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DISADVANTAGEOUS SYNDICATES IN EXCHANGE ECONOMIES

by

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1. INTRODUCTION

A number of authors [e.g. 2, 7, 14] have used an atomless measure on a space of agents' characteristics as an economic model. In such a model any one agent or trader has a negligible influence on the outcome as is presumed in the idea of perfect competition. There is a natural extension of this model to the case of imperfectly competitive economies, i.e., economies containing some agents whose actions have some non-negligible effect on prices or supply of commodities. If we consider on the set of agent's characteristics those measures containing atoms, we can then interpret an atom as a group of traders, or a syndicate, which acts as a single entity. Shitovitz [13] and Gabszewicz and Mertens [10] (as well as others) have used such models to investigate the circumstances under which a syndicate, or syndicates, can exploit those traders not in the syndicate.

The core and the set of competitive allocations (defined in chapter 2) are the solution concepts commonly used to determine reasonable outcomes in economies. The competitive allocations are independent of syndicate formation (where atoms are used to represent a syndicate) in the sense that the definition is personal to individual agents regardless of whether or not syndicates exist. The core on the other hand depends on what coalitions are allowed to form. Aumann [2], Vind [14], and others have shown (under mild assumptions) that for atomless economies, the core is the same as the set of competitive allocations. If we restrict the coalitions which can form (by creating syndicates), blocking is more difficult and the core will be no smaller than if all coalitions are allowed. Shitovitz [13] and Gabszewicz and Mertens [10] were interested in determining what kind of syndicates could form such that the equivalence of the competitive allocations and the core would still be true. There is in Shitovitz an open problem: in a market with one syndicate and a continuum of traders, for any core allocation x, must there be a competitive allocation y whose utility to the syndicate is no greater than that of x? Aumann [1] has shown the answer to this question to be no. His examples will be discussed in chapter 3.

That a syndicate's formation may allow points to enter the core which are worse than all previous core points for the syndicate is somewhat non-intuitive. We shall say that a syndicate whose formation enlarges the core in this manner is disadvantageous. In this paper, we shall investigate this phenomenon and try to characterize it in several ways. We will show first that any number of types(greater than or equal to two) of agents may be present in a economy in which a syndicate may be disadvantageous (in the sense of Aumann's examples). We will then show that any type of agent can be embedded in an economy such that the traders of that type will be a disadvantageous syndicate. We next show that an agent's similarity,or dissimilarity, to other agents in an economy will affect whether or not there is a syndicate which is disadvantageous. Lastly, we show that the set of economies in which the phenomenon of the existence of disadvantageous syndicates is abundant in a certain sense.

The mathematical model and a discussion of it are presented in chapter 2.

Results of other people which will be used and one of Aumann's examples are presented in chapter 3. We give our results in chapter 4 and discuss them in chapter 5.

2. THE MATHEMATICAL MODEL

We will consider only pure exchange economies with M commodities. In the absence of production, an economic agent a is characterized by his needs, his tastes, and his ownership of resources. These characteristics will be specified by

-- a consumption set $\Omega = R_+^M$, the non-negative orthant of M-dimensional Euclidean space (the same for all agents)

-- a preference relation $\succsim_a$ on Ω

-- an endowment vector $\omega(a) \in \text{int}(\Omega)$

Throughout, we will make the following assumptions on $\succsim_a$:

i. $\succsim_a$ is a complete preorder (i.e., transitive and reflexive)

ii. continuity -- for any $x \in \Omega$

$\{y \in \Omega \mid y \succsim_a x\}$ and $\{y \in \Omega \mid x \succsim_a y\}$ are closed

iii. convexity -- $x \succsim_a y \Rightarrow \lambda x + (1 - \lambda)y \succsim_a y$ for $\lambda \in [0,1]$

iv. monotonicity (or desirability) -- $x \geq y \Rightarrow x \succ_a y$ (where we write $x > y$ to mean $x^i > y^i$ $i = 1,\ldots,M$, $x \geqq y$ to mean $x^i \geqq y^i$ $i = 1,\ldots, M$, $x \geq y$ to mean $x \geqq y$ but not $x = y$).

