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**Costly Information Acquisition, Price Volatility,  
and Efficiency in Competitive Market Games**

by

Matthew O. Jackson\* and James Peck\*\*

Preliminary Draft: December 1993

Revision: March 1994

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*Abstract:* We examine price formation in a simple static model with asymmetric information, a countable number of risk neutral traders and without noise traders. Prices can exhibit excess volatility (the variance of prices exceeds the variance of dividends), even in such a simple model. More generally, we show that for an open set of parameter values no equilibrium has prices which turn out to equal the value of dividends state by state, while for another open set of parameter values there exist equilibria such that equilibrium prices equal the value of dividends state by state. When information collection is endogenous and costly, expected prices exhibit a “V-shape” as a function of the cost of information: They are maximized when information is either costless so that everyone acquires it, or else is so costly that no one chooses to acquire it. Prices are depressed if information is cheap enough so that some agents become informed, while others do not. If the model is altered so that information is useful in making productive decisions, then the V-shape is altered, reducing the attractiveness of prohibitively high costs.

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## 1. Introduction

Milgrom (1981) and Dubey, Geanakoplos, and Shubik (1987) offer rational expectations equilibrium as a tool for investigating price behavior and information acquisition. The core of the criticism is that in order to understand how prices come to reveal information (and thus to understand the value of and incentives to acquire asymmetric information) one has to model the mechanism through which prices are formed. Well known paradoxes, such as those discussed by Grossman (1976) and Grossman and Stiglitz (1980), are symptomatic of the fact that rational expectations equilibrium is a limiting notion, designed to capture the point at which no further adjustment in actions or prices is necessary, but not well suited for understanding how prices come to incorporate information. If one takes the rational expectations definition literally, then one must reconcile the assumption that all traders can condition their actions on the realized price, with the fact that prices result from those actions.<sup>1</sup>

Here, following this criticism, we examine price formation in a simple static model. Trade occurs according to the rules of the market game of Shapley and Shubik (1977). Specifying the trading rules allows us to explore price behavior, information acquisition, and efficiency. In particular, we study the equilibrium acquisition of information and its resulting welfare implications, as well as equilibrium price volatility relative to the volatility of the payoff of the underlying asset.

We consider an infinite number of agents, so that the model is competitive: no agent can influence the price. The terminal payoff (dividend) of the risky asset is unknown and individuals observe a signal which is correlated with the payoff. Signals are independently and identically distributed across individuals, conditional on the dividend. In contrast to the rational expectations equilibria of the same economy, agents must choose their actions only based on their private information; bids and offers cannot be made contingent upon the realized price. This is similar to the information one has available when deciding what to bid in a standard auction, such as the Vickrey auctions considered in Milgrom (1981).<sup>2</sup>

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<sup>1</sup> More formally, prices should be measurable with respect to agents' equilibrium actions, which is not commonly part of the definition, and is only possible if one adds some noise (e.g. noise traders) to the model.

<sup>2</sup> The market game, as an alternative to an auction, avoids indivisibilities and allows agents to decide on how much to offer, as well as how much to bid. With complete informa-

When the various bids and offers are aggregated to form a price, the resulting price is equal to the realized value of the asset only by chance. We show that for an open set of parameter values no equilibrium has prices which turn out to equal the value of dividends state by state, while for another open set of parameter values there exist equilibria such that equilibrium prices equal the value of dividends state by state. In fact, as we shall show, the price may be more or less variable than the underlying asset, depending on the details of the environment.

With regards to information acquisition and welfare implications our results may be summarized as follows. When information is costly to acquire, the number of agents who acquire information depends on the cost relative to its informativeness. As one would expect, holding all else equal, the fraction of agents acquiring information is decreasing in the cost of that information. Since equilibrium information is valuable, prices cannot equal the value of dividends state-by-state, otherwise agents could free-ride off that information. What is more interesting is that equilibrium expected prices exhibit a “V-shape” as a function of the cost of information: They are maximized when information is either costless so that everyone acquires it, or else is so costly that no one chooses to acquire it. Expected prices are depressed if information is cheap enough so that some agents become informed, while others do not. The degree to which the prices are depressed exactly offsets the resources spent on acquiring information. The resulting welfare implications are straightforward: since resources spent on information are lost to the economy, and since the equilibrium has no redistributive effect, it is best to either have costless information, or information so costly that no one chooses to acquire it. This would suggest taxing information acquisition to make it prohibitively costly, if it is not already so.

What is missing from the above discussion is the idea that information may have some productive value, in that it allows for more efficient investment decisions. The last section of the paper examines the model when information helps in making productive decisions. There, the V-shape of the expected price is altered, reducing the attractiveness of prohibitively high costs. This implies that one may, in fact, want to subsidize information

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tion, the equilibria of the market game coincide with the Walrasian equilibria [see Dubey, Geanakoplos, and Shubik (1987)]. The results we present in this paper would also obtain in standard auction settings where the amount to be sold is exogenously fixed, as we discuss in Section 8.

acquisition.

The advantage of modelling the behavior and utilities of all traders is that welfare analysis is possible. One can evaluate the welfare implications of subsidizing/taxing information acquisition, of imposing disclosure rules or insider trading rules, and so on. Furthermore, since explicit treatment of price formation can yield equilibria quite different from any rational expectations equilibrium, this work suggests that further examination of price formation and resulting welfare and policy issues are warranted. Our analysis can only be seen as suggestive: one must venture beyond our tractable model to games corresponding to actual or proposed trading institutions.

In addition to the study of information acquisition, the model allows for a close look at the characteristics of the equilibrium price. Most notably, equilibrium prices exhibit characteristics which are inconsistent with standard competitive models with symmetric information, but which are consistent with historical observations. For a non-degenerate set of parameters, the unique equilibrium prices of the market game exhibit excess volatility (the variance of prices exceeds the variance of the asset's dividend).<sup>3</sup>

A popular alternative to the approach taken here is to examine rational expectations equilibrium with some noise in the model, so that equilibrium prices are not fully revealing and the paradoxes are avoided. Information is valuable in such a model, which allows one to study the acquisition of information [for instance, Grossman and Stiglitz (1980), Verchick (1982), Kyle (1989)]. Such models also allow for excess volatility in prices [Campbell and Kyle (1993) and Wang (1993)]. One difference of our approach is that we can examine welfare implications, without having noise traders absorb losses due to information asymmetries. Another difference is that in our model the excess volatility in the prices is fully attributable to the structure of information and the rules by which prices are formed,

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<sup>3</sup> Some of the first work exposing the excess volatility pricing anomaly is by LeRoy and Porter (1981) and Shiller (1981). [A survey of the literature is offered by Flood and Hodrick (1991).] This early work has fallen under a number of criticisms concerning both the methodology [e.g. Kleidon (1986)] and the specification of the model [e.g. Marsh and Merton (1986)]. More recent studies [see for example Campbell and Shiller (1988), West (1988), and Cochrane (1992)] admit richer models and use stronger econometric techniques. Particular focus has been on finding a way to generate a time varying discount rate which would generate sufficient volatility in prices. The requirements on the volatility of this discount rate seem to be too large to justify with a reasonable (symmetric information competitive) model.

rather than depending on the degree of noise present. In this way our analysis complements previous studies, providing insight helping to separate the value of information and the price characteristics attributable to the information structure and trading rules, from that attributable to the inherent noise in an economy.

The sunspots literature also admits excess volatility (and is consistent with Shiller's alternative hypothesis). [See, for instance, Shell (1977), Azariadis (1981), Cass and Shell (1983).] Recent papers in that area have developed richer models for this alternative hypothesis of speculative bubbles. For example, in Jackson and Peck (1991) it is shown that there exist speculative equilibria in which fully rational agents use private information in bidding for assets. The private information is not correlated with the assets' dividend processes, but is correlated with the information of other agents. Agents earn strictly positive rents from using this information in their bids. In these equilibria the price may be more volatile than the underlying dividend process and there may be high premia associated with the assets, even without risk aversion.

The main contrast between the sunspots literature and the model considered here is that actions are only contingent on information directly correlated with the final payoff of the asset here, rather than speculative (extrinsic) information. [The static setting and the absence of correlating devices in the model which we consider here lead to unique equilibrium prices, and so what is "fundamental" here is never an issue.] Although the previous research shows that speculation is compatible with full rationality, it still requires that agents solve a possibly large coordination problem in identifying what information to use. Furthermore, the implications derived from a theory based on fundamental information are different from those based on speculative information. A fundamental theory ties the degree of excess volatility to the information structure of the model, information which is likely to be observable. Developing a richer understanding of price behavior in models with asymmetric information will help lead to more powerful tests and more confident conclusions.

## **2. The Basic Model with Endowed Information**

We begin with an analysis where the information structure is exogenously given. Agents privately observe signals which are independently and identically distributed. In subsequent

sections (6–8), we will extend the model so that information is costly and only some agents will choose to become informed in equilibrium.

A countable number of risk-neutral agents are indexed by  $i \in \mathbb{N} = \{1, 2, 3, \dots\}$ . (The countable number assures that markets are competitive, without running into the technical problems with regards to the law of large numbers experienced by models with a continuum of agents.) There is a single consumption good. A risky asset pays dividends of the consumption good. Agents are each endowed with 1 unit of the consumption good and 1 unit of the risky asset. Agents observe information at time 0, agents trade at time 1, and the state is revealed and agents consume at time 2.

#### INFORMATION.

There are a finite number,  $n$ , of possible dividend realizations. These are denoted  $d^S$ , indexed by  $S \in \{1, \dots, n\}$ .  $S$  is referred to as the dividend state. (This distinguishes it from a state of the world, which also specifies realizations of all private information.) The probability of dividend state  $S$  is  $\pi(S)$ . Information takes the form of signals  $s \in \{1, \dots, m\}$ . The probability of observing signal  $s$  conditional on  $S$  is  $\pi(s|S)$ . The probability of realizing  $S$  conditional on observing signal  $s$  is denoted  $\pi(S|s)$ , and is defined according to Bayes' Rule. Let  $s^i$  denote the signal observed by the  $i$ -th agent. Signals are independently and identically distributed across agents.

#### THE MARKET GAME.

Trade occurs according to the market game of Shapley and Shubik (1977). Agents simultaneously make bids and offers in the market for the risky asset. Offers are amounts of the risky asset to be sold, and bids are amounts of the consumption good to be paid. The total amount of the consumption good bid is allocated to agents in proportion to the size of the offers they made. Similarly, the total amount of the risky asset offered is allocated to agents in proportion to the size of the bids they made.

The strategy space for agent  $i$  is  $(b^i, q^i) \in \mathbb{R}_+ \times \mathbb{R}_+$ , where  $b^i$  denotes the amount of the consumption good bid by agent  $i$  and  $q^i$  denotes the amount of the risky asset offered by agent  $i$ . Notice that agents are permitted to bid and offer at the same time.

