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To Be The First or To Be The Best: New Product Quality and Timing in R&D Competition

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Abstract

Better quality products usually command bigger market shares and higher profits for the firm. However, developing better products also requires higher R&D expenditures and longer development times, during which a competing firm may develop and introduce its product first and capture a share of the market. In equilibrium, competing firms must balance these two effects in determining their product quality and timing decisions. We model this R&D competition between two dissimilar firms as a stochastic stopping game and investigate the nature of its equilibrium. In equilibrium, each firm sets a reservation level of the product quality it aims to develop. The technologically stronger firm is shown to set a higher quality target and command a higher market share and profit. Competition to be the first induces each firm to introduce lower quality products earlier than it would as a monopolist. However, the net social benefit is shown to be higher with competition than that with just the weaker firm as a monopolist, although the stronger firm as a monopolist is shown to yield an even more desirable outcome, which in fact turns out to be socially optimal. Finally, if the firms are identical, this socially optimal outcome is also attainable with competition, and in fact it is attained at pace faster than that with the strong monopolist or centrally controlled R&D.

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1. Introduction

New product quality and the speed with which it is developed and introduced into the market are major strategic variables for firms competing for market share. If the consumers' utility function is increasing in the quality of the product, the firms that introduce better products would enjoy higher demand at given prices (at least after a consumer information/perception lag). Moreover, the consumers' time preference for earlier consumption also implies greater damand for products introduced earlier into the market. Thus, a firm that develops and introduces *better* quality products *faster* than the competition would enjoy a significant competitive advantage. Some supporting empirical evidence and qualitative discussion may be found in Wheelwright and Clark [18].

However, the firm's R&D process of developing new products involves the*technological uncertainty* about the time and resources required to achieve a certain quality. In addition, the firm also faces the *market uncertainty* about the quality and timing of new products that other firms may develop and introduce into the market. Although introducing a better product would gain a higher market share, the product development process may also take longer, thereby increasing the risk that a competitor may develop and introduce his product first to capture a share of the market. In equilibrium, all competing firms must take these tradeoffs and uncertainties into account in determining their product quality and market timing decisions.

In Deshmukh and Chikte [4] we proposed and analyzed a *decision-theoretic* model of a single firm's problem of determining when to stop its R&D and introduce the product of quality developed so far. We also permitted the technological uncertainty in the firm's R&D process to be controlled by its dynamic resource allocation decisions over time. However, the market uncertainty about the appearance of competing products was taken to be given exogenously. The resulting stochastic evolution of the firm's product quality, as measured relative to its competition, was then modeled as a continuous-time Markov process that is controlled by the firm's dynamic resource allocation decisions. We showed that, during the progress of the R&D project, the firm should (a) stop and introduce its product into the market as soon as its relative product quality exceeds a certain target level, (b) allocate more resources to speed up the development as its product quality approaches this target, and (c) abandon the project if its relative product quality falls below a certain minimum threshold. In this paper, we formulate and analyze a *game-theoretic* model in which two competing firms choose their product development and market timing strategies. Thus the market uncertainty about introduction of competing products will emerge endogenously through the strategic interaction among the competing firms. Within this richer framework we will study the effect of competition on the firms' product quality and market timing strategies. However, in order to focus on the quality and timing decisions alone, we will abstract from the firms' dynamic resource allocation decisions by assuming constant rates of R&D expenditures. Also, for concreteness and analytical tractability, we will consider a much simpler stochastic model of the firms' product development processes.

In particular, as often found in the R&D literature (see, for example, Reinganum [16], and Telser [17]), each firm's product developement process will be modeled as sampling from a known probability distribution at a fixed cost; in addition we will also permit the time intervals between the successive draws to be random variables. The technological uncertainty in each firm's R&D process is thus modeled by random variables representing the time, cost, and magnitude of product quality improvement. The firms' cost rates and probability distributions then define their technological capabilities, which we allow to be different but known to both the firms. At any time during the R&D process, either firm may stop the development and introduce its product into the market, or it may continue to improve the product with the concomitant risk that the other firm may develop and decide to introduce its product first.

On the demand side, we will assume that the product is infinitely durable (or enjoys a total brand loyalty), so that a firm that introduces its product first permanently captures a share of the market. A better quality product will be assumed to "scoop" a larger market share and yield a higher profit for the firm. However, if one firm has already introduced its product, the quality of the other firm's product will be required to be at least as high in order for that firm to gain any share of the residual market. Furthermore, the higher the follower's relative quality advantage, the higher will be its share of the residual market. We also permit the follower to quit R&D if it seems economically unprofitable to try to "leapfrog" the quality of the leader's product that is already on the market. In the one shot game that we will consider, each firm will be allowed to introduce its product only once (although we will later indicate an extension to multiple introductions). For example, there may be a very high fixed cost of new product introduction, or the firm's reputation for quality may get permanently associated with the quality of the product it introduces first, so that any subsequent product introductions are immaterial.

The general problem addressed here was considered in a preliminary paper by Judd [8]. He considered *identical* firms who race to develop a least cost product, where (as in patent races) only the first one to introduce the product gets the *entire* market - *the followers get nothing* - and the game ends. In our model, (a) the firms are allowed to be dissimilar, and (b) the follower may also get a share of the market, provided he develops and introduces a product that is better than the one on the market. This permits us to (a) study the effect of a firm's product development capability on its product quality and timing decisions, and (b) allow the follower a natural choice of quitting the race or continuing R&D in the hope of overtaking the leader's product quality. Within our framework, we are also able to address social welfare implications of R&D competition, as in Kamien and Schwartz [9] and Loury [12].

The competitive search component of our R&D model is similar – with two important differences regarding the information and reward structures – to the one by Reinganum [16], which was also analyzed later by Mamer [13] as an example of his theory of monotone stopping games. In that model, neither firm knows when the other stops R&D, and hence the follower cannot modify his strategy after the other firm has stopped. In our model, the follower knows when and what quality product the leader has introduced into the market, and accordingly modifies his quality target, thus permitting the firms to employ closed loop strategies based on the market information. Using the terminilogy from stochastic duels, as in Mamer [13], theirs is a "silent" duel, while ours is a "noisy" one. Secondly, in their model, the firms' R&D processes are aimed at reducing the production cost parameters in the Cournot duopoly game that ensues *after both* firms have stopped their R&D. The firms' terminal payoffs in the R&D phase are then (the Cournot profit) functions of their cost parameters developed, but not of the *times* at which they were attained. In essence, during the R&D phase, the firms compete on product quality but not on the timing of its introduction, which, as noted above, is not common knowledge. In our model, on the other hand, being the first to introduce a given quality product has a distinct advantage of immediately capturing the "low hanging" share of the market, leaving fewer and more selective consumers for the follower to go after. Our particularly simple modeling of the firms' demand and profit functions enables us to analyze their closed loop strategies for determining product quality *and* timing decisions, *both* of which affect their profits.