These are fairly standard assumptions.[1/] Some of the assumptions can be weakened. We have not done so since the weakened forms make the proofs of our results considerably more obscure.

An alternative formulation of preferences $\succsim_a$, is to represent them by the graph

$$P(a) = \{(x,y) \in \Omega \times \Omega \mid x \succsim_a y\}$$

of the preference relation. An agent a can then be thought of as a point $a = (P(a),\omega(a))$ in $\mathcal{A} = \mathcal{P} \times \Omega$, where $\mathcal{P}$ is the set of graphs which correspond to preferences satisfying i - iv.

We will sometimes find it convenient to use utility functions to express preferences. Following Kannai [11] we will define a function $U : \mathcal{A} \times \Omega \to R^1_+$. For every $a \in \mathcal{A}$ and every $x \in \Omega$ there is a unique y on the diagonal of R^M_+ such that $x \sim_a y$. That such a y exists and is unique follows from monotonicity and continuity. Let $U(a,x) = |y|$, the Euclidean norm of y. Hildenbrand [7] has shown that U is continuous.

We will want to describe similarity of agents characteristics. To get such a concept, let $d_1(\omega(a),\omega(a'))$ be the Euclidean metric on initial endowments, and let $d_2(P(a),P(a')) = \max_{x \in \Omega} \frac{|U(a,x) - U(a',x)|}{1 + |x|^2}$.

Then d_2 is a metric (see Kannai [11]) which induces the minimal topology on $\mathcal{P}$ which makes $A = \{(x,y,P):(x,y) \in P\}$ closed in $\Omega \times \Omega \times \mathcal{P}$ (see Hildenbrand [7]). This and other continuity properties (see Kannai [11] and Hildenbrand [7]) justify our choice of this topology. We obtain a metric (and induced topology) on $\mathcal{A}$ by defining $d(a,a') = d_1(\omega(a),\omega(a')) + d_2(P(a),P(a'))$.

A pure exchange economy can conveniently be represented by a measure ν on $\mathcal{A}$ with $\nu(\mathcal{A}) = 1$ (where the σ-algebra, $\mathcal{B}(\mathcal{A})$, is the collection of Borel sets). This measure can be thought of as describing the distribution of the agents composing the economy. In the case of a finite economy consisting of the agents $a_1, \ldots, a_n$, the measure can be taken to be that which puts mass $\frac{1}{n}$ at each of the points $a_1, \ldots, a_n$. As in Hildenbrand [7], we will find it necessary to replace $\mathcal{A}$ by $\mathcal{A} \times R^1$ to allow us to consider groups of agents having identical characteristics. 2/ The support of a measure μ, written $\text{supp}(\mu)$, is the smallest closed subset in $\mathcal{A}$ with measure equal to 1. We will denote the agents in $\text{supp}(\mu)$ by a^{μ}_{α}, α in some index set. For technical reasons we will restrict outselves to measures with compact support. Note that this includes all finite economies. We denote the set of these measures by $\mathcal{E}$ 3/ and shall refer to them as economies.

An economy μ where no agent can by his own actions affect the aggregate outcome of the economy can be described by a measure such that $\mu(a) = 0$ for all $a \in \text{supp}(\mu)$. Economies, or portions of economies, with this property will be called atomless. A set $B \in \mathcal{B}(\mathcal{A})$ is null if $\mu(B) = 0$; a property is said to hold almost everywhere (written a.e.), if it is true except on a null set.

