Discussion Paper No. 421

Preplay Negotiations in Non-Cooperative Games

by

Ehud Kalai*

April 1980

* This research was supported by the National Science Foundation Grant #SOC-7907542 and by the J. C. Kellogg Center for Advanced Studies in Managerial Economics.

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1. Introduction

In a one shot normal form non-cooperative game, often the outcome is not desirable from the point of view of all the players. The classical example of such a situation is the prisoners' dilemma game where the players get trapped into an unfavorable situation because of their inability to communicate binding agreements to each other.

This problem partially disappears in infinitely repeated games. In such games cooperative outcomes can be at equilibrium due to the fact that future plays may be conditioned on past plays. That is, unlike a one-shot game, a player who does not cooperate in one play of the game may be punished for it in later plays. These threat strategies, when played, tend to bring about a cooperative outcome. However even in these games, some outcomes which are at equilibrium can be uniformly dominated (for all the players) by another feasible outcome. For an extensive discussion and a survey of this literature, see Aumann [1978].

The purpose of this paper is to suggest an example of a formal preplay negotiation procedure. The purpose of such a procedure is to improve outcomes in a normal form game by introducing formal process of early communications. When we test this procedure on a prisoners' dilemma game we observe that with any positive number of preplays all the perfect equilibrium of the resulting extended game yield the cooperative payoff. In another example we show that the perfect equilibria payoffs converge to be Pareto optimal as the number of preplays increase.

The generality of these types of results for our proposed procedure or for modified versions of it is an open problem at this time. Another interesting open question is--what would be the effect of formal preplay negotiations on games of incomplete information.

The preplay procedure that we suggest here is attractive for a few reasons. It is easy to explain to the players and the payoff computations (unlike Nash [1953] or Kalai-Rosenthal [1978] procedures) are trivial. Also (unlike Nash [1953]) these computations do not depend on any knowledge of the parameters of the game being available to the referee who conducts the preplay game. Another nice feature of the procedure is that the players, individually or simultaneously, have the option (under the procedure rules) to play in a way that will guarantee them a payoff at least as big as the one shot payoff. Because of this property we can expect that they would be willing to participate in this extended game.

For an intuitive motivation of this procedure one may think of the customary preplay game involved in buying a house in the U.S. Usually in such a procedure the players, alternately, sign one side of a binding contract until both sides of the same contract are signed. We modified this procedure in order to make it symmetric (instead of one player moving first) and by fixing in advance the number of preplays that will take place.

2. The Preplay Procedure and its Perfect Equilibria

For a positive integer m we let $M = \{1,2,\ldots m\}$ denote the set of players. For each $i \in M$ let S_i denote a finite set of strategies of player i, $S = \overset{m}{\underset{i=1}{X}} S_i$, and $f^i\colon S \to R$ denote the von-Neumann Morgenstern utility payoff of player i. Thus this is a standard m-person non-cooperative normal form game. The extended game with the preplays is described verbally as follow. Let n be a positive integer; n denotes the number of preplays agreed upon in advance and which is common knowledge. Initially every player i chooses a strategy $s_i \in S_i$ yielding an m-tuple $s^n \in S$. These choices are made simultaneously by all players and they are announced publicly after the choices have been made. The players may use randomization in choosing their strategies. Now there will be n preplays numbered $n, n-1, \ldots 2, 1$. For every preplay j there will be two strategy m-tuples $s^j \varepsilon S$ and $s^{j-1} \varepsilon S$. s^j is the strategy leading into this preplay, which is common knowledge, and s^{j-1} will be the strategy tuple coming out of the preplay. s^{j-1} will be the entering strategy into the j-1 preplay. At the end of preplay 1 strategy $s^0 \varepsilon S$ will result and every player i will be paid $f^i(s^0)$.

All the preplays are identical and are described as follows. Let s^j be the entering strategy leading into the j^{th} preplay. Now all the players simultaneously choose new strategies $s^{j,1} = (s_1^{j,1}, s_2^{j,1}, \ldots, s_m^{j,1})$. If $s^{j,1} = s^j$, i.e. no changes took place, then the jth preplay ends and s^{j-1} is defined to be s^j. However if some changes took place then let C^1 denote the set of players who changed, i.e. $C^1 = \{i \varepsilon M : s_i^{j,1} \neq s_i^j\}$, and define $s_i^{j-1} = s_i^{j,1}$ for every $i \varepsilon C^1$. In other words the revising players will have their revised strategy as their entering strategies into the j-1 preplay. Now the players who did not revise, i.e. those who belong to $M - C^1$, simultaneously announce strategies $s_i^{j,2}$. These choices are made public and the same procedure follows. Let C^2 denote the players that made a revision on this second trial, i.e. $C^2 = \{i \varepsilon M - C^1 : s_i^{j,2} \neq s_i^{j,1}\}$. If $C^2 = \emptyset$ then for every $i \varepsilon M - C^1$ $s_i^{j-1} = s_i^j$ and the jth preplay ends. Otherwise

let $s_i^{j-1} = s_i^{j,2}$ for every $i \varepsilon C^2$ and repeat the same procedure for the players in $M - C^1 - C^2$. Continuing this process, the sequence of trials must end in at most m trials and s^{j-1} is determined and is public knowledge for the next preplay.

