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TIME TO BUILD AND THE PERSISTENCE OF UNEMPLOYMENT*

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The equilibrium growth model is modified and used to explain the cyclical variances of a set of economic time series, the covariances between real output and the other series, and the autocovariance of output. The model is estimated using quarterly data for the post-war U.S. economy. The crucial feature of the model that gives rise to the persistence of cyclical output is the assumption that more than one time period is required for the construction of new productive capital. The fit is surprisingly good in light of the model's simplicity and the small number of free paramaters.

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1. Introduction

That wine is not made in a day has been recognized by economists (e.g., Böhm-Bawerk [1891]) for a long time. But, neither are ships nor factories built in a day. The thesis of this essay is that the assumption of multiple-period construction is crucial for explaining the serial correlation properties of the cyclical component of real output. A model of economic fluctuations is developed and estimated using U.S. quarterly data for the post-war period. The serial correlations of cyclical output for the model match well with those observed for the U.S. economy in that period. The standard deviations and correlations with output of the other variables determined by the model are also consistent with the U.S. data.

Our approach integrates growth and business cycle theory. Like standard growth theory, a representative infinitely-lived household is assumed. As fluctuations in employment are central to the business cycle, the stand-in consumer values not only consumption but also leisure. The most important modification to the standard growth model is that multiple periods are required to build new capital goods and only finished capital goods are part of the productive capital stock. Each stage of production requires a period and utilizes resources. Half-finished ships and factories are not part of the productive capital stock. Section 2 contains a short critique of the commonly used investment technologies, and presents evidence that single period production, even with adjustment costs, is inadequate. The preference-technology-information structure of the model is presented in Section 3. The exogenous stochastic components in the model are shocks to technology and imperfect indicators of productivity. The two technology shocks differ in their persistence.

The steady state for the model is determined in Section 4, and quadratic approximations are made which result in an "indirect" quadratic utility function that values leisure, the capital goods, and the negative of investments. Most of the relatively small number of parameters are estimated using findings in other applied areas of economics or steady state considerations. For example, the degree of risk aversion is selected to be consistent with the market price of non-diversifiable risk reported in the finance literature; the number of periods required to build new productive capital is of the magnitude reported by business; the parameters of preferences for leisure and consumption streams are not inconsistent with the finding of labor economics. The small set of free parameters imposes considerable discipline upon the inquiry. The estimated model and the comparison of its predictions with the empirical regularities of interest are in Section 5. The final section contains concluding comments.

2. A Critique of Conventional Aggregate Investment Technologies

There are two basic technologies that have been adopted in empirical studies of aggregate investment behavior. The first assumes a constant-returns-to-scale neoclassical production function F with labor L and capital K as the inputs. Total output F(K,L) constrains the sum of investment and consumption, or

$$C + I \leq F(K,L) ,$$

where $C,I,K,L \geq 0$, the rate of change of capital $\dot{K}$ is investment less depreciation and depreciation is proportional with factor δ to the capital stock:

$$\dot{K} = I - \delta K.$$

This is the technology underlying the work of Jorgenson (1965) on investment behavior.

An implication of this technology is that the relative price of the investment and consumption good will be a constant independent of the relative outputs of the consumption and investment good.[1] It also implies that the shadow price of existing capital will be the same as the price of the investment good.[2] There is a sizable empirical literature that has found a strong association between the level of investment and a shadow price of capital

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[1] This, of course, assumes neither C nor I is zero. Sargent (1979a) within a growth context with shocks to both preference and technologies has at a theoretical level analyzed the equilibrium with corners. Only when investment was zero did the price of the investment good relative to that of the consumption good become different from one and then it was less than one. This was not an empirical study and Sargent states that there currently are no computationally practical econometric methods for conducting an empirical investigation within that theoretical framework.

[2] The shadow price of capital has been emphasized by Brunner and Meltzer (1972) and Tobin (1969) in their aggregate models.

obtained from stock market data.[1] This finding is inconsistent with this assumed technology as is the fact that this shadow price varies considerably over the business cycle.

The alternative technology, which is consistent with these findings, is the single capital good adjustment cost technology.[2] Much of that literature is based upon the problem facing the firm and the aggregation problem receives little attention. This has led some to distinguish between internal and external adjustment costs. For aggregate investment theory this is not an issue (see Mussa [1977]) though for other questions it will be. Labor resources are needed to install capital whether the acquiring or supplying firm installs the equipment. With competitive equilibrium it is the aggregate production possibility set that matters. That is, if the Y_j are the production possibility sets of the firms associated with a given industrial organization and Y_j' for some other industrial organization, the same aggregate supply behavior results if $\Sigma\, Y_j = \Sigma\, Y_j'$.

The adjustment cost model, rather than assuming a linear product transformation curve between the investment and consumption goods, imposes curvature. This can be represented by the following technology:

$$G(C,I) \leq F(K,L)\,,$$

$$\dot{K} = I - \delta K\,,$$

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[1] See Malkiel, von Furstenberg, and Watson (1979).

[2] See Abel (1979) and Hayashi (1980) for recent empirical studies based on this technology.

where G like F is increasing, concave and homogeneous of degree one. Letting the price of the consumption good be one, the price of the investment good q_t, the rental price of capital r_t, and the wage rate w_t, the firm's problem is to maximize real profits

$$C_t + q_t I_t - w_t L_t - r_t K_t$$

subject to the production constraint. As constant returns to scale are assumed, the distribution of capital does not matter, and one can proceed as if there were a single price-taking firm. Assuming an interior solution, given that this technology displays constant returns to scale and that the technology is separable between inputs and outputs,

$$I_t = F(K_t, L_t)h(q_t) \equiv Z_t h(q_t),$$

where Z_t is defined to be aggregate output. The function h is increasing so high investment-output ratios are associated with a high price of the investment good relative to the consumption good. Figure 1 depicts the investment-consumption product transformation curve and Figure 2 the function h(q).

