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Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice

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Abstract

We analyze a market where the consumer must rely on experts to identify the correct type of service. Medical services, repair services and various types of consulting and advisory services belong to this broad category. Our focus is on situations where the diagnosis of the consumer's needs is costly and the expert's effort is unobservable. We develop a model where experts offer competing contracts and consumers may gather multiple opinions. In various contractual settings, we explore the incentives that a competitive sampling of prices and opinions provides for experts to exert effort. We find that there is a tension between price competition and the quality of the advise provided in equilibrium. Under all the contracting scenarios considered, the equilibrium fails to realize the second best welfare optimum. In some of the cases, no gains from trade are realized. On the other hand, limiting price competition via price control increases total welfare.

1. Introduction

This paper analyzes a model of the provision of 'credence' services. A credence service has the property that the agent who purchases the service (the *principal*) must rely on *experts* to identify the correct type of service. Medical services, repair services and various types of consulting and advisory services belong to this broad category. Our focus are situations where the diagnosis of the consumer's needs is costly and the expert's effort is unobservable. We develop a model where experts offer competing contracts and consumers may gather multiple opinions. In various contractual settings we explore the incentives that a competitive sampling of prices and opinions provides for experts to exert effort.

The provision of credence services is beset by a number of information problems. Here, we are concerned with a class of scenarios in which the effort invested by the expert in diagnosing the principal's need is unobservable to the principal and the final success of the treatment is not contractible (say, because it is not easily or objectively measurable). We also focus on a the role of a specific mechanism—the gathering of multiple opinions—in mitigating the information problem and disciplining the expert's behavior.

There are of course other potential information problems[1] and other important forces[2] that work to mitigate these problems. We do not underestimate the importance of these missing elements. Rather, we put them aside in the interest of isolating particular forces.

As a motivating example for the situation we have in mind, consider a home repair scenario. The principal realizes that his home is not heated properly. He contacts a plumber and requests his advice. The plumber sends a person who examines the problem and later recommends a course of action. Incurring a high cost, the plumber can send a competent person. Alternatively, the plumber can send an incompetent person at a low cost. The principal does not know the level of expertise of the technician. However, he can reduce his uncertainty by sampling additional firms.

We model the basic scenario as follows. A principal recognizes that he needs the service in question, but he is uncertain as to which of a continuum of possible types of service he should get. There is one right choice for the principal which would give him a payoff of $V > 0$; each of the other choices gives a payoff 0. The set of possible choices is modeled as a continuum to assure that an unguided guess will not yield the right choice with positive probability. There are experts who can identify the right choice by incurring a cost c. The principal can consult such experts, but he does not observe whether or not the expert actually incurred the cost.

[1] E.g., the actions of an expert in the treatment stage may be unobservable (as is often the case in various repair services); the innate ability of an expert may also be unobservable.

[2] Reputation effects provide an important inducement by substituting for the difficulty to objectively measure the success of of the service.

The principal samples experts sequentially out of a large population of experts. A sampled expert offers the principal an initial contract stipulating a diagnosis (or consultation) fee and prices to be paid in different contingencies to be mentioned below. Then, the principal decides whether to consult this expert or to continue sampling. If the principal decides to consult an expert, he incurs a cost representing the time and inconvenience associated with such interaction. Then the expert decides whether or not to invest the required effort in the diagnosis. In both cases the expert provides a recommendation. At any point in the process the principal may decide to stop and acquire the service in question from one of the previously sampled experts.

A contract between the principle and an expert may only depend on observable actions of those two agents. Specifically, we do not allow contracts that use information revealed by third parties, i.e., other experts. Such contracts would always be subject to collusion between a subset of the parties and hence may not be sustainable.

Two characteristics of the environment determine which contracts are feasible. The first characteristic is the *observability* of the service that the principal ends up purchasing (the *performed service*). If the performed service is observable, then the contract may stipulate that the expert must be paid even if he did not perform the service but made the same recommendation as the performed service. The second characteristic is the *appropriability* of the expert's information. The experts' information is appropriable if the recommendation does not disclose all the essential information for the performance of the service. In other words, given a correct recommendation, the principal himself or another expert that he will enlist cannot simply carry out that recommendation, without further diagnosis.

These two dimensions capture what is essentially all the *hard information* available in this situation, i.e., they capture all the feasible *bilateral* contracting environments. Together they span four alternative scenarios differing in the information available to condition payments upon. Scenario 1 is such that the performed service is unobservable but the information is appropriable. In this case, besides the diagnosis fee, the expert can be paid only for the actual provision of the service. In Scenario 2 the performed service is observable, but the information is not appropriable. In this case the expert effectively sells only the diagnosis and payments depend on whether the principal adopt it (even if the expert does not perform the service). In Scenario 3 both the performed service is observable and the information is appropriable. Here the payment depend on whether the recommendation is adopted, and on whether the expert is chosen to perform the service. Finally, Scenario 0 is the trivial scenario in which the performed service is unobservable and the information is not appropriable. In this case payments can only take the form of a fixed fee.

The equilibrium analysis yields the following results. In Scenario 2, unrestricted competition results in a non-degenerate equilibrium in which experts invest in the diagnosis. In the other three scenarios, unrestricted competition leads to a degenerate equilibrium in which experts do not invest in the diagnosis of their prospective principals. In Scenario 0 it is impossible to reward the expert for exerting effort. In Scenario 1, the result is driven by a straightforward form of price undercutting. The experts bid down the price with the aim of capturing the business in the contingency that the principal searches for matching recommendations. In this manner the price is bid down to a level that destroys the incentives to exert effort. In scenario 3 the explanation is more complicated, as it also involves the role of contracts in providing incentives. Price competition implies that the price of the service must be equal to the price of the advise plus the cost of performing the service. But a putative equilibrium satisfying this condition is undermined by a deviation to a contract with a slightly higher price for the performance of the service. Such a contract provides the expert with stronger incentives to exert effort and thus enables her to charge a higher price.

In the first best outcome the principal consults only one expert who makes the required effort. Obviously, this outcome is not sustainable in equilibrium or by any form of price regulation. The second best is the welfare maximizing outcome subject to the informational constraints. In our model, this corresponds to the maximum surplus that can be achieved by appropriately designing the price system. In scenarios 1 and 3 the second best is achieved when the price of performing the service is maximal (subject to the participation constraint of the consumer). In this case, the probability that the expert exerts effort is maximized. In scenario 2 the second best price is between the equilibrium price and the maximal price. Hence in all the above scenarios, the equilibrium fails to sustain the second best welfare optimum because prices are too low. An intervention that imposes minimum prices can yield better allocations than unrestricted competition. In Scenarios 1 and 3, a lower bound on the price is necessary to induce experts to exert any effort; but also in Scenario 2, a minimum price improves welfare.

The inefficiency of the equilibrium, even in the second best sense, is perhaps the main qualitative insight that emerges from this analysis. The point is that, to provide the right incentives for the second best outcome, the prices have to be sufficiently high. But such prices are incompatible with the competition among experts. This competition is a necessary by-product of the process of gathering multiple opinions.

The related theoretical literature on markets for credence goods or services is not very large[3]. The formal models that we are aware of address the incentives that experts may have to misrepresent minor problems as major ones in order to profit at the expense of principals. As explained above, the present paper focuses on a rather different aspect of the problem. In a less direct way, this paper is also related to the literature on product quality provision under conditions of asymmetric information, particularly to the moral hazard strand of this literature[4].

2. The Model

In this section we present a base model. As the subsequent analysis will show, the results may be sensitive to the exact specification of the contractible variables and we will therefore consider a number of scenarios with respect to these specifications. However, to keep the exposition as clear as possible, we start with a single scenario (called Scenario 1) which is presented in this section and analyzed in the subsequent one. In later parts we will introduce other possible scenarios.

The principal is uncertain as to which of a continuum of possible types of service he should get. The range of possible service types is the set $[0, 1]$. The principal needs a specific service $\alpha \in [0, 1]$. His utility of getting service a is

$$\begin{cases} V \text{ if } a = \alpha \\ 0 \text{ if } a \neq \alpha \end{cases}$$

where $V > 0$. That is, the principal benefits from this service only if it exactly matches his need. The principal does not know his own type α and has a uniform prior on $[0, 1]$. The set of possible types is modeled as a continuum to assure that an unguided guess will not yield the right choice with positive probability.

There is a large population of identical *experts*, indexed by $k \in [0, 1]$, whom the principal may consult to learn about the appropriate choice of service. By

[3] See, e.g., Arrow [1963], Darby and Karni [1973] Pitchik and Schotter [1987, 1989], Wolinsky [1993, 1995] , Glazer and McGuire [1991] and Emmons [1994].

[4] In this literature (see, e.g., Wolinsky [1983]) two better informed sellers face less informed buyers and the analysis explores how search or reputation interact with the competition to determine prices and quality levels. Some important features separate this paper from that literature and give rise to different form of competition and different sets of relevant contracts.. For example, multiple opinions here may share common information, whereas in the traditional analyses of product quality provision sellers posses independent information.

incurring a cost $c > 0$ an expert can identify with certainty the type of service needed. If an expert does not incur the cost, she learns about the correct service type with small probability $\varepsilon > 0$; with probability $(1 - \varepsilon)$ she does not learn anything beyond the prior. The principal does *not* observe whether or not the expert actually incurred the cost. The cost of performing the service itself is independent of whether the type of service is the correct one, it is assumed equal across all types and is normalized to 0.

The process of eliciting opinions takes place over time which is divided into discrete periods. Within each period events unfold in the following order:

1. The expert offers a contract $(d, p) \in [-s, \infty) \times [0, \infty)$, where d is the fee the principal will pay for the diagnosis and p is the price he will pay for the treatment itself should he choose this expert to perform it.

2. The principal decides on one of the following actions: (i) to be diagnosed by the recently sampled expert; (ii) to sample a new expert; (iii) to transact with one of the previously sampled experts; (iv) to quit the process without purchase. The decision to acquire the service and the decision to quit terminate this process.

3. If the principal chose (i), he pays the fee d and incurs a cost $s > 0$. Then, the expert may offer a lower treatment price than the one she offered originally, i.e., $p' \leq p$.

4. Next, the expert chooses an effort level $e \in \{0, 1\}$ where $e = 1$ is the level required for a correct diagnosis and it involves the cost c.

5. Finally, the expert provides a recommendation $r \in [0, 1]$. If the expert invests in the information, r is the correct recommendation; if she does not invest, r is correct with probability ε and is a random recommendation with probability $1 - \varepsilon$.

Notice that we are not including the experts' reporting decisions as part of the strategy. Instead, we assume directly that, if the expert exerts effort on the diagnosis (i.e., $e = 1$), she reports her finding *truthfully*; otherwise she picks a recommendation according to a uniform distribution on $[0, 1]$.

The expert cannot observe at any point the history of a principal prior to their interaction. The principal cannot observe whether or not the expert invested in getting the information.

Before proceeding, let us comment briefly on some of the above features of the model. Notice that (3) specifies that all the principal's decisions take place at one point after a new expert was observed. Alternatively, the decisions could be split and the purchase decision could be moved to the end of the period after getting the expert's recommendation (and before sampling a new one). We choose this formulation for two reasons. First, it simplifies the formal description. Second, it allows us to introduce a simple form of competitive pressure since the principal can always observe an additional contract offer before he makes a decision.

