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ON STOCKHOLDER UNANIMITY TOWARDS CHANGES IN PRODUCTION PLANS

by

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1. Introduction

If a firm is owned by several stockholders each of whom has different risk preferences, then whose risk preferences determine whether the firm accepts or rejects a risky, proposed change in its production plan? It might appear that this is a political problem whose resolution is necessarily through political means such as proxy fights. This appearance, however, is only partially accurate. Arrow [1], Ekern [4], Ekern and Wilson [5], Leland [10], Baron [3], and others have developed a theory showing that if a sufficient variety of securities are traded on the stock market, then conflict among stockholders does not occur. The reason is that if an economy has enough different securities, then in equilibrium each stockholder's preferences towards risk is aligned with every other's stockholder's preferences. Consequently stockholders are unanimous in their evaluations of risky alternatives.

The purpose of this paper is to inquire if any reason exists why one should expect a sufficient variety of securities to be traded on the stockmarket to achieve stockholder unanimity. The answer I propose is of a mixed character. An incentive does exist within an economy to introduce a variety of securities onto the market that is sufficient to assure stockholder unanimity for certain classes of firms' decisions. A decision, for example, to increase production capacity for a currently marketed, successful product falls into this group. But it is impossible for a variety of securities to exist within an economy that is sufficient to assure that stockholder unanimity exists for all classes of firms' decisions. A decision, for example, to be the first firm to invest in a radically different production technology is not assured of unanimity.

This paper is organized as follows. The next section, Section 2, describes the model that is used throughout. It is a standard two-period, one good model of an economy with firms and consumers. The one unusual feature that it contains is a distinction between market states of nature and individualized states of nature. Individualized states are a finer partition of the states of nature than are the market states. In addition to setting up the model, the section includes a substantial discussion of how this formal model relates to reality. The next three sections contain the formal analysis of the model. In Section 3 the idea of spanning is introduced. In Section 4 I show that an incentive exists for entrepreneurs to introduce new securities into the economy as long as the set of market states are not spanned. That such an incentive exists implies that the economy is not in full, long run equilibrium unless the set of market states is spanned.

In Section 5 I show that stockholder unanimity exists towards a change in a firm's production plan if the set of market states is spanned and if the returns that the proposed change in production plan yields are only a function of market states. If the returns of the change depends on the identity of the individualized states as well as the market states, then stockholder unanimity is not assured. Section 6, the last section, summarizes the results: spanning of the market states is a likely occurrence within an economy and, consequently, stockholder unanimity is likely for certain classes of decisions. Based on these formally derived results, the section concludes with an informal characterization of the types of decisions for which stockholder unanimity is likely and the types for which it is unlikely.

2. The Model

Consider a one consumption good, two period economy with $\mathcal{I} = \{1,\ldots,I\}$ consumers, $\mathcal{F} = \{1,\ldots,F\}$ firms, and $\mathcal{K} = \{1,\ldots,K\}$ states of nature.[1] Let firms be large with ownership separated from management. Assume that managers act in the interests of stockholders whenever stockholders' interests are well defined.[2] In the first period individuals and firms are ignorant of the state of nature. At the beginning of the second period they learn either the true state of nature or some subset of $\mathcal{K}$ within which the true state is contained. Where convenient and where confusion is unlikely the first period is referred to as now and the second period as then. Transaction costs are assumed to be zero.

In the first period individuals may trade their initial endowments of the single consumption good and the firms' stocks. In the second period no trade takes place. Each individual consumes his second period consumption good endowment and the returns on his stockholdings. The endowment of consumption good individual i receives now is $\omega_1^i > 0$ and the endowment he receives then if state $k \in \mathcal{K}$ occurs is $\omega_2^i(k) > 0$. His consumption of the consumption good is x_1^i now and $x_2^i(k)$ then if state k occurs. The consumption good can not be carried over from now to then. The endowment of firm f's stock that individual i receives now is $\bar{z}_f^i$. Adapt the convention that, for all $f \in \mathcal{F}$, $\Sigma_{\mathcal{J}} \bar{z}_f^i = 1$. Individual i's holding of firm f's stock now after all trades are complete is z_f^i. A negative z_f^i is permissible and corresponds to a short sale of the stock. An individual i is called an initial stockholder of firm f if $\bar{z}_f^i > 0$ and a final stockholder if $z_f^i > 0$.