On another track of the literature on innovation races, the competing firms choose dynamic resource allocation strategies to attain a *given target*, as in Loury [12], Lee and Wilde [11], Dasgupta and Stiglitz [2], Reinganum [14, 15], Harris and Vickers [6,7], and Fudenberg, et al [5]. In contrast to the winner-take-all races with prespecified targets featured in that literature, firms in this paper *choose* the "finish lines" they wish to attain, although they do not control the resource allocations required to attain them. A more comprehensive model would permit the firms to choose both optimal stopping *and* dynamic resource allocation strategies in a stochastic game, which would then be a full game-theoretic generalization of our earlier paper [4].

Our present model may be described in a lighter vein in terms of two fruit pickers of differing strengths who are jumping on a (stochastic) trampoline by a tall apple tree with fruits hanging at different heights. Picker 1 requires the energy equivalent of one apple per jump, while picker 2 - being weaker - requires two apples worth of energy to make one jump. Upon reaching any height, either picker may stop and collect all apples below that level, each of which he can sell at a unit price, or he may continue jumping in the hope of reaching higher fruits. Once a picker claims the low hanging fruit below a certain level, the other picker may continue jumping, trying to reach the remaining fruit above that level, or he may quit if the remaining fruit seems too high in relation to his jumping strength. We ask (a) what level of fruit each picker would aim to reach for, (b) what his net (of energy consumption) fruit collection and profit be, (c) how these compare to the case in which he is the only picker, and (d) how these depend upon the pickers' relative strengths. Finally, suppose social welfare involves taking down the most fruit from the tree (net of the pickers' energy consumption) in the shortest amount of time. We then ask whether the society should let the two pickers compete or cooperate, or employ only one of them, and in that case which one.

In section 2, we formalize the model and introduce the notation used throughout the paper. In section 3, we summarize the solution of the follower's problem, which is a variant of the usual single person search problem. This solution enters the equilibrium analysis of the two firm problem analyzed in section 4. We characterize the reservation quality level equilibrium and show that time competition to introduce the product first prompts both firms to set lower quality targets than in the absence of competition. In section 5, we examine how the firms' relative technological strengths affect their equilibrium payoffs and strategies; we show that the stronger firm will set a higher quality target and expect to get a higher market share and a higher payoff. In section 6, we consider socially optimal product quality and timing decisions, and compare these to the competitive solution. We show that with competition, while lower quality products are introduced into the market sooner, in the end the competition yields a higher net social benefit than with a technologically weaker monopolist, although a technologically stronger monopolist would yield an even more desirable outcome which is in fact socially optimal. Furthermore, if the firms are identical, the socially optimal outcome is also attainable, *and* more speedily, with competition than with monopoly or centrally controlled R&D.

2. The Model

Suppose two firms, denoted as i and j = 1, 2 (with $i \neq j$), are competing to develop and introduce similar products into a finite market. The products are assumed to be infinitely durable and the total market size will be taken to be 1. Suppose the firms compete on both product quality and the timing of its introduction into the market, although not on price. We measure product quality in terms of the number of consumers who are willing to purchase that product at the given price. In particular, if firm i introduces a product of quality $q_i \in [0,1]$ first, it will capture a market share q_i forever, and earn profit which is also equal to q_i. We may think of a continuum of consumers uniformly distributed on the unit interval [0, 1], where the consumer of type q has the reservation quality level of q, so that, at the given price, she will only purchase a product of quality q or better. Consumers of higher types q are choosier about the product quality, and are willing to wait longer for the firms to develop the better products; in effect, they are willing to pay more in terms of a higher cost of postponed consumption. Thus all consumers in $[0, q_i]$ will purchase a product of quality q_i, so that q_i is also the demand for the product of quality q_i introduced by firm i. Taking the selling price to be unity and the variable cost of production to be zero, firm i's profit (exclusive of the R&D costs) will then also be equal to q_i. However, if firm j has already introduced a product of quality q_j into the market, the follower (firm i) must introduce a product of quality q_j or better in order to capture any of the residual market $(1 - q_j)$. If it introduces a product of quality q_i, it will capture q_i portion of that market, and earn a profit equal to Max $(q_i - q_j, 0)$.

The R&D technology for developing new products involves time, cost, and uncertainty, and the two firms may differ in their technological capabilities. Suppose that firm i expends its resources at a constant rate c_i per unit time in order to conduct its product development activity. Suppose λ_i is the probabilistic (hazard) rate of discoveries by firm i, so that the time T_i between successive discoveries by firm i is an exponentially distributed random variable with mean $1/\lambda_i$. The magnitude of a discovery determines the quality of the product developed by the end of this time, which is a random variable Q_i drawn from a known absolutely continuous probability distribution function F_i on [0,1]. Then $\lambda_i[1 - F_i(q)]$ is the probabilistic rate (i.e., the parameter of the exponential distribution of the time) at which firm i develops products of quality q or better. A (stochastically) dominant firm will require lower developmental costs, and/or is able to make (stochastically) faster discoveries of better products. Formally, we will use the following

<u>Definition:</u>* *Firm 1 is said to be (technologically) stronger than firm 2 if $c_1 \leq c_2$ and/or $\lambda_1[1 - F_1(q)] \geq \lambda_2[1 - F_2(q)]$ for all $q \in [0, 1]$.

As usual, if firm 1 is stronger than firm 2, and if $g:[0, 1]\rightarrow\Re$ is a nondecreasing (profit) function, then the above stochastic dominance implies $E[g(Q_1)] \geq E[g(Q_2)]$. To avoid trivial solutions, we will assume that $E[Q_i] \geq c_i/\lambda_i$, for i = 1, 2; otherwise, it is never optimal for firm i to pursue R&D. We also assume that the firms' R&D processes are mutually independent.