In (2) we restrict the service fee to be larger than $-s$. The rationale for this assumption is that, with $d < -s$, a visit to the expert would become profitable for principals who have no interest in the service in question. If we added to the model a fringe of such principals, such offers will be unprofitable for experts under any circumstances. Thus, instead of complicating the model in this fashion, we simply rule out this possibility by assumption.

The feature that experts may revise their price downwards, as described in (4) above, will be referred to as *ex-post* price revision. This feature is probably not objectionable on grounds of lack of realism. However, its main purpose in the model is to prevent informational issues from artificially getting in the way of price competition. We will comment on it below.

The feature that 0 effort still yields a correct diagnosis with probability $\varepsilon > 0$ can be attributed to the 0 level being just a normalization for a minimal effort level which is not totally useless. However, this assumption also has a technical role. We comment on it further in Section 6 below.

The utility for the principal of type α who obtains the service a at the price p after having sampled n experts whose fees were $d_1, ..., d_n$ is

$$\begin{cases} V - p - \sum_{i=1}^{n} d_i - ns & \text{if } \alpha = a \\ -p - \sum_{i=1}^{n} d_i - ns & \text{if } \alpha \neq a \end{cases}$$

The utility to the principal who never obtains the service is 0. Observe that the principal gets zero benefit from a service that does not match his exact need. The choice of 0 rather than any other constant smaller than V does not have a qualitative effect on the analysis. The principal seeks to maximize his expected utility.

An expert who operates under the contract (d, p) and exerts effort $e \in \{0, 1\}$, receives the following payoff

$$\begin{cases} d - e \cdot c + p & \text{if the principal purchases the service from this expert } \textit{in some period.} \\ d - e \cdot c & \text{if the principal does not purchase the service from this expert } \textit{in any period.} \end{cases}$$

Observe that the cost of providing the service is zero. This normalization has no qualitative effect on the analysis. An expert seeks to maximize the expected profit.

The relevant past *history* of the principal records the sequence of experts who diagnosed him, their initial offers, revised prices and the recommendations they made. This is then a finite sequence of the form $\{(d_i, p_i, p_i', r_i)\}_{i=1}^n$. Let H_n denote the set of all possible histories of length n, where H_0 is the empty set, and let $\mathcal{H}^K = \bigcup_{i=0}^{K} H_i$.

Every period after a new expert was sampled the principal chooses a probability distribution over the available options: to transact with one of the previously sampled experts, to be diagnosed by the newly sampled experts, or to continue sampling. Formally, the principal's action in period n is a vector $\lambda = (\lambda_0, \ldots, \lambda_{n+1})$ where

$$\lambda \in \Delta^{n+2} = \{(\lambda_0, \ldots, \lambda_{n+1}) : \lambda_i \geq 0, \sum_{i=0}^{n+1} \lambda_i \leq 1\}.$$

The interpretation is that with probability λ_0 the principal quits the process, with probability $\lambda_i, 1 \leq i \leq n$,the principal purchases from the previously sampled expert labelled i (recorded in the history h), with probability λ_{n+1}, the principal chooses to be diagnosed by the newly sampled expert (who offers (d, p)), and with probability $1 - \sum_{i=0}^{n+1} \lambda_i$ he skips the newly sampled expert and continues sampling. Note that a principal who never chooses to be diagnosed does not incur any cost or benefit and hence this strategy is equivalent to quitting the process.

The principal's strategy σ is a sequence of functions $\sigma = (\sigma_n)_{n=0}^{\infty}$, where $\sigma_n : H_n \times [-s, \infty) \times \mathbf{R}_+ \rightarrow \Delta^{n+2}$.

Experts do not observe the history of the principal. Therefore the initial contract offer (d, p) is independent of the history. The revised price may depend on the initial contract (d, p), and the effort decision depends on the initial contract and the revised price (d, p, p'). A *strategy* for expert $k \in [0, 1]$ is therefore a four-tuple (d_k, p_k, p_k', x_k), where (d_k, p_k) is the initial offer, $p'(d_k, p_k) \leq p_k$ is the revised price offer, and $x_k(d,p,p') \in [0,1]$ is the probability of making the diagnostic effort. We denote a strategy profile by $\tau = \{d_k, p_k, p'_k, x_k\}_{k\in[0,1]}$, where (d_k, p_k, p'_k, x_k) is assumed to be a measurable function of k.

Notice that the strategies prescribe a deterministic choice for the prices (that are continuous variables) and a randomized choice for the discrete decisions, like a choice of an expert. This is done to convexify the set of actions.

The *beliefs* of an expert is a probability measure over the set $\mathcal{H}^\infty$ of possible past histories of the principal. Let $\beta(H)$ denote an expert's belief upon being sampled (but before being selected) that the history of the principal belongs to the set $H \subseteq \mathcal{H}^\infty$. Let $\beta(H \mid d,p)$ denote the belief of an expert who offers (d,p), conditional on being *selected* by the principal for a diagnosis. Consider a principal's strategy σ and experts' strategies τ. Given these strategies, let β^K denote the beliefs of an expert (over $H \subseteq \mathcal{H}^K$), conditional on being one of the first K experts to be sampled at random.[5] Clearly, these beliefs are well defined and identical for every expert as long as $0 < K < \infty$. We will say that the beliefs β are *consistent* with σ and τ, if $\beta^K \to \beta$ as $K \to \infty$.[6]

An *equilibrium* is a strategy σ for the principal, a strategy profile τ for experts, and beliefs β such that: (i) σ maximizes the principal's utility after any possible history. (ii) For each $k \in [0,1]$, (d_k, p_k, p'_k, x_k) maximizes the expert's profit, given the beliefs β. (iii) β is *consistent* with σ and τ.

The objective of our formulation of beliefs is to capture a situation where experts are have no information as to their position in the sampling process. With finitely many experts and periods this is described by experts having a uniform distribution over the K spots in the sampling procedure. Our consistency requirement captures the same feature for the infinite model. We choose the infinite population to avoid the complications associated with the small probability event that the principal's search exhausts all the available experts.

A *symmetric equilibrium* an equilibrium with the additional property that (d_i, p_i, p'_i, x_i) is the same for all $i \in [0,1]$.

In the following we restrict attention to symmetric equilibria and refer to them simply as equilibria. We discuss the limitation of our restriction to symmetric equilibria in Section 6 below.

Finally, to remain focused on the cases of interest in which there is sufficient potential surplus that might possibly induce the investments in diagnosis and

[5] More precisely, β^K is the belief over the principal's history conditional on the event that the expert is sampled with probability $1/K$ as the i−th expert for every $i = 1, ..., K$.

[6] Convergence of beliefs is in distribution.

search, we assume that V is sufficiently large relative to $c+s$ and that ε is small. Specifically, we assume that $V > 2c + 2s$ and $\varepsilon < s/V$. In words, the value of the correct service is greater than the full cost of getting diagnosed twice, and the total expected surplus associated with a random transaction with expert who does not invest in the diagnosis, $\varepsilon V - s$, is negative.

3. Equilibrium

Let (d, p, p', x) describe the equilibrium strategy of all experts. Observe that, if there exists an equilibrium in which the experts offer an ex-post price reduction, i.e., $p'(d,p) = \tilde{p} < p$, then obviously there is also an equilibrium with $p = \tilde{p} = p'(d, \tilde{p})$. Therefore, without loss of generality, we may assume that $p'(d,p) = p$.

We first characterize the principal's best response to this experts' strategy. The symmetry of the profile means that only the expert recommendations may vary along a search history. The principal's best response is an optimal stopping rule applied to sequences of recommendations. First, since two matching recommendations necessarily reveal the correct diagnosis and since search itself is not beneficial, it is optimal to stop after any history with two such recommendations. Second, the uniformity assumption guarantees that, if it is optimal to stop after some history with n non-matching recommendations, then it is optimal to stop after any history with n non-matching recommendations. Thus, if there exists an optimal stopping rule, there is one of the form S_n below.

Rule S_n : Stop after two matching recommendations
or after n recommendations, whichever occurs first.

where S_0 means no participation and S_∞ means searching for two matching recommendations. After stopping, the rule specifies either purchase or quitting. Without loss of generality, we may assume that when stopping occurs due the accumulation of two matching recommendations, then S_n prescribes purchase from one of the two experts who provided these recommendations. Similarly, S_1 also prescribes purchase after stopping. No loss is involved, since when an optimal rule prescribes no purchase after two matching observations, it is not more beneficial than S_0. However, when stopping occurs after a history with $n > 1$ non-matching recommendations, the optimal decision might be either purchase or quitting.

Lemma 1. *Suppose experts' behavior is given by the symmetric profile (d, p, x). If $d > -s$, then the best response of the principal is one of the following three* *strategies: S_0, S_1 followed by a purchase, and S_∞ followed by a purchase from one of the two experts who provided the matching recommendations.*

If $d = -s$, then, in addition, continued search for a finite number of periods after receiving two matching recommendations is a best response. If $d = -s$ and $p = V$, then any plan of search with purchase after two or more matching recommendations or without purchase is a best response.

Proof. First, we demonstrate that for $V > p$ the principal either searches once or searches at least until two matching recommendations.

Let $z = x + \varepsilon(1-x)$, i.e., z is the probability that an expert who chooses to invest with probability x will generate the correct diagnosis. Consider the principal's problem after a history consisting of $n \geq 1$ non-matching recommendations. Given this sample of n observations, the probability that a randomly drawn expert out of these n has the correct diagnosis is

$$\varphi(n) = \frac{(1-z)^{n-1}z}{(1-z)^n + n(1-z)^{n-1}z} = \frac{z}{1+(n-1)z}.$$

Now assume that the rule $S_n, n \geq 2$ is optimal. First, we consider the case where the principal plans to purchase after n different recommendations. In this case,

$$0 \leq \varphi(n)V - p < \varphi(n-1)V - p.$$

Since S_n is optimal, it follows that when the expert has a sample of $n-1$ different recommendations he (weakly) prefers to sample one additional time. Since we may assume that the principal purchases from the expert sampled last, the incremental value of the last recommendation is

$$(z - \varphi(n-1))\, V - (s+d) \geq 0.$$

But the incremental value of the $n+1$st sample would be

$$(z - \varphi(n))\, V - (s+d) > (z - \varphi(n-1))\, V - (s+d) \geq 0$$

and hence S_n cannot be optimal if the consumer purchases after n different recommendations.

Therefore, assume that the consumer only purchases after two matching recommendations. In this case, the incremental value of the last recommendation is

$$
\begin{aligned}
& z\varphi(n-1)(n-1)(V-p)-(s+d)-\max\{\varphi(n-1)V-p,0\} \\
\leq\; & z\varphi(n-1)(n-1)(V-p)-(s+d) \\
=\; & \frac{(n-1)z^2}{1+(n-2)z}(V-p)-(s+d) \\
<\; & \frac{nz^2}{1+(n-1)z}(V-p)-(s+d) \\
=\; & z\varphi(n)n(V-p)-(s+d)
\end{aligned}
$$

where the last expression is the incremental value of an additional recommendation. Hence, also in this case it cannot be optimal to stop search after $n \geq 2$ non-matching recommendations when $V > p$.

