Discussion Paper No. 412

# REGULATING A MONOPOLIST WITH UNKNOWN COSTS

by

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December 1979

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Abstract. We consider the problem of how to regulate a monopolistic firm whose costs are unknown to the regulator. The regulator's objective is to maximize a linear social welfare function of the consumers' surplus and on the firm's profit. In the optimal regulatory policy, prices and subsidies are designed as functions of the firm's cost report so that expected social welfare is maximized, subject to the constraints that the firm has nonnegative profit and has no incentive to misrepresent its costs. We explicitly derive the optimal policy and analyze its properties.

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## 1. Introduction

In their classic papers Dupuit (1952) and Hotelling (1938) considered pricing policies for a bridge that had a fixed cost of construction and zero marginal cost. They demonstrated that the pricing policy that maximizes consumer well-being is to set price equal to marginal cost and to provide a subsidy to the supplier equal to the fixed cost, so that a firm would be willing to supply the bridge. This first-best solution is based on a number of informational assumptions. First, the demand function is assumed to be known to both the regulator and to the firm. While the assumption of complete information may be too strong, the assumption that information about demand is as available to the regulator as it is to the firm does not seem unnatural. A second informational assumption is that the regulator has complete information about the cost of the firm or at least has the same information about cost as does the firm. This assumption is unlikely to be met in reality, since the firm would be expected to have better information about costs than would the regulator. As Weitzman (1978) has stated,

> "An essential feature of the regulatory environment I am trying to describe is uncertainty about the exact specification of each firm's cost function. In most cases even the managers and engineers most closely associated with production will be unable to precisely specify beforehand the cheapest way to generate various hypothetical output levels. Because they are yet removed from the production process, the regulators are likely to be vaguer still about a firm's cost function." (p. 684)

As this observation suggests, it is natural to expect that a firm would have better information regarding its costs that would a regulator. The purpose of this paper is to develop an optimal regulatory policy for the case in which the regulator does not know the costs of the firm.

One strategy that a regulator could use in the absence of full information about costs is to give the firm the title to the total social surplus and to delegate the pricing decision to the firm. In pursuing its own interests, which would then be to maximize the total social surplus, the firm would adopt the same marginal cost pricing strategy that the regulator would have imposed if the regulator had known the costs of the firm. This approach has been proposed by Loeb and Magat (1979) but leaves the equity issue unresolved, since the firm receives all the social surplus and consumers receive none. To resolve the equity issue, Loeb and Magat propose that the right to the monopoly franchise be auctioned among competing firms as a means of transferring surplus from producers to consumers. However, if there are no other producers capable of supplying the product efficiently, an auction will not be effective. Thus, in this paper we will not assume that an efficient auction could be conducted. In the absence of the auction possibility, it is clear that consumers would be better off by allowing the firm to operate as a monopolist rather than transferring the total surplus to the firm, since in that case consumers would at least receive some benefit from the firm's output. Another approach that might be considered to transfer surplus from producers to consumers would be to levy a lump-sum tax against the firm. When the regulator does not know the cost, however, it runs the risk that if the tax is set too high the firm may decline to supply the good.

The approach taken in this paper to regulation under asymmetric information is based on the work of Myerson (1979) (1981) and involves the design of a regulatory policy which recognizes that the firm may have an incentive to misreport its cost in order to obtain a more favorable price. An incentive-compatible regulatory policy in which the firm has no incentive to misreport its cost can, however, be shown to be at least as good as any non-incentive-compatible regulatory policy, so the regulator need only consider incentive-compatible policies. That is, since the regulator does not know the firm's costs, the regulator must set the firm's price and subsidy as a function of some cost report from the firm, and the regulatory policy must satisfy the constraint that the firm should have an incentive to report the information needed by the regulator. Because of this constraint, the regulatory policy can be optimal only in a constrained sense, and a welfare loss results from the informational asymmetry.

The optimal regulatory policy necessarily depends on the regulator's prior information about the firm's costs. If it is optimal for the firm to produce, the optimal pricing rule will be shown to depend only on the regulator's information about costs. As with the first-best solution, the optimal regulatory policy under asymmetric information is such that production is warranted only if the social benefit resulting from the optimal pricing rule is at least as great as the "adjusted" fixed cost. In order to implement a regulatory policy, it is necessary to provide the firm with a fair rate of return, and in Dupuit's and Hotelling's complete-information case with a constant marginal cost and a fixed cost, a subsidy equal to the fixed cost is used to induce the firm to produce. In the regulatory policy considered here, a subsidy is used both to reward the firm sufficiently so that it will produce and to induce the firm to reveal its costs.

In Section 2 we define the basic model used to describe the regulator's problem, and in Section 3 we analyze that problem and derive the optimal regulatory policy. In Section 4 the general properties of this optimal policy are discussed. The special cases of known fixed costs and of known marginal costs are discussed in Sections 5 and 6.

## 2. Basic Structures

To model the problem of regulating a natural monopoly when its cost structure is not known to the regulator, we could let the monopolistic firm have costs determined by some function $C(q, \theta)$, where q is the quantity produced and $\theta$ is a cost parameter that is unknown to the regulator. To keep the problem mathematically tractable, however, we shall assume that the firm's cost function is bilinear in q and $\theta$, of the form

$$C(q,\theta) = (c_0 + c_1\theta)q + (k_0 + k_1\theta) \quad \text{if } q>0, \quad \text{and } C(0,\theta) = 0, \tag{1}$$

where $c_0$, $c_1$, $k_0$, $k_1$ are known constants satisfying $c_1 \geq 0$ and $k_1 \geq 0$. For mathematical simplicity, we assume that the range of possible $\theta$ is bounded within some known interval from $\theta_0$ to $\theta_1$ $(\theta_0 < \theta_1)$.

To interpret this cost function, observe that $k_0 + k_1\theta$ represents a fixed cost incurred to produce any positive output, and $c_0 + c_1\theta$ re-represent the marginal cost of producing each unit after the first. For example, this formulation is general enough to include, as special cases, the case of unknown marginal costs $(C(q,\theta) = k_0 + \theta q)$ and of unknown fixed costs $(C(q,\theta) = \theta + c_0 q)$, which will be discussed in Sections 5 and 6, respectively.

We assume that the firm knowsthe true value of its cost parameter $\theta$, but that $\theta$ is not known to the regulator. Furthermore, the regulator is not assumed to be able to audit the cost actually incurred by the firm, so that the regulatory policy cannot be based on the true cost of the firm.

Thus, if the regulator asks for a cost report from the firm, we must anticipate that the firm would misreport its cost function whenever this was to its advantage.

The regulator's problem is to decide how the firm's regulated price and subsidy should be determined, as functions of some cost report from the firm. The following observation is central to the analysis of the regulator's problem:

> <u>Proposition</u>. (<u>The revelation principle</u>.) Without any loss of generality, the regulator may be restricted to regulatory policies which require the firm to report its cost parameter $\theta$ and which give the firm no incentive to lie.

In different contexts this revelation priciple has been discussed in several other recent papers (see Dasgupta, Hammond, and Maskin (1979), Gibbard (1973), and Harris and Townsend (1981), and Myerson (1979)). To see why it is true, suppose that the regulator chose some general regulatory policy, not of the form described in the proposition. For each possible value of $\theta$, let $\Psi(\theta)$ be the cost report that the firm would submit if its true cost parameter were $\theta$. That is, $\Psi(\theta)$ maximizes the firm's expected profit, when it is confronted with this regulatory policy and its true cost parameter is $\theta$. Now consider the following new regulatory policy: ask the firm to report its cost parameter $\theta$; then compute $\Psi(\theta)$; and then enforce the regulations that would have been enforced in the original regulatory policy if $\Psi(\theta)$ had been reported there. It is easy to see that the firm never has any incentive to lie to the regulator in the new policy. (Otherwise it would have had some incentive to lie to itself in the originally given policy.) Thus, the new policy is of the form described in the proposition, and it always gives the same outcomes as the original policy.