Figure 1

Investment-Consumption Transformation Curve for Adjustment Cost Theory

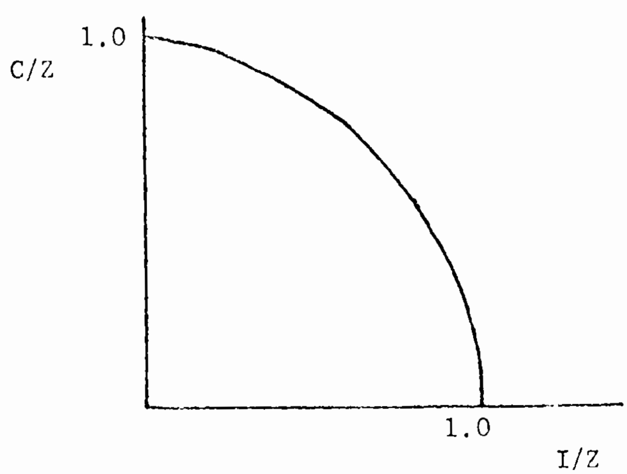
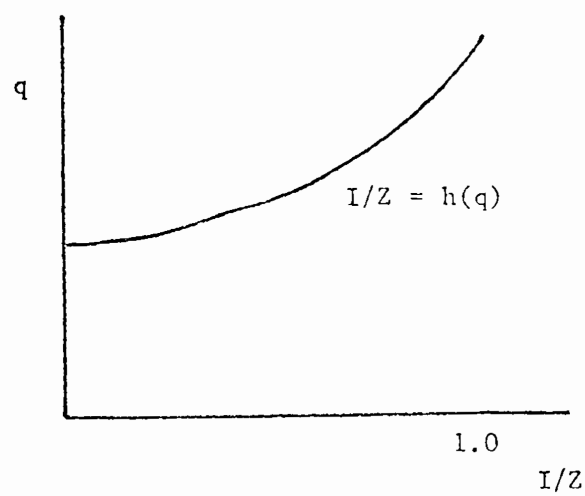


Figure 2

Investment-Supply Curve



For any I/Z, the negative of the slope of the transformation curve in Figure 1 is the height of the curve in Figure 2. This establishes that a higher q will be associated with higher investment for this technology. This restriction of the theory is consistent with the empirical findings previously cited.

There are other predictions of this theory, however, which are questionable. If we think of the q-investment curve h depicted in Figure 2 as a supply curve, the short- and the long-run supply elasticities will be equal. Typically economists argue that there are specialized resources which cannot be instantaneously and costlessly transferred between industries and that even though short-run elasticities may be low, in the long run supply elasticities are high. As there are no specialized resources for the adjustment cost technology, such considerations are absent and there are no penalties resulting from rapid adjustment in the relative outputs of the consumption and investment good.

To test whether the theory is a reasonable approximation, we examined cross-section state data. The correlations between the ratios of commercial construction to either state personal income or state employment and price per square foot[1] are both -0.35. With perfectly elastic supply and uncorrelated supply and demand errors, this correlation can not be positive. To explain this large negative correlation, one needs a combination of high variability in the cross-sectional supply relative to cross-sectional demand plus a positive slope for the supply curve. Our view is that, given mobility of resources, it seems more plausible that the demand is the more variable. Admitting potential data problems, this cross-sectional result casts some doubt upon the adequacy of the single capital good adjustment cost model.

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[1] The data on commercial construction and price per square foot was for 1978 and was obtained from F.W. Dodge Division of McGraw-Hill.

At the aggregate level an implication of the single capital good adjustment cost model is that when the investment-output ratio is regressed on current and lagged q, only current q should matter.[1] The findings of Malkiel, von Furstenberg, and Watson (1979) are counter to this prediction.

In summary, our view is that neither the neoclassical nor the adjustment cost technologies are adequate. The neoclassical structure is inconsistent with the positive assocation between the shadow prices of capital and investment activity. The adjustment cost technology is consistent with this observation but inconsistent with cross-sectional data and the association of investment with the lagged as well as the current capital shadow prices. In addition, the implication that long- and short-run supply elasticities are equal is one which we think a technology should not have.

Most destructive of all to the adjustment-cost technology, however, is the finding that the time required to complete investment projects is not short relative to the business cycle. Mayer (1960), on the basis of a survey, found that the average time (weighted by the size of the project) between the decision to undertake an investment project and the completion of it was twenty-one months. Similarly, Hall (1977) found the average lag between the design of a project and when it becomes productive to be about two years. It is the thesis of this essay that periods this long or even half that long have important effects upon the serial correlation properties of the cyclical component of output.

The technological requirement that there are multiple stages of production is not the delivery lag problem considered by Jorgenson (1965). He theorized at the firm level and imposed no consistency of behavior requirement

[1]This observation is due to Hayashi (1980).

for suppliers and demanders of the investment good. His was not a market equilibrium analysis and there was no theory accounting for the delivery lag. Developing such a market theory with information asymmetries, queues, rationing, and the like is a challenging problem confronting students of industrial organization.

Our technology assumes that a single period is required for each stage of construction or that the time required to build new capital is a constant. This is not to argue that there are not alternative technologies with different construction periods, patterns of resource use, and total costs. We have found no evidence that the capital goods are built significantly more rapidly when total investment activity is higher or lower. Lengthening delivery lags (see Carlton [1979]) in periods of high activity may be a matter of longer queues and actual construction times may be shorter. Premiums paid for earlier delivery could very well be for a more advanced position in the queue than for a more rapidly constructed factory. These are, of course, empirical questions and important cyclical variation in the construction period would necessitate an alternative technology.

Our time-to-build technology is consistent with short-run fluctuations in the shadow price of capital because in the short run capital is supplied inelastically. It also implies that the long-run supply is infinitely elastic so on average the relative price of the investment good is independent of the investment-output ratio.

3. The Model

Technology

The technology assumes time is required to build new productive capital. Let s_{jt} be the number of projects j stages or j periods from completion for $j = 1,\ldots, J-1$, where J periods are required to build new productive capacity. New investment projects initiated in period t are s_{Jt}. The recursive representation of the laws of motion of these capital stocks is

(3.1) $$k_{t+1} = (1-\delta)k_t + s_{1t},$$

(3.2) $$s_{j,t+1} = s_{j+1,t} \quad \text{for} \quad j = 1,\ldots,J-1.$$

Here δ is the depreciation rate. The element s_{Jt} is a decision variable for period t.

The final capital good is the inventory stock y_t inherited from the previous period.[1] Thus in this economy there are J + 1 types of capital: inventories y_t, productive capital k_t, and the capital stocks j stages from completion for $j = 1,\ldots,J-1$. These variables summarize the effect of past decisions upon current and future production possibilities.

Let φ_j for $j = 1,\ldots,J$ be the fraction of the resources allocated to the investment project in the j^{th} stage from the last. Total non-inventory investment in period t is

$$\sum_{j=1}^{J} \varphi_j s_{jt}.$$

Total investment i_t is this amount plus inventory investment, $y_{t+1} - y_t$, and consequently

(3.3) $$i_t = \sum_{j=1}^{J} \varphi_j s_{jt} + y_{t+1} - y_t.$$

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[1] All stocks are beginning-of-the-period stocks.