If firm i succeeds first in developing a product of quality q_i, it may decide to stop the R&D process, introduce the product into the market, and earn q_i, or it may continue R&D in the hope of improving its product quality and profits. If, however, in the meanwhile, the other firm j first introduces a product of quality q_j, the follower firm i may choose to quit R&D and get nothing, or it may continue to improve its product quality to some higher level $q_i' \geq q_j$ in hope of receiving $(q_i' - q_j)$. Thus we permit the follower to modify his strategy when the other firm introduces its product first. The game ends when both firms have introduced their products, or when one introduces its product and the other quits.

For analytical convenience, we will ignore the firms' discount rate of time preference. Of course, each firm prefers to introduce its product earlier in order to save on product development costs and to capture a less discriminating share of the market before its competitor does. Similarly, consumers who demand better quality products are more patient in waiting for the firms to develop them.

Throughout the paper, for typographical convenience, we will employ the often used notation $E[g(Q); A]$ to denote the partial expectation $\int_A g(q)\, dF(q)$ of any (bounded measurable) function $g:[0, 1]\rightarrow\Re$ over a (Borel) subset A of [0, 1]. For example, with this notation, the conditional expectation $E[g(Q) \mid Q \geq q]$ of a firm's profit given a positive probability event $A = \{Q \geq q\}$ is computed as $E[Q; Q \geq q]/[1 - F(q)]$. Note that for any constant $\bar{q}$, $E(\bar{q}; A) = \bar{q}P(A)$.

3. <u>The Follower's Problem</u>

The follower's problem is a variant of the usual single person search problem (see, for example, Chow, Robbins and Siegmund [1], DeGroot [3], Ch. 13, or Kohn and Shavell [10]), with a slight modification in the follower's payoff function, which is now reduced by the share of the market that the leader has already usurped. The analysis is based on standard martingale methods in probability theory; we review it here, since the same approach will enable us to characterize the firms' equilibrium strategies later in the paper; it also has a natural economic interpretation.

Suppose firm j has already introduced its product of quality q_j and suppose firm i's best product quality developed so far over the past n draws is q_{in} at the R&D cost C_n. If firm i stops R&D, it may choose to introduce q_{in} or quit, and thus receive the payoff

$$M_n = \text{Max}\ (q_{in} - q_j, 0) - C_n$$

If firm i continues R&D for exactly one more observation, it will have to wait for an exponentially distributed length of time with parameter λ_i for the $(n+1)^{st}$ draw, during which it will incur the expected sum of c_i/λ_i, and the draw will result in sampling Q_i from the distribution $F_i(\cdot)$. At that point, again firm i may either introduce the old product of quality q_{in}, or the newly developed product quality Q_i, or quit, whichever happens to be the best alternative, and thus receive the expected net payoff of

$$M_{n+1} = \text{Max}\ (Q_i - q_j, q_{in} - q_j,\ 0) - C_{n+1}$$

Hence, the expected improvement from continuing R&D for exactly one more draw is

$$\mathbf{E[M_{n+1} - M_n \mid M_1,...M_n] = E[Max\ (Q_i - max\ (q_{in}, q_j), 0)] - c_i/\lambda_i} \qquad \mathbf{(1)}$$

which is nonincreasing in n, since q_{in}, the best draw over the past n observations, is nondecreasing in n, and Max $(\cdot)$ is a nondecreasing function. Hence if $E[M_{n+1} - M_n \mid M_1,...M_n] \leq 0$ for some n, then it is also true that $E[M_{m+1} - M_m \mid M_1, ...M_m] \leq 0$, for all $m \geq n$, i.e., $\{M_m, m \geq n\}$ is a *supermartingale*, and we are in the "monotone case" of Chow, Robbins and Siegmund [1]. Therefore by their Theorem 3.3 on page 55, this inequality will continue to hold even if m is replaced by a (Markov) *stopping time* N. (Recall that a *stopping time* of a sequence of random variables $\{M_n, n=1, 2, ...\}$ is a random variable N such that for any n the occurrence or nonoccurrence of the event $\{N \leq n\}$ is determinable at time n only on the basis of information contained in $\{M_1, M_2,...M_n\}$.) Alternatively, note that the net expected one stage improvement is uniformly bounded by $|1 - c_i/\lambda_i|$, so that by Theorem 1 on page 361 of Degroot [3], $\{M_m, m \geq n\}$ is a *regular* supermartingale, i.e., $E(M_N) \leq M_n$ for all stopping times $N \geq n$. Hence, by Theorem 1 on page 367 of DeGroot [3], the optimal stopping time is

$$N^* = \text{Inf}\,\{n: E[M_{n+1} - M_n \mid M_1,\ldots\ldots M_n] \le 0\}$$

i.e., it is optimal to stop as soon as the expected one stage improvement is less than the expected cost of sampling. Hence, we may define a *reservation level* q_i^o as the solution, if one exists, of

$$\lambda_i E\{\text{Max}\ \ [Q_i - \ \text{max}\ (q_{in}, q_j),\ 0]\} = c_i \tag{2}$$

and the optimal stopping time T_i^* as the time of first breakthrough $N^* = n$ at which $q_{in} \ge q_i^o$. (Note that q_{in}, the best quality developed so far, will exceed the reservation level q_i^o as soon as the most recent draw Q_i does so, i.e., the firm need not "recall" a previously developed product quality q_{in} that it had chosen not to introduce earlier.) Equation (2) simply balances the firm's expected marginal rate of return from quality improvement over the next draw against the search cost rate. It is easy to verify that, for a given value of q_j, equation (2) has a solution q_i^o if and only if $\lambda_i E[\text{Max}\ (Q_i - q_j, 0)] \ge c_i$, and in that case q_i^o is also the unique solution of

$$\lambda_i E[\text{Max}\ \ (Q_i - q_i^o,\ 0)] = c_i \tag{3}$$

If $q_j \ge q_i^o$, firm i should quit R&D, because overtaking the leader's quality q_j is not worth the R&D costs involved. If $q_j \le q_i^o$, the optimum reservation level is the unique solution $q_i^o \ge q_j$ of equation (2), since the LHS of equation (2) is convex decreasing in $q_{in} \in [q_j, 1]$. With the reservation level q_i^o, the follower's net expected payoff until quality q_i^o or better is developed (which takes a random amount of time that is exponentially distributed with parameter $\lambda_i[1 - F_i(q_i^o)]$) is given by $\{\lambda_i E[Q_i - q_j;\ Q_i \ge q_i^o] - c_i\} / \{\lambda_i[1 - F_i(q_i^o)]\}$, which, by virtue of equation (3), equals $q_i^o - q_j$. As is well-known, by monotonicity of $\lambda_i E[\text{Max}\ (Q_i - q_{in}, 0)]$, it is easy to show that a stronger follower will aim for a higher quality target and expect a higher profit. We summarize these basic results as

