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INFORMATION, TRADE, AND COMMON KNOWLEDGE

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ABSTRACT

In any voluntary trading process, if agents have rational expectations, then it is common knowledge among them that the equilibrium trade is feasible and individually rational. This condition is used to show that when risk-averse traders begin at a Pareto optimum (relative to their prior beliefs) and then receive private information (which disturbs the marginal conditions), they can never agree to any non-null trade. This result has implications for the nature of the information transferred among agents in any voluntary exchange. In particular, if trading occurs on competitive markets before and after agents receive private information, there always exists a fully revealing rational expectations equilibrium *ex post*, in which the *change* in relative prices (rather than the prices themselves) reveals a sufficient statistic for all agents' information. Moreover, even at equilibria that are less than fully revealing, the information conveyed by price changes "swamps" each trader's private information.

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1. Introduction

Halfway through the growing season a grain trader receives a private report on the state of the crop. Should he use this information to speculate in grain futures? Or should he assume futures prices already impound so much information that his own information is valueless, and on that basis refrain from speculating?

Generally, how do traders who have rational expectations respond to new, private information? We investigate this question using a general model of voluntary trade, so that our results apply to, but are not limited to, competitive markets.

Our central result is that, regardless of the institutional structure, if the initial allocation is *ex ante* Pareto-optimal (as occurs, for example, when it is the outcome of a prior round of trading on complete, competitive markets), then the receipt of private information cannot create any incentives to trade.

From one perspective this no-trade result may seem surprising. The receipt of private information will generally lead the traders to hold

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different posterior beliefs, even if their prior beliefs are identical. This, in turn, will result in an inequality of the traders' marginal rates of substitution for wealth across states of the world. One might expect this to create incentives for trade. Why then, does no trade takes place? Since the initial allocation is Pareto optimal, there can be no valid insurance motive or transactions motive for trading -- a trader's only motive is his hope of finding an advantageous bet. Therefore, the mere willingness of the other traders to accept their parts of the bet is evidence to at least one trader that his own part is unfavorable. Hence no trade can be found that is acceptable to all traders.

This no-trade result depends crucially on the assumption that it is <u>common knowledge</u> when a trade is carried out that it is feasible and that it is mutually acceptable to all of the participants. Informally, a fact or an event is common knowledge among members of a group if it is known by each of them, if each knows that it is known by each of them, if each knows that each knows that it is known..., etc. An example is used below to illustrate why trading can be consistent with more limited kinds of inference, but not with rational expectations.

We then examine the information conveyed by equilibrium prices when there are markets both before and after traders receive private information. In Theorem 2 we show that a fully revealing equilibrium on <u>ex post</u> markets always exists. At this equilibrium the change in relative prices is a sufficient statistic for all agents' private information taken jointly (although, of course, no trade takes place). In Theorem 3 we show that at any <u>ex post</u> equilibrium, even if it is not fully revealing, the information conveyed by the change in relative prices "swamps" each agent's private information taken individually. That is, each agent's posterior beliefs given both price changes and his private signal depend only on the price changes. It is also shown that the change in relative prices is a purely informational phenomenon, i.e., the change is independent of traders' endowments, preferences, and prior beliefs.

This independence result is in contrast to the conclusions for static rational expectations models. In the static models most often studied in the literature, traders are assumed to make inferences based on the price vector observed at a single market date. To do this, a trader must know a great deal about underlying supply and demand conditions, since prices depend on these as well as on the various traders' information. In a dynamic model, on the other hand, relative prices change over time in a simple way in response to new information, so that traders can easily make inferences from price changes without knowing anything about other traders' endownments, preferences, or prior beliefs. The dynamic rational expectations model studied here is in this sense simpler, and perhaps more plausible, than many static models. (Note that price changes are in fact commonly studied by securities' traders to guide their investment decisions.)

In Section 2, common knowledge is formally defined and the no-trade theorem is proved. An example is used to illustrate the importance of the role of common knowledge. In Section 3, Theorems 2 and 3, which deal with the information revealed by equilibrium prices on _ex post_ markets, are presented. The relationship between our results and previous work is discussed in Section 4.