Total output, that is, the sum of consumption c_t and investment, is constrained as follows:

$$c_t + i_t \leq f(\lambda_t, k_t, n_t, y_t), \tag{3.4}$$

where n_t is labor input, λ_t a shock to technology and f a constant-returns-to-scale production function to be parameterized subsequently.

Treating inventories as a factor of production warrants some discussion. With larger inventories, stores can economize on labor resources allocated to restocking. Firms, by making larger production runs, reduce equipment down time associated with shifting from producing one type of good to another. Besides considerations such as these, analytic considerations necessitated this approach. If inventories were not a factor of production, it would be impossible to locally approximate the economy using a quadratic objective and linear constraints. Without such an approximation no practical computational method currently exists for computing the equilibrium process of the model.

The production function is assumed to have the form

$$f(\lambda, k, n, y) = \lambda n^{\theta}[(1 - \sigma)k^{-\nu} + \sigma y^{-\nu}]^{-\frac{1-\theta}{\nu}} \tag{3.5}$$

where $0 < \theta < 1$, $0 < \sigma < 1$, and $-1 \leq \nu < \infty$. This form was selected because, among other things, it results in a share θ for labor in the steady state. The elasticity of substitution between capital and inventory is $1/(1+\nu)$. This substitution parameter is probably less than one which requires ν to be positive.

Preferences

The preference function whose expected value the representative household maximizes has the form

$$\Sigma\ \beta^t u[c_t,\ \alpha(L)\ell_t],\ 0 < \beta < 1,$$

where β is the discount factor, ℓ_t leisure, L the lag operator, and

$$\alpha(L) = \sum_{i=0}^{\infty} \alpha_i L^i.$$

This structure admits intertemporal substitution of leisure. Normalizing so one is the endowment of time, we let $n_t = 1 - \ell_t$ be the time allocated to market activity. The polynomial lag operator is restricted so that the α_i sum to one and

$$\alpha_i = (1 - \eta)^{i-1}\ \alpha_1,\ i \geq 1, \text{ where } 0 < \eta \leq 1.$$

With these restrictions

$$\alpha(L)\ell_t = 1 - \alpha(L)n_t = 1 - \alpha_o n_t - (1 - \alpha_o)\eta \sum_{i=1}^{\infty} (1 - \eta)^{i-1}\ n_{t-i}.$$

By defining the variable

$$a_t = \sum_{i=1}^{\infty} (1 - \eta)^{i-1}\ n_{t-i}\ ,$$

the distributed lag has the following recursive representation

$$\alpha(L)\ell_t = 1 - \alpha_o n_t - \eta(1 - \alpha_o)a_t,$$

$$a_{t+1} = (1 - \eta)a_t + n_t. \tag{3.6}$$

The variable a_t summarizes the effects of all past leisure choices on current and future preferences. If $n_s = n_t$ for all $s \leq t$, then $a_t = n_t/\eta$ and the distributed lag is simply $1 - n_t$.

The parameters α_o and η determine the degree to which leisure is intertemporally substitutable. We require $0 < \eta \leq 1$ and $0 < \alpha_o \leq 1$. The nearer α_o is to one, the less is the intertemporal substitution of leisure.

For α_0 equal to one, a time-separable utility results. With η equal one, a_t equals n_{t-1}. This is the structure employed by Sargent (1979b). As η approaches zero, past leisure choices have greater effect upon current utility flows.

Non-time-separable utility functions are implicit in the empirical study of aggregate labor supply by Lucas and Rapping (1969). Grossman (1973) and Lucas (1977) discuss why a non-time-separable utility function is needed to explain the business cycle fluctuations in employment and consumption. Cross-sectional evidence of households' willingness to redistribute labor supply over time is the lumpiness of that supply. There are vacations and movements of household members into and out of the labor force for extended periods which are not in response to large movements in the real wage. Another observation suggesting high intertemporal substitutability of leisure is the large seasonal variation in hours of market employment. Finally, the failure of Abowd and Ashenfelter (1981) to find a significant wage premium for jobs with more variable employment and earnings patterns is further evidence. In summary, it is difficult to rationalize the observed patterns of employment and wages (actually marginal products of labor) with a standard time-separable utility function.

The utility function in our model is assumed to have the form

$$u[c_t,\alpha(L)\ell_t] = \frac{1}{\gamma}[c_t^{1/3}(\alpha(L)\ell_t)^{2/3}]^{\gamma},$$

where $\gamma < 1$ and $\gamma \neq 0$. If the term in the square brackets is interpreted as a composite commodity, then this is the constant-relative-risk-aversion utility function with the relative degree of risk aversion being $1 - \gamma$. We thought this composite commodity should be homogeneous of degree one as is the case when there is a single good. The relative size of the two exponents inside the brackets is motivated by the fact that households' allocation of time to non-market activities is about twice as large as the allocation to market activities.

Information Structure

We assume that the technology parameter is subject to a stochastic process with components of differing persistence. The productivity parameter is not observed but the stand-in consumer does observe an indicator or noisy measure of this parameter at the beginning of the period. This might be due to errors in reporting data or just the fact that there are errors in the best or consensus forecast of what productivity will be for the period. On the basis of the indicator and knowledge of the economy-wide state variables, decisions of how many new investment projects to initiate and of how much of the time endowment to allocate to the production of marketed goods are made. Subsequent to observing aggregate output, the consumption level is chosen with inventory investment being aggregate output less fixed investment and consumption.

Specifically, the technology shock λ_t is the sum of a permanent component λ_{1t} and a transitory component λ_{2t}:

$$\lambda_t = \lambda_{1t} + \lambda_{2t} + \bar{\lambda}. \tag{3.7}$$

In the spirit of the Friedman-Muth permanent-income model[1], the permanent component is highly persistent so

$$\lambda_{1,t+1} = \rho\lambda_{1t} + \zeta_{1t} , \tag{3.8}$$

where ρ is less than but near one and ζ_{1t} is a permanent shock. The transitory component equals the transitory shock so

$$\lambda_{2,t+1} = \zeta_{2t} . \tag{3.9}$$

The indicator of productivity π_t is the sum of actual productivity λ_t and a third shock ζ_{3t}.

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[1] Brunner, Cukierman, and Meltzer (1980) emphasize the importance of permanent and transitory shocks in studying macro fluctuations.

(3.10) $\pi_t = \lambda_t + \zeta_{3t} = \lambda_{1t} + \lambda_{2t} + \zeta_{3t} + \bar{\lambda}$.