We will want a topology on these measures which represent economies so as to be able to examine similar economies. The metric we use (again following Hildenbrand [7]) is the Prohorov metric ρ defined as $\rho(\mu,\nu) = \inf \{\varepsilon > 0 \mid \text{for every } E \in \mathcal{B}(\mathcal{A}), \nu(E) \leq \mu(B_{\varepsilon}(E)) + \varepsilon$ and

$\mu(E) \leq \nu(B_\epsilon(E)) + \epsilon\}$. ($B_\epsilon(E)$ denotes the ϵ-neighborhood of E with respect to our metric d on agents' characteristics, i.e. $B_\epsilon(E) = \bigcup_{x \epsilon E} \{y \mid d(x,y) < \epsilon\}$). Intuitively, we are saying two economies are close if they assign similar weights to similar coalitions. The topology induced by this metric is the weak topology (see Billingsley [4]). It is characterized by the property that $\mu_n \to \mu$ if and only if

$$\int f d\mu_n \to \int f d\mu \quad \text{for every bounded, continuous function } f.$$

For a given economy μ, the coalitions which can form are the sets in the Borel σ-algebra of $\mathcal{A}$. A syndicate will be a group of traders such that either none of the traders belong to any coalition or all of them do. When a given non-null coalition $C \epsilon \mathcal{B}(\mathcal{A})$ forms a syndicate we will then allow only the coalitions which contain either all of C or none of C to form, i.e., our new σ-algebra is $\mathcal{B}_C = \{A \epsilon \mathcal{B} \mid C \subset A$ or $C \cap A = \emptyset\}$. We will designate by μ_C the economy which has the same distribution of agents as μ and in which the coalition C has formed a syndicate. The set C is an atom in the measure μ_C, since for any set $D \epsilon \mathcal{B}_C$ s.t. $D \subset C, \mu_C(D) = \mu_C(C)$ or $\mu_C(D) = 0$. We will write $\{\tilde{a}\}$ to represent the coalition or syndicate of all agents with the same characteristics as $\tilde{a} \epsilon \mathcal{A}$. An economy μ may or may not have a syndicate; it will be explicitly stated whether or not it contains one when necessary. When possible we will use a subscript to indicate the presence of a syndicate.

An atomless economy μ and the economy μ_C in which the syndicate

C has formed are identical when μ is restricted to the σ-algebra $\mathcal{B}_C$. In this paper we will only consider syndicates of traders of the same type (though not <u>all</u> traders of a type need belong to the syndicate). Furthermore we will consider only the case where a unique syndicate arises from an atomless economy.

An allocation for the economy μ is a μ-integrable function f of $\mathrm{supp}(\mu)$ into Ω, such that $\int_{\mathcal{A}} f d\mu = \int_{\mathcal{A}} \omega d\mu$ Note that the integrability of f implies it must then be constant over an atom. We are considering only syndicates of like agents so this is no problem. (That is since all agents are identical, consumption bundles which are identical is a reasonable distribution of the syndicates share among themselves. The aggregate bundle is all that is of importance in determining dis-advantageousness.) If we allowed arbitrary syndicates to form, we would want to allow differential treatment of syndicate members.

$\int_B f d\mu$ will <u>not</u> mean the aggregate bundle to the coalition B, but loosely speaking, it is related to the aggregate bundle in the sense that $\int_B f d\mu = \int_B g d\mu$ means that if f(a) is allocated to every agent $a \in B$, then it is possible to redistribute the commodities allocated to the coalition B among its members so as to enable each $a \in B$ to obtain g(a). We will write $\int f$ for $\int f d\mu$ when no confusion will arise.

An allocation f is blocked in μ via a coalition $B \in \tilde{\mathcal{B}}$ (where $\tilde{\mathcal{B}}$ is the relevant σ-algebra for μ) if there exists an allocation g such that

i) $g(a) >_a f(a)$ a.e. in B

ii) $\mu(B) > 0$

iii) $\int_B g d\mu = \int_B \omega d\mu$

Intuitively, a non-null coalition can block an allocation f, if by a redistribution of its initial resources it can assure all its members (except possibly for a null set) a higher utility than at f. The set of unblocked feasible allocations for μ is called the core of μ, written $\text{core}(\mu)$. An allocation which is not blocked by $\text{supp}(\mu)$ is called Pareto optimal, (P.O.). If an allocation g is such that $g(a) \succsim_a \omega(a)$ a.e. $a \in \text{supp}(\mu)$, it is said to be individually rational (i.r.).

