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JOB DIFFERENTIATION AND WAGES

by

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I. Introduction

The introduction of interchangeable parts during the industrial revolution made mass production possible, bringing with it enormous increases in productivity. The use of interchangeable parts is not confined to manufactured goods, however. In many instances, labor, too, is highly standardized. Looking inside a firm, one typically finds many workers performing identical tasks - assembly line workers, clerks, telephone operators, etc. Any such group within a firm can also be viewed as a set of "interchangeable parts."

Organizing work in this way has the advantage that both supervision of the work process and comparison of performance among workers are easier when there are such groups of "homogeneous labor." However, it has the disadvantage that fully utilizing the talents and skills of individual workers is more difficult. Hence, we may expect the pressure toward standardization to be stronger for larger firms, where organizational problems are more severe and the degree of specialization of labor is greater, and weaker in smaller firms, where the reverse is true.

If work is standardized within each firm, a "job" can be thought of as a certain set of tasks. We may ask, then, what determines the types of jobs and the corresponding wage rates firms offer.

A model of imperfect competition in the labor market will be used here to examine how job descriptions and wages are determined. For simplicity, a job description will consist of a single parameter, which can be thought of as a level of skill.

Workers acquire skill through training, which is assumed to have a positive cost. In order to hold a particular job, a worker must have attained a skill

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level at least as high as the one specified for that job, and if he takes the job, he is paid the corresponding wage rate. It is assumed that any worker can acquire any level of skill, and hence can hold any job, but that training costs differ among workers. Skill is assumed to be non-firm-specific, so that the worker bears his own cost of training.

Given the menu of skill-wage offers made by all firms, each worker selects the job that maximizes his wage net of training costs. Since training costs differ among workers, job choices differ as well. Consequently, although workers differ from each other *ex ante*, *ex post* any two workers who hold the same job are equally skilled (since no worker will choose to acquire skill above the minimum required for the job he selects) and receive equal wages.

The problem facing each firm is to select a skill level and wage rate. These two parameters determine, given the actions of other firms, how many workers it will attract, and, consequently, its profit level.

The behavior of firms in such a world is examined below. First, competition in wages alone is discussed, under the assumption that each employer's skill level is exogenously fixed. Then competition in both wages and skills is discussed. It is shown that for any number of profit-maximizing firms, if skill levels are fixed there is a unique Nash equilibrium in pure wage strategies. However, if firms choose both skill levels and wage rates, there is no Nash equilibrium in pure strategies.

II. The Model

A. Production Technology

The production technologies of all firms are assumed to be identical, and to be linear in labor input. Thus, by choosing units appropriately "skill" and "productivity" can be taken as identical, so that "s" is the marginal (and average) product of individuals with skill level "s". The difference between the skill level and the wage rate set by a firm is its profit margin per worker. A job offer will be called *viable* if it specifies a wage rate no greater than the associated skill level, i.e., if it specifies a non-negative profit margin.

Initially, it will be assumed that each firm can make only one job offer. Since the firm's total profit is linear in the number of workers, it hires all workers seeking a job who have the required qualifications.

B. Workers

There is a continuum of workers, with each characterized by a parameter $z \in [0,Z]$ that reflects his training costs. The parameter is defined so that costs are decreasing in z. Costs might vary among individuals because of differences in ability, access to capital markets, etc.

The cost to a z-worker of acquiring skill level s is given by $f(z)c(s)$ satisfying:

$$f(z) > 0, \ f'(z) < 0, \qquad 0 \leq z \leq Z, \tag{1}$$

$$c(s) > 0, \ c'(s) > 0, \ c''(s) > 0, \qquad 0 < s, \tag{2a}$$

$$c(0) = 0, \quad c'(0)\, f(Z) < 1 \tag{2b}$$

Therefore, the productivity net of training costs of a z-worker at skill level s is $s-f(z)c(s)$. Assumption (2b) is sufficient to guarantee that at least some individuals have a positive net product over some ranges of skills.

Furthermore, since costs are strictly convex in skill, each worker's net product is strictly concave in skill. Let $\sigma(z)$, defined by

$$f(z)\, c'[\sigma(z)] = 1 \quad , \quad \text{if } f(z)c'(0) \leq 1$$

$$\sigma(z) = 0 \quad , \quad \text{if } f(z)c'(0) \geq 1$$

denote a z-worker's most productive skill level. The interval $[\sigma(0), \sigma(Z)]$ will be called the efficient range of skills, since at any skill level outside this range all workers' net productivities could be raised either by increasing the required skill to $\sigma(0)$ or by decreasing it to $\sigma(Z)$.

C. The Allocation of Labor

Suppose that there are n firms, indexed by $i = 1,\ldots n$, making skill-wage offers $[(s^i,w^i),\ i=1,\ldots,n]$. Assume that the firms are indexed so that s^i is increasing in i. Each worker chooses the job for which his wage, net of training costs, is maximized (see Figure I). If $w^i \leq w^j$ for any $j < i$, then obviously firm i will attract no workers. Hence we may assume that w^i is also increasing in i.

First, it can be established that under the assumptions in (1) and (2), workers' choices increase monotonically in z. For suppose that $j < i$, and that z chooses firm j and z' chooses firm i. Then

$$f(z)\,[c(s^i) - c(s^j)] > w^i - w^j > f(z')\,[c(s^i) - c(s^j)]. \tag{3}$$

The higher wage offered by firm i compensates z', but not z, for the higher training costs. From (3) we see that $f(z) > f(z')$, which implies that $z' > z$.

Next, we will show that even if wage offers increase with skill requirements, some firms may attract no workers. However, if we impose the restriction that each s^i lie in the efficient range, then for any viable wage rates offered by its competitors, each firm can make a viable wage offer that is high enough to attract some workers.

