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MARKET STRUCTURE AND INNOVATION

by

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I. Introduction:

Since Schumpeter's monumental *Capitalism, Socialism, and Democracy*, considerable attention has been directed toward examining the relationship between the market structure in a given industry and incentives for firms to invest in innovative activity. The problem may be summarized as follows: In an actively competitive market in which no firm has dominance, firms struggle to maintain their market share and feel pressure to invest heavily in research and development so as to beat their rivals to a profitable innovation. At the same time however, intense competition necessarily lowers the expected return to innovative activity. At the other extreme a monopolist can appropriate all of the gains from successful innovation without fear of rival precedence. The monopolist, however, faces less external pressure to innovate, since he is already earning monopoly rents and his market position is more secure than that of a competitive firm.

Considerations such as these have led economists to investigate both theoretically and empirically the impact of firm size and industry concentration on R and D investments.\(^1\) Cross-industry empirical analysis, exemplified by the

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\(^{1}\)Author is Assistant Professor of Economics and Urban Affairs. He has considerably benefited from discussions on this topic with F.N. Scherer, and with Mort Kamien and Nancy Schwartz. He alone, however, is responsible for any remaining errors.

\(^{1}\)This literature has been thoroughly reviewed by Kamien and Schwartz [2].
work of Mansfield [5], Williamson [9], and Scherer [6], suggests that the vigor of research and development efforts increases with industry concentration up to a point, but declines thereafter.²

Moreover, various writers have attempted to construct models of the profit maximizing firm which rationalize these findings. Noteworthy among these efforts is the work of Kamien and Schwartz [3], [4]. The primary thrust of their work is the analysis of the optimal timing decision under rivalry for a firm contemplating the introduction of an innovation. The firm can save R and D costs by pursuing a longer-lived development strategy, but only at the expense of increasing the probability that a rival will introduce the innovation first. Kamien and Schwartz then show (in [4]) that more intense rivalry, characterized by a decreased expected time of rival introduction, will first elicit a greater R and D investment by the expected profit maximizing firm, but will eventually cause the optimal intensity of innovative activity to decline. These comparative static results are consistent with the empirical findings reported above.

However, this analysis does not provide an adequate answer to the initially posed problem for several reasons. In the first place it is a partial equilibrium analysis, studying the behavior of an individual firm which views market conditions parametrically. In a given industry every firm is a rival of every other firm. Thus, the likelihood of rivals' precedence depends on the R and D strategies of other market participants, and cannot be varied independently of these decisions. Secondly, one cannot infer the change in aggregate innovative activity from the change in a single firm's investment intensity when rivalry has increased. The reason is that if greater rivalry means more firms competing for the same prize

²Using the market share of the largest four firms as a measure of concentration, Scherer [6] sees this turning point at almost 55%, while Williamson [9] concludes that a 5 - 30% market share yields greatest innovative activity.
then a lower investment by each one of them could well be outweighed by their increased number. Finally, the policy relevance of these conclusions concerning the impact of increased rivalry on R and D investments is far from clear. Unanswered is the question of whether or not the greater investment in innovation which more competition might elicit is actually in the social interest.

This first point concerning the interdependency of firm investment strategies has been recognized for some time. An insightful earlier analysis by Scherer [7] studied the problem of firm R and D expenditures as a Cournot game. Each firm took account of other firms' investment intensities when formulating its optimal strategy, but believed other firms' actions would be unaffected by its own decisions. Scherer found that symmetrically increasing the number of firms, and hence reducing the "representative" would-be innovator's initial market share, led to a greater marginal payoff to R and D investment for each one of them. However, he could not be sure that the overall profitability of the R and D project would remain non-negative as the number of firms increased. He was thus quite correct in qualifying his conclusion that atomistic competition would provide the greatest incentive for innovative activity. An additional limitation of his analysis is the assumed deterministic relationship between R and D expenditure and the time at which the innovation becomes available. R and D projects are inherently risky undertakings, though risk may be reduced through greater expenditure.