Let S be a topological space, T a compact subset of Ω. A mapping $F: S \to T$ such that for each $s \in S$, $F(s)$ is a non-empty subset of T, is a correspondence. The correspondence F is said to be upper semi-continuous at the point $x_o \in S$ if for each $\mathcal{O}$ open in T such that $F(x_o) \subset \mathcal{O}$ $\exists$ a neighborhood $\mathcal{U}(x_o) \ni x \in \mathcal{U}(x_o) \Rightarrow F(x) \subset \mathcal{O}$. F is called lower semi-continuous if for each $\mathcal{O}$ open in T such that $\mathcal{O} \cap F(x_o) \neq \emptyset$ $\exists$ a neighborhood $\mathcal{U}(x_o)$ such that $x \in \mathcal{U}(x_o) \Rightarrow F(x) \cap \mathcal{O} \neq \emptyset$. For $\mu \in \mathcal{S}$, let $\mathcal{L}_F = \{f \mid f: S \to T, f$ is μ-integrable and $f(s) \in F(s)$ a.e. $s \in S\}$ and let $\int F = \{\int f \mid f \in \mathcal{L}_F\}$. We will find useful the correspondence $G: \mathcal{A} \times \Omega \to \Omega$ defined by $G(a,x) = \{y - \omega(a) \mid y \in \Omega, y \succ_a x\}$.

The budget set $\gamma(a,p) = \{x \in \Omega \mid px \leq p\omega(a)\}$ is a correspondence from $\mathcal{A} \times S \to \Omega$ where S is the M-dimensional simplex

$\{p \in \Omega \mid \sum_{i=1}^{M} p^i = 1\}$. We will call elements of S prices. The demand correspondence ξ of $\mathcal{A} \times S^o \to \Omega$ (where S^o is the interior of S), assigns to each $a \in \mathcal{A}$ and $p \in S^o$ the elements of $\gamma(a,p)$ which are maximal with respect to P(a). That the demand correspondence is non-empty for each a follows from the compactness of γ and continuity of preferences. For agent a, we will define

$$\bigcup_{p \in S^o} \xi(a,p) \text{ to be the offer curve for agent } a.$$

A competitive equilibrium for the economy μ is a feasible allocation f and a price p such that $f(a) \in \xi(a,p)$ a.e. $a \in \text{supp}(\mu)$. The equilibrium correspondence $\overline{W}$ assigns to each $\mu \in \mathcal{S}$, the set of competitive equilibria for μ. We call an allocation x competitive if there exists a $p \in S$ such that (p,x) is a competitive equilibrium. We will denote by W the correspondence which assigns to each $\mu \in \mathcal{S}$ the set of competitive allocations for μ. A pair (p,f) is called an efficiency equilibrium for the economy μ if f(a) is maximal with respect to P(a) over $B(f(a),p) = \{x \mid p\,x \leq pf(a)\}$ a.e. $a \in \text{supp}(\mu)$. B(f (a),p) is called the efficiency budget set and f is called an efficiency allocation. If f is an efficiency allocation $\varphi(f)$ will denote the prices which support f as an efficiency equilibrium. Note that the only difference between efficiency equilibria and competitive equilibria is that $p(f(a) - \omega(a))$ need not equal 0 for f to be an efficiency equilibrium; if $p(f(a) - \omega(a)) = 0$ a.e. $a \in \text{supp}(\mu)$ at an efficiency equilibrium, then it is a competitive equilibrium.

