].

[8]Under the Revenue Act of 1954, three depreciation formulas including the sum-of-years-digits were allowed for tax purposes. For a wide range

of the tax life and the interest rate, the sum-of-years-digits formula dominates the other two depreciation formulas in that it gives the highest value of z. See Hall and Jorgenson [10] for more details on the depreciation formulas in the postwar U.S. Actual depreciation practice by the U.S. corporations are much more complicated than our assumption presumes. More fact-finding empirical research on this is highly desirable.

[9] A detailed description of the calculation along with a FORTRAN program for the calculation is available upon request from the author.

[11] This does not include land and inventories. Capital stock is calculated by the perpetual inventory method which does not take the installation costs into account. A convenient interpretation is to say that the installation costs are submerged in physical depreciation. Anyway, capital stock is a smooth time series and would not play a crucial role in the discussion below.

[12] In obtaining quarterly series, von Furstenberg took the moving average of two successive end-of-quarter values of average q. To obtain the beginning-of-year value, we take the simple average of the last quarter value of the previous year and the first quarter value of the current year.

[13] However, this is not a channel through which inflation depresses investment. To see this, suppose a change occurs in the corporate tax law which now allows a firm to evaluate the existing assets at current prices in calculating depreciation allowances. Then, A_t will immediately change to reflect current asset prices, but that change will be exactly offset by a re-evaluation of the firm in the stock market, leaving modified q unchanged.

TABLE I.

year	(1) A	(2) $p_I K$	(3) a	(4) z	(5) k	(6) h	(7) $\tilde{q}$	(8) $p_I I$
1952	36.74	172.27	.213	.298	.0	.564	.499	22.58
1953	36.75	182.63	.201	.294	.0	.596	.559	24.88
1954	44.35	191.57	.232	.309	.0	.587	.515	24.59
1955	49.74	202.25	.246	.316	.0	.711	.680	27.75
1956	54.60	227.18	.240	.318	.0	.774	.783	32.46
1957	58.81	250.49	.235	.314	.0	.727	.718	34.78
1958	65.52	261.59	.250	.321	.0	.674	.624	30.09
1959	69.25	270.36	.256	.330	.0	.807	.822	32.66
1960	73.86	278.43	.265	.329	.0	.842	.859	35.22
1961	79.16	286.35	.276	.329	.0	.853	.859	35.04
1962	83.30	295.30	.282	.330	.0216	.954	1.036	38.68
1963	88.01	307.90	.288	.332	.0276	.898	.956	40.58
1964	91.98	322.59	.285	.354	.0289	1.009	1.174	45.58
1965	96.87	342.97	.282	.354	.0314	1.062	1.268	54.71
1966	100.70	376.77	.267	.345	.0319	1.116	1.363	62.94
1967	106.27	418.58	.254	.339	.0325	.919	1.058	63.79
1968	109.82	461.67	.238	.332	.0341	.949	1.121	69.91
1969	113.10	514.89	.220	.322	.0244	.979	1.162	78.23
1970	115.85	577.54	.201	.327	.0110	.789	.888	78.75
1971	123.28	637.27	.193	.332	.0196	.761	.875	83.34
1972	129.83	689.65	.188	.336	.0321	.794	.959	94.15
1973	136.33	756.54	.180	.336	.0339	.869	1.094	110.72
1974	141.95	903.56	.157	.327	.0347	.753	.933	121.64
1975	149.12	1074.71	.139	.317	.0548	.581	.704	117.81
1976	159.50	1143.43	.139	.327	.0692	.623	.800	130.30
1977	172.29	1244.44	.138	.335	.0724	n.a.	n.a.	152.08
1978	184.58	1386.22	.133	.328	.0706	n.a.	n.a.	178.48

Note: See the text for definitions and data sources.

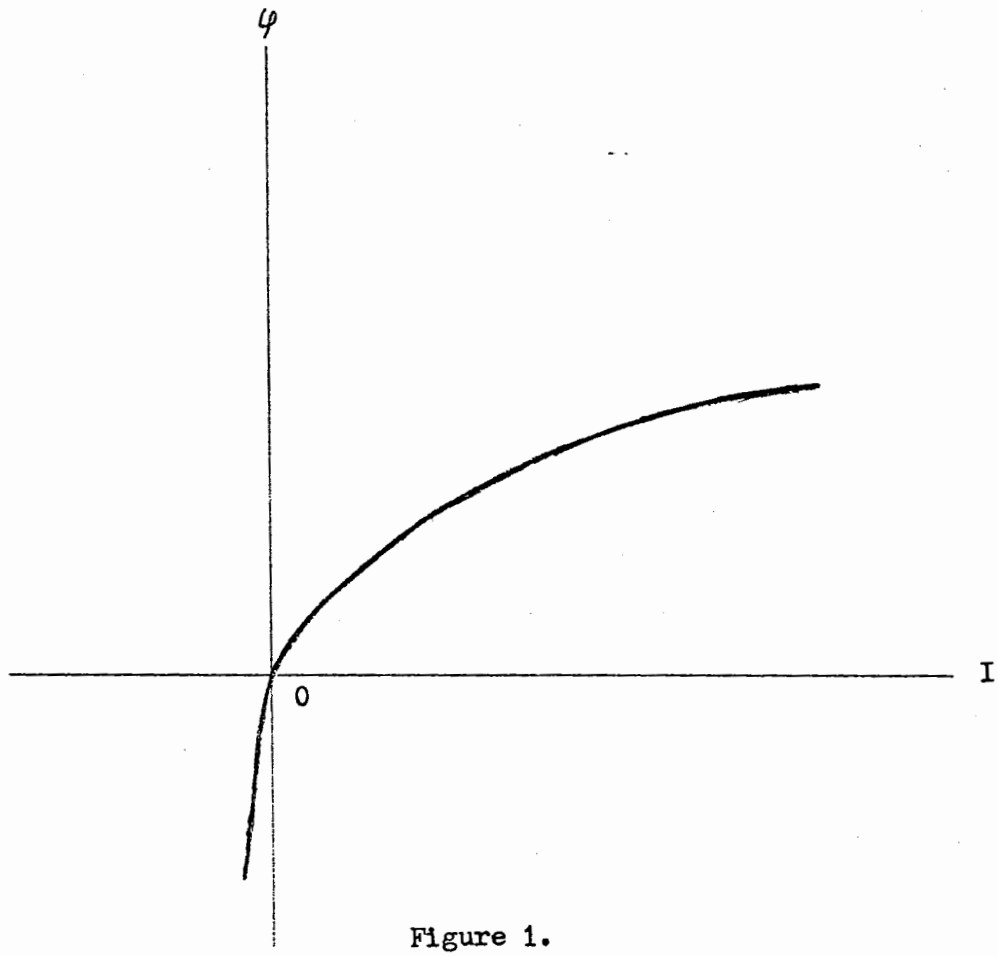

Figure 1.