When $d > -s$, search is costly so that continuing beyond two matching observations is obviously suboptimal. This proves the lemma. ■

The additional best responses in the case of $d = -s$ do not add anything to the analysis. Therefore, in what follows we ignore them and for the case $d = -s$ also consider only the three options described in the lemma. To further avoid inconsequential discussion, we also assume that, if the principal is indifferent between quitting and participation with the intent of making a purchase, he always chooses the latter.

These assumptions and Lemma 1 imply that we may characterize the principal's behavior along the path of a symmetric profile by the participation decision and by a probability f that he stops after the first recommendation (with probability $1-f$ the principal searches until he gets two matching recommendations). Thus, letting $U(S_n)$ denote the expected payoff associated with rule S_n, the principal participates if $\max[U(S_1), U(S_\infty)] \geq 0$ and then $f = 0$, 1 or some value in $[0,1]$ depending on whether $U(S_1) \gtreqless U(S_\infty)$.

To derive the experts' best response, let $B(n), n = 0, 1, \ldots$denote the expert's belief that the principal has previously sampled n times. That is, $B(n) = \beta(H)$ where H is the set of all histories of length n with n non-matching observations. Now, if the expert invests in the diagnosis (i.e., chooses $e = 1$), she will first incur the cost c and then will get the price p in two contingencies: (i) if this is his first sample and he decides to stop; (ii) if he is searching for two matching recommendations and will eventually pick this expert over the other one. The probability of (i) is $f \cdot B(0)$. Since by assumption the principal's strategy does not depend on the name of the expert, the probability that this expert will be chosen over the other matching recommendation is $1/2$ and hence the probability of (ii) is $(1 - fB(0))/2$.

$$d + p \cdot f \cdot B(0) + (1 - f \cdot B(0)) \frac{p}{2} - c \tag{3.1}$$

On the other hand, if the expert does not invest in the diagnosis, she will still get p in the above two contingencies. But while the probability of (i) remains the same, the probability of (ii) is $\varepsilon(1 - fB(0))p/2$, since this expert will make the correct diagnosis only with probability ε. Thus, the expected profit in this case is

$$d + p \cdot f \cdot B(0) + \varepsilon(1 - fB(0))\frac{p}{2} \tag{3.2}$$

Thus, given the equilibrium contract offer (d, p), the expert's optimal effort decision is $e = 0$ or 1 depending on whether (3.2) is greater or smaller than (3.1).

It is immediately obvious that there exist degenerate equilibria in which the experts do not invest in the diagnosis (i.e., $x = 0$) and the principal does not select any expert. In such an equilibrium, on and off the path, an expert expects other experts not to invest in the diagnosis and hence she has no incentive to do so. In this equilibrium none of the potential gains from trade are realized. The following proposition shows that this is true for all equilibria.

Proposition 1. *There is no equilibrium in which the principal participates (gets diagnosed anywhere) and* $x > 0$.

Proof. Suppose that the path of a symmetric equilibrium is described by (d, p, x, f) with $x > 0$. This implies that (3.2) is less than or equal to (3.1). Hence

$$0 \leq (1 - \varepsilon)(1 - f \cdot B(0)) \frac{p}{2} - c$$

It follows that $f < 1$, since otherwise $B(0) = 1$ and the inequality fails to hold. Note that $f < 1$ implies $U(S_1) \leq U(S_\infty)$, i.e.,

$$zV - p - (s + d) \leq V - p - 2\frac{s + d}{z} \tag{3.3}$$

where as above, $z = x + \varepsilon(1 - x)$. Consider a possible expert's deviation to an ex-post price revision $p' < p$. Let z' denote the probability that, in the equilibrium of the continuation, the deviant expert's diagnosis is correct, $z' \geq \varepsilon > 0$. Clearly, if the deviant expert's recommendation matches the recommendation of a previously sampled expert, the principal purchases from the deviant expert since her price is lower. Let us analyze the behavior of the principal after a history consisting of $n+1$ non-matching observations: one from the deviant expert and $n \geq 0$ from other experts (some of whom might have been sampled before the deviant expert was encountered and some after that). Observe that, given this sample of $n+1$ observations, the probability that the deviant expert's diagnosis is correct is

$$\begin{aligned}\gamma' &= \frac{(1-z)^n z'}{(1-z)^n(1-z') + nz(1-z)^{n-1}(1-z') + (1-z)^n z'} \\ &= \frac{(1-z)z'}{1-z+nz(1-z')} \leq z'.\end{aligned}$$

If $n \geq 1$, the probability that a randomly drawn expert out of these n has the correct diagnosis is

$$\begin{aligned}\gamma &= \frac{z(1-z)^{n-1}(1-z')}{(1-z)^n(1-z') + nz(1-z)^{n-1}(1-z') + (1-z)^n z'} \\ &= \frac{z(1-z')}{1-z+nz(1-z')} < z\end{aligned}$$

Thus, the principal's incremental (ignoring the past costs) expected payoff from stopping and making a purchase is $\gamma' V - p'$ or $\gamma V - p$ depending on whether he purchases from the deviant or one of the others. The principal's incremental expected payoff from continuing the search for a matching recommendation is

$$V - (1-\gamma'-n\gamma)(p + 2\frac{s+d}{z}) - \gamma'(p' + \frac{s+d}{z}) - n\gamma(p + \frac{s+d}{z})$$

First rearranging and then using the fact $\gamma' + n\gamma \geq z$ and (3.3), we get

$$\begin{aligned}& V - (1-\gamma'-n\gamma)(p + 2\frac{s+d}{z}) - \gamma'(p' + \frac{s+d}{z}) - n\gamma(p + \frac{s+d}{z}) \\ = \ & V + \gamma'(p-p') - p - (2-\gamma'-n\gamma)\frac{s+d}{z} \\ > \ & V + \gamma'(p-p') - p - (2-z)\frac{s+d}{z} \\ > \ & zV - p \geq \gamma V - p\end{aligned}$$

Thus, the incremental payoff associated with continuing to search for two matching recommendations is strictly higher than the incremental payoff associated with stopping and making a purchase from one of the n other experts. Obviously, the payoff of continuing is also strictly greater than the RHS of (3.3), which by the participation of the principal in this equilibrium is nonnegative. Thus, it is suboptimal for the principal to quit.

It follows that, after encountering the deviant expert, the principal either stops and purchases immediately from this expert, or continues to search until he gets two matching observations. Therefore, if the deviant expert invests in the diagnosis, the principal will surely purchase from her, since in the event he continues to search the price p' will be lower than the price p of the matching recommendation. Thus, by choosing $p' < p$ such that

$$p' > p \cdot f \cdot B(0) + (1 - f \cdot B(0)) \frac{p}{2}$$

and investing in the diagnosis, the expert's profit $p' - c$ will be larger than the equilibrium profit $p \cdot f \cdot B(0) + (1 - f \cdot B(0)) \frac{p}{2} - c$ and hence the deviation is profitable. ■

Thus, equilibria fail to realize any of the potential gains from trade. This market failure owes to the tension between price competition and quality provision: price competition destroys the experts' incentives to exert effort. The reason for this is clear. An expert will only invest effort if it increases the probability of a sale by a sufficient amount *and* if the sale generates profit. But the principal for whom the expert's effort decision may make a difference must have an identical recommendation from another expert. This means that two experts are competing in a Bertrand fashion for the provision of the service. As a consequence, the experts make zero profit conditional on a sale to this principal. Moreover, in equilibrium the expert only sells to half of the principals with matching recommendations. Since the cost of effort must be incurred prior to the sale, it follows that the expert makes a loss by exerting effort. Put in other words, to create incentives for investment in effort, the price has to be sufficiently high. But the very process of information gathering forces stiff price competition as the principal is collecting bids and advice at the same time. This competition does not allow high prices that are necessary for the sustenance of the incentives in equilibrium.

In some of the literature on quality provision under asymmetric information[7], price mark-ups play a similar role of inducing sellers to provide high quality. In

[7] See, e.g., Wolinsky (1983).

these models the price mark-ups are sustained in the face of price competition by adverse out-of-equilibrium beliefs. That is, a firm that tries to undercut the equilibrium price is believed to be offering inferior quality. This effect is prevented in the current model by allowing the experts to issue a revised price offer after the principal has incurred the search cost. In the absence of that ex-post price cutting it would be possible to construct an equilibrium with $x > 0$. To sustain such an equilibrium it must be assumed that, if an expert deviated to a lower price, a principal who samples this expert is believed to be new in the market. Recall that, when the expert decides on the effort, he does not know the principal's history. This out of equilibrium belief will give rise to an equilibrium in the continuation game that would be indeed unattractive for the principal.

4. Alternative contracts

So far we assumed that payments to an expert can be conditioned only on: (1) whether or not the principal chose to be diagnosed by this expert and (2) whether or not the principal obtained the service from this expert. Obviously, if these are the only facts that can be contracted upon, then the contract considered above is the only one to consider. While this scenario is not without interest, it is also possible to imagine circumstances under which more (or less) information can be contracted on.

In what follows we expand the set of scenarios considered. To do so in a systematic way, we distinguish two characteristics of the environment that determine which contracts are enforceable. The first is the *observability* of the service that the principal ends up purchasing (the *performed service*). If the performed service is observable then it can be contracted upon with any expert, not only the one who ends up serving this principal. Thus, the observability means that the advice can be sold separately from the service and yet the payment may depend on whether the advise was adopted or not (which indirectly reflects its accuracy). In the model of Section 2 we assumed that the performed service cannot be contracted on.

The second characteristic is the *appropriability* of the expert's information. The experts' information is appropriable if on the basis of an expert's recommendation alone the principal (or another expert) cannot learn all the essential information for the performance of the treatment. In other words, the principal cannot simply carry out an expert's recommendation and another expert will not be able to do so without further diagnosis. Notice that, if we take the model very literally, then the information might appear as not appropriable, since the experts' recommendations convey their exact diagnosis (a point in $[0, 1]$) and principals understand them well enough to decide when two recommendations match. However, we may adopt an alternative interpretation whereby the information contained in the recommendation is not sufficient to identify the required service completely (formally, we may think of the diagnosis as two dimensional and of the recommendation as revealing only one of the dimensions), so that principals' ability to search for matching recommendations does not contradict the appropriability of the information[8].

The four possible combinations of these two characteristics give rise to four different scenarios.

Scenario 0 The performed service is unobservable and the information is not appropriable. In this scenario it is impossible to condition the payments to experts on anything other than the principal's visit to this expert. Therefore, the only possible equilibrium is the degenerate one in which no expert invests in information.

Scenario 1 The performed service is unobservable but the information is appropriable. The appropriability of the information implies that a principal would have to get the service from one of the experts who diagnosed him. The unobservability of the performed service implies that payments to an expert can be conditioned on a principal's decision only through the acquisition of the service. This is the scenario considered in Section 2 above.

Scenario 2 The performed service is observable but the information is not appropriable. Here payments from the principal to an expert can depend on whether the principal ended up adopting the expert's recommendation. In this case it is irrelevant who performs the service.

Scenario 3 The performed service is observable and the information is appropriable. Here payments from the principal to an expert can be conditioned both on whether the principal ended up adopting the expert's recommendation and on whether the principal was actually served by the expert.

