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Abstract

We augment the standard Crawford-Sobel (*Econometrica* 1982) model of cheap talk communication by allowing the informed party to use both costless and costly messages. The issues on which we focus are the consequences for cheap talk signaling of the option to use a costly signal ("burned money"); the circumstances under which both cheap talk and burned money are used to signal information; and the extent to which burning money is the preferred instrument for information transmission.

1 Introduction

Legend has it that in 509 B.C. Mucius Scaevola was caught trying to kill Lars Porsena, the king of Clusium, who was besieging Rome. When brought before Porsena, Scaevola revealed that he was but the first of three hundred Romans who had sworn to kill him. Porsena threatened to torture Scaevola

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unless he divulged details of the Roman plot. Scaevola refused, claiming he would never betray his fellows. Recognizing that such a claim was the epitome of cheap talk, Scaevola proceeded to push his right hand into the altar fire and hold it there until it burned off. Porsena found the signal credible, released Scaevola (which, not incidentally, means "left-handed") and made peace with Rome [13, 468].

Although somewhat extreme, Mucius Scaevola's behaviour clearly illustrates that in many cases cheap talk is not the only means of communication. In particular, informed parties typically have the opportunity to impose costs on themselves. While we expect most people to draw the line short of self-immolation, we also expect that most people have the willingness and ability to accept some direct loss in utility to transmit information in a more credible manner than employing cheap talk alone. A common euphemism for such self-imposed losses in utility (e.g. [18]), and the one we adopt here, is that of "burned money".

The canonic model for strategic cheap talk communication is due to Crawford and Sobel [6]. Subsequent literature applying variants of the Crawford/Sobel model is growing and varied.[1] But, as suggested above, purely costless signaling is a polar case. Thus, the extent to which results derived from applications of the polar case to substantive problems are robust, depends on the extent to which the polar case offers a good approximation to situations in which there is also some possibility of costly signaling. In the sequel, we address the robustness issue by augmenting a particular version of the standard Crawford/Sobel model of cheap talk communication in the presence of asymmetric information, allowing the informed party to use both costless and costly messages. The first main result is that the set of equilibria can be dramatically increased when costly signals can be used. In particular, given a sufficient budget, the set of perfect Bayesian equilibria to the strategic communication game *always* contains a continuum of semi-pooling equilibria, with the separating equilibrium at one end of the continuum and the pooling equilibrium at the other. The second main result is that the availability of costly signals can improve the precision of cheap talk communication. On the other hand, whether the availability of costly signals can induce influential cheap talk in situations other than those in which such signaling is possible without burned money, depends on the details of the environment. We provide two results on when the possibility of burning money does not affect the opportunities for cheap talk commu-

[1] See, *inter alia*, [8]; [9]; [14]; [10]; [11]; [1]; [2]; [12].

nication, and an example in which influential cheap talk is possible only if costly signals can also be used.

Apart from substantive applications, there is also increasing attention being paid to equilibrium refinements for cheap talk games.[2] Although this paper does not concern such refinements directly, some of our results bear on the use of efficiency criteria to select the most informative equilibrium in particular games. Specifically, Crawford and Sobel [6], Gilligan and Krehbiel [10] and others have argued for using (loosely speaking) the most informative available equilibrium when looking for predictions from the model in applied settings. And this selection is justified in most cases both because it is the unique *ex ante* efficient equilibrium and because it defines the upper bound on credible information transmission. However, the possibility of using burned money to signal information leads to multiple *ex ante* efficient equilibria and drives a wedge between the two selection criteria.

Section 2 presents the model. Section 3 examines equilibria to the model, focussing on the consequences for cheap talk signaling of the option to burn money and on the circumstances under which both cheap talk and burned money are used to signal information. Section 4 illustrates the results with an example much-used in the applied literature and, for this example, develops some welfare comparisons. Section 5 concludes.

2 Model

The basic setup is due to Crawford and Sobel [6]. At the start of the game, Nature privately reveals the value of a parameter $t \in [0,1]$ to the sender; having observed t, the sender transmits a signal to the receiver, who then takes an action $a \in \Re$; payoffs are then distributed to both agents. The sender's signal may be costless (cheap talk) or costly (burned money). The critical distinction between the two is that only the latter directly enters the sender's utility function. Specifically, let M be an uncountable and otherwise arbitrary set of costless signals (messages), with generic element m, and let $b \in \Re_+$ denote a costly signal. So costly signals are associated with the positive reals. Then given any 4-tuple $(a,t,m,b) \in \Re \times [0,1] \times M \times \Re_+$, the sender's and the receiver's preferences are (respectively) described by:

$$U^S(a,t,m,b) = u^S(a,t,x) - b \tag{1}$$

$$U^R(a,t,m,b) = u^R(a,t) \tag{2}$$

[2]For example, [7]; [15]; [16]; [4]; [5].

where $x \in (0, \infty)$ is a scalar describing the extent to which the sender and the receiver share common preferences over (a, t)-pairs. In particular, assume that, for every (a, t)-pair, $\lim_{x \to 0} u^S(a, t, x) = u^R(a, t)$. We also assume u^S and u^R are at least twice differentiable in all arguments with finite first derivatives at all $(a, t) \in \Re \times (0, 1)$, strictly concave in a with, for every $t \in [0, 1]$, $\arg\max_{a \in \Re} u^S(a, t)$ and $\arg\max_{a \in \Re} u^R(a, t, x)$ finite, and that the cross partials, u_{12}^S, u_{12}^R, and u_{13}^S are all strictly positive.[3] It follows that for any $x > 0$ and all $t \in [0, 1]$, $\arg\max_{a \in \Re} u^R(a, t) < \arg\max_{a \in \Re} u^S(a, t, x)$ and the extent to which the sender's most preferred action exceeds that of the receiver's is increasing in x for any t.

In general the receiver chooses her action under uncertainty over the value of t. Assume the receiver and the sender share a common prior on t, described by the smooth probability density, $h(t)$, with support $[0, 1]$. Let $g(\cdot|m, b)$ denote the receiver's posterior beliefs over $[0, 1]$ conditional on hearing a message $m \in M$ and a signal $b \in \Re_+$.

Let $\sum \equiv M \times \Re_+$. Then the sender's strategy is given by:

$$\sigma : [0, 1] \to \sum \tag{3}$$

where we write $\sigma(t) = (m(t), b(t))$. The receiver's strategy is given by:

$$\alpha : \sum \to \Re. \tag{4}$$

By virtue of M being uncountable and preferences being strictly concave in actions and increasing in the parameter t, the restriction to pure strategies in (3) and (4) is without loss of generality.

For any signal $(m, b) \in \sum$, let $T((m, b); \sigma) = \{t : \sigma(t) = (m, b)\}$.

Definition 1 *An equilibrium to the sender/receiver game described above is a list of strategies (σ^*, α^*) and posterior beliefs for the receiver $g(\cdot|m, b)$ such that:*

(e.1) $\forall t \in [0, 1]$, $\sigma^*(t) \in \arg\max_{(m,b) \in \Sigma} U^S(\alpha^*(m, b), t, m, b)$;
(e.2) $\forall (m, b) \in \sum$, $\alpha^*(m, b) \in \arg\max_{a \in \Re} \int_0^1 U^R(a, t, m, b) g(t|m, b) dt$;
(e.3) If $T((m, b); \sigma) \neq \emptyset$ *then*

$$\begin{aligned} g(t|m, b) &= \frac{h(t)}{\int_{T(\cdot)} h(\tau) d\tau}, \ \forall t \in T((m, b); \sigma); \ \textit{and} \\ g(t|m, b) &= 0 \ \textit{otherwise}. \end{aligned}$$

[3] Crawford and Sobel [6] only impose the assumption $u_{13}^S > 0$ for their welfare results. Although the assumption is mild in the present context, the characterization of equilibria in their model does not depend upon it.