The shock vectors $\zeta_t = (\zeta_{1t}, \zeta_{2t}, \zeta_{3t})$ are independent multivariate normal with mean vector zero and diagonal covariance matrix.

The period-t labor supply decision n_t and new investment project decision s_{Jt} are made contingent upon the past history of productivity shocks, the λ_k for $k < t$, the indicator of productivity π_t, the stocks of capital inherited from the past, and variable a_t. These decisions cannot be contingent upon λ_t for it is not observed or deducible at the time of these decisions. The consumption-inventory investment decision, however, is contingent upon λ_t for aggregate output is observed prior to this decision and λ_t can be deduced from aggregate output and knowledge of inputs.

The state space is an appropriate formalism for representing this recursive information structure. Because of the two-stage decision process, it is not a direct application of Kalman filtering. Like that approach the separation of estimation and control is exploited. The general structure assumes an unobservable state vector, say x_t, that follows a vector autoregressive process with independent multivariate normal innovations:

(3.11) $x_{t+1} = Ax_t + \epsilon_{0t}$, where $\epsilon_{0t} \sim N(0,V_0)$.

Observed prior to selecting the first set of decisions is

(3.12) $p_{1t} = B_1x_t + \epsilon_{1t}$, where $\epsilon_{1t} \sim N(0,V_1)$.

The element B_1 is a matrix and the ϵ_{1t} are independent over time. Observed prior to the second set of decisions and subsequent to the first set is

$$(3.13) \qquad p_{2t} = B_2 x_t + \epsilon_{2t}, \text{ where } \epsilon_{2t} \sim N(0,V_2).$$

Equations (3.11)-(3.13) define the general information structure.

To map our information structure into the general formulation, let $x_t' = (\lambda_{1t},\lambda_{2t})$, $B_1 = (1\ 1)$, $B_2 = (1\ 1)$,

$$A = \begin{bmatrix} \rho & 0 \\ 0 & 0 \end{bmatrix} \qquad V_0 = \begin{bmatrix} \mathrm{var}(\zeta_1) & 0 \\ 0 & \mathrm{var}(\zeta_2) \end{bmatrix},$$

$V_1 = [\mathrm{var}\ (\zeta_3)]$ and $V_2 = [0]$. With these definitions, the information structure (3.7)-(3.10) viewed as deviations from the mean and the general representation (3.11)-(3.13) are equivalent.

Let m_{0t} be the expected value and Σ_0 the covariance of the distribution of x_t conditional upon the $p_k = (p_{1k}, p_{2k})$ for $k < t$. Using the conditional probability laws for the multivariate normal distribution (see Mood and Graybill [1966], p.208) and letting m_{1t} and Σ_1 be the mean and covariance of x_t conditional upon p_{1t} as well, we obtain

$$(3.14) \qquad m_{1t} = m_{0t} + (B_1\Sigma_0)'(B_1\Sigma_0 B_1' + V_1)^{-1}(p_{1t} - B_1 m_{0t})$$

and

$$(3.15) \qquad \Sigma_1 = \Sigma_0 - (B_1\Sigma_0)'(B_1\Sigma_0 B_1' + V_1)^{-1}(B_1\Sigma_0).$$

Similarly, the mean vector m_{2t} and covariance matrix Σ_2 conditional upon p_{2t} as well are

$$(3.16) \qquad m_{2t} = m_{1t} + (B_2\Sigma_1)'(B_2\Sigma_1 B_2' + V_2)^{-1}(p_{2t} - B_2 m_{1t})$$

and

(3.17) $$\Sigma_2 = \Sigma_1 - (B_2\Sigma_1)'(B_2\Sigma_1B_2' + V_2)^{-1}B_2\Sigma_1.$$

Finally, from (3.11),

(3.18) $$m_{0,t+1} = Am_{2t}$$

and

(3.19) $$\Sigma_0 = A\Sigma_2A' + V_0.$$

The covariances Σ_0, Σ_1 and Σ_2 are defined recursively by (3.15), (3.17), and (3.19). The matrix V_0 being of full rank along with the stability of A is sufficient to insure that the method of successive approximations converges exponentially fast to a unique solution.

The covariance elements Σ_0, Σ_1 and Σ_2 do not change over time and are therefore not part of the information set. The m_{0t}, m_{1t} and m_{2t} do change but are sufficient relative to the relevant histories for forecasting future values of both the unobserved state and the observable p_t and for estimating the current unobserved state.

Equilibrium

To determine the equilibrium process for this model, we exploit the well-known result that in the absence of externalities, competitive equilibria are Pareto optima. With homogeneous individuals, the relevant Pareto optimum is the one which maximizes the welfare of the stand-in consumer subject to the technology constraints and the information structure. Thus, the problem is to

$$\text{maximize } E\ \Sigma_{t=0}^{\infty}\beta^t u[c_t,\ 1 - \alpha_0 n_t - \eta(1 - \alpha_0)a_t]$$

subject to constraints (3.1)-(3.4), (3.6), and (3.11)-(3.13), given

k_0, $s_{10},\ldots,s_{j-1,0}$, a_0, and that $x_0 \sim N(m_0, \Sigma_0)$. The decision variables at time t are n_t, s_{Jt}, c_t, and y_{t+1}. Further, n_t and s_{Jt} cannot be contingent upon p_{2t} for it is observed subsequent to these decisions.

This is a standard discounted dynamic programming problem. There are optimal time-invariant or stationary rules of the form

$$n_t = n(k_t, s_{1t}, s_{2t},\ldots,s_{J-1,t}, y_t, a_t, m_{1t})$$
$$s_{Jt} = s(k_t, s_{1t}, s_{2t},\ldots,s_{J-1,t}, y_t, a_t, m_{1t})$$
$$c_t = c(k_t, s_{1t}, s_{2t},\ldots,s_{Jt}, y_t, a_t, n_t, m_{2t})$$
$$y_{t+1} = y(k_t, s_{1t}, s_{2t},\ldots,s_{Jt}, y_t, a_t, n_t, m_{2t})$$

It is important to note that the second pair of decisions are contingent upon m_{2t} rather than m_{1t} and that they are contingent also upon the first set of decisions s_{Jt} and n_t.

The existence of such decision rules and the connection with the competitive allocation is established in Prescott and Mehra (1980). But, approximations are necessary before equilibrium decision rules can be computed. Our approach is to determine the steady state for the model with no shocks to technology. Next, quadratic approximations are made in the neighborhood of the steady state. Equilibrium decision rules for the resulting approximate economy are then computed. These rules are linear so in equilibrium the approximate economy is generated by a system of stochastic difference equations for which covariances are easily determined.