Each firm f has a production plan that specifies what actions it plans to take both now and, contingent on the state of nature, then. Given its production plan, firm f pays a return then of $a_f(k)$ units of consumption good if state k occurs. Thus at the beginning of the second period, a final stockholder i receives $z_f^i\ a_f(k)$ of consumption good if state k occurs. He pays $z_f^i\ a_f(k)$ of consumption good of he is a short seller of firm f.

Let the price now of firm f's stock be p_f. Every individual i is a price taker and picks his vector $z^i = [z_1^i,\ldots,z_F^i]$ of stockholdings and vector $x^i = [x_1^i, x_2^i(1),\ldots,x_2^i(K)]$ of consumption so as to maximize his utility

$$U^i[x_1^i,x_2^i(1),\ldots,x_2^i(K)] \tag{1}$$

subject to the K + 1 budget constraints:

$$x_1^i + \Sigma_{\mathcal{F}}\, z_f^i\, p_f \leq \omega_1^i + \Sigma_{\mathcal{F}}\, \bar{z}_f^i\, p_f \;;$$
$$x_2^i(k) \leq \omega_2^i(k) + \Sigma_{\mathcal{F}}\, z_f^i\, a_f(k) \;;\quad k = 1,\ldots,K.$$

U^i is i's strictly monotonic, continuously differentiable utility function. I assume that for any set of strictly positive prices $p = \{p_1,\ldots,p_F\}$ the maximizing consumption bundle $x^i = \{x_1^i,x_2^i(1),\ldots,x_2^i(K)\}$ has strictly positive components.

Let the only securities traded be the stocks of the F firms. A vector of prices $p = (p_1,\ldots,p_F)$, a vector of consumption plans $x = (x^1,\ldots,x^I)$, and a vector of stockholdings $z = (z^1,\ldots,z^I)$ is a $\mathcal{F}$-equilibrium if (a) for all $i \in \mathcal{I}$, the plan (x^i,z^i) maximizes i's utility given the budget constraints and (b) the market for each firm's stock clears: $\Sigma_{\mathcal{I}}\, z_f^i = 1$ for all $f \in \mathcal{F}$. I defer for the moment defining equilibrium when the set of securities traded is not fixed with the set $\mathcal{F}$.

Let each state of nature $k \in \mathcal{K}$ be represented as an ordered pair k = (st) where $s \in \mathcal{S} = \{1,\ldots,S\}$, $t \in \mathcal{T} = \{1,\ldots,T\}$, and $k = (s-1)T + t$. The first component, s, taken by itself is called the market state. Arrow and Lind [2] introduced this distinction and Malinvaud [11,12] has analyzed its implications for the optimal sharing of individual risks through insurance.

A market state $s' \in \mathcal{S}$ identifies the state of the world in sufficient detail to determine how much each firm, given its present production plan, will earn in period two if an individualized st occurs such that $s = s'$. Market states are observable by both firms and investors. The information that distinguishes different market states are the values of variables that are observable and verifiable to all investors. This information includes macro variables such as the money supply, OPEC's pricing policies, and Congressional action on regulatory policy. It also includes micro variables such as consumer demand for a new product that some firm f will introduce in period two as part of its production plan. This last example is observable since at the end of period two consumers will have revealed their preferences towards the product, which in turn affect the firm's reported earnings. Therefore, given firm f's current production plan, both managers and investors are able to estimate what the firm's returns will be conditional upon the market state s. This means that a firm's returns a_f may be written as $a_f(s\,1) = a_f(s\,2) = \ldots = a_f(sT) \equiv a_f(s)$ where $a_f(s)$ is introduced as a convenient notation.