2. Common Knowledge and Trade

In rational expectations models it is assumed that each agent infers whatever information he can from the market variables he observes, as well as from the non-market signals to which he has access. Furthermore, in these models each agent believes -- and is justified in believing -- that all other agents also make full use of the information available to them. Since prices (or other market signals) are potentially an important source of information in rational expectations settings, it is important to know what kind of information they convey.

Certainly, at an equilibrium of any voluntary trading process, in addition to his private information, each agent knows that the equilibrium trade is feasible and is acceptable to the other agents. Moreover, since each agent knows that all the other agents are rational, agent i knows that all agents $j \neq i$ know that the trade is feasible and is acceptable to the others, and that all know that all know that the trade is feasible and acceptable, etc. Under rational expectations this "etc." consists of an infinite sequence of statements.

These statements can be expressed very concisely using Aumann's [1976] definition of common knowledge. Let the state of the world be described by $\omega \in \Omega$, and let each agent's information be represented by a partition on Ω. Let P_i denote agent i's partition, for $i = 1,...n$, and for any $\omega \in \Omega$ define $P_i(\omega)$ to be the element of P_i that contains ω. This is to be interpreted as follows: when the state of the world is ω, trader i knows only that the state is in $P_i(\omega)$. Thus, trader i knows that an event A has occurred if $P_i(\omega) \subset A$. Let R be the meet of the partitions $P_1,...,P_n$, and for any $\omega \in \Omega$ define $R(\omega)$ to be the element of R that contains ω. (The meet of a collection of partitions is their finest common coarsening.)

Definition[1] (Aumann): An event A is common knowledge at ω among agents $1,...,n$ if $R(\omega) \subset A$.

In what follows, we distinguish between the information available to a trader at two nearby points in time. Trader i's information just prior to trading is represented by the partition $\hat{P}_i$. His information at the time of trading, including whatever he can infer from prices or from the behavior of other traders, is represented by P_i.

At a rational expectations equilibrium of any voluntary trading process, it is common knowledge among all agents at the time of trading that the agreed-upon trade is feasible and mutually acceptable. As will be shown below, the fact that this market information is common knowledge is sufficient to preclude trading based solely on differences in private information.

Consider a pure exchange economy with n traders in an uncertain environment. Let Ω be the (finite) set of possible states of the world, with generic element ω. For our purposes, it is convenient to think of ω as consisting of two components. Let $\Omega = \Theta \times X$ and $\omega = (\theta,x)$. The set Θ will be called the set of payoff-relevant events; endowments and utility functions may depend on θ. The set X consists of payoff-irrelevant events; these events do not affect endowments or tastes directly. However, x and θ may be statistically related.

There are ℓ commodities in each state of the world, and for simplicity we assume that the consumption set of each trader in each state of the world is R_+^ℓ. Each trader i is described by:

a) his endowment, $e_i:\Theta \to R_+^\ell$;

b) his utility function, $U_i:\Theta \times R_+^\ell \to R$;

c) his (subjective) prior beliefs about ω, $p_i(\cdot)$; and

d) his (prior) informational partition, $\hat{P}_i$.

It is assumed that $U_i(\theta, \ ): R_+^\ell \to R$ is increasing for all i,θ. If $U^i(\theta, \ )$ is concave (resp. strictly concave) for all θ, trader i is said to be _weakly_ (resp. _strictly_) _risk-averse._

A _trade_ $t = (t_1,\ldots,t_n)$ is a function from Ω to $R^{n\ell}$, where $t_i(\omega)$ describes trader i's net trade of physical commodities in state ω. If the trade t can be described by function from Θ to $R^{n\ell}$, it is called a _θ-contingent trade_. A trade is _feasible_ if:

$$e_i(\theta) + t_i(\theta,x) \geq 0 \ , \qquad \forall i,\theta,x; \quad (1)$$

$$\sum_{i=1}^{n} t_i(\theta,x) \leq 0 \ , \qquad \forall \theta,x; \quad (2)$$

Assume that $p_i(\omega) > 0$ for every state ω and every trader i, and let $E_i[\ ]$ denote the expectation under p_i. If

$$p_1(x|\theta) = \ldots = p_n(x|\theta) \ , \qquad \forall x,\theta, \quad (3)$$

we will say that beliefs are _concordant._ When one thinks of x as information about θ, concordant beliefs mean (roughly) that the traders agree about how this information should be interpreted. Concordant beliefs arise naturally in statistical problems where θ is an unknown parameter about which traders may hold different views, and x is a statistic whose conditional distribution is objectively determined.

