DISCUSSION PAPER NO. 44

THE INCENTIVES FOR PRICE-TAKING BEHAVIOR IN LARGE ECONOMIES*

by

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December 1973
(Twice Revised)

* The authors wish to thank John Ledyard and Hugo Sonnenschein for their most helpful comments.

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1. Introduction

Economists typically assume that consumers in large economies will adopt price-taking behavior. Yet as long as there are but a finite number of agents, the demand of each will have some impact on price formation. By recognizing this impact and appropriately altering his offers to buy and sell from their competitive values, an agent may be able to manipulate prices to his benefit. He would then have an incentive to adopt such non-competitive behavior.

Recently, Hurwicz [5] has shown that this incentive problem is not limited to the competitive mechanism, but rather is quite pervasive. Specifically, he has shown that there cannot exist any system for allocating resources which yields individually rational, Pareto optimal outcomes and which has the property that no agent can ever benefit from departing from the specified behavioral rules of the system, given that the other agents adhere to these rules.[1] This result, which applies in particular to the competitive price system, is all the more striking since Hurwicz does not allow arbitrary departures from the specified rules. Rather, he restricts the cheating to misrepresentation of preferences, which would be undetectable by an agency unable to observe preferences directly.[2]

Despite this impossibility theorem, one would expect that the incentive to deviate from competitive price-taking behavior would be very small in large enough economies. As the number of agents increases and the demand of any single agent becomes a decreasing portion of the aggregate, his ability to influence price formation and the possible gains from non-competitive behavior should be reduced. The plausibility of this conjecture is enhanced by consideration of the case of a very large exchange economy, i.e., one with a continuum of infinitesimal agents. In this context, no consumer can benefit from deviating from passive price-taking behavior.

The purpose of this paper is to study this conjecture that the incentive for an individual to adopt non-competitive behavior decreases to zero as the economy becomes large. By this we mean that, even knowing the amounts demanded and supplied by the other agents at each possible price, if these quantities are given, the utility gain that any agent can achieve by manipulating prices rather than taking them as given goes to zero as the number of agents goes to infinity.

If one were able to establish this conjecture for "most" sequences of economies, one would have a basis for support of the common assumption of competitive behavior. If the gain any individual can hope to realize by price manipulation can be expected to become arbitrarily small in large enough economies, then one might well assume that each agent will in fact take prices as given. This assumption would be even more acceptable if one also realizes that there may be differential costs involved in finding profitable non-competitive behavior over acting as a price-taker. If these costs do not also go to zero, then the net gain from deviating from competitive behavior would be zero or negative in large economies.

To study this conjecture, we define an exchange economy as a collection of agents, defined in terms of preferences and endowments, and an assignment to each agent of a correspondence from prices into net trades. For each such specification, a (possibly empty) set of market clearing prices results: if the correspondence assigned to each agent is his competitive excess demand correspondence, these prices are the competitive equilibrium prices. In general, we think of these correspondences as indicating the amounts each agent offers to buy or sell at each price. By his choice of such a correspondence, that is, by his choice of non-competitive behavior, an agent is able to influence the prices at which exchange occurs. We then consider the gains that any one agent can obtain, given the behavior of the other agents, by adopting such non-competitive behavior. In Section 2 this model is defined formally. In Section 3, we investigate the conjecture in the context of replica economies in which the other agents use their true competitive demand correspondences. We are able to show in this special case that the gain from non-competitive individual behavior does in fact go to zero. Then, in Section 4, we seek to extend this result to arbitrary sequences of exchange economies in which the number of agents goes to infinity while the endowment of each agent in per capita terms goes to zero. A simple example, which appears to show no obvious pathologies, indicates that this is not always possible. In this example, we exhibit a sequence of economies with the property that one given agent, by departing from competitive behavior, is able to bring about the same alteration in prices and consequent improvement in his well-being in each economy in the sequence. However, one readily shows that this example is essentially exceptional. In theorem 2 we present a simple sufficient condition for limiting incentive compatibility of the competitive response, even when the other agents are not constrained to be acting competitively.

This condition is that the correspondence assigning to each economy its set of market-clearing prices be continuous. This continuity condition is known to hold in a very large class of economies.

These results then provide one possible justification for the assumption of competitive behavior in large exchange economies. An alternative justification arises from the literature on the relationship between core and competitive allocations and, in particular, from the work of Bewley [1]. The final section of this paper contains a brief statement of the relevant results from this literature and a comparison of the nature of the justifications of the competitive assumption arising from that work and from the present analysis.