Define

$$\int b^i di = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{b^i}{N}$$

and

$$\int q^i di = \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{q^i}{N}.$$

No trade takes place if either  $\int q^i di$  or  $\int b^i di$  is zero or not well defined. Otherwise, the price associated with the market game is

$$P = \frac{\int b^i di}{\int q^i di}.$$

An agent offering  $q^i$  units of the risky asset for sale receives  $q^i P$  units of the consumption good, and an agent bidding  $b^i$  units of the consumption good receives  $b^i/P$  units of the risky asset.<sup>4</sup>

We consider (Bayesian) Nash equilibria of the market game, where agents choose bids and offers after having observed information, and where the payoff to agent  $i$  is the utility of final consumption. Agents' strategies are functions from observed signals into bids and offers, which maximize expected utility conditional on information and the strategies of the other agents. (See Harsanyi (1967, 1968) for more detail.) We focus on pure strategies, so agent  $i$ 's strategy space is represented by  $b^i : \{1, \dots, m\} \rightarrow \mathbb{R}_+$  and  $q^i : \{1, \dots, m\} \rightarrow \mathbb{R}_+$ . Given risk-neutrality and the convexity of the action spaces, considering mixed strategies would not substantively change matters, but would lead to considerable technical complications.<sup>5</sup>

A strong version of the law of large numbers assures that prices have a simple representation (almost surely) as a function of  $S$ , under the following condition.

*Convergent Bids and Offers:*

$$B(s) = \int b^i(s) di$$

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<sup>4</sup> For more discussion of this market game see Shapley (1976), Shubik (1973) (1977), Mas Colell (1982), Dubey, Geanakoplos, and Shubik (1987), and Peck and Shell (1985).

<sup>5</sup> A mixed strategy would map signals into the some set of probability measures over  $(b, q) \in \mathbb{R}_+ \times \mathbb{R}_+$ . Careful restrictions on the admissible measures are necessary so that the resulting measures over the product space are well defined over particular realizations.



is well defined for each  $s$  and

$$\sum_{i=1}^{\infty} \frac{\text{var}[b^i|S]}{i^2} < \infty$$

for each  $S$ . Similarly,

$$Q(s) = \int q^i(s) di$$

is well defined for each  $s$  and

$$\sum_{i=1}^{\infty} \frac{\text{var}[q^i|S]}{i^2} < \infty.$$

for each  $S$ .

Notice that this condition is trivially satisfied in any symmetric situation, where all but a finite number of agents bid and offer according to the same function of their signals. The condition is also satisfied when agents bid and offer according to different functions, as long as the variance of the functions is bounded across agents (or does not grow too fast) and the limit of the average bids and offers contingent on any dividend state is well defined.

Under the convergent bids and offers condition (see the appendix for a detailed argument), prices may be written as

$$P(S) = \frac{\sum_s \pi(s|S)B(s)}{\sum_s \pi(s|S)Q(s)}. \quad (1)$$

### 3. When Can Prices Reflect the Value of Dividends?

In this section, we consider when it is possible to have realized prices equal to dividends for each dividend realization. It turns out that there are open sets in the parameter space for which it is impossible to have realized prices equal to realized dividends, as well as open sets for which they can.

Notice that equilibrium prices may turn out to be fully revealing (*ex post*), in the sense that they differ across different states, even when they are not always equal to realized dividends. The fact that prices may be revealing *ex post* is irrelevant to traders, since their bids and offers are not made contingent on realized prices. This contrasts with rational expectations equilibria, where agents' actions can be made fully contingent on the realized prices as they submit demand schedules (hence, the source of well known paradoxes). Observed practices, such as limit orders, allow for partially contingent actions, but arguably not all. The question of whether an uninformed agent could trade in equilibrium without

suffering losses (a strong efficient markets hypothesis) is thus a question of whether prices equal dividends for all possible realizations, rather than whether prices turn out to be ex post revealing.

For prices to equal dividends in each dividend state, it must be that for each  $S$

$$P(S) = \frac{\sum_s \pi(s|S)B(s)}{\sum_s \pi(s|S)Q(s)} = d^S$$

or that for each  $S$

$$\sum_s \pi(s|S)B(s) - \sum_s \pi(s|S)Q(s)d^S = 0 \quad (2)$$

Given a number of dividend states  $n$  and a number of signals  $m$ , for the purposes of finding solutions to (2), our model is parametrized by the selections of  $\pi(s|S)$  and  $d^S$ . Let  $\mathcal{A}$  denote the set of all such selections. One needs to specify  $\pi(s|S)$  for each  $1 \leq S \leq n$  and each  $1 \leq s \leq m-1$ , and one needs to specify  $d^1, \dots, d^n$ . For any  $d^1, \dots, d^n \in \mathbb{R}_+^n$  and equilibrium, we can rescale  $d^1, \dots, d^n \in \mathbb{R}_+^n$  to be in  $[0, 1]^n$  and find an equivalent equilibrium by rescaling bids by the same scalar. Then  $\mathcal{A}$  can be viewed as the subset of  $[0, 1]^{nm}$  such that  $\sum_{i=(S-1)m+1}^{Sm-1} a_i \leq 1$  for each  $S \in \{1, \dots, n\}$ .

**PROPOSITION 1.** *If  $m \leq \frac{n}{2}$  then generically<sup>6</sup> there are no equilibria where realized prices equal dividends. If  $m > \frac{n}{2}$ , then there is an open set in  $\mathcal{A}$  such that each selection of parameters in that open set has no equilibrium for which realized prices equal dividends. If  $m > \frac{n}{2}$ , then there is also an open set in  $\mathcal{A}$  such that each selection of parameters in that open set has at least one equilibrium for which realized prices equal dividends.*

The result that there is an open set in  $\mathcal{A}$  such that each selection of parameters in that open set has no equilibrium for which realized prices equal dividends, is not true only of parameter choices with some dividend whose realization is close to 0. It is also true, for instance, if there are two different dividend realizations which are relatively close to each other and a third which is far away. The basic intuition is that in any dividend state there will be agents observing different signals. Only in some circumstances are the probabilities and dividend states such that strategies can be found which when weighted according to the various conditional probabilities precisely equal dividend realizations in each state.

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<sup>6</sup> On an open set and dense set (using the topology induced by the Euclidean metric) on  $\mathcal{A}$ .

#### 4. Excess Volatility and Comparative Statics.

The next question we consider the question is: When does variance of equilibrium prices exceed the variance of the underlying dividends? We examine how excess volatility depends the size of dividends and the informativeness of signals. In this section, we continue to assume that each agent is endowed with a signal and specialize the model as follows.

There are three dividend states  $S \in \{0, 1, 2\}$ . The probabilities of the states are  $\pi(0) = \pi(2) = .25$  and  $\pi(1) = .5$ . Dividends are  $d^0 = 0$ ,  $d^1 = \delta$  and  $d^2 = 1$ . In this case, the expected dividend is  $\delta/2 + 1/4$ . There are two possible signals  $s = 0$  and  $s = 1$ . The probability of seeing signals conditional on  $S$  is

$$\begin{aligned} \pi(s = 0|S = 0) &= z & \pi(s = 1|S = 0) &= 1 - z \\ \pi(s = 0|S = 1) &= .5 & \pi(s = 1|S = 1) &= .5 \\ \pi(s = 0|S = 2) &= 1 - z & \pi(s = 1|S = 2) &= z, \end{aligned}$$

where  $z \in [.5, 1]$ . Given this probability structure, Bayes' rule defines the following probabilities of a dividend state given a signal

$$\begin{aligned} \pi(S = 0|s = 0) &= \frac{z}{2} & \pi(S = 1|s = 0) &= \frac{1}{2} & \pi(S = 2|s = 0) &= \frac{1 - z}{2} \\ \pi(S = 0|s = 1) &= \frac{1 - z}{2} & \pi(S = 1|s = 1) &= \frac{1}{2} & \pi(S = 2|s = 1) &= \frac{z}{2} \end{aligned}$$

Again, there are two types of equilibria. The first are “no-trade” equilibria in which the average bids and offers are zero (or are not well defined). The second are “trading” equilibria, in which positive bids and offers occur.

**PROPOSITION 2.** *Consider any trading equilibrium with convergent bids and offers. Equilibrium prices are uniquely determined. Aggregate equilibrium bids and offers contingent on any signal are unique up to a rescaling. Furthermore:*

*For  $z = .5$  (uninformative signals), equilibrium prices are always equal to the unconditional expected dividend and there is no price variance.*

*For  $z > .5$  (informative signals):*

If  $\delta \leq \frac{1-z}{2z-1}$ , then the (unique) equilibrium expected price and price variance are

$$E[P] = \frac{1 + \delta - z}{8z(1 - z)}$$

$$\text{Var}[P] = \frac{(1 + \delta - z)^2(2z - 1)^2(3 - 4z + 4z^2)}{(8z(1 - z))^2}.$$

If  $\delta \geq \frac{1-z}{2z-1}$ , then the (unique) equilibrium expected price and price variance are

$$E[P] = E[d] = 1/4 + \delta/2$$

$$\text{Var}[P] = \frac{(1 + \delta - z)(2 + \delta - z) + 3/2 - 2\delta z - \sqrt{(2 + \delta - z)^2 - 4\delta z}}{8}.$$

The proof of Proposition 2 is presented in the appendix.

Clearly the expressions presented in Proposition 2 are hard to interpret directly. They are more easily understood graphically, as illustrated in Figures 1 and 2. Figure 1 shows that the equilibria described in Proposition 2 exhibit excess volatility for some values of  $\delta$  and  $z$ . Excess volatility is highest for middle values of  $\delta$ . When  $\delta$  is very low, conditional on the low signal the expected value of the dividend is relatively low, and the value conditional on the high signal is also relatively low. Prices are all relatively low. Similarly when  $\delta$  is close to 1, the prices are all relatively high, without much variance. If  $z$  is very low (near .5), then signals are not very informative and the variance of prices is low. As  $z$  gets higher, the variance of the prices grows and exceeds the variance of the dividend. This is easiest to see for a fixed  $\delta$  as in Figure 2, where  $\delta = .5$ . Once  $z$  nears 1, the signals become very informative and the prices converge to the actual dividend values, and the excess volatility disappears.

We remark that the expected price is sometimes less than the expected dividend (when  $z > .5$  and  $\delta < \frac{1-z}{2z-1}$ ). This is still consistent with an expected return of 1, since the decision to buy depends on the expected ratio of dividend over price. (Essentially, Jensen's inequality is at work here.)

Notice that, although "no-trade" theorems (see, for example, Milgrom and Stokey (1982)) apply to the situation analyzed in this section, trade occurs. No-trade theorems state that it be common knowledge that some agents anticipate being strictly better off as

a result of voluntary trade. Here agents are indifferent between trading and not trading at the equilibrium prices. The not-trade theorem makes our excessive volatility result all the more striking. We have not restricted bids and offers, so unlimited long and short positions are admissible. However, since a trader is indifferent in equilibrium, rescaling bids and offers and capping their size, does not affect the equilibrium price.

The next section explores excess volatility and the value of information when agents are not indifferent in equilibrium.

## 5. Costs of Information.

In this section, we analyze price formation when information is costly. This will require us to modify the basic model of previous sections. In order to have an equilibrium in which some agents pay to become informed, there must be some value to information in equilibrium. Previously, the initial allocation was Pareto optimal, so the no-trade theorem implied that no agent could expect to profit from equilibrium trade.