4. Steady State, Approximation, and Computation of Equilibrium

Variables without subscript denote steady state values. The steady state interest rate is

$$r = (1 - \beta)/\beta$$

and the steady state price of (non-inventory) capital

$$q = \sum_{j=1}^{J} (1 + r)^{j-1} \varphi_j .$$

The latter is obtained by observing that φ_1 units of consumption must be foregone in the current period, φ_2 units the period before, etc., in order to obtain one additional unit of capital for use next period.

Two steady state conditions are obtained by equating marginal products to rental rates:

$$f_y = r \quad \text{and} \quad f_k = q(r + \delta)$$

so

$$f_k/f_y = q(r + \delta)/r.$$

For production function (3.5), this reduces to

$$(4.1) \qquad y = \left[\frac{r + \delta}{r} q \frac{\sigma}{1 - \sigma}\right]^{\frac{1}{\nu+1}} k \equiv b_1 k.$$

Differentiating the production function with respect to capital, substituting for y from (4.1), and equating to the steady-state rental price, one obtains

$$(1 - \theta)(1 - \sigma) b_2^{-\frac{1-\theta}{\nu} - 1} \lambda n^{\theta} k^{-\theta} = q(r + \delta),$$

where $b_2 = 1 - \sigma + \sigma b_1^{-\nu}$. Solving for k as a function of n yields

$$(4.2) \qquad k = \left[\frac{(1-\theta)(1-\sigma)}{q(r+\delta)} b_2^{-\frac{1-\theta}{\nu}} - 1\right]^{1/\theta} \lambda^{1/\theta} n \equiv b_3 \lambda^{1/\theta} n.$$

Steady-state output as a function of n is $f = b_2^{-(1-\theta)/\nu} b_3^{1-\theta} \lambda^{1/\theta} n \equiv b_4 \lambda^{1/\theta} n$. In the steady state, net investment is zero, so:

$$(4.3) \qquad c = b_4 \lambda^{1/\theta} n - \delta k = (b_4 - \delta b_3) \lambda^{1/\theta} n.$$

The steady-state values of c, k, and y are all proportional to $\lambda^{1/\theta} n$. We also note that the capital-output ratio is b_3/b_4, and that consumption's share of total steady-state output is $1 - \delta \frac{b_3}{b_4}$.

Turning now to the consumer's problem and letting μ be the Lagrange multiplier for the budget constraint and w_t the real wage, first-order conditions are

$$\frac{1}{3} c_t^{\gamma/3-1} [\alpha(L)\ell_t]^{2\gamma/3} = \mu$$

and

$$\frac{2}{3} \sum_{i=0}^{\infty} \beta^i \alpha_i c_{t+i}^{\gamma/3} [\alpha(L)\ell_{t+i}]^{(2\gamma/3)-1} = \mu w_t.$$

In the steady state, $c_t = c$, $\ell_t = \ell$, and $w_t = w$ for all t. Making these substitutions and using the fact that the α_i sum to one, these expressions simplify to

$$\frac{1}{3}(c^{1/3}\ell^{2/3})^{\gamma} = \mu c \text{ and } \frac{2}{3}(c^{1/3}\ell^{2/3})^{\gamma} \sum_{i=0}^{\infty} \beta^i \alpha_i = \mu w \ell.$$

Thus,

$$2c \sum_{i=0}^{\infty} \beta^i \alpha_i = w\ell.$$

But $\sum_{i=0}^{\infty} \beta^i \alpha_i = \alpha_0 + \frac{(1-\alpha_0)\eta}{r+\eta}$ and $\ell = 1 - n$, so

$$(4.4) \qquad 2c\left[\alpha_0 + \frac{(1-\alpha_0)\eta}{r+\eta}\right] = w(1 - n).$$

Returning to the production side, the marginal product of labor equals the real wage:

$$(4.5) \qquad w = f_n = \frac{\theta}{n} f = \theta\, b_4\, \lambda^{1/\theta}.$$

Using (4.3) and (4.5), we can solve (4.4) for n:

$$n = \left[1 + 2\frac{\alpha_0 r+\eta}{\theta(r+\eta)}\left(1 - \delta\frac{b_3}{b_4}\right)\right]^{-1}.$$

That n does not depend upon average λ matches well with the American experience over the last thirty years. During this period, output per man-hour has increased by a few hundred percent, yet man-hours per person in the 16-65 age group has changed but a few percent.

Approximation About the Steady State

If the utility function u were quadratic and the production function f linear, there would be no need for approximations. In equilibrium, consumption must be equal to output minus investment. We exploit this fact to eliminate the nonlinearity in the constraint set by substituting $f(\lambda, k, n, y) - i$ for c in the utility function to obtain

$$u[f(\lambda, k, n, y) - i, n, a].$$

The next step is to approximate this function by a quadratic in the neighborhood of the model's steady state. As investment i is linear in the decision and state variables, it can be eliminated subsequent to the approximation and still preserve a quadratic objective.

Consider the general problem of approximating function $u(x)$ near $\bar{x}$. The approximate quadratic function is denoted by $U(x)$.

$$U(x) = u(\bar{x}) + b'(x - \bar{x}) + (x - \bar{x})'Q(x - \bar{x})$$

where $x, b \in \mathbb{R}^n$ and Q an $n \times n$ symmetric matric. Let $\bar{x}_i \pm z_i$ be the range over which the approximation is to hold for each of the n components of x. The element z^i denotes the projection of z onto the i^{th} component (so $z^i_i = z_i$, and $z^i_j = 0$ for $j \neq i$).

The elements b_i and q_{ii} are selected so that the approximation error is zero at $\bar{x} + z^i$ and $\bar{x} - z^i$. This requires

$$b_i = \frac{u(\bar{x} + z^i) - u(\bar{x} - z^i)}{2z_i}$$

and

$$q_{ii} = \frac{u(\bar{x} + z^i) - u(\bar{x}) + u(\bar{x} - z^i) - u(\bar{x})}{2z_i^2} .$$

The elements q_{ij}, $i \neq j$, are selected to minimize the sum of the squared approximation errors at $\bar{x} + z^i + z^j$, $\bar{x} + z^i - z^j$, $\bar{x} - z^i + z^j$ and $\bar{x} + z^i - z^j$. The approximation error at the first point is

$$u(\bar{x} + z^i + z^j) - u(\bar{x}) - b_i z_i - b_j z_j - q_{ii} z_i^2 - q_{ij} z_j^2 - 2q_{ij} z_i z_j .$$

Summing over the square of this error and the three others, differentiating with respect to q_{ij}, setting the resulting expression equal to zero and solving for q_{ij}, we obtain

$$q_{ij} = \frac{u(\bar{x} + z^i + z^j) - u(\bar{x} + z^i - z^j) - u(\bar{x} - z^i + z^j) + u(\bar{x} - z^i - z^j)}{8 z_i z_j}$$

for $i \neq j$.