First, consider any pair of firms k and j, with $s^k < s^j$ and $w^k < w^j$. From the argument above, we know that if any worker chooses firm j over firm k, then worker Z also chooses j. Hence, if

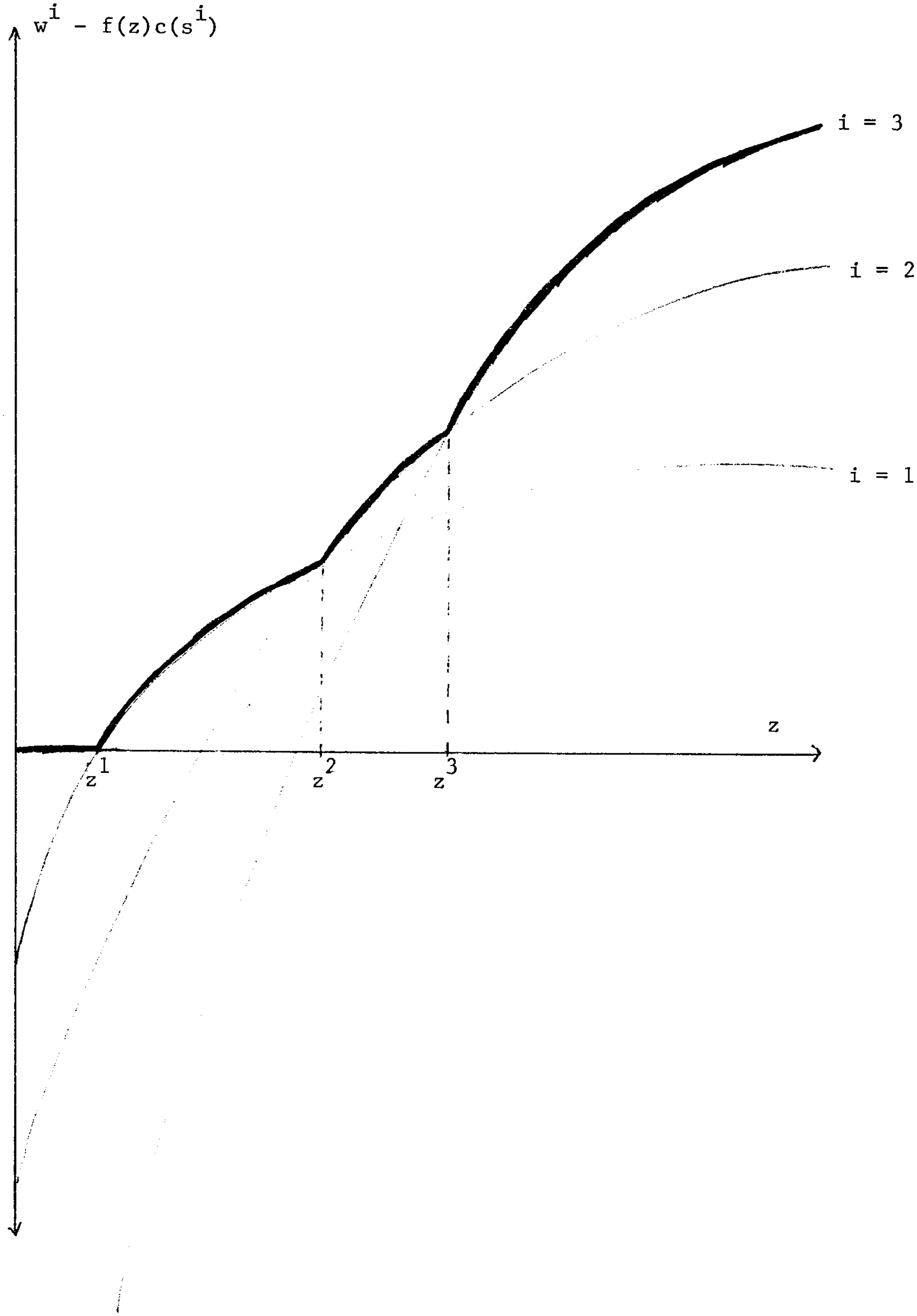
$$f(Z)\,[c(s^j) - c(s^k)] \geqslant w^j - w^k,$$

then all workers prefer k to j, and firm j attracts no workers. However, if s^j is in the efficient range, since $w^k \leq s^k$,

$$f[\sigma^{-1}(s^j)]\cdot[c(s^j) - c(s^k)] < s^j - s^k \leq s^j - w^k.$$

By offering a wage of $s^j - \varepsilon$, for ε sufficiently small firm j can attract

FIGURE I



workers with z-parameter in a neighborhood of $\sigma^{-1}(s^j)$ (see Figure II).

Next, consider any three firms i, j and k, all requiring skills in the efficient range, and all making viable wage offers (or unemployment, $s^i = w^i = 0$, may be one "offer").

$$\sigma(0) \leq s^i < s^j < s^k \leq \sigma(Z),$$

$$w^i < w^j < w^k,$$

$$w^\alpha \leq s^\alpha, \alpha = i, j, k.$$

Those workers satisfying:

$$w^i - f(z)\, c\, (s^i) < w^j - f(z)\, c\, (s^j)$$

and

$$w^k - f(z)\, c\, (s^k) < w^j - f(z)\, c\, (s^j)$$

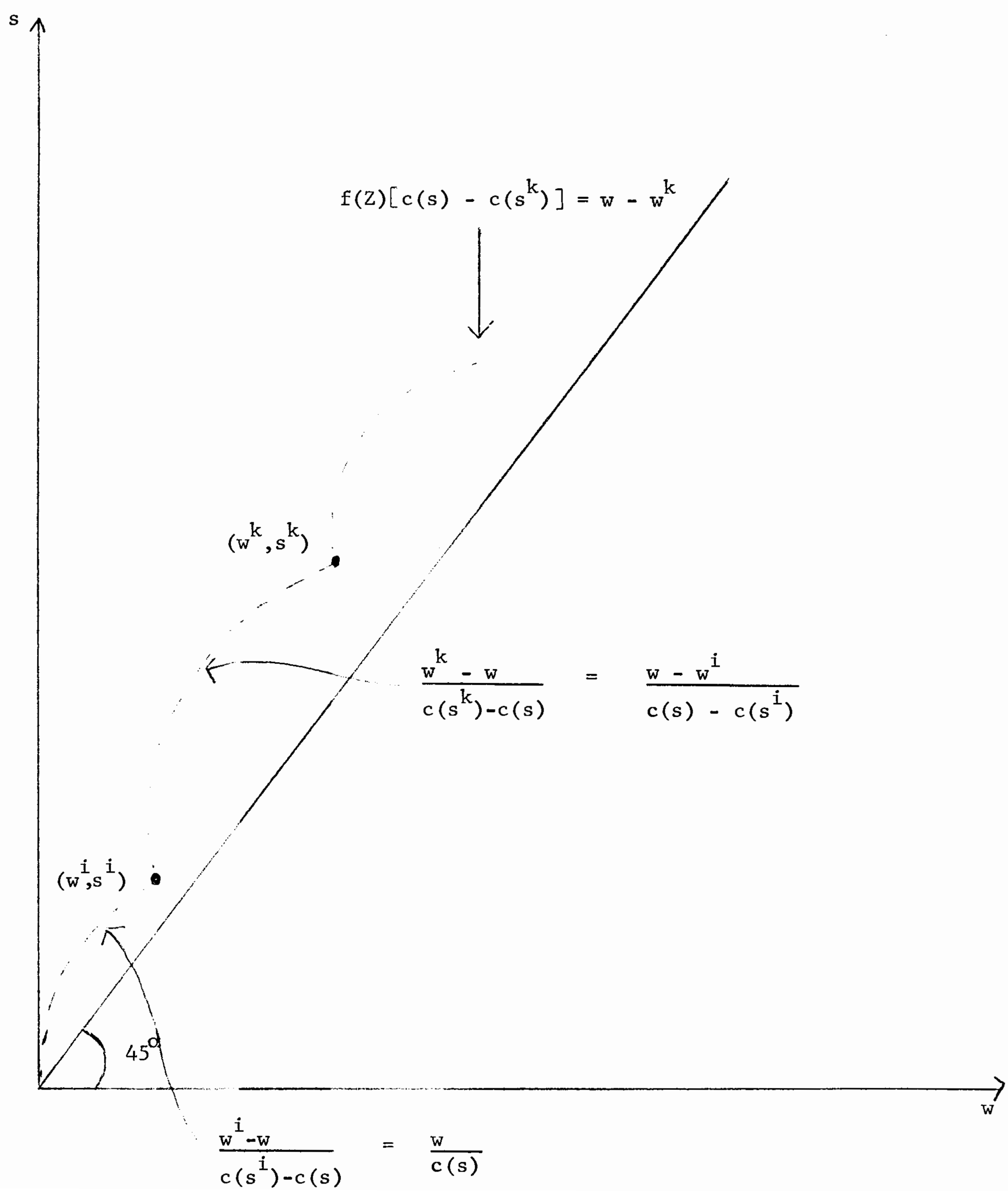
Choose firm j. Rearranging terms, this is equivalent to:

$$\frac{w^k - w^j}{c(s^k) - c(s^j)} < f(z) < \frac{w^j - w^i}{c(s^j) - c(s^i)} \tag{4}$$

Now suppose i and k offer the highest wages that are viable, i.e., suppose that $w^i = s^i$ and $w^k = s^k$. Then if firm i offers a wage $w^j = s^j$, (4) is satisfied for $z = \sigma^{-1}(s^j)$. Hence for ϵ sufficiently small, (4) is also satisfied if $w^j = s^j - \epsilon$, for all viable offers by the two competitors, for z in a neighborhood of $\sigma^{-1}(s^j)$ (see Figure II).

Putting these results together, we see that if its own skill requirement is in the efficient range, and if all other firms are making viable wage offers, any firm can make strictly positive profits. The first argument applies to the firm n, which competes with only one "neighboring" firm; the second applies to all others, which compete with two "neighbors". Hence, if all firms have skill requirements in the efficient range, all can make strictly positive profits.

FIGURE II


s
$f(Z)[c(s) - c(s^k)] = w - w^k$
(w^k, s^k)
$\frac{w^k - w}{c(s^k)-c(s)} = \frac{w - w^i}{c(s) - c(s^i)}$
(w^i, s^i)
45^o
w
$\frac{w^i - w}{c(s^i)-c(s)} = \frac{w}{c(s)}$

Assume now that the skill-wage offers of firms are such that each attracts some workers. Since workers' choices are monotone in z, the allocation of labor among firms is described by the vector $(z^1, z^2, \ldots, z^n)$, where the i^{th} firm's labor force consists of $z \in (z^i, z^{i+1})$. The values $(z^1, \ldots, z^{n+1})$ are defined by:

$$w^i - w^{i-1} \equiv \Delta w^i = f(z^i)\,[c(s^i) - c(s^{i-1})],\ i = 1, \ldots, n,$$

$$s^0 \equiv w^0 \equiv 0,$$

$$z^{n+1} \equiv Z$$

III. Wage Competition

Consider first the situation in a market with n firms, each with a predesignated skill requirement, competing by making wage offers. Given $(s^1, \ldots s^n)$, let $\alpha(\cdot)$ be the function relating z^i to $w^i - w^{i-1}$:

$$\Delta w^i \equiv f[\alpha^i(\Delta w^i)]\cdot[c(s^i) - c(s^{i-1})], \tag{5}$$

and let

$$\alpha_1^i \equiv \left.\frac{d\alpha(\Delta w)}{d(\Delta w)}\right|_{\Delta w^i} = \frac{1}{f'(\alpha^i)[c(s^i) - c(s^{i-1})]} < 0,$$

$$\alpha_{11}^i \equiv \left.\frac{d^2\alpha(\Delta w)}{d(\Delta w)^2}\right|_{\Delta w^i} = \frac{-f''(\alpha^i)(\alpha_1^i)^2}{f'(\alpha^i)}$$

Furthermore, let $m(\cdot)$ be the density function for z in the population, and recall that the parameter z is an artificial index, so that the choice of scale is arbitrary. Hence when training costs have the form in (1) - (2), it can be assumed without loss of generality that z is scaled so that:

$$m'(z)\,f'(z) - m(z)f''(z) = 0, \qquad 0 \leq z\,. \tag{6}$$

(The proof is in the Appendix).