Following the Bayesian approach, we assume that the regulator has some subjective prior probability distribution for the unknown parameter $\theta$, prior to receiving any cost reports from the firm. We let $f(\cdot)$ be the density function for this probability distribution, and we assume that $f(\theta)$ is a continuous function of $\theta$ with $f(\theta) > 0$ over the interval $[\theta_0, \theta_1]$ and with $F(\theta)$ denoting the cumulative distribution function for $\theta$.

The demand function is assumed to be known by both the firm and the regulator. We let $P(\cdot)$ denote the inverse demand function, so that $P(q)$ is the price at which the consumers demand the output q.

Ignoring income effects, the total value $V(q)$ to consumers of an output quantity q is the area under the demand curve, given by

$$V(q) = \int_0^q P(\tilde{q})d\tilde{q}. \tag{2}$$

The consumers' surplus is then $V(q) - qP(q)$.

We assume that the regulator has consumer and producer surplus objectives, and has three basic regulatory instruments available to achieve its objective:

1) the regulator can decide whether to allow the firm to do business at all;
2) if the firm is in business, then its price or quantity of output may be regulated; and
3) the firm may be given a subsidy or charged a tax.

Now, using the revelation principle, we may consider only regulatory policies under which the firm's cost report will reveal its cost parameter $\theta$, so the regulatory instruments can be chosen as functions of $\theta$. Thus, we shall describe a regulatory policy by four outcome functions $(r,p,q,s)$, to be interpreted as follows. For any $\hat{\theta}$ in $[\theta_0,\theta_1]$, if the firm reports that its cost parameter is $\hat{\theta}$, then $r(\hat{\theta})$ is the probability that the regulator will permit the firm to do business at all.[1] Since $r(\hat{\theta})$ is a probability, it must satisfy

$$0 \leq r(\hat{\theta}) \leq 1. \tag{3}$$

If the firm does go into business after reporting $\hat{\theta}$, then $p(\hat{\theta})$ will be its regulated price, and $q(\hat{\theta})$ will be the corresponding quantity of output, satisfying[2]

$$p(\hat{\theta}) = P(q(\hat{\theta})). \tag{4}$$

Finally, $s(\hat{\theta})$ will be the expected subsidy paid to the firm if it reports cost parameter $\hat{\theta}$. For example, if the firm would get a subsidy $s^*(\hat{\theta})$ if it were allowed to go into business, but would get no subsidy if it were not allowed to go into business, then the expected subsidy is $s(\hat{\theta}) = r(\hat{\theta})s^*(\hat{\theta})$. If $s(\hat{\theta})$ is negative, then it represents a tax on the firm.

The firm is assumed to be risk neutral. Thus, given a regulatory policy $(r,p,q,s)$, if the firm's cost parameter is $\theta$, and if the firm reports $\theta$ honestly, its expected profit $\pi(\theta)$ is

$$\pi(\theta) = [p(\theta)q(\theta) - (c_0 + c_1\theta)q(\theta) - k_0 - k_1\theta]r(\theta) + s(\theta). \tag{5}$$

If the firm were to misrepresent its cost and report $\hat{\theta}$, when $\theta$ is its true cost parameter, its expected profit would be

$$\pi^*(\hat{\theta},\theta) = [p(\hat{\theta})q(\hat{\theta}) - (c_0 + c_1\theta)q(\hat{\theta}) - k_0 - k_1\theta]r(\hat{\theta}) + s(\hat{\theta}) \tag{6}$$

Thus, to guarantee that the firm has no incentive to misrepresent its cost, we must have

$$\pi(\theta) = \underset{\hat{\theta}}{\text{maximum}}\ \pi^*(\hat{\theta},\theta) \tag{7}$$

for all $\theta$ in $[\theta_0, \theta_1]$.

We assume that the regulator cannot force the firm to operate if it expects a negative profit. So the regulatory policy must also satisfy the individual rationality condition

$$\pi(\theta) \geq 0 \tag{8}$$

for all $\theta$ in $[\theta_0,\theta_1]$.

We say that a regulatory policy (r,p,q,s) is <u>feasible</u> if it satisfies the four constraints (3), (4), (7) and (8) for all $\theta$ in $[\theta_0,\theta_1]$. Thus, when the regulator uses a feasible regulatory policy, the firm will be willing to submit honest cost reports and to operate whenever permitted. The regulator's problem is to find a feasible regulatory policy that maximizes social welfare, which would be specified next.

If the consumers are risk-neutral and have additively separable utility for money and the firm's product, the net expected gain for the consumers from a regulatory policy (r,p,q,s) would be[3]

$$\int_{\theta_0}^{\theta_1} ([V(q(\theta)) - p(\theta)q(\theta)]r(\theta) - s(\theta))f(\theta)d\theta.$$

That is, the consumers' expected gain is the expected consumers' surplus from the marketplace minus the firm's expected subsidy, which must be paid by the consumers through their taxes. The regulator's expectation of the firm's profit (before $\theta$ is known) is

$$\int_{\theta_0}^{\theta_1} \pi(\theta)f(\theta)d\theta.$$

We assume that the regulator maximizes a weighted sum of the expected gains to consumers plus the expected profit for the firm. Specifically, we assume that there is some number $\alpha$, satisfying

$$0 \leq \alpha \leq 1,$$

such that the regulator's objective is to maximize

$$\int_{\theta_0}^{\theta_1} ([V(q(\theta)) - p(\theta)q(\theta)]r(\theta) - s(\theta))f(\theta)d\theta \;+\; \alpha\int_{\theta_0}^{\theta_1}\pi(\theta)f(\theta)d\theta. \tag{9}$$

## 3. Derivation of the Optimal Policy

We first state and prove two lemmas which provide a more useful characterization of the regulator's problem than the definitions given in the preceding section.

Lemma 1. A regulatory policy is feasible if and only if it satisfies the following conditions for all $\theta$ in $[\theta_0,\theta_1]$

$$0 \leq r(\theta) \leq 1, \tag{3}$$

$$p(\theta) = P(q(\theta)), \tag{4}$$

$$\pi(\theta) = \pi(\theta_1) + \int_{\theta}^{\theta_1} r(\tilde{\theta})(c_1 q(\tilde{\theta}) + k_1)d\tilde{\theta}, \tag{10}$$

$$\pi(\theta_1) \geq 0, \text{ and} \tag{11}$$

$$r(\theta)\ (c_1 q(\theta) + k_1) \geq r(\hat{\theta})(c_1 q(\hat{\theta}) + k_1) \text{ for all } \hat{\theta} \geq \theta. \tag{12}$$

Proof. First we show that feasibility (defined by conditions (3), (4), (7), (8)) implies the conditions in the lemma. Since (3) and (4) are simply repeated from the definition and (11) is implied by (8), we only need to show (10) and (12).

From (7) for any $\theta$ and $\hat{\theta}$

$$\pi(\theta) \geq \pi^*(\hat{\theta},\theta) = \pi(\hat{\theta}) + r(\hat{\theta})(c_1 q(\hat{\theta}) + k_1)(\hat{\theta} - \theta), \tag{13}$$

using the definitions (5) and (6). Thus

$$r(\hat{\theta})(c_1 q(\hat{\theta}) + k_1)(\hat{\theta} - \theta) \leq \pi(\theta) - \pi(\hat{\theta}) \leq r(\theta)(c_1 q(\theta) + k_1)(\hat{\theta} - \theta) \tag{14}$$

where the second inequality follows from the analogue of (13) with the roles of $\theta$ and $\hat{\theta}$ reversed. Then (12) follows from (14), when $\hat{\theta} > \theta$.