There are several methods of introducing the gains from trade necessary for information to be valuable in equilibrium. One method is to consider an overlapping generations model, where risky assets are infinitely lived and sold from generation to generation. In such a model, there are always some agents constrained to sell their assets. As a proxy for the overlapping generations dynamics, but in keeping with a simple and static model, we assume that  $q$  units of the risky asset (per agent) are exogenously offered for sale and that agents can bid but not offer. Thus, one can think of our traders as the younger generation of an OLG model, and the risky asset as being sold by the older generation.<sup>7</sup>

Before we proceed with the equilibrium analysis, let us remark that if information is costly, then it will be impossible to have equilibrium prices equal to the realized value of dividends, for each possible dividend realization (assuming that dividends can take on at least two distinct values). This contrasts with the analysis of Proposition 1 where, for an open set of parameters, it was possible to have equilibrium prices equal to the realized value of dividends, for each possible dividend realization. The reasoning is straightforward. Suppose to the contrary that equilibrium prices are equal the realized value of dividends,

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<sup>7</sup> An alternative route would be to introduce risk aversion and heterogeneous endowments. We have worked out some examples with risk aversion exhibiting excess volatility, however, closed form solutions and comparative statics are more difficult.

for each possible dividend realization. It is necessarily true that some agents are expending resources to acquire information. Since equilibrium prices are equal to the realized value of dividends, an agent is indifferent between bidding and not bidding. Any agent acquiring information could deviate and not acquire information or bid, thereby raising their utility.

Let  $c \in (0, 1]$  denote the cost of information (in units of the consumption good). The timing of the game is as follows. First, traders simultaneously choose whether or not to become informed. Each trader starts with an endowment of 1 unit of the consumption good, so informed traders are left with resources of  $(1 - c)$  when the market opens. Next, traders bid any amount less than or equal to their resources, so  $p = \int b^i di/q$ .<sup>8</sup> We assume that traders do not observe whether or not others are informed.<sup>9</sup>

To further simplify the analysis, we specialize to the signal structure of section 4, setting  $z = 1$ . Thus there are three dividend states  $S \in \{0, 1, 2\}$ . The probabilities of the states are  $\pi(0) = \pi(2) = .25$  and  $\pi(1) = .5$ . Dividends are  $d^0 = 0$ ,  $d^1 = \delta$  and  $d^2 = 1$ . In this case, the expected dividend is  $E[d] = 1/4 + \delta/2$ . There are two possible signals  $s = 0$  and  $s = 1$ . The probability of seeing signals conditional on  $S$  is

$$\begin{aligned} \pi(s = 0|S = 0) &= 1 & \pi(s = 1|S = 0) &= 0 \\ \pi(s = 0|S = 1) &= .5 & \pi(s = 1|S = 1) &= .5 \\ \pi(s = 0|S = 2) &= 0 & \pi(s = 1|S = 2) &= 1, \end{aligned}$$

Bayes' rule defines the following probabilities of a dividend state given a signal

$$\begin{aligned} \pi(S = 0|s = 0) &= \frac{1}{2} & \pi(S = 1|s = 0) &= \frac{1}{2} & \pi(S = 2|s = 0) &= 0 \\ \pi(S = 0|s = 1) &= 0 & \pi(S = 1|s = 1) &= \frac{1}{2} & \pi(S = 2|s = 1) &= \frac{1}{2} \end{aligned}$$

For trading equilibria that are symmetric among informed and symmetric among uninformed, denote each informed trader's bid function as  $b(1)$  and  $b(0)$ , and denote each un-

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<sup>8</sup> Allowing short sales complicates the analysis, but does not change the flavor of the results, provided there is something which limits agents' desire to bid infinite amounts. For instance, risk aversion or imperfect competition will suffice.

<sup>9</sup> Thus, (Bayesian) Nash equilibrium is still our equilibrium notion. We could allow agent's to observe which other agents have become informed, in which case the game would be sequential. One would then use some notion of perfection. The equilibrium we are examining would still hold.

informed trader's bid as  $bu$ . Also, let the fraction of informed traders (when it exists) be denoted as  $F$ . Thus  $F = \int x^i di$ , where  $x^i = 1$  if  $i$  is informed and 0 if  $i$  is not informed.

PROPOSITION 3. *If the quantity of the asset offered is not too large<sup>10</sup> ( $q < \frac{2(1+c)}{1+2\delta}$ ), then unique trading equilibrium which is symmetric among informed and symmetric among uninformed is characterized as follows:*

(i) *If  $c \leq 1/(3 + 8\delta)$ , then  $0 < F < 1$ , uninformed bidders do not trade, and informed bidders bid only when they observe a high signal.*

(ii) *If  $1/(3 + 8\delta) < c \leq 1/(3 + 4\delta)$ , then  $0 \leq F < 1$ , uninformed bidders participate in the bidding, and informed bidders bid only when they observe a high signal.*

(iii) *If  $1/(3 + 4\delta) < c$ , then  $F = 0$  and uninformed bidders bid the (per share) expected value of dividends.*

PROPOSITION 4. *For the equilibria outlined in Proposition 3, the expected price has a "V" shape: when  $c = 0$ , the expected price equals the expected dividend; as  $c$  increases, expected price falls, reaches a minimum (kink), and then rises. Furthermore, for small enough  $c$  and large enough  $\delta$  (close to 1), prices have higher variance than dividends.*

Proposition 3 and 4 are illustrated in figures 3 through 5. The figures specialize to the case where  $\delta = 1$ . There, when costs are small enough, agents end up overbidding in the highest dividend state ( $P(2)$  is close to  $3/2$ ) and underbidding in the middle dividend state ( $P(1)$  is close to  $3/4$ ). Informed traders are actually hoping that the dividend state is 1; they are willing to bid with the high signal because the losses in dividend state 2 are compensated by gains in dividend state 1. As  $c$  increases, informed traders must be compensated for higher costs, so prices fall. Expected price and excess volatility therefore fall. When  $c = 1/11$ , there are few enough informed traders that the uninformed start to enter. Here is where expected price is at its minimum. Naturally, price volatility continues to fall since the uninformed make the same bid in all states. As  $c$  continues to rise, fewer

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<sup>10</sup> If  $q$  is large, then given the cap on bids, prices will be low. This means that expected profits are possible, above the costs of becoming informed. Although such equilibria are interesting, they seem to be an artifact of the limits on bidding. For this reason, we do not focus on them. The proof makes clear how to find the (unique symmetric) equilibria for large  $q$ .

traders become informed, the “winner’s curse” effect becomes less severe, and expected price rises. When  $c > 1/7$ , the cost is too great to justify anyone becoming informed, and everyone bids the expected dividend for each share on the market,  $q(1 + 2\delta)/4$ .

## 6. Investment Decisions.

From the sellers’ points of view, the model in the previous section suggests that they would receive the highest expected revenue when information is either costless or so costly that no one chooses to become informed. Also notice that information is not serving any productive purpose; resources spent gathering information are simply lost to the economy. When some agents pay a positive cost to become informed, total societal welfare would improve if the government placed a prohibitively high tax on information. This conclusion depends critically on the fact that information does not affect the resources available.

In this section, we alter the model to introduce an investment decision made by a manager. This investment stage brings into the model the idea that information helps to allocate resources efficiently. The payoff of the risky asset is now given by

$$2DI - I^2,$$

where  $D$  takes on the values 0,  $\delta$ , and 1. If the manager knows  $D$ , then the optimal investment choice and the value of a share of the firm is  $D$ . Therefore, when the optimal investment is undertaken for each state, the risky asset pays 0 in state  $S = 0$ ,  $\delta^2$  in state  $S = 1$ , and 1 in state  $S = 2$ .<sup>11</sup>

The timing of the game is as follows. First, agents simultaneously choose whether or not to become informed. Next, informed agents choose bids not greater than  $1 - c$ , and uninformed agents choose bids not greater than 1. After trading occurs, the manager observes the equilibrium price and chooses an investment level to maximize expected returns. As in the previous section, we specialize to the case where  $z = 1$ .

**PROPOSITION 5.** *For small enough  $q$ , ( $q < \frac{2(1+c)}{1+2\delta^2}$ ) The unique trading equilibrium which is symmetric among informed and symmetric among uninformed is characterized as follows:*

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<sup>11</sup> The shape of the investment function is not important, provided that it is strictly concave (so that infinite investment is not optimal) and depends on  $D$ . This shape was chosen for its obvious tractability.



(i) If  $c \leq 1/(3 + 8\delta^2)$ , then  $0 < F < 1$ , uninformed bidders do not trade, and informed bidders bid only when they observe a high signal.

(ii) If  $1/(3 + 8\delta^2) < c \leq 1/(3 + 4\delta^2)$ , then  $0 \leq F < 1$ , uninformed bidders participate in the bidding, and informed bidders bid only when they observe a high signal.

(iii) If  $1/(3 + 4\delta^2) < c \leq 1/(2 + 2\delta)$ , then there is no symmetric trading equilibrium.

(iv) if  $1/(2 + 2\delta) \leq c$ , then no one is informed,  $F = 0$ . Equilibrium bids are  $b^u = q(\delta/2 + 1/4)$ , and inefficient investment is made.

When the efficient investment for each state is undertaken, the endogenously determined dividend is the same as the exogenous dividend of the previous section (substituting  $\delta^2$  for  $\delta$ ), so the “V” shape of the expected price and the possibility of excess volatility (Proposition 4) carry over. For very low costs, few resources are expended on information acquisition, most people become informed, and the expected price is high (Figure 6 illustrates the case where  $\delta = 1$ ). As costs become higher, more resources are spent on information acquisition, fewer people become informed, and the expected price falls until it reaches a minimum (kink) at  $c = 1/(3 + 8\delta^2)$ . As  $c$  increases further, still fewer people become informed, allowing uninformed people to enter the market and the expected price rises once again until reaching its maximum at  $c = 1/(3 + 4\delta^2)$ . At that point it is too costly to become informed and there are only uninformed agents in equilibrium (the interim non-existence region is discussed below), and the price drops since investment is less efficient which leads to lower dividend payoffs.

The reason for the range in which equilibrium does not exist is the discontinuity in investment that arises as the fraction of informed traders approaches zero. As  $c$  approaches  $1/(3 + 4\delta^2)$  from the left, the fraction of informed traders approaches zero, but prices still reveal the state of nature, and efficient investment occurs. When we have  $c \in [1/(3 + 4\delta^2), 1/(2 + 2\delta))$ , then no one will want to become informed if investment is efficient (and the uninformed bid accordingly). However, if investment is inefficient, then there is an incentive for someone to deviate and become informed, taking advantage of the lower prices. If prices are expected to be informative, then no one has an incentive to become informed; if prices are expected not to be informative, then there is a profitable deviation to gather information.

The non-existence of symmetric trading equilibria includes mixed-strategies. In fact, it does not appear that asymmetric trading equilibria exist either. What “wants” to happen is

to have an arbitrarily small positive fraction of consumers correlate on a coin flip, becoming informed with some probability. When the group becomes informed, prices are slightly different in the three states, so efficient investment occurs. The possibility of the group not becoming informed allows the required wedge between uninformed utility of zero and informed gross profits of  $c$ .