The reaction function of firm i, if it makes the Nash assumption of no response by its competitors, is defined as the solution of:

$$\max_{w^i} \Pi^i(w^{i-1}, w^i, w^{i+1}) = (s^i - w^i) \int_{\alpha^i}^{\alpha^{i+1}} m(z)dz, \tag{7}$$

Note that firm i's response depends only on the actions of its near neighbors. Differentiating (7) the condition for a maximum is:

$$\frac{\partial \pi^i}{\partial w^i} = -\int_{\alpha^i}^{\alpha^{i+1}} m(z)dz - (s^i - w^i)[m(\alpha^{i+1})\alpha_i^{i+1} + m(\alpha^i)\alpha_1^i] = 0, \tag{8}$$

and since, using (6),

$$\frac{\partial^2 \pi^i}{(\partial w^i)^2} = 2[m(\alpha^{i+1})\alpha_1^{i+1} + m(\alpha^i)\alpha_1^i] < 0, \; w^{i-1} \leq w^i \leq w^{i+1},$$

The solution in (8) is unique.

Totally differentiating (8), we find that firm i's response to wage changes by its neighbors to be:

$$dw^i = \frac{m(\alpha^{i+1})\alpha_1^{i+1} \, dw^{i+1} + m(\alpha^i)\alpha_1^i \, dw^{i-1}}{2\,[m(\alpha^{i+1})\,\alpha_1^{i+1} + m(\alpha^i)\,\alpha_1^i]} \tag{9}$$

Hence, equal wage changes by both of firm i's near competitors induce a reaction in the same direction, but only one-half as large, in firm i's wage.

Using these results about reaction functions, the existence and uniqueness of a Nash equilibrium in pure wage strategies can be proved.

Theorem 1: Given n firms, requiring skill levels $s^1, \ldots, s^n$, with $\sigma(0) \leq s^1 < s^2 < \ldots < s^{n-1} < s^n \leq \sigma(Z)$, there exists a Nash equilibrium in pure wage strategies.

Proof: A Nash equilibrium can be constructed as follows. Let $\underset{\sim}{w}(0) = \underset{\sim}{0}$, and let $w^i(t)$ be firm i's best response against $\underset{\sim}{w}(t-1)_{/i}$, for $t = 1, 2, \ldots$. The sequence of wage vectors $\{\underset{\sim}{w}(t), t = 0, 1, 2, \ldots\}$ is strictly increasing. To see this note that at step $t = 0$ each firm makes zero profit, since all workers will prefer to remain unemployed. At step $t = 1$ each firm will offer a positive wage, expecting to attract some workers and to make a positive profit. Hence all wage rates increase at step $t = 1$. Inspecting (9) we see that at step $t = 2$, since each firm is reacting against strictly higher wage offers by its competitors, its own response is also higher. By induction, then, each firm's response also increases at every later step. Since the sequence $\{\underset{\sim}{w}(t)\}$ is increasing and is obviously bounded above by $\underset{\sim}{s}$, it converges.

It is also obvious that each firm's reaction function is continuous. Therefore, since $\lim_{t\to\infty} [\underset{\sim}{w}(t) - \underset{\sim}{w}(t-1)] = \underset{\sim}{0}$, $\lim_{t\to\infty} \underset{\sim}{w}(t)$ is a Nash equilibrium. Q.E.D.

Theorem 2: The Nash equilibrium of Theorem I is unique.

Proof: Suppose that there are two Nash equilibrium wage vectors, $\underset{\sim}{w}$ and $\hat{\underset{\sim}{w}}$. Consider

$$\max_i |w^i - \hat{w}^i|,$$

i.e., the firm (or any one of the firms), call it m, whose wage offers differ by the greatest amount between the two equilibria. At any equilibrium all firms attract some workers, so that w^m depends only on w^{m-1} and w^{m+1}. (If m=1, w^m depends only on w^2, and if m=n, only on w^{n-1}. In these cases the obvious modifications in the argument must be made.) But, from (9) we see that

$$|w^m - \hat{w}^m| < 1/2 \operatorname{Max} \{|w^{m-1} - \hat{w}^{m-1}|, |w^{m+1} - \hat{w}^{m+1}|\},$$

contradicting the selection of m. Q.E.D.

From (9) it is easy to see that the entry of a new firm has the expected effect on the equilibrium wage offers of all firms. The wage rates of the two closest neighbors of the entrant are increased by the direct effect of competition from the new firm, and there is then a ripple effect on all other firms, with the wage increases becoming more and more damped for more distant firms.

Suppose now that each firm makes several job offers. Let there be G firms, indexed by g=1, ... G, offering a total of $n \geq G$ jobs, indexed by i = 1, ..., n. As before, assume that $\sigma(0) \leq s^1 \leq s^2 \leq \ldots \leq s^n$. Furthermore, let I^g denote the index set for jobs offered by firm g.

$$I^g \cap I^{g'} = \emptyset \qquad \forall\ g \neq g'$$

$$\bigcup_{g=1}^{G} I^g = \{1,2,3,\ldots, n\}$$

Given the wage rates offered by all other firms, firm g chooses $(w^i,\ i \in I^g)$

as the solution of:

$$\underset{(w^i, i \in I^g)}{\text{MAX}} \quad \Pi g = \sum_{i \in I^g} (s^i - w^i) \int_{\alpha^i}^{\alpha^{i+1}} m(z)\, dz.$$

where the α^i's are, as before, determined by (5).