2. Definitions and the Model

We will consider only pure exchange situations with a fixed number, N, of commodities. In such situations, an economic agent is characterized by his needs, his tastes and his ownership of resources. These characteristics are specified mathematically by a (non-empty) consumption set $X \subset R^N$,[3] a preference relation $\succsim$ on X and an endowment vector $w \in R^N$. Throughout, we will assume that $\succsim$ is a continuous complete preorder on X and that X is closed.

An alternative, but also standard, formalization is to represent the consumer's needs and tastes by the graph

$$G = \{(x,x') \in X \times X \mid x \succsim x'\} \subset R^N \times R^N$$

of his preference relation. When employing this formalization we will make the continuity assumption that G is closed. (This assumption insures that a continuous utility function always exists for each consumer.) Then, letting $\mathscr{L}$ denote the collection of all those subsets of $R^M \times R^M$ which are the graphs of continuous preference preorders on closed consumption sets, we can identify a consumer's characteristics with a point in $A \equiv (\mathscr{L} \times R^N)$. We will write $\succsim(a)$, $X(a)$ and $w(a)$, respectively, for the preferences, consumption set and endowment of an agent a.

Let P denote the standard unit simplex in R^N_+. The competitive, price-taking response gives, for each possible agent and each price, the set of net trades which are preference-maximizing for the agent given those prices. Formally, given $a \in A$ and $p \in P$,

$$C(a,p) \equiv \{z \in R^N | z = x - w(a) \text{ and } x \text{ is } \succsim(a) \text{ - maximal on } X(a) \text{ subject to } px \leqq pw(a)\}.$$

[Although we do not assume $C(a,p) \neq \emptyset$, conditions sufficient for non-emptiness of $C(a,p)$ are well-known. Note that $C(a,p)$ is closed for all $a \in A$ and $p \in P$].

To allow for deviations from passive price-taking behavior, we permit agents to select responses to prices other than those specified by the competitive response. Specifically, let $\mathscr{S}(a)$ denote the collection of all correspondences S from P into R^N with the properties that, for all $p \in P$, $S(p) + \{w(a)\} \subset X(a)$ and, if $z \in S(p)$, then $pz \leqq 0$. We think of the G correspondence as possible strategies an agent can employ in departing from the competitive rules. An important particular case is that considered by Hurwicz [5], where these strategies are limited to being misrepresentations of the agent's characteristics; i.e., the choices by agent a from $\mathscr{S}(a)$ are limited to those S for which $S(\cdot) = C(a',\cdot)$ for some $a' \in A$ with $w(a') = w(a)$ and $X(a') = X(a)$. This requirement is a very useful one, since it supplies valuable structure to the problem. However, since we wish to obtain positive results rather than an impossibility theorem, it is appropriate here to make deviations from price-taking as easy as possible. Thus, we do not adopt this requirement.[4]

A finite exchange economy E is a finite collection of agents from A and an assignment of a response correspondence from $\mathcal{S}$ to each agent; that is, E is a finite collection of points of the form (a,S,i), where $a \in A$, and $S \in \mathcal{S}(a)$, and $i \in [0,1]$ is an index used to distinguish otherwise identical agents. Given an economy E consisting of agents $a_1,\ldots,a_M$ with response functions $S_1,\ldots,S_M$, we say a price p is market-clearing if there exists $z_m \in S_m(p)$ such that

$$0 = \sum_{m=1}^{M} z_m.$$

Denote by Q(E) the set of all market-clearing prices for E. Note that Q(E) is determined by the correspondences S_m. If the response correspondence assigned to each agent is his competitive response, then Q(E) is the usual set of competitive equilibrium prices.

We now wish to consider the possibilities for an individual to manipulate price formation by his choice of a response correspondence S, given the choices by the other agents in the economy. Given an economy, E, consisting of agents $a_1,\ldots,a_M$ with response correspondences $S_1,\ldots,S_m$, and given a particular agent belonging to E, say a_i, we say that a price $\overline{p}$ is attainable by a_i in E if there exists some $S^* \in \mathcal{S}(a_i)$ such that, when a_i uses S^* as his response,

$$0 \in \sum_{m \neq i} S_m(\overline{p}) + S_i^*(\overline{p}).$$

Denote the set of prices attainable by a in E by H(a,E), and note that $Q(E) \subset H(a,E)$. If p is attainable by a in E and z is a trade he can then make, we will say that $x(a) \equiv z + w(a)$ is an attainable consumption vector for a in E. If a consumption vector is attainable by a in E by using his competitive response $C(a,\cdot)$, we will call x a competitive consumption for a in E.