***Proposition 1**. If the leader j's product quality is $q_j \le q_i^o$, where q_i^o is the unique solution of equation (3), the follower i should set its optimum reservation quality level at q_i^o, yielding the expected profit equal to $(q_i^o - q_j)$. If $q_j \ge q_i^o$ the follower should quit and get nothing. Thus, the follower's optimal expected payoff will be Max $(q_i^o - q_j,\ 0)$. If firm 1 is stronger than firm 2, then $q_1^o \ge q_2^o$.*

The simplicity of the follower's problem and its solution follow from our particularly simple modeling of the firms' market shares and profit functions in terms of their product quality and timing. The solutions q_1^o and q_2^o in this noncompetitive phase of the game will play a critical role in determining the equilibrium payoffs and strategies in the competitive phase.

4. Equilibrium Quality Targets with Competition

Suppose firm j adopts a reservation level strategy q_j, so that it will introduce the product as soon as its quality exceeds q_j. We first show that firm i's unique best response among *all* possible stopping rules is also given by a reservation level strategy, which can be characterized as in the preceding section. We also show that with competition, the firms will set lower reservation quality levels than without competition.

Lemma 1. ***In the competitive phase, if firm j follows an R&D stopping strategy specified by a reservation quality level q_j, firm i's best response is to set a reservation level $q_i^*(q_j)$, which is the unique solution of***

$$\lambda_i E[Max\ (Q_i - q_i, 0)] = c_i + \lambda_j E[q_i - Max\ (q_i^o - Q_j, 0);\ Q_j \geq q_j] \qquad (4)$$

Moreover, $q_i^*(q_j) \leq q_i^o$. The best response function $q_i^*:[0, 1] \rightarrow [0, q_i^o]$ is continuous on $[0, 1]$ and nondecreasing on $[q_i^o, 1]$.

Proof: Let $M_n = q_{in} - C_n$ be firm i's payoff if it were to introduce its best quality developed by the n^{th} draw, given that firm j has not yet introduced its product. If firm i waits to introduce its product until the next draw, it will receive $M_{n+1} = Max(Q_i, q_{in}) - C_{n+1}$ provided firm j has still not developed its product of quality q_j or better by then, which happens with probability $\lambda_i/\{\lambda_i + \lambda_j[1 - F_j(q_j)]\}$. On the other hand, with probability $\lambda_j[1 - F_j(q_j)] / \{\lambda_i + \lambda_j[1 - F_j(q_j)]\}$, firm j will be the first one develop and introduce quality $Q_j \geq q_j$, and in that case, firm i will receive a payoff equal to $M_{n+1} = Max(q_{in} - Q_j, q_i^o - Q_j, 0) - C_{n+1}$ by either introducing its best product q_{in} developed so far, or continuing optimally, as in the previous section. Therefore, after the n^{th} draw, the expected marginal improvement from continuing R&D until the next decision point (which arrives after an exponentially distributed length of time with parameter $\{\lambda_i + \lambda_j[1 - F_j(q_j)]\}$) is

$$E[M_{n+1} - M_n \mid M_1,...M_n] = \{\lambda_i E[Max(Q_i - q_{in}, 0)] + \lambda_j E[Max(q_{in} - Q_j, q_i^o - Q_j, 0) - q_{in}; Q_j \geq q_j] - c_i\}/\{\lambda_i + \lambda_j[1 - F_j(q_j)]\}$$

Since q_{in} is nondecreasing in n, if the RHS ≤ 0 for some n, it will continue to be so for all $m \geq n$. Hence, as in the previous section, we may define the reservation level $q_i^*(q_j)$ as the minimum value of q_{in} at which this happens, which, upon simplification, yields

$$\lambda_i E[Max(Q_i - q_{in}, 0)] = c_i + \lambda_j E[q_{in} - Max(q_{in} - Q_j, q_i^o - Q_j, 0); Q_j \geq q_j] \quad (5)$$

which would reduce to equation (4) if the solution of (5) satisfied $q_i^*(q_j) \leq q_i^o$. To show this, consider both sides of equation (5) as functions in q_{in}. It is straightforward to verify that the LHS of equation (6) is convex and decreasing in q_{in}, while the RHS is linear increasing (with slope $\lambda_j[1 - F_j(q_j)]$) for $q_{in} \in [0, Max\ (q_i^o, q_j)]$, and concave increasing for higher values of q_{in} (with slope $\lambda_j[1 - F_j(q_{in})]$). Moreover, at $q_{in} = q_i^o$, we have

$$LHS = \lambda_i E[Max(Q_i - q_i^o, 0)] \leq c_i + \lambda_j E[Min(Q_j, q_i^o); Q_j \geq q_j] = RHS,$$

since the LHS = c_i by equation (3), and the second term of the RHS is nonnegative. Hence, equation (5) has a unique solution $q_i^*(q_j) \leq q_i^o$, so that equation (5) reduces to equation (4). Thus, firm i's best response function $q_i^*:[0, 1] \to [0, q_i^o]$ is well defined as the solution of equation (4). As above, the LHS of equation (4) is convex decreasing and the RHS is linearly increasing in $q_{in} \in [0, q_i^o]$ with slope $\lambda_j[1-F_j(q_j)]$, which is decreasing in q_j, thus implying monotonicity of q_i^*. Finally, with the absolute continuity of $F_j(\cdot)$, it follows that $q_i^*(q_j)$ is continuous in q_j.

QED

Thus the threat of appearance of a competing product induces firms to set lower quality targets as an insurance against losing the less discriminating share of the consumer pool to the competitor. Time competition to be the first thus prompts firms to aim at introducing inferior quality products earlier than they would in the absence of competition. The next result shows that the best response function also represents the firm's expected profit function.