This paper contributes to a resolution of these issues by combining the approaches of Scherer and Kamien and Schwartz. Each firm is assumed to face a stochastic technological relationship between the level of its R and D investment and the time at which it is ready to introduce an innovation. Moreover, each firm also faces market uncertainty, not knowing when any rival's R and D
efforts will be successful. Firms have a probability distribution for the random date at which a rival will introduce the innovation. Firms are interdependent because the market uncertainty about a rival's introduction date which each firm faces is derived from rival investment decisions and the technologically uncertain relationship between those investments and the time of introduction of the innovation. Given the industry's market structure, equilibrium occurs when each firm's investment decision maximizes its expected discounted profits, subject to the other firms' R and D investment strategies. Rivalry is taken to be greater when the number of competing firms increases.

II. The Model

Imagine a world in which n identical firms compete for the constant, known perpetual flow of rewards V which will become available only to the first firm which introduces an innovation. Assume for the moment indefinite patent protection so that related innovators get no net rewards. Assume further that firm i, by making a contractual commitment to R and D with an implied present value of costs x_i, in effect purchases a random variable \( \tau(x_i) \), which represents the uncertain date at which the R and D project will be successfully completed. This commitment is assumed binding so that the costs of carrying out an investment project may be taken as known at the initial moment, independent of subsequent developments. Moreover, assume the following technological relationship:

\[
\Pr[ \tau(x_i) \leq t] = 1 - e^{-h(x_i)t}
\]

That is, \( \tau(x_i) \) is exponentially distributed with the expected time of introduction given by

\[
E \tau(x) = h(x)^{-1}.
\]

This, by making an investment valued at x, the firm "produces" the constant instantaneous probability h(x) that the innovation will be ready for the market at any subsequent moment.
Here $h(.)$ is taken to be twice continuously differentiable, strictly increasing, and satisfying

$$h(0) = \lim_{x \to \infty} h'(x) = 0,$$

$$h''(x) \geq 0 \text{ as } x \leq \bar{x},$$

with $\bar{x}$ possibly equal to zero. (3) expresses the assumption that while there may be an initial range of increasing returns to scale in the R and D technology, eventually diminishing returns are encountered. Denoted by $\bar{x}$ the point where $h(x)/x$ is greatest. (See Figure 1)

To express the $i^{th}$ firm's market uncertainty regarding the time at which any rival will introduce the innovation, define $\tau_{ij}$ as the random variable representing this unknown date. If firms' expectations are rational as we shall assume, then $\tau_{ij}$ is related to the behavior of other firms by

$$\tau_{ij} = \min_{1 \leq j \neq i} \{\tau(j)\} .$$

Let us assume now that there are no externalities in the R and D process (no theft of trade secrets for example), so that the random variables $\tau(x_j)$, $i = 1, \ldots, n$, may reasonably be taken as independent. Then

$$\Pr[\tau_{ij} \leq t] = 1 - \exp(-at) h(x_j) = 1 - e^{-at \int x_j}$$

where

$$a = \frac{1}{\int x_j}.$$

and $a$ is taken as constant by the $i^{th}$ firm. Let the race at which firms discount future receipts be $r$. At any time $t \geq 0$ the $i^{th}$ firm earns a revenue flow $r$ in the event that $\tau(x_j) \leq \min(\tau_{ij}, t)$. This is because in order to earn the reward flow at $t$, a firm must have already introduced the innovation earlier. Moreover, it must also be the case that no other firm "beat it to the market" with the innovation. Integrating the joint density of $(\tau(x_j), \tau_{ij})$ over the relevant region
we have

\[ \Pr(\tau(x_i) \leq \min_{\tau_i} (\tau_i, \tau)) = e^{-at} \frac{\rho(x_i)}{\rho(\tau) + \rho(\tau_i)} \]

\[ = e^{-at} \frac{\rho(x_i)}{\rho(\tau) + \rho(\tau_i)} \left( 1 - \exp(-\tau[s + \rho(\tau_i)]) \right). \]

Assuming that the \( i \)-th firm chooses \( x_i \), given \( s, r, \) and \( V \), to maximize expected discounted profits, it must solve the following problem:

\[ \max_{x} \left[ \mathbb{E} \sum_{t=0}^{\infty} e^{-rt} (1 - \exp(-\tau[s + \rho(\tau_i)]) \right] \]

or equivalently

\[ \max_{x} \left[ \mathbb{E} \sum_{t=0}^{\infty} e^{-rt} (1 - \exp(-\tau[s + \rho(\tau_i)]) \right] \]

From our assumptions (3) it is clear that a global interior maximum will exist so long as expected profits are non-negative at some \( x > 0 \). We will assume this to be true for \( s = 0 \) (i.e. in the absence of rivalry), and deduce below further conditions assuring the non-negativity of profits at a solution to the first order condition for (7) when \( a > 0 \). The assumption is not restrictive, for the problem would not be very interesting if innovation were unattractive even in the absence of rivalry.