We define the competitive response as individually incentive compatible for an agent a in E if, for any consumption vector x attainable by a in E, there exists a competitive consumption y for a in E such that $y \succsim(a)\ x$.

Thus, for the competitive mechanism to be individually incentive compatible for a in E, any consumption vector a can generate for himself by departing for competitive behavior must be dominated by some competitive consumption. In the case of a unique equilibrium, this requires that the best the agent could achieve by manipulation is no better than the competitive outcome. This definition is very stringent, and it is perhaps not surprising that incentive compatibility may be difficult to realize. Yet, in the context of a continuum of infinitesimal agents, it does obtain. Within such a model, no agent can influence price formation, since his endowment and demand are completely negligible relative to the aggregate amounts. Any misrepresentation of his demand then leaves his budget set unaltered and cannot benefit him.

Thus, incentive compatibility holds for infinite economies but not, in general, for finite ones. This leads one to wonder if, in some sense, the incentives improve as the number of agents increase. That this should be true is intuitively very appealing. If we think of the agent as manipulating prices via his choice of the correspondence

S, then presumably the larger the economy in which he finds himself, and the smaller a part his endowment and demand are of the whole, then the less will be his relative influence on price formation and the smaller will be his ability to alter the equilibrium. It is to this question that we now turn.

3. Replica Economies

A simple first approach to this question is to consider the impact of numbers on the incentive for any one agent to deviate from competitive behavior when all other agents are using their true competitive responses and the numbers increase by replication. Thus, throughout this Section we will consider only economies in which the response correspondence specified for each agent a is his competitive response, $C(a,\cdot)$,

Given such an economy, E, composed of M agents with characteristics $a_1,\ldots,a_M$, the k-fold replica of E, denoted $E^{(k)}$, has kM agents, k of whom have characteristics a_m, $m=1,\ldots,M$. We will always assume $E \subset E^{(k)} \subset E^{(k+1)}$. Note that, given that the response correspondences are the competitive responses, the equilibrium prices are invariant under replications if the $C(a_m,\cdot)$ are convex-valued. Consequently, given any utility function representing a consumer's tastes, the utility values of the competitive consumptions to him are also unaffected by replication.

Consider the prices that are attainable by a given agent in two replications, $E^{(k)}$ and $^{(k')}$, of a given economy, where $k > k'$. Suppose the agent can balance the demand arising from the rest of the economy at a price p in $E^{(k)}$. Then, if his consumption set is convex, he can surely also balance that in the smaller economy $E^{(k')}$, where the total aggregate demand is a fraction of that in $E^{(k)}$. Further, if his consumption set is bounded from below, given any price which is not an equilibrium price, meeting the demand at that price must, for large enough k, eventually carry him outside his consumption set. Thus, the sets of prices he can attain in each economy form a decreasing sequence whose intersection is the true competitive prices. These statements are verified by the following lemma.

Lemma: Let E be an economy with M agents and suppose X(a) is convex and $\succsim(a)$ is weakly convex for each $a \in E$. Then $H(a,E^{(k)}) \subset H(a,E^{(k')})$ for each a in E. Suppose that, for each a in E, $p \equiv (p_1,\ldots,p_N) \in P$ and $p_n = 0$ implies $z_n > 0$ for all $z = (z_1,\ldots,z_N) \in C(a,p)$. Then, if X(a) is lower-bounded and $p \notin W(E)$ there exists k* such that $k > k^*$ implies $p \notin H(a,E^{(k)})$.

Proof: The proof is straightforward, consisting solely of formalizing the arguments in the previous paragraph, and is left to the reader.

Given this result, the set of prices any agent can make appear to be equilibria can be seen to shrink monotonically down to the actual competitive prices. Unfortunately, we cannot conclude from this that the incentive to misrepresent preferences, as measured (say) by the possible utility gain, decreases correspondingly, since the net trades actually available to an agent at a given price differ as k changes. We can, however, obtain a more limited result.

Definition: Let $\langle E^k \rangle$ be a sequence of economies and let a be an agent belonging to each E^k. Then the competitive mechanism is limiting individually incentive compatible for a in $\langle E^k \rangle$ if for any continuous utility U for $\succsim(a)$ and any $\epsilon > 0$ there exists k* such that $k > k^*$ implies that for each x attainable by a in E^k there exists a competitive allocation y to a in E^k such that $U(y) > U(x) - \epsilon$.