The conditions for a maximum are:

$$\Pi_i^g \equiv \frac{\partial \Pi^g}{\partial w^i} = -\int_{\alpha^i}^{\alpha^{i+1}} m(z)dz + m(\alpha^{i+1})\, \alpha_1^{i+1} \cdot [X_g^{i+1} e^{i+1} - e^i] - m(\alpha^i)\, \alpha_1^i \cdot (e^i - X_g^{i-1} e^{i-1}) = 0, \qquad i \in I^g \tag{10}$$

where $X_g^j \equiv \begin{cases} 1 \text{ if } j \in I^g \\ 0 \text{ if } j \notin I^g \end{cases}$, $\qquad i = 1,\ldots,n;\ g = 1,\ldots,G,$

and $e^i \equiv s^i - w^i$, $\quad i = 1, \ldots n.$

Since $[\Pi_{ij}^g]$ is negative definite, the solution is unique, and each firm's reaction function is single-valued. The existence and uniqueness of a Nash equilibrium in pure strategies can be shown using the same arguments as above.

Furthermore, some qualitative remarks can be made about the nature of the equilibrium as the number of firms or the pattern of offers (the index sets I^g) is varied.

It is clear from (10) that only actions by adjacent competitors affect a firm's optimal wage offers. Hence, if there are two firms making multiple skill-wage offers, but their skill levels alternate at every step - i.e., if $I^1 = \{1,3,5,\ldots,n\text{-}1 \text{ or } n\}$ and $I^2 = \{2,4,6,\ldots,n\text{-}1 \text{ or } n\}$ - then the Nash equilibrium vector of wage offers is exactly the same as if there were n different firms. Only if a firm offers jobs at two or more adjacent skill levels can it exercise an extra degree of monopsony power.

In fact, the Nash equilibrium wage vector depends only on the pattern of competition in the following sense. Suppose there are n types of jobs, $(s^1, s^2, \ldots, s^n)$, and let $\mathcal{J}(\cdot)$ be the function mapping $\{1,2,\ldots,n\}$ into the appropriate index sets.

$$i \in I^g \Rightarrow \mathcal{J}(i) = I^g \quad , \; i = 1,\ldots,n.$$

Next, let $\underset{\sim}{v}$ be the (n-1) - dimensional vector defined as follows:

$$v_i = \begin{cases} 0 \text{ if } \mathcal{J}(i) = \mathcal{J}(i+1) \\ 1 \text{ if } \mathcal{J}(i) \neq \mathcal{J}(i+1) \end{cases}$$

$$i = 1,\ldots, n-1.$$

The equilibrium wage vector clearly depends only on the vector $\underset{\sim}{v}$, and thus only indirectly on the number of firms or the index sets I^g. That is, $\underset{\sim}{v}$ conveys complete information about the "degree of competition" in the market.

For any fixed number of firms and any fixed set of skill levels $(s^1,\ldots,s^n)$, varying the index sets I^g unambiguously increases (decreases) all wage rates if and only if competition among adjascent neighbors in unambiguously increased (decreased).

For example, given $(I^g$, g=1, ..., G) , choose any $\bar{i} \in I^{\bar{g}}$ and $\bar{g}' \neq \bar{g}$, and let:

$$\hat{I}^{\bar{g}} = I^{\bar{g}}/\{\bar{i}\}$$

$$\hat{I}^{\bar{g}'} = I^{\bar{g}} \cup \{\bar{i}\}$$

$$\hat{I}^g = I^g, \qquad\qquad g \neq \bar{g}, \bar{g}'.$$

The Nash equilibrium wage rates associated with the market described by the sets $[\hat{I}^g]$ are all unambiguously higher than those in the market described by the sets $[I^g]$, if and only if $\underset{\sim}{\hat{v}} > \underset{\sim}{v}$, i.e., if and only if one of the following conditions is satisfied:

1) $\bar{i} - 1 \in I^{\bar{g}}$ and $i + 1 \notin I^{\bar{g}'}$ $\quad (v_{i-1} = 0,\ \hat{v}_{i-1} = \hat{v}_i = 1).$
2) $\bar{i} + 1 \in I^{\bar{g}}$ and $i - 1 \notin I^{\bar{g}'}.$ $\quad (v_i = 0,\ \hat{v}_{i-1} = \hat{v}_i = 1).$

It can also be shown that the firm's ability to exploit workers (to pay them less than their marginal product) is greater for skill levels that are more insulated from outside competition. For suppose that firm g offers jobs at skill levels s^i, s^{i+1}, ..., s^j. From (10) we see that the conditions for profit-maximazation include:

$$\Pi^g_k = -\int_{\alpha^k}^{\alpha^{k+1}} m(z)dz + m(\alpha^{k+1})\,\alpha_1^{k+1}\cdot(e^{k+1} - e^k) - \tag{11}$$

$$m(\alpha^k)\,\alpha_1^k \cdot (e^k - e^{k-1}) = 0$$

$$k = i+1, \ldots, j-1,$$

Suppose that the firm's profit margin per worker were higher for skill level k than at both adjascent skill levels, i.e., that

$$e^{k-1} > e^k < e^{k+1}, \text{ for some } k = i+1, \ldots, j-1. \tag{12}$$

Then, from (11) we see that every term of Π^g_k would be negative, and the necessary condition for an optimum could not be satisfied. Hence (12) cannot hold for an optimal set of wage offers. For a single monopsonist making n job offers, the same argument implies that $e^1 < e^2 < \ldots < e^n$.

Next, let us consider the efficiency properties of the equilibrium. It is easy to show that given any fixed set of skill requirements, assuming that there is a free labor market governed by private incentives, the allocation of labor among jobs is efficient if and only if for each job the wage rate is equal to the marginal product of labor.