Individualized states are not fully observable by either managers or investors. They are distinguished one from another by very detailed information about what is technically feasible and what are each individual's circumstances, talents, preferences, etc. Three types of information contained by individualized states are particularly important for this paper's purposes. First, individualized states contain information about how individual i's endowment varies with the individualized state. For example, if $k' = s't'$ is a state where individual i gets promoted and state $k'' = s't''$ is a state where individual i does not get promoted, then $x_2^i(k') > x_2^i(k'')$. Yet $a^f(k') = a^f(k'') \equiv a^f(s')$ because whether it is person i or person j who gets promoted is of no appreciable consequence to the earnings of any firm. Second, individualized states contain information about how individual consumers would react to radically different products that no firm is offering now or, given its current production plan, is planning to offer on the market during the second period. Notice that if two individualized states are differentiated from each other solely by different consumer reactions to a product that is not marketed currently and is not scheduled for marketing in the second period, then they are indistinguishable unless some firm should change its production plan and schedule the product's introduction for the second period.[3] Third, the individualized states contain information about the feasibility and costliness of technologies that no firm is either currently using or planning to use in the second period. Exactly as in the case of the unmarketed product, if no firm changes its production plan to include a test of the untried technology, then an individualized state where that technology is feasible and profitable is indistinguishable from a state where that technology is a failure.

2. Equilibrium, Implicit Prices, and Spanning

Let $\mathcal{L}$ be the LaGrange expression formed from i's maximization problem (1). Given a set of prices $p = (p_1,\dots,p_F)$, the first order conditions for i's stockholdings $z^i = (z_1^i,\dots,z_F^i)$ and consumption plan $x^i = \{x_1^i, x_2^i(11),\dots,x_2^i(ST)\}$ to be maximal are:

$$\frac{\partial \mathcal{L}}{\partial x_1^i} = \frac{\partial U^i[x^i]}{\partial x_1^i} - \lambda^i = 0; \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial x_2^i(st)} = \frac{\partial U^i[x^i]}{\partial x_2^i(st)} - \delta_{st}^i = 0, \tag{3}$$

$$s = 1,\dots,S \text{ and } t = 1,\dots,T;$$

$$\frac{\partial \mathcal{L}}{\partial z_f^i} = -\lambda^i p_f + \sum_{\mathcal{S}} \sum_{\mathcal{T}} \delta_{st}^i a_f(s) = 0, \tag{4}$$

$$f = 1,\dots,F.$$

The ST + 1 LaGrange multipliers λ^i and δ^i_{st} have the usual interpretations: λ^i is the marginal utility for i of a unit of consumption good now and δ^i_{st} is the marginal utility for i of a unit of consumption good then if state st occurs. Define $\psi^i_{st} = \delta^i_{st}/\lambda^i$ to be i's implicit price now for consumption good then if state st occurs. It is how much consumption good now i is willing to pay for one unit of consumption good then if state st occurs.

For each individual i, the F equations (4) may be rewritten as:

$$\begin{aligned}
\Sigma_{\mathcal{T}}\, \psi^i_{1t}\, a_1(1) + \Sigma_{\mathcal{T}}\, \psi^i_{2t}\, a_1(2) + \ldots + \Sigma_{\mathcal{T}}\, \psi^i_{St}\, a_1(S) &= p_1 \\
\Sigma_{\mathcal{T}}\, \psi^i_{1t}\, a_2(1) + \Sigma_{\mathcal{T}}\, \psi^i_{2t}\, a_2(2) + \ldots + \Sigma_{\mathcal{T}}\, \psi^i_{St}\, a_2(S) &= p_2 \\
&\;\;\ldots \\
\Sigma_{\mathcal{T}}\, \psi^i_{1t}\, a_F(1) + \Sigma_{\mathcal{T}}\, \psi^i_{2t}\, a_F(2) + \ldots + \Sigma_{\mathcal{T}}\, \psi^i_{St}\, a_F(S) &= p_F \, .
\end{aligned} \tag{5}$$