The idea of this definition is a simple one: that the incentive to mispresent, as measured by the gain from misrepresentation,, becomes arbitrarily small in large economies. Note that the concept of limiting individual incentive compatibility does not depend on the particular utility function chosen, nor is the definition limited to sequences generated by replication or to those in which the responses are specified to be the competitive ones.

Theorem 1: Suppose that in some finite economy E the following conditions are met:

1) X(a) is lower-bounded for each a in E;
2) $C(a,\cdot)$ is a closed correspondence for each a in E on

$$P_E \equiv \{p \in P \mid \sum_{a \in E} C(a,p) \neq \emptyset\}; \text{ and}$$

3) if $p \in P \sim P_E$, then for any $B \in R$ there exists a neighborhood N(p) of p in P and an agent a in E such that, for any $p' \in N(p)$, if $x \in C(a,p')$ then $x_n > B$ for some n, $n=1,\ldots,N$, (i.e., if aggregate excess demand is undefined at p, then near to p some agent's excess demand becomes arbitrarily large).

Let $\bar{a}$ be an agent in E, let U be a continuous utility function representing $\succsim(a)$, and suppose that the corresponding inverse utility function, V, is continuous at the points in Q(E). Then, if the conclusions of the Lemma hold for this agent, the competitive mechanism is limiting individually incentive compatible for $\bar{a}$ in the sequence $\langle E^k \rangle$, where $E^k = E^{(k)}$.

Proof: Conditions 1), 2) and 3) imply that $H(a,E^{(n)})$ is closed for each a in E and each $n \geqq 2$. Now, suppose that limiting individual incentive compatibility does not hold for $\bar{a}$. Then there exists a sequence $\langle x^k \rangle$ of consumption vectors with x^k attainable for a in $E^{(n_k)}$ such that for each competitive allocation x to $\bar{a}$, $U(x^k) > U(x) + \epsilon$, where $\epsilon > 0$. Let $p^k \in H(a,E^{(n_k)}) \subset P_E$ be a price at which x^k is attained. Since the $H(a,E^{(n)})$ are nested, $p^k \in H(a,E^{(n)})$, $k \geqq n$. The sequence $\langle p^k \rangle$ is bounded, and so contains a subsequence, which we again denote $\langle p^k \rangle$, converging to some $\bar{p}$. Then, since $H(a,E^{(n)})$ is closed for $n \geqq 2$, $\bar{p} \in \bigcap_{n=1}^{\infty} H(\bar{a},E^{(n)}) = W(E)$. Thus, $p^k \to \bar{p} \in W(E)$ and V is continuous at $\bar{p}$, while $V(p^k) \geqq U(x^k)$, so $\limsup U(x^k) \leq V(\bar{p}) = U(x)$ for some $x \in C(\bar{a},\bar{p}) + w(\bar{a})$. This contradiction establishes the result.

One would hope to be able somehow to sharpen this result to say that the only utility levels an agent could always achieve would be competitive. This sharpening is clearly not possible, as can be seen by considering an economy with three types. The excess demand from the first two types together in $E^{(k)}$ is $(kf(p_1,p_2), kg(p_1,p_2))$, $p_i > 0$, where $f(p_1,p_2) = p_2 - p_1$ and $g(p_1,p_2) = (p^2 - p_1p_2)/p_2$. Each agent of the third type holds one unit of each good and has preferences given by $U(x_1,x_2) = \min(x_1,x_2)$. Thus, the excess demand from the third type is (0,0) if both prices are positive. The unique equilibrium is $p_1 = p_2 = 1/2$. Eventually the price $\left(\frac{k+1}{2k+1}, \frac{k}{2k+1} \right)$ is in $H(a,E^{(k)})$ for any agent of the third type, yielding a consumption bundle to him of $\left(\frac{3k+1}{2k+1}, \frac{k}{2k+1} \right)$. However, for each k, this bundle has lower utility value to him than the unique competitive allocation to him of (1,1). Essentially, there is no way to keep an agent from throwing away utility, although one would not expect him to do so.

3. General Sequences of Economics

We would like to obtain parallel results for situations in which the number of agents increases in an arbitrary fashion. This is especially so since we wish to allow all of the agents to employ responses other than the competitive ones. For example, these responses might be those corresponding to some solution of the game in which each agent's strategies are defined by a choice of a response correspondence. Under such an interpretation, there is no reason to suppose, even if the sequence of "true" economies is generated by replication, that the solution strategies will be invariant under replication. Clearly, any monotonicity result is too much to hope for, but we would still like to obtain a result interpretable in terms of the incentive for any single agent to deviate individually from competitive behavior going to zero as the number of agents goes to infinity.