The current game only allows the manager to look at the equilibrium price. If an informed stockholder could report his/her signal to the manager, then it is easier to deviate from an equilibrium where no one is informed. The range of nonexistence would be slightly larger.

The model of this section captures the tradeoff between resources spent gathering information and the fact that markets could reveal information and allow managers to make better decisions. Increasing the cost of information could be harmful if uninformed (“small traders”?) are afraid to enter the market, so fewer informed traders mean lower prices. Increasing the cost of information further could be beneficial if prices continue to be informative while fewer resources are spent on information gathering. Put another way, making information more costly allows uninformed traders to enter the market in greater numbers, and efficient investments continue to be made. However, making information prohibitively costly interferes with the ability of markets to efficiently allocate resources.

Although we are hesitant to use this model to make policy recommendations on how to regulate information gathering on markets, we feel that the approach taken here, of specifying the price-formation mechanism, is the right way to pose the question. There is a tradeoff between allocative efficiency of information gathering and the cost of information gathering; a tradeoff that is impossible to capture in “price-taking” and “noise-trader” models. One direction for future work would be to model the information in a more sophisticated way. Here, a higher fraction of traders becoming informed does not affect the total information possessed by the market. An implication is that the smallest positive  $F$  is welfare-maximizing. A richer information structure, such as one where (a finite number of) agents choose the precision of their information, would allow the optimal amount of information gathering to continuously influence payoffs, and the discontinuity problem mentioned above would probably not arise.

## 7. Choices in Modelling

We wish to emphasize that it is the methodology of this paper, rather than the particulars of the model choice, which is responsible for the results we have obtained. Qualitatively similar results hold if the particular rules of the game are changed, or if we allow risk aversion, a different signal structure, or a finite number of agents.<sup>12</sup> We chose to use the market game, because it models price formation and still allows a competitive analysis when information is asymmetric<sup>13</sup>, it has unique symmetric trading equilibria, allows for bids and offers, and permits agents to buy or sell variable amounts.<sup>14</sup> The risk neutrality of agents, and the three dividend state uncertainty structure were chosen for their analytic tractability. These assumptions have allowed us to perform comparative statics on the excess volatility, endogenize information acquisition, and analyze the role of equilibrium information revelation in productive efficiency. In sum, the specificity of the model allows for a very extensive analysis. What we have lost, of course, is the ability to develop a general understanding of how sensitive these results are to broader changes in the structure of the environment. This leaves ample room for future research.

The model we have discussed in this paper is a static one. This provides for unique equilibria, simplifying the analysis. We have argued that the results obtained extend to dynamic models. We should clarify some of the issues which would arise in such a dynamic version. If dividends are independent across time, and agents only have information about the next period's dividend, the model extends most easily. When there is cross correlation, the model becomes more complicated, since price histories convey information. In any case,

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<sup>12</sup> A simple example of a second price auction with two bidders illustrates these changes all at once. The terminal value of the risky asset, denoted  $d$ , is equal to the realization of the average of two random variables  $X^1$  and  $X^2$ , which are independently and identically, uniformly distributed over the interval  $(0,1)$ . Agent 1 knows the value of  $X^1$  (but not the value of  $X^2$ ). Similarly, agent 2 knows the value of  $X^2$  (but not the value of  $X^1$ ). There is a symmetric equilibrium in which agent  $i$  bids  $b^i = x^i$ , where  $x^i$  is the realized value of  $X^i$ . The (unconditional) expected value  $d$  is  $E(d) = 1/2$  and the price of the asset is  $p = \min(X^1, X^2)$ , where  $E(p) = 1/3$  (1/6 less than the expected dividend). The ratio of the variance of the price to the variance of the dividend is 4/3. Thus, there is excess volatility, since the price is more variable than the payoff of the underlying asset.

<sup>13</sup> McAfee and McMillan (1987) provide a nice motivation for paying close attention to price formation when there is asymmetric information, and for being able to do so in a competitive setting.

<sup>14</sup> Alternatives to the market game, such as a share auction (see Wilson (1979)), or a game where agents submit demand schedules (as in Jackson (1991)), generally have an enormous multiplicity of equilibria.

as long as information is arriving over time, the price phenomena we have outlined here are likely to persist.

## References

- AZARIADIS, C. (1981), "Self-Fulfilling Prophecies," *Journal of Economic Theory*, **25**, pp. 380-396.
- BEJA, A. (1977), "The Limits of Price Information in Market Processes," Working Paper 61, Research Program in Finance, University of California at Berkeley.
- CAMPBELL, J. AND R. SHILLER (1988), "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, **1**, pp. 195-228.
- CASS, D. AND K. SHELL (1983), "Do Sunspots Matter?," *Journal of Political Economy*, **91**, pp. 193-227.
- COCHRANE, J. (1992), "Explaining the Variance of Price Dividend Ratios," *Review of Financial Studies*, **5**, pp. 243-280.
- DUBEY, P., J. GEANAKOPOLOS, AND M. SHUBIK (1987), "The Revelation of Information in Strategic Market Games: A Critique of Rational Expectations Equilibria," *Journal of Mathematical Economics*, **16**, pp. 105-137.
- FLOOD, R. AND R. HODRICK (1990), "On Testing for Speculative Bubbles," *Journal of Economic Perspectives*, **4**, pp. 85-101.
- GROSSMAN, S. (1976), "On the Efficiency of Competitive Stock Markets Where Agents have Diverse Information," *Journal of Finance*, **31**, pp. 573-585.
- GROSSMAN, S. AND J. STIGLITZ (1980), "On the Impossibility of Informationally Efficient Markets," *The American Economic Review*, **70**, pp. 393-408.
- GROSSMAN, S. AND R. SHILLER (1981), "The Determinants of the Variability of Stock Market Prices," *American Economic Review*, **71**, pp. 222-227.
- HANSEN, L. AND R. JAGGANNATHAN (1991), "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy*, **99**, pp. 225-262.
- JACKSON M. (1991), "Equilibrium, Price Formation, and the Value of Private Information," *Review of Financial Studies*, **4**, pp. 1-16.
- JACKSON, M. AND J. PECK (1991), "Speculation and Price Fluctuations with Private, Extrinsic Signals," *Journal of Economic Theory*, **55**, pp. 274-295.
- KYLE, A. (1989), "Informed Speculation with Imperfect Competition," *Review of Economic Studies*, **56**, pp. 317-356.
- LEROY, S. AND R. PORTER (1981), "The Present Value Relation: Tests Based on Implied Variance Bounds," *Econometrica*, **49**, pp. 555-574.
- MARSH, T. AND R. MERTON (1986), "Dividend Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," *American Economic Review*, **76**, pp. 483-498.
- MAS-COLELL, A. (1982), "The Cournotian Foundations of Walrasian Equilibrium Theory: An Exposition of Recent Theory." In W. Hildenbrand, *Advances in Economic Theory*, Cambridge: Cambridge University Press.

- MCAFEE, P. AND J. MCMILLAN (1987), "Auctions and Bidding," *Journal of Economic Literature*, **25**, pp. 699-738.
- MILGROM, P. (1981), "Rational Expectations, Information Acquisition, and Competitive Bidding," *Econometrica*, **49**, pp. 921-944.
- MILGROM, P. AND N. STOKEY (1982), "Information, Trade, and Common Knowledge," *Journal of Economic Theory*, **26**, pp. 17-27.
- MILGROM, P. AND R. WEBER (1982), "A Theory of Auctions and Competitive Bidding," *Econometrica*, **50**, pp. 1089-1122.
- PECK, J. AND K. SHELL (1985), "Market Uncertainty: Sunspot Equilibrium in Imperfectly Competitive Economies," Caress WP # 85-21, University of Pennsylvania.
- SHELL, K. (1977), "Monnaie et Allocation Intertemporale," mimeo.
- SHILLER, R. (1981), "Do Stock Prices Move Too much to be Justified by Subsequent Changes in Dividends?," *American Economic Review*, **71**, pp. 421-426.
- SHAPLEY, L. (1976), "Noncooperative General Exchange." In S. Lin, *Theory and Measurement of Economic Externalities*, New York: Academic Press.
- SHAPLEY, L. AND M. SHUBIK (1977), "Trade Using One Commodity as a Means of Payment," *Journal of Political Economy*, **85**, pp. 937-968.
- SHUBIK, M. (1973), "A Theory of Money and Financial Institutions: Commodity, Money, Oligopoly, Credit and Bankruptcy in a General Equilibrium Model," *Western Economic Journal*, **11**, pp. 24-38.
- SHUBIK, M. (1977), "A Price-Quantity Buy-Sell Market with and without Contingent Bids," Cowles Foundation DP # 455, Yale University.
- TIROLE, J. (1982), "On the Possibility of Speculation Under Rational Expectations," *Econometrica*, **50**, pp. 1163-1182.
- VERRECCHIA, R. (1982), "Information Acquisition in a Noisy Rational Expectations Economy," *Econometrica*, **50**, pp. 1415-1430.
- WEST, K. (1988), "Dividend Innovations and Stock Price Volatility," *Econometrica*, **56**, pp. 37-61.
- WILSON, R. (1979), "Auctions of Shares," *Quarterly Journal of Economics*, **94**, pp. 675-689.

## Appendix

### Derivation of (1)

Suppose that state  $S$  is realized. Under the convergent bids and offers condition,

$$B(s) = \int b^i(s) di$$

is well defined for each  $s$  and

$$\sum_{i=1}^{\infty} \frac{\text{var}[b^i|S]}{i^2} < \infty.$$

Similarly,

$$Q(s) = \int q^i(s) di$$

is well defined for each  $s$  and

$$\sum_{i=1}^{\infty} \frac{\text{var}[q^i|S]}{i^2} < \infty.$$

By definition

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|S]}{N} = \lim_{N \rightarrow \infty} \sum_{i=1}^N \sum_s \frac{\pi(s|S) b^i(s)}{N}$$

and thus

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|S]}{N} = \lim_{N \rightarrow \infty} \sum_s \pi(s|S) \sum_{i=1}^N \frac{b^i(s)}{N}.$$

Since  $B(s)$  is well defined and thus finite for each  $s$ , and since  $b^i(s) \geq 0$  for all  $i$  and  $s$  and the number of  $s$  is finite,

$$\lim_{N \rightarrow \infty} \sum_s \pi(s|S) \sum_{i=1}^N \frac{b^i(s)}{N} = \sum_s \pi(s|S) \lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{b^i(s)}{N} = \sum_s \pi(s|S) B(s).$$

Thus

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|S]}{N} = \sum_s \pi(s|S) B(s).$$

Similarly,

$$\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[q^i|S]}{N} = \sum_s \pi(s|S) Q(s).$$

We have established that  $\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|S]}{N}$  exists and is finite and  $\sum_{i=1}^{\infty} \frac{\text{var}[b^i|S]}{i^2} < \infty$ , and that,  $\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{E[q^i|S]}{N}$  exists and is finite and  $\sum_{i=1}^{\infty} \frac{\text{var}[q^i|S]}{i^2} < \infty$ . By Theorem D of Holmos (1988), page 204, it follows that  $\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{b^i - E[b^i|S]}{N} = 0$

almost surely and that  $\lim_{N \rightarrow \infty} \sum_{i=1}^N \frac{q^i - E[q^i|S]}{N} = 0$  almost surely. This implies that  $\int b^i d\mathbf{i} = \lim_{n \rightarrow \infty} \sum_{i=1}^N \frac{E[b^i|S]}{N}$  almost surely and  $\int q^i d\mathbf{i} = \lim_{n \rightarrow \infty} \sum_{i=1}^N \frac{E[q^i|S]}{N}$  almost surely. It follows that

$$P(S) = \frac{\int E[b^i|S] d\mathbf{i}}{\int E[q^i|S] d\mathbf{i}} = \frac{\sum_s \pi(s|S) B(s)}{\sum_s \pi(s|S) Q(s)}. \quad (1),$$

almost surely.