Define $\psi^i_s = \Sigma_{\mathcal{T}}\, \psi^i_{st}$ to be i's implicit price now for consumption then if market state s occurs. It, analogous to ψ^i_{st}, is how much i is willing to pay now for one unit of consumption then if market state s occurs. System (5) may be re-written in matrix form:

$$A\, \psi^i_{\mathcal{S}} = p' \tag{6}$$

where

$$A = \begin{bmatrix} a_1(1) & a_1(2) & \ldots & a_1(S) \\ a_2(1) & a_2(2) & \ldots & a_2(S) \\ \ldots & & \ldots & \ldots \\ a_F(1) & a_F(2) & \ldots & a_F(S) \end{bmatrix}, \quad \psi^i_{\mathcal{S}} = \begin{bmatrix} \psi^i_1 \\ \psi^i_2 \\ \ldots \\ \psi^i_S \end{bmatrix}, \quad p' = \begin{bmatrix} p_1 \\ p_2 \\ \ldots \\ p_F \end{bmatrix} .$$

The set of market states $\mathcal{S} = \{1,\ldots,S\}$ is <u>spanned</u> if rank A = S. In other words, the set of market states is spanned if there are at least as many firms with independent return vectors a_f as market states of nature.

Given the returns matrix A and the price vector p', equation (6) may be interpreted as an equilibrium restriction on each individual's implicit prices for market states. The assumption that each individual's utility function is strictly monotonic implies that all implicit prices ψ^i_{st} and ψ^i_s are strictly positive. Therefore the set of column vector y that satisfy (8) in an admissible manner is $\Omega^{S-F} = \{y | y \in R^S_+ \text{ and } Ay = p'\}$ where R^S_+ is the S-dimensional nonnegative orthant. This is a convex subset of R^S_+ with dimensionality S-F that consists of a unique point only if rank A = S, i.e. all individuals necessarily have identical implicit prices for each market state, s, only if the market states are spanned. If the market states are not spanned, then two individuals i and j who are in equilibrium may have different market state implicit prices $\psi^i_{\mathcal{S}} \neq \psi^j_{\mathcal{S}}$. Note that even if the set of market states is spanned, the implicit prices for individualized states st may not be identical, i.e. there may exist (st) $\in \mathcal{K}$ and i,j $\in \mathcal{J}$ such that $\psi^i_{st} \neq \psi^j_{st}$. Only market completeness, not spanning, guarantees that implicit prices for individualized states as well as market states are identical across individuals. Completeness requires that the number of securities with linearly independent return vectors equal ST, the number of individualized states of the world. Existence of ST linearly independent securities would require that the returns of securities vary not only with respect to the market state, but also with respect to the unobservable individualized states.

4. The Incentive to Achieve Spanning

This section shows that an economy is not in full equilibrium unless the set of market states is spanned. Specifically if the set of market states is not spanned, then entrepreneurs can make a riskless profit by introducing new securities. Therefore, if one is willing to assume that the economy does tend towards equilibrium, then spanning of market states is a consequence of the market process, not an assumption that is sometimes arbitrarily made about the market process.

Permit any individual i to issue a new security labeled g, subject to the requirements that its returns (a) be nonnegative and (b) be a function only of market states and not of individualized states. Thus, exactly as for the stocks of the F firms, $a_g(s) \equiv a_g(s1) = a_g(s2) = \ldots = a_g(sT) \geq 0$ for all $s \in \mathcal{S}$. Issuance of such a security is feasible for an individual because the market states are observable and thus contracts can be made contingent on them.[4]

An $\mathcal{F}$-equilibrium is a full equilibrium only if no individual can make a riskless profit by introducing a new security g onto the market. Thus if a $\mathcal{F}$-equilibrium is a full equilibrium, then each individual i has exhausted his opportunities for maximization with respect to his consumption plan x^i, his trading plan z^i, and the possibilities of introducing new securities. In a $\mathcal{F}$-equilibrium each individual takes the set of securities as given and maximizes only with respect to his plans x^i and z^i. The result I prove is: in the absence of transaction costs, a necessary condition for a $\mathcal{F}$-equilibrium to be a full equilibrium is that the stocks of the F firms span the market states.[5] The only exception to this occurs when the market states are not spanned and, at some $\mathcal{F}$-equilibrium, all individuals by chance have the same implicit prices. This is an unlikely occurrence if individuals have heterogeneous utility functions and endowment streams.