For an economy μ_B, if there exists an allocation $f \in \text{core}(\mu_B)$ such that for every $g \in W(\mu)$ $g(a) >_a f(a)$ a.e. $a \in B$, then the syndicate B is said to be disadvantageous. The term disadvantageous is applied because had the coalition B not formed a syndicate, the core would be equal to the set of competitive allocations. Recall μ has no atoms. Having formed, there is now a core allocation strictly less preferred by all members than the worst core allocation when they do not form. The allocation f is called a disadvantageous allocation for B, or simply disadvantageous. Note that there may also be $h \in \text{core}(\mu_B)$ with $h(a) >_a g(a)$ $\forall\, a \in B$ and $\forall\, g \in W(\mu)$. Our definition of disadvantageousness is therefore a weak one and should not be taken as a normative judgement that a syndicate should not form. Those core points which are added may not arise for some reason external to our considerations, and only core allocations which are better may arise. One of Aumann's examples is especially interesting, though, for the reason that <u>all</u> the new core allocations are disadvantageous for the syndicate which forms.

One last comment is appropriate; the use of the core in investigating these problems is not beyond question (see Aumann [1]). Its use is an attempt to analyze coalition formation in a symmetric manner, i.e., the use of the core imposes the same a priori rules for the traders in syndicates and those traders who are not in syndicates. If there are fundamental asymmetries in the workings of an economy other than the syndicate structure, this approach will not prove enlightening, for it specifically rules out such asymmetries.

3. PREVIOUS RESULTS

In this chapter we will first state several results proved by others which we will use at various times.

Theorem S (Shitovitz [13]).

Let μ be an economy, x an individually rational allocation. Then x is not blocked by any coalition that contains all the atoms if and only if there exists a price p such that:

i) (p,x) is an efficiency equilibrium

ii) $p \cdot \omega(a) \geq p \cdot x(a)$ a.e. $a \in$ atomless part of $\text{supp}(\mu)$

We will use this result to characterize the core of an economy containing a single atom. The following form will be used.

Theorem S'.

Let μ_B be an economy with B the sole syndicate. Then an individually rational allocation x is in the core if and only if

i) $o \notin \int_{A'} G(a,x(a))$ for all $A' \subset \text{supp}(\mu_B) \setminus B$ such that $\mu(A') > 0$

ii) there exists a price p such that (p,x) is an efficiency equilibrium with $p \cdot \omega(a) \geq p \cdot x(a)$ a.e. $a \notin B$

Proof: Suppose i and ii are true.

By theorem S we know that x is not blocked by any coalition containing B. Hence if x is blocked, it must be blocked by a coalition $C \subset B^c$, $\mu_B(C) > 0$, and an allocation y. Then $y(a) >_a x(a)$ a.e. $a \in C \Rightarrow y(a) - \omega(a) \in G(a,x(a))$ a.e. $a \in C$. Hence we have $\int_C [y(a) - \omega(a)] = o \in \int_C G(a,x(a))$. Conversely suppose $x \in \text{core}(\mu_B)$ and

$o \in \int_C G(a,x(a))$, $\mu(C) > 0$. Then $\exists\ g \in \mathcal{L}_G$ such that $\int_C g = \int_C \omega$ and $g(a) \succ_a x(a)$ $a \in C$; hence x is blocked by C, which is a contradiction. Lastly we know by Theorem S that $x \in$ core $\Rightarrow$ ii holds. ||

Shitovitz obtains as a corollary of this theorem a result first obtained for general measure space economies by Hildenbrand. 4/

Theorem H1 An i.r. allocation x is Pareto optimal if and only if there exists a price p such that (p,x) is an efficiency equilibrium. ||

We will also use a result of Hildenbrand and Mertens [8] concerning the continuity of $W(\mu)$.

Theorem H2 Let C be a compact subset of $\mathcal{A}$. The equilibrium set correspondence μ is compact-valued and upper semi-continuous.

We now present one of Aumann's examples somewhat modified. The economy consists of three types, a_1, a_2 and a_3 with measures $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$ respectively. Sample indifference curves for a_1 are indicated for a_1 in Figure 1. (The left sides have slope of -1.) The indifference curves are over net trades, i.e., the initial endowments are subtracted from the allocation. Note that this is merely a translation of the indifference curves so that the initial endowments coincide with the origin. Whenever we speak of one net trade being preferred (less preferred, indifferent) we will mean that when the initial endowment is added to the net trades, the indicated relationships holds. All other indifference curves are parallel to these and have their vertices on the line segment bc. The offer curve for a_1 is then the line segment bc (which has slope 1)plus portions of price lines coinciding with the sides of the indifference curves. Sample indifference curves for a_2 and a_3 are shown in figure 1 in the fourth quadrant. All other indifference curves are again parallel to these. The locus of vertices for a_2 is the line going through f, d and g(where $d = -c$). For a_3 the locus is the line through f, d and h.

