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Anticipations and Endogenous Present Bias

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JEL Classification: D01, D01, D03, D90, D91

Keywords: Anticipation, Cognitive Skill, Discounting, Human Capital, Impatience, Present Bias, Time Preference

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– Very Preliminary and Incomplete –

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1 Motivation

In a seminal paper, Becker and Mulligan (1997) argued that economic agents had the power to affect their discount factor through various means, such as investing in education, and studied how this investment depended on such factors as personal wealth. In a recent paper (Galperti and Strulovici, 2014), we argued that an agent's subjective discount factor naturally concerns the agent's future well-being (or future preferences) rather than his future instantaneous utility from consumption per se. This distinction turns out to have important consequences, which we explore in this paper.

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The first distinction concerns the object of investment. The Becker-Mulligan approach presumes that the agent invests in what may be best described as a psychological discount factor concerning future consumption utility. It is unclear, however, why the agent's investment "in the future" should translate, specifically, into a geometric discounting of consumption utility. While convenient, this particular functional form may seem rather arbitrary in their framework. In fact, Becker and Mulligan first consider a two-period model in which the agent invests in the "future," which is obviously the second period. They then move on to a more general multi-period model in which the investment has a geometrically decaying impact on future utility and seems less well founded.

Our approach is conceptually closer to the simpler present/future dichotomy motivating the Becker-Mulligan approach. However, because it is applied to future well-being rather than consumption utility, it generates a geometric decay of the discount factor similar to the on assumed by Becker and Mulligan. More specifically, it generates a quasi-hyperbolic discounting model of consumption utility, whose parameters are determined by the agent's preference over future well-being.

By modifying his evaluation of the present/future trade-off in terms of well-being, the agent affects the entire representation of his preferences in terms of consumption utility, which includes both a present-bias parameter and discounting parameter. This has two consequences.

Firstly, the present-bias parameter ("beta") is now endogenous. The agent can exert some control over the magnitude of his present bias and, hence, time inconsistency. By modifying his present bias, the agent is thus also affecting his need or value for commitment.

Secondly, our approach to "investing in the future" generates a positive correlation between present bias and impatience: in our framework, an agent investing in the future becomes at the same time less time inconsistent and more patient. This positive correlation is consistent with the empirical findings of Burks et al. (2009), and indeed provides a novel explanation for these findings.

2 Model

Consider the following simple model. At each point in time, the agent's preference over consumption streams $c = (c_0, c_1, \ldots)$, with $c_t \geq 0$ for all t, is determined by the utility representations

$$U(c) = u(c_0) + \sum_{t=1}^{\infty} \alpha^t \gamma U({}_tc) = u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t),$$

where $\gamma \in (0, \frac{1-\alpha}{\alpha})$, $\alpha \in (0,1)$, $\beta = \frac{\gamma}{1+\gamma}$, $\delta = \alpha(1+\gamma) < 1$ (the mathematical equivalent between the two representations has been derived and discussed by Saez-Martin and Weibull (2005) and Galperti and Strulovici (2014)). We interpret α as the agent's subjective discount factor of current utility from future well-being (i.e., $U({}_tc)$), which is given by the anticipatory utility $G(U) = \gamma U$, and γ as the vividness or "imaginability" of future well-being. Suppose that u is twice differentiable with $u' > 0$, $u'' < 0$, and satisfies the usual Inada conditions to ensure interior solutions.

Becker and Mulligan (1997) argued that economic agents had the power to affect their discount factor through various means, such as investing in education, and studied how this investment depended on such factors as personal wealth. In a recent paper (Galperti and Strulovici, 2014), however, we argue that an agent's subjective discount factor concerns his future well-being (or future preferences) rather than his future instantaneous utility from consumption.[1] Therefore, investments in one's perception of the future should directly affect the coefficient γ rather than the coefficients β and δ, which only arise as a result of a fortuitous algebraic manipulation transforming future well-being or preferences into consumption utilities.[2]

Another potential concern with the Becker-Mulligan approach is that the agent invests in what may best be described as a psychological discount factor. It is unclear, however, why the agent's investment would translate into a geometric discounting of consumption. Becker and Mulligan distinguish between a two-period model, in which the agent invests in the "future" which is obviously the second period, and a more general multi-period model in which the

[1]Indeed, how economic agents perceive the future depends on more than their narrow consumption events: it involves anticipations of these consumption events, anticipations of future anticipations and more generally, many other considerations which enter their future well-being.