**Proof of Proposition 1:** (2) can be written as  $AX = 0$  for some  $X \geq 0$  such that  $X \neq 0$ , where  $A$  and  $X$  are defined by

$$A = \begin{pmatrix} \pi(s=1|S=1) & \dots & \pi(s=m|S=1) & -d^1 \pi(s=1|S=1) & \dots & -d^1 \pi(s=m|S=1) \\ \pi(s=1|S=2) & \dots & \pi(s=m|S=2) & -d^2 \pi(s=1|S=2) & \dots & -d^2 \pi(s=m|S=2) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \pi(s=1|S=n) & \dots & \pi(s=m|S=n) & -d^n \pi(s=1|S=n) & \dots & -d^n \pi(s=m|S=n) \end{pmatrix}$$

$$X = \begin{pmatrix} B(1) \\ \vdots \\ B(m) \\ Q(1) \\ \vdots \\ Q(m) \end{pmatrix}$$

It follows that there exists an equilibrium for which realized prices equal dividends, if and only if the intersection of the null space of  $A$  and  $\mathbb{R}_+^{2m}$  is larger than  $\{0\}$ . Letting  $r$  denote the number of independent rows of  $A$ , the null space of  $A$  has dimension  $2m - r$  (recall that it is always true that  $r \leq 2m$ ).

The result for  $m \leq \frac{n}{2}$  follows from the fact that generically (in  $\mathcal{A}$ ) the number of independent rows  $r$  will be  $2m$  and so the null space will be of dimension 0.

Next, suppose that  $m > \frac{n}{2}$  and consider  $A$  with full row rank ( $r = n$ ). Also suppose that there exists a dividend state  $S'$  such that  $d^{S'} = 0$ , there exists a dividend state  $S''$  such that  $d^{S''} > 0$ , and that  $\pi(s|S) > 0$  for all  $s$  and  $S$ . For  $X \neq 0$  to solve  $AX = 0$  (in particular the equation corresponding to state  $S''$ ) it must be that  $X_h > 0$  for some  $h$ ,



$1 \leq h \leq m$ . Since  $d^{S^i} = 0$  and  $A_{S^j} > 0$  for all  $j$ ,  $1 \leq j \leq m$ , we must have  $X_i < 0$  for some  $i$ ,  $1 \leq i \leq m$  (in particular, in order to solve the equation corresponding to state  $S^i$ ). Since negative bids are not allowed, there is no equilibrium where prices equal dividends state by state. Given the full row rank of the matrix  $A$ , the null space varies continuously at  $A$  (see below for details), so prices cannot equal dividends state by state, for an open set of matrices of  $A \in \mathcal{A}$ .

Finally, suppose that  $m > \frac{n}{2}$  and construct  $A$  as follows. For  $i \leq m$ , let  $d^i = i$ ,  $\pi(s = i | S = i) = 1$  and  $\pi(s = j | S = i) = 0$  for  $j \neq i$ . For  $m < i$ , let  $d^i = (2i + 1)/2$ ,  $\pi(s = i - m | S = i) = 1/2 = \pi(s = i - m + 1 | S = i)$  and  $\pi(s = j | S = i) = 0$  for  $j \neq i - m$ ,  $j \neq i - m + 1$ . (Since  $n < 2m$ ,  $i < 2m$  and so the above definitions make sense.) Consider  $X$  such that  $B(s) = s$  and  $Q(s) = 1$ . It is easily checked that  $AX = 0$ . Given the full row rank of the matrix  $A$ ,<sup>15</sup> the null space varies continuously at  $A$ . Thus there exists  $X'$ , which is positive in all elements, in the null space of each  $A'$  in some neighborhood of  $A$ .

### Continuity of the Null Space.

Consider  $A$ . First we show that the null space varies upper hemi-continuously at  $A$ . Consider a sequence  $A^k \rightarrow A$  and consider  $X^k \rightarrow X$  where  $X^k$  is in the null space of  $A^k$  for each  $k$ . Suppose that  $X$  is not in the null space of  $A$ , so  $AX = b \neq 0$ . Find  $\epsilon$  such that if  $\|b' - b\| \leq \epsilon$  then  $b' \neq 0$ . There exists  $K$  such that for  $k \geq K$   $\|A^k X - b\| \leq \epsilon/2$  and  $\|A^k X^k - A^k X\| \leq \epsilon/2$ . Thus  $\|A^k X^k - b\| \leq \epsilon$  and so  $A^k X^k \neq 0$ , which is a contradiction. Next notice that the set of matrices with less than full row rank is a closed set. Thus, since  $A$  has full row rank, it follows that there is a neighborhood of  $A$  consisting of matrices of full rank. Thus all matrices in that neighborhood have the same dimensional null space. Given the upper hemi-continuity and the fact that the null spaces are of the same dimension, the null spaces must vary continuously at  $A$ .

### Proof of Proposition 2:

To characterize trading equilibria, we begin by examining an agent's expected wealth as a function of  $b^i$  and  $q^i$ , conditional on seeing  $s = 0$  or  $s = 1$ . Under the convergent bids

<sup>15</sup>  $A$  can be diagonalized through elementary operations. Rows  $i \leq m$  are already diagonalized. For each row  $i > m$  the leading entry is  $1/2$ , so subtract  $1/2$  of rows  $i - m$  and  $i - m + 1$ . The  $i$ -th entry (now the leading entry) of row  $i > m$  will be  $-1/4$ .

and offers condition  $P(0)$ ,  $P(1)$ , and  $P(2)$  are well defined and so

$$E[W^i | s^i = 0] = 1 + \frac{1-z}{2} \left[ 1 - q^i + \frac{b^i}{P(2)} - b^i + q^i P(2) \right] \\ + \frac{1}{2} \left[ d \left( 1 - q^i + \frac{b^i}{P(1)} \right) - b^i + q^i P(1) \right] + \frac{z}{2} [-b^i + q^i P(0)]$$

$$E[W^i | s^i = 1] = 1 + \frac{z}{2} \left[ 1 - q^i + \frac{b^i}{P(2)} - b^i + q^i P(2) \right] \\ + \frac{1}{2} \left[ d \left( 1 - q^i + \frac{b^i}{P(1)} \right) - b^i + q^i P(1) \right] + \frac{1-z}{2} [-b^i + q^i P(0)]$$

$P(0)$ ,  $P(1)$ , and  $P(2)$  are not affected by a change in  $b^i$ ,  $q^i$  by a single agent. The first order conditions in bids and offers conditional on information lead to

$$\frac{(1-z)}{P(2)} + \frac{d}{P(1)} \leq 2 \quad (= \text{if } B(0) > 0) \quad (3)$$

$$\frac{z}{P(2)} + \frac{d}{P(1)} \leq 2 \quad (= \text{if } B(1) > 0) \quad (4)$$

$$(1-z)P(2) + P(1) + zP(0) \leq 1 - z + d \quad (= \text{if } Q(0) > 0) \quad (5)$$

$$zP(2) + P(1) + (1-z)P(0) \leq z + d \quad (= \text{if } Q(1) > 0) \quad (6)$$

Notice that if  $z = .5$  then by (1),

$$P(0) = P(1) = P(2) = \frac{B(0) + B(1)}{Q(0) + Q(1)}.$$

Also notice that (4) is equivalent to (3) and (6) to (5). Since there is trade, at least one  $B(0)$  and  $B(1)$  are greater than 0, and similarly at least one  $Q(0)$  and  $Q(1)$  are greater than 0. (3), (4), (5) and (6) become

$$(.5 + d) \frac{Q(0) + Q(1)}{B(0) + B(1)} = 2$$

or

$$\frac{B(0) + B(1)}{Q(0) + Q(1)} = \frac{.5 + d}{2}.$$

So from now on assume that  $z > .5$  It must be that  $B(1) > 0$  and  $B(0) = 0$ , since (4) implies that (3) holds strictly. Normalize  $B(1) = 1$ . Thus (3), (4), (5), and (6) can be rewritten as

$$zQ(1) + (1-z)Q(0) + d(Q(1) + Q(0)) = 2 \quad (7)$$

$$\frac{(1-z)z}{zQ(1) + (1-z)Q(0)} + \frac{1}{Q(1) + Q(0)} + \frac{z(1-z)}{(1-z)Q(1) + zQ(0)} \leq 1 - z + d \quad (= \text{if } Q(0) > 0) \quad (8)$$

$$\frac{z^2}{zQ(1) + (1-z)Q(0)} + \frac{1}{Q(1) + Q(0)} + \frac{(1-z)^2}{(1-z)Q(1) + zQ(0)} \leq z + d \quad (= \text{if } Q(1) > 0) \quad (9)$$

where at least one of (8) and (9) must hold with equality. Trading equilibria can then be partitioned into situations where (i)  $Q(0) = 0$ ,  $Q(1) > 0$ , (ii)  $Q(0) > 0$ ,  $Q(1) = 0$ , or (iii)  $Q(0) > 0$ ,  $Q(1) > 0$ . We consider these in turn.

(i)  $Q(0) = 0$ ,  $Q(1) > 0$ . (8) and (9) become

$$\frac{2}{Q(1)} \leq 1 - z + d \quad (10)$$

$$\frac{2}{Q(1)} = z + d \quad (11)$$

Since  $z > .5$ , then it is clear that there is no solution to (10) and (11).

(ii)  $Q(0) > 0$ ,  $Q(1) = 0$ . In this case, (7), (8), and (9) become

$$(1 - z + d)Q(0) = 2 \quad (12)$$

$$\frac{2}{Q(0)} = 1 - z + d \quad (13)$$

$$\frac{z^2}{(1-z)Q(0)} + \frac{1}{Q(0)} + \frac{(1-z)^2}{zQ(0)} \leq z + d \quad (14)$$

To have a solution to these equations, it follows from (12) and (13) that  $Q(0) = \frac{2}{d+1-z}$ .

Verifying (14) then leads to

$$\frac{z^2}{(1-z)} + 1 + \frac{(1-z)^2}{z} \leq \frac{2(z+d)}{d+1-z}$$

which reduces to  $d \leq \frac{1-z}{2z-1}$ . Substituting equilibrium bids and offers into prices (1) provides

$P(0) = \frac{(1-z)(d+1-z)}{2z}$ ,  $P(1) = \frac{d+1-z}{2}$  and  $P(2) = \frac{z(d+1-z)}{2(1-z)}$ . Direct calculation of the

expectation and variance leads to the claimed expressions.