It follows that necessary conditions for \( x \) to be an interior solution to (7) are:

\[ \frac{h'(\hat{x})}{(s + r + h(x))^2} = \frac{r}{V} = 0 \]

and

\[ h''(x)(s + r + h(\hat{x})) - 2h'(\hat{x})^2 \leq 0 \]

Equation (8) defines \( \lambda = \lambda(a, r, V) \) explicitly. \( \lambda \) is the expected profit maximizing investment in research and development for a firm which presumes that the instan-
taneous probability of rival introduction is $a$. Note from (8) and (9) that $\hat{x}$ is increasing in $V$ and decreasing in $r$, as one would expect. Now the symmetry of our firms dictates that, in equilibrium, they pursue the same investment strategies. Moreover, since their expectations are rational and each is investing $x^*$ in equilibrium, we must have $a = (n-1)h(x^*)$, or from (8)

$$x^* = x(n-1)h(x^*), \quad (r, V).$$

Equation (10) implicitly defines the equilibrium level of firm $R$ and $D$ investment $x^* = x(n, r, V)$. We note that an equilibrium exists as long as $R$ and $D$ is profitable in the absence of rivalry. Notice also that we define equilibrium relative to a fixed market structure. Entry is considered in the next section.

We can now study the impact of greater rivalry on a firm's innovative activity by studying the dependency of $x^*$ on $n$.

Before pursuing this, however, we will first examine why a partial equilibrium analysis of this problem gives misleading results. The method of Kamien and Schwartz [4] was effectively to calculate $\Delta x/\Delta a$. They conclude that greater rivalry stimulates $R$ and $D$ activity if $\Delta x/\Delta a > 0$. Now it is easily seen from (8) and (9) that $\Delta x/\Delta a < 0$ if $h(x) \approx a + r$. Consult Figure 2. There are two cases: (i) $h^{-1}(r) \geq x(o, r, V)$ or (ii) $h^{-1}(r) < x(o, r, V)$. Case (i) implies that greater rivalry always reduces investment. Moreover, since it is clear that $\lim h(o, r, V) = 0$, the only possible pattern in case (ii) is that indicated in the figure - namely that investment is a single peaked function of the degree of rivalry, increasing initially but eventually declining thereafter.

Now these are precisely the conclusions reached in [4]. Yet these results do not stand once rival behavior is made endogenous. For when $n \geq 2$, $\Delta x/\Delta a$ is necessarily negative at any equilibrium. One then has the following proposition:
Proposition I: As the number of firms in the industry (i.e. the extent of rivalry) increases, the equilibrium level of firm investment declines.

Proof: Regarding \( n \) as a continuous variable, totally differentiate (10) to find (for \( n \geq 1 \)) that

\[
\frac{\partial x^*}{\partial n} = \frac{\frac{\partial x}{\partial a} h'(x^*)}{1 - (n-1)h'(x^*)}\frac{\partial x}{\partial a} < 0. \quad \text{QED.}
\]

Thus we have found the implication of profit maximization, rational expectations, and Cournot behavior to be that increasing the extent of rivalry unambiguously reduces an individual firm’s incentive to invest in \( R \) and \( D \).

It does not follow from Proposition I that a more competitive market structure means a later expected introduction date for the innovation. Define the random variable \( \tau(n) = \min \{ \tau(x^*) \} \), the stochastic time at which the innovation becomes available to society. Notice that in equilibrium we have (suppressing dependency on \( r \) and \( v \))

\[(11) \quad E(\tau(n)) = \left(\frac{\partial b(x^*(n))}{\partial a} \right)^{-1}.\]

The following proposition shows that given a reasonable stability condition, increasing the number of competitors in an industry reduces the expected time which society has to wait for the innovation, despite the fact that each competitor invests less in \( R \) and \( D \).