In figure 2 we have shown with a dashed line the negative of the offer curve for a_1. Since the measure of $\{a_1\}$ and $\{a_2\} \cup \{a_3\}$ is each $\frac{1}{2}$, a point x on the offer curve for a_1 will be part of a competitive allocation if and only if $-x$ is the average of the demands for a_2 and a_3. It is easy to see that the allocation of c to a_1 and d to a_2 and a_3 will be competitive. It is easy to verify by elementary geometry that for any $\tilde{p}$ such that $c \succ_{a_1} \xi(a_1, \tilde{p})$ (see figure 2), the average demand for a_2 and a_3 is greater than $-\xi(a_1, \tilde{p})$. Hence c will be the least preferred(by a_1) competitive allocation (in fact it is unique).

We will next show that if $\{a_1\}$ forms a syndicate there will be a point in the core which yields less utility than c for all $a \in \{a_1\}$. We let r be a point on the interior of the line segment bc and s and t points on the loci of the vertices for a_2 and a_3 respectively (as shown in figure 3) such that the average of s and t is $-r$. This then constitutes an allocation (i.e.the net trades add up to 0). Call it $\hat{x}$. Each trader is maximizing over his budget set at price $\hat{p}$, hence this is an efficiency equilibrium. In addition, $\hat{x}$ is individually rational and $\hat{p}\hat{x}(a) < \hat{p}\omega(a)$ for $a \in \{a_2\}$ and $a \in \{a_3\}$. Lastly it is clear that $0 \notin \int_A G(a, \hat{x}(a))$ for non-null $A \subset \{a_2\} \cup \{a_3\}$. Thus by theorem S', $\hat{x}$

is in the core of this economy if $\{a_1\}$ forms a syndicate. Since $c \succ_{a_1} \hat{x}(a_1)$, $\{a_1\}$ is a disadvantageous syndicate. For further details of this example and specific utility functions yielding preferences as indicated, see Aumann [1].

A differentiable version of this example can be constructed by smoothing in the neighborhood of the vertices of the indifference curves. We can do this to yield the same core point. New competitive allocations may be added but if the smoothing is close to the original indifference curves, any new competitve allocations will not be significantly far from $\hat{x}$, and the phenomenon will still hold. Aumann [1] gives a more detailed explanation of such an example.

4. RESULTS

SECTION 4.1

In all of Aumann's examples of disadvantageous syndicates, there were three types of traders. It is clear that we can construct such examples with arbitrarily large numbers of types, since we can change an agent's preferences in some region of Ω which will have no effect on the core points or the competitive equilibria. Equally clear is the fact that with one type there can be no gains from trade, hence no disadvantageous syndicates. The unanswered case is when there are two types of traders. Is is possible to have a disadvantageous syndicate in an economy with two types? The answer is yes as shown in the example below. The example has the interesting property that it treats the atomless part of the economy unequally; in a sense it treats one type as though it were in fact two types. If we rule out this unequal treatment, proposition 1 shows that a syndicate consisting of all traders of one type can not be disadvantageous in an economy with two types.

Lemma 1.1: Let μ be a finite economy with n traders, $U_1,\dots,U_n$ continuous utility functions representing their preferences. Then for any trader i, $N_1^i = \{r \in R \mid \exists\ x \in \text{P.O. with } U_i(x^i) = r\} = \{r \in R \mid \exists\ x$ a feasible allocation with $U_i(x^i) = r\} = N_2^i$

Proof: By the symmetry of the problem, we need only establish this for any agent, so let us consider trader number 2. It is obvious that $N_1^2 \subset N_2^2$ since Pareto optimal points must be allocations.