This method rather than the second order Taylor series expansion was chosen because we wanted the approximation to hold over the entire range $\bar{x}_i \pm z_i$. If the z_i are all small, then, of course, our approximation is very near the second order Taylor series expansion at $\bar{x}$.

Computation of Equilibrium

The equilibrium process for the approximate economy maximizes the welfare of the representative household subject to the technological and informational constraints as there are no externalities. This simplifies the determination of the equilibrium process by reducing it to solving a linear-quadratic maximization problem. For such mathematical structure there is a separation of estimation and control. Consequently, the first step in determining the equilibrium decision rules for the approximate economy is to solve the following deterministic problem:

$$\text{Max} \sum_{t=0}^{\infty} \beta^t U(k_t, n_t, y_t, \lambda_t, i_t, a_t)$$

subject to

(4.6) $k_{t+1} = (1 - \delta)k_t + s_{1t}$

(4.7) $s_{j,t+1} = s_{j+1,t} \quad , \quad j = 1,\dots,J-1$

(4.8) $x_{t+1} = Ax_t$

(4.9) $a_{t+1} = (1 - \eta)a_t + n_t$

(4.10) $i_t = \sum_{j=1}^{J} \varphi_j s_{jt} + y_{t+1} - y_t$

(4.11) $\lambda_t = x_{1t} + x_{2t}$.

At this stage the fact that there is an additive stochastic term in the equation determining x_{t+1} is ignored as is the fact that x_t is not observed for our economy. Constraints (4.6)-(4.9) are the laws of motion for the state variables. The free decision variables are n_t, s_{Jt} and y_{t+1}. It was convenient to use inventories taken into the subsequent period, y_{t+1}, as a period t decision variable rather than i_t because the decisions on inventory carry-over and consumption are made subsequent to the labor supply, n_t, and new project, s_{Jt}, decisions.

For notational simplicity we let the set of state variables other than the unobserved x_t be $z_t = (k_t, y_t, a_t, s_{1t},\ldots,s_{J-1,t})$ and the set of decision variables $d_t = (n_t, s_{Jt}, y_{t+1})$. The unobserved state variables $x_t = (x_{1t}, x_{2t})$ are the permanent and transitory shocks to technology. Finally, $v(x, z)$ is the value of the deterministic problem if the initial state is (x, z). It differs from the value function for the stochastic problem by a constant.

The value function $v(x, z)$ is computed by the method of successive approximations or value iteration. Using constraints (4.10) and (4.11) to substitute for i_t and λ_t in the utility function, the resulting utility function $U(x, z, d)$ is negative definite in (z, d). This along with the stability of the A matrix is sufficient to insure that the method of successive approximations will converge to the optimum. If $v_j(x, z)$ is the j^{th} approximation, then

$$v_{j+1}(x_t, z_t) = \max_{d_t}\{U(x_t, z_t, d_t) + \beta v_j(x_{t+1}, z_{t+1})\}$$

subject to constraints (4.6)-(4.9). The initial approximation, $v_0(x, z)$, is that function which is identically zero.

The function U is quadratic and the constraints are linear. Then, if v_j is quadratic, v_{j+1} must be quadratic. As v_0 is trivially quadratic, all the v_j are quadratic and therefore easily computable. We found that the sequence of quadratic functions converged fairly quickly.

The next step is to determine the optimal inventory carry-over decision rule. Let $y_{t+1} = y(x_t, z_t, n_t, s_{Jt})$ be the <u>linear</u> function which solves

$$(4.12) \qquad \underset{y_{t+1}}{\text{Max}}\{U(x_t, z_t, n_t, s_{Jt}, y_{t+1}) + \beta v(x_{t+1}, z_{t+1})\}$$

subject to (4.6)-(4.9) and both n_t and s_{Jt} given. Finally, the solution to the program

$$\underset{s_{Jt}, n_t}{\text{Max}} \; v_2(x_t, z_t, n_t, s_{Jt}),$$

where v_2 is the value of maximization of (4.12), is determined. The optimal decision rules, $s_{Jt} = s(x_t, z_t)$ and $n_t = n(x_t, z_t)$, are both linear.

Because of the separation of estimation and control in our model, these decision rules can be used to determine the motion of the stochastic economy. In each period t, a conditional expectation, m_{0t}, is formed on the basis of observations in previous periods. An indicator of the technology shock is observed, which is the sum of a permanent and a transitory component as well as an indicator shock. The conditional expectation, m_{1t}, of the unobserved x_t is computed according to equation (3.13), and s_{Jt} and n_t are determined from

$$(4.13) \qquad s_{Jt} = s(m_{1t}, z_t),$$

$$(4.14) \qquad n_t = n(m_{1t}, z_t),$$

where x_t has been replaced by m_{1t}. Then the technology shock, λ_t, is observed, which changes the conditional expectation of x_t. From (3.15), this expectation is m_{2t}, and the inventory carry-over is determined from

$$y_{t+1} = y(m_{2t}, z_t, s_{Jt}, n_t). \tag{4.15}$$

To summarize, the equilibrium process governing the evolution of our economy is given by (3.1)-(3.3), (3.6), (3.11)-(3.14), (3.16), (3.18), and (4.13)-(4.15).

5. Model Estimation

The advantages of formulating the model as we did and then constructing an approximate model for which the equilibrium decision rules are linear are twofold. First, the specifications of preferences and technology are close to those used in many applied studies. This facilitates checks of reasonableness of many parameter values. Second, our approach facilitates the selection of parameter values for which the model steady-state values are near average values for the American economy during the period being explained. These two considerations reduce dramatically the number of free parameters that will be varied when searching for a set that results in covariances near those observed. In explaining the covariances of the cyclical components, there are only seven free parameters, with the range of two of them being severely constrained a priori.

Capital for our model reflects all tangible capital including stocks of plant and equipment, consumer durables and housing. Consumption does not include the purchase of durables but does include the services from the stock of durables. Different types of capital have different construction periods and pattern of resource requirements. The findings summarized in Section 2 suggest an average construction period of nearly two years for plants. Consumer durables, however, have much shorter average construction period. Having but one type of capital, as a compromise, we assume four quarters are required with one-quarter of the value put in place occurring in each quarter. Thus $J = 4$ and $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = 0.25$.

