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Default Risk, Homemade Leverage, and the Modigliani-Miller Theorem

by

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ABSTRACT

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Using stochastic dominance arguments, the M-M theorem that the value of a firm is independent of its financing is shown to be valid if investors are able to borrow at the same nominal interest rate as firms or if all investors hold bonds of levered firms. If investors are required to pledge collateral for personal borrowing, the nominal interest rate paid by the investor may be less than the nominal interest rate paid by levered firms. The values of the levered and unlevered firms will be equal if all economic agents are risk neutral, but if investors and lenders are risk averse, the value of the levered firm may be greater than the value of the unlevered firm.

Default Risk, Homemade Leverage, and the Modigliani-Miller Theorem

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Franco Modigliani and Merton Miller (MM) have shown in their classic paper that in the absence of taxes and transactions costs the value of a firm does not depend on its debt-equity ratio, since an individual investor may create his own homemade leverage to offset any particular debt-equity ratio chosen by the firm. The M-M theorem recently has been reexamined by Joseph Stiglitz and by Vernon Smith, who consider the implications of less restrictive assumptions than those in the original proof. Stiglitz has indicated that the M-M theorem holds under more general conditions, while Smith focused on preferences of risk averse, expected utility maximizing investors for various debt-equity ratios for firms given that the probability of default on corporate bonds is positive. Here, the effects of default risk and differences in the nominal interest rates for firm and individual borrowing are considered for classes of expected utility maximizing investors who may or may not be risk averse. The paper provides an interpretation of the M-M theorem using the concept of stochastic dominance studied by Josef Hadar and William Russell, Giora Hanoch and Haim Levy, and the authors cited therein.

A distribution function G_1 is said to dominate a distribution function G_2 for the class of increasing utility functions if $G_1(z) \leq G_2(z)$ for all z and $G_1(z^o) < G_2(z^o)$ for some z^o. This dominance criterion implies that for all strictly increasing utility functions U the expected utility with G_1 is greater than that with G_2 or

$$\int_{-\infty}^{\infty} U(t)dG_1(t) > \int_{-\infty}^{\infty} U(t)dG_2(t).$$

Every individual with such a utility function prefers G_1 to G_2, and if all individuals with increasing utility functions prefer G_1 to G_2, then $G_1(z) \leq G_2(z)$ for all z and $G_1(z^o) < G_2(z^o)$ for some z^o. A distribution G_1 dominates G_2 for the class of nondecreasing, concave utility functions if and only if $\int_{-\infty}^{z} [G_1(t)-G_2(t)]dt \leq 0$ for all z and $G_1 \neq G_2$ for some z_o. Then no risk averse or risk neutral individual prefers G_2 to G_1.

I. The M-M Theorem and Default Risk

To investigate the M-M Theorem, consider two firms in the same risk class[1] with identical probability distributions of gross earnings[2] X such that for any state that occurs both firms have the same earnings. The probability of bankruptcy ($X < 0$) is assumed to be positive (but less than one), and hence, the probability of default on any corporate bonds issued is positive. Stochastic constant returns to scale are assumed, and the distribution function of gross earnings is denoted by $F(X)$. If suffices to assume that one firm is financed solely by equity and the other by debt and equity. The value V_1 of the unlevered firm is $V_1 = E_1$ where E_1 is the value of the firm's shares of common stock, while the value V_2 of the levered firm is the sum of the value E_2 of its equity and the value of its outstanding bonds D_2 or $V_2 = E_2 + D_2$. The gross (nominal) interest rate $r(\geq 1)$ on bonds already issued is assumed to be a constant. If gross earnings X are less than the interest plus the repayment of the borrowing, the earnings accrue to bondholders. The firms' investments are assumed to be fixed so that the effects of financing decisions may be isolated. In the absence of transactions costs and taxes, the values of both firms may be shown to be equal, independent of the capital structure of the firm, if investors and the firms may borrow at the same rate r or if all equity owners of the levered firm hold bonds with interest rate r of firms in the same risk class. The only restriction placed on investors' utility functions is that they be increasing in the return from investment.