[8]As an example, consider the case of a medical service. One part of the diagnosis is of the identification of the correct disease. However, for every disease there may be many possible treatments and hence the second dimension of the diagnosis is the specification of the best treatment for the particular patient. Only the disease is communicated to the consumer but both pieces of information are necessary to perform the correct service.

The two dimensions of contractibility that span the above four scenarios capture the possibilities of conditioning payments on what is all the *"hard information"* available in this situation. By this we mean information pertaining to the actions of the contracting parties. However, they do not exhaust all the dimensions that, in principle, can be contracted upon. For example, payments to experts might be conditioned on the diagnoses of other experts, i.e., information that depends on *third parties* actions. Contracting on such "soft information" is problematic since it makes contracts susceptible to collusion between some subset of players. For example, consider a contract that relies on the opinions of other experts. First, such a contract requires an agreement on who qualifies as a legitimate expert. Then, there may be a temptation, e.g., for the principal and dishonest experts to avoid payment by misrepresenting information; alternatively, two dishonest experts may collude to avoid penalties. For these reasons, we do not allow the possibility of contracting on soft information and focus on contracting on hard information instead.

4.1. Scenario 2: observability without appropriability

In this scenario, the performed service is observable but the information is not appropriable. The observability means that the payment to an expert may also depend on the actual service performed on the principal, whether or not it was performed by the particular expert in question[9]. The inappropriability of the information means that the good that is being sold here is just the diagnosis. That is, once the principal obtains the diagnosis, he can either perform the treatment himself or take it to another expert who would be willing to perform the prescribed treatment at a price equal to its cost (we implicitly assume that there is a competitive fringe of experts who are willing to perform the service on demand). Therefore, the price that an expert charges for the service cannot differ from the price she charges for an adopted recommendation by more than the cost of providing the service which was normalized to 0.

Thus, the contract offered by the experts in this scenario is a pair (d, p), as before, except that p is the price that the expert gets if her advice is adopted. The notions of history, strategy, beliefs and equilibrium remain unchanged from

[9]Notice that letting the payment to an expert depend on the adoption of her advice does not necessarily require the customer's decision to be publically observable. Such contracts might also be implemented by shifting the burden of the proof to the customer, for example, by making the customer pay for an advice, unless he can prove that he acquired a different type of service.

the previous section except for taking into account the fact that a principal must pay *all* experts who recommended the performed service.

The analysis follows closely the development of the previous section. As before, the expert strategy is (d, p, p', x), where $p'(d, p) \leq p$ are the ex-post price revisions that the expert may offer and $x(d, p, p')$ is the probability with which the expert makes effort. Suppose that (d, p, p', x) is an equilibrium strategy. As before, without loss of generality we may assume that $p'(d, p) = p$.

It is straightforward to extend Lemma 1 to this situation. Here too the principal's best response is again characterized by the participation decision and by the probability f of stopping after the first diagnosis (rule S_1) as opposed to searching for two matching opinions (rule S_∞). Recall that we use the notation $z = x + \varepsilon(1 - x)$ and use $U(S_n)$ to denote the principal's expected utility of search rule S_n. Here

$$U(S_\infty) = V - 2p - 2\frac{s + d}{z} \tag{4.1}$$

and

$$U(S_1) = zV - p - d - s \tag{4.2a}$$

Note that $U(S_\infty)$ accounts for the fact that the principal must pay both of the experts who provided matching opinions.

Given the beliefs B of the expert and the strategy of the principal the profit of an expert who chooses $e = 1$ (the counterpart of (3.1)) is now

$$d + p \cdot f \cdot B(0) + (1 - f \cdot B(0))\, p - c \tag{4.3}$$

and the profit for an expert who chooses $e = 0$ is

$$d + p \cdot f \cdot B(0) + \varepsilon\,(1 - f \cdot B(0))\, p$$

As before, suppose that (d, p, x, f) describe the path of an equilibrium such that $x > 0$. Then the optimality of the expert's effort decision requires

$$p \cdot f \cdot B(0) + \varepsilon\,(1 - f \cdot B(0))\, p \leq p \cdot f \cdot B(0) + (1 - f \cdot B(0))\, p - c. \tag{4.4}$$

Note that $f = 1$ implies the principal follows rule S_1 with certainty. Hence $B(0) = 1$ and (4.4) cannot hold. Therefore, $x > 0$ is possible only if $f < 1$. This in turn implies that $U(S_\infty) \geq U(S_1)$, i.e.,

$$zV - p - (s + d) \leq V - 2p - 2\frac{s + d}{z} \tag{4.5}$$

In addition, individual rationality of experts and the principal requires

$$d + p \cdot f \cdot B(0) + (1 - f \cdot B(0))\, p - c \geq 0 \tag{4.6}$$

and

$$V - 2p - 2\frac{s+d}{z} \geq 0 \tag{4.7}$$

Thus, on the path of a symmetric equilibrium with $x > 0$, $f < 1$ and the system (4.4)-(4.7) holds.

Given f, x we may also characterize consistent beliefs of experts. Conditional on the event that the principal is ever diagnosed, the expected number of diagnoses is given by

$$1 + (1 - f) \sum_{n=1}^{\infty} \left((1-z)^n + n(1-z)^{n-1} z\right) = \left(f + 2\frac{1-f}{z}\right)$$

and hence

$$B(0) = \frac{1}{f + 2\frac{1-f}{z}} = \frac{z}{fz + 2(1-f)}. \tag{4.8}$$

Since experts here can be compensated for a correct advice, one may expect that the competition does not play the same destructive role as it does in the previous scenario. Indeed, the next proposition demonstrates that in this scenario there is an equilibrium in which experts get informed with positive probability.

Proposition 2. *There exists an equilibrium with $x > 0$. In all such equilibria $d = -s, p = c + s$, $z = (V - c - s)/V$.*

Proof. First, observe that with $d = -s$ and $p = c + s$, $z = (V - c - s)/V$ and

$$f = \frac{2s - 2c\frac{\varepsilon}{1-\varepsilon}}{cz + 2s - 2c\frac{\varepsilon}{1-\varepsilon}},$$

(4.4) and (4.5) hold as equalities. Recall that we are assuming throughout that $V > 2(c + s)$ and $\varepsilon < s/V$. These assumptions guarantee that $f > 0$. Let us now verify that this is an equilibrium. First note that $p + d = c$ and hence, given $x > 0$, the sum of the price and the (negative) diagnosis fee cannot be any lower without the expert making a loss. Second, consider the deviation of an expert to $(\tilde{d}, \tilde{p})$, such that $\tilde{d} \geq -s$, and $\tilde{p} + \tilde{d} \geq c$. To show that this deviation is unprofitable, let us complete the description of the equilibrium after this deviation as follows. The deviant expert believes that, if the principal responds to the offer, he must be starting the search, i.e., $B(0 \mid \tilde{d}, \tilde{p}) = 1$. The expert then gets informed with probability $\tilde{x}$ which makes the principal with this history exactly indifferent between stopping and continuing to search for a matching recommendation. The principal in turn chooses to stop with probability $\tilde{f} < 1$. Clearly, since after the empty history the principal is just indifferent between stopping and continuing to search, it follows that after any longer history the principal would strictly prefer to continue. Now, observe that the payoff to principal who continues searching after sampling the deviating expert is

$$V - c - (\tilde{p} + \tilde{d}) - 2s$$

in case this expert gets informed, and it is

$$V - 2(c + s) - (s + \tilde{d})$$

in case she does not get informed. Obviously, both of these are smaller than or equal to

$$V - 2c - 2s$$

which is what the principal gets by skipping the deviant, and if either $\tilde{d} \neq -s$ or $\tilde{p} \neq c + s$, then one of these inequalities must be strict and the principal is strictly worse off by sampling the deviating expert. Since, regardless of his history, the principal prefers (at least weakly) to continue searching, he does not benefit from being diagnosed by the deviant expert. It is therefore optimal for the principal to skip this expert. hence, there is no profitable deviation.

Let us verify now that this is the only symmetric equilibrium. First, we show that in any symmetric equilibrium $d = -s$. If $d > -s$, then consider the deviation to $(\tilde{d}, \tilde{p}) = (-s, p + d + s)$. Recall that the probability $\tilde{z}$ that this expert will provide the correct diagnosis is positive, $\tilde{z} \geq \varepsilon > 0$. Now recall that, after the principal decides to stop, he still gets to hear an additional price offer. Thus, if the principal who would otherwise stop after the first recommendation sampled $(\tilde{d}, \tilde{p})$ before departing, he would strictly prefer to continue. This implies that $(\tilde{d}, \tilde{p})$ strictly increases the expected profit of the deviating expert.

To complete the argument note that, given $d = -s$, it cannot be the case that $p > c + s$, since then a deviation to $(-s, \tilde{p})$, such that $c + s < \tilde{p} < p$ would also attract the principal in the event that he was about to depart. By the same reasoning as above, if $\tilde{p}$ is sufficiently close to p, the expert's profit will strictly increase. ■

Notice that the uniqueness of this equilibrium was established using the assumption that, before stopping the search, the principal gets to observe a last contract offer. This introduces a simple form of a competitive pressure despite the fact that experts are sampled sequentially, one at a time. In the absence of some such pressure, there would be multiple equilibria in this scenario. However, the required competitive pressure could also be achieved by assuming that in each period the principal chooses from a sample of two or more competing contract offers. This alternative form would not alter any of the other results in this paper and we chose not to adopt it only for the sake of expositional simplicity.

The existence of a non-trivial equilibrium is not entirely surprising. In the previous scenario, the incentives to invest in the diagnosis are destroyed by the price competition. We noted there that the imposition of a price floor can restore the incentives. It turns out now that the observability of the performed service prevents the competition from reducing the prices by too much and hence it has a similar effect in maintaining the incentives.

4.2. Scenario 3: observability and appropriability

In this scenario the performed service is observable and the information is appropriable. This means that payments from the principal to an expert can be conditioned both on whether the principal ended up adopting the expert's recommendation and on whether the principal was actually served by the expert. The contract offered by the experts in this scenario takes the form of a triple (d, p_1, p_2) of prices paid by the principal to the expert in the following contingencies: d is the diagnosis fee[10], p_1 is paid if the principal acquires the service from this expert, p_2 is the price paid by the principal if he ended up acquiring elsewhere the type of service recommended by this expert. Thus, the overall payment from the principal to this expert is:

1. $d + p_1$ if he performs the recommended service with the expert;
2. $d + p_2$ if he performs the recommended service with some other expert;
3. d if he performs a different service, or no service.

[10] We continue to assume that $d \geq -s$. As before, this assumption might be motivated by the existence of a fringe of customers who are not interested in the service, but would take advantage of prices that allow them to simply collect money.

Other features of the model and the notions of history, strategy, beliefs and equilibrium remain unchanged from the previous section. Thus, the only difference is that p_1 and p_2 need not be equal. The expert strategy is now $(d, p_1, p_2, p'_1, p'_2, x)$, where $p'_i(d, p_1, p_2) \leq p_i$ are the ex-post price revisions that the expert may offer and $x(d, p_1, p_2, p'_1, p'_2)$ is the probability with which the expert makes effort. Suppose that $(d, p_1, p_2, p'_1, p'_2, x)$ is an equilibrium strategy. As before, without loss of generality we may assume that $p'_i(d, p_1, p_2) = p_i$.