We might, for example, conjecture that if the number of agents in E^k goes to infinity in such a way that the endowment of each agent becomes arbitrarily small relative to the aggregate, the competitive mechanism will enjoy limiting individual incentive compatibility. The results of the previous section indicate that this is true when the sequence $\langle E^k \rangle$ is generated by replication. However, the following counterexample indicates that such a conjecture is not true in general.

We consider a sequence of economies $\langle E^k \rangle$ in each of which there are three types of agents. Denote these types by T_1, T_2 and T_3^k. In the first economy, E^1, there is one agent of type T_1, two of type T_2 and one of type T_3^1. In the economy E^k there will be one of type T_1, 2k of type T_2 and (2k-1) of type T_3^k. Thus, the number of agents in E^k is 4k.

The box diagram of Figure 1 represents the economy E^1. The initial allocation is at I. The origin for the T_1 and T_3^1 agents is the lower left hand corner, that for the T_2 agents is the upper right. The T_1 and T_3^1 agents have the same endowment, but differ in their preferences.

The curves IAB, ICD and IF^1G^1 are portions of the offer curves for the T_1 agent, a T_2 agent and the T_3^1 agent. Three points z_1, z_2, z_3^1, on IAB, ICD and IF^1G^1 which are colinear with each other and with I can represent competitive allocation if $d(z_1,z_2) = d(z_2,z_3^1)$, so that the trades balance.

The three points A,D,F^1 represent the unique competitive allocation in E^1. Now, by acting as if this offer curve were IA′B′, the T_1 agent can guarantee that B',C,G^1 becomes the unique competitive equilibrium with this misrepresentation. Thus, he gains by at least the utility value to him of B′ minus that of A.

We now indicate how to generate the sequence $\langle E^k \rangle$. This sequence will have the property that for each k, A continues to be the unique competitive consumption for the T_1 agent, while for each k, he will be able to achieve B′ by using this same strategy. Thus the gain from departing from competitive behavior does not diminish for the T_1 agent as the number of agents in the economy goes to infinity.

Specifically, E^2 is generated by adding two more T_2 agents and by replacing the T_3^1 agent by three agents, each of whom have the same endowment as the T_1 agent but whose offer curves are chosen to balance A,D,F^2 and $B'C,G^2$. This process is illustrated in Figure 2, with IF^2G^2 being the offer curve for each of the T_3^2 agents. In this case, three points z_1,z_2,z_3^2 colinear with I with $d(I,z_1) < d(I,z_2) < d(I,z_3)$ balance if $c(I,z_1) + 3d(I,z_3) = 4d(I,z_2)$. Thus, the trade represented by A,D,F^2 is a competitive net trade in E^2, while B',C,G^2 represents a competitive net trade with the misrepresentation of his offer curve by the T_1 agent.

This example appears to exhibit no obvious pathologies: no agent monopolizes some commodity, the holdings of any agent are strictly positive and remain bounded in all the economies and the preferences are convex and strictly monotone. Moreoever it would be fairly simple to alter the example so that E^k was contained in E^{k+1}, nor would imposing a strict convexity requirement on preferences prevent construction of a similar example. Yet, the agent of type T_1 can bring about the same alteration in prices and in his welfare independent of the size of the economy in which he is placed. Moreover, he can do so by using the same response correspondence at each stage, and this correspondence is one which could be rationalized as being competitive, so that Hurwicz's criterion is met.

This example thus indicates that we cannot hope to obtain an immediate, complete generalization of Theorem 1. However, the example also indicates the nature of the problem with achieving such an extension and the type of condition which will be needed to obtain limiting incentive compatibility in this general case. As will be seen, a sufficient condition is the continuity of the Q correspondence between economies and prices. This condition may be expected to be met in a wide class of situations, so that the example is essentially the exception rather than the rule. However, to state this condition formally we must first introduce some further structure, including, of course, a topology on economies.