To show that the values of the levered and unlevered firms are equal, first assume that the value V_1 of the unlevered firm exceeds the value V_2 of the levered firm and that an investor holds an α proportion of the equity $V_1 = E_1$ of the unlevered firm. The return Y_1 on the investment αV_1 is

$$(1) \qquad Y_1 = \begin{cases} 0 & \text{if } X \leq 0 \\ \alpha X & \text{if } X > 0 \end{cases}$$

If the investor sells his equity αV_1 in the unlevered firm and invests the proceeds in the equity and bonds of the levered firm in proportions E_2/V_2 and D_2/V_2, respectively, the return Y_2 from the levered firm is[3]

$$(2) \qquad Y_2 = \begin{cases} 0 & \text{if } X \leq 0 \\ \alpha(V_1/V_2)X & \text{if } X > 0 \end{cases}$$

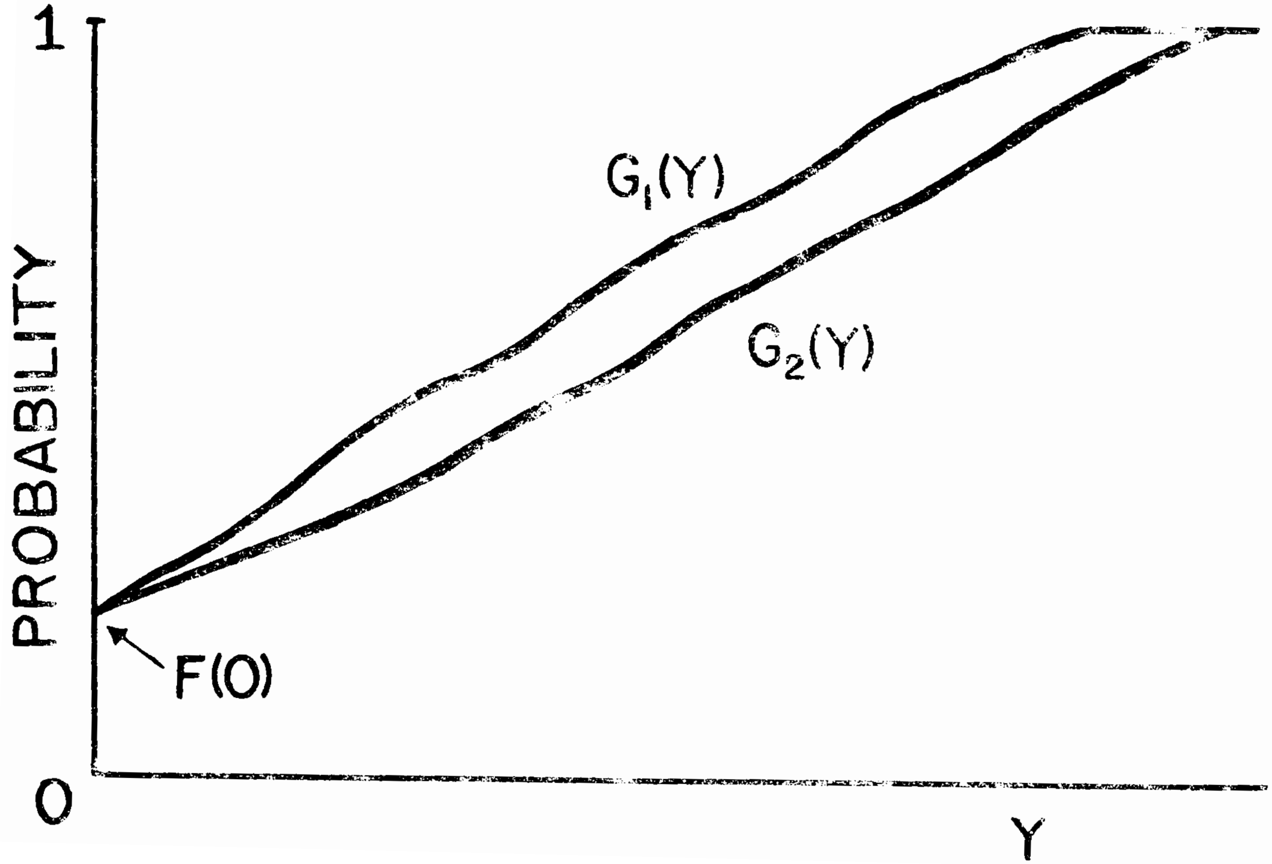
Let the distribution functions of Y_1 and Y_2 be denoted by G_1 and G_2. respectively. If the firms become bankrupt or have zero earnings, the investor's return on either investment is zero, and hence, the probabilities $G_1(0)$ and $G_2(0)$ of no return are equal. With positive earnings and $V_1 > V_2$, the probability of obtaining less than some return Y is less with G_2 than with G_1 for all $Y > 0$ $[G_2(Y) = F((Y/\alpha)(V_2/V_1)) < G_1(Y) = F(Y/\alpha)]$, so the distribution G_2 dominates the distribution of G_1 for the class of increasing utility functions[4] as indicated in Figure 1. If the value of the unlevered firm exceeds that of the levered firm, all investors with increasing utility functions prefer to sell their securities in the unlevered firm and purchase equity and bonds in the levered firm. The value V_2 of the levered firm will be bid up until it is at least as great as the value of the unlevered firm.