The principal's utility from searching for two matching opinions is

$$U(S_\infty) = V - p_1 - p_2 - 2\frac{s+d}{z} \tag{4.9}$$

and the utility from stopping after the first recommendation is

$$U(S_1) = zV - p_1 - d - s \tag{4.10a}$$

The principal's best response is again described by the participation decision and a probability f of stopping after the first recommendation. Given (d, p_1, p_2, x, f), the belief $B(0)$ is given by (4.8) above. The profit of an expert who chooses $e = 1$ is

$$d + p_1 \cdot f \cdot B(0) + (1 - f \cdot B(0)) \frac{p_1 + p_2}{2} - c \tag{4.11}$$

and the profit for an expert who chooses $e = 0$ is

$$d + p_1 \cdot f \cdot B(0) + \varepsilon (1 - f \cdot B(0)) \frac{p_1 + p_2}{2}$$

As before, suppose that (d, p_1, p_2, x, f) describe the path of an equilibrium such that $x > 0$. Then $f < 1$ and the following system holds

$$\begin{aligned} & p_1 \cdot f \cdot B(0) + \varepsilon (1 - f \cdot B(0)) \frac{p_1 + p_2}{2} \\ \leq \;& p_1 \cdot f \cdot B(0) + (1 - f \cdot B(0)) \frac{p_1 + p_2}{2} - c. \end{aligned} \tag{4.12}$$

$$zV - p_1 - (s + d) \leq V - p_1 - p_2 - 2\frac{s+d}{z} \tag{4.13}$$

$$d + p_1 \cdot f \cdot B(0) + (1 - f \cdot B(0)) \frac{p_1 + p_2}{2} - c \geq 0 \tag{4.14}$$

$$V - p_1 - p_2 - 2\frac{s+d}{z} \geq 0 \tag{4.15}$$

Intuitively, one would expect that the same reasons that made it possible to sustain a non-trivial equilibrium in the previous scenario would make it possible here. The fact that, in addition to the observability of the performed service, the information here is also appropriable, should reinforce this intuition even further. However, it turns out that this intuition is false in the following sense.

Proposition 3. *There is no symmetric equilibrium with* $x > 0$.

The proof is relegated to the appendix. The result of Proposition 3 is surprising since intuitive reasoning might suggest that in Scenario 3 it should be easier than in Scenario 2 to sustain equilibria in which experts exert effort. After all, experts can still be rewarded for a correct diagnosis in the same manner as in Scenario 2. What interferes with this logic here is that the appropriability of the information also adds a new dimension to the competition since in Scenario 3 it may matter which expert is chosen to perform the service. In other words, while in Scenario 2 the only competitive role of the contract is to attract the principal for a diagnosis, here the contract can be used to compete over the principal's choice where to have the service performed. This added dimension intensifies the competition and hence makes it harder to sustain an equilibrium with the required mark-up.

Roughly speaking the proof proceeds as follows. First, there cannot be an equilibrium with $p_1 \neq p_2$, since this will be undermined by simple ex-post undercutting of the higher of the two prices. Such undercutting is aimed at shifting the expert's transactions from the lower priced to the higher priced option. Second, a configuration with $p_1 = p_2 = p > c/(1-\varepsilon)$ (such as the equilibrium outcome of scenario 2) cannot be sustained here as an equilibrium, since an expert can profitably deviate to an offer with $p_1 \neq p_2$, such that p_2 is slightly lower than p and p_1 is higher than p. These prices manipulate the equilibrium in the continuation subgame in a way that increases the expert's effort. This, in turn, makes the principal willing to pay the higher p_1, which renders the deviation profitable. Finally, the competition in diagnosis fees implies that in a non degenerate equilibrium $d = -s$. This rules out a non-degenerate equilibrium with $p_1 = p_2 = p = c/(1-\varepsilon)$, since cost coverage requires $p \geq c + s$ which is higher than $c/(1-\varepsilon)$.

The price undercutting argument that rules out an equilibrium with $p_1 \neq p_2$ is similar to the undercutting considerations in Scenario 1. However, since here an expert who makes the effort is assured to be paid, once $p_1 = p_2$ there is no incentive for further ex-post undercutting, and so the ex-post undercutting does not destroy the incentives as it does in Scenario 1. The deviation to unequal prices is not of the simple undercutting variety. In fact, the deviant profits from attracting transactions to a *higher* price. This higher price induces herself to exert effort with a higher probability.

In Section 6 below we describe a variation on the basic model in which the diagnosis fee d is restricted to be nonnegative. In that version, Scenario 3 has an equilibrium in which experts exert effort and where $p_1 = p_2 = p = c/(1-\varepsilon)$. As noted above, these prices cannot be sustained in equilibrium of the main version since a larger mark-up is required to cover the negative diagnosis fees that the competition leads to.

5. Welfare

The main qualitative insight in this paper concerns the conflict between price competition and the quality of expert advice. In this section we demonstrate that, in all the scenarios, the equilibria do not attain the second best welfare optimum level.

Welfare will be measured by the sum of expected payoffs (the principal's expected payoff and experts' profits). Since prices here are just transfers, the endogenous variables on which welfare depends are x and f. The former captures the extent to which experts acquire information and the latter captures the intensity of the principal's search. Let W denote the total expected payoff, then

$$W = f\,(zV - s - xc) + (1-f)\left(V - 2\frac{s}{z} - 2c\frac{x}{z}\right) \tag{5.1}$$

where as before $z = x + \varepsilon(1-x)$. The term $2cx/z$ captures the expected diagnosis costs associated with searching for two matching recommendations. If ε were 0, this cost would be $2c$, but $\varepsilon > 0$ means that the correct diagnosis can be sometimes obtained at no cost and hence the expected total cost is less than $2c$.

The "first best" outcome is such that the correct diagnosis is obtained at the minimal cost of search and diagnosis. Since ε is assumed small, in a first best world we have that $x = z = 1$ and $f = 1$. In lesser worlds, the magnitudes of x and f are optimal responses of the experts and the principal to the incentives created by the prevailing contracts. We will use the term "second best" to refer outcomes that maximize welfare from among all those that can be sustained by prices that respect the informational constraints. To make this notion precise, define a *fixed price equilibrium* as follows. For scenarios 1 and 2 a fixed price equilibrium is a profile (d, p, x, f) and beliefs B such that (i) the experts' effort decision is optimal given B, (ii) the beliefs B are consistent, and (iii) the principal's strategy is optimal given (d, p, x). Similarly, in scenario 3 a fixed price equilibrium is a profile (d, p_1, p_2, x, f) and beliefs B such that the experts' effort decision and the principal's strategy are optimal and the beliefs are consistent. Now, the second best outcome maximizes welfare over all those sustainable by a fixed price equilibrium.

5.1. Scenario 2 (observability without appropriability):

We begin our analysis with Scenario 2, i.e., the case where the service is contractible but the information is not appropriable. Recall that in this case, if the principal searches for a matching recommendation, he must pay the price twice.

Let $W(d, p)$ denote the maximum welfare level attained in a fixed-price equilibrium supported by (d, p). Thus, the second best problem is to find the fixed price equilibrium (d, p) that maximizes $W(d, p)$.

The derivation of the optimum is complicated by the individual rationality constraints which require to treat different sub-regions of the parameter space separately. For our purposes it would be sufficient to derive the optimum fro a subset of the parameter space for which there is an interior solution, and in so doing we will avoid some tedious but straightforward technical discussion. Thus, we restrict further the set of admissible parameters to assure that V is sufficiently large, s is sufficiently small and ε is sufficiently small relative to s. The following proposition shows that for this range of the parameters, the second best contract has a price which is larger than $c + s$, the equilibrium price under competition. In addition, the price is smaller than the maximum feasible price V. Finally, the second best contract has diagnosis fee $-s$ as in the competitive equilibrium.

Proposition 4. *Suppose $V > 5c/2$. Then, there is an $\bar{s} > 0$, and a $\bar{\varepsilon} > 0$ such that for $0 < s < \bar{s}$, $\varepsilon/s < \bar{\varepsilon}$, the second best contract (d, p) is such that $d = -s$ and $p \in (c + s, V)$. In the resulting fixed price equilibrium we have that $x \in (0, 1)$.*

The proof is relegated to the appendix. Notice that the role of the assumption on the parameters is to assure that the individual rationality constraints are not binding. That is, that the unconstrained solution for p obtained in the proof of Proposition 4 lies in the interval $[c + s, (V - s)/2]$. When the individual rationality constraints are binding, the analysis is somewhat more complicated. When the unconstrained solution for p is smaller than $c + s$, then at the optimum $p + d = c$.

Observe that, although the equilibrium of Proposition 2 realizes some of the gains from trade, its outcome is not second best. At least in the range of the parameters covered by Proposition 4, the equilibrium price is lower than the second best price. Thus, the imposition of a price floor can raise welfare.

5.2. Scenarios 1 and 3 (appropriability without and with observability):

First, we consider scenario 3 in which p_1 is not constrained to be equal to p_2. Clearly, the second best allocation here is superior to its counterpart in Scenario 2. As before, welfare is captured by (5.1) and $W(d, p_1, p_2)$ denotes the maximum surplus attained in a fixed-price equilibrium supported by the prices (d, p_1, p_2). The following proposition characterizes the second best contract—the maximizer of $W(d, p_1, p_2)$.

Proposition 5. *In scenario 3, the second best contract is* $(d, p_1, p_2) = (-s, V, 0)$. *In the resulting fixed price equilibrium* $x = 1$ *and* $f = (V - 2c)/(V - c)$.

Since in the optimum $p_2 = 0$, it follows that this is also the welfare maximum in Scenario 1 in which payments can be conditioned only on treatment. Thus Proposition 5 has the following corollary for scenario 1:

Corollary 1. *In scenario 1, the second best contract is* $(d, p) = (-s, V)$. *At these optimal prices* $x = 1$ *and* $f = (V - 2c)/(V - c)$.

These results demonstrate that the optimal price-fee pair is such that the principal gets reimbursed for his search activity. The fee $d = -s$ makes the principal willing to search even though experts exert effort with certainty ($x = 1$). Since p is the highest price consistent with individual rationality on the principal side, it gives experts the strongest possible inducement to ensure that a sale is made. This, in turn, minimizes the probability, $1 - f$, with which the principal has to engage in search in order to keep experts from choosing low effort.

Obviously, the equilibria of Scenarios 1 and 3 do not achieve the second best optimum. Since the second best is achieved here through the imposition of a minimum price on the provision of the service itself, only appropriability is required and hence the second best is also attainable in the environment of Scenario 1.

6. Discussion

The contracts considered in Scenario 1 correspond to contracts that prevail in actual markets. In contrast, we are not aware of obvious examples of actual markets in which contracts of the type considered in Scenarios 2 and 3 prevail. It might be that in actual markets the ability to observe the decisions made by principals is sufficiently limited to rule out the enforceability of the contracts offered in Scenarios 2 and 3. Thus, the analysis of those scenarios is not motivated by an attempt to explain particular institutions, but rather by the need to explore how all the potentially available information might be used.