Often we will refer to a strategy pair (σ, α) as an equilibrium, leaving the restrictions on beliefs implicit. Say that an equilibrium is *informative* if the receiver's posterior belief over $[0, 1]$ conditional on receiving some equilibrium signal is distinct from her prior belief; and say that an equilibrium is *influential* if the receiver's (equilibrium) strategy is not constant in (equilibrium) signals. Clearly, influential equilibria must be informative but the converse is not true. Finally, if $\alpha(\sigma(t)) = a$ for some $t \in T((m, b); \sigma)$, say that the pair (σ, α) *elicits* the action a. Thus at least two actions are elicited in any influential equilibrium, and we can identify one equilibrium as being more influential than another in terms of the relative number of distinct actions elicited in the two. For any equilibrium (σ, α) let

$$A(\sigma, \alpha) \equiv \cup_{t=0}^{t=1} \alpha(\sigma(t))$$

denote the set of actions elicited by (σ, α).

Because we are interested in the particular roles of cheap talk and costly signals, it is useful to disentangle their respective effects on the elicited actions. Evidently, for cheap talk *per se* to be influential at least two actions must be elicited in equilibrium by cheap talk messsages alone. Were this not the case then one can imagine an equilibrium (σ, α) in which $\sigma(t) = (m^\circ, 0)$ for all $t \leq \hat{t}$ and $\sigma(t) = (m_t, b(t))$ for all $t > \hat{t}$, $m_t \neq m^\circ$, and $b(t)$ strictly increasing in t. But in this case the cheap talk messages, while different for some pairs of types, are irrelevant for the receiver's decision. Similarly, for burned money *per se* to be influential at least two actions must be elicited in equilibrium by costly signals themselves. These remarks motivate the following:

Definition 2 *An equilibrium (σ, α) exhibits*

(i) influential cheap talk if $\exists t, t' \in [0, 1]$ such that $m(t) \neq m(t')$, $b(t) = b(t')$, and $\alpha(\sigma(t)) \neq \alpha(\sigma(t'))$;

(ii) influential costly signals if $\exists t, t' \in [0, 1]$ such that $b(t) \neq b(t')$ and $\alpha(\sigma(t)) \neq \alpha(\sigma(t'))$.

Note that an equilibrium exhibits both influential cheap talk and influential costly signals only if it elicits at least three actions.

The definition above treats cheap talk and burned money asymmetrically. Whereas no restrictions are imposed on the cheap talk component when defining influential costly signals, there is a restriction on the burned money component in the definition of influential cheap talk. To see why, suppose (m_1, b_1) and (m_2, b_2) are two signals sent in some equilibrium (σ, α)

with $m_1 \neq m_2$ and $b_1 \neq b_2$. Then there exists an equilibrium (σ', α') with $A(\sigma, \alpha) = A(\sigma', \alpha')$ in which types sending (m_2, b_2) in (σ, α) send (m_1, b_2) in (σ', α'); but there need not exist any equilibrium (σ', α') in which types sending (m_2, b_2) in (σ, α) send (m_2, b_1) in (σ', α'), or those sending (m_1, b_1) in (σ, α) send (m_1, b_2) in (σ', α'). For example, suppose (σ, α) is an equilibrium in which $\sigma(t) = (m_1, b)$ for all $t \in [0, t_1)$, $\sigma(t) = (m_2, b)$ for all $t \in [t_1, t_2)$ and $\sigma(t) = (m_3, b')$ for all $t \in [t_2, 1]$. Then (σ, α) exhibits influential cheap talk if $m_1 \neq m_2$, and (σ, α) exhibits influential costly signals if $b' \neq b$. Furthermore, there exists another equilibrium where $m_3 = m_2$ (or m_1) and all else stays the same.

3 Equilibrium

One class of equilibria involves the receiver ignoring any costly signal and any equilibrium in this class is a Crawford and Sobel cheap talk equilibrium (hereafter, CS equilibrium). Crawford and Sobel [6, 1437] show that "essentially" all such equilibria are of the following form: there is a finite partition $< t_0 \equiv 0, t_1, ..., t_N \equiv 1 >$ of the type space $[0, 1]$ such that, for all $i = 0, ..., N-1$ and all types $t \in (t_i, t_{i+1}]$, $\sigma(t) = m_i$ and $i \neq j$ implies $m_i \neq m_j$. The partitions are implicitly defined by the incentive compatibility conditions in $(e.1)$ which require boundary types t_i to be indifferent between sending the message associated with $(t_{i-1}, t_i]$ and that associated with $(t_i, t_{i+1}]$. Specifically, since no type uses burned money in a CS equilibrium, $(e.1)$ implies

$$\forall i = 1, ..., N-1, \; u^S(\alpha^*(m_{i-1}, 0), t_i, x) = u^S(\alpha^*(m_i, 0), t_i, x). \qquad (5)$$

Crawford and Sobel show that, for any $x > 0$, there is an equilibrium in which $N = 1$ and the equilibrium is neither informative nor influential. Further, for any $x > 0$ there exists a most influential equilibrium identified by the largest integer N such that $t_1 > 0$ and (5) holds; let $N(x)$ be this integer. Then Crawford and Sobel show that as $x \to 0$, $N(x) \to \infty$ and $t_i - t_{i-1} \to 0$, $i = 1, ..., N$, and that there exists some finite $\hat{x}$ such that $N(x) = 1$ for all $x > \hat{x}$. That is, if the sender's and the receiver's preferences coincide then the most influential equilibrium is fully separating in type whereas, if $x > \hat{x}$, the only CS equilibrium is wholly uninformative.

We argued in the Introduction that cheap talk messages are rarely the only feasible means of communication between a sender and a receiver.

What might be equilibria for this case? To begin, we derive some general properties of equilibria.

Given receiver beliefs $g(s|\cdot)$ and any pair of types $t \leq t'$, let

$$y(t,t') \equiv \arg\max_{a\in\Re} \int_t^{t'} u^R(a,s)g(s|\cdot)ds$$

and write $y(t) = y(t,t)$ to save notation. Under the assumptions on u^R, $y(t,t')$ is strictly increasing in both arguments.

Lemma 1 *Let (σ,α) be an equilibrium. Then:*

(i) $\alpha(\sigma(t)) = \alpha(\sigma(t'))$ $\forall t,t' \in [\underline{s},\bar{s}]$ implies $b(t) = b(t')$ $\forall t,t' \in [\underline{s},\bar{s}]$;

(ii) $\alpha(\sigma(t))$ strictly increasing in t on $[\underline{s},\bar{s}]$ implies $b(t)$ strictly increasing in t on $[\underline{s},\bar{s}]$;

(iii) $t > t'$ implies $\alpha(\sigma(t)) \geq \alpha(\sigma(t'))$.