Approximately ten percent of national income account GNP is the capital consumption allowance and another ten percent excise tax. To GNP should be added the depreciation of consumer durables which has the effect of increasing the share of output going to owners of capital. In 1976 compensation to employees plus proprietary income was approximately 64 percent of the quantity GNP plus consumer durable depreciation less indirect profit tax, while owners of capital received about 36 percent. As labor share is θ, we set $\theta = .64$.

Different types of capital depreciate more rapidly than others, with durables depreciating more rapidly than plant and housing, and land not depreciating at all. As a compromise we set the depreciation rate equal to 10% per year. A real interest rate of four percent per year is assumed. This implies a steady-state capital to annual output ratio of 2.4. Of total output 64% is wages, 24% depreciation and 12% return on capital which includes consumer durables.

The remaining parameters of technology are average λ which we normalize to one by measuring output in the appropriate units, and parameters σ and ν. which determine the shares of and substitution between inventories and capital. Inventories are about one-fourth of annual GNP so we require ν and σ be such that $k/y = 10$. A priori reasoning indicates the substitution opportunities between capital and inventory are small suggesting ν should be considerably larger than zero. We restricted it to be no less than two, but it is otherwise a free parameter in our search for a model to explain the covariances and autocovariances of aggregate variables. Given ν and the value of $b_1 = y/k$, σ is implied. From (4.1) it is

$$\sigma = \left[1 + \frac{q(r+\delta)}{rb_1^{\nu+1}}\right]^{-1}.$$

For purposes of explaining the covariances of the percentage deviation from steady state values ν is the only free parameter associated with technology.

The parameter β is implied by the assumed four-percent steady-state interest rate which is close to estimates of the average return on all capital (see Ibbotson and Fall [1979]). The parameters α_0 and η which affect intertemporal substitutability of leisure will be treated as free parameters for we could find no estimate for them in the labor economics literature. The degree of risk aversion, γ, is also a free parameter. As stated previously, the steady-state labor supply is independent of the productivity parameter λ.

The remaining parameters are those specifying the process on λ_t and the variance of the indicator. These three parameters are var(ζ_1), var(ζ_2) and var(ζ_3). In summary, the parameters that are estimated from the variance-covariance properties of the model are these variances plus the parameter ν determining substitutability of inventories and capital, the parameters α_0 and η determining intertemporal substitutability of leisure, and the risk aversion parameter γ. For each set of parameter values, our model of the approximate economy is used to compute several statistics which summarize the serial correlation and covariance properties. These statistics are compared with those of the actual U.S. data for the period 1950:1 to 1979:2 as reported in Hodrick and Prescott (1980). The set of parameter values is chosen which fits the actual data well. Having only seven degrees of freedom to explain the observed pattern of the covariances imposes considerable discipline upon the analysis.

A key part of this procedure is the computation of dynamic competitive equilibrium for each combination of parameter values. Because the conditional forecasting can be separated from control in this model, the dynamic equilibrium decision rules need only be computed for each new combination of the parameters, ν, α_0, η, and γ. Similarly, the conditional expectations of the permanent and transitory shocks which enter the decision rules depend only on the variances of the three shocks and not upon the parameters of preferences and technology.

For each set of parameter values a large number of observations are generated from the equilibrium process of the model using random number generator for the three innovations per period. We do not have a detailed model of the factors of growth (see e.g., Denison [1974]). Because of the permanent shocks, however, there can be considerable drift away from the steady state. We did not find it reasonable to include this drift in the cyclical component, which is thought of as describing sharper, more rapid movements. The observations from the model are therefore decomposed into a cyclical or rapidly varying component and a growth or slowly varying component in the same way as in Hodrick and Prescott (1980).

For each set of parameter values the following statistics are computed: the autocorrelation of output for up to six periods, standard deviations of the variables of interest, and their correlations with output. As is the case in Hodrick and Prescott's study, only the comovements of the different variables with output are emphasized. In their paper, the variables (except interest rates) are measured in logs while we use the values rather than the logs. This is of consequence only in the measurement of amplitudes, so in order to make our results comparable to theirs, our standard deviations (except for interest rates) are divided by the steady states of the respective variables. One can then interpret the cyclical components essentially as percentage deviations as in Hodrick and Prescott.

Some initial evaluations of parameter points revealed over what ranges it made sense to search and how fine to make the grid on the various parameters. For example, the search over shock variances was done in terms of the variances of ζ_1 and ζ_2 divided by the sum of the three variances, $\mathrm{var}(\zeta_1)/\sum_{j=1}^{3} \mathrm{var}(\zeta_j)$ and $\mathrm{var}(\zeta_2)/\sum_{j=1}^{3} \mathrm{var}(\zeta_j)$. The sum of the variances was selected so that the estimate of the variance of cyclical output for the model equalled that of cyclical output for the U.S. economy during the sample period. An initial search was made over the entire range between zero and one for the two relative variances, but it quickly became clear that there was no reason to go beyond a value of 0.2 for $\mathrm{var}(\zeta_2)/\sum_{j=1}^{3} \mathrm{var}(\zeta_j)$. Later, the upper limit of that range was further reduced to 0.05. In the case of ν, we tried the values 2, 3, 4, and 5. For α_0 we used a grid of 0.1 for the range of 0.2 to 0.6, and for η a grid of 0.05 for the range of 0.05 to 0.2. The risk parameter γ was not subject to extensive search. Blume and Friend (1975), using cross-sectional data, estimated γ to be -1.0, though that was with respect to an indirect utility function of wealth. There is some tradition in the finance literature to use the logarithmic function as the utility function. That would correspond to the limiting case of $\gamma = 0$ in our model. We considered only $\gamma = -1$, -0.5, and -0.1.

The selected values of the parameters are reported in Table 1.

<u>Tests of the Model</u>

All reported statistics refer to the cyclically or rapidly varying components for both the model and the U.S. economy. Estimated autocorrelations of real output for our model along with sample values for the U.S. economy in the post-war period are reported in Table 2. The fit is very good, particularly in light of the model's simplicity.

Table 3 contains estimates of standard deviations and correlations with output for the model's variables. Table 4 contains sample values of statistics for the post-war U.S. economy as reported in Hodrick and Prescott (1980).

The variables in our model do not correspond perfectly to those available for the U.S. economy so care must be taken in making comparisons. A second problem is that there may be measurement errors that seriously bias the estimated correlations and standard deviations. A final problem is that the estimates for the U.S. economy are subject to sampling error. As a guide to the magnitude of this variability, we estimated the standard deviation of the model's estimates which, like the estimate for the U.S. economy, uses only 118 observations. These numbers are the second numbers in the parentheses in Tables 2 and 3.