Perhaps the main qualitative insight of the above analysis is the inefficiency of the equilibrium even in the second best sense. This is very pronounced in Scenarios 1 and 3, but it also arises in the equilibrium of Scenario 2 in which experts do exert some effort. The idea is that attainment of the second best requires to maintain sufficiently high prices, but such prices are incompatible with the competition. In this context, the fact that Scenarios 2 and 3 might be assuming the availability of too much information just strengthens the result. Finally, this insight seems quite robust in the sense that it would survive the extensions outlined later on in this section..

In the remainder of this section we revisit some of the central modeling choices, discuss further their possible justification and consider some alternative variations.

Weakened Price Competition through Product Differentiation This paper demonstrates that price competition may destroy the incentives to exert effort. In our setting, experts are homogenous and hence Bertrand style price competition is operating at full force. We could modify the environment and analyze experts who are differentiated along some dimension (say, location or personality traits). In this case, price mark-ups could be sustained simply because of product differentiation. We conjecture that these mark-ups would enhance the incentives for experts to invest in the diagnosis. However, such mark-ups need not be sufficient to achieve the second best, so that the inefficiency that arises in the present model would not disappear. We chose not to incorporate such differentiation into the model since the added insight does not justify the substantial additional complications that this modification would add to the analysis.

Diagnosis fee Recall that, in some of the results derived in the above analysis, the diagnosis fee d ended up being $-s$. That is, the diagnosis fee fully compensates principals for their search cost. This was the case in the constrained welfare optima of Scenarios 1,2 and 3 as well as in the non-degenerate equilibria of Scenario 2. In practice it is not unusual for experts to charge a diagnosis fee below cost. Consider, for example, a plumber who comes to the homeowner and inspects the problem at no charge. Since the cost of diagnosis is positive in this example, a zero diagnosis fee corresponds to a negative diagnosis fee in our model.

Nevertheless, it may sometimes be more plausible to think of d as being strictly above $-s$ or even nonnegative. First, such a restriction might reflect a convention of behavior that does not allow experts to attract potential principals by offering them direct monetary payments. In such a case experts might still try to provide amenities that reduce the principal's costs, but it this might be insufficient to eliminate these costs completely. Second, the restriction on d might reflect the danger of abuse by outsiders with low search cost. This point refers to the possibility, mentioned before in connection with the restriction $d \geq -s$, that rewarding principals for the visit might attract low search cost people who are not interested in the service and who participate only to profit from the collection of the rewards. For these reasons, it is of interest to examine the extent to which the above analysis relies on the possibility $d = -s$, and how the results might change under the restriction $d \geq 0$. The following discussion outlines the consequences of this modification. The main difference is that, since principals now are forced to bear the search costs, configurations with $x = 1$ are precluded from being sustained as equilibria or second best welfare optima.

The characterizations of the second best results change as follows. In Scenario 3, the result of Proposition 5 is replaced by the following result (the proof is relegated to the appendix).

Proposition 6. *If d is restricted to be nonnegative in Scenario 3, then $W(d, p_1, p_2)$ is maximized at $d = 0, p_2 = 0$ and $p_1 = V - 2s/z$. With these prices $x \in (0, 1)$.*

As a corollary, we get that the second best result for Scenario 1 (the counterpart of the corollary to Proposition 5).

Proposition 7. *If d is restricted to be nonnegative in Scenario 1, then $W(d, p)$ is maximized at $d = 0$, and $p = V - 2s/z$. At these optimal prices $x \in (0, 1)$.*

Consider now the equilibrium analysis. The equilibrium result of Scenario 1 reported in Proposition 1 remains in tact. That is, the only symmetric equilibrium in this scenario is with $x = 0$. This can be seen by simply following the steps of the proof. In Scenario 2, we have an equilibrium with $x > 0$ in the present variation as well. The counterpart of the equilibrium of the main version is with $d = 0$, $p = \widetilde{c}$, where $\widetilde{c} = c/(1 - \varepsilon)$ and $f = 0$. But, for a subset of the parameters, there also exists an equilibrium with $p > \widetilde{c}$. The following proposition (proved in the appendix) describes these equilibria.

Proposition 8. *There exist $\overline{s} > 0$, such that, for all $s \leq \overline{s}$, there exist equilibria with $x > 0$. All the possible equilibrium outcomes are as follows. (i) For all $s \leq \overline{s}$, the following is an equilibrium outcome: $d = 0$, $p = \widetilde{c}$, $z = [V - \widetilde{c} - s + \sqrt{(V - \widetilde{c} - s)^2 - 8sV}]/2V$, $f = 0$. (For s in a subinterval of $[0, \overline{s}]$, there also exists an equilibrium with the same outcome except that $z = [V - \widetilde{c} - s - \sqrt{(V - \widetilde{c} - s)^2 - 8sV}]/2V$) (ii) If c is not too large, then there is an interval $[s', s'']$, $s' > 0$, $s'' < \overline{s}$, such that, for $s \in [s', s'']$, the following is an equilibrium outcome: $d = 0$, $p = V - 3\sqrt{sV} + s > \widetilde{c}$, $z = \sqrt{s/V}$, $f = 2(p - \widetilde{c})/[2(p - \widetilde{c}) + \widetilde{c}z]$.*

The restriction on d changes the result of Scenario 3 qualitatively. In contrast to the result of Proposition 3, Scenario 3 now has a non-degenerate equilibrium (with $x > 0$): the equilibrium outcome of Scenario 2 with $p = \widetilde{c}$ is also an equilibrium outcome here. The reason for the different conclusion is that, with $p_1 = p_2 = c/(1 - \varepsilon)$, a deviation $(d, \widetilde{p}_1, \widetilde{p}_2)$ of the sort that prevents the original Scenario 2 equilibrium from prevailing in the unconstrained Scenario 3 requires $\widetilde{p}_1 > c/(1 - \varepsilon) > \widetilde{p}_2$. However, $\widetilde{p}_2 < c/(1 - \varepsilon)$ induces the expert not to make any effort and is thus not part of a credible deviation.

Normalizations The model embodies two normalizations. The principal's benefit from an improperly diagnosed service is normalized to zero and so is the experts' cost of providing the service. These normalizations do not have any qualitative consequences. They just simplify the exposition by economizing on the notation. If we assumed instead that the principal's benefit from an improperly diagnosed service is L such that $s < L < V$, then the degenerate equilibrium with $x = 0$ would involve trade. If there was a positive cost for the provision of the service, then the only effect on the results will be that the equilibrium of Scenario 2 would have two prices p_1 and p_2 rather than just one and the differential $p_1 - p_2$ would just be equal to the cost.

Price competition Price competition enters the models of this paper in two ways. First, when the principal searches for a confirming opinion, the sampled firms are naturally engaged in price competition. Second, there is also ex-ante price competition, in the sense that the prices can be used to attract the principal. The latter aspect of the competition enters through the feature of the model that, before stopping the search, the principal gets to observe a last contract offer. This ex-ante price competition plays a role only in ensuring the uniqueness of the equilibrium in Scenario 2 – in the absence of some competitive pressure, there would be multiple equilibria in that scenario. However, as we noted there, the required competitive pressure need not necessarily take this form. The same effect would be achieved by modeling the price competition in a more standard Bertrand style fashion whereby in each period the principal chooses from a sample of two or more competing contract offers. This alternative form would not alter any of the results in this paper and we chose not to adopt it only for the sake of expositional simplicity.

Symmetry While the basic model allows to consider asymmetric configurations in which experts employ different strategies, the analysis focused only on symmetric equilibria. In some contexts focusing on symmetric equilibria might be justified by the symmetry of the underlying situation and it does not seem to entail a significant loss of insights. The problem here is that two of our observations are negative results establishing the impossibility of sustaining an equilibrium in which experts invest the required effort. Thus, by leaving out the possibility of asymmetric equilibria, the question of whether the failure to realize the gains from trade is a necessary consequence of the situation is only partly answered. So far, we have not been able rule out the possibility of asymmetric equilibria in either of the scenarios. However, we believe that the result of Proposition 1 in Scenario 1 is quite robust. The simple undercutting argument that gives rise to this result is very strong and it is hard to see how any sort of equilibrium can avoid it. Let us explain heuristically why we hold this belief and why it is difficult to prove this assertion formally. Suppose that Scenario 1 has an asymmetric equilibrium in which experts make some effort. Let $\overline{p}$ denote the maximal equilibrium price and let $\overline{x}$ denote the equilibrium effort of an expert who charges $\overline{p}$. Now $\overline{x} > 0$ implies that, conditional on this expert being sampled, with positive probability the principal searches for a matching recommendations but still ends up transacting with this expert. But in such a case the matching recommendation that the principal must be from another expert who charges $\overline{p}$. Now it may seem that the ex-post price undercutting argument of Proposition 1 should work with respect to the $\overline{p}$ as well, thus establishing the result for the asymmetric case as well. However, there is a difficulty in applying this argument here. We cannot rule out the possibility that, after being offered a lower ex-post price by a $\overline{p}$ expert, the principal will decide to concentrate his search on experts who charge lower prices. In a such a case, upon getting a matching recommendation, the principal will not go back to transact with the $\overline{p}$ expert and hence after deviating this expert has no incentive to invest effort in the diagnosis. Finally, as an alternative robustness argument, observe that in a variation on the basic model in which whenever the principal obtains two matching opinions he can get the two experts to bid against each other, the result of Proposition 1 would be obvious.

The role of ε The feature that 0 effort still yields a correct diagnosis with probability $\varepsilon > 0$ is probably not objectionable on grounds of lack of realism. After all, the 0 effort level can be viewed just as a normalization for a minimal effort level which is not totally useless. However, this assumption plays has a technical role in avoiding the need to deal with some issues associated with zero probability events which might be encountered off the equilibrium path. If we assumed instead that $\varepsilon = 0$, there will appear a new type of equilibrium in which price competition is deterred by allowing irrelevant price offers to trigger significant changes in the principal's search behavior. This behavior will disappear in the presence of some noise and the assumption $\varepsilon > 0$ provides such noise that fits naturally into the model.

The following example describes an equilibrium that appears in Scenario 1 in the in the case $\varepsilon = 0$, but is ruled out by the assumption $\varepsilon > 0$. Consider Scenario 1 and the configuration $d = -s$, $p = V$, $x = 1$, $f = 0$. That is, all experts get informed and the principal always searches for a matching recommendation. Suppose that, if the principal encountered a price deviation (whether in the announced price or in the form of ex-post price revision), he would quit the process and hence not purchase from that expert. Suppose also that an expert who deviated chooses effort 0.

Note that the above described behavior, after a possible deviation, constitutes an equilibrium in the continuation. Given the principal's response, it is optimal for the deviant expert to choose effort 0. This and the fact that the principal is exactly indifferent between quitting and staying on in the process imply that the principal's decision to quit is indeed optimal. Now, this behavior in the continuation sustains the configuration $d = -s$, $p = V$, $x = 1$, $f = 0$ as an equilibrium. Observe that price undercutting is deterred in this equilibrium by the principal's "threat" to quit. In contrast, in the case of $\varepsilon > 0$, any price undercutting in this configuration will enhance the benefit of the principal's participation and search, so that price undercutting cannot be deterred by such "threats."