Proof. By incentive compatibility, for all $t,t' \in [0,1]$, $t > t'$:

$$\begin{aligned} u^S(\alpha(\sigma(t)),t,x) - b(t) &\geq u^S(\alpha(\sigma(t')),t,x) - b(t') \\ u^S(\alpha(\sigma(t')),t',x) - b(t') &\geq u^S(\alpha(\sigma(t)),t',x) - b(t). \end{aligned}$$

Claim (i) follows immediately. To prove (ii), first note that since, by supposition, $\alpha(\sigma(\cdot))$ is strictly increasing on $[\underline{s},\bar{s}]$, $\sigma(\cdot)$ is separating on $[\underline{s},\bar{s}]$. By [6, Lemma 1], $x > 0$ and $\sigma(\cdot)$ separating imply $b(t) \neq b(t')$ $\forall t,t' \in [\underline{s},\bar{s}]$, $t \neq t'$ (with the cheap talk component of σ on the interval being irrelevant). In particular, $b(t)$ must be strictly monotone on $[\underline{s},\bar{s}]$ and so, by Royden [17], differentiable almost everywhere on the interval. Hence, given $\alpha(\cdot)$, $(e.1)$ implies that $\forall t \in (\underline{s},\bar{s})$,

$$\left.\frac{\partial[u^S(\alpha(\cdot,b(s)),t,x) - b(s)]}{\partial s}\right|_{s=t} = 0.$$

By $(e.2)$ and $\sigma(\cdot)$ separating on $[\underline{s},\bar{s}]$, $\alpha(\cdot,b(s)) = y(s)$. Substituting for $\alpha(\cdot)$ and doing the calculus, we find

$$u_1^S(y(t),t,x)y'(t) = b'(t).$$

By assumption, $u_{13}^S > 0$; so, for all $s \in [0,1]$ and $x > 0$,

$$y(s) < \arg\max_{a\in\Re} u^S(a,s,x) < \infty.$$

Hence, $u_{11}^S < 0$ and $u_{12}^R > 0$ imply $u_1^S(y(t), t, x)y'(t) > 0$ and, therefore, the equality implies $b'(t) > 0$ as claimed. Finally, adding the inequalities above and collecting terms yields

$$u^S(\alpha(\sigma(t)), t, x) - u^S(\alpha(\sigma(t)), t', x) \geq u^S(\alpha(\sigma(t')), t, x) - u^S(\alpha(\sigma(t')), t', x).$$

By assumption, $u_{12}^S > 0$ or, equivalently, $u^S(a, t, x)$ is supermodular in a and t. Claim (iii) now follows from $t > t'$.□

There are two implications of Lemma 1 worth making explicit. The first implication is that for any equilibrium (σ, α), the set of types eliciting a given action is convex. To see this, let $Z(a; \sigma, \alpha) = \{t : \alpha(\sigma(t)) = a, a \in \Re\}$. Then the claim is trivially true if $Z(a; \sigma, \alpha)$ is empty or singleton; if $t, t' \in Z(a; \sigma, \alpha)$, $t > t'$, then Lemma 1(iii) implies that for all $t^\circ \in (t, t')$, $\alpha(\sigma(t^\circ)) = a$ and hence $t^\circ \in Z(a; \sigma, \alpha)$. It is evident that for all $t, t' \in Z(a; \sigma, \alpha)$, we must have $b(t) = b(t')$; that is, $\sigma(t)$ can differ from $\sigma(t')$ in at most the cheap talk component of the signal. Therefore, by [6], for any equilibrium (σ, α), there exists an equilibrium (σ^*, α^*) in which $\sigma^*(t) = \sigma^*(t')$ for all $t, t' \in Z(a; \sigma^*, \alpha^*)$ and, for all $t \in [0, 1]$, $\alpha^*(\sigma^*(t)) = \alpha(\sigma(t))$. That is, if under (σ, α) there exist multiple cheap talk messages associated with $t \in Z(a; \sigma, \alpha)$, so all such messages elicit the action a, then there exists another equilibrium in which all $t \in Z(a; \sigma, \alpha)$ send the same message and elicit the action a. As with Crawford and Sobel, "essentially" all equilibria have a partition structure in which types in any element of the partition either pool together by choosing the same signal and eliciting the same action, or all separate by choosing distinct costly signals and eliciting distinct actions.

The second implication of Lemma 1 is that the receiver's equilibrium strategy $\alpha(\sigma(\cdot))$, being monotonic and having range $[0, 1]$, must be differentiable almost everywhere on $[0, 1]$ ([17]). Since the sender's equilibrium payoff $U^S(\cdot, t, \cdot)$ is clearly continuous in t, this in turn implies that the burned money component of the sender's equilibrium strategy, $b(\cdot)$, must likewise be differentiable almost everywhere on $[0, 1]$ (although not necessarily monotonic). In particular, Lemma 1 gives that the only points at which, in equilibrium, $b(\cdot)$ can be decreasing are discontinuity points (which only occur at the boundaries of partition segments). Moreover, if at some $t' > 0$ the sender's equilibrium strategy moves from a pooling to a separating segment as t increases, then necessarily $\lim_{\varepsilon \downarrow 0} b(t' - \varepsilon) < \lim_{\varepsilon \downarrow 0} b(t' + \varepsilon)$. To see this, for $s \in (t, t')$ with $t < t'$, let $\bar{y} = \alpha(\sigma(s)) = \alpha(m, b)$ and suppose $\sigma(\cdot)$ is separating on $(t', t'']$, $t'' \geq t'$; then $s' \in (t', t'']$ implies $\alpha(\sigma(s')) = y(s')$.

Suppose $\lim_{\varepsilon\downarrow 0} b(t'-\varepsilon) = b \geq b' = \lim_{\varepsilon\downarrow 0} b(t'+\varepsilon)$. Since $x > 0$ and $t' > 0$, our assumptions on preferences give

$$\bar{y} < y(t') < \arg\max_{a\in\Re} u^S(a, t', x),$$

so $u^S(y(t'), t', x) > u^S(\bar{y}, t', x)$. But, by continuity, the incentive compatibility conditions implied by $(e.1)$ require

$$u^S(\bar{y}, t', x) - b = u^S(y(t'), t', x) - b',$$

which is impossible when $b \geq b'$.

The following result shows that we can "squeeze in" separating segments at the far end of *any* CS equilibrium partition.

Theorem 1 *Let (σ, α) be a CS equilibrium with supporting partition $< t_0 \equiv 0, t_1, ..., t_N \equiv 1 >$. Then for all $\hat{t} \in [0, t_1]$ there exists a partition $< s_0 \equiv 0, s_1 = \hat{t}, s_2, ..., s_N, s_{N+1} \equiv 1 >$ supporting an equilibrium $(\sigma, \alpha)(\hat{t})$ such that:*

$$\begin{aligned} \forall i &= 0, ..., N-1,\ \forall t \in [s_i, s_{i+1}),\ \sigma(t) = (m_i, 0),\ m_i \neq m_j\ \forall i \neq j; \\ \forall t &\in [s_N, 1],\ \sigma(t) = (m^\circ, b(t)), \end{aligned}$$

where $b(t) = \int_0^t u_1^S(y(s), s, x) y'(s) ds + C(s_N)$ and

$$C(s_N) = u^S(y(s_N), s_N, x) - u^S(y(s_{N-1}, s_N), s_N, x) - \int_0^{s_N} u_1^S(y(s), s, x) y'(s) ds.$$

Proof. Let (σ, α) denote the CS equilibrium in which N actions are elicited. Let $< t_0 \equiv 0, t_1, ..., t_N \equiv 1 >$ be the partition supporting (σ, α). This partition is defined by (5). Let $\hat{t} \in [0, t_1]$ and define the partition $< s_0, s_1, ..., s_{N+1} >$ by $s_0 = 0$, $s_1 = \hat{t}$, $s_{N+1} = 1$ and, $\forall i = 1, ..., N-1$,

$$u^S(y(s_{i-1}, s_i), s_i, x) = u^S(y(s_i, s_{i+1}), s_i, x).$$

To see that such a partition exists for any $\hat{t} < t_1$, fix $\hat{t} = s_1 \in [0, t_1)$. By definition of a CS equilibrium, $u^S(y(0, t_1), t_1, x) = u^S(y(t_1, t_2), t_1, x)$ and, therefore, $u^S(y(0, t_1), s_1, x) > u^S(y(t_1, t_2), s_1, x)$. By assumptions on the sender's and the receiver's preferences, $y(0, s_1) < \min\{\arg\max_{a\in\Re} u^S(a, s_1), y(0, t_1)\}$ and, by continuity and monotonicity of $y(t, t')$ in both arguments, there exists a type $s_2 > s_1$ such that $y(s_1, s_2) > y(0, s_1)$ and $u^S(y(0, s_1), s_1, x) = u^S(y(s_1, s_2), s_1, x)$. For any $s' < s < t$ let

$$V(s', s, t) = u^S(y(s', s), s, x) - u^S(y(s, t), s, x).$$

Holding s' fixed and setting $V(\cdot) \equiv 0$, implicitly differentiate V to obtain

$$\left.\frac{dt}{ds}\right|_{s'} = \frac{u_2^S(y(s',s),s,x) - u_2^S(y(s,t),s,x)}{u_1^S(y(s,t),s,x)y_2(s,t)}. \tag{6}$$