[2]More generally, Galperti and Strulovici (2014) axiomatize a preference representation of the form $U(c) = u(c_0) + \sum_{t\geq 1} \alpha^t G(U({}_tc))$, where G is the anticipatory utility from future well-being. This representation translates into an additively separable representation in terms of consumption only when G is linear.

investment has a geometrically discounting impact on future utility.

Our approach is conceptually closer to the simpler present/future dichotomy motivating the Becker-Mulligan approach. However, because it is applied to well-being rather than consumption, it generates a geometric decay of the discount factor on consumption utility. Moreover, it also generates endogenous present bias.

Specifically, we want to allow the agent to affect his γ by investing time, effort, and resources in human capital which allows him to better imagine his future well-being implied by current actions. Therefore, let $e \geq 0$ be the agent's investment and $\gamma(e)$ be the resulting vividness of future well-being with $\gamma(0) > 0$, so the the agent is never fully myopic. Assume that $\gamma(\cdot)$ is twice differentiable with $\gamma' > 0$ and $\gamma'' < 0$. It is immediate that by investing in vividness, the agent reduces both his degree of present bias β and his long-run impatience δ. As shown by Galperti and Strulovici (2014), the model is only well-specified as a representation of anticipations preferences if $\gamma(e)$ is less than $\frac{1-\alpha}{\alpha}$ (e.g., the 'effective cost' of increasing γ becomes arbitrarily prohibitive as $\gamma \to \frac{1-\alpha}{\alpha}$).

Under these assumptions, we have a "meta-preference" of the form

$$U(c, e) = u(c_0) + \sum_{t=1}^{\infty} \alpha^t \gamma(e) U({}_t c).$$

We can consider different cases:

- the agent chooses (c, e) once and for all with commitment;
- the agent chooses (c_t, e_t) in each period with full sophistication and we look for a steady state in which $(c_t, e_t) = (c^*, e^*)$ in every period;[3]
- the investment e is persistent with some decaying rate ρ so that investing e_t in period t endows the agent at $t+1$ with $e_{t+1} = (1-\rho)e_t$.

For now, assume that α is fixed exogenously.

[3]This approach would required the use of a generalized Euler equation à la Harris and Laibson (2001).

3 Single choice with commitment

Let I be the present-discounted value (at some gross market rate R) of the agent's lifetime income. Then, the agent's problem is to maximize

$$u(c_0) + \sum_{t=1}^{\infty} \alpha^t \gamma(e) U({}_t c) \tag{1}$$

subject to

$$ke + \sum_{t=0}^{\infty} c_t \leq I.$$

The parameter $k > 0$ captures in a very simple way the 'price' of human-capital investment e relative to consumption. One possible interpretation of k is in terms of cognitive skills: higher cognitive skills and intelligence may allow the agent to imagine the future more easily, thus reducing the cost is terms of forgone consumption of increasing γ (see, e.g., Burks et al. (2008)).

We first manipulate the agent's objective (1) as follows. For any amount $b \geq 0$, let

$$\begin{aligned} W(b,e) &= \max_{\sum_{\tau=1}^{\infty} c_\tau \leq b} \sum_{t=1}^{\infty} \alpha^t \gamma(e) U({}_t c) \\ &= \max_{\sum_{\tau=1}^{\infty} c_\tau \leq b} \beta(e) \sum_{t=1}^{\infty} [\delta(e)]^t u(c_t) \\ &= \beta(e) \max_{\sum_{\tau=1}^{\infty} c_\tau \leq b} \sum_{t=1}^{\infty} [\delta(e)]^t u(c_t) \end{aligned} \tag{2}$$

where $\beta(e) = \frac{\gamma(e)}{1+\gamma(e)}$ and $\delta(e) = \alpha(1+\gamma(e))$. Note that the solution to this problem must satisfy $\sum_{\tau=1}^{\infty} c_\tau = b$. Then, we can rewrite the agent's problem as

$$\max_{ke+c_0+b \leq I} u(c_0) + W(b,e) = \max_{b,e \geq 0} u(I - b - ke) + W(b,e). \tag{3}$$

Let us first consider the solution to (2). Letting $\sigma \geq 0$ be the Lagrange multiplier associated with b, we have that for any t and s

$$[\delta(e)]^t u'(c_t) = \sigma$$

$$[\delta(e)]^s u'(c_s) = \sigma$$

and hence $u'(c_{t+1}) = \frac{1}{\delta(e)} u'(c_t)$ for all t. So, in particular, a higher investment e, by increasing $\delta(e)$ increases the optimal ratio $\frac{u'(c_t)}{u'(c_{t+1})}$. If, for instance, $u = \ln$, this corresponds to a higher ratio $\frac{c_{t+1}}{c_t}$, i.e., higher consumption growth.