The proof is this. Assume, contrary to the result, that the economy is in full equilibrium but the set of market states is not spanned. Therefore Ω^{S-F} contains a multiplicity of points and, unless individuals have identical utility functions and endowment streams, almost certainly a pair of individuals i,j exist who have unequal implicit prices over market states: $\psi^i_{\mathcal{J}} \neq \psi^j_{\mathcal{J}}$.

Since the vectors $\psi^i_{\mathcal{J}}$ and $\psi^j_{\mathcal{J}}$ are distinct points in R^S_+, they are also disjoint convex sets. Therefore a hyperplane exists that separates them, i.e. a vector $a_g = [a_g(1) \ldots a_g(S)] > 0$ and scalars p_g^+ and p_g^- exist such that either

$$0 < \Sigma_{\mathcal{J}} a_g(s)\psi^i_s < p_g^- < p_g^+ < \Sigma_{\mathcal{J}} a_g(s)\psi^j_s \tag{7}$$

or

$$0 < \Sigma_{\mathcal{J}} a_g(s)\psi^j_s < p_g^- < p_g^+ < \Sigma_{\mathcal{J}} a_g(s)\psi^i_s \quad . \tag{8}$$

All the components of a_g may be chosen to be nonnegative because $\psi^i_{\mathcal{J}}$ and $\psi^j_{\mathcal{J}}$ are single points within R^S_+. Assume without loss of generality that (7) is satisfied. Suppose a third individual $k \in \mathcal{J}$ offers to sell at price p_g^+ and buy at price p_g^- the vector of returns $a_g(s)$. Let this return vector be called security g. Given the offer price of p_g^- individual i wants to sell individual k some quantity of security g because the marginal utility he attaches to the purchase of one unit of g is

$$\lambda^i \Sigma_{\mathcal{J}} \psi^i_s \, a_g(s) - \lambda^i \, p_g^- \; < \; 0 \; . \tag{9}$$

That the left hand side represents i's marginal utility for g follows from (4) and the definition of ψ^i_s. The direction of the inequality follows from (7) because $\lambda^i > 0$. Similarly, individual j wants to buy from individual k some quantity of g at price p_g^+. If k astutely selects the prices p_g^+ and p_g^-, then the quantities that i wants to sell and j wants to buy will be equal and k can make a riskless profit of $y(p_g^+ - p_g^-) > 0$ where y is the positive quantity traded. Therefore, the economy is not in full equilibrium because k has an incentive to introduce a new security. This contradicts the original assumption that the market is in equilibrium and proves the result.

After some entrepreneur has introduced security g, then a new $\mathcal{F}$-equilibrium may be achieved with individuals trading the F stocks plus the new security g.[6] Exactly as before, a necessary condition for this new $\mathcal{F}$-equilibrium to be a full equilibrium is that the F+1 securities being traded span the set of market states. If they do not span the set of market states, then entrepreneurs have an incentive to introduce another security g'. Clearly this process continues until a sufficient variety of securities are traded so that the set of market states is spanned.[7]

5. Spanning and Stockholder Unanimity[8]

The preceding section has shown that the market process does tend to insure that the set of market states is spanned. This section shows that spanning guarantees that for certain well defined types of proposed changes in a firm's production plan stockholders will be either unanimously in favor of the change or unanimously against the change. For other types of proposed changes, however, stockholders will not be unanimous.

Suppose that the set of market states is spanned. Take the endowment streams of consumption goods for individuals $[\omega_1^i, \omega_2^i(11),\ldots,\omega_2^i(ST)]$ as fixed. Also suppose that individuals' initial endowments of stock $[\bar{z}^i_1,\ldots,\bar{z}^i_F]$ are such that together with their consumption good endowments, they constitute an equilibrium allocation, i.e. when the market opens no trades at all take place. Let $\hat{p} = (\hat{p}_1,\ldots,\hat{p}_F)$ be the equilibrium price vector for this initial situation.[9] Because market states are spanned and the economy is in equilibrium, all consumers have identical implicit prices for market states, i.e. $\psi^i_{\mathcal{J}} = \psi^j_{\mathcal{J}}$ for all $i,j \in \mathcal{J}$.