Since $V(s',s,t) \equiv 0$, $y(s',s) < \arg\max_{a\in\Re} u^R(a,s) < y(s,t)$; by $u_{11}^S(\cdot) < 0$, therefore, the denominator of (6) is strictly negative. And by assumption, $u_{12}^S(\cdot) > 0$ so the numerator of (6) is strictly negative also. Hence, $dt/ds|_{s'} > 0$, in which case $s_1 < t_1$ and $u^S(y(0,s_1),s_1,x) = u^S(y(s_1,s_2),s_1,x)$ imply $s_1 < s_2 < t_2$. Now fixing s_1 and s_2 and, *mutatis mutandis*, repeating the preceding argument, we find there exists $s_3 < t_3$ such that $u^S(y(s_1,s_2),s_2,x) = u^S(y(s_2,s_3),s_2,x)$; and so forth for $s_4,\ldots,s_N < 1$. By construction, for all $i = 0,...,N-1$ and for all $t \in [s_i,s_{i+1})$, the strategy $\sigma(t) = (m_i,0)$ is a best response to the receiver's strategy

$$\alpha(m_i,0) = \arg\max_{a\in\Re} \int_{s_i}^{s_{i+1}} u^R(a,s)g(s|m_i,0)ds \equiv y(s_i,s_{i+1}).$$

Similarly, this receiver strategy is a best response to $\sigma(t)$ on $[0,s_N)$.

For any $t \in [s_N,1]$, let $b(t) = \beta(t) + C(s_N)$ where

$$\beta(t) = \int_0^t u_1^S(y(s),s,x)y'(s)ds;$$

then for these types, $\sigma(t) = (m^\circ,\beta(t)+C(s_N))$. By earlier reasoning, $\beta(t) > 0$ and strictly increasing in $t \in [s_N,1)$. Suppose the receiver sees a signal $(m^\circ,b) \in M \times [\beta(s_N)+C(s_N),\beta(1)+C(s_N)]$. Given $\sigma(\cdot)$, $(e.3)$ implies $g(t|m^\circ,\beta(t)+C(s_N)) = 1$, all $t \in [s_N,1]$. Hence, by (2), $\alpha(m^\circ,b) = y(t)$ is the unique best response. For any out-of-equilibrium signal (m,b) with $m \neq m^\circ$ and $b \in [\beta(s_N)+C(s_N),\beta(1)+C(s_N)] \equiv [b(s_N),b(1)]$, the receiver is free to ignore the message m and set $\alpha(m,b) = \alpha(m^\circ,b)$. And for any out-of-equilibrium signal (m,b) with $b \notin [\beta(s_N)+C(s_N),\beta(1)+C(s_N)]$, the receiver is free to adopt identical out-of-equilibrium beliefs to those induced by the equilibrium signal $(m_0,0)$. Hence $\alpha(m,b) = y(0,s_1)$ is a best response in this case.

Now fix $\alpha(\cdot)$ as described above and consider the signaling strategy $\sigma(t)$ for $t \in [s_N,1]$. By $u_{12}^S(\cdot) > 0$ and $\alpha(m,b) \in \{\alpha(m_0,b),y(0,s_1)\}$ for all signals $(m,b) \in \sum$ such that $m \neq m^\circ$ or $b \notin [b(s_N),b(1)]$, $\sigma(t)$ is a best response if

$$(i)\ U^S(\alpha(\sigma(t)),t,m(t),b(t)) \geq U^S(\alpha(m_{N-1},0),t,m_{N-1},0),$$

and

$$(ii)\ \forall t' \in [s_N, 1], U^S(\alpha(\sigma(t)), t, m(t), b(t)) \geq U^S(\alpha(\sigma(t')), t, m(t'), b(t')).$$

Inequality (i) is equivalent to

$$u^S(y(t), t, x) - \int_0^t u_1^S(y(s), s, x)y'(s)ds - C(s_N) \geq u^S(y(s_{N-1}, s_N)), t, x).$$

By definition of $C(s_N)$ and $u_{12}^S(\cdot) > 0$, this inequality holds strictly for all $t \in (s_N, 1]$ and holds with equality at $t = s_N$. Hence every sender type $t > s_N$ prefers to send $\sigma(t)$ to any signal (m, b) such that $m \neq m^\circ$ or $b \notin [b(s_N), b(1)]$. Given this and $\beta'(s) > 0$ on $[s_N, 1]$, the optimization problem of the sender of type t is equivalent to choosing:

$$b \in \arg\max_{b \geq 0} u^S(y(\beta^{-1}(b - C(s_N))), t, x) - b.$$

By strict concavity, the solution to this problem is uniquely solved by $b \geq 0$ such that

$$u_1^S(y(\beta^{-1}(b - C(s_N))), t, x)y'(\cdot)\frac{\partial \beta^{-1}(\cdot)}{\partial b} = 1.$$

By the inverse function rule, this equation is equivalent to

$$b'(s) = u_1^S(y(s), t, x)y'(s)$$

which, by definition of $b(\cdot)$, is solved at $s = t$. Hence, inequality *(ii)* holds and $\sigma(\cdot)$ is a best response to $\alpha(\cdot)$ as required.□

Figure 1 illustrates the theorem.

Figure 1 here

Theorem 1 immediately implies that although the same *number* of cheap talk messages are sent in the equilibrium $(\sigma, \alpha)(\hat{t})$ as in the benchmark CS equilibrium, as $\hat{t}$ goes to zero the inferences the receiver draws from at least one such message become increasingly precise. In other words, the precision of cheap talk messages in the presence of burned money can increase relative to that when burned money is unavailable. Moreover, it is immediate from the theorem that if there exists a CS equilibrium that elicits $N \geq 1$ actions, there exists an equilibrium $(\sigma, \alpha)(\hat{t})$ in which N actions are elicited through influential cheap talk and at least one action is elicited through influential burned money. In particular, because Theorem 1 goes through for $N = 1$, we have the following corollary.

Corollary 1 *For any* $t \in [0,1]$ *let* $\beta(t) = \int_0^t u_1^S(y(s), s, x) y'(s) ds$ *and, for any* $\hat{t} \in [0,1]$, *let*

$$C(\hat{t}) = u^S(y(\hat{t}), \hat{t}, x) - u^S(y(0, \hat{t}), \hat{t}, x) - \int_0^{\hat{t}} u_1^S(y(s), s, x) y'(s) ds.$$

Then for all $x > 0$ *and for all* $\hat{t} \in [0,1]$ *there exists an equilibrium such that, for all* $t \in [0, \hat{t}]$, $\sigma(t) = (m^\circ, 0)$ *and, for all* $t \in (\hat{t}, 1]$, $\sigma(t) = (m^\circ, \beta(t) + C(\hat{t}))$.