Since $r(\theta)(c_1 q(\theta) + k_1)$ is a nonincreasing function of $\theta$, it must be continuous almost everywhere in $[\theta_0, \theta_1]$. Thus, if we divide (14) by $(\theta - \hat{\theta})$ and take the limit as $\hat{\theta} \to \theta$, we obtain

$$\pi'(\theta) = -r(\theta)(c_1(\theta) + k_1)$$

for almost all $\theta$. Integrating implies that (10) must hold for any feasible regulatory policy.

Conversely, we must show that conditions (7) and (8) are implied by the conditions in the lemma. Condition (8) follows easily from (10) and (11), since $c_1 \geq 0$ and $k_1 \geq 0$ by assumption. To prove (7), observe that (10) implies

$$\pi^*(\hat{\theta},\theta) = \pi(\hat{\theta}) + r(\hat{\theta})(c_1 q(\hat{\theta}) + k_1)(\hat{\theta} - \theta)$$
$$= \pi(\theta) - \int_{\theta}^{\hat{\theta}} [r(\tilde{\theta})(c_1 q(\tilde{\theta}) + k_1) - r(\hat{\theta})(c_1 q(\hat{\theta}) + k_1)]d\tilde{\theta}.$$

If $\hat{\theta} > \theta$ then the integrand is nonnegative (since $\tilde{\theta} < \hat{\theta}$) by (12), so $\pi^*(\hat{\theta},\theta) \leq \pi(\theta)$. If $\hat{\theta} < \theta$ then the integrand is nonpositive, but then the integral is nonnegative (since the direction of integration is backwards), so that $\pi^*(\hat{\theta},\theta) \leq \pi(\theta)$ still holds, as (7) requires. Q.E.D.

<u>Lemma 2</u>. For any feasible regulatory policy, the social welfare function (9) is equal to

$$\int_{\theta_0}^{\theta_1} [V(q(\theta)) - (c_0 + c_1 z_\alpha(\theta))q(\theta) - k_0 - k_1 z_\alpha(\theta)]r(\theta)f(\theta)d\theta \quad - (1 - \alpha)\pi(\theta_1) \tag{15}$$

where

$$z_\alpha(\theta) = \theta + (1-\alpha)\frac{F(\theta)}{f(\theta)}. \tag{16}$$

<u>Proof</u>. From the definition of $\pi(\theta)$ in (5), we obtain

$$p(\theta)q(\theta)r(\theta) + s(\theta) = \pi(\theta) + ((c_0 + c_1\theta)q(\theta) + k_0 + k_1\theta)r(\theta). \tag{17}$$

Also, using (10) from Lemma 1,

$$\int_{\theta_0}^{\theta_1} \pi(\theta)f(\theta)d\theta$$

$$= \int_{\theta_0}^{\theta_1} \left(\int_{\theta}^{\theta_1} r(\tilde{\theta})(c_1 q(\tilde{\theta}) + k_1)d\tilde{\theta} + \pi(\theta_1)\right)f(\theta)d\theta$$

$$= \int_{\theta_0}^{\theta_1} r(\tilde{\theta})(c_1 q(\tilde{\theta}) + k_1)\int_{\theta_0}^{\tilde{\theta}} f(\theta)d\theta d\tilde{\theta} + \pi(\theta_1)$$

$$= \int_{\theta_0}^{\theta_1} r(\theta)(c_1 q(\theta) + k_1)F(\theta)d\theta + \pi(\theta_1). \tag{18}$$

Substituting (17) and (18) into (9) yields

$$\int_{\theta_0}^{\theta_1} ([V(q(\theta)) - p(\theta)q(\theta)]r(\theta) - s(\theta) + \alpha\pi(\theta))f(\theta)d\theta$$

$$= \int_{\theta_0}^{\theta_1}([V(q(\theta)) - (c_0 + c_1\theta)q(\theta) - k_0 - k_1\theta]r(\theta) - (1 - \alpha)\pi(\theta))f(\theta)d\theta$$

$$= \int_{\theta_0}^{\theta_1} [V(q(\theta)) - (c_0 + c_1\theta)q(\theta) - k_0 - k_1\theta]r(\theta)f(\theta)d\theta$$

$$- (1 - \alpha) \int_{\theta_0}^{\theta_1} F(\theta)(c_1 q(\theta) + k_1)r(\theta)d\theta - (1 - \alpha)\pi(\theta_1).$$

Formula (15) then follows by straightforward simplification. Q.E.D.

Lemma 2 gives a strong suggestion as to what the optimal policy should be. The integrand in (15) is maximized for each $\theta$ by choosing $q(\theta)$ to maximize $V(q(\theta)) - (c_0 + c_1 z_\alpha(\theta))q(\theta)$, and by letting $r(\theta)$ equal one or zero depending on whether the bracketed expression is positive or negative. Then the subsidy can be chosen so that $\pi$ satisfies conditions (10) and (11). But this solution will not be feasible unless the monotonicity condition (12) is also satisfied, and this condition implies that $z_\alpha(\theta)$ must be nondecreasing in $\theta$. Unfortunately, for some densities $f(\cdot)$, (16) need not yield a monotone $z_\alpha(\cdot)$ function. With some carefully chosen definitions, therefore, we now construct another function which is closely related to $z_\alpha(\cdot)$, but which is always monotone nondecreasing.

Given $z_\alpha(\cdot)$ as in (16), let

$$h_\alpha(\phi) = z_\alpha(F^{-1}(\phi)) \tag{19}$$

for any $\phi$ between 0 and 1. (Notice that the cumulative distribution function $F(\theta)$ is strictly increasing, so that it is indeed invertible). Let

$$H_\alpha(\phi) = \int_0^\phi h(\tilde{\phi})d\bar{\phi}. \tag{20}$$

Next, using the notation of Rockafeller (1979, page 36), let

$$\bar{H}_\alpha(\phi) = \text{conv } H_\alpha(\phi). \tag{21}$$

That is, $\bar{H}_\alpha(\cdot)$ is the highest convex function on the interval $[0,1]$ satisfying $\bar{H}_\alpha(\phi) \leq H_\alpha(\phi)$ for all $\phi\in[0,1]$. Since $\bar{H}_\alpha$ is convex, it is differentiable almost everywhere. Then let

$$\bar{h}_\alpha(\phi) = \bar{H}'_\alpha(\phi) \tag{22}$$

whenever this derivative is defined, and extend $\bar{h}_\alpha(\phi)$ by right-continuity to all $0 \leq \phi \leq 1$. Finally, let

$$\bar{z}_\alpha(\theta) = \bar{h}_\alpha(F(\theta)). \tag{23}$$

The following lemma summarizes the properties of this $\bar{z}_\alpha(\cdot)$ function that are needed to derive the optimal policy,

<u>Lemma 3</u>. There exists a continuous function $G_\alpha:[\theta_0,\theta_1]\to\mathbb{R}$ such that $G_\alpha(\theta) \geq 0$ for all $\theta$, $\bar{z}_\alpha(\theta)$ is locally constant whenever $G_\alpha(\theta) > 0$, and

$$\int_{\theta_0}^{\theta_1} A(\theta)z_\alpha(\theta)f(\theta)d\theta = \int_{\theta_0}^{\theta_1} A(\theta)\bar{z}_\alpha(\theta)f(\theta)d\theta - \int_{\theta=\theta_0}^{\theta_1} G_\alpha(\theta)dA(\theta) \tag{24}$$

for any monotone function $A(\cdot)$. Furthermore, $\overline{z}_{\alpha}(\theta)$ is a nondecreasing function of $\theta$, and if $z_{\alpha}(\theta)$ is a nondecreasing function of $\theta$ then $\overline{z}_{\alpha}(\theta) = z_{\alpha}(\theta)$ for all $\theta$.