(iii)  $Q(0) > 0$ ,  $Q(1) > 0$ . (7) implies that

$$Q(0) = \frac{2 - (z+d)Q(1)}{d+1-z} \quad (15)$$

Since (8) and (9) hold with equality, substituting from (15), (8) and (9) become

$$\begin{aligned} \frac{(1-z)z}{zQ(1) + (1-z)\frac{2-(z+d)Q(1)}{d+1-z}} + \frac{1}{Q(1) + \frac{2-(z+d)Q(1)}{d+1-z}} \\ + \frac{z(1-z)}{(1-z)Q(1) + z\frac{2-(z+d)Q(1)}{d+1-z}} = 1 - z + d \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{z^2}{zQ(1) + (1-z)\frac{2-(z+d)Q(1)}{d+1-z}} + \frac{1}{Q(1) + \frac{2-(z+d)Q(1)}{d+1-z}} \\ + \frac{(1-z)^2}{(1-z)Q(1) + z\frac{2-(z+d)Q(1)}{d+1-z}} = z + d \end{aligned} \quad (17)$$

Solving (16) for  $Q(1)$  provides two possible solutions:

$$Q(1) = \frac{(2-d-z)(1+d-z) - 4dz + (1+d-z)\sqrt{(2+d-z)^2 - 4dz}}{2d(1+d)(2z-1)}$$

and

$$Q(1) = \frac{(2-d-z)(1+d-z) - 4dz - (1+d-z)\sqrt{(2+d-z)^2 - 4dz}}{2d(1+d)(2z-1)}$$

The corresponding solutions for  $Q(0)$  according to (15) are

$$Q(0) = \frac{z(2d+2-z) - d(d+2) - (d+z)\sqrt{(2+d-z)^2 - 4dz}}{2d(1+d)(2z-1)}$$

and

$$Q(0) = \frac{z(2d+2-z) - d(d+2) + (d+z)\sqrt{(2+d-z)^2 - 4dz}}{2d(1+d)(2z-1)}$$

Since the first of these two solutions for  $Q(0)$  is always negative, it follows that

$$Q(1) = \frac{(2-d-z)(1+d-z) - 4dz - (1+d-z)\sqrt{(2+d-z)^2 - 4dz}}{2d(1+d)(2z-1)}$$

and

$$Q(0) = \frac{z(2d+2-z) - d(d+2) + (d+z)\sqrt{(2+d-z)^2 - 4dz}}{2d(1+d)(2z-1)}$$

A tedious but straightforward calculation shows that (17) is also satisfied. Notice that  $Q(1) \geq 0$  if and only if  $d \geq \frac{1-z}{2z-1}$ , while  $Q(0) \geq 0$  for any  $z \in [.5, 1]$ . To obtain the expected price and price variance expressions, one can substitute the equilibrium bids and offers into (1) to obtain equilibrium prices. An alternative (and computationally easier) method, is to

simultaneously solve (3), (5) and (6) directly. (3) shows that  $P(2) = P(1)z/(2P(1) - d)$ . Substituting and solving (5) and (6) for  $P(0)$  and  $P(1)$  one finds

$$P(0) = \frac{-2 + d - z + \sqrt{(2 + d - z)^2 - 4dz}}{4}$$

$$P(1) = \frac{2 + 3d - z - \sqrt{(2 + d - z)^2 - 4dz}}{4}$$

Then subtracting (6) from (5) provides  $P(2) = P(0) + 1$  (one could also use  $P(2) = P(1)z/(2P(1) - d)$  which is more tedious), so

$$P(2) = \frac{2 + d - z + \sqrt{(2 + d - z)^2 - 4dz}}{4}$$

Direct calculations of the expectation and variance lead to the claimed expressions.

**Proof of Proposition 3:** First, notice that  $F$  must be well-defined in any trading equilibrium which is symmetric among informed and symmetric among uninformed. Otherwise, for prices to be well defined it would have to be that the informed and uninformed are bidding the same whenever trade occurs, which is inconsistent with an equilibrium in which some agents pay to become informed.

Consider any agent who purchases information in equilibrium. It must be that such an agent bids  $1 - c$  conditional on one signal and zero conditional on the other signal. (Informed agents must expect to profit conditional on receiving at least one signal, or they would not acquire information. Since no single agent is large enough to affect the price, if they expected to profit conditional on one signal and not to lose conditional on the other signal, then an informed agent would be better off bidding all of his/her wealth without purchasing the signal.) Since the dividend has an equal chance of being  $\delta$  or  $1$  conditional on  $s = 1$  and dividend has an equal chance of being  $\delta$  or  $0$  conditional on  $s = 0$ , uninformed agents bids are the same in all states, informed agents always have a higher conditional expected return from investing conditional on  $s = 1$  than  $s = 0$ . Thus, informed agents must be bidding  $1 - c$  conditional on  $s = 1$  and bidding  $0$  conditional on  $s = 0$ . The expected utility of an uninformed agent who bids  $b^u$  is

$$U^{uninf} = 1 + b^u \left( \frac{1}{4P(2)} + \frac{\delta}{2P(1)} - 1 \right). \quad (18)$$

The ex ante expected utility of informed traders, who bid  $1 - c$  given the high signal and 0 given the low signal, is

$$U^{inf} = (1 - c) \left( \frac{1}{2} + \frac{1}{2} \left( \frac{\delta}{2P(1)} + \frac{1}{2P(2)} \right) \right), \quad (19)$$

We can represent prices as follows:

$$P(0) = (1 - F)b^u/q, \quad (20)$$

$$P(1) = (1 - c)F/2q + (1 - F)b^u/q, \quad (21)$$

and

$$P(2) = (1 - c)F/q + (1 - F)b^u/q. \quad (22)$$

We first divide potential equilibria into two categories. (I) where the (ex ante) expected utility of all agents is larger than one, and (II) where the (ex ante) expected utility of all agents is equal to one. First, we show that (I) is only possible when  $q > \frac{2(1+c)}{1+2\delta}$ . Then we will outline the types of equilibria (corresponding to those in Proposition 3) which satisfy (II).

We subdivide (I) into three categories. (a)  $F = 1$  and  $U^{inf} > 1$  and  $U^{uninf} < U^{inf}$  when  $b^u = 1$ . (b)  $F = 0$ ,  $U^{uninf} > 1$  and  $U^{inf} < U^{uninf}$ , and  $b^u = 1$ . (c)  $0 < F < 1$ ,  $U^{uninf} > 1$  and  $U^{inf} = U^{uninf}$ , and  $b^u = 1$ .<sup>16</sup>

Using (18)–(22), the requirement of  $U^{inf} > 1$  under (a) reduces to  $q > \frac{2(1+c)}{1+2\delta}$ . The requirement that  $U^{uninf} > 1$  when  $b^u = 1$  under (b) reduces to  $q > \frac{4}{1+2\delta}$  which is always as large as  $\frac{2(1+c)}{1+2\delta}$ . Under (c)  $P(1) > (1 - c)/2q$  and  $P(2) > (1 - c)/q$ . Thus  $U^{inf} < \frac{1-c}{2} + q \left( \frac{1+2\delta}{4} \right)$ . Since  $U^{inf} > 1$ , this implies that  $\frac{1-c}{2} + q \left( \frac{1+2\delta}{4} \right) > 1$  or that  $q > \frac{2(1+c)}{1+2\delta}$ .

Thus we have shown that under the requirement that  $q \leq \frac{2(1+c)}{1+2\delta}$ , only (II) is possible. Now let us examine the existence of equilibria under (II). Again we subdivide into three categories Type (i):  $U^{inf} = 1$  and  $U^{uninf} < 1$  whenever  $b^u > 0$ . Type (ii):  $U^{inf} = 1 = U^{uninf}$  and  $b^u > 0$ . Type (iii):  $U^{inf} < 1 = U^{uninf}$  and  $b^u > 0$ .

For a type (i) equilibrium,  $b^u = 0$ ,  $U^{inf} = 1$  and  $U^{uninf} < 1$  whenever  $b^u > 0$  (thus when  $b^u = 1$ ). In this case from (20)–(22), we know that  $P(0) = 0$   $P(1) = (1 - c)F/2q$

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<sup>16</sup> Notice that for  $U^{uninf} > 1$ , one needs  $b^u > 0$ , and then it must be that the unconditional expected return from bidding is greater than zero so uninformed traders should set  $b^u = 1$ .

and  $P(2) = (1 - c)F/q$ . Solving  $U^{inf} = 1$  provides  $F = \frac{2q}{1+c} \left(\frac{\delta}{2} + \frac{1}{4}\right)$ . for  $F \leq 1$  we need  $\frac{2q}{1+c} \left(\frac{\delta}{2} + \frac{1}{4}\right) \leq 1$ , which is guaranteed when, as in Proposition 3,  $q < \frac{2(1+c)}{1+2\delta}$ . Solving  $U^{uninf} < 1$  when  $b^u = 1$ , provides  $c \leq 1/(3 + 8\delta)$ .

Type (ii):  $U^{inf} = 1 = U^{uninf}$  and  $b^u > 0$ . Solving  $U^{inf} = 1 = U^{uninf}$  from (18) and (19) provides  $P(1) = (1 - c)\delta/(2 - 6c)$  and  $P(2) = (1 - c)/8c$ . Then simultaneously solving (21) and (22) provides  $F = \frac{q}{c} \left(\frac{1-3c-4c\delta}{4-12c}\right)$  and  $b^u = \frac{(1-c)(1-3c-8c\delta)}{2(q-3cq-4c\delta q-4c+12c^2)}$ . The inequalities  $0 \leq F < 1$  and  $0 < b^u$ , are satisfied when  $\frac{1}{3+8\delta} < c \leq \frac{1}{3+4\delta}$ .  $b^u \leq 1$  corresponds to  $q \leq 8c(1 - 3c)/((1 + c)(1 - 3c) - 8c^2\delta)$ , which is guaranteed when, as in Proposition 3,  $q < \frac{2(1+c)}{1+2\delta}$ .<sup>17</sup>

For type (iii) equilibria,  $U^{inf} < 1 = U^{uninf}$  and  $b^u > 0$ . Here  $F = 0$  and  $P(0) = P(1) = P(2) = b^u/q$ . Solving  $1 = U^{uninf}$  provides  $b^u = q \left(\frac{1}{4} + \frac{\delta}{2}\right)$ . Thus  $b^u \leq 1$  whenever  $q \leq 1/\left(\frac{1}{4} + \frac{\delta}{2}\right)$  which is guaranteed when, as in Proposition 3,  $q < \frac{2(1+c)}{1+2\delta}$ .  $U^{inf} < 1$  becomes  $(1 - c) \left(\frac{1}{2} + (\delta + 1)/(2\delta + 1)\right) < 1$  which simplifies to  $\frac{1}{3+4\delta} < c$ . We have thus established Proposition 3.