Proof. For any $x > 0$, there exists an uninformative CS equilibrium that elicits exactly one action. All such equilibria are supported by the degenerate partition $< t_0 \equiv 0, t_1 \equiv 1 >$. Let $< 0, \hat{t}, 1 >$ be a binary partition of $[0,1]$ with $\hat{t} \in [0,1]$. Define the strategies $(\sigma, \alpha)(\hat{t})$ as in Theorem 1 with $N = 1$, save having the boundary type $\hat{t}$ pool with $t < \hat{t}$ rather than separate. By Theorem 1, $(\sigma, \alpha)(\hat{t})$ is an equilibrium, completing the proof.□

When $\hat{t} = 0$, the equilibrium $(\sigma, \alpha)(0)$ is fully separating in t on $[0,1]$; and when $\hat{t} = 1$, the equilibrium $(\sigma, \alpha)(1)$ is pooling in t on $[0,1]$. Therefore, because the most influential cheap talk equilibrium when $x = 0$ is separating, Corollary 1 shows that there exists a separating equilibrium to the sender/receiver game for *every* value of x (conditional on the sender having a sufficiently lax budget constraint). In particular, for all $x \geq 0$, the equilibria $(\sigma, \alpha)(0)$ and $(\sigma, \alpha)(1)$ are the extremes of a continuum of semi-pooling equilibria: $(\sigma, \alpha)(\hat{t})$, $\hat{t} \in [0,1]$.

Theorem 1 implies that a sufficient condition for there to exist equilibria exhibiting both influential cheap talk and influential costly signals, is that there exist influential CS equilibria. In some settings this is also a necessary condition (see Proposition 1, below). More generally, in any influential CS equilibrium the lowest two actions (i.e. the first two distinct actions elicited by the lowest types) are by definition elicited by influential cheap talk. Hence Theorem 1 claims that, when such a CS equilibrium exists, there is an equilibrium in which the lowest two actions are also elicited by influential cheap talk and at least one higher action is elicited by burned money. Conversely, the next result implies that if there is any equilibrium in which the lowest two actions are elicited by influential cheap talk, then there is an influential CS equilibrium.

Definition 3 *Let the partition* $< 0, t_1, \ldots, t_{N-1}, 1 >$ *support an equilibrium* (σ, α). *Say that* (σ, α) *is a left-pooling influential equilibrium if* $t_1 > 0$ *and*

(i) $N \geq 2$;

(ii) $\forall t \in [0, t_1)$, $\sigma(t) = (m, b)$;

(iii) $b \geq \lim_{\varepsilon \downarrow 0} b(t_1 + \varepsilon)$.

As we argued earlier, if the sender's equilibrium strategy is pooling on some interval (t, t') and separating on the adjacent interval $(t', t'']$, then necessarily the burned money component of the sender's strategy must have a discontinuous upwards jump at t'. Thus an influential equilibrium is left-pooling *either* if $b = \lim_{\varepsilon \downarrow 0} b(t_1 + \varepsilon)$, in which case the lowest two equilibrium actions are elicited by influential cheap talk, *or* if the first change in the equilibrium level of burned money as t increases from zero is discontinuous downwards at t_1 to another pooling segment. In either case, that is, types in (t_1, t_2) must pool on the same signal, say $(m', b') \neq (m, b)$.

Theorem 2 *There exists a left-pooling influential equilibrium if and only if there exists an influential CS equilibrium.*

Proof. Theorem 1 establishes sufficiency. To prove necessity, let (σ, α) be a left-pooling influential equilibrium. Then $N \geq 2$ and $0 < t_1 < t_2 \leq 1$. Let $\lim_{\varepsilon \downarrow 0} b(t_1 + \varepsilon) = b'$. By continuity and incentive compatibility,

$$u^S(y(0, t_1), t_1, x) - b = u^S(y(t_1, t_2), t_1, x) - b',$$

where, for all $t \in [0, t_1)$, $\alpha(\sigma(t)) = \alpha(m, b) = y(0, t_1)$ and, for all $t \in (t_1, t_2]$, $\alpha(\sigma(t)) = \alpha(m', b') = y(t_1, t_2)$, where $(m', b') \neq (m, b)$. By assumption, $b \geq b'$ so that this equality implies

$$V(0, t_1, t_2) = u^S(y(0, t_1), t_1, x) - u^S(y(t_1, t_2), t_1, x) \geq 0.$$

Recall $y(r, s)$ is strictly increasing in both arguments and, by assumption, $u^S_{12} > 0$. Hence,

$$V(0, t_1, 1) = u^S(y(0, t_1), t_1, x) - u^S(y(t_1, 1), t_1, x) \geq 0,$$

with strict inequality if $t_2 < 1$. Similarly, $y(t) < \arg\max_{a \in \Re} u^S(a, t, x)$ for all $t \in [0, 1]$ implies $V(0, 1, 1) < 0$. Therefore, by continuity of best-response actions and equilibrium payoffs in type, $y(0, t_1) < y(0, 1) < y(1)$ implies there exists some $t^* \in [t_1, 1)$ such that $V(0, t^*, 1) = 0$. Now, for all $s \in [0, t^*]$, let $\hat{\sigma}(s) = (m, 0)$; for all $s \in (t^*, 1]$, let $\hat{\sigma}(s) = (m', 0)$ with $m' \neq m$; and let $\hat{\alpha}(m,0) = y(0,t^*)$ and $\hat{\alpha}(m',0) = y(t^*,1)$. Then, by construction, $(\hat{\sigma},\hat{\alpha})$ is an influential CS equilibrium (with out-of-equilibrium messages identified by the receiver with, say, the equilibrium message m).□

Theorems 1 and 2 say that, in the present model, the existence of burned money can improve the precision of cheap talk communication but may not expand the set of environments in which cheap talk is credible. However, Theorem 2 cannot be extended to cover all influential equilibria: there are circumstances in which equilibria exist exhibiting both influential cheap talk and influential burned money, but there is no influential CS equilibrium (of course, such equilibria cannot be left-pooling influential equilibria). To see this, let the prior distribution on types, $h(\cdot)$, be a beta distribution on $[0,1]$ with parameters (μ, v) and assume preferences are quadratic:

$$\begin{aligned} U^S(a,t,m,b) &= -(x+t-a)^2 - b; \\ U^R(a,t,m,b) &= -(t-a)^2. \end{aligned}$$

Then for all t, $\arg\max_{a\in\Re} U^S(a,t,\cdot) = t + x > y(t) = t$. Assume the following parameterization obtains:

$$\begin{aligned} x &= 0.1157 \\ (\mu, v) &= (10, 2). \end{aligned}$$

Then (to four decimal places), $y(0,1) = 0.1667$ and it can be checked that there exists no influential CS equilibrium. However, the following (again to four decimal places) describes an influential equilibrium (σ, α):

$$\begin{aligned} \forall t &\in [0, 0.15),\ \sigma(t) = (m, 2xt) \text{ and } \alpha(\sigma(t)) = t; \\ \forall t &\in [0.15, 0.2),\ \sigma(t) = (m', 0.0397) \text{ and } \alpha(\sigma(t)) = 0.1739; \\ \forall t &\in [0.2, 1],\ \sigma(t) = (m'', 0.0397) \text{ and } \alpha(\sigma(t)) = 0.2889, \end{aligned}$$

where $m' \neq m''$. (Notice that the maximal burned money by any separating type is strictly smaller than the amount sent by types distinguished by their cheap talk signal; i.e. $2x(0.15) = 0.0347 < 0.0397$.) Figure 2 illustrates the equilibrium.

Figure 2 here

The example shows there are situations in which the existence of burned money induces influential equilibrium cheap talk when it would otherwise be impossible.[4] Although we have been unable to provide a general theorem characterizing such situations (and it seems unlikely that one is readily available), Proposition 1, below, covering the special case of $(\mu, \upsilon) = (1, 1)$ (i.e. $h(\cdot)$ uniform on $[0, 1]$) suggests that some sort of asymmetry is necessary. Furthermore, the uniform assumption on the distribution of types, along with that of quadratic preferences, has been much used in applied theoretical work involving cheap talk information transmission. Consequently, we consider it in some detail.

4 Example

Apart from providing a useful and salient illustration of the preceding results, the quadratic/uniform specification allows us to say something about the welfare properties of the equilibria identified above. And since these properties are the basis of the equilibrium selections made in the applied literature to permit comparative static results (e.g. [10]), checking the robustness of the properties to the introduction of burned money is important.