<u>Proof</u>. The function $G_{\alpha}$ in the lemma is

$$G_{\alpha}(\theta) = H_{\alpha}(F(\theta)) - \overline{H}_{\alpha}(F(\theta)).$$

$G_{\alpha}$ is continuous, since $H_{\alpha}$ and $\overline{H}_{\alpha}$ are continuous functions. By construction of $\overline{H}_{\alpha}$, $H_{\alpha} \geq \overline{H}_{\alpha}$, and $\overline{H}_{\alpha}$ is flat (so that $\overline{H}_{\alpha}' = \overline{h}_{\alpha}$ is locally constant) whenever $H_{\alpha} > \overline{H}_{\alpha}$. To derive equation (24), use integration by parts to get

$$\int_{\theta_0}^{\theta_1} A(\theta)(z_{\alpha}(\theta) - \overline{z}_{\alpha}(\theta))f(\theta)d\theta$$

$$= \int_{\theta=\theta_0}^{\theta_1} A(\theta)d[H_{\alpha}(F(\theta)) - \overline{H}_{\alpha}(F(\theta))] = \int_{\theta=\theta_0}^{\theta_1} A(\theta)dG_{\alpha}(\theta)$$

$$= G_{\alpha}(\theta_1)A(\theta_1) - G_{\alpha}(\theta_0)A(\theta_0) - \int_{\theta=\theta_0}^{\theta_1} G_{\alpha}(\theta)dA(\theta).$$

Then observe that $\overline{H}_{\alpha}(0) = H_{\alpha}(0)$ and $\overline{H}_{\alpha}(1) = H_{\alpha}(1)$, so that $G_{\alpha}(\theta_0) = G_{\alpha}(\theta_1) = 0$, because the convex hull of a continuous function always equals the function at the endpoints of the domain in $\mathbb{R}$. $\overline{z}_{\alpha}(\theta)$ is nondecreasing because $\overline{h}_{\alpha}$ is the nondecreasing derivative of a convex function. If $z_{\alpha}(\theta)$ were nondecreasing, then $H_{\alpha}$ would be convex, so that $\overline{H}_{\alpha} = H_{\alpha}$ and $\overline{h}_{\alpha} = h_{\alpha}$ and $\overline{z}_{\alpha} = z_{\alpha}$. Q.E.D.

We can now state the optimal regulatory policy. Let $\overline{p}(\theta)$ and $\overline{q}(\theta)$ be defined by

$$\overline{p}(\theta) = c_0 + c_1\overline{z}_{\alpha}(\theta), \tag{25}$$

$$P(\overline{q}(\theta)) = \overline{p}(\theta). \tag{26}$$

Let $\overline{r}(\theta)$ satisfy

$$\overline{r}(\theta) = \begin{cases} 1 & \text{if } V(\overline{q}(\theta)) - \overline{p}(\theta)\overline{q}(\theta) \geq k_0 + k_1\overline{z}_{\alpha}(\theta), \\ 0 & \text{if } V(\overline{q}(\theta)) - \overline{p}(\theta)\overline{q}(\theta) < k_0 + k_1\overline{z}_{\alpha}(\theta). \end{cases} \tag{27}$$

and let

$$\bar{s}(\theta) = [(c_o + c_1\theta)\bar{q}(\theta) + k_0 + k_1\theta - \bar{p}(\theta)\bar{q}(\theta)]\bar{r}(\theta)$$

$$+ \int_{\theta}^{\theta_1} \bar{r}(\tilde{\theta})(c_1\bar{q}(\tilde{\theta}) + k_1)d\tilde{\theta}. \qquad (28)$$

The following theorem establishes the optimality of this policy.

Theorem. The regulatory policy $(\bar{r},\bar{p},\bar{q},\bar{s})$ given in (25) - (28) is feasible and maximizes the social welfare function (9) among all feasible regulatory policies.

Proof. First we check that the regulatory policy is feasible, using Lemma 1. Conditions (3) and (4) are obviously satisfied. To check conditions (10) and (11), we substitute (28) into (5) to obtain

$$\pi(\theta) = \int_{\theta}^{\theta_1} \bar{r}(\theta)(c_1 q(\tilde{\theta}) + k_1)d\tilde{\theta}, \text{ and } \pi(\theta_1) = 0.$$

Since $\bar{z}_\alpha(\theta)$ is nondecreasing, $\bar{p}(\theta)$ is nondecreasing, and so $\bar{q}(\theta)$ is nonincreasing in $\theta$. Notice that

$$\frac{\partial}{\partial q} [V(q) - P(q)q] = -P'(q)q > 0$$

since $V'(q) = P(q)$. (Recall (2).) Thus, the consumers' surplus $V(\bar{q}(\theta))-\bar{p}(\theta)\bar{q}(\theta)$ is nonincreasing in $\theta$, since $\bar{q}(\theta)$ is nonincreasing, and so $\bar{r}(\theta)$ is also nonincreasing in $\theta$. Thus (12) is satisfied.

Now we show that the regulatory policy is optimal. When we substitute equation (24) into formula (15), using $A(\theta) = -r(\theta)(c_1q(\theta) + k_1)$, we find that the regulator's social welfare function (9) is equal to

$$\int_{\theta_0}^{\theta_1} [V(q(\theta)) - (c_0 + c_1\bar{z}_\alpha(\theta))q(\theta) - k_0 - k_1\bar{z}_\alpha(\theta)]r(\theta)f(\theta)d\theta$$

$$- \int_{\theta=\theta_0}^{\theta_1} G_\alpha(\theta)d[-r(\theta)(c_1q(\theta) + k_1)] \quad - (1-\alpha)\pi(\theta_1), \qquad (29)$$

for any feasible regulatory policy. Since $G_\alpha(\theta) \geq 0$ and $[-r(\theta)(c_1 q(\theta) + k_1)]$ is nondecreasing, the second integral in (29) must be nonnegative for any feasible policy; but this integral equals zero (its optimal value) for the policy $(\bar{r},\bar{p},\bar{q},\bar{s})$, because $\bar{z}_\alpha(\theta)$, $\bar{q}(\theta)$, and $\bar{r}(\theta)$ are locally constant whenever $G_\alpha(\theta) > 0$. In the third term in (29), $(1-\alpha)\pi(\theta_1) \geq 0$ for any feasible policy (since $\alpha \leq 1$), but this term equals zero (again, its optimal value) at the policy $(\bar{r},\bar{p},\bar{q},\bar{s})$. Finally, to optimize the first integral in (29), we want to choose each $q(\theta)$ so that

$$0 = V'(q(\theta)) - (c_0 + c_1\bar{z}_\alpha(\theta))$$

and we want to choose each $r(\theta)$ so that

$$r(\theta) = \begin{cases} 1 & \text{if } V(q(\theta)) - (c_0 + c_1\bar{z}_\alpha(\theta))q(\theta) \geq k_0 + k_1\bar{z}_\alpha(\theta), \\ 0 & \text{if } V(q(\theta)) - (c_0 + c_1\bar{z}_\alpha(\theta))q(\theta) < k_0 + k_1\bar{z}_\alpha(\theta). \end{cases}$$

But these equations are equivalent to (25)-(27), since $V'(q(\theta)) = P(q(\theta))$, so $(\bar{r},\bar{p},\bar{q},\bar{s})$ maximizes the first integral in (29) among all feasible policies. So $(\bar{r},\bar{p},\bar{q},\bar{s})$ maximizes (29), which is equivalent to maximizing (9). Q.E.D.