**Proof of Proposition 4:** The proof above provides expressions for  $P(0)$ ,  $P(1)$ , and  $P(2)$  in each of (i)–(iii). (In (ii),  $P(0)$  needs to be calculated using (20) and the expressions obtained for  $F$  and  $b^u$ . This provides  $P(0) = (1 - c)(8c\delta + 3c - 1)/(8c(1 - 3c))$ .) Straight-forward calculations show that the expected price always equals  $P(1)$ , and that for type (i) equilibria, the price variance equals  $\frac{(1-c)^2 \left(\frac{1}{4} + \frac{\delta}{2}\right)^2}{2(1+c)^2}$ , for type (ii) the price variance equals  $\frac{(4cd - (1-3c))^2}{2} \left(\frac{1-c}{8c(1-3c)}\right)^2$ , and for type (iii) equilibria, the price variance equals zero. These provide Proposition 4.

**Proof of Proposition 5:** Whenever a positive fraction of agents become informed, the argument of Proposition 3 applies. Parts (i) and (ii) follow immediately (substituting  $\delta^2$  for  $\delta$ ). To have an equilibrium where no one becomes informed, we must have  $U^{inf} \leq 1 = U^{uninf}$  and  $b^u > 0$ . However, since prices now do not reveal the state of nature, the manager will maximize the expected value of the investment leading to an optimal  $I^* = E(D) = \frac{\delta}{2} + \frac{1}{4}$ , fixed across states. The relevant “dividends” are now  $-E(D)^2$ ,  $E(D)(2\delta - E(D))$ , and  $E(D)(2 - E(D))$  in states  $S = 0$ ,  $S = 1$  and  $S = 2$ . The expressions

<sup>17</sup>  $8c(1 - 3c)/((1 + c)(1 - 3c) - 8c^2\delta) > \frac{2(1+c)}{1+2\delta}$  reduces to  $c > \frac{1}{3+8\delta}$ .

for  $U^{inf}$  and  $U^{uninf}$  are no longer the same as in the proof of Proposition 3. Noting that the (constant) price is  $b^u/q$ , we now have

$$1 = U^{uninf} = 1 - b^u + q \left[ \frac{E(D)}{2}(2\delta - E(D)) + \frac{E(D)}{4}(2 - E(D)) - \frac{E(D)^2}{4} \right].$$

Thus  $b^u = qE(D)^2$ ,<sup>18</sup> and so the constant price must be  $E(D)^2$ . Thus

$$U^{inf} = \frac{1-c}{2} \left( 1 + \frac{2\delta - E(D)}{2E(D)} + \frac{2 - E(D)}{2E(D)} \right).$$

Then  $U^{inf} \leq 1$  becomes  $\frac{1-c}{2} \left( 2 + \frac{1}{E(D)} \right) \leq 1$ . It now follows that we have an equilibrium with no informed traders if and only if  $c \geq 1/(2 + 2\delta)$  holds. Parts (iii) and (iv) follow immediately.

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<sup>18</sup> Notice that  $b^u < 1$ , since  $q < 2(1+c)/(1+2\delta^2)$  and  $c \leq 1$ .