Throughout this section, assume the prior distribution over types, $h(\cdot)$, is uniform on $[0, 1]$ and that preferences are quadratic as defined above. Then for all $s \le t$, $y(s, t) = [s + t]/2$ (and, as before, $\arg\max_{a \in \Re} U^S(a, t, \cdot) = t + x > y(t) = t$). Applying (5), [6, 1440-4] show the arbitrage conditions characterizing any CS equilibrium here are

$$t_0 = 0; t_N = 1; \text{ and } \forall i = 1, ..., N,\ t_i = t_1 i + 2i(i-1)x. \tag{7}$$

Given x and $t_N = 1$, these equations imply $t_1 \equiv t_1(N) = [1 - 2N(N-1)x]/N$. Thus, the number of actions elicited in the most influential equilibrium, $N(x)$, identified by the largest integer N such that $t_1(N) > 0$, goes to

[4] An associate editor offered the following finite game example. The sender may be one of three equally likely types, $\{t_1, t_2, t_3\}$, and the receiver's action set contains three alternatives, $\{a_1, a_2, a_3\}$. Dollar payoffs, $(v^S(a, t), v^R(a, t))$, without burned money are as follows:

	a_1	a_2	a_3
t_1	1, 1	0, 0	2, −10
t_2	0, 0	1, 2	2, −10
t_3	0, 0	1, −10	100, 1

It is easy to check that there is no influential cheap talk equilibrium. If, however, burned money is available, there is an equilibrium in which the type-t_3 sender burns two dollars and types t_1 and t_2 separate with cheap talk. Clearly, the asymmetry in payoffs across types is important here.

infinity as x goes to zero and equals one for all $x > 1/4$; i.e. if $x > 1/4$ the only CS equilibrium is wholly uninformative.

Now permit burned money to be used and consider the equilibrium constructed in Theorem 1. Suppose the partition $< t_0 \equiv 0, t_1, ..., t_N \equiv 1 >$ supports a CS equilibrium in which N actions are elicited. Then for any $\hat{t} \in [0, t_1]$, the partition $< s_0, s_1, ..., s_{N+1} >$ defined in the proof to Theorem 1 is characterized by

$$s_0 = 0; s_1 = \hat{t}; s_{N+1} = 1; \text{ and } \forall i = 2, ..., N, \; s_i = s_1 i + 2i(i-1)x. \qquad (8)$$

Clearly, $s_N \leq 1$ with strict inequality if $\hat{t} < t_1$. Let $\sigma(\cdot)$ be as defined in the theorem for the partition $< s_0, s_1, ..., s_{N+1} >$; then $(e.2)$ and the uniform prior imply $\alpha(m_i, 0) = [s_{i+1} + s_i]/2$ for all $i = 0, 1, \ldots, N-1$. Doing the calculations, similarly derive $\sigma(t) = (m^\circ, b(t))$ and $\alpha(m^\circ, b(t)) = t$ for all $t \in [s_N, 1]$, where

$$\begin{aligned} b(t) &= 2xt + C(s_N) \\ &= 2xt + [s_N + s_{N-1}][\frac{1}{4}(s_N - s_{N-1}) - x]. \end{aligned}$$

Given the CS equilibrium, equations (7) imply $(t_i - t_{i-1}) = t_1 + 4x(i-1)$, all $i = 1, ..., N$. Similarly, equations (8) above imply that for any $s_1 \in [0, t_1)$, $(s_i - s_{i-1}) = s_1 + 4x(i-1)$. Thus, for all $i = 0, ..., N-1$, the length of the interval of types sending any given message $(m_i, 0)$ shrinks as s_1 goes to zero. So although the same *number* of cheap talk messages are sent in the equilibrium supported by (8) as in the benchmark CS equilibrium defined by (7), as s_1 goes to zero the inference the receiver draws froms *any* such message becomes increasingly precise. Furthermore, with a uniform prior and quadratic preferences, Theorem 2 can be strengthened.

Proposition 1 *Suppose preferences are quadratic and the distribution of types is uniform. Then there exists an equilibrium exhibiting influential cheap talk if and only if there exists an influential CS equilibrium.*

Proof. Sufficiency follows from Theorem 1. To check necessity, first recall that, by an earlier observation, there exists an influential CS equilibrium if and only if $x < 1/4$. Now suppose (σ, α) is an equilibrium exhibiting influential cheap talk. Then $\exists t, t'$ such that $\sigma(t) = (m, b)$, $\sigma(t') = (m', b)$ and $\alpha(\sigma(t)) \neq \alpha(\sigma(t'))$. Let $a_i = \alpha(\sigma(t))$ and $a_j = \alpha(\sigma(t'))$. Let $Z_\ell \equiv Z(a_\ell; \sigma, \alpha)$, $\ell = i, j$. By earlier arguments, we know that Z_i and Z_j are disjoint convex sets. Let $t_\ell = \inf Z_\ell$, $t_{\ell+1} = \sup Z_\ell$, $\ell = i, j$. Without loss of generality, assume $t_{i+1} \leq t_j$. There are two cases: (i) $t_{i+1} = t_j$, or (ii) $t_{i+1} < t_j$.

(i) Suppose $t_{i+1} = t_j$, so $j = i+1$. Then by $(e.2)$ and $(e.3)$, $a_i = [t_i + t_{i+1}]/2$ and $a_j = [t_{i+1} + t_{i+2}]/2$. Hence, by continuity of equilibrium $U^S(\cdot, t, \cdot)$ in t, incentive compatibility requires:

$$-(x + t_{i+1} - a_i)^2 = -(x + t_{i+1} - a_j)^2.$$

And this equality holds if and only if $x = [t_i + t_{i+1} - 2t_{i+1}]/4$. But $0 \leq t_i < t_{i+1} < t_{i+2} \leq 1$ implies $\sup\{[t_i + t_{i+2} - 2t_{i+1}]/4\} = 1/4$.

(ii) Suppose $t_{i+1} < t_j$, so $j > i+1$. If $b(\cdot)$ is weakly increasing on (t_i, t_{j+1}), then $b(t) = b \ \forall t \in (t_i, t_{j+1})$ in which case, letting $a_k \in A(\sigma, \alpha)$ be such that $\inf Z(a_k; \sigma, \alpha) = t_{i+1}$, setting $j = k$ gives us situation (i). Therefore, $b(\cdot)$ must be decreasing somewhere on (t_i, t_{j+1}). Thus, by Lemma 1, $\exists t_k \in (t_i, t_{j+1})$ such that $b(\cdot)$ is discontinuous at t_k and, $\forall \epsilon > 0$ sufficiently small, $b(t_k - \epsilon) > b(t_k + \epsilon)$. Let $b_1 = \lim_{\varepsilon \downarrow 0} b(t_k - \epsilon)$, $b_2 = \lim_{\varepsilon \downarrow 0} b(t_k + \epsilon)$ and, without loss of generality, assume $\sigma(t_k) = (m_1, b_1)$. There are then two possibilities: either $b(\cdot)$ is separating or $b(\cdot)$ is pooling on some subinterval (t_k, t_{k+1}). Let $\bar{a} = \alpha(\sigma(t_k))$. If $b(\cdot)$ is separating on (t_k, t_{k+1}), continuity of equilibrium $U^S(\cdot, t, \cdot)$ in t and incentive compatibility require t_k indifferent between eliciting $\bar{a}$ and eliciting $y(t_k) = t_k$. Therefore,

$$(x + t_k - \bar{a})^2 - x^2 = b_2 - b_1.$$

By $Z(\bar{a}; \sigma, \alpha)$ convex, $(t_k - \bar{a}) \geq 0$. Hence the LHS of this equality is nonnegative but $b_1 > b_2$. So $b(\cdot)$ separating on (t_k, t_{k+1}) is not possible. Assume $b(\cdot)$ is pooling on (t_k, t_{k+1}); then, for $t \in (t_k, t_{k+1})$, $(e.2)$ and $(e.3)$ imply $\alpha(\sigma(t)) = [t_k + t_{k+1}]/2$. Therefore, by continuity of equilibrium $U^S(\cdot, t, \cdot)$ in t, incentive compatibility requires:

$$-(x + t_k - \bar{a})^2 - b_1 = -(x + t_k - [t_k + t_{k+1}]/2)^2 - b_2,$$

which implies

$$x = \frac{(t_k - t_{k+1})^2}{4(t_k + t_{k+1} - 2\bar{a})} - \frac{\Delta + (t_k - \bar{a})^2}{(t_k + t_{k+1} - 2\bar{a})}$$

where $\Delta = (b_1 - b_2)$. Since $\bar{a} \leq t_k < t_{k+1} \leq 1$ and $\Delta > 0$, the RHS of the equation is strictly less than $1/4$. This completes the proof of necessity.□