Lemma 2. ***If firm j follows a reservation level strategy q_j, then firm i's optimal reservation level $q_i^*(q_j)$ also equals its expected optimal payoff.***

Proof: Suppose Π_i is firm i's expected payoff following its best response reservation level strategy $q_i^*(q_j)$ when firm j follows its reservation level strategy q_j. Starting at time 0, firm i waits for an exponentially distributed length of time with parameter $\lambda_i[1 - F_i(q_i^*(q_j))]$ until it develops a product better than $q_i^*(q_j)$, i, j = 1, 2. The time until *either* firm succeeds in attaining its target is also exponentially distributed with parameter

$$\Lambda = \{\lambda_i[1-F_i(q_i^*(q_j))] + \lambda_j[1 - F_j(q_j)]\}$$

By that time, firm i expects to have spent c_i/Λ, and with probability $\lambda_i[1-F_i(q_i^*(q_j))] / \Lambda$ it will be the first to introduce its product and receive the conditional expected profit equal to

$$E[Q_i \mid Q_i \geq q_i^*(q_j)] = E[Q_i; Q_i \geq q_i^*(q_j)] / [1 - F_i(q_i^*(q_j))],$$

while with probability $\lambda_j[1 - F_j(q_j)] / \Lambda$ firm j will succeed first, in which case firm i would expect to receive payoff of

$$E[\text{Max}\,(q_i^o - Q_j, 0) \mid Q_j \geq q_j] = E[\text{Max}\,(q_i^o - Q_j, 0); Q_j \geq q_j] / [1 - F_j(q_j)].$$

Thus firm i's expected profit at time 0, following strategy $q_i^*(q_j)$ in response to q_j, is

$$\Pi_i = \{\lambda_i[1-F_i(q_i^*(q_j))]\, E[Q_i \mid Q_i \geq q_i^*(q_j)] + \lambda_j[1 - F_j(q_j)]\, E[\text{Max}\,(q_i^o - Q_j, 0) \mid Q_j \geq q_j] - c_i\} / \Lambda$$

which upon simplification yields

$$\lambda_i\, E[Q_i - \Pi_i; Q_i \geq q_i^*(q_j)\,)] = c_i + \lambda_j\, E[\Pi_i - \text{Max}\,(q_i^o - Q_j, 0); Q_j \geq q_j].$$

However, from equation (4) of ***Lemma 1***, $\Pi_i = q_i^*(q_j)$ is the unique solution of this equation.

QED

We may now characterize the two firms' equilibrium strategies and expected payoffs.

Proposition 2. *There exists a Nash equilibrium* (q_1^*, q_2^*) *in the space of reservation level strategies, where* (q_1^*, q_2^*) *is a solution of equations:*

$$\lambda_1 E[Max\ (Q_1 - q_1^*, 0)] = c_1 + \lambda_2 E[q_1^* - Max\ (q_1^o - Q_2, 0);\ Q_2 \geq q_2^*] \quad (6)$$

$$\lambda_2 E[Max\ (Q_2 - q_2^*, 0)] = c_2 + \lambda_1\ E[q_2^* - Max\ (q_2^o - Q_1, 0);\ Q_1 \geq q_1^*] \quad (7)$$

In equilibrium, q_i^* *is also firm i's profit, and* $q_i^* \leq q_i^o$, $i = 1, 2$.

Proof: In light of ***Lemmas 1 and 2***, only the existence of a solution (q_1^*, q_2^*) to equations (6) and (7) remains to be proved. By ***Lemma 1***, the firms' reaction functions $q_i^*: [0, 1] \rightarrow [0, 1]$, $i = 1, 2$, are well-defined and continuous, so that the composite function $q^* = q_i^* \circ q_j^*: [0, 1] \rightarrow [0, 1]$ is also continuous. Hence, by the Brouwer's fixed point theorem, q^* has a fixed point $q_i^* \in [0, 1]$, and we can then define $q_j^* = q_j^*(q_i^*)$.

QED

Equations (6) and (7) have a natural interpretation. The LHS is the expected marginal rate of improvement in firm i's profit from continuing to improve quality beyond q_i^*, while the RHS is the firm i's direct R&D cost rate plus the expected rate of loss in the firm's profit due to possible preemption by firm j's discovery of a product better than q_j^*. In equilibrium, the two firms select their reservation levels so as to equate their expected marginal costs and benefits. With competition each firm sets a lower quality target, and expects to get smaller profit than it would if it were a monopolist.

5. Technological Strength As Competitive Advantage

In the absence of competition, as ***Proposition 1***, if firm 1 is stronger than firm 2 (in the sense of the ***Definition***), we have $q_1^o \geq q_2^o$, so that the stronger firm aims to develop a higher quality product and expects to get a higher profit. In this section, we show that this is true in equilibrium in the competitive phase as well. To prove this, we need the following important result, the economic significance of which will be discussed in the next section.

Lemma 3. *If firm 1 is stronger than firm 2, then* $q_2^o \leq q_1^* + q_2^* \leq q_1^o$.

Proof: In the following we will need to recall from ***Propositions 1 and Lemma 1*** that $q_2^* \le q_2^o \le q_1^o$, and $q_1^* \le q_1^o$. Now substituting for c_i from equation (3) we may rewrite equation (4) with $q_i = q_i^*$ and $q_j = q_j^*$ as

$$\lambda_i E[\mathrm{Max}(Q_i - q_i^*, 0)] - \lambda_i E[\mathrm{Max}(Q_i - q_i^o, 0)] = \lambda_j E[q_i^* - \mathrm{Max}(q_i^o - Q_j, 0); Q_j \ge q_j^*],\ i, j = 1, 2.$$

Rewriting the LHS, and using $q_i^* \le q_i^o$ to decompose the first integral on the LHS we get

$$\begin{aligned} \text{LHS} &= \lambda_i E[(Q_i - q_i^*); Q_i \ge q_i^*] - \lambda_i E\,[(Q_i - q_i^o); Q_i \ge q_i^o] \\ &= \lambda_i E[(Q_i - q_i^*); q_i^o \ge Q_i \ge q_i^*] + \lambda_i E[(Q_i - q_i^*); Q_i \ge q_i^o] - \lambda_i E[(Q_i - q_i^o); Q_i \ge q_i^o] \\ &= \lambda_i E[(Q_i - q_i^*); q_i^o \ge Q_i \ge q_i^*] + \lambda_i E[(q_i^o - q_i^*); Q_i \ge q_i^o] \\ &= \lambda_i E[\mathrm{Min}(Q_i, q_i^o) - q_i^*; Q_i \ge q_i^*], \text{ while} \end{aligned}$$

$$\text{RHS} = \lambda_j E[\mathrm{Min}(Q_j, q_i^o) + q_i^* - q_i^o; Q_j \ge q_j^*].$$