[Insert Figure 1 here.]

The rest of this paper deals with the issue of whether or not the value of the levered firm may exceed the value of the unlevered firm. To demonstrate that the values will be equal when the default risk is positive, assume that $V_2 > V_1$ and that all equity investors in the levered firm also hold bonds with a nominal interest rate r of firms in the same risk class as the firms in question. If an investor holds an α fraction of the equity of the levered firm and α^*D_2 bonds, the return on the investment in the levered firm is

$$(3) \qquad Y_2 = \begin{cases} 0 & \text{if } X \leq 0 \\ \alpha^* X & \text{if } 0 < X \leq rD_2 \\ \alpha X + (\alpha^* - \alpha) rD_2 & \text{if } X > rD_2 \end{cases}$$

FIGURE 1



Let $\alpha^o = \min\{\alpha, \alpha^*\}$, and consider selling $\alpha^o(E_2 + D_2) = \alpha^o V_2$ of the investment in the levered firm and buying an $(\alpha^o V_2/V_1)$ portion of the unlevered firm. The return Y_1^o on the investments in firms two and one then is

$$(4) \qquad Y_1^o = \begin{cases} 0 & \text{if } X \leq 0 \\ \alpha^* X + \alpha^o X((V_2/V_1)-1) & \text{if } 0 < X \leq rD_2 \\ \alpha X + (\alpha^*-\alpha)rD_2 + \alpha^o X((V_2/V_1)-1) & \text{if } X > rD_2 \end{cases}$$

The probabilities $G_1(0)$ and $G_2(0)$ of no return are equal, but if $V_2 > V_1$ and earnings are positive, $G_1(Y) < G_2(Y)$ for $Y > 0$, since investment in the unlevered firm yields a greater share of the earnings than does investment in the bonds and shares of the unlevered firm. The distribution of Y_1 dominates that for Y_2 for the class of increasing utility functions, so all equity investors holding bonds of firms in the same risk class prefer to sell bonds and shares of the levered firm and to purchase shares in the unlevered firm if $V_2 > V_1$. If all equity investors hold some bonds of firms in the same risk class, then $V_2 \not> V_1$, and the value of the levered and the unlevered firms will be equal independent of their financing.

If all equity investors in the levered firm do not hold bonds, the M-M theorem is valid given a positive probability of bankruptcy if investors may borrow at the same nominal interest rate as the levered firm. Assume that the value V_2 of the levered firm is greater than the value V_1 of the unlevered firm and that an investor owns an α fraction of the equity of the levered firm and no bonds of firms in that risk class. The total return Y_2 is

$$(5) \qquad Y_2 = \begin{cases} 0 & \text{if } X \leq rD_2 = X_2^* \\ \alpha(X-rD_2) & \text{if } X > X_2^* \end{cases}$$

If the investor sells his equity αE_2 of firm two, borrows an amount αD_2 at an interest rate r, and invests $\alpha(E_2+D_2)$ in the equity of the unlevered firm, the investor owns an $\alpha(E_2+D_2)/E_1 = \alpha V_2/V_1$ fraction of the equity of the unlevered firm.[5] Assuming that the investor's equity value αV_2 in the unlevered firm may be used as the sole collateral[6] for the personal borrowing αD_2, the investor's total return Y_1 from the unlevered firm is

$$(6) \qquad Y_1 = \begin{cases} 0 & \text{if } X \leq rD_2(V_1/V_2) = X_1^* \\ \alpha((V_2/V_1)X - rD_2) & \text{if } X > X_1^* \end{cases}$$

If $V_2/V_1 > 1$, then $X_1^* < X_2^*$, and the probability $G_1(0) = F(X_1^*)$ of no return from the holdings of shares of the unlevered firm is less than the probability $G_2(0) = F(X_2^*)$ of no earnings from holding shares in the levered firm. For $Y > 0$, $G_1(Y) = F((V_1/V_2)(Y/\alpha+rD_2)) < G_2(Y) = F(Y/\alpha+rD_2)$ if $V_2 > V_1$, since the investor is able to purchase a larger share of the earnings of the unlevered firm. The distribution function G_1 dominates G_2 for the class of increasing utility functions, so all investors prefer to sell shares of the levered firm, to create homemade leverage by borrowing at an interest rate r, and to invest in the shares of the unlevered firm. Consequently, the values of the levered and unlevered firms are equal, and the cost of capital is independent of the debt-equity ratio.