Alternative interpretation of the model The model and the analysis were developed in the context of a single principal who samples sequentially from a population of experts. It is possible to embed this basic scenario in a market setting in which the principal side consists of a population of such principals. In such a model we envision the market as operating over time without beginning or end. At any period the principals who have just obtained the service depart from the market and there is a flow of new principals into the market. Thus, at any time the population consists of principals who have experienced different search histories. In a steady state the distribution of histories over the principal population remains constant over time (although the principals themselves change). A steady state equilibrium of this model would correspond to the equilibrium of the model analyzed above, where the beliefs coincide with the equilibrium steady state distribution of the principal population.

7. Appendix

Proof of Proposition 3. Suppose there exists a symmetric equilibrium (d, p_1, p_2, f, B) in which $x > 0$. Following are a number of observations concerning such an equilibrium.

Observation 1: $p_2 \geq c$.

Suppose to the contrary that $p_2 < c$. Then $x > 0$ implies $p_1 > c$, since otherwise the expert is better off with $x = 0$. Now suppose that an expert undercuts the price p_1 slightly once the principal has chosen to sample him. I.e., he chooses an ex-post price $\widetilde{p}_1' < p_1$. Since there is a positive probability (at least $\varepsilon > 0$) that this expert's diagnosis is correct, by a similar argument to the one given in Proposition 1, the principal's best response is either to purchase immediately from this expert or to keep searching to obtain two matching recommendations. Hence, by investing the cost c, the expert can guarantee a profit of $d + \widetilde{p}_1' - c$, which for $\widetilde{p}_1'$ sufficiently close to p_1 is larger than the equilibrium profit given by the LHS of (4.14). In other words, by shedding p_1, the expert switches all transactions to a price near p_1 instead of having some fraction at p_2.

Observation 2: $x < 1$.

Observe that since $p_2 > 0$ and $d \geq -s$, (4.13) holds only if $x < 1$.

Observation 3: $p_1 = p_2 = c + s$

Suppose that $p_1 > p_2$. Consider an ex-post price undercutting $\widetilde{p}_1' < p_1$ such that $\widetilde{p}_1' > p_2$. The same argument as the one used in Observation 1 above implies that, by choosing to always invest in the diagnosis, the deviant expert's profit is at least $d + \widetilde{p}_1' - c$, which for $\widetilde{p}_1'$ sufficiently close to p_1 is larger than the equilibrium profit.

Suppose that $p_2 > p_1$. Consider an ex-post price undercutting $\widetilde{p}_2' < p_2$ such that $\widetilde{p}_2' > p_1$. Let $\widetilde{x}$ denote the deviant expert's decision in the subgame following this deviation. Obviously, is $\tilde{x} \in (0, 1)$. Let w be the probability that the principal would stop and transact with this expert. (i.e., w is the counterpart of $B(0)f$). The counterpart of (4.12) is

$$p_1 w + \varepsilon (1 - w) \widetilde{p}_2' = p_1 w + (1 - w) \widetilde{p}_2' - c$$

Notice that since $\widetilde{p}_2' < p_2$ all those who are diagnosed correctly and continue searching end up paying the deviant expert the price $\widetilde{p}_2'$. This equality and (4.12) imply

$$(1 - w) \widetilde{p}_2' = c/(1 - \varepsilon) = (1 - B(0)f) \frac{p_1 + p_2}{2}$$

This equality has the following three implications. First, if $\widetilde{p}_2'$ is sufficiently close to p_2, so that $\widetilde{p}_2' > (p_1 + p_2)/2$, then $w > B(0)f$. Second, the deviant expert's profit, $d + p_1 w + \varepsilon(1 - w)\widetilde{p}_2'$, is equal to $d + p_1 w + \varepsilon c/(1 - \varepsilon)$. Similarly, the equilibrium profit, $d + p_1 B(0)f + \varepsilon (1 - B(0)f) \frac{p_1 + p_2}{2}$, is equal to $d + p_1 B(0)f + \varepsilon c/(1 - \varepsilon)$. Thus, the deviant's profit is larger than the equilibrium profit and so the deviation is profitable.

Thus $p_1 = p_2$. Finally, $p_1 = p_2 = c + s$ follows from the argument presented in Proposition 2.

Observation 4. $d = -s$.

If $d > -s$, then consider the deviation to $(\tilde{d}, \tilde{p}, \tilde{p}) = (-s, p + d + s, p + d + s)$. Since this expert provides the correct diagnosis with positive probability (at least ε), the principal would choose to be diagnosed by her. Furthermore, recall that, after the principal decides to stop, he still gets to hear an additional price offer. Thus, if the event that the principal intends to stop after the first recommendation but observes $(\tilde{d}, \tilde{p}, \tilde{p})$ before departing, he would strictly prefer to continue. This implies that $(\tilde{d}, \tilde{p}, \tilde{p})$ strictly increases the expected profit of the deviating expert.

Observation 5: A candidate equilibrium $(-s, p, p)$ is undermined by the deviation $(-s, \tilde{p}_1, \tilde{p}_2)$, where $\tilde{p}_1 > p > \tilde{p}_2$ (and where $\tilde{p}_1$ and $\tilde{p}_2$ are sufficiently close to p).

First observe that $p \geq c + s > c$. Let $\tilde{x}$ denote the deviant expert's decision in the continuation equilibrium. We claim that $0 < \tilde{x} < 1$. To see this observe that $\tilde{x}$ may not be equal to 0, since then the expert's payoff is $-s + \varepsilon \tilde{p}_2$, but by choosing $e = 1$ the expert can get $-s + \tilde{p}_2 - c$. Also, $\tilde{x}$ may not be equal to 1, since then the principal's best response is to stop and hence the expert profits from choosing $\tilde{x} = 0$.

Therefore, the expert is indifferent regarding the effort decision. Let w be the probability that, if the principal samples the deviant expert, he would stop and transact with this expert. As above

$$\tilde{p}_1 w + \varepsilon(1-w)\tilde{p}_2 = \tilde{p}_1 w + [1-w]\tilde{p}_2 - c \tag{7.1}$$

Solving (7.1) we get

$$w = \frac{\tilde{p}_2 - \frac{c}{1-\varepsilon}}{\tilde{p}_2} \tag{7.2}$$

The profit of the deviant would be

$$-s + w(\tilde{p}_1 - c) + (1-w)(\tilde{p}_2 - c)$$

Substituting w from (7.2), the deviant expert's profit is

$$-s + \frac{\tilde{p}_2 - \frac{c}{1-\varepsilon}}{\tilde{p}_2}(\tilde{p}_1 - c) + \frac{\frac{c}{1-\varepsilon}}{\tilde{p}_2}(\tilde{p}_2 - c)$$

which upon rearrangement yields

$$-s + \frac{\tilde{p}_2 - c}{\tilde{p}_2(1-\varepsilon)}\tilde{p}_1 - \frac{\varepsilon}{1-\varepsilon}(\tilde{p}_1 - c)$$

Since ε is small, and since $p = c + s$ is sufficiently smaller than V, there is an appropriate choice of $\tilde{p}_1$ and $\tilde{p}_2$, that make this profit is larger than the equilibrium profit, $-s + p - c$.

Hence, this deviant offer is indeed attractive for the principal after any history on the path. Therefore, the configuration (d, p, p) cannot be an equilibrium.

Observations 2 and 5 together imply that there is no equilibrium with $x > 0$, since observation 2 implies that in such an equilibrium $p_1 = p_2$, while observation 5 implies that such a configuration is not immune to deviations. ■

Proof of Proposition 4. The value of f for which $W(d, p)$ is attained is such that (4.4) holds with equality. To see this suppose that, at the optimum, (4.4) holds with strict inequality. Then $z = x = 1$ and hence

$$W(d, p) = f(V - s - c) + (1 - f)(V - 2s - 2c)$$

This expression is increasing in f. Now, since (4.4) implies $f < 1$ and since by the supposition (4.4) is not binding, an increase in f will not violate the constraints. Therefore, there must be a fixed price equilibrium with a higher f and hence higher W, contrary to the presumed optimality of the prices.

Let $\widetilde{c} = c/(1-\varepsilon)$. Rearranging the equation (4.4), we get

$$f = \frac{2p - 2c/(1-\varepsilon)}{2p - 2c/(1-\varepsilon) + cz/(1-\varepsilon)} = 2\frac{p - \tilde{c}}{2p - 2\tilde{c} + \tilde{c}z}$$

Upon substitution in $W = f\,(zV - s - xc) + (1-f)\left(V - 2s/z - 2\frac{x}{z}c\right)$ and $x = \frac{z-\varepsilon}{1-\varepsilon}$ we get

$$\begin{aligned} W &= f\,(zV - s - xc) + (1-f)\left(V - 2\frac{s+xc}{z}\right) \\ &= f\,(zV - s - (z-\varepsilon)\tilde{c}) + (1-f)\left(V - 2\frac{s + z\tilde{c} - \varepsilon\tilde{c}}{z}\right) \\ &= \frac{zV(2p - \tilde{c}) - 2p(s + \tilde{c}z) + 2p\tilde{c}\varepsilon}{2p - 2\tilde{c} + \tilde{c}z} \end{aligned}$$

$$\begin{aligned} \frac{\partial W}{\partial x} &= \frac{\partial}{\partial z}\left(\frac{zV(2p - \tilde{c}) - 2p(s + \tilde{c}z) + 2p\tilde{c}\varepsilon}{2p - 2\tilde{c} + \tilde{c}z}\right)(1-\varepsilon) \\ &= 2\frac{(p-c)\,(2Vp - Vc - 2pc) + cps - pc^2\varepsilon}{(2p - 2c + cz)^2}(1-\varepsilon) > 0 \end{aligned}$$

where the inequality owes to $V \geq 2p \geq c$ and ε small (relative to cs).