With the example of the previous section, in which preferences are quadratic but the distribution of types is asymmetric, Proposition 1 indicates that the possibility of burned money inducing influential cheap talk depends in some way on asymmetries in the environment. Specifically, there has to be sufficient difference between the payoffs high types can expect to achieve relative to low types, for otherwise the high types would be unwilling to burn the necessary money to dissuade low types from pooling with them in equilibrium.

For the family of equilibria identified in Corollary 1, in which only burned money is influential, the costly component of the signaling strategy for types greater than $\hat{t}$ is simply $b(t) = 2xt + \frac{\hat{t}}{4}[\hat{t} - 4x]$. Thus a semi-pooling equilibrium of this form exists for $\hat{t} \in [0, 1]$ if and only if the sender has a budget of at least $b(1) = 2x + \frac{\hat{t}}{4}[\hat{t} - 4x]$ to burn; in particular, a fully separating equilibrium in burned money (i.e. where $\hat{t} = 0$) exists if and only if the sender has a budget of at least $2x$.

Now consider some welfare properties of the equilibria for the quadratic preference and uniform prior case. Often a criterion of *ex ante* (i.e. before Nature reveals the sender's type t) efficiency is invoked to justify focussing on the most influential rather than any less influential equilibrium (e.g. [6]; [10]; [11]; [2]): only the most influential equilibrium is *ex ante* efficient and, moreover, it uniquely defines the most that cheap talk can achieve in the game. In this context, the following results, for the quadratic preference and uniform prior specification, are of some interest (the proofs of which, largely being tedious algebra, are omitted and available from the authors on request).

For any equilibrium $\eta = (\sigma, \alpha)$, let $\bar{u}^S(t, x; \eta)$ denote the η-equilibrium payoff to the sender of type $t \in [0, 1]$ given $x > 0$, and let $\bar{u}^R(t; \eta)$ denote the η-equilibrium payoff to the receiver given the sender is type t. Let $CS(N)$ denote a CS equilibrium in which N actions are elicited (so $CS(N(x))$ is the (unique) most influential CS equilibrium at x) and, for any CS equilibrium (σ, α), let $(\sigma, \alpha)(\hat{t})$ be the equilibrium strategy pair constructed in the proof to Theorem 1.

Proposition 2 *Suppose preferences are quadratic and the distribution of types is uniform. Suppose $t_1 > 0$ in the partition supporting the $CS(N(x))$ equilibrium for $x > 0$. Then for all $\hat{t} \in [0, t_1)$,*

$$\int_0^1 \bar{u}^S(t, x; CS(N(x)))dt > \int_0^1 \bar{u}^S(t, x; (\sigma, \alpha)(\hat{t}))dt;$$

$$\int_0^1 \bar{u}^R(t; CS(N(x)))dt < \int_0^1 \bar{u}^R(t; (\sigma, \alpha)(\hat{t}))dt.$$

The benefit to the sender from playing an equilibrium $(\sigma, \alpha)(\hat{t})$ over the most influential CS equilibrium, $CS(N(x))$, is in the reduction of variance in final payoffs that $(\sigma, \alpha)(\hat{t})$ affords; the cost is in terms of the expected costly signal conditional on the realization of t being sufficiently high. When preferences are quadratic and there is a uniform prior on the unknown parameter (t), therefore, the proposition shows that the expected cost of burned money dominates the expected gain from more precise cheap talk. In the presence of costly signals, therefore, the *ex ante* selection criterion no longer yields a unique equilibrium. It follows that results in the applied literature that exploit such a selection need further qualification.

More generally, the *ex ante* welfare criterion is suspect since it is sensitive to monotonic type-specific transformations of the sender's utility schedule. Specifically, suppose we rescale utilities so that

$$\tilde{U}^S(a, t, m, b) = -v(t)[(x + t - a)^2 + b]$$

with $v(t) > 0$ all $t \in [0, 1]$. Then for every $t \in [0, 1]$, the sender's *interim* optimal behaviour (i.e. once t is revealed) is invariant to the choice of $v(t)$. But it is easy to see that any *ex ante* welfare calculation is certainly not invariant to the choice of $v(t)$ across t. It is of some interest, therefore, to identify circumstances under which *interim* and *ex ante* calculations yield the same prediction. In particular, because the receiver is clearly best off in the separating equilibrium with costly signals, $(\sigma, \alpha)(0)$, identified in Corollary 1, the interesting questions involve the sender's welfare.

An equilibrium (σ, α) is said to be $SP(\hat{t})$ if and only if σ is as defined in Corollary 1, $\hat{t} \in [0, 1]$. Then $SP(0)$ and $SP(1)$ are, respectively, the fully separating equilibrium and the fully pooling equilibrium.

Proposition 3 *Suppose preferences are quadratic and the distribution of types is uniform. For all $x > 0$ and $t \in [0, 1]$, $\bar{u}^S(t, x; CS(N(x))) \geq \bar{u}^S(t, x; SP(0))$ with strict inequality for a set of types with strictly positive measure.*

Proposition 3 says that all sender types prefer the most influential CS equilibrium, $CS(N(x))$, to the most influential (separating) equilibrium, $SP(0)$. (Of course, the receiver's preferences are the reverse.) Consequently, the proposition immediately yields,

Corollary 2 *Let* $\tilde{U}^S(a,t,m,b) = -v(t)[(x+t-a)^2+b]$ *be the sender's payoff,* $v(t) > 0$ *for all* $t \in [0,1]$ *and assume the distribution of types is uniform. Then, for all* $x \geq 0$*, the sender ex ante strictly prefers* $CS(N(x))$ *to* $SP(0)$.

Because Proposition 3 obtains when $N(x) = 1$, a plausible conjecture is that for any semi-pooling equilibrium $SP(\hat{t})$, $\hat{t} \in (0,1)$, all sender types likewise prefer the pooling equilibrium $SP(1)$ to any $SP(\hat{t})$ equilibrium, and prefer any $SP(\hat{t})$ equilibrium to the separating equilibrium $SP(0)$. However, so long as $x < 1/2$ (in the current specification) this conjecture is false in general. In particular, it can be shown that for any $CS(N)$ equilibrium (σ,α), there exists a $(\sigma,\alpha)(\hat{t})$ equilibrium under which some sender types strictly prefer the $(\sigma,\alpha)(\hat{t})$ equilibrium to the $CS(N)$ equilibrium [3]. For example, if $N = 1$ and $x \in (1/4, 1/2)$ the relevant $(\sigma,\alpha)(\hat{t})$ equilibria are the semi-pooling equilibria $SP(\hat{t})$ and, for any $\hat{t} \in (4x-1, 1)$, only moderate types strictly prefer the $CS(1)$ (equivalently, the $SP(1)$) to the $SP(\hat{t})$ equilibrium. Furthermore, since a positive measure of both high and low extreme types hold the opposite strict preference to that of the moderates, the set of types strictly preferring the $SP(\hat{t})$ equilibrium to the $CS(1)$ equilibrium here is not convex. Thus *ex ante* welfare calculations for the model are not always invariant to how payoffs are scaled across types.