## 4. Analysis of the Optimal Solution.

If the regulator had complete information about the firm's costs, the optimal policy would be to set price equal to marginal cost and to subsidize the firm by an amount equal to its fixed cost, unless this subsidy exceeded the consumers' surplus in which case the firm would not produce. That is, if $\theta$ were known to the regulator, the complete-information solution would be

$$p(\theta) = c_0 + c_1\theta\ ,\ q(\theta) = P^{-1}(p(\theta)) \tag{31}$$

$$r(\theta) = \begin{cases} 1 & \text{if } V(q(\theta)) - p(\theta)q(\theta) \geq k_0 + k_1\theta \\ 0 & \text{if } V(q(\theta)) - p(\theta)q(\theta) < k_0 + k_1\theta \end{cases} \tag{31}$$

$$s(\theta) = (k_0 + k_1\theta)r(\theta) \tag{32}$$

Of course, this policy is not feasible for the regulator when $\theta$ is unknown, because it does not satisfy the incentive-compatibility constraint (8). The firm would have positive incentives to misrepresent its costs, by reporting costs higher then the true $\theta$. However, it is instructive to compare our optimal policy (25) - (28) to this complete-information solution (30) - (32). The optimal $\overline{p}(\theta)$, $\overline{q}(\theta)$ and $\overline{r}(\theta)$ are chosen as if the regulator were applying the complete information solution to $\overline{z}_\alpha(\theta)$ rather then to $\theta$. Since $\overline{z}_\alpha(\theta)$ is greater then $\theta$, this transformation from $\theta$ to $\overline{z}_\alpha(\theta)$ may be viewed as an accommodation to the firm's incentive to overstate its costs in the complete information solution. There is no obvious relationship between the optimal subsidy $\overline{s}(\theta)$ in (28) and the complete-information subsidy in (32), because $\overline{s}(\theta)$ is determined by the need to prevent the firm from misrepresenting its costs, whereas the subsidy in (32) was only designed to cover the firm's fixed costs.

Another parallel between the optimal regulatory policy under uncertainty and the complete-information solution is that both $\overline{p}(\theta)$ in (25) and $p(\theta)$ in (30) are determined independently of the demand curve. That is, in both cases the optimal regulatory price depends only on the regulator's information about the firm's costs.

Since the optimal regulated price $\overline{p}(\theta)$ is generally strictly higher than the firm's marginal costs $(c_0 + c_1\theta)$, and since $\overline{p}(\theta)$ does not depend on the demand curve, the optimal regulated price $\overline{p}(\theta)$ may in some cases be higher than the unregulated monopoly price $p_M(\theta)$ determined by the usual MR = MC condition. To see that this can indeed happen, suppose $k_0 = k_1 = 0$ (so fixed costs are zero) and consider a marginal cost $c_0 + c_1\theta$ and the corresponding price $\overline{p}(\theta)$. Since $\overline{p}(\theta)$ is independent of the demand function, a demand function can be chosen that intersects the price axis between marginal cost and the regulated price $\overline{p}(\theta)$. Clearly, the monopoly price must be lower than $\overline{p}(\theta)$ in this example, since demand is zero at $\overline{p}(\theta)$.

From an ex post point of view, it may seem inefficient and paradoxical for the regulator to ever force the firm to charge a price higher than the unregulated monopoly price. To understand why this may be optimal, observe that the regulator wants to encourage the firm to admit that it has low costs, whenever this is true, so that a low price can be set to generate a large consumers' surplus. But to prevent the firm from misrepresenting its costs when it has low costs, the regulator either must reward the firm with subsidies for announcing low costs or must somehow punish the firm for announcing high costs. Such punishments may take the form of forcing the firm to charge a price above the monopoly price when its costs are high or of not permitting the firm to produce ($\overline{r}(\theta) = 0$). From this point of view supermonopoly prices may be seen as a less extreme punishment than complete shut-down, since they still generate some consumers' surplus.

In general, all the regulator's instruments $(\overline{r},\overline{p},\overline{q},\overline{s})$ are used together to guide the firm to honestly report its cost parameter while generating the highest possible social welfare. The optimal regulatory price $\overline{p}(\theta)$ is a nondecreasing function of $\theta$, while the quantity produced $\overline{q}(\theta)$ is nonincreasing in $\theta$. From (27), the function $\overline{r}(\theta)$ is nonincreasing in $\theta$, with $\overline{r}(\theta) = 1$ for all $\theta$ below the critical value $\theta^*$ at which

$$V(\bar{q}(\theta^*)) - \bar{p}(\theta^*)\bar{q}(\theta^*) = k_0 + k_1\, \bar{z}_\alpha(\theta^*)),$$

and with $\bar{r}(\theta) = 0$ (denoting shut-down) for all $\theta$ above $\theta^*$. Differentiating (28) in the interval where $\bar{r}(\theta) = 1$, yields

$$\bar{s}'(\theta) = \bar{q}'(\theta) \cdot ([c_0 + c_1\theta] - [P(\bar{q}(\theta)) + \bar{q}(\theta)\, P'(\bar{q}(\theta))]). \tag{33}$$

Since $\bar{q}'(\theta) \leq 0$, and since the second factor in (33) is just marginal cost minus marginal revenue at $\bar{q}(\theta)$, $\bar{s}(\theta)$ is decreasing in $\theta$ when the regulated price $\bar{p}(\theta)$ is below the monopoly price $p_M(\theta)$, and $\bar{s}(\theta)$ is increasing in $\theta$ when $\bar{p}(\theta) > p_M(\theta)$. To understand these results, observe that the difference between $\bar{p}(\theta)$ and $p_M(\theta)$ tends to give the firm some incentive to misrepresent its costs in order to obtain a price closer to the monopoly price. The subsidy $\bar{s}(\theta)$ then must vary with $\theta$ ao as to offset this incentive.

However, whether the subsidy is increasing or decreasing $\theta$, the firm's expected profit is always decreasing in $\theta$ when $\bar{r}(\theta) = 1$, since by (10)

$$\pi'(\theta) = -\bar{r}(\theta)(c_1\bar{q}(\theta) + k_1).$$

Consequently, if the firm has a low cost parameter, it will be allowed to earn a greater profit than if it had a high cost parameter in order to provide a reward for reporting its lower costs. The profit $\pi(\theta_1)$ of a firm with the highest possible cost is zero, since there is no need to reward such a firm.

Let us now see how our optimal solution varies with $\alpha$, the weight given in the social welfare function to the firm's profits. First we must establish the following basic mathematical result, a corollary of Lemma 3.

<u>Corollary</u>. For any $\theta$ in $[\theta_0, \theta_1]$, $\bar{z}_\alpha(\theta)$ is a nonincreasing function of $\alpha$.