Theorem 3: Given $\underset{\sim}{s} = (s^1, s^2, \ldots, s^n)$, the skill levels required for each of n types of jobs, the allocation of labor is efficient if and only if:

$$w^i = s^i, \qquad i = 1, \ldots, n$$

Proof: For any given $\underset{\sim}{s}$ and $\underset{\sim}{w}$, the net output of the economy is:

$$Y = \sum_{i=1}^{n} \int_{\alpha^i}^{\alpha^{i+1}} [s^i - f(z)c(s^i)]m(z)dz,$$

where $(\alpha^1, \ldots, \alpha^{n+1})$ are, as before, defined by (5). Differentiating Y with respect to $\underset{\sim}{w}$, the conditions for a maximum are:

$$\frac{\partial \gamma}{\partial w^i} = [s^{i+1} - s^i - f(\alpha^{i+1})[c(s^{i+1}) - c(s^i)]] \frac{\partial \alpha^{i+1}}{\partial (\Delta w^{i+1})} -$$

$$[s^i - s^{i-1} - f(\alpha^i)[c(s^i) - c(s^{i-1})]] \frac{\partial \alpha^i}{\partial (\Delta w^i)} = 0 \tag{13}$$

$$i = 1, \ldots, n-1,$$

$$\frac{\partial \gamma}{\partial w^n} = -[s^n - s^{n-1} - f(\alpha^n)[c(s^n) - c(s^{n-1})]] \frac{\partial \alpha^n}{\partial (\Delta w^n)} = 0.$$

Using the last equation above together with the last equation in (5) implies that:

$$s^n - s^{n-1} = f(\alpha^n)[c(s^n) - c(s^{n-1})] = w^n - w^{n-1}.$$

Using this result together with the $(n-1)^{st}$ equations in (5) and (13) then gives

$$s^{n-1} - s^{n-2} = f(\alpha^{n-1})[c(s^{n-1}) - c(s^{n-2})] = w^{n-1} - w^{n-2}$$

Continuing by induction:

$$s^i - s^{i-1} = w^i - w^{i-1}, \quad i = 1, \ldots, n. \tag{14}$$

Since $w^0 = s^0 = 0$, the first equation in (14) implies that $w^1 = s^1$, and, by induction,

$$w^i = s^i, \quad i = 1, \ldots, n.$$

Q.E.D. [1]

Not surprisingly, the Nash equilibrium wage vector is, in general, inefficient. What is perhaps less obvious is the degree of competition required to achieve efficiency. Consider firm i, with fixed skill requirement s^i. For a given sequence of viable actions by its closest competitors, i.e., a sequence

$$[(s^{i-1}(t), w^{k-1}(t)), (s^{i+1}(t), w^{i+1}(t)), t = 1,2,\ldots],$$

satisfying

$$s^{i-1}(t) < s^i < s^{i+1}(t),$$

$$w^{i-1}(t) \leq s^{i-1}(t), \; w^{i+1}(t) \leq s^{i+1}(t), \; t = 1,2,\ldots,$$

let $[w^i(t),\ t = 1,2,\ldots]$ be firm i's sequence of optimal responses. Suppose that $w^i(t)$ converges, and that $\lim_{t\to\infty} w^i(t) = s^i$. What can be said about the sequence $[(s^{i-1}, w^{i-1}), (s^{i+1}, w^{i+1})]$? The answer is given by

<u>Theorem 4</u>: The optimal wage of firm i approaches the efficient (competitive) level, $\lim_{t\to\infty} w^i(t) = s^i$, if and only if

1. The wage offers of both neighboring competitors approach their competitive levels, <u>and</u>

2. The skill requirements of both neighboring competitors approach s^i,

i.e., if and only if

$$\lim_{t\to\infty} w^{i-1}(t) = \lim_{t\to\infty} s^{i-1}(t) = s^i,$$

and

$$\lim_{t\to\infty} w^{i+1}(t) = \lim_{t\to\infty} s^{i+1}(t) = s^i .$$

<u>Proof</u>: Consider any sequence of skill-wage offers by firms i-1 and i+1, and assume that the sequence of optimal responses by firm i satisfies

$$\lim_{t\to\infty} w^i(t) = s^i$$

From (8), this implies that

$$\lim_{t\to\infty} \frac{\partial \pi^i}{\partial w^i} = -\lim_{t\to\infty} \int_{\alpha^i}^{\alpha^{i+1}} m(z)dz = 0$$

$$\Rightarrow \quad \lim_{t\to\infty} \alpha^i(t) = \lim_{t\to\infty} \alpha^{i+1}(t) = \sigma^{-1}(s^i),$$

where the last equality follows from the fact that workers with $z = \sigma^{-1}(s^i)$ prefer the offer $(s=s^i,\ w=s^i)$ to any other viable offer.

Using (5) and the definition of $\sigma(z)$, this in turn implies that:

$$\lim_{t\to\infty} \frac{c(s^i) - c[s^{i-1}(t)]}{s^i - w^{i-1}(t)} = \lim_{t\to\infty} \frac{c[s^{i+1}(t) - c(s^i)]}{w^{i+1}(t) - s^i} = \frac{1}{f[\sigma^{-1}(s^i)]} = \frac{1}{c'(s^i)} \qquad (15)$$

Furthermore, the restriction

$$w^{i-1}(t) \leq s^{i-1}(t) < s^{i}, \quad t = 1,2,\ldots$$

implies that

$$\frac{c(s^{i}) - c[s^{i-1}(t)]}{s^{i} - w^{i-1}(t)} \leq \frac{c(s^{i}) - c[s^{i-1}(t)]}{s^{i} - s^{i-1}(t)} < \frac{1}{c'(s^{i})},$$

$$t = 1,2,\ldots$$

Hence (15) is satisfied if and only if

$$\lim_{t\to\infty} s^{i-1}(t) = \lim_{t\to\infty} w^{i-1}(t) = s^{i}$$

Reversing the inequalities, an exactly analogous argument establishes that

$$\lim_{t\to\infty} s^{i+1}(t) = \lim_{t\to\infty} w^{i+1}(t) = s^{i}.$$

Q.E.D.

Using Theorem 4, the conditions under which the Nash equilibrium approaches the competitive equilibrium ($\tilde{w} = \tilde{s}$) are clear.

Theorem 5: The Nash equilibrium vector of wage offers approaches the competitive equilibrium if and only if:

1. the number of job types increases on every sub-interval of $[\sigma(0), \sigma(Z)]$, and
2. no single firm retains control of all job types in any sub-interval of $[\sigma(0), \sigma(Z)]$.