As noted earlier, in determining the market-clearing prices of an economy E, it is sufficient to consider only the response correspondences. Denote by $\mathcal{S}$ the set of all correspondences S from P to R^N such that $S \in \mathcal{S}(a)$ for some a in A and endow $\mathcal{S}$ with a metric topology.[5] Let the response correspondences of the M agents in an economy E be $S_1,\ldots,S_M$. Then, for purposes of examining Q(E), it is actually sufficient to describe E by the simple measure μ on $\mathcal{S}$ defined by

$$\mu(F) = \frac{\#[F \cap \{S_1,\ldots,S_M\}]}{M}$$

where F is any Borel subset of $\mathcal{S}$.[6] Thus, each response correspondence actually in E is assigned weight 1/M. Now, consider the collection $\mathcal{M}$ of all Borel probability measures on $\mathcal{S}$ which have compact support.[7] We can think of elements this collection as describing (abstract) economies. Now, if we fix a particular price $\overline{p}$, we define a correspondence $\varphi(\cdot,\overline{p})$ on $\mathcal{S}$ given by $\varphi(S,\overline{p}) = S(\overline{p})$ for each S. The condition that $\overline{p}$ be a market-clearing price for the economy described by the measure $\mu \in \mathcal{M}$ then becomes

$$0 \in \int_{\mathcal{S}} \varphi(S,\overline{p})d\mu \equiv \int_{\mathcal{S}} S(\overline{p})d\mu.$$

We may then think of Q as a correspondence on $\mathcal{M}$ into P, with $Q(\mu)$ being the set of market clearing prices for the economy described by μ. If we now endow $\mathcal{M}$ with the topology of weak convergence of measures, we are able to speak of the continuity properties of Q viewed as a correspondence between two topological spaces.[8]

With the frame-work, let us re-examine the example. If we consider a sequence $\langle\mu^k\rangle$ of measures describing the sequence of economies when the T_1 agent uses his true preferences, the Q correspondence is constant and single-valued on this sequence. But if we consider the limit μ of the sequence $\langle\mu^k\rangle$, $Q(\mu)$ contains all prices in P which are the normals of planes passing through I and any point on CD. Thus, Q "blows up" at the limit: it is upper-hemi-continuous at μ but not lower hemi-continuous.[9] It is precisely this lack of full continuity that is the key to the example.

Theorem 2: Consider a sequence $\langle E^k\rangle$ of finite economies such that $\#E^k \to \infty$. Suppose that the sequence $\langle\mu^k\rangle$ of simple measures describing $\langle E^k\rangle$ converges to a measure μ and that the correspondence Q is continuous at μ. Suppose $\bar{a}$ belongs to E^k for all k and that an inverse utility function for $\bar{a}$ exists and is continuous in a neighborhood of $Q(\mu)$. Then the competitive response is limiting individually incentive compatible for $\bar{a}$ in $\langle E^k\rangle$.

Proof: Let E^k be described by the simple measure μ^k, suppose $\bar{a}$ belongs to each E^k, and suppose S_k is the response used by $\bar{a}$ in E^k. The choice of a response S_k', different from S_k, by $\bar{a}$ in E^k defines a new "apparent economy" described by a simple measure ν^k, where

$$\nu^k(F) = \frac{\#\{F \cap [(\text{supp } \mu^k) \cup (S_k') \sim (S_k)]\}}{\#\{\text{supp } \mu^k\}}$$

for all Borel sets F of $\mathscr{S}$.

As $\langle\mu^k\rangle$ converges to μ, so too will the sequence $\langle\nu^k\rangle$, since the corresponding measures differ on only a point whose measure goes to zero. Then, by the continuity of Q, given any $\epsilon > 0$, there exists k^* such that $k \geqq k^*$ implies

$$d(Q(\mu), Q(\mu^k)) < \varepsilon$$

and

$$d(Q(\mu), Q(\nu^k)) < \varepsilon,$$

where d is the Hausdorff metric on subsets of S. Thus, by the triangle inequality,

$$d(Q(\mu^k), Q(\nu^k)) < 2\ \varepsilon.$$

Thus, for large enough k, any equilibrium price of the kth "apparent economy" is arbitrarily close to an equilibrium price of E^k, while both lie within the neighborhood of $Q(\mu)$ on which the indirect utility function V is continuous. If we now suppose that $S^k(\cdot) = C(\bar{a}, \cdot)$, the conclusion follows easily.