We are interested in the payoffs that are associated with Selten's [1975] perfect equilibrium strategies of the extended preplay game. By Selten's result the existence of such an equilibrium is guaranteed. We will use only two necessary conditions that perfect equilibrium strategies possess in order to derive necessary conditions on the perfect equilibrium payoffs of the preplay game. Thus our results in the following examples are correct for any equilibrium concept which satisfies these two conditions.

The first property is that <u>weakly dominated strategies are used with probability 0</u> at a perfect equilibrium. That is for every player $i \in M$ consider a pair of strategies $s_i, t_i \in S_i$ with the property that $f^i(s_1,\ldots,s_{i-1}, s_i, s_{i+1},\ldots,s_m) \geq f^i(s_1,\ldots,s_{i-1}, t_i, s_{i+1},\ldots,s_m)$ for every $(s_1,\ldots,s_{i-1}, s_{i+1},\ldots,s_m) \in \underset{j \neq i}{X} S_j$ and such that the inequality is strict in some cases, then t_i will be used with probability zero at every perfect equilibrium.

The second property is <u>subgame perfectness</u> (see Selten [1975]). It states that if we look at equilibrium strategies for the whole game and look at the strategies that they induce on a "subgame" (see Selten [1975]) then the subgame strategies should be in equilibrium in the subgame. This property, in addition to its game theoretic rationality, makes computation of equilibrium much easier because it enables us to use "backwards induction" in our computations.

Consider for example the preplay game described earlier. Suppose we want to compute the perfect equilibrium payoffs associated with the game with two preplays left and when the entering strategy to preplay 2 being s^2. In order to compute perfect equilibrium strategies and payoffs in the second preplay we have to know only the perfect equilibrium payoffs associated with preplay 1 with its various entering strategies. Similarly we use our results for preplay 2 in order to compute preplay 3 and so on. We use this technique in the following examples.

3. <u>Preplays and the Prisoners' Dilemma</u>

We consider the following two person prisoner's dilemma game

Player II

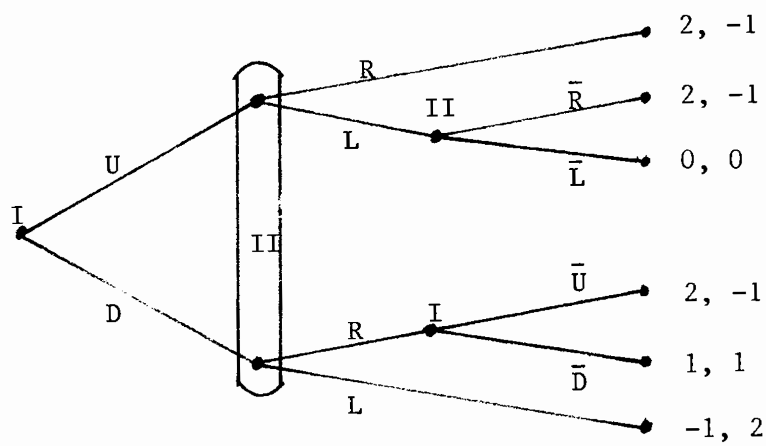
Player I		L	R
	U	0,0	2,-1
	D	-1,2	1, 1

Our computation of the perfect equilibrium payoffs are given in the following table.

Perfect equilibrium payoffs

with entering strategies:				in preplay number:
U,L	U,R	D,L	D,R	
0,0	2,-1	-1,2	1,1	0
0,0	0,0	0,0	1,1	1
1,1	1,1	1,1	1,1	2
1,1	1,1	1,1	1,1	n>2

The row associated with preplay 0 is just the payoffs from the original matrix. Now consider for example the 0,0 entry in the D,L column and preplay 1. The extensive form description of the resulting game is the following



For this game it is clear that $(U,\bar{U})$, $(L,\bar{L})$ are the only perfect equilibrium strategies and these yield the payoffs of (0,0).

Thus if the players play the game with one preplay only then in their choice of initial strategis they face the following matrix--

	L	R
U	0,0	0,0
D	0,0	1,1

Clearly, by the weak domination property, the only perfect equilibrium strategies are (D,R) yielding a payoff of (1,1).

If they play the game with 2 or more preplays then in their initial choice of strategies they face the matrix--

	L	R
U	1,1	1,1
D	1,1	1,1

4. A Game of Coordination

We consider the following 2-person game:

		Player II L	Player II R
Player I	U	1,1	0,0
	D	0,0	1,1

Perfect equilibrium payoffs associated with--

entering strategies: UL	UR	DL	DR	at preplay:
1,1	0,0	0,0	1,1	0
1,1	1,1 or 1/2,1/2	1,1 or 1/2,1/2	1,1	1
1,1	1,1 or 3/4,3/4	1,1 or 3/4,3/4	1,1	2
1,1	1,1 or $\frac{2^n-1}{2^n}, \frac{2^n-1}{2^n}$	1,1 or $\frac{2^n-1}{2^n}, \frac{2^n-1}{2^n}$	1,1	$n \geq 2$

It follows that with n preplays the only perfect equilibrium payoffs of this game are either 1,1 or $\frac{2^{n+1}-1}{2^{n+1}}, \frac{2^{n+1}-1}{2^{n}+1}$ and thus a very fast convergence to Pareto optimality is achieved.

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