5 Conclusion

The Crawford and Sobel model of cheap talk communication has been widely applied, and the extent to which results from such applications are robust depend in part on the extent to which the polar case of cheap talk only is a good approximation to a world in which both cheap talk and burned money might be used to signal information. The equilibrium results reported here suggest that if the sender has sufficient resources the polar case may be misleading, and so care should be exercised in interpreting applied results that rest on this case. In particular, Theorem 1 and its corollary showed how the burning money option can be used to signal essentially any amount of information, up to and including separation. But more importantly, we have also demonstrated a more indirect effect of this option, namely the ability to signal through cheap talk: while the conditions for influential cheap talk to exist in the Crawford/Sobel model are equivalent to those for a class of equilibria in the presence of burning money (Theorem 2), an example shows how burning money can actually allow for influential cheap talk when it could not otherwise exist. We also proved how the environments in which this indirect effect can occur must necessarily depart from the standard uniform/quadratic specification of the model (Proposition 1).

In many applied problems the focus is on the most influential of the available equilibria. Justifications for such a focus typically rest on *ex ante* efficiency arguments or on identifying the upper bound on credible information transmission. Our results indicate that, from the informed party's perspective, the ability to send a costly signal with burned money generates a conflict between these two rationales. Specifically, Proposition 3 demonstrates that (at least in the quadratic preference and uniform prior environment) even at the interim stage, the informed party invariably prefers the most influential cheap talk equilibrium to the fully separating (and hence fully informative) equilibrium. On the other hand, the remarks following the proposition indicate that a mix of cheap talk and separation through burned money can be preferred by a subset of informed sender types to *any* cheap talk equilibrium.

References

[1] Austen-Smith, D. 1990. Information transmission in debate. *American Journal of Political Science* 34, 124-52.

[2] Austen-Smith, D. 1993. Interested experts and policy advice: multiple referrals under open rule. *Games and Economic Behavior* 5, 3-43.

[3] Austen-Smith, D. and J. Banks. 1995. Cheap talk and burned money. UR Working Paper.

[4] Blume, A. 1994. Equilibrium refinements in sender-receiver games. *Journal of Economic Theory* 64, 66-77.

[5] Blume, A., Y-G. Kim and J. Sobel 1993. Evolutionary stability in games of communication. *Games and Economic Behavior* 5, 547-76.

[6] Crawford, V. and J. Sobel 1982. Strategic information transmission. *Econometrica* 50, 1431-51.

[7] Farrell, J. 1993. Meaning and credibility in cheap talk games. *Games and Economic Behavior* 5, 514-31.

[8] Farrell, J. and R. Gibbons 1989a. Cheap talk with two audiences. *American Economic Review* 79, 1214-23.

[9] Farrell, J. and R. Gibbons 1989b. Cheap talk can matter in bargaining. *Journal of Economic Theory* 48, 221-37.

[10] Gilligan, T. and K. Krehbiel 1987. Collective decision making and standing committees: an informational rationale for restictive amendment procedures. *Journal of Law, Economics and Organization* 3, 145-193.

[11] Gilligan, T. and K. Krehbiel. 1989. Asymmetric information and legislative rules with a heterogenous committee. *American Journal of Political Science* 33, 459-490.

[12] Harrington, J. 1992. The revelation of information through the electoral process: an exploratory analysis. *Economics and Politics* 4, 255-76.

[13] Johnston, B. (ed) 1987. *Collier's Encyclopedia v.20.* New York: MacMillan.

[14] Matthews, S. 1989. Veto threats: rhetoric in a bargaining game. *Quarterly Journal of Economics* 104, 347-69.

[15] Matthews, S., M. Okuno-Fujiwara and A. Postlewaite 1991. Refining cheap talk equilibria. *Journal of Economic Theory* 55, 247-73.

[16] Rabin, M. 1990. Communication between rational agents. *Journal of Economic Theory* 51, 144-70.

[17] Royden, H.L. 1968. *Real Analysis (2e)* New York: Collier MacMillan.

[18] van Damme, E. 1989. Stable equilibria and forward induction. *Journal of Economic Theory* 48, 476-96.

Figure 1

Illustration of Theorem 1 for a binary CS equilibrium supported by partition $< 0, t_1, 1 >$

For all $t \in [\tau_{i-1}, \tau_i)$, $\tau = s, t$ and $i = 1, 2$, $\sigma(t) = (m_{i-1}, 0)$

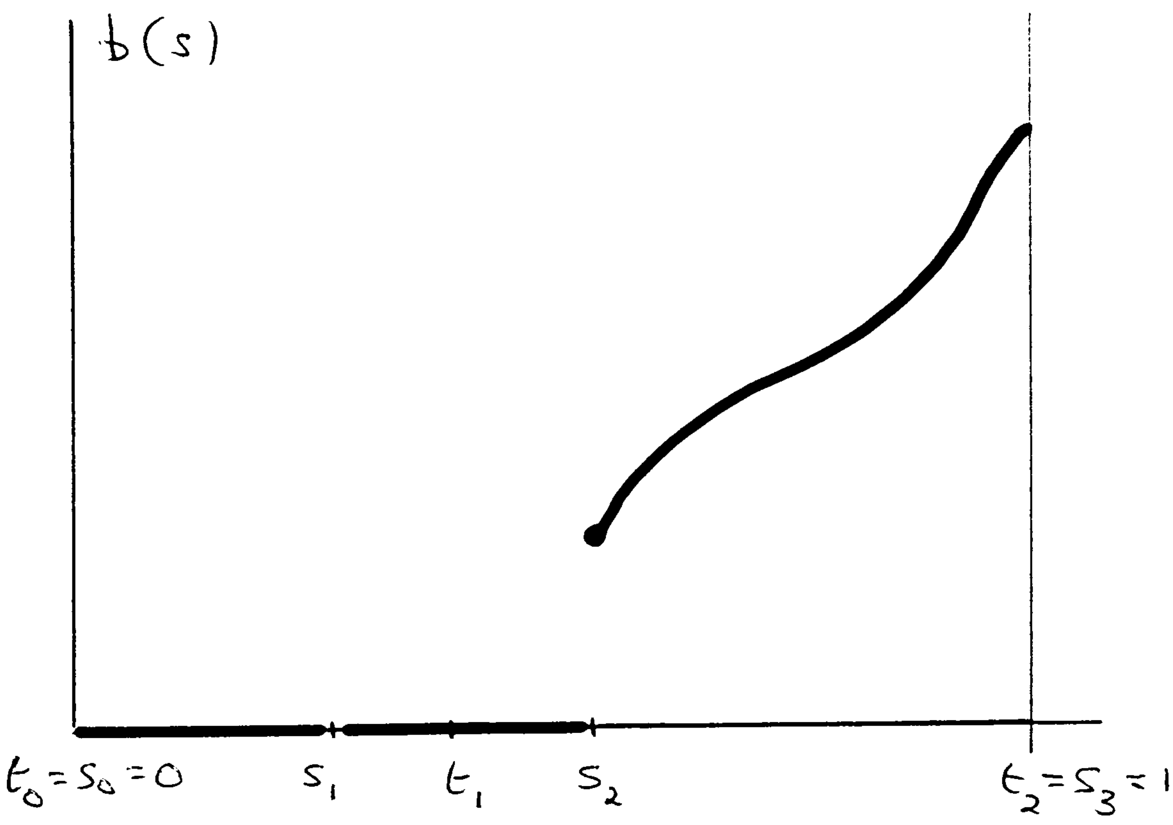

Figure 2

Burned money can induce influential cheap talk signaling