Thus, for any p in this range, it is optimal to increase x as much as possible. Since

$$z = \frac{V - p + s + d + \sqrt{(V - p + s + d)^2 - 8V(s+d)}}{2V}$$

and $x = (z-\varepsilon)/(1-\varepsilon)$, we have $\partial x/\partial d < 0$, and this implies that it is optimal to set $d = -s$. Now, for $d = -s$, $z = \frac{V-p}{V}$ and upon substituting into W we get

$$W = \frac{(V-p)(2p - \tilde{c}) - 2p(s + \tilde{c}\frac{V-p}{V}) + 2p\tilde{c}\varepsilon}{2p - 2\tilde{c} + \tilde{c}\frac{V-p}{V}}$$

Now if ε is sufficiently small, the first order condition is approximately:

$$\frac{\partial}{\partial p}\frac{(V-p)(2p - \tilde{c}) - 2p(s + \tilde{c}\frac{V-p}{V})}{2p - 2\tilde{c} + \tilde{c}\frac{V-p}{V}} \approx 0$$

which yields

$$p \approx V\frac{\tilde{c}(V - \tilde{c}) + \sqrt{\tilde{c}(V - \tilde{c})(\tilde{c}(V - \tilde{c}) + s(V - \tilde{c})}}{(2V - \tilde{c})(V - \tilde{c})}$$

and if s is sufficiently small, p is near

$$V\frac{2\tilde{c}}{2V-\tilde{c}}$$

This together with V being sufficiently large relative to c implies $c+s<p<(V-s)/2$, which means that p covers the experts' cost and satisfies the principal's participation constraint, $c+s\leq p\leq xV-s=V-p-s$ ■

Proof of Proposition 5. The analogous argument to that given in the proof of Proposition 4 above yields

$$f=\frac{p_1+p_2-2\tilde{c}}{p_1+p_2-2\tilde{c}+\tilde{c}z}$$

Therefore,

$$\begin{aligned} W &= f\left(zV-s-xc\right)+(1-f)\left(V-2s/z-2c\frac{x}{z}\right) \qquad (7.3)\\ &= \frac{(p_1+p_2-2\tilde{c})[zV-s-xc]+\tilde{c}z(V-\frac{2s}{z}-2c\frac{x}{z})}{p_1+p_2-2\tilde{c}+\tilde{c}z} \\ &= \frac{(p_1+p_2-2\tilde{c})[zV-s-z\tilde{c}]+\tilde{c}z(V-\frac{2s}{z}-2\tilde{c})+\varepsilon\tilde{c}[p_1+p_2]}{p_1+p_2-2\tilde{c}+\tilde{c}z}\end{aligned}$$

Next observe from (4.12) and (4.15) that $2\tilde{c}\leq p_1+p_2\leq V$. The following calculation shows that W is increasing in x for p_1+p_2 in this range,

$$\begin{aligned} &\frac{\partial W/(1-\varepsilon)}{\partial x}=\frac{\partial W}{\partial z}= \\ &\frac{(p_1+p_2-2\tilde{c})\left((p_1+p_2-2\tilde{c})V+\tilde{c}(V-p_1-p_2)\right)+(p_1+p_2)s\tilde{c}+\varepsilon\tilde{c}^2(p_1+p_2)}{\left(2p-2\tilde{c}+z\tilde{c}\right)^2} \\ &> 0\end{aligned}$$

Also observe that for $x=1$, W is increasing in p_1+p_2. Thus, an upper bound on W is obtained at $x=1$ and $p_1+p_2=V$. It remains to verify that there is a feasible choice of (d,p_1,p_2) such that $p_1+p_2=V$ and that supports a fixed price equilibrium with $x=1$.

Now (4.13) implies that $x=1$ can be attained if and only if $d=-s$ and $p_2=0$. Thus, $(d,p_1,p_2)=(-s,V,0)$ maximizes W. ■

Proof of Proposition 6. The analysis proceeds as in the proof of Proposition 5 recognizing that W is increasing in x. However, since $d\geq 0$, (4.13) cannot hold for $x = 1$. Therefore, in the optimum $x < 1$. This implies that in the optimum (4.13) holds with equality, since otherwise x can be increased further. Thus,

$$zV - p_1 - (s+d) = V - p_1 - p_2 - 2\frac{s+d}{z} \tag{7.4}$$

yielding

$$z = \frac{V - p_2 + s + d + \sqrt{(V - p_2 + s + d)^2 - 8V(s+d)}}{2V}. \tag{7.5}$$

Since $x = (z - \varepsilon)/(1 - \varepsilon)$, it follows that $\partial x/\partial d < 0$, implying that it is optimal to set $d = 0$. Now, since p_1 and p_2 appear as a sum everywhere except in the expressions for z and hence x, the optimum involves setting $p_2 = 0$ to maximize x and setting p_1 to the desired level. Observe from (7.3) that

$$\begin{aligned} &\frac{\partial W}{\partial p_1} \\ = \;& \frac{\partial}{\partial p_1}\left(\frac{(p_1 + p_2 - 2\widetilde{c})(zV - s - z\widetilde{c}) + \widetilde{c}z(V - \frac{2s}{z} - 2\widetilde{c}) + \varepsilon\widetilde{c}(p_1 + p_2)}{p_1 + p_2 - 2\widetilde{c} + \widetilde{c}z}\right) \\ = \;& \frac{\widetilde{c}z\left(zV - s - z\widetilde{c} - \left(V - 2\frac{s}{z} - 2\widetilde{c}\right) - \varepsilon(\widetilde{c} - \frac{2}{z})\right)}{(p_1 + p_2 - 2\widetilde{c} + \widetilde{c}z)^2}. \end{aligned}$$

With $d = 0$ and $p_2 = 0$, (7.4) implies

$$zV - s = V - 2\frac{s}{z}.$$

Therefore, $\partial W/\partial p_1 > 0$, so p_1 is set to the maximum possible, given the constraints, which is equal to $V - 2\frac{s}{z}$. ■

Proof of Proposition 8. Let (d, p, x), such that $d = 0$, $p \geq \widetilde{c}$ and $x > 0$, be the experts' equilibrium strategy. Consider a deviation $(\widehat{d}, \widehat{p})$, with $\widehat{p} > \widehat{c}$. Clearly, if $\widehat{p} + \widehat{d} + s \geq 2p + 2s/z$, then this deviation is unattractive to the principal under any circumstances and hence does not undermine the equilibrium. Suppose therefore that this is not the case. Assume that the expert believes that, if the principal responds, he has no previous observations, i.e., $B(0 \mid \widehat{d}, \widehat{p}) = 1$. Let $\widehat{x}$ denote the deviant expert's decision in the continuation equilibrium, let $\widehat{z} = \widehat{x} + \varepsilon(1 - \widehat{x})$ and let $\widehat{f}$ denote the probability that the principal will stop after the sampling the deviant, conditional on this being his first sampling. Obviously, as we have shown in similar situations earlier (see, e.g., the proof of Proposition 3), $0 < \widehat{x} < 1$. This together with $\widehat{p} > \widetilde{c}$ implies that $\widehat{f} \in (0, 1)$. Hence,

$$\widehat{z}V - \widehat{p} = V - [2(1-\widehat{z}) + \widehat{z}]\frac{s}{z} - p - [\widehat{z}\widehat{p} + (1-\widehat{z})p]$$

That is,

$$\widehat{z} = \frac{V - 2s/z + \widehat{p} - 2p}{V - s/z + \widehat{p} - p} \tag{7.6}$$

Consider the principal after a history consisting of $n \geq 1$ non-matching recommendations. Recall from Section 3 the probability $\varphi(n)$ that a randomly drawn expert out of these n has the correct diagnosis, $\varphi(n) = z/[1 + (n-1)z]$. Observe that, after such history, the principal's incremental payoff from getting diagnosed by the deviant expert and continuing the search in the event that her recommendation still does not match any of the previous ones is

$$V - \widehat{d} - [1 + \frac{2 - \widehat{z} - n\varphi(n)}{z}]s - p - [\widehat{z}\widehat{p} + (1-\widehat{z})p]$$

Thus, the difference in incremental benefit between this plan and continuing to search among the other experts is

$$\frac{\widehat{z} - z}{z}s + \widehat{z}(p - \widehat{p}) - \widehat{d} \tag{7.7}$$

Notice that this expression is independent of n. Since $\widehat{z}$ is such that, with $n = 0$ previous recommendations, the principal is just indifferent between stopping and continuing, then with any number of previous recommendations, the principal prefers to continue searching after getting a non-matching recommendation from the deviant expert. Thus, (7.7) captures the gain associated with being diagnosed by the deviant expert, for all the relevant principal's history. Therefore, for the configuration described in the proposition to be an equilibrium, the expression in (7.7) must be maximized over $(\widehat{d}, \widehat{p})$ at $\widehat{p} = p$ and $\widehat{d} = 0$.

Clearly, since $\widehat{d} \geq 0$, maximization requires $\widehat{d} = 0$. The first order condition with respect to $\widehat{p}$ is

$$\left(\frac{s}{z} + (p - \widehat{p})\right)\frac{\partial \widehat{z}}{\partial \widehat{p}} - \widehat{z} \leq 0 \text{ with equality holding for } p > \widetilde{c} \tag{7.8}$$

where the inequality version is reserved for $p = \widetilde{c}$ Since the second derivative is negative everywhere, the solution to the first order condition is the global maximum.

Now, the configuration (d, p, x) under consideration is an equilibrium outcome if it satisfies (7.8) and the system (4.4)-(4.7). Notice further that (4.5) must hold with equality, i.e., $zV - s - p = V - 2s/z - 2p$. If it held with strict inequality instead, then $\lim_{\widehat{p} \to \widetilde{c}} \widehat{z} > z$. That is, a slight upward price deviation would be associated with a discrete increase in z and this would necessarily undermine the equilibrium.

Part (i). Observe that $p = \widetilde{c}$ together with $x > 0$ imply $f = 0$. Conditions (4.5) (holding with equality) and (7.8) evaluated at $\widehat{p} = \widetilde{c}$ yield.

$$z = \frac{V - \widetilde{c} - s \pm \sqrt{(V - \widetilde{c} - s)^2 - 8sV}}{2V}$$

$$z \geq \sqrt{\frac{s}{V}} \tag{7.9}$$

Let z^+ and z^- denote the large and small z roots (with the $+$ and $-$) respectively. Let s_1 be the maximal s for which $(V - \widetilde{c} - s)^2 \geq 8sV$, let s_2 be the maximal s for which $z^+V \geq s + \widetilde{c}$ and let $\overline{s} = \min[s_1, s_2]$. Now, it is straightforward to verify that any $s \leq \overline{s}$ satisfies (7.9) and by the choice of $\overline{s}$ it also satisfies (4.7). Thus, the configuration with $d = 0, p = \widetilde{c}, z = z^+$ and $f = 0$ is an equilibrium.

Similarly, there is also a range of s (a subset of $[0, \overline{s}]$) for which this configuration with $z = z^-$ instead of z^+ is also an equilibrium.

Part (ii). Conditions (7.8), evaluated at $\widehat{p} = p$, and (4.5), both holding as equalities have a unique solution such that $p > 0$.

$$p = V - 3\sqrt{sV} + s \qquad z = \sqrt{s/V} \tag{7.10}$$

By construction, a configuration with these p and z is immune against deviations and it satisfies (4.5). To be an equilibrium outcome, this configuration has to satisfy the rest of system (4.4)-(4.7). The choice of $f = 2(p - \widetilde{c})/[2(p - \widetilde{c}) + \widetilde{c}z]$ guarantees that (4.4) holds. Condition (4.6) is

$$0 \leq p - \widetilde{c} \equiv V - 3\sqrt{sV} + s - \widetilde{c}$$

and condition (4.7) is

$$0 \leq z^+V - s - p \equiv 4\sqrt{sV} - 2s - V$$

It can be verified that, if $\widetilde{c} \leq (\sqrt{2} - 1)V/2$ and $s \in [s', s''] = [V(3 - 2\sqrt{2})/2, 7V + 2\widetilde{c} - \sqrt{5V + 4\widetilde{c}}]$, then those conditions are satisfied (the condition on $\widetilde{c}$ guarantees that $[s', s'']$ is non-empty) and hence this configuration is an equilibrium outcome. It can also be verified that $s'' < \overline{s}$.

Finally, it follows from the arguments of Parts (i) and (ii) above that the p, z combinations described there are the only ones possible in equilibrium. Notice also that we may not have such an equilibrium with $d > 0$, since then a slight undercutting of d would be profitable for an expert. Therefore, the above described configurations are the only possible. ■

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