Thus equations (6) and (7) may be rewritten as

$$\lambda_1 E[\mathrm{Min}(Q_1, q_1^o) - q_1^*; Q_1 \ge q_1^*] = \lambda_2 E[\mathrm{Min}(Q_2, q_1^o) + q_1^* - q_1^o; Q_2 \ge q_2^*] \quad (8)$$

$$\lambda_2 E[\mathrm{Min}(Q_2, q_2^o) - q_2^*; Q_2 \ge q_2^*] = \lambda_1 E[\mathrm{Min}(Q_1, q_2^o) + q_2^* - q_2^o; Q_1 \ge q_1^*] \quad (9)$$

Adding equations (8) and (9) and rearranging terms yields

$$\begin{aligned} &\lambda_1 E[\mathrm{Min}(Q_1, q_1^o) - \mathrm{Min}(Q_1, q_2^o); Q_1 \ge q_1^*] - \lambda_2 E[\mathrm{Min}(Q_2, q_1^o) - \mathrm{Min}(Q_2, q_2^o); Q_2 \ge q_2^*] \\ &= \lambda_1 (q_1^* + q_2^* - q_2^o)\,[1 - F_1(q_1^*)] - \lambda_2 (q_1^o - q_1^* - q_2^*)\,[1 - F_2(q_2^*)] \end{aligned} \quad (10)$$

Now add $\lambda_2(q_1^* + q_2^* - q_2^o)\,[1 - F_2(q_2^*)]$ to both sides of equation (10), define the nonnegative constant (the equilibrium probabilistic rate at which one of the firms will introduce its product)

$$\Lambda^* = \lambda_1[1 - F_1(q_1^*)] + \lambda_2[1 - F_2(q_2^*)] \quad (11)$$

rearrange and combine terms to yield

$$\begin{aligned}(q_1^* + q_2^* - q_2^o)\Lambda^* &= \lambda_1 E[\mathrm{Min}(Q_1, q_1^o) - \mathrm{Min}(Q_1, q_2^o); Q_1 \geq q_1^*] \\ &\quad + \lambda_2(q_1^o - q_2^o)[1 - F_2(q_2^*)] - \lambda_2 E[\mathrm{Min}(Q_2, q_1^o) - \mathrm{Min}(Q_2, q_2^o); Q_2 \geq q_2^*] \\ &= \lambda_1 E[\mathrm{Min}(Q_1, q_1^o) - \mathrm{Min}(Q_1, q_2^o); Q_1 \geq q_1^*] \\ &\quad + \lambda_2 E[q_1^o - q_2^o - \mathrm{Min}(Q_2, q_1^o) + \mathrm{Min}(Q_2, q_2^o); Q_2 \geq q_2^*],\end{aligned}$$

which we need to show is nonnegative. Since $q_1^o \geq q_2^o$, the first term on the RHS is nonnegative, and, since $q_2^o \geq q_2^*$, we may decompose the second term into two terms as

$$\begin{aligned}&\lambda_2 E[q_1^o - q_2^o - \mathrm{Min}(Q_2, q_1^o) + \mathrm{Min}(Q_2, q_2^o); q_2^o \geq Q_2 \geq q_2^*] \\ &+ \lambda_2 E[q_1^o - q_2^o - \mathrm{Min}(Q_2, q_1^o) + \mathrm{Min}(Q_2, q_2^o); Q_2 \geq q_2^o] \\ &= \lambda_2 E[q_1^o - q_2^o; q_2^o \geq Q_2 \geq q_2^*] \\ &+ \lambda_2 E[q_1^o - \mathrm{Min}(Q_2, q_1^o); Q_2 \geq q_2^o],\end{aligned}$$

each of which is also nonnegative, because $q_1^o \geq q_2^o$ and $q_1^o \geq \mathrm{Min}(Q_2, q_1^o)$.

Similarly, to show $q_1^* + q_2^* \leq q_1^o$, add $\lambda_1(q_1^o - q_1^* - q_2^*)[1 - F_1(q_1^*)]$ to both sides of equation (10),rearrange, and combine terms to yield

$$\begin{aligned}(q_1^o - q_1^* - q_2^*)\Lambda^* &= \lambda_2 E[\mathrm{Min}(Q_2, q_1^o) - \mathrm{Min}(Q_2, q_2^o); Q_2 \geq q_2^*] \\ &\quad + \lambda_1 E[q_1^o - q_2^o - \mathrm{Min}(Q_1, q_1^o) + \mathrm{Min}(Q_1, q_2^o); Q_1 \geq q_1^*].\end{aligned}$$

Nonnegativity of the first term on rhe RHS again follows from $q_1^o \geq q_2^o$, but to show nonnegativity of the second term we need to consider two possible cases, depending on whether $q_1^* \geq q_2^o$, or $q_1^* \leq q_2^o$. In the former case, on the set $\{Q_1 \geq q_1^*\}$we also have $Q_1 \geq q_2^o$, so

$$\lambda_1 E[q_1^o - q_2^o - \mathrm{Min}(Q_1, q_1^o) + \mathrm{Min}(Q_1, q_2^o); Q_1 \geq q_1^*] = \lambda_1 E[q_1^o - \mathrm{Min}(Q_1, q_1^o); Q_1 \geq q_1^*],$$

which is nonnegative. If $q_2^o \geq q_1^*$, we may split the term in question and use $q_1^o \geq q_2^o$ to yield

$$\begin{aligned}&\lambda_1 E[q_1^o - q_2^o - \mathrm{Min}(Q_1, q_1^o) + \mathrm{Min}(Q_1, q_2^o); q_2^o \geq Q_1 \geq q_1^*] \\ &+ \lambda_1 E[q_1^o - q_2^o - \mathrm{Min}(Q_1, q_1^o) + \mathrm{Min}(Q_1, q_2^o); Q_1 \geq q_2^o] \\ &= \lambda_1 E[q_1^o - q_2^o; q_2^o \geq Q_1 \geq q_1^*] + \lambda_1 E[q_1^o - \mathrm{Min}(Q_1, q_1^o); Q_1 \geq q_2^o],\end{aligned}$$

each of which is nonnegative.