Proposition II: Suppose that with the industry in equilibrium, a unit increase in \( R \) and \( D \) investment by any single firm causes the investment of each other firm to fall by less than a unit. Then increasing the number of firms always reduces the expected industry introduction date.
Proof: Industry expected introduction date declines with the number of firms if and only if \( \frac{d}{dn} (nh(x^*(n))) > 0 \). Now

\[
\frac{d}{dn} (nh(x^*(n))) = h'(x^*(n)) + nh'(x^*(n)) \frac{dx^*}{dn}
\]

\[
= h(x^*(n)) \left[ 1 + \frac{nh'(x^*(n))}{1-(n-1)h'(x^*(n))} \right] \frac{dx^*}{dn}
\]

from the proof of Proposition I. Thus

\[
\frac{d}{dn} (nh(x^*(n))) \lessapprox 0 \Rightarrow 1 \lessapprox -h'(x^*) \frac{dx^*}{dn}.
\]

Suppose the industry is in equilibrium and some firm raises investment one unit. Then each other firm sees an increase in \( a \) of \( h'(x^*) \), and hence reduces investment by the amount \( -h'(x^*) \frac{dx^*}{dn} \). \( QED \)

III. Competitive Entry and Socially Optimal Market Structure:

The foregoing discussion has examined optimal firm investment in \( R \) and \( D \) with the market structure \( (n) \) given. In the absence of barriers to entry, we may expect additional firms to enter the innovation race as long as expected profits are strictly positive. Using (7) and (8) we may write the equilibrium expected profits of a representative firm as:

\[
\pi(a; x, r, V) = \frac{h(x^*)}{h'(x^*)} \left\{ \frac{(a+r+h(x^*))}{(a+r)} \right\} - x^*
\]

where equilibrium requires \( a = (n-1) h(x^*) \). Now if \( h \) is a concave function then \( \frac{h(x)}{x} \geq h'(x) \), and expected profits are always positive. This gives the following result:

**Proposition III:** If the technology for innovation exhibits diminishing returns to scale throughout, in the sense that \( h'' < 0 \), then expected profits
are driven to zero only in the limit as the number of firms approaches infinity.

Thus atomistic competition would be the natural outcome with continuously diminishing returns and zero entry costs. In this limiting case, each firm would invest an infinitesimal amount. Given our stability condition however, aggregate innovative activity would be greater than that forthcoming in equilibrium with any finite number of firms.

More interesting is the case with an initial range of increasing returns. Here there are two exhaustive possibilities. Either entry continues until equilibrium expected profits have been driven to zero with a finite (though perhaps non-integral) number of firms in the industry, or expected profits approach zero asymptotically as the number of firms goes to infinity. In either case continued entry causes a monotonic decrease in equilibrium expected profits. Moreover, the zero expected profit equilibrium will involve firms operating with "excess capacity" in the sense that they will not exploit all of the scale economies in the innovation technology. These results are summarized in the following proposition.

**Proposition IV:** The equilibrium expected profits of a representative firm decrease as additional firms enter the industry. With initial increasing returns entry eventually drives profits to zero, possibly with a finite number of firms in the industry. Industry equilibrium with increasing returns and zero expected profits always involves "excess capacity" in the R and D technology.

**Proof:** Supressing dependence on \( r \) and \( V \) we have from (7) that \( \pi = \pi(a,x) \).

In equilibrium \( a = (n-1)h(x^*) \), and (10) gives \( x^* \) as a function of \( n \).
Thus

\[ \frac{d\pi}{dn} \text{ [eq.]} = \frac{\partial \pi}{\partial x} \frac{\partial x}{\partial n} + h'(x) \frac{\partial x}{\partial n} \]

It is obvious from (7) that \( \frac{\partial \pi}{\partial x} < 0 \), while (8) is the requirement

\[ \frac{\partial \pi}{\partial x} = 0. \] Hence

\[ \frac{d\pi}{dn} < 0 \quad \text{as} \quad \frac{-(n-1)h'(x^*)}{-x^*} \frac{\partial x}{\partial n} + 1 < 1. \]

Thus equilibrium profits decrease in \( n \). Now suppose profits are positive for all finite \( n \). Then (10) implies \( \lim_{n \to \infty} x^*(n) = 0 \). Moreover, from (12) and the fact that \( \lim_{x \to 0} h(x)/x = h'(0) \) it follows that

\[ \lim_{n \to \infty} (n-1)h(x^*(n)), x^*(n)) \to 0. \]

On the other hand, if there exists \( n_0 < \infty \) for which equilibrium expected profits are zero, then (12) implies

\[ \frac{h(x^*(n_0))}{x^*(n_0)} = \frac{h(x^*(n_0))}{x^*(n_0)} + r \]

Clearly \( h(x^*(r_0))/x^*(n_0) < h'(x^*(n_0)) \). Thus \( x^*(n_0) < \overline{x} \), and there is excess capacity. QED.