The above arguments demonstrate that the M-M theorem holds if the probability of firm bankruptcy is positive if all equity investors in the levered firm hold bonds yielding r of firms in the same risk class or if they may borrow at the same rate as firms and the borrowing is secured solely by the shares of the unlevered firm.[7] This result is in contrast to Smith's Theorem 2 if all investors hold bonds yielding r of firms in the same risk class. The result is equivalent to Stiglitz's extension (Sec. III, p. 788) of the M-M theorem and to Smith's Theorem 3 for the case in which the investor may borrow at the same rate as firms. Risk aversion is not assumed here, however.

Stiglitz's extension is based on the assumption that if an investor buys, on margin, shares in the unlevered firm using the shares as collateral he can borrow at the same interest rate r as the levered firm. Smith has argued that margin borrowing will be possible only at a higher interest rate than that at which the firm borrows. To show that the investor can borrow at a rate r using the shares in the unlevered firm as collateral, consider a lender[8] with initial wealth W* who invests αD_2 in bonds of the levered firm. His contingent terminal wealth is

$$W = \begin{cases} W^* - \alpha D_2 & \text{if } X \leq 0 \\ W^* - \alpha D_2 + \alpha X & \text{if } 0 < X \leq rD_2 \\ W^* + \alpha D_2(r-1) & \text{if } X > rD_2 \end{cases} \tag{7}$$

Next suppose that the lender lends αD_2 to the investor who invests $\alpha(D_2+E_2)$ in the unlevered firm and pledges the shares as collateral. The collateral arrangement is such that if earnings X are insufficient to cover the debt obligation $\alpha D_2 r$ the return $\alpha(V_2/V_1)X$ (for $X > 0$) goes to the lender. The lender's contingent wealth W' then is

$$W' = \begin{cases} W^* - \alpha D_2 & \text{if } X \leq 0 \\ W^* - \alpha D_2 + \alpha(V_2/V_1)X & \text{if } 0 < X \leq rD_2 \\ W^* + \alpha D_2(r-1) & \text{if } X > rD_2 \end{cases} \tag{8}$$

If $V_2 > V_1$, the lender prefers to lend to the investor rather than hold bonds of the levered firm, since the distribution of W' dominates the distribution of W for the class of increasing utility functions. For $V_2 = V_1$, W and W' are identical, so the investor is able to borrow at the same nominal interest rate as the firm if the above collateral arrangement may be made.[9] Consequently, if $V_2 > V_1$ and the margin permitted is at least D_2/V_2, investors will create homemade leverage by borrowing at a rate r and will purchase shares in the unlevered firm rather than hold shares of the levered firm. This process will equate the values of the firms.

II. Differences in Firm and Investor Nominal Borrowing Rates

If investors are not able to borrow the proportion D_2/V_2 of the investment in the unlevered firm because of margin restrictions, limitations on short sales, or market imperfections, for example, the investor seeking to create homemade leverage will be required to secure personal borrowing with collateral or to pay a higher nominal interest rate than r. The nominal interest rate r^* at which an investor may borrow to create homemade leverage depends on the collateral and may differ from the rate r for the firm. Personal borrowing creates a new security, and the implications of this new security for the M-M theorem are investigated in this section.

Suppose that collateral αC, satisfying $0 \leq \alpha C \leq \alpha r^* D_2$, is pledged against the amount borrowed αD_2, where the amount C may be determined by a specific collateral arrangement or by personal bankruptcy laws.[10] If earnings plus the collateral are insufficient to cover the debt obligations ($X \leq (r^* D_2 - C)(V_1/V_2)$), the investor is assumed to pay the lender $(\alpha(V_2/V_1)X + \alpha C)$ (with probability one). If $X > (V_1/V_2)(r^* D_2 - C)$, the return plus the collateral covers the debt obligations. A lender's contingent wealth W_1' with such a loan is

$$(9) \qquad W_1' = \begin{cases} W^* - \alpha D_2 + \alpha C & \text{if } X \leq 0 \\ W^* - \alpha D_2 + \alpha(C+(V_2/V_1)X) & \text{if } 0 < X \leq (V_1/V_2)(r^*D_2-C) \\ W^* + \alpha D_2(r^*-1) & \text{if } X > (V_1/V_2)(r^*D_2-C) \end{cases}$$

If $r^* \geq r$, $V_2 > V_1$, and $C > 0$, $W_1' > W$ (in (7)) for all $X < r^*D_2$ and $W_1' = W$ for all $X \geq r^*D_2$. The distribution of W_1' dominates the distribution of W, so all lenders with increasing utility functions prefer to lend to the investor rather than hold bonds in the levered firm.