QED

If the two firms are technologically identical (in the sense of $\lambda_1 = \lambda_2$, $F_1 = F_2$, and $c_1 = c_2$, then in ***Proposition 1***, we have $q_1^o = q_2^o$, and by the above ***Lemma 3***, $q_1^* + q_2^* = q_1^o = q_2^o$, yielding

Corollary: ***With identical firms in a symmetric equilibrium,*** $q_i^* = q_i^o/2$, $i = 1, 2$.

In the asymmetric firm case, we can now show that, in equilibrium, the stronger firm will set a higher quality target and expect to capture a higher market share and profit.

Proposition 3. ***If firm 1 is stronger than firm 2, then*** $q_1^* \geq q_2^*$.

Proof: Suppose to the contrary that $q_2^* \geq q_1^*$, and we will get a contradiction. Using ***Lemma 3*** we obtain $2q_2^* \geq q_1^* + q_2^* \geq q_2^o$. Now consider equation (9) and use firm 1's stochastic dominance over firm 2, together with monotonicity of Min (Q_2, q_2^o) to get the inequality below:

$$\begin{aligned}\lambda_2 E[\mathrm{Min}(Q_2, q_2^o) - q_2^*\,;\, Q_2 \geq q_2^*] \quad &= \lambda_1 E[\mathrm{Min}(Q_1, q_2^o) + q_2^* - q_2^o;\, Q_1 \geq q_1^*] \\ &\geq \lambda_2 E[\mathrm{Min}(Q_2, q_2^o) + q_2^* - q_2^o;\, Q_2 \geq q_1^*] \\ &= \lambda_2 E[\mathrm{Min}(Q_2, q_2^o) + q_2^* - q_2^o;\, q_2^* \geq Q_2 \geq q_1^*] \\ &\quad + \lambda_2 E[\mathrm{Min}(Q_2, q_2^o) + q_2^* - q_2^o;\, Q_2 \geq q_2^*],\end{aligned}$$

where we have used the supposition $q_1^* \leq q_2^*$ to split the integral and obtain the last equality. Now dividing by λ_2 and combining terms yields

$$\begin{aligned}& E[q_2^o - 2q_2^*;\, Q_2 \geq q_2^*] && \geq E[\mathrm{Min}(Q_2, q_2^o) + q_2^* - q_2^o;\, q_2^* \geq Q_2 \geq q_1^*] \\ \text{i.e.,}\quad & (q_2^o - 2q_2^*)\,[1 - F_2(q_2^*)] && \geq E[Q_2 + q_2^* - q_2^o;\, q_2^* \geq Q_2 \geq q_1^*],\end{aligned}$$

since $Q_2 \leq q_2^* \leq q_2^o$ over the range of integration on the RHS. However, as shown above, the LHS of this inequality is nonpositive, whereas by ***Lemma 3*** we have $q_1^* + q_2^* \geq q_2^o$, so that the integrand on the RHS is nonnegative over the range of integration $\{q_2^* \geq Q_2 \geq q_1^*\}$, thereby yielding a contradiction.

QED

6. Social Optimality

If fast, low cost development of high quality products is socially desirable, it is natural to ask how competition fares vis-a-vis monopoly or centralized control of R&D. In terms of the *speed* of innovation alone, as measured by the expected time required to achieve a certain quality level, it is obviously better to have two firms competing in parallel to develop that quality rather than having either one firm working alone. This follows since the total probabilistic rate $\Lambda(\bar{q}) = \{\lambda_1[1 - F_1(\bar{q})] + \lambda_2[1 - F_2(\bar{q})]\}$ at which one of them succeeds in developing a product of quality $\bar{q}$ or better is greater than that for either one, $\lambda_i[1 - F_i(\bar{q})]$, $i = 1,2$. On the other hand, in terms of the *quality* of the product that the firms aim to develop, we have already seen (*Lemma 1*) that $q_i^* \leq q_i^0$, so competition induces firms to lower their individual targets.

As a measure of social benefit that involves both the product quality and the developmental time, we consider the expected *final* product quality that will be eventually introduced (or, equivalently, the expected total consumer demand that will be eventually satisfied), *net* of the expected total R&D costs during the development period. With this measure, we can show that it is better to have the two competing firms than the weaker firm pursuing R&D alone, but the stronger firm doing R&D alone is even better, and in fact the latter coincides with a socially optimal strategy in the centrally planned setting. However, if the two firms are identical, competition also yields this socially optimal outcome, and it does so faster than monopoly or centrally controlled R&D. To prove these results, we start with the following result which seems intuitively obvious.

***Lemma 4.* With the two competing firms in equilibrium (q_1^*, q_2^*), the expected net social benefit equals $q_1^* + q_2^*$.**

Proof: Starting at time 0, and following the equilibrium strategies (q_1^*, q_2^*), the time T until one of the firms introduces its product is exponentially distributed with parameter Λ^* given by equation (11), and during this time the two firms together will have spent $(c_1 + c_2)/\Lambda^*$. At time T, with probability $\lambda_j[1 - F_j(q_j^*)]/\Lambda^*$ firm j will be the first one to introduce its product of quality $Q_j \geq q_j^*$.

and in that case the other firm i will continue optimally as a follower with the net expected benefit of Max $(q_j^o - Q_i, 0)$, as in ***Proposition 1***, i, j = 1, 2. Hence, the total expected benefit is

$$\{\lambda_1[1 - F_1(q_1^*)]\, E[Q_1 + \text{Max}\,(q_2^o - Q_1, 0) \mid Q_1 \geq q_1^*] +$$
$$\lambda_2[1 - F_2(q_2^*)]\, E[Q_2 + \text{Max}\,(q_1^o - Q_2, 0) \mid Q_2 \geq q_2^*] - (c_1 + c_2)\}/\Lambda^*$$
$$= \{\lambda_1 E[Q_1 + \text{Max}\,(q_2^o - Q_1, 0);\, Q_1 \geq q_1^*] + \lambda_2\, E[Q_2 + \text{Max}\,(q_1^o - Q_2, 0);\, Q_2 \geq q_2^*] - (c_1 + c_2)\}/\Lambda^*$$
$$= \{\lambda_1 E[\text{Max}\,(Q_1, q_2^o);\, Q_1 \geq q_1^*] + \lambda_2 E[\text{Max}\,(Q_2, q_1^o);\, Q_2 \geq q_2^*] - (c_1 + c_2)\}/\Lambda^*$$