<u>Proof</u>. Pick any $\alpha$ and $\beta$ such that $0 \leq \alpha < \beta \leq 1$. Let $\Delta(\theta) = \bar{z}_\beta(\theta) - \bar{z}_\alpha(\theta)$

and let $\Delta^+(\theta) = \max\,[0,\Delta(\theta)]$. From Lemma 3, we obtain

$$\int_{\theta_0}^{\theta_1} \Delta^+(\theta)\;(z_\beta(\theta) - z_\alpha(\theta))d\theta$$

$$= \int_{\theta_0}^{\theta_1} \Delta^+(\theta)\Delta(\theta)d\theta + \int_{\theta=\theta_0}^{\theta_1} (G_\alpha(\theta) - G_\beta(\theta))d[\Delta^+(\theta)].$$

The integrand $\Delta^+(\theta)\Delta(\theta)$ is obviously nonnegative for all $\theta$. Whenever $\Delta^+(\theta)$ is increasing in $\theta$, $\bar{z}_\beta(\theta)$ must be increasing in $\theta$, and so $G_\beta(\theta) = 0$. Similarly, whenever $\Delta^+(\theta)$ is decreasing in $\theta$, $\bar{z}_\alpha(\theta)$ must be increasing, and so $G_\alpha(\theta) = 0$. Thus $\int(G_\alpha - G_\beta)d\Delta^+ \geq 0$, and so $\int(\Delta^+)(z_\beta - z_\alpha)d\theta \geq 0$. But $\Delta^+(\theta) \geq 0$ and

$$z_\beta(\theta) - z_\alpha(\theta) = (\alpha-\beta)F(\theta)/f(\theta) < 0$$

for all $\theta > \theta_0$, so $\Delta^+(\theta) = 0$ for all $\theta$, which implies $\bar{z}_\alpha(\theta) \geq \bar{z}_\beta(\theta)$. Q.E.D.

To get a more intuitive understanding of this result, observe that, in the special case when $z_\alpha(\theta)$ is increasing in $\theta$, we have $\bar{z}_\alpha(\theta) = z_\alpha(\theta)$. Then, $z_\alpha(\theta) = \theta + (1-\alpha)\;F(\theta)/f(\theta)$ is seen to be decreasing in $\alpha$.

The optimal regulated price $\bar{p}(\theta) = c_0 + c_1\bar{z}_\alpha(\theta)$ is thus a decreasing function of $\alpha$, while $\bar{q}(\theta)$ is an increasing function of $\alpha$. This feature of the optimal solution may seem counterintuitive, but it is due to the incentive problem created by the asymmetry of information. To interpret the welfare implication, substitute (1) and (5) into the social function (9) to obtain

$$\int_{\theta_0}^{\theta_1} [V(q(\theta)) - C(q(\theta),\theta))r(\theta) - (1-\alpha)\,\pi(\theta)]f(\theta)d\theta.$$

The term $(V(q(\theta)) - C(q(\theta),\theta))r(\theta)$ is the gross surplus, and $(1-\alpha)\pi(\theta)$ may be interpreted as the welfare omission resulting from a weight smaller than one given to the firm's interests. As $\alpha$ approaches one, the welfare omission goes to zero, and the optimal regulated price decreases towards the marginal cost, which in the limit maximizes the gross surplus.

The range of cost parameters for which the firm is allowed to produce increases with $\alpha$; that is, $\bar{r}(\theta)$ is a nondecreasing function of $\alpha$, for any $\theta$. To see this, recall the definition of $\bar{r}(\theta)$ in (27), and observe that

$$\frac{\partial}{\partial\alpha}\ (V(\bar{q}(\theta)) - P(\bar{q}(\theta))\bar{q}(\theta))$$

$$= -P'(\bar{q}(\theta))q(\theta)\ \frac{\partial\bar{q}(\theta)}{\partial\alpha} \geq 0,$$

while $\frac{\partial}{\partial\alpha}\ (k_0 + k_1\ \bar{z}_\alpha(\theta)) \leq 0$. Thus $\theta^*$ is an increasing function of $\alpha$, where

$$\theta^* = \max\{\theta \mid \bar{r}(\theta) = 1\}.$$

The profit of the firm may be written as

$$\pi(\theta) = \int_{\theta}^{\theta^*} (\ c_1\ \bar{q}(\tilde{\theta}) + k_1)d\,\tilde{\theta}\,.$$

Since $\bar{q}$ and $\theta^*$ are both increasing in $\alpha$, $\pi(\theta)$ is an increasing function of $\alpha$, for any fixed $\theta$. Thus, although the consumers are paying lower prices as $\alpha$ increases, the firm's total revenue must be increasing in $\alpha$. For any $\theta$, either the firm's subsidy $\bar{s}(\theta)$ must be increasing in $\alpha$, or the price reduction must be associated with an increase in operating profit $\bar{p}(\theta)\bar{q}(\theta)-C(\bar{q}(\theta),\ \theta)$. The latter condition happens only in those cases when $\bar{p}(\theta)$ is higher than the unregulated monopoly price $p_M(\theta)$, so we should expect that $\bar{s}(\theta)$ is usually (but not always) increasing in $\alpha$.

The net expected gain to consumers,

$$\int_{\theta_0}^{\theta_1} [(V(\bar{q}(\theta)) - \bar{p}(\theta)\bar{q}(\theta))\, \bar{r}(\theta) - \bar{s}(\theta)]f(\theta)d\theta,$$

is a decreasing function of $\alpha$, because the regulator is decreasing the relative weight given to this term in his objective function as $\alpha$ increases.

To give an overall measure of how the optimal regulated prices vary with $\alpha$, we can compute the expected price $E\bar{p}(\theta)$:

$$\begin{aligned} Ep(\theta) &= c_0 + c_1 \int_{\theta_0}^{\theta_1} \bar{z}_\alpha(\theta)f(\theta)d\theta \\ &= c_0 + c_1 \int_{\theta_0}^{\theta_1} z_\alpha(\theta)f(\theta)d\theta \\ &= c_0 + c_1 \left[\int_{\theta_0}^{\theta_1} \theta f(\theta)d\theta + (1-\alpha) \int_{\theta_0}^{\theta_1} F(\theta)d\theta\right] \\ &= c_0 + c_1 \,(\alpha E\theta + (1-\alpha)\theta_1), \end{aligned}$$

where $E\theta$ is the expected value of the cost parameter. (In the above derivation, the second equality follows from Lemma 3 with $A(\theta) = 1$; the last equality follows from integration by parts.) When the firm's interests are given no weight ($\alpha=0$), the expected price is equal to the highest possible marginal cost, $c_0 + c_1\theta_1$. When the firm's interests are given equal weight with the consumer's interests ($\alpha=1$), the expected price equals expected marginal cost. Between these extremes, the expected price decreases linearly in $\alpha$.

For the case of $\alpha=1$, we get $z_1(\theta) = \theta$, which is increasing, so by Lemma 3,

$$\bar{z}_1(\theta) = z_1(\theta) = \theta.$$

Thus, price is always equal to marginal cost $c_0 + c_1\theta$ when $\alpha=1$, and our optimal solution coincides with the solution proposed by Loeb and Magat (1979).

## 5. The Case of Known Fixed Costs: An Example

To illustrate our optimal solution, let us consider an example with known fixed costs. Let $k_1 = c_0 = 0$ and $c_1 = 1$, so that $C(q,\theta) = k_0 + \theta q$ and $\theta$ represents the unknown marginal cost. Suppose that $\theta$ is uniformly distributed on $[\theta_0,\theta_1]$, so $f(\theta) = 1/(\theta_1-\theta_0)$. The optimal price function is then

$$\bar{p}(\theta) = \theta + (1-\alpha)(\theta - \theta_0) = (2-\alpha)\theta - (1-\alpha)\theta_0 , \quad \text{for } \theta\varepsilon[\theta_0,\theta_1],$$

which is increasing in $\theta$ and has range $[\theta_0, \theta_1 + (1-\alpha)(\theta_1-\theta_0)]$.

Let us assume a linear demand function of the form

$$q = P^{-1}(p) = a - bp, \quad \text{where } b > 0 \text{ and } a > 2b\theta_1.$$

Then the quantity $\bar{q}(\theta)$ that the firm will sell if its marginal cost is $\theta$ is given by

$$\bar{q}(\theta) = a - b[(2-\alpha)\theta - (1-\alpha)\theta_0].$$

This function may be interpreted as an adjusted (inverse) demand function expressed as a function of the marginal cost instead of the price.