Theorem 5 points out a rather fundamental inefficiency under a competitive market. Wage rates converge to their efficient levels only as the number of job types gets larger (and even then, only if competition among closest neighbors is maintained). On the other hand, if there are fixed costs associated with maintaining different types of jobs -- organizational costs, costs of supervision, etc.--then the efficient number of job types may be rather small. The inefficiency arising from non-competitive wages is eliminated only by adding more job types, a change which--past a point--creates inefficiencies of its own.

The problem would be especially severe in a "thin" labor market, like a small, geographically isolated town, where the range of worker types would be wide relative to the total number of workers. Stated the other way around, a "thick" market has two advantages: the efficient number of job types is larger, so that on the average jobs and workers could be more closely matched; and wage rates will be bid up toward their competitive levels, so that workers will, in fact, select jobs for which they are better suited.

IV. Wage-Skill Competition

Suppose now that firms may choose both skill requirements and wage rates. Then it is easy to show that there is no Nash equilibrium in pure strategies.

Theorem 6: For any $n \geq 2$ firms choosing both skill requirements and wage rates, no Nash equilibrium in pure strategies exists.

Proof: The proof will be given for n=2; the extension for a larger number of firms will be obvious.

Let $\Pi\ (s,w,\hat{s},\hat{w})$ denote the profit of a firm offering (s,w) when its competitor offers $(\hat{s},\hat{w})$. Suppose that $[(\bar{s}^1, \bar{w}^1), (\bar{s}^2, \bar{w}^2)]$ is a Nash equilibrium. Then by assumption:

$$\Pi\ (\bar{s}^1, \bar{w}^1, \bar{s}^2, \bar{w}^2) \geq \Pi\ (\bar{s} - \epsilon, \bar{w}^2 + \epsilon, \bar{s}^2, \bar{w}^2), \forall\ \epsilon.$$

and

$$\Pi\ (\bar{s}^2, \bar{w}^2, \bar{s}^1, \bar{w}^1) \geq \Pi\ (\bar{s}^1 - \epsilon, \bar{w}^1 + \epsilon, \bar{s}^1, \bar{w}^1), \forall\ \epsilon.$$

Since every worker prefers a job that requires less skill and pays a higher wage, such a firm making such an offer will attract all workers who choose to take a job. Hence:

$$\Pi(\bar{s}^2 - \epsilon, \bar{w}^2 + \epsilon, \bar{s}^2, \bar{w}^2) = \Pi(\bar{s}^2 - \epsilon, \bar{w}^2 + \epsilon, 0, 0), \forall\ \epsilon > 0.$$

and

$$\Pi(\bar{s} - \epsilon, \bar{w}^1 + \epsilon, \bar{s}^1, \bar{w}^1) = \Pi(\bar{s}^1 - \epsilon, \bar{w}^1 + \epsilon, 0, 0), \forall\ \epsilon > 0.$$

Consequently, since the profit function $\Pi(s,w,s^1,w^1)$ is continuous except where $s=s^1$ and $w=w^1$,

$$\lim_{\epsilon_+ \to 0} \Pi(\bar{s}^2 - \epsilon, \bar{w}^2 + \epsilon, \bar{s}^2, \bar{w}^2) = \lim_{\epsilon_+ \to 0} \Pi(\bar{s}^2 - \epsilon, \bar{w}^2 + \epsilon, 0, 0) =$$

$$\Pi(\bar{s}^2, \bar{w}^2, 0, 0) \geq \Pi(\bar{s}^2, \bar{w}^2, \bar{s}^1, \bar{w}^1).$$

Similarly

$$\lim_{\epsilon_+ \to 0} (\bar{s}^1 - \epsilon, \bar{w}^1 - \epsilon, \bar{s}^1, \bar{w}^1) = \lim_{\epsilon_+ \to 0} \Pi(\bar{s}^1 - \epsilon, \bar{w}^1 + \epsilon, 0, 0) =$$

$$\Pi(\bar{s}^1, \bar{w}^1, 0, 0) \geq \Pi(\bar{s}^1, \bar{w}^1, \bar{s}^2, \bar{w}^2).$$

Putting the inequalities together implies:

$$\Pi(\bar{s}^1, \bar{w}^1, \bar{s}^2, \bar{w}^2) \geq \Pi(\bar{s}^2, \bar{w}^2, 0, 0) \geq \Pi(\bar{s}^2, \bar{w}^2, \bar{s}^1, \bar{w}^1) \geq$$

$$\Pi(\bar{s}^1, \bar{w}^1, 0, 0) \geq \Pi(\bar{s}^1, \bar{w}^1, \bar{s}^2, \bar{w}^2).$$

The second (fourth) inequality must be strict if firm 2 (firm 1) loses some workers when its competitor makes its Nash equilibrium offer instead of a zero offer. Hence the second (fourth) inequality must be strict if $\bar{s}^2 \leq \bar{s}^1$ $(\bar{s}^1 \leq \bar{s}^2)$.

Q.E.D.

Intuitively, the proof says that for any pair of competing (neighboring) firms, located at different points in s-w space, at least one will be able to increase its profit by abandoning its own position and instead undercutting its competitor by the smallest possible amount.

V. Conclusions

The model examined above shares several features with the signalling model developed by Spence [1,2]. Here, as in Spence's models, firms offer wage schedules, with wage rates dependent on "skill" or "education".[2] Then, given the menu of alternatives offered by firms, heterogeneous workers choose how much "skill" to acquire.

However, the model here differs from Spence's in several other respects. It is assumed here that a worker's productivity depends only on the level of skill he attains, and not on his personal characteristic (z-parameter). A job can be thought of as being so highly regimented that there is no room for performing it "better" or "worse". Rather, the alternatives are only "successful" or "unsuccessful" completion of the tasks. Furthermore, adequate performance is certain if the worker possesses the required level of skill, and inadequate performance is certain if he does not. Consequently, the signalling function of educational attainment disappears: since the z-parameter (ability, family background, or whatever) does not affect productivity directly, firms have no interest in acquiring information about that parameter. Only the worker's "education" (which should be thought of as "highest level of training completed" rather than "years of training") affects his productivity, and that is directly and perfectly observable. Hence, the problem of adverse selection that arises under differential information does not arise here.