The results of Proposition IV may be illustrated graphically as a special case of Chamberlin's monopolistic competition equilibrium [1]. Denote by \( C(\cdot) \) the inverse of the function \( h(\cdot) \). Then \( C(h) \) is the present value of costs incurred in producing the constant instantaneous probability of introduction \( h \). Now with initial increasing returns, the average cost function \( C(h)/h \) will be L-shaped. Moreover, average and marginal revenues may be expressed as functions of \( h \) for the representative firm with \( a, r, \) and \( V \) given. These relationships are noted in Figure 3. Expected profits are maximized where \( MR = MC \). Now the
argument to prove $\frac{d}{dn} \bigg|_{eq} < 0$ may be employed to show that $\frac{d}{dn} [(n-1)h(x^*(n))] > 0$, without recourse to the stability assumption. Thus, entry raises a few the representative firm in equilibrium, causing the marginal and average revenue curves of Figure 3 to decline. This process continues until the average cost curve is just tangent to the average revenue curve at a level of $h$ (and hence $x$) where marginal revenue equals marginal costs. (Figure 4) Since average revenue is downward sloping, marginal revenue lies below it everywhere. Consequently, when profits are maximized and simultaneously equal to zero, it must be that $MC < AC$. Thus there will be excess capacity.

Let us turn now to a consideration of the efficiency properties of these equilibria. The classical arguments for failure in the market for investments hinge upon the inability of innovators to appropriate the entire social benefits of their innovations ([8], Chp. 15). We wish to argue here that when innovation takes place under conditions of rivalry, another important source of misallocation arises as a consequence of competition. So as to focus on this phenomenon, we assume that the private reward flow $V$ accurately measures the social value of the innovation. This is equivalent to assuming that the innovator will be a perfectly discriminating monopolist, post introduction. While somewhat unrealistic, this assumption permits additional insights of value to be had.

Suppose first that the industry’s market structure (i.e., number of identical firms) is fixed. Each firm chooses an investment level to maximize $\eta(a, x)$. When all firms make the same investment we have $a = (n-1)h(x)$. Now firms are completely symmetric, and hence each firm has an equal chance of winning the innovation race. That is, each firm has probability $1/n$ of being the first to introduce the innovation. But society is indifferent as to which of the $n$ firms is first. Thus the net social payoff to having each firm invest $x$ is $n\eta((n-1)h(x), x)$. 
It is clear then that firms tend to over-invest in $R$ and $D$ because they do not take account of the parallel nature of their efforts.

**Proposition V:** Given a fixed market structure, industry equilibrium will have each firm investing more in $R$ and $D$ than is socially optimal.

**Proof:** Firms set $\frac{\partial \Pi}{\partial a} (a, x) = 0$ when they maximize profits, while equilibrium requires $a = (n-1)h(x)$. Moreover, second order conditions require $\frac{\partial^2 \Pi}{\partial a^2} < 0$. Now a social planner seeks to maximize $\Pi(a, x)$ subject to $a = (n-1)h(x)$. This requires

$$\frac{\partial^2 \Pi}{\partial a^2} (n-1)h'(x) + \frac{\partial \Pi}{\partial x} = 0.$$

But $\frac{\partial \Pi}{\partial a} < 0$, implying $\frac{\partial \Pi}{\partial x} > 0$ at a social optimum. Hence private firms are investing more than optimal in equilibrium. QED.

Consider now the case in which a social planner may choose the number of parallel $R$ and $D$ projects as well as their intensity. (We neglect the indivisibility of such projects, treating $n$ as though it were continuous.) Then the planning problem is

$$\max_{x, n} \left[ n \Pi(a, x; \Psi, \tau) \right] \text{ s.t. } a = (n-1)h(x)$$

Let $n^*$ denote the socially optimal market structure and, as before, let $n_o$ be the zero equilibrium expected profit market structure. Then we have the following important result:

**Proposition VI:** When the $R$ and $D$ technology is characterized by diminishing returns throughout, then the zero profit equilibrium is socially optimal. When there are initial scale economies however, competitive entry leads to too many firms joining the innovation race.
Proof: From (7), first order conditions for a social optimum are
\[
\frac{h(x)}{x} = \frac{(nh(x) + r)^2}{V} = h'(x).
\]
Hence the optimal investment level at the socially optimal market structure is \( \bar{x} \) (see Figure 1). With diminishing returns everywhere \( \bar{x} = 0 \) and \( n^* = m \), which is the asymptotic competitive outcome.