If the investor sells his equity αE_2 in the levered firm, borrows αD_2 at r^* by pledging collateral αC under the above terms, and invests $\alpha(E_2 + D_2)$ in shares of the unlevered firm, the return Y_1^* is

$$(10) \qquad Y_1^* = \begin{cases} -\alpha C & \text{if } X \leq (V_1/V_2)(r^*D_2-C) \\ \alpha((V_2/V_1)X-r^*D_2) & \text{if } X > (V_1/V_2)(r^*D_2-C). \end{cases}$$

For any nominal interest rate r^* the distribution of Y_1^* does not dominate, for the class of increasing utility functions, the distribution of the return Y_2 in (5) on equity ownership in the levered firm. Consequently, the value of the levered firm may be greater than the value of the unlevered firm, since all equity owners of the levered firm may not prefer to create homemade leverage and switch to equity ownership in the unlevered firm.

If all lenders have concave utility functions, a further condition on the nominal interest rate r^* may be obtained using the concept of dominance for the class of concave utility functions. For $r^* < r$ and for $X <(>)\ X^* = (V_1/V_2)(r^*D_2-C)$ the probability $H(W^o)$ that $W \leq W^o$ is greater (less) than the probability $H_1(W^o)$ that $W_1' \leq W^o$ for all $W^o <(>)$ $W + \alpha D_2(r^*-1)$[11] as indicated in Figure 2. Given this property, Hanoch and Levy (Theorem 3) have shown that all lenders with concave utility functions prefer the distribution of W_1' to W if and only if $E(W_1') - E(W) \geq 0$, where E denotes expectation. All lenders thus prefer to lend to the investor rather than hold the bonds of the levered firm if and only if[12]

$$(11) \quad E(W_1') - E(W) = \alpha CF(X^*) + \int_0^{X^*} \alpha X(V_2/V_1 - 1)dF(X) - \int_{X^*}^{rD_2} \alpha XdF(X)$$