The demand function and the adjusted demand function are represented in Figure 1. Let us assume that the fixed cost $k_0$ satisfies

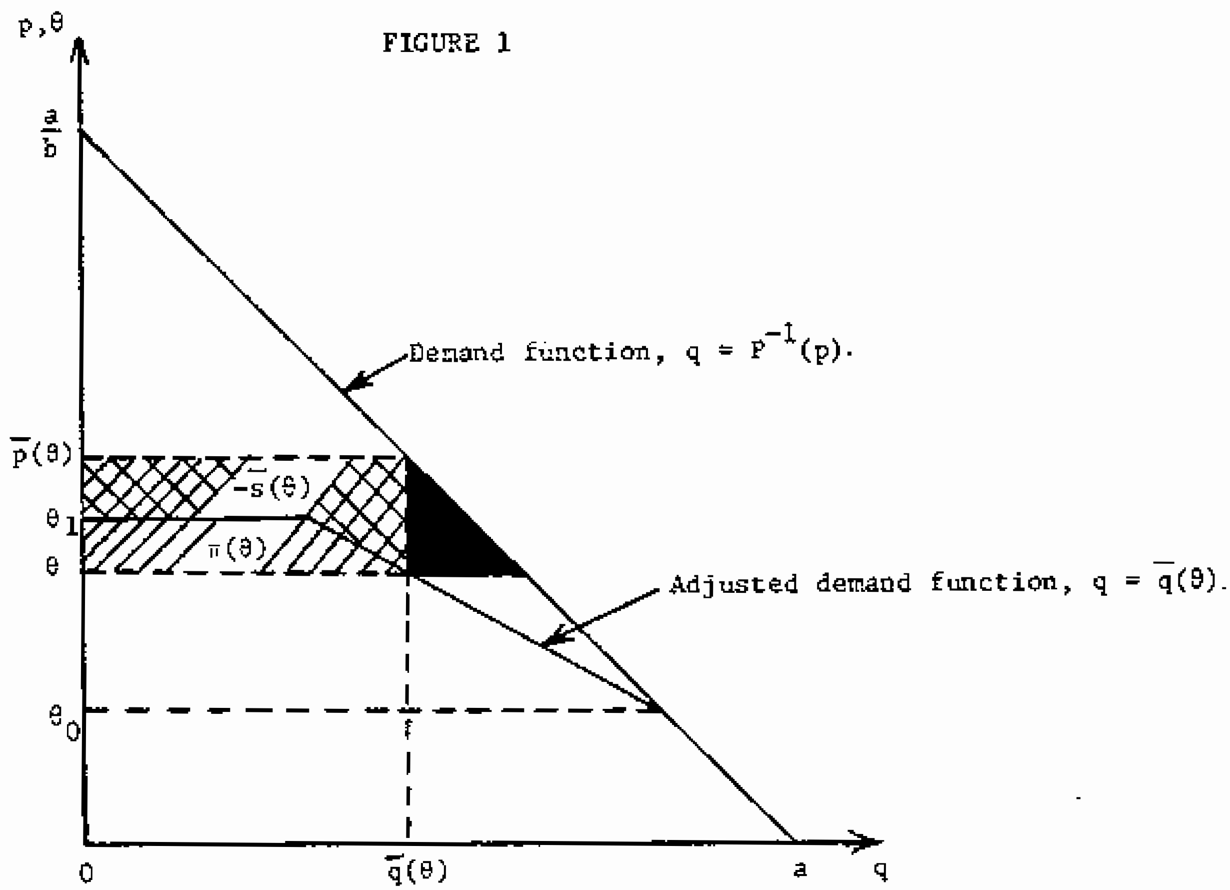
$$V(\bar{q}(\theta_1)) - \bar{p}(\theta_1)\bar{q}(\theta_1) \geq k_0,$$

so that the firm will produce for any $\theta\varepsilon[\theta_0,\theta_1]$. The profit of the firm is

$$\bar{\pi}(\theta) = \int_{\theta}^{\theta_1} \bar{q}(\tilde{\theta})\, d\tilde{\theta} ,$$

which is positive for $\theta<\theta_1$ and is represented by the slashed area below the adjusted demand function and above the horizontal line at $\theta$ in Figure 1. Thus, the firm's profit from the optimal regulatory policy is equal to what the consumers' surplus would be if demand were shifted to the

FIGURE 1


p,θ
a/b
Demand function, q = P^-1(p).
p̄(θ)
-s̄(θ)
θ1
π(θ)
θ
Adjusted demand function, q = q̄(θ).
θ0
0
q̄(θ)
a
q

adjusted demand function and if price were set at marginal cost. That is, from the firm's perspective, the optimal regulatory policy looks like the policy of Loeb and Magat (1979) (in which the subsidy equals the consumers' surplus) except that the demand curve has been effectively shifted by the regulator.

The subsidy $\bar{s}(\theta)$ paid by consumers to the firm is from (28)

$$\bar{s}(\theta) = \int_{\theta}^{\theta_1} \bar{q}(\tilde{\theta})d\tilde{\theta} - ((\bar{p}(\theta) - \theta)\bar{q}(\theta) - k_0)$$

where the last term is the operating profit of the firm. If $k_0 = 0$, the subsidy for the example is negative and $-\bar{s}(\theta)$ is represented by the cross-hatched area in Figure 1. The net gain to consumers $(V(\bar{q}(\theta)) - \bar{p}(\theta)\bar{q}(\theta) - \bar{s}(\theta))$ is thus the upper triangle above $\bar{p}(\theta)$ plus the tax $(-\bar{s}(\theta))$ levied on the firm. The welfare loss that results, because of the need to screen the possible marginal costs that the firm might have, is the solid triangle represented by the difference between the price $\bar{p}(\theta)$ and the marginal cost $\theta$.

As the weight $\alpha$ accorded the firm's interests in the social welfare function is increased, the price $\bar{p}(\theta)$ decreases and equals $\theta$ at $\alpha=1$. The adjusted demand function rotates upward as $\alpha$ is increased and coincides with the demand function for $\theta \in [\theta_0,\theta_1]$ when $\alpha=1$. The subsidy paid by consumers to the firm is then

$$\bar{s}(\theta) = \int_{\theta}^{\theta_1}(a-b\tilde{\theta})d\tilde{\theta},$$

so the firm is paid the entire surplus represented by the prices between $\theta$ and $\theta_1$ when $\alpha=1$. The welfare loss $L(\theta)$ in our example is

$$L(\theta) = \int_{\theta}^{\bar{p}(\theta)} [ P^{-1}(p) - \bar{q}(\theta)] \, dp = \frac{1}{2}b((1-\alpha)(\theta - \theta_0))^2.$$

Thus, as $\alpha$ is increased, the welfare loss is reduced, but also the net consumer surplus is reduced because of the greater subsidy paid to the firm. For $\alpha=1$ the welfare loss is eliminated, and the solution given here is essentially the solution proposed by Loeb and Magat (1979) in which consumers surrender all of the surplus corresponding to the possible marginal costs that the firm might have.