Note that if the agents are risk-averse and have concordant beliefs, then for any feasible trade t, the θ-contingent trade $t^* \equiv (E_1[t_1|\theta],\ldots,E_n[t_n|\theta])$ is feasible and weakly preferred by each trader.[2] Intuitively, t differs from t^* only in that it includes side bets abut x. Risk-averse traders with concordant beliefs find such bets to be unattractive when the markets for θ-contingent claims are complete. It is only when these markets are incomplete that the side bets may become attractive as imperfect surrogates for θ-contingent trades.

<u>Theorem 1</u>: Suppose that all traders are weakly risk-averse, that the initial allocation $e = (e_1,\ldots,e_n)$ is Pareto-optimal relative to θ-trades, that agents' prior beliefs are concordant, and that each trader i observes the private information conveyed by the partition $\hat{P}_i$. If it is common knowledge at ω that t is a feasible θ-trade and that each trader weakly prefers t to the zero trade, then every agent is indifferent between t and the zero trade. If all agents are strictly risk-averse then t is the zero trade.[3]

<u>Proof</u>: Recall that P_i denotes the information partition of trader i which includes including whatever information is conveyed at equilibrium by the trading process, and that R is the meet of $P_1,\ldots,P_n$. Suppose that it is common knowledge at $\omega' \equiv (\theta',x')$ that t is a feasible, mutually acceptable θ-trade. Then for every i and every $\omega\varepsilon R(\omega')$

$$E_i[U_i(\theta,e_i + t_i)|P_i(\omega)] \geqslant E_i[U_i(\theta,e_i)|P_i(\omega)] \qquad (4)$$

Suppose that the inequality in (4) is strict for trader j at ω', and consider the (θ,x)-trade t* defined by:

$$t_i^* \equiv t_i\, 1_{R(\omega')}\ , \qquad \forall i.$$

(where $1_{R(\omega')} = 1$ if $\omega\ \varepsilon\ R(\omega')$, and $1_{R(\omega')} = 0$ otherwise). Since t is feasible, so is t*. Also, viewing t* ex ante, we find that for each trader i

$$E_i[U_i(\theta, e_i + t_i^*)]$$

$$= E_i[E_i[U_i(\theta, e_i + t_i\ 1_{R(\omega')}|P_i]]$$

$$= E_i[E_i[U_i(\theta, e_i)\ 1_{R^c(\omega')}|P_i]] + E_i[E_i[U_i(\theta, e_i + t_i)\ 1_{R(\omega')}|P_i]]$$

Since R is coarser than P_i, this expression is equal to:

$$= E_i[1_{R^c(\omega')} E_i[U_i(\theta, e_i)|P_i]] + E_i[1_{R(\omega')} E_i[U_i(\theta, e_i + t_i)|P_i]]$$

$$\geq E_i[1_{R^c(\omega')} E_i[U_i(\theta, e_i)|P_i]] + E_i[1_{R(\omega')} E_i[U_i(\theta, e_i)|P_i]]$$

$$= E_i[U_i(\theta, e_i)],$$

where R^c denotes the complement of R, and the inequality follows from (4). Moreover, the inequality is strict for trader j. Hence t* is feasible and ex ante is strictly Pareto-superior to the null trade. Since agents' beliefs satisfy (3), the θ-trade t** = E[t*|θ] is feasible, and ex ante is strictly Pareto-superior to the null trade, contrary to our hypothesis about the initial allocation.

If traders are strictly risk averse, if t satisfies (4), and if t is not null, then 1/2 t** is a Pareto-improving θ-trade, contrary to the assumption that the initial allocation is Pareto-optimal. Q.E.D.