Table 1

Model Parameters[a]

Preference Parameters	Technology Parameters
$\alpha_0 = .50$	$\nu = 4.0$
$\eta = .10$	$\theta = .64$
$\gamma = -.50$	$\sigma = .28 \times 10^{-5}$
$\beta = .99$	$\varphi_1 = .25$
	$\varphi_2 = .25$
Shock Variances	$\varphi_3 = .25$
$var(\zeta_1) = (.0091)^2$	$\varphi_4 = .25$
$var(\zeta_2) = (.0013)^2$	$\delta = .025$
$var(\zeta_3) = (.0090)^2$	$\bar{\lambda} = 1.0$

a. For parameters with a time dimension, the unit of time is a quarter of a year.

Table 2

Autocorrelations of Output[a]

Order	Model Estimated Value	U.S. Economy Period 1950.1-1979.2 Sample Value
1	.72 (.01, .06)	.84
2	.45 (.01, .13)	.57
3	.28 (.02, .13)	.27
4	.19 (.02, .13)	-.01
5	.02 (.02, .13)	-.20
6	-.13 (.02, .14)	-.30

a. Standard errors of estimates are the first element in parentheses. The second element is the standard error for the model if only 118 observations were available.

Table 3

Model's Standard Deviations and Correlations

with Real Output[a]

Variable	Estimated Standard Deviations (percent)	Estimated Correlations
Real Output	1.80 (.04, .27)	-
Consumption	.64 (.01, .11)	.94 (.003, .015)
Investment	6.44 (.10, .64)	.80 (.006, .050)
Inventories	.89 (.01, .07)	-.16 (.014, .106)
Inventories plus	1.99 (.03, .19)	.39 (.008, .047)
Capital Stock	.63 (.01, .09)	-.07 (.008, .057)
Hours	1.05 (.02, .13)	.93 (.003, .014)
Productivity	.90 (.02, .13)	.90 (.003, .026)
Real Interest Rate (Annual)	.22 (.003, .02)	.48 (.013, .092)

a. Standard errors of estimates are the first element in parentheses. The second element is the standard error for the model if only 118 observations were available.

Table 4

Standard Deviations and Correlations

with Real Output

U.S. Economy 1950.1 - 1979.2

	Standard Deviations (percent)	Correlations with Real Output
Output	1.8	-
Total Consumption	1.3	.74
Services	.7	.62
Non-Durables	1.2	.71
Durables	5.6	.57
Investment Fixed	5.1	.71
Capital Stock		
Durable Mfg.	1.2	-.21
Non-durable Mfg.	.7	-.24
Inventories	1.7	.51
Hours	2.0	.85
Productivity	1.0	.10

The model is consistent with the large (percentage) variability in investment and low variability in consumption and their high correlations with real output. The model's negative correlation between the capital stock and output is consistent with the data though its magnitude is somewhat smaller.

Inventories for our model correspond to finished and nearly finished goods while the inventories in Table 4 refer to goods in process as well. We added half the value of uncompleted capital goods to the model's inventory variable to obtain what we call inventories plus. This corresponds more closely to the U.S. inventory stock variable with its standard deviation and correlation with real output being consistent with the U.S. data.

In Table 3 we include results for the implicit real interest rate given by the expression

$$r_t = \frac{\partial u/\partial c_t}{\beta E(\partial u/\partial c_{t+1})} - 1.$$

The expectation is conditional on the information known when the allocation between consumption and inventory carry-over is made.

The most troublesome anomalies are the model's low variability of hours and the related problem of high correlation of productivity with output. Part of this discrepancy might be due to measurement errors. For example, all members of the household may not be equally productive, say due to differing stocks of human capital. If there is a greater representation in the work force of the less productive, for example youth, when output is high, hours would be overestimated. The effects of such errors would be to bias the variability of employment upwards and the correlation between productivity and output downwards. Measurement errors in employment that are independent of the cycle would have a similar effect on the correlation between output and productivity.

A more likely explanation might be the oversimplicity of the model. The shocks to technology, given our production function, are pure productivity shocks. Some shocks to technology alter the transformation between the consumption and investment goods. For example, investment tax credits, accelerated depreciation, and the like have such effects, and so do some technological changes. Further, some technological change may be embodied in new capital, and only after the capital becomes productive is there the increment to measured productivity. Such shocks induce variation in investment and employment without the variability in productivity. This is a question that warrants further research.

Sensitivity of Results to Parameter Selection

All the considered values of the risk aversion parameter, γ, and the inventory-capital substitution parameter, ν, yield similar results. This is true for somewhat smaller values of the intertemporal-leisure-substitution parameters, η and α_0, as well. The results are sensitive to the relative variances of the shocks. Only if the variance of the transitory shock to technology is small relative to the sum of the three variances and the size of the other two variances comparable are the model's serial correlation properties consistent with those for the U.S. post-war economy. In other words, the confounding of the permanent shock to the technology with the noisy indicator is crucial to the model.

In other words, they reduce the elasticity of responses of employment to perceived temporary increases in the real wage, while to explain the business cycle this elasticity must be large.

There are several refinements which should improve the performance of the model. In particular, we conjecture that introducing the hours per week that productive capital is employed as a decision variable, with agents having preferences defined on hours worked per week, should help. Introducing more than a single type of productive capital with different types requiring different periods for construction and having different patterns of resource requirement is feasible. It would then be possible to distinguish between plant, equipment, housing and consumer durables investments. This would also have the advantage of permitting the introduction of features of our tax system which affect transformation opportunities facing the economic agents.[1] Another refinement is in the estimation procedure. But, in spite of the considerable advances recently made by Hansen and Sargent (1980), computational considerations still preclude the application of maximum likelihood or Bayesian estimation techniques.

Models such as the one considered in this paper could be used to predict the consequence of a particular policy rule upon the operating characteristics of the economy. As we estimate the preference-technology structure, our structural parameters will be invariant to the policy rule selected even though the behavioral equations are not. There are computational problems, however, associated with determining the equilibrium behavorial equations

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[1] See e.g. Hall and Jorgenson (1967).

of the economy when feedback policy rules, that is, rules that depend on the aggregate state of the economy, are used. The competitive equilibrium, then, will not maximize the welfare of the stand-in consumer, so a particular maximization problem cannot be solved to find the equilibrium behavior of the economy. Instead, methods such as those developed in Kydland (1980) to analyze policy rules in competitive environments will be needed.

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