Suppose we require that the consumption sets be the positive orthant, that each agent hold a positive amount of each commodity and that the S correspondences from which an agent can choose are limited to being continuously differentiable functions f on the interior of P obeying Walras law with equality and the desireability condition that if $p_n \to p$, a boundary point of P, then $\| f(p_n) \| \to \infty$. If we metrize this space $\mathcal{D}$ of functions by requiring uniform convergence of the functions and their first derivatives on compact sets, the set of economies (described by measures, as above) on which the Q correspondence is continuous is open and dense in the topology of weak convergence. For a proof of this, see Delbaen [2] or K. Hildenbrand's appendix to Part II, Chapter 2 of [4]. Moreover, if we drop the requirement that the functions be continuously differentiable, requiring only continuity, we still have that the set of economies on which Q is continuous is a dense subset [2]. In fact, it is a "residual set," that is, the countable intersection of open dense sets [3]. Thus, at least for these important special cases, we can say that the competitive response is "usually" limiting individually incentive compatible.

5. The Core and the Competitive Assumption

We noted in the introduction that, while our results provide one justification of the competitive assumption, the literature on the core and competitive equilibria points to another possible justification.[10] (A precise statement of the central results in this literature is given in [4].) In this Section we will briefly sketch this argument and compare the two approaches.

As is now well-known, the core and competitive allocations coincide in economies described by a non-atomic measure on the agents characteristics, while in the case of replication, the core shrinks monotonically down to the competitive allocations. In this latter case, it is easy to show, in fact, that each core allocation is arbitrary close to some competitive equilibrium allocation if the number of agents is large enough (see [4]). Thus, in large replica economies, all core allocations are very close to ones that would arise from agents taking prices as given. The parallel result for more general sequences of economies has been achieved by T. Bewley [1]. In particular, he has shown that in large enough exchange economies, if there are many agents similar to any one agent in their preferences and endowments, then the core allocations are all approximately competitive in the sense that for any core allocation there is a price such that the demand of each agent in the economy at this price is within any specified neighborhood of the consumption he gets at the core allocation. This price may be taken to be an equilibrium price for the economy that is limit of the sequence of finite economies. Then, if the equilibrium price correspondence is continuous at this limit, the given price will then be close to some equilibrium price for the finite economy.

Thus, every core allocation is almost decentralized by a price that is almost an equilibrium price. If we then believe that exchange takes place in such a way that the resultant allocation is in the core, we may as well assume that these allocations actually arise from price-taking behavior under the workings of the competitive mechanism. The difficulty with this justification, however, is that there may be no obvious reason for believing that the allocations arising in an economy will actually belong to the core.

The results in this paper offer the basis for a somewhat different justification of the competitive assumption in exchange situations. We assume explicitly that exchange is guided by prices, but that consumers will attempt to manipulate these prices by altering their offers to buy and sell at various prices from their true competitive values. Under the assumptions of Theorems 1 or 2 the gain an agent can hope to achieve by such behavior goes to zero. One might then assume that, since there is so little to gain, each agent acts competitively.

The strength of this approach is that it posits allocation via prices and that it explicitly recognizes the impact that agents can have on demand. In this, the present approach to justifying the competitive assumption is somewhat more like that which arises by considering the impact of numbers on the Cournot equilibrium. In this model, which deals with firms rather than consumers, one assumes that agents will not act as price-takers, but then shows that the outcomes of their behavior approach the competitive outcome.[11] This assumption of the allocation process being directed by prices seems to be a valuable one for treating the question at hand.

The chief weakness of the present approach would appear to be its concentration on individual action. Theorem 2 allows that the agents in the rest of the economy may not be acting as price-takers, and, implicitly, that their choices of responses may be coordinated. However, we have offered no analysis of how these choices might be made, or of the payoffs to them. This question is open, and may be worthy of investigation.

Footnotes

1. An excellent overview of this work and of the related literature is provided in Hurwicz's Ely Lecture [6].

2. See the paper by Ledyard [7], in which the pairs of economies and resource allocation mechanisms for which correct preference relevation is consistent with individual self-interest are characterized. Ledyard shows that even a single-valued core in utility space is not generally sufficient to establish this incentive-compatibility for general resource allocation mechanisms.

3. We use R^N to denote Euclidean N-space and R^N_+ to denote its non-negative orthant.

4. For the same reasons, it is also appropriate to allow individual agents to know the demands of the others and the outcomes resulting from any manipulation of prices they attempt. Expecting agents to have this much information obviously is not realistic, but any limiting of the information they have would presumably make adherence to passive competitive behavior even more attractive.

5. We do not specify a particular topology here. In fact, it may be necessary to restrict the choices from $\mathscr{A}(a)$ to some subset of this space (e.g., the set of correspondences in $\mathscr{A}(a)$ with closed graphs) and to topologize this subset in order to obtain useful results.