After the agents observe $\hat{P}_1(\omega),\ldots,\hat{P}_n(\omega)$ respectively their posterior beliefs about θ do, in general, differ. Still they do not trade. Intuitively, if any agent is willing to accept a trade, he reveals something about the signal he has observed. If a trade takes place all agents must know that the claims balance and that each agent regards the trade as beneficial to himself. Theorem 1 shows that in some situations this common knowledge is enough to preclude trade completely. (Note that trader i need not observe either prices or the net trades of others. Of course, he does know that the net trades of the others sum to $-t_i$.)

If agents do not have rational expectations, each agent may know that a proposed trade is feasible and is acceptable to all agents, yet those facts may not be common knowledge. The distinction is illustrated by the following example.

Suppose that two agents hold the prior beliefs about the pair (θ,x) given in Table 1.

TABLE 1

	$\theta=1$	$\theta=2$
$x=1$	.20	.05
$x=2$	.05	.15
$x=3$	.05	.05
$x=4$	.15	.05
$x=5$	.05	.20

Let the information structures for agents 1 and 2 respectively be described by the following partitions on X:

$\hat{P}_1$: {x = 1 or 2}, {x = 3 or 4}, {x = 5}

$\hat{P}_2$: {x = 1}, {x = 2 or 3}, {x = 4 or 5}

Assume that both agents are risk-neutral and suppose that the following bet is proposed: if $\theta=1$ agent 2 pays one dollar to agent 1, if $\theta=2$ agent 1 pays one dollar to agent 2. Suppose that $x=3$ occurs. Consider the following types of behavior which might occur.

CASE A: Naive behavior

Since at $x=3$, $p(\theta=1|P_1) = 2/3 > 1/2$, agent 1 accepts the bet. Similarly, since at $x=3$, $p(\theta=2|P_2) = 2/3 > 1/2$, agent 2 accepts the bet.

CASE B: First-order sophistication

Agent 1 reasons as follows:

> "I know that either $x=3$ or $x=4$. If $x=3$, $p(\theta=2|P_2) = 2/3 > 1/2$, so I could expect agent 2 to accept the bet. If $x=4$, $p(\theta=2|P_2) = 5/9 > 1/2$, so I could expect him to accept the bet. Therefore, the fact that agent 2 accepts the bet tells me nothing new. Since $p(\theta=1|P_1) = 2/3 > 1/2$, I will accept the bet."

Agent 2 reasons similarly, and also accepts the bet.

CASE C: Rational expectations

Agent 1 reasons as follows:

> "If $x=1$, agent 2 knows that $x=1$ and will refuse the bet. Hence if agent 2 accepts the bet $x\neq1$. Therefore, if I observe the partition element {1,2} and if agent 2 accepts the bet, then $x=2$. If $x=2$ the bet is disadvantageous to me. Hence if I observe {1,2} I should

refuse to bet. Agent 2 will use a similar line of reasoning to conclude that he should refuse the bet if he observes {4,5}.

Hence I will refuse the bet if I observe {1,2} or {5}, and agent 2 will refuse the bet if he observes {1} or {4,5}. Since I am risk-neutral, I am indifferent between accepting and rejecting the bet if I observe {3,4} , and it doesn't matter to me what agent 2 does when he observes {2,3}."

Agent 2 will use a similar line of reasoning to conclude that he should refuse the bet if he observes {1} or {4,5}. Hence the bet is accepted by both only if x=3.

If both agents are slightly risk-averse, the analysis is unchanged in Cases A and B, but in Case C agent 1 will decline the bet if he observes {3,4} (since it can take effect only if x=3) and agent 2 will decline the bet if he observes {2,3}.

If all information is public, beliefs need not be concordant to preclude trade from an initial position which was <u>ex ante</u> Pareto optimal relative to θ - trades. For concave, differentiable utility functions, if x is publicly announced, then further trade is precluded if and only if:

$$\frac{p_1(x|\theta)}{p_1(x|\theta')} = \cdots = \frac{p_n(x|\theta)}{p_n(x|\theta')}, \qquad \forall x,\theta,\theta'. \quad (5)$$

3. What Prices Reveal

Suppose that before any information about (θ,x) is revealed, a round of trading is conducted using a market mechanism. Let e denote the competitive equilibrium allocation and let $q(\theta) \in R_+^\ell$ denote the prices supporting e. Let Q be the join of $\hat{P}_1,\ldots,\hat{P}_n$. (The join of a collection of partitions is their coarsest common refinement.) Thus, Q conveys all of the information contained in $P_1,\ldots, P_n$, but no more.