## 6. The Case of Known Marginal Cost

Consider now the case in which the regulator knows the marginal cost but does not know the fixed cost. Let $c_1 = k_0 = 0$ and $k_1 = 1$, so that $C(q,\theta) = \theta + c_0 q$ and $\theta$ is the unknown fixed cost. Then

$$\bar{p}(\theta) = c_0 \text{ , and}$$

$$\bar{r}(\theta) = \begin{cases} 1 & \text{if } V_0 \geq \bar{z}_\alpha(\theta) \\ 0 & \text{if } V_0 < \bar{z}_\alpha(\theta), \end{cases}$$

where

$$V_0 = V(P^{-1}(c_0)) - c_0 P^{-1}(c_0).$$

The term $V_0 = V(P^{-1}(c_0)) - c_0 P^{-1}(c_0)$ is the consumer surplus resulting from a price equal to marginal cost. Since $\bar{z}_\alpha(\theta)$ is nondecreasing in $\theta$, there exists an $\theta^*$ such that

$$\bar{r}(\theta) = \begin{cases} 1 & \text{if } \theta \leq \theta^* \\ 0 & \text{if } \theta > \theta^*, \end{cases}$$

where [4]

$$\theta^* = \bar{z}_\alpha^{-1}(V_0).$$

The subsidy paid to the firm is

$$\bar{s}(\theta) = \begin{cases} \theta + (\theta^* - \theta) = \theta^* & \text{if } \theta \leq \theta^* \\ 0 & \text{if } \theta > \theta^*, \end{cases}$$

and the profit of the firm is

$$\pi(\theta) = \begin{cases} (\theta^* - \theta) & \text{if } \theta \le \theta^* \\ 0 & \text{if } \theta > \theta^*. \end{cases}$$

Notice that $\theta^*$ is nondecreasing in $\alpha$, by the Corollary in Section 4.

This regulatory policy may be interpreted as an auction in which the regulator offers to pay $\theta^*$ to the firm if it will produce and sell its output at the marginal cost $c_0$. The offer will be accepted if the firm has a cost parameter at least as low as $\theta^*$ and will otherwise be rejected. A welfare loss can result because the firm is not allowed to produce if it has a cost parameter $\theta$ between $\theta^*$ and $V_0$ even though the consumer surplus exceeds the fixed cost. The welfare loss resulting in our optimal policy is zero if $\theta \ge V_0$ because even in the complete information solution the firm would not have produced. If $\theta < \theta^*$, the welfare loss is the difference between the subsidy $\bar{s}(\theta)$ and the complete-information subsidy $\theta$ less the proportion $\alpha$ of profit included in the welfare function. The difference $\Delta S$ in the subsidy is

$$\Delta S = (\theta^* - \theta) = \pi(\theta),$$

which is the profit of the firm under the optimal policy, and the welfare loss is thus $(1-\alpha)\pi(\theta)$. If the cost parameter satisfies $\theta^* < \theta < V_0$, so that the firm does not produce under our policy while it would under the complete-information solution, the welfare loss is the consumer surplus $V_0$ less the subsidy $\theta$ that would be paid in the complete information solution. The welfare loss $L(\theta)$ is thus

$$L(\theta) = \begin{cases} (1-\alpha)(\theta^* - \theta) & \text{if } \theta \le \theta^* \\ V_0 - \theta & \text{if } \theta^* < \theta < V_0 \\ 0 & \text{if } \theta \ge V_0. \end{cases}$$

The expected welfare loss is then obtained by taking the expectation of $L(\theta)$.

## 7. General Two-Parameter Uncertainty

In this paper, we have allowed that the regulator may be uncertain about both the marginal cost and fixed cost of the firm, provided that these two unknowns vary collinearly. (Recall (1).) More generally, one may try to compute optimal regulatory policies for cost functions of the form

$$C(q;c,k) = \begin{cases} c\,q + k & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

where c and k are random cost parameters (known by the firm) having some general probability distribution on $\mathbb{R}^2_+$. Although we have not been able to extend the optimal solution explicitly to this general two-parameter case, we expect that most of the qualitative results discussed here should still be valid. However, at least two of our more technical results do not extend to the general case. We can show examples in which the optimal regulatory policy does involve proper randomization with respect to shutting down the firm, so that r is strictly between 0 and 1 for some values of the cost parameters. Also, the result that the optimal regulated price is independent of the demand curve does not extend to the general two-parameter case.

For example, suppose that the two cost parameters (c,k) could be (1,0) (low costs), or (1,4) (high fixed cost), or (3,0) (high marginal cost), all with equal probability 1/3. Let demand be $P(q) = 7 - 3q$. For $\alpha = 0$, the optimal regulatory policy is[5]

$$\bar{p}(1,0) = 1, \quad \bar{q}(1,0) = 2, \quad \bar{r}(1,0) = 1, \quad \bar{s}(1,0) = 2$$

$$\bar{p}(1,4) = 1, \quad \bar{q}(1,4) = 2, \quad \bar{r}(1,4) = .5, \quad \bar{s}(1,4) = 2$$

$$\bar{p}(3,0) = 4, \quad \bar{q}(3,0) = 1, \quad \bar{r}(3,0) = 1, \quad \bar{s}(3,0) = -1$$

Notice that, with high fixed costs, the regulator must randomize over whether to let the firm go into business. However, if we raise the demand curve to $P(q) = 8 - 3q$, then the optimal regulatory policy changes to

$$\bar{p}(1,0) = 1, \quad \bar{q}(1,0) = 2.33, \quad \bar{r}(1,0) = 1, \quad \bar{s}(1,0) = 4$$

$$\bar{p}(1,4) = 1, \quad \bar{q}(1,4) = 2.33, \quad \bar{r}(1,4) = 1, \quad \bar{s}(1,4) = 4$$

$$\bar{p}(3,0) = 3, \quad \bar{q}(3,0) = 1.67, \quad \bar{r}(3,0) = 1, \quad \bar{s}(3,0) = 0$$

Notice that the regulated price for a firm with high marginal cost changes from 4 to 3 as the demand curve shifts. With the higher demand, it becomes more worthwhile to keep the (1,4)-type in business, even though this requires a higher subsidy to the (1,0)-type. Then, with a higher subsidy to the (1,0)-type, it is no longer necessary to raise the price for the (3,0)-type to screen it from the (1,0)-type.

Essentially the two-parameter problem is more complicated because there are incentive constraints in two directions to worry about. For example, a low-cost firm (1,0) must not be able to gain by reporting high fixed cost (1,4), and it must also not be able to gain by reporting high marginal cost (3,0). Of these two constraints, both are binding in our example with the lower demand curve, but only the first of the two constraints is binding with the higher demand curve. The greater difficulty in solving the general case of two-parameter cost functions arises because of this ambiguity as to which of these directional incentive-compatibility constraints may be binding in the regulator's optimization problem.

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FOOTNOTES

* Graduate School of Business, Stanford University, and J.L. Kellogg Graduate School of Management, Northwestern University. The first author's work has been supported by a grant from the National Science Foundation, Grant No. SOC 77-07251.

1. A regulatory policy that has a positive probability that there will be no output may seem unrealistic, but in the optimal regulatory policy $r(\theta)$ will equal one unless the consumer surplus is less than an "adjusted" fixed cost, in which case $r(\theta) = 0$.

2. It is easy to show that if the firm is risk-neutral then randomized pricing policies cannot be optimal. On the other hand, if there were uncertainty about the demand curve, then the regulator would have to choose between regulating price and letting quantity be random, or regulating quantity and letting price be random. Weitzman (1974) has studied this issue in a similar context. If consumers are homogeneous then nonlinear pricing policies like those of Spence (1977) are not relevant.

3. See Schmalensee (1972) for an analysis of the expected consumer surplus as a measure of welfare.

4. If $V_0 > \bar{z}_\alpha(\theta_1)$ then let $\theta^* = \theta_1$. If $V_0 < \bar{z}_\alpha(\theta_0)$ then let $\theta^* = \theta_0$.

5. These solutions can be verified by standard Lagrangean techniques. The key step is to linearize the incentive-compatibility and individual-rationality constraints by writing them in terms of $q^o = r \cdot q$, $s^o = s + p \cdot q \cdot r$, and $r$.