Now suppose firm f considers a change in its production plan such that its vector of returns over market states changes from $a_1 = [a_1(11),\ldots,a_1(st),$ $\ldots,a_1(ST]$ to $a_f + b = [a_f(11) + b(11),\ldots,a_f(ST) + b(ST)]$ where b is a vector in ST-dimensional Euclidean space. Let b have two characteristics. First, let b, as does a_f, depend only on the market state $s \in S$. Thus, for all $s \in S$, $b(s) \equiv b(s1) = b(s2) = \ldots = b(sT)$. Second, let the magnitude of b be small enough compared to the economy as a whole to justify price taking behavior. In particular, assume that every individual takes his equilibrium implicit prices $\psi^i_{\mathcal{S}}$ as invariant with respect to a small change in firm one's production plan. Given these assumptions, all initial stockholders of the firm are unanimous in approving or disapproving this change.

This is seen by picking an arbitrary initial stockholder i and recalling that the LaGrange multiplier δ^i_{st} from that individual's maximization problem (1) represents the marginal utility he obtains from one additional unit of consumption then if state st occurs. Therefore, since b is small and since $\bar{z}^i_f b$ is how much his consumption then will change if the proposed change b is adopted, adoption of b causes i's utility to change by the quantity ΔU^i:

$$\Delta U^i = \bar{z}^i_f \, \Sigma_{\mathcal{S}} \, \Sigma_{\mathcal{T}} \, \delta^i_{st} b(s). \tag{10}$$

Individual i approves of b if $\Delta U^i > 0$. Recall that: $\psi^i_{st} = \delta^i_{st}/\lambda^i$, $\psi^i_s = \Sigma_{\mathcal{T}} \psi^i_{st}$, $\delta^i_{st} > 0$, $\lambda^i_{st} > 0$, and $\psi^i_s > 0$. Therefore

$$\Delta U^i = \lambda^i \, \bar{z}^i_f \, \Sigma_{\mathcal{S}} \, \psi^i_s b(s). \tag{11}$$

The quantity ΔU^i has the same sign for all initial stockholders because $\bar{z}^i_f > 0$ for all stockholders i, $\lambda^i > 0$ for all i, and, since the market states are spanned, $\psi^i_{\mathcal{S}} = \psi^j_{\mathcal{S}}$ for all pairs of individuals $i,j \in \mathcal{I}$. Therefore the stockholders of the firm unanimously approve or disapprove the change. If, however, the set of market states were not spanned, then unanimity would not have been assured because $\psi^{i}_{\mathcal{S}}$ would not necessarily equal $\psi^{j}_{\mathcal{S}}$ for every pair $i,j \in \mathcal{I}$ of initial stockholders.

Crucial to the derivation of stockholder unanimity under spanning is that the components of b only vary with the market states $s \in \mathcal{S}$. If a firm proposes a production change b such that $b(st') \neq b(st'')$ for two individualized states (st'), $(st'') \in \mathcal{X}$, then unanimity is not necessarily achieved because spanning does not guarantee that the implicit prices ψ^{i}_{st} for individualized states are identical across all individuals. Therefore whether stockholder unanimity exists with respect to a proposed change b depends on whether b is a function only of the market states $\mathcal{S}$ or are also functions of the individualized states $\mathcal{X} = \mathcal{S} \times \mathcal{T}$.

6. Conclusions

Formally I have shown two results. First, if the market states are initially not spanned, then in a world of no transaction costs an incentive exists for individual entrepreneurs to introduce new securities in sufficient variety to span the market states. Second, given that the market states are spanned, stockholders of a firm are unanimous concerning the acceptance or rejection of any production plan change whose returns are a function only of market states. Stockholders, however, may not be unanimous concerning production plan changes whose returns vary with the individualized states as well as the market states.[10]

Given the interpretation that I gave market states and individualized states in Section 2, what is an appropriate interpretation of these formal results? In particular, how can firms' decisions for which stockholder unanimity is likely to exist be distinguished from firms' decisions for which stockholder unanimity is not likely to exist? The answer I propose is this. Stockholders are likely to approve or disapprove unanimously routine investment decisions involving (a) expansion or contraction of capacity for existing product lines or (b) the introduction of a new product that is only marginally different from already marketed products. The reason is that the success or failure of such investments is relatively predictable conditional upon the observable variables that define market states. Speaking intuitively, investors are already placing comparable bets on existing production plans of firms; therefore the existing odds apply to the proposed change in production plan.