If the signals $P_1(\omega),\ldots,P_n(\omega)$, are revealed to agents $1,\ldots,n$, respectively and markets reopen, we know from Theorem 1 that e is still a competitive equilibrium allocation. Theorem 2 concerns the price vector that supports e as a revealing rational expectations equilibrium.

<u>Theorem 2</u>: Let $e = (e_1,\ldots,e_n)$ be an <u>ex ante</u> Pareto-optimal allocation relative to θ-trades, supported by the prices $q(\theta)$, and assume that agents' prior beliefs satisfy (3). If the signals $\hat{P}_1(\omega),\ldots,\hat{P}_n(\omega)$ are revealed to agents $1,\ldots,n$, respectively and markets are reopened, the price vector $\hat{q}(\theta|x)$ given by:

$$\hat{q}(\theta|x) = q(\theta)p(Q(\omega)|\theta) ,$$

together with the initial allocation, constitutes a fully revealing rational expectations equilibrium.

<u>Proof</u>: Since each agent observes $p(Q(\omega)|\theta)$ through the change in prices, using Bayes' Theorem, agent i's posterior for θ is given by:

$$p_i(\theta|Q(\omega)) = p(Q(\omega)|\theta)p_i(\theta)/p_i(Q(\omega)),$$

so that the price vector is a sufficient statistic for all the private signals. It is straightforward to check that the initial allocation is a competitive equilibrium relative to the new prices and fully revealed information.

Q.E.D.

Notice that it is the <u>change</u> in prices that reveals all of the information about θ available to all traders (i.e., $\hat{q}(\theta|x)/q(\theta)$ is a sufficient statistic for $P_1(\omega),\ldots,P_n(\omega)$). It does this in a very simple, easily interpretable way, and in a way that does not depend on any trader's preferences. In the usual rational expectations model, if traders' preferences are unknown, each trader must attempt to sort out θ-relevant information from information about preferences as he scrutinizes market prices.

There may be other rational expectations equilibria as well -- equilibria that are less than fully revealing. However, the following theorem shows that in <u>any</u> equilibrium, information from an agent's private signal is "swamped" by price information, just as it is in the fully revealing equilibrium.

<u>Theorem 3</u>: Assume that all agents are strictly risk-averse and have continuously differentiable utility functions $U_i(\theta, \;)$; that an *ex ante* round of trade on competitive θ-markets leaves agents at a Pareto-optimal allocation e supported by the price vector q; that $e_i \in R^{\ell}_{++}$, for all i; and that agents' prior beliefs satisfy (3). Suppose that agents $1,\ldots,n$ observe the private signals $P_1(\omega),\ldots,P_n(\omega)$, respectively, and θ-markets reopen. By Theorem 1, e is still a competitive equilibrium allocation; let $\hat{q}(\theta|x)$ be any price vector supporting it. Then

$$p_i(\theta|P_i(\omega),\hat{q}) = p_i(\theta|\hat{q}), \;, \qquad \forall\theta,\ i. \qquad (6)$$

Proof: For simplicity of notation take $\ell = 1$, and let U_i' denote the marginal utility of consumption. Since e is a competitive equilibrium allocation *ex ante*,

$$\frac{p_i(\theta)\; U_i'(\theta,\; e_i(\theta))}{p_i(\theta')\; U_i'(\theta, e_i(\theta'))} = \frac{q(\theta)}{q(\theta')}$$

and since it is still an equilibrium allocation *ex post*,

$$\frac{p_i(\theta|P_i(\omega),\hat{q})U_2'(e_i(\theta))}{p_i(\theta'|P_i(\omega),\hat{q})U_2'(e_i(\theta'))} = \frac{\hat{q}(\theta|Q(\omega))}{\hat{q}(\theta'|Q(\omega))} \qquad \forall \theta,\theta',\ i.$$