6. The Borel sets of a metric space are the smallest σ-algebra of subsets containing the open sets.

7. The support of a measure is the smallest closed set of full measure. We use "supp ν" to denote the support of ν.

8. See [4] for a fuller developments of these techniques.

9. A correspondence W from one metric space, X, into another, Y, is continuous at a point $x \in X$ if $W(x) \neq \emptyset$, if for any open set U meeting W(x) there is a neighborhood V of x such that, for each x′ in V, W(x′) meets U and if for each open set U containing W(x) there is a neighborhood V of x such that $W(x') \subset U$ for all $x' \in V$.

10. The core of an exchange economy is the set of allocations with the property no coalition of agents can, by trading only their own endowments within the coalition, achieve a consumption for each member of the coalition which he prefers to what he received under the given allocation. By competitive equilibrium allocations, we mean those which arise when every agent uses his competitive response.

11. For a recent discussion of the Cournot approach, see [8].

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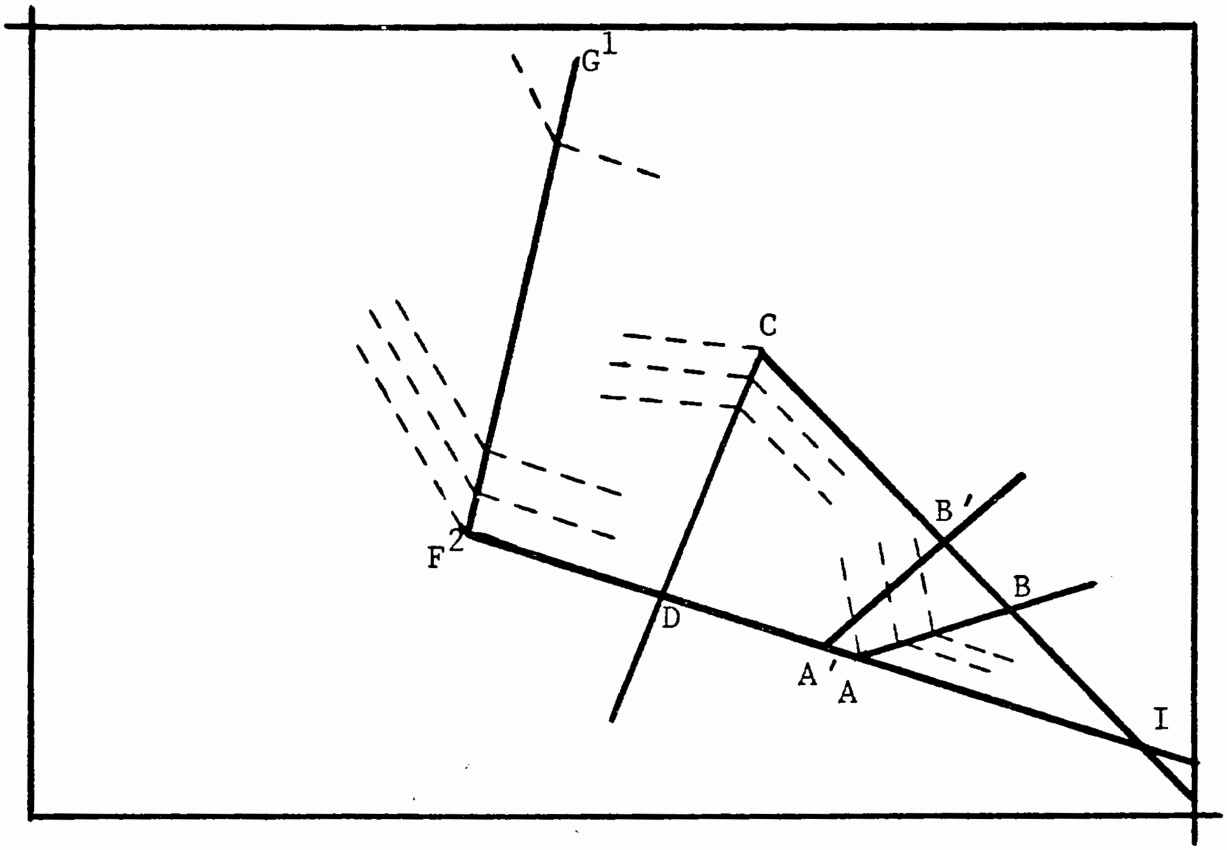

FIGURE 1

THE ECONOMY E^1