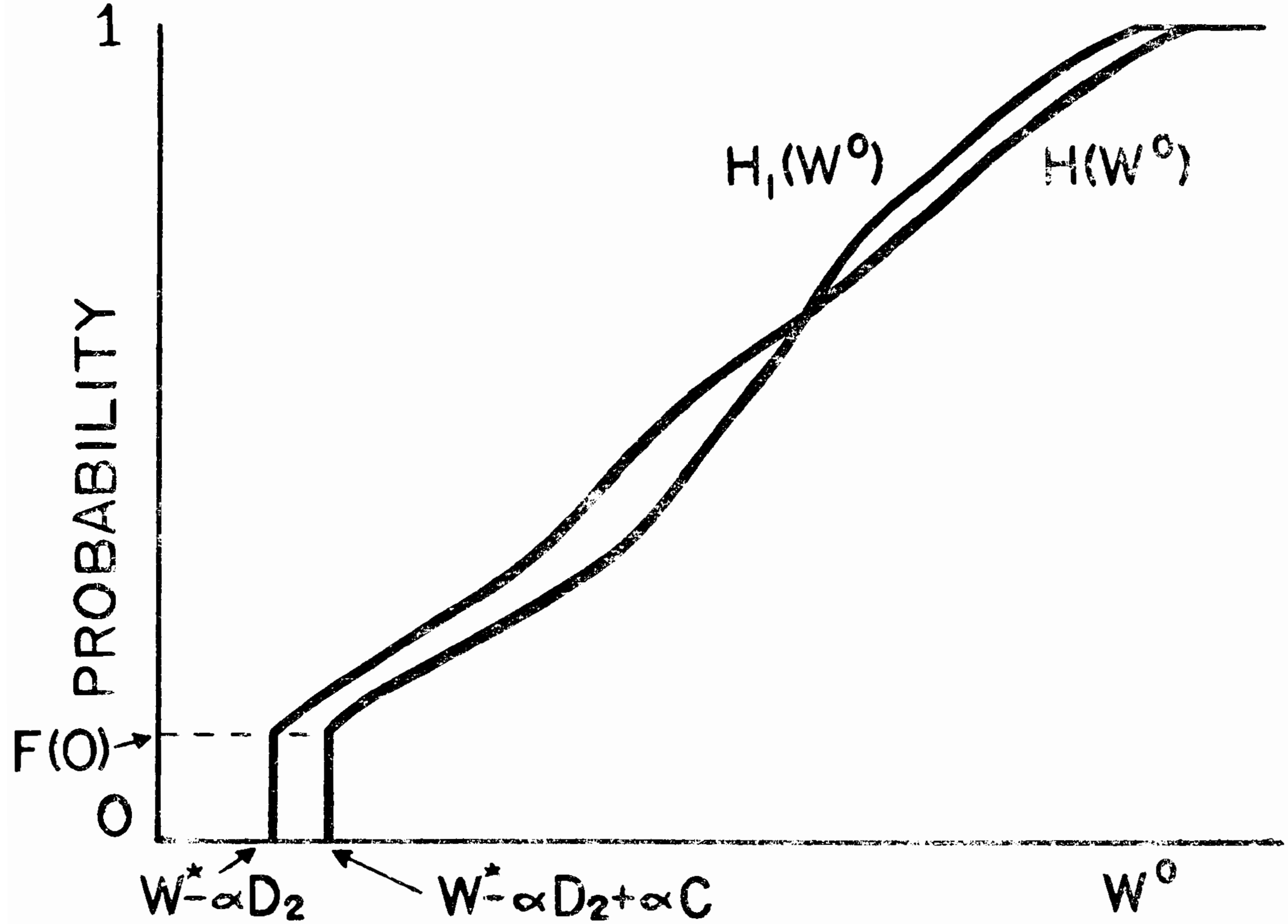
$$+ \alpha D_2(r^*(1-F(X^*)) - r(1-F(rD_2))) \geq 0.$$

Let r^{**}[13] be defined as the value of r^* such that $E(W_1') - E(W) = 0$. If F is absolutely continuous, $\frac{\partial[E(W_1')-E(W)]}{\partial r^*} = \alpha D_2(1-F(X^*)) > 0$, so for $r^* > r^{**}$ the difference in the expected wealth levels is positive.[14] The interpretation of this result is that the distribution of W_1' is "less risky" than the distribution of W if $r^* \geq r^{**}$, and hence, all risk averse investors prefer H_1 to H. Consequently, if the expected return from lending to investors seeking to create homemade leverage is at least as great as that from holding bonds, all risk averse lenders prefer to make secured loans at a nominal interest rate of r^{**} or higher rather than to hold bonds yielding r. To determine if there is any demand for secured loans at a nominal interest rate r^*, the investor's opportunities must be investigated.

[Insert Figure 2 here.]

If all investors have concave utility functions, all investors may prefer the distribution of Y_2 in (5) to that of Y_1^* in (10), since the distributions of Y_1^* and Y_2 satisfy the conditions of Hanoch and Levy's Theorem 3. All risk averse investors prefer Y_2 to Y_1^* if and only if $E(Y_2) - E(Y_1^*) \geq 0$. Evaluating the difference in the expected returns yields[15]

FIGURE 2



$$(12)\ E(Y_2) - E(Y_1^*) = \alpha CF(X^*) - \alpha \int_{X^*}^{rD_2} (V_2/V_1)XdF(X) + \int_{rD_2}^{\infty} X(1-V_2/V_1)dF(X)$$

$$+ \alpha D_2(r^*(1-F(X^*)) - r(1-F(rD_2))),$$

where $X^* = (V_1/V_2)(r^*D_2-C)$. If $E(Y_2) - E(Y_1^*) \geq 0$, all investors with concave utility functions prefer to hold their equity in the levered firm rather than to create homemade leverage and switch to equity ownership in the unlevered firm.