Together these conditions imply that:

$$\frac{p_i(\theta|P_i(\omega),\hat{q})}{p_i(\theta'|P_i(\omega),\hat{q})} = \frac{p_i(\theta)}{p_i(\theta')} \; \frac{\hat{q}(\theta|Q(\omega))/q(\theta)}{\hat{q}(\theta'|Q(\omega))/q(\theta')} \quad \forall\ \theta,\theta',\ i; \ (7)$$

The posterior probabilities $p_i(\theta|P_i(\omega),\hat{q})$ are completely determined by their ratios and the condition that the probabilities must sum to one. Then since the right hand side of (7) depends on $Q(\omega)$ only through $\hat{q}$, equation (6) follows.

Q.E.D.

Equation (7) shows that any *ex post* equilibrium prices supporting e, even if they are not fully revealing, have the following important property: the change in relative prices is independent of agents' endowments, utility functions, and prior beliefs, and is also independent of the initial allocation e.

5. Conclusions

The results above cast light on two issues that have been widely discussed in the literature on trade under uncertainty. The first is the value of private and public information when trading takes place *ex ante* and *ex post*. Marshall (1974) shows that if all agents hold identical prior beliefs and if both *ex ante* and *ex post* markets are available, then the release of public information has neither private nor social value and leads to no further trading. Marshall also claims that although private information is socially valueless, it is valuable to the individual who receives it. As Hirshleifer [1971] did, Marshall argues that an individual who receives private information can speculate profitably at (virtually) unchanged market prices. This argument rests on the assumption that an individual is "small" relative to the market.

Theorem 1 above shows that this argument is invalid if traders' expectations are rational; a trader with new information is never "small". On the contrary, any attempt to speculate on the basis of new information must result in that information becoming impounded in prices, so that profitable speculation is impossible. Hence, if beliefs are concordant, private information has neither private nor public value. The argument used in Theorem 1 can also be used to show that (5) is a necessary and sufficient condition for public information to be valueless.

The second issue addressed is the nature of the information revealed by prices in a rational expectations equilibrium. This is a question that has been addressed by Grossman [1977], Radner [1979], Allen [1978], and others. It is a question that has proven hard to answer, at least in part because it is usually difficult to interpret the informational assumptions. In the work just cited, information about all agents endowments, utility functions, and prior beliefs is commingled in market variables with information about their private signals about the state of the world.

Theorems 2 and 3 above show that when markets are available both before and after information is released, it is the change in relative prices that reveals information. This seems to us a more appealing conclusion than the claim that price levels reveal information. Moreover, Theorem 2 shows that when beliefs are concordant, the change in relative prices has a purely informational character: the changes do not depend on any characteristics of agents or on the initial allocation (except that it must be Pareto-optimal).

Finally, Theorem 3 shows that the information conveyed by any *ex post* equilibrium prices, whether or not they are fully revealing, "swamps" the private signal received by any agent. That is, after the equilibrium prices are formed each agent can afford to forget the signal he observed; to compute his posterior beliefs he only needs to remember how prices have changed.

Our results concerning rational expectations market equilibria raise anew the disturbing questions expressed by Beja (1977), Grossman and Stiglitz (1980), and Tirole (1980): Why do traders bother to gather information if they cannot profit from it? How does information come to be reflected in prices if informed traders do not trade or if they ignore their private information in making inferences? These questions can be answered satisfactorily only in the context of models of the price formation process[4]; and our central result, the no-trade theorem, applies to all such models when rational expectations are assumed.

FOOTNOTES

[1]An axiomatic characterization of common knowledge is given in Milgrom (1981).

[2]Marshall (1974) shows that this is true if traders have identical prior beliefs, i.e., if $p_i(\omega) = p_j(\omega)$, $\forall i, j$, $\forall \omega$.

[3]Milton Harris has called this the Groucho Marx Theorem since it is reminiscent of Groucho's remark: "I'd never join any club that would have me for a member."

[4]Recent papers by Dubey and Shubik (1979) and Wilson (1978) address the question of how prices are formed. Glosten (1979) and Milgrom (1981a) study models of price formation where the traders have unequal access to information.

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