With initial increasing returns, Proposition IV showed that \( \bar{x}(n_0) < \bar{x} \).

Now by Proposition V, \( \bar{x}(n^*) > \bar{x} \). Moreover by Proposition I, \( \partial \bar{x}/\partial n < 0 \). Hence \( n_0 > n^* \).

The meaning of Proposition VI is clear. When entry is unimpeded, the technology possesses initial economies of scale, and innovating firms struggle for the entire social payoff, there will be too much competition. Intuitively this may be seen as follows. Economies of scale are always fully exploited in a socially optimal allocation. This implies a finite number of firms, each operating at the efficient scale, earning positive expected profits. Now the social payoff is obtained when any one of these firms is successful, and each firm has an equal chance of success. Thus the social net gain is proportional to the private net gain. Yet private profits attract entry. When all of the private profits have been competed away and the number of firms is finite, then the net social gain has vanished as well. When the number of firms is infinite in the zero profit equilibrium, net social gain could be positive, but could hardly be maximal. For in this instance no scale economies are being exploited, and "mergers" of parallel R and D efforts would obviously improve performance.

As a final point we note that a public authority could induce private firms to sustain the socially optimal investment level and market structure through a judicious use of licensing fees and variable patent life. Suppose that upon expiration of patent protection imitation is immediate and complete,
and that the total flow of quasi-rents \( V \) would be thereby competed away. If the length of the patent were \( T \) (taken as infinite in the preceding analysis) then the equivalent quasi-rent flow in perpetuity would be \( (1-e^{-rT})V \). The optimal patent life is then set such that at the socially optimal market structure (as derived above) firms, in equilibrium, desire to invest \( x \). Lump sum licensing fees are then set at the level of firm expected profits (possibly negative) so that competitive entry will assure just the right market structure.

IV. Conclusion:

An equilibrium model of investment in R and D under rivalry has been constructed. In this model firms are assumed to maximize their expected profits under conditions of technological and market uncertainty. Their perceived market risks are not social risks however, and this leads to a basic failure of the competitive mechanism. It is seen that more competition (rivalry) reduces individual firm investment incentives in equilibrium, yet leads (under certain reasonable conditions) to an increased probability that the innovation will be introduced by any future date.

However, more competition is not necessarily socially desirable. We show that with continuously diminishing returns to R and D investment, atomistic competition is the market structure giving optimal innovative activity. This structure is approached under competitive conditions with costless entry. However, in the more realistic case of initial scale economies, the optimal market structure involves a finite number of firms. Yet, if entry is again costless and occurs until no firm expects positive profit in equilibrium, more firms will enter the innovation race than is socially optimal. Given any market structure, competing firms invest more in R and D than would be optimal, not taking account of the parallel nature of their efforts. The nature of the
market failure is quite similar to that which occurs in common pool resource problems. Social welfare can be maximized by appropriately limiting entry and firm investments with licensing fees and finite patent life.

The model studied is highly simplified, and several qualifications are in order. We have omitted the possibility of imitation lowering the flow of rewards to the innovating firm subsequent to introduction of the innovation. Imitation does not affect the socially optimal allocation, but does reduce private investment incentives. Thus, the result that competitive firms over invest may not hold in that case. Similarly, the zero expected profit equilibrium market structure would seem to involve fewer firms with the possibility of imitation. Thus, the result that entry barriers can improve welfare may also fail when the possibility of imitation is introduced.

Another important short coming of the model is that competing firms lose nothing but their R and D investment when a rival beats them to the innovation. In reality the market shares of competing firms are constantly changing as new innovations attract competitors' customers. Again, these gains and losses of market share involve private, but not social payoffs. Their inclusion will affect the above stated results on the relationship between equilibrium and optimal allocations. Given these observations, the agenda for future work should be clear.
References:


Figure 1: \( \hat{x} = \tilde{x} = 0 \) and \( \hat{x} > \tilde{x} > 0 \)

Figure 2: \( h^{-1}(s+r) \) and \( h^{-1}(s) \)