To investigate firm valuation when collateral is pledged to create homemade leverage, one must determine if investors holding equity in the levered firm will wish to borrow at r^* and lenders will wish to lend at r^*. All investors and all lenders are assumed to have concave utility functions. The differences in expected earnings opportunities for investors and expected wealth opportunities for lenders are related by $E(Y_2) - E(Y_1^*) = E(W_1') - E(W) + \alpha(1-V_2/V_1) \int_0^{\infty} XdF(X)$. If $V_2 = V_1$ and $r^* = r^{**}$, $E(Y_2) - E(Y_1^*) = E(W_1') - E(W) = 0$. The derivative with respect to r^* of the differences is

$$(13)\quad \left.\frac{\partial[E(Y_2)-E(Y_1^*)]}{\partial r^*}\right|_{V_2=V_1} = \left.\frac{\partial[E(W_1')-E(W)]}{\partial r^*}\right|_{V_2=V_1} = \alpha D_2(1-F(X^*)) > 0.$$

All holders of bonds in the levered firm prefer to lend to equity owners of the levered firm at interest rates $r^* \geq r^{**}$ rather than hold the bonds of the levered firm, but none of those equity owners prefer to borrow, sell their equity in the levered firm, and purchase equity in the unlevered firm. For $r^* < r^{**}$ and $V_2 = V_1$, $E(Y_2)-E(Y_1^*) = E(W_1') - E(W) < 0$. All equity owners of the levered firm may not prefer to hold their equity, and some may prefer to borrow at $r^* < r^{**}$ to create homemade leverage and switch to ownership in the unlevered firm. For $r^* < r^{**}$ all lenders may not prefer to lend to the equity owners in the levered firm and hence some may prefer to hold bonds in the levered firm.

Alternatively, if $r^* = r^{**}$ with r^{**} defined at $V_2 = V_1$, a greater value V_2 increases $E(W_1') - E(W)$ and decreases $E(Y_2) - E(Y_1^*)$, since for all r^*

$$\left.\frac{\partial[E(W_1')-E(W)]}{\partial V_2}\right|_{V_2=V_1} = \int_0^{X^*} (\alpha/V_1)XdF(X) > 0$$

(14)

$$\left.\frac{\partial[E(Y_2)-E(Y_1^*)]}{\partial V_2}\right|_{V_2=V_1} = -(\alpha/V_1)\int_{X^*}^{\infty} XdF(X) < 0.$$

For $V_2 > V_1$ and $r^* = r^{**}$ (defined at $V_2 = V_1$) all bondholders with concave utility functions prefer to lend to equity holders in the levered firm, and there may be some equity owners in the levered firm who prefer to borrow to create homemade leverage, sell their equity, and purchase equity in the unlevered firm. The same may occur for certain nominal interest rates $r^* > r^{**}$ if $V_2 > V_1$. This process may stop with the value of the levered firm greater than the value of the unlevered firm if investors and lenders may have any concave utility functions. Determination of the equilibrium values requires further assumptions regarding utility functions and/or probability distributions.

As an example, assume that all investors and lenders are risk neutral. If $V_2 = V_1$ and $r^* = r^{**}$, no equity holder in the levered firm prefers to sell his equity, borrow, and switch to the unlevered firm, and no bondholder in the levered firm prefers to lend to equity holders in the levered firm. If $V_2 = V_1$ and $r^* <(>)\ r^{**}$, then all (no) risk neutral lenders prefer to hold bonds in the levered firm and all (no) risk neutral equity owners in the levered firm prefer to borrow to create their own leverage. Consequently, if $V_2 = V_1$ no transactions will take place and $V_2 = V_1$ and $r^* = r^{**}$ yields an equilibrium. If $V_2 > V_1$, then there exists an $r^* > r^{**}$, r^{**} defined for the particular V_2 and V_1 in question, such that $E(W_1') - E(W) > 0$ and $E(Y_2) - E(Y_1^*) = E(W_1') - E(W) + \alpha(1-V_2/V_1)\int_0^\infty XdF(X) < 0$. For example, at $r^* = r^{**}$ and $V_2 > V_1$, $E(W_1') - E(W) = 0$ and $E(Y_2) - E(Y_1^*) = \alpha(1-V_2/V_1)\int_0^\infty XdF(X) < 0$. Since $E(W_1') - E(W)$ is continuous in r^*, an $r^* > r^{**}$ may be found such that all bondholders prefer to lend rather than to hold bonds of the levered firm and all investors prefer to create their own leverage and switch from equity holding in the levered firm to equity holdings in the unlevered firm. This process continues until $V_2 = V_1$ and $r^* = r^{**}$.

If some or all investors or lenders are risk averse, the process may terminate with the value of the levered firm being greater than the value of the unlevered firm. Some risk averse investors may prefer to be protected by the limited liability of the levered firm and will not create homemade leverage by pledging collateral in order to purchase equity in the unlevered firm. Furthermore, some holders of the bonds of the levered firm may not prefer to lend at a low enough interest rate to generate sufficient sales of the shares of the levered firm and purchases of the shares of the unlevered firm to equate the values of the firms.