The structure of competition, on the other hand, is more richly spelled out here. While Spence considers only perfect competition, the emphasis here has been to study the properties of equilibria when the number of firms is small, so that each exercises some degree of monopsony power.

Consequently some of the conclusions reached here are quite different. Spence found that at the competitive equilibrium, either each worker receives a wage equal to his marginal product, or else all indistinguishable members of a group receive a wage equal to the average marginal product of that group's members. However, because of the signalling function of education, there is a tendency toward overinvestment in it, so that the competitive outcome may be inefficient.

In the model examined here, given the skill requirements of firms, setting a wage equal to the marginal product for each skill level would be efficient. Since the inefficiency in Spence's model arises because of the signalling function of education, this is not surprising. However, the wage rate offered for a particular skill level, s^i, approaches the marginal product for that skill level if and only if both neighboring competitors offer jobs with skill requirements approaching s^i and wage rates approaching the competitive levels. If all jobs' skill requirements differ significantly from each other, all wage rates will fall short of the corresponding marginal products. Hence the equilibrium wage configuration in this model reflects the use of monopsony power to depress wages.

Furthermore, a basic inefficiency in monopolistically competitive markets is revealed. Inefficiencies in the allocation of labor are eliminated only if wage rates are bid up to their competitive levels. However, this occurs only if the number of competing job offers increases. Hence, if there are fixed costs in the underlying technology, increasing the number of job offers may also be inefficient. This problem would be most severe in a "thin" market.

In Section IV it was shown that if firms compete in both wages and skills, then there is no Nash equilibrium. This result is similar to a non-existence proposition of Spence [1, Proposition 8]. Using a model where there are only two types of workers, he showed that if firms can carry on production using only one "type" of worker, then an equilibrium wage configuration may not exist. Non-existence occurs whenever there is a "one-step" wage contract (i.e., a contract that leads to equal education and equal wages for both "types") that both types of workers prefer to the efficient contract (i.e., the contract that leads to the efficient level of training for each group, and sets the wage equal to the marginal product for each group). Here it was shown that if there is a continuum of "types" of workers, and if firms are limited to making a finite number of job offers, an equilibrium never exists. The non-existence theorem in Section IV eliminates from further consideration one potential explanation of how the constellation of job offers found in the market is determined.

Finally, note that the results here can be extended in a straightforward way to a situation of imperfect competition among firms selling a differentiated product. The product should be thought of as one of which a consumer purchases one and only one unit. Let s^i denote the quality (equal to cost of production) of firm i's product and p^i the price charged by firm i. Let $g(z)v(s)$ denote a z-consumer's valuation of a product of quality s. Each consumer chooses the product i for which his net surplus is greatest.

$$\max_i \, [g(z)v(s^i) - p^i]$$

For $g(\cdot)$ and $v(\cdot)$ satisfying:

$$v(s) > 0,\ v'(s) < 0\ ,\ v''(s) < 0\ , \qquad s \geq 0;$$

$$g(z) > 0,\ g'(z) > 0\ , \qquad z \geq 0\ ,$$

Theorems 1-6 remain valid.

Appendix

Given

$m(z) > 0$, $f(z) > 0$, $f'(z) < 0$, $0 \leq z \leq Z$,

let $\gamma(z)$ be the solution of the differential equation:

$$\gamma'(z) = a \cdot \frac{f'(z)}{m(z)}$$

$$\gamma(0) = 0,$$

where $a < 0$ is an arbitrary constant. Thus $\gamma(\cdot)$ is merely a rescaling of z, with $\gamma' > 0$. Straightforward calculation shows that:

$$\frac{\gamma''}{\gamma'} = \frac{f''}{f'} = \frac{m'}{m}$$

Next, define $\varphi(\cdot)$ and $\mu(\cdot)$ as the cost and density functions for the rescaled parameter:

$$\varphi(x) \equiv f[\gamma^{-1}(x)],$$

$$\mu(x) \equiv m[\gamma^{-1}(x)], \qquad 0 \leq x \leq \gamma(Z),$$

with

$$\varphi'[\gamma(z)] = \frac{f'(z)}{\gamma'(z)},$$

$$\varphi''[\gamma(z)] = \frac{1}{[\gamma'(z)]^2} \cdot [f''(z) - f'(z)\frac{\gamma''(z)}{\gamma'(z)}],$$

$$\mu'[\gamma(z)] = \frac{m'(z)}{\gamma'(z)}, \qquad 0 \leq z \leq Z.$$

It is easily verified that the cost and density functions associated with the rescaled parameter γ satisfy (6).

$$\mu'[\gamma(z)]\,\varphi'[\gamma(z)] - \mu[\gamma(z)]\,\varphi''[\gamma(z)] =$$

Appendix (Cont'd.)

$$\frac{m'f'}{(\gamma')^2} - \frac{m}{(\gamma')^2}\left[f'' - f'\frac{\gamma''}{\gamma'}\right] =$$

$$\frac{1}{(\gamma')^2} \cdot \left[m'f' - mf'' + mf'\left(\frac{f''}{f'} - \frac{m'}{m}\right)\right] = 0.$$

Consequently the assumption in (6) involves no loss of generality.

Footnotes

[1] This assumes that some workers choose to remain unemployed. If all workers are employed, any wage schedule satisfying

$$s^i - s^{i-1} = w^i - w^{i-1}, \qquad i = 2,\ldots,n$$

leads to an efficient allocation of labor.

[2] Spence's step-function wage schedules in [1] are exactly equivalent to the "point" offers in the model here, since only left-hand end-points of the "steps" are ever chosen by workers.

References

Spence, Michael, "Competition in Salaries, Credentials, and Signalling Prerequisites for Jobs", _Quarterly Journal of Economics_ XC (Feb., 1976), 51-74

____________________, "Competitive and Optimal Responses to Signals: An Analysis of Efficiency and Distribution", _Journal of Economic Theory_, VIII (March, 1974), 296-332

____________________, _Market Signalling: Information Transfer in Hiring Related Processes_ (Cambridge: Harvard University Press, 1973).