Stockholders may not be unanimous concerning investment decisions that involve a major commitment to (a) a new, untested technology or (b) a product that is radically different than anything currently on the market. The reason is that the returns of such investments depend on the individualized states as well as the market states. Consider case (a) first. If a firm changes its product plan to include investment in a technology that no other firm is using or is planning to use, then for the first time the feasibility of that technology becomes an additional observable characteristic of the state of nature. Because previously that characteristic had been unobservable, investors had been unable to make bets conditional on it. Therefore no market odds exist by which investors and firm managers can evaluate whether an investment in that technology should be made. Generally, some stockholders will be in favor of the investment and others will be against. Case (b) involving the introduction of a new product is similar except that consumer reaction to the new product is the characteristic of the state of nature that becomes observable for the first time.

In summary, this theoretical discussion suggests two conclusions regarding stockholder unanimity. First, stockholder unanimity is likely for investment decisions such as plant expansion that essentially involve more of the same. Second, decisions that involve substantial innovation such as the implementation of a radical technology are likely to create division among stockholders. This latter conclusion places absolute limits on the extent that the market can mediate among stockholders diverse risk preferences and subjective probabilities. Within a dynamic economy decisions of this latter, nonroutine type are constantly facing firms as new technologies are discovered and new products are conceived. Stockholders inevitably will disagree over which of these ideas are worth substantial investment. It is their very newness that makes it impossible for an established price to exist on the market by which their profitability can be evaluated.

Footnotes

[1]All the conclusions of this paper remain valid in a multiperiod, many good, rational expectations model of the type that, for example, Hart [7] used.

[2]Thus I assume that the moral hazard problem of providing managers with incentives to act in the interest of stockholders has been solved. For a discussion of this problem see, for example, Jensen and Meckling [8].

[3]A product ℓ is radically different if it is so differentiated from currently marketed or planned products that it is impossible to do a Lancasterian [9] analysis of the demand for ℓ's attributes.

[4]The creation of new securities by entrepreneurs is a frequent occurrence in United States' security markets. Two examples from recent history are the creation of options markets for certain common stocks and the creation of new futures markets for some commodities. See Ross [13] for an analysis of how the creation of an options market increases the number of linearly independent securities even though options are based on existing securities. See Sandor [14] for a historical account of how the Chicago Board of Trade and the professional traders who compose it established the market in plywood futures.

[5]Spanning of the set of market states does not assure a full equilibrium because individuals may still have an incentive to enter into insurance contracts with each other. For example, whether individual i lives or dies in period two probably does not affect any firm's returns appreciably and thus is information that is not necessary to distinguish one market state from another. But whether i lives or dies is an observable event and groups of individuals may find it desirable to pool their risk through a set of life insurance contracts. See Malinvaud [11,12] for a detailed discussion of the important role insurance plays in the bearing of individualized risks.

[6]Since security g is purely a set of transfers from one individual to another and not a claim on the real returns of a firm, the market clearing condition for security g is $\Sigma_{\mathcal{I}} z_g^i = 0$, not $\Sigma_{\mathcal{I}} z_g^i = 1$.

[7]In the presence of transaction costs this process would stop at a point short of spanning.

[8]The case I treat here is called ex post stockholder unanimity in the literature. If one is willing to make the strong assumption of perfect foresignt with respect to equilibrium stock prices and implicit prices (as is the case in rational expectations equilibria), then ex ante unanimity can also be shown. Baron [3] provides a clear exposition and discussion of the intracacies of ex ante unanimity versus ex post unanimity.

[9]This equilibrium initial situation is the result of previous trading activity that is not explicitly included within this model.

[10]An economy whose market states are spanned does not in general have any particularly attractive optimality properties. See Hart [7] and Grossman [6] for full discussions of optimality within incomplete markets.

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