Letting

$$\begin{aligned} V(b, \delta) &= \max_{\sum_{\tau=1}^{\infty} c_\tau \leq b} \sum_{t=1}^{\infty} \delta^t u(c_t) \\ &= \delta \max_{0 \leq c_1 \leq b} \{u(c_1) + \delta V(b - c_1, \delta)\}, \end{aligned}$$

we can write the agent's problem as

$$\max_{b,e \geq 0} u(I - b - ke) + \beta(e) V(b, \delta(e)). \tag{4}$$

Assuming differentiability of V in both arguments, we have the first-order conditions

$$\begin{aligned} u'(I - b - ke) &= \beta(e) V_1(b, \delta(e)) \\ &= \beta(e)\sigma > 0 \end{aligned}$$

$$\begin{aligned} ku'(I - b - ke) &= \beta'(e) V(b, \delta(e)) + \beta(e)\delta'(e) V_2(b, \delta(e)) \\ &= \underbrace{\beta'(e) V(b, \delta(e))}_{\text{New effect}} + \beta(e) \underbrace{\delta'(e) \sum_{t=1}^{\infty} [\delta(e)]^{t-1} u(c_t^*)}_{\text{(Discounted) Becker-Mulligan's effect}} \end{aligned}$$

We note that Becker-Mulligan's effect is weaker here because $\beta < 1$: this is the effect of e on the agent's degree of long-run impatience. The new effect comes from the fact that investing in the "vividness" of future well-being also makes the agent less present-biased. Hence, the agent cares more about his overall future payoff V. Combining the two first-order conditions, we get

$$k = \frac{\beta'(e) V(b, \delta(e)) + \beta(e)\delta'(e) V_2(b, \delta(e))}{\beta(e) V_1(b, \delta(e))}.$$

Consider the cross partial derivatives of the objective in (4):

$$\frac{\partial^2}{\partial e \partial k} = keu''(I - b - ke) \leq 0$$

$$\frac{\partial^2}{\partial e \partial I} = -ku''(I - b - ke) > 0$$

$$\frac{\partial^2}{\partial b \partial k} = eu''(I - b - ke) \leq 0$$

$$\frac{\partial^2}{\partial b \partial I} = -u''(I - b - ke) > 0$$

$$\frac{\partial^2}{\partial b \partial e} = ku''(I - b - ke) + \beta(e)V_1(b, \delta(e)) + \beta'(e)V(b, \delta(e)) + \beta(e)\delta'(e)V_2(b, \delta(e))$$

The first four formulas suggest the following comparative static conclusions (if the last formula is positive):

- A lower k (cognitive cost) implies higher investment e in vividness, which in turn implies *both* higher β (less present bias) *and* higher δ (more long-run patience). This matches what Burks et al. (2009) find in their data.

- A higher I (wealth) implies higher investment e in vividness, which implies again *both* higher β (less present bias) *and* higher δ (more long-run patience). This predicts that wealth should be positively correlated with both a lower degree of present bias and a higher long-run patience. In particular, the model predicts a possible causal relation *from* wealth to both present bias and long-run patience.

- A lower k (cognitive cost) implies higher b, i.e., savings for future periods. This is intuitive given the combined effects on β and δ which make the future more valuable.

- A higher I (wealth) implies higher savings in absolute terms. This is of course because of the decreasing marginal utility of current consumption, but here also because of the positive indirect effect on both β and δ.

- It is interesting to consider the savings *rate*, rather than its absolute value. Let this rate be $s = b/I$ so that $I - b = I(1 - s)$. Then

$$\frac{\partial^2}{\partial s \partial k} = Ieu''(I(1 - s) - ke) < 0,$$

which means that a decrease in cognitive cost would increase the savings rate (people with higher cognitive skills should tend to have higher savings rates). Also,

$$\begin{aligned}\frac{\partial^2}{\partial s \partial I} &= -I(1-s)u''(I(1-s)-ke) - u'(I(1-s)-ke) \\ &= -u'(I(1-s)-ke)\left[1 + \frac{I(1-s)u''(I(1-s)-ke)}{u'(I(1-s)-ke)}\right].\end{aligned}$$

This cannot be signed unambiguously and hence higher wealth need not have a clear effect on the savings rate.

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