To prove that this equals $(q_1^* + q_2^*)$, we need to show that

$$\lambda_1 E[\text{Max}\,(Q_1, q_2^o) - (q_1^* + q_2^*);\, Q_1 \geq q_1^*] + \lambda_2 E[\text{Max}\,(Q_2, q_1^o) - (q_1^* + q_2^*);\, Q_2 \geq q_2^*] = (c_1 + c_2)$$

However, this equation follows immediately if we rewrite equations (6) and (7) as

$$\lambda_1 E[Q_1 - q_1^*;\, Q_1 \geq q_1^*] = c_1 + \lambda_2 q_1^*[1 - F_2(q_2^*)] - \lambda_2 E[\text{Max}(Q_2, q_1^o) - Q_2;\, Q_2 \geq q_2^*], \text{ and}$$

$$\lambda_2 E[Q_2 - q_2^*;\, Q_2 \geq q_2^*] = c_2 + \lambda_1 q_2^*[1 - F_1(q_1^*)] - \lambda_1 E[\text{Max}(Q_1, q_2^o) - Q_1;\, Q_1 \geq q_1^*].$$

add them, and rearrange terms.

QED

It also seems clear that with a centrally controlled R&D, permitting free transfer of knowledge between the firms, a socially optimal strategy would be to operate only the stronger firm:

Lemma 5. ***With a centally planned R&D, if firm 1 is stronger than firm 2, the socially optimal strategy would be to let only firm 1 carry out the product development, yielding the resulting net benefit of q_1^o.***

Proof: Starting with both firms pursuing R&D, we wish to determine the time T at which the planner should stop one of the processes, and also determine which one. We show that process 2 should be stopped at T = 0. In the second phase over [T, ∞), given that only one process will be operated, if q is the best quality developed so far, continuing process 1 is expected to yield

$\text{Max}(q, q_1^o)$, which is greater than or equal to $\text{Max}(q, q_2^o)$, since $q_1^o \geq q_2^o$. Thus if one of the processes is to be turned off, it should be process 2, and it now remains to be determined when to do so.

In the first phase during [0, T), running the two independent R&D processes simultaneously is equivalent to a single R&D process with the probabilistic discovery rate equal to $\lambda = (\lambda_1 + \lambda_2)$, the probability distribution of the size of a discovery given by the convex combination $F = (\lambda_1 F_1 + \lambda_2 F_2)/(\lambda_1 + \lambda_2)$, the total R&D cost rate of $c = c_1 + c_2$, and the terminal reward function $R(q) = \text{Max}(q, q_1^o)$. Following the approach of section 3 with these parameters, if q_n is the best quality developed by *either* process by stage n of discovery, and C_n is the total R&D cost, the expected net benefit of terminating process 2 is $M_n = \text{Max}(q, q_1^o) - C_n$, while continuing for exactly one more stage would yield $M_{n+1} = \text{Max}(Q, q, q_1^o) - C_{n+1}$. Hence, the net expected improvement (taken with respect to the distribution F of Q) from waiting for one more stage is

$$\mathbf{E[M_{n+1} - M_n \,|M_1,....M_n] = E\{Max\ [Q - max\ (q,\ q_1^o),\ 0]\} - c/\lambda} \qquad (12)$$

which we claim to nonpositive for all q, so that $\{M_n, n=1, 2,...\}$ is a supermartingale, and hence the optimal stopping time is T = 0. Since the RHS of equation (12) is nonincreasing in q, it suffices to show that it is nonpositive for q = 0. Now writing it out explicitly yields

$$\{\int_{[q_1^o,\, 1]} (q - q_1^o)[\lambda_1 dF_1(q) + \lambda_2 dF_2(q)] - (c_1 + c_2)\}/(\lambda_1 + \lambda_2)$$

$$= \{\int_{[q_1^o,\, 1]} (q - q_1^o)\, \lambda_2 dF_2(q) - c_2\}/(\lambda_1 + \lambda_2),$$

since $\int_{[q_1^o,\, 1]} (q - q_1^o)\, \lambda_1 dF_1(q) = c_1$, by equation (3) for i =1. However, since $q_1^o \geq q_2^o$ and $\int_{[q_1^o,\, 1]} (q - \bar{q})\, \lambda_2 dF_2(q)$ is decreasing in $\bar{q}$, we have $\int_{[q_1^o,\, 1]} (q - q_1^o)\, \lambda_2 dF_2(q) \leq \int_{[q_1^o,\, 1]} (q - q_2^o)\, \lambda_2 dF_2(q) = c_2$, where the equality follows again by equation (3) with i = 2.

QED

We may now cosolidate the contents of ***Lemmas 3, its corollary, Lemmas, 4, and 5*** as

Proposition 4. With the competing firms in equilibrium at (q_1^*, q_2^*), the net social benefit $q_1^* + q_2^*$ is greater than q_2^o with only the weaker firm pursuing R&D, but less than q_1^o with only the stronger firm pursuing R&D, which is also the socially optimal outcome in a centrally planned R&D. An efficient equilibrium with competing firms is a solution of the problem: Maximize $(q_1^* + q_2^*)$ subject to equations (6) and (7). If the competing firms are identical, the equilibrium outcome q_1^o is not only socially optimal, but also attained at twice the speed that either firm would as a monopolist.

7. Remarks

We have proposed and analyzed a simple stopping game model of R&D competition among dissimilar firms whose profits depend upon both the quality and the timing of products introduced into the market. We have characterized the equilibrium as a solution of a pair of algebraic equations with a natural economic interpretation. We have also analyzed how a firm's R&D capability affects its strategy and profits. Finally, we have considered the implications for social optimality.

In terms of extensions, the tractability of our model seems to depend critically on the assumption of two firms, and incorporating discounting appears to complicate the analysis beyond reason. Our model also assumes that each firm can introduce at most one product, while an extension would permit the firms to introduce a sequence of innovations, each one of an increasingly higher quality. Our preliminary results along these lines indicate that, upon each successful draw, the firms progressively capture an additional share of the residual market, and the qualitative nature of the firms'equilibrium stopping rules continues to hold, although the firms' payoffs do change. Finally, it would be important to model the firms' profit - and the society's welfare - functions as more general functions that are increasing in the product quality and decreasing in the time of its introduction. Our objective has been try and capture essential elements of product quality and timing tradeoffs in a simple competitive model, in the hope of achieving an optimal balance in this paper.

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