III. Conclusions

Using stochastic dominance arguments, the M-M theorem has been shown to be valid if all investors are able to borrow at the same nominal interest rate as firms by pledging the securities in the unlevered firm as collateral or if investors in the equity of levered firms also hold bonds yielding r in firms of the same risk class. If because of margin restrictions, for example, the investor is required to pledge additional collateral, the default risk to the lender is reduced, and the nominal interest rate paid by the investor may be less than the nominal interest rate paid by the levered firm. The values of the levered and unlevered firms will be equal if all economic agents are risk neutral, but if investors and lenders are risk averse, the value of the levered firm may be greater than the value of the unlevered firm.

Footnotes

1. If X_1 and X_2 denote the gross earnings of two firms, respectively, then both are in the same risk class if $X_1 = kX_2$ with probability one. To simplify the notation, assume $k = 1$, and let $X_1 = X_2 = X$.

2. This representation of the earnings of firms is discussed by Modigliani and Miller and by Jan Mossin (p.65).

3. For $X > rD_2$, $Y_2 = \alpha V_1(E_2/V_2)\ (X-rD_2)/E_2 + \alpha V_1(D_2/V_2)\ (rD_2/D_2) = \alpha(V_1/V_2)X$. For $0 < X \leq rD_2$ the profit X accrues to the bondholders, and the investor's share is $\alpha V_1(D_2/V_2)$, so $Y_2 = \alpha V_1(D_2/V_2)X/D_2 = \alpha(V_1/V_2)X$ if $X > 0$.

4. Hadar and Russell (1971, Theorem 4) prove that if $(V_1/V_2) > 1$ and $Y_1 \geq 0$, the distribution of $Y_2 = (V_1/V_2)Y_1$ dominates that for Y_1 for the class of increasing utility functions.

5. The margin requirement necessary to permit sufficient borrowing is (D_2/V_2).

6. That is, if the investor defaults, he forfeits his shares of the unlevered firm and none of his other assets.

7. Stiglitz indicates that the M-M theorem holds if instead of personal borrowing the investor may sell short the shares of the levered firm pledging the new shares of the unlevered firm as the sole collateral.

8. The distinction between lenders and investors is only made for expository purposes. Any lender may also hold shares of the firms.

9. The same result obtains if both firms are levered and the investor invests an (E_1/V_1) fraction in shares and a (D_1/V_1) fraction in bonds.

10. An analysis analogous to that in this section can be made for the case in which no collateral is pledged and the individual borrows at a nominal interest rate greater than r.

11. This condition implies that the distribution functions of W and W_1' cross only once.

12. The condition in (11) holds for $X^* \leq rD_2$ (equivalently, $r^* \leq (V_2/V_1)r+C/D_2$). If $X^* > rD_2$, then $r^* > r$ and the distribution of W_1' dominates the distribution of W for the class of increasing utility functions implying that $E(W_1') - E(W) > 0$.

13. The nominal interest rate r^{**} is a function of $(V_2, V_1, C, r, F, D_2, \alpha)$.

14. The interest rate r^{**} is less than r, since at $V_2 = V_1$ and $r^{**} \geq r$, $E(W_1') - E(W) > 0$, and $E(W_1') - E(W)$ is increasing in r^* and in V_2 as indicated in (14) below.

15. The expression in (12) holds for $r^* \leq (V_2/V_1)r + C/D_2$. If $r^* > (V_2/V_1)r + C/D_2$, a similar expression can be determined.

References

J. Hadar and W. R. Russell, "Rules for Ordering Uncertain Prospects," Amer. Econ. Rev., 59 (1969), 25-34.

J. Hadar and W. R. Russell, "Stochastic Dominance and Diversification," J. Econ. Theory, 3 (1971), 288-305.

G. Hanoch and H. Levy, "The Efficiency Analysis of Choices Involving Risk," Rev. Econ. Stud., 36 (1969), 335-346.

F. Modigliani and M. H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," Amer. Econ. Rev., 48 (1958), 261-97.

J. Mossin, Theory of Financial Markets, Prentice-Hall, 1973.

V. L. Smith, "Default Risk, Scale, and the Homemade Leverage Theorem," Amer. Econ. Rev., 62 (1972), 66-76.

J. E. Stiglitz, "A Re-Examination of the Modigliani-Miller Theorem," Amer. Econ. Rev., 59 (1969), 784-793.