Figure 1

$\text{Max}[.9, \text{Var}(P)/\text{Var}(d)]$

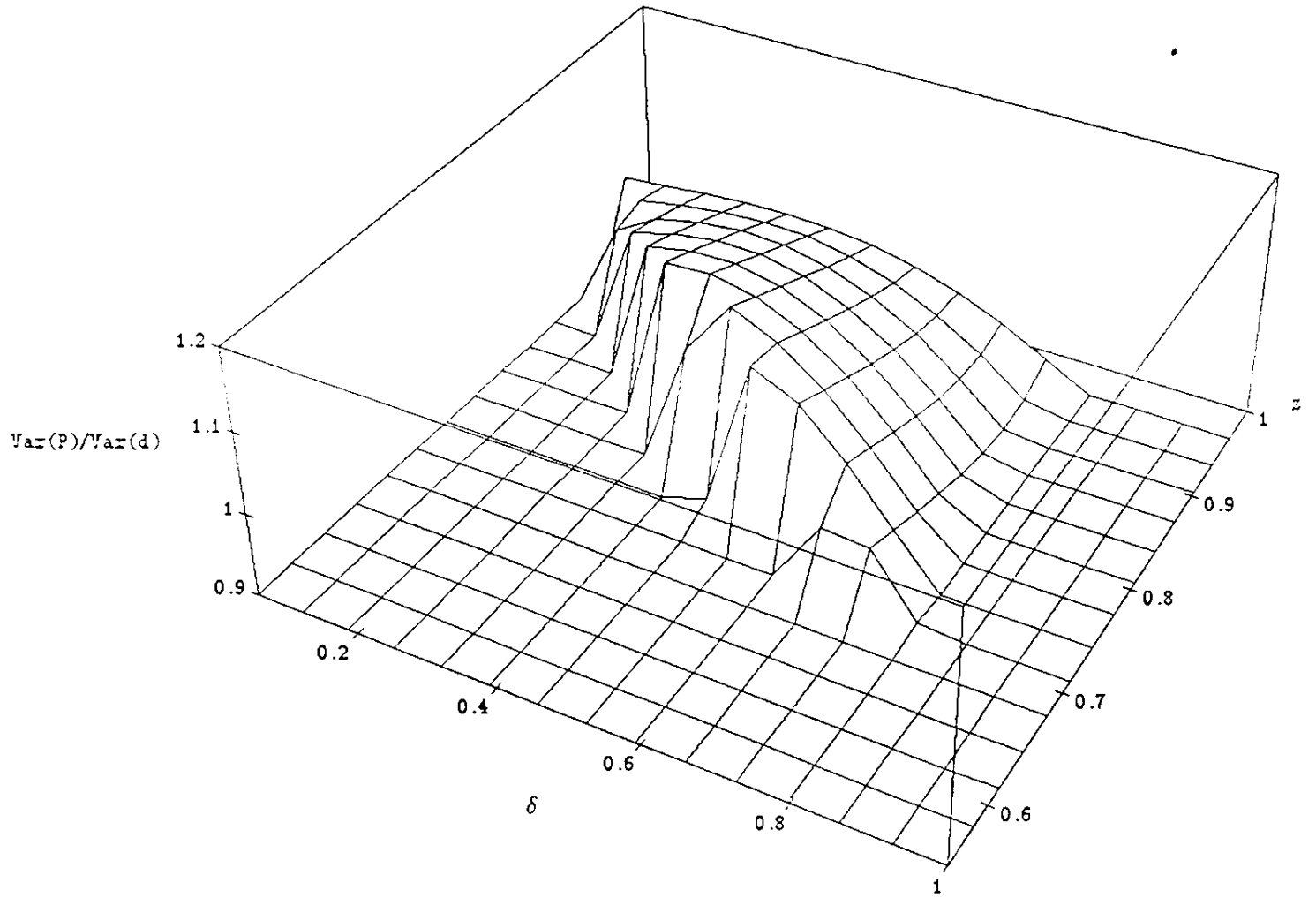


Figure 2

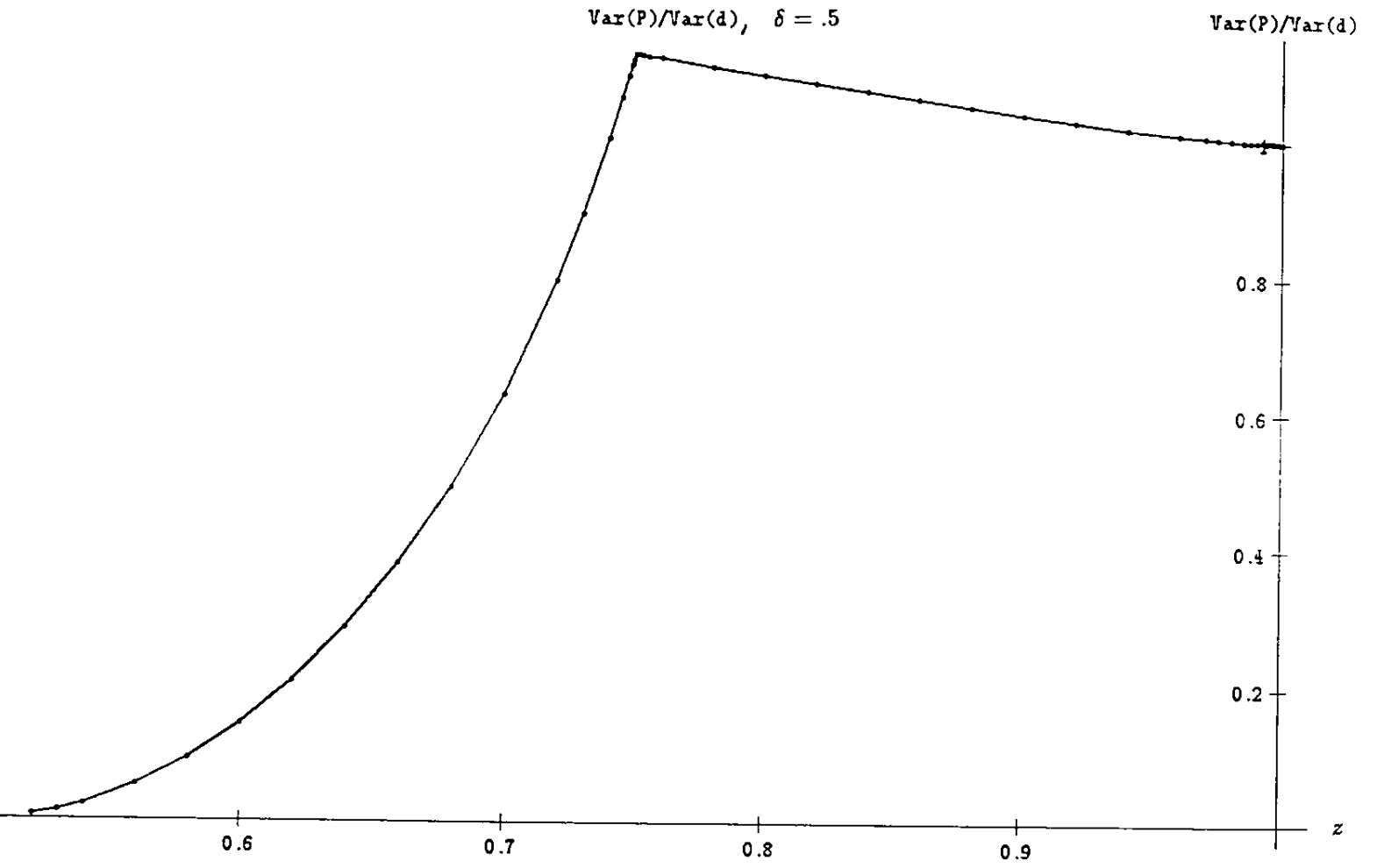


Figure 3

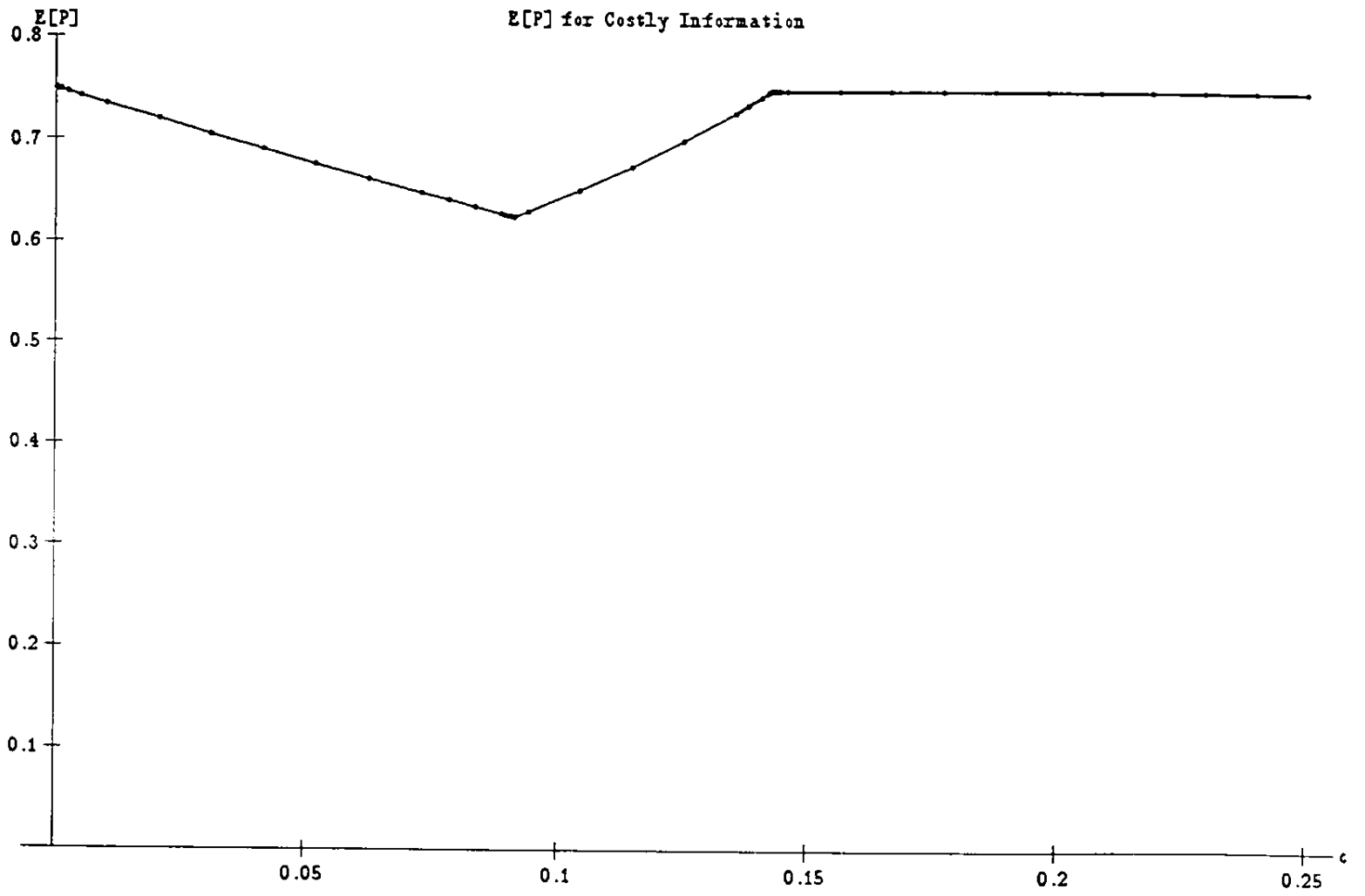


Figure 4

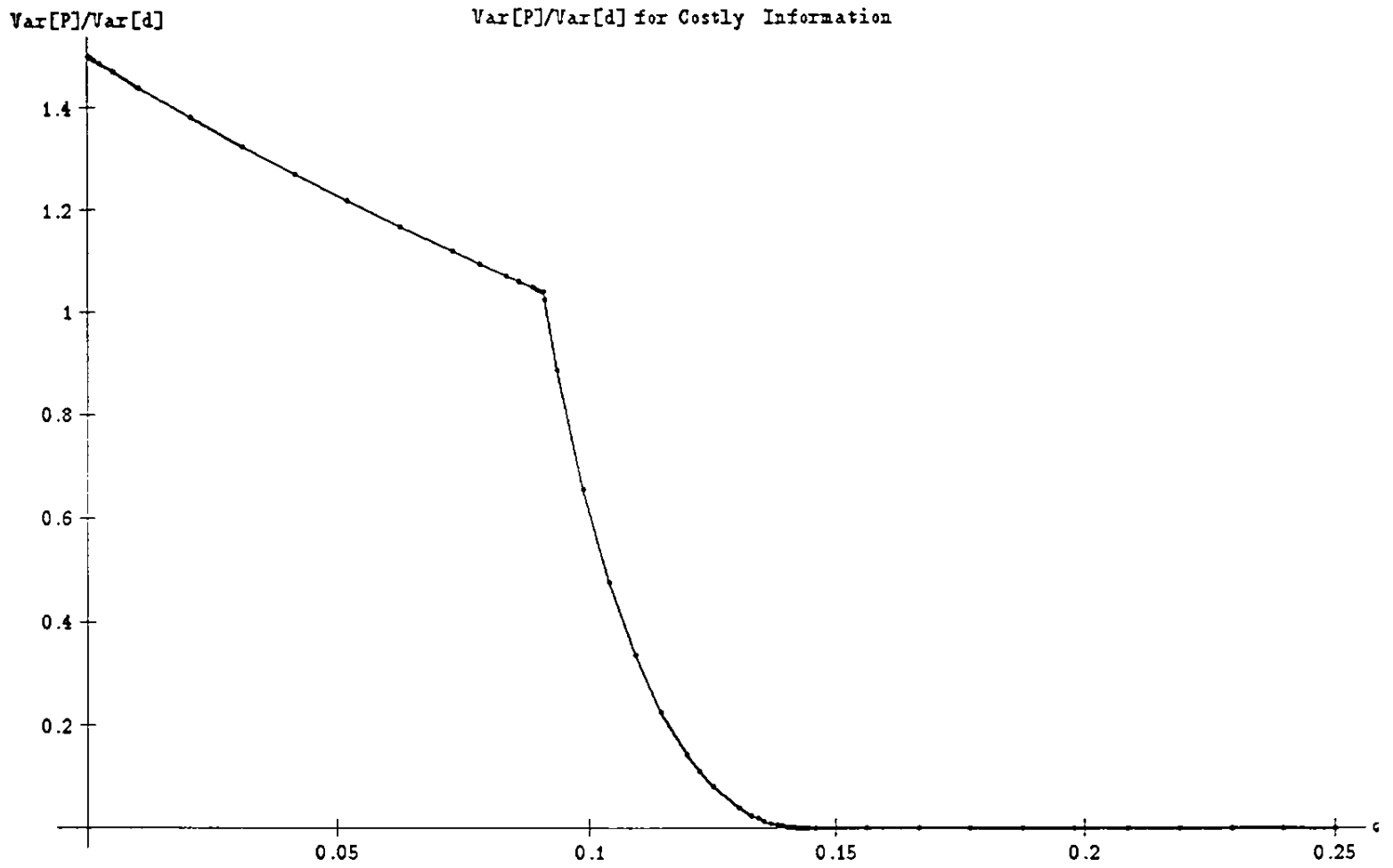


Figure 5

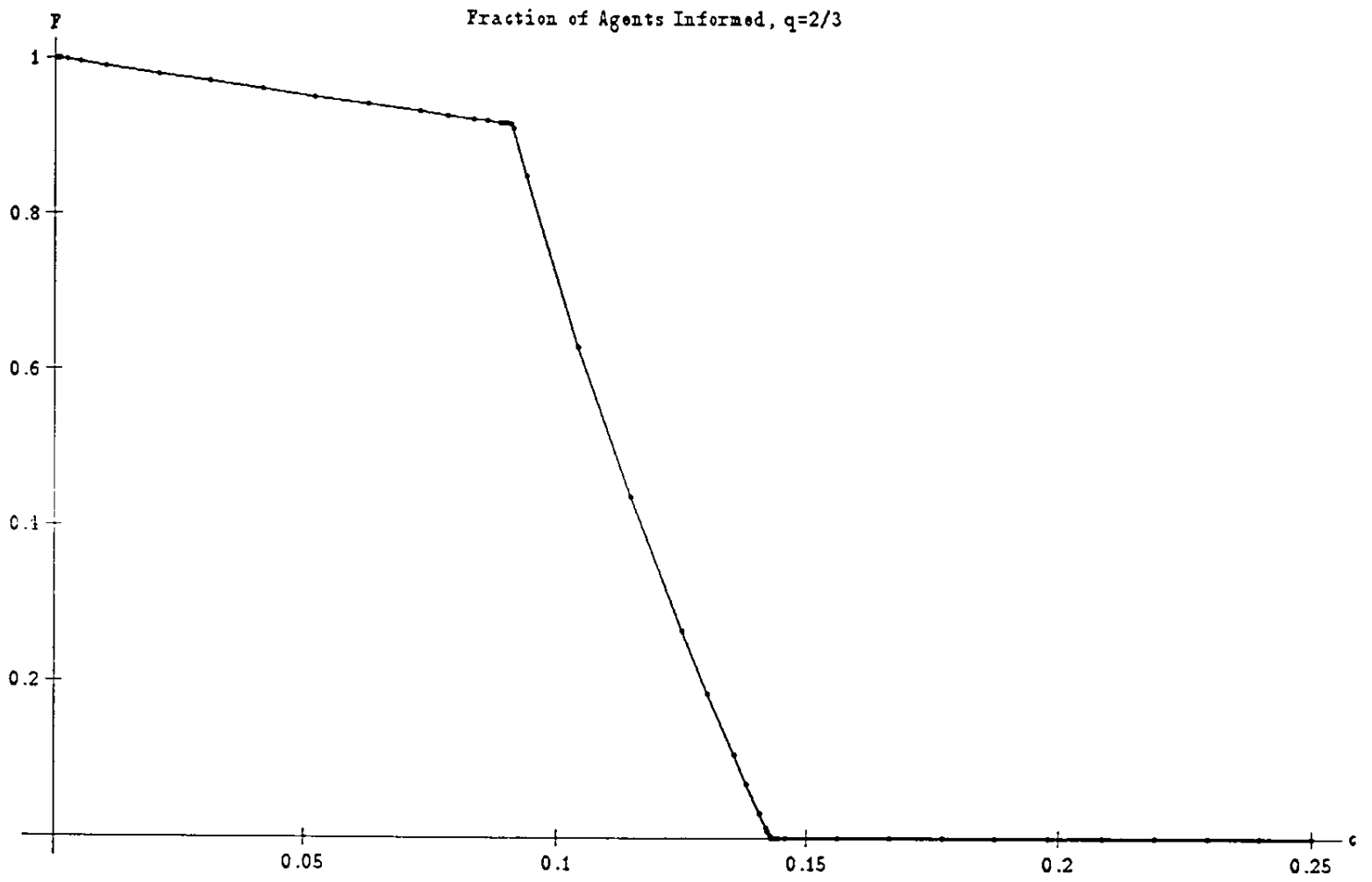


Figure 6