To show $N_2^2 \subset N_1^2$, let $\bar{x}$ be a feasible allocation. We will hold the utilities fixed for traders $2,3,\ldots,n$ at $U_i(\bar{x}^i)$ and maximize the utility of trader 1. $\bar{S}_i = \{x \in \Omega \mid U_i(x^i) \geq U_i(\bar{x}^i)\}$ is closed for each $i = 1,\ldots,n$ since U_i is continuous, hence $\bar{S} = \prod_{i=1}^{n} \bar{S}_i$ is closed. Let $\tilde{S} = \{(x^1,x^2,\ldots,x^n) \in S \mid \sum_{i=1}^{n} x^i = \sum_{i=1}^{n} \omega^i\}$. It is easy to see that $\tilde{S}$ is closed and bounded, hence $\tilde{S}_1$, the projection of $\tilde{S}$, is closed and bounded, thus compact. Since U_1 is continuous, a maximum is attained over $\tilde{S}_1$ at some point say $\hat{x}^1$, part of a feasible allocation $\hat{x}$.

$\hat{x}$ is Pareto optimal since if there were an allocation such that some trader j preferred his share to $\hat{x}^j$, there would be an allocation which gives j slightly less (by continuity) and trader 1 slightly more. But monotonicity implies this allocation would yield trader 1 a higher utility than $\hat{x}^1$ contrary to the assumption. Similarly we see that all traders except for 1 must receive the minimum utility, $U_i(\bar{x}^i)$. In particular, player 2 receives $U_2(\bar{x}^2)$ at a Pareto optimum. ||

<u>Proposition 1:</u> Let $\mu_{\{a_1\}}$ be an economy with two types of traders, a_1 and a_2, with $\mu(\{a_1\}), \mu(\{a_2\}) > 0$. Then any core allocation $\bar{x}(a)$ such that $\bar{x}(a)$ is identical for all $a \in \{a_2\}$ is not disadvantageous for the atom $\{a_1\}$.

<u>Proof:</u>

We will show that there exists a competitive allocation $\tilde{x}(a)$ such that $\bar{x}(a) \succsim_a \tilde{x}(a)$ a.e. $a \in \{a_1\}$.

Since the allocation $\bar{x}$ take on only two values, $\bar{x}(a_1)$ and $\bar{x}(a_2)$, we will denote them by $\bar{x}^1$ and $\bar{x}^2$ respectively. Since any competitive equilibrium treats almost all agents of the same type equally (up to indifference), convexity of preferences implies that if $y \in W$, then

$$\tilde{y}(a) = \frac{1}{\mu(\{a_i\})} \int_{\{a_i\}} y(a) \qquad a \in \{a_i\} \quad i = 1,2$$

is a competitive allocation indifferent to $y(a)$ for almost all a. Thus for any competitive allocation, there is another competitive allocation indifferent for almost all agents which takes on only two values. Hence, without loss of generality we may write y^1 and y^2 to denote the allocation y to agents a_1 and a_2 respectively. For the remainder of the proof then, we will consider only allocations taking on only two values.

By theorem S, there exists a price p such that $(p,\bar{x})$ is an efficiency equilibrium and $p \cdot \bar{x}^1 \geq p \cdot \omega^1$. If $p \cdot \int_{\{a_1\}} \bar{x} = p \cdot \int_{\{a_1\}} \omega$ then $p \cdot \int_{\{a_2\}} \bar{x} = p \cdot \int_{\{a_2\}} \omega$. But $p \cdot \int_{\{a_2\}} \bar{x} = p \cdot \int_{\{a_2\}} \omega$ and the fact that all traders in $\{a_2\}$ are treated equally at $\bar{x}$, implies that $p \cdot \bar{x}(a) = p \cdot \omega(a)$ for all a, and hence that $\bar{x}(\cdot)$ is a competitive allocation.

Suppose then, that $p \cdot \int_{\{a_1\}} \bar{x} > p \cdot \int_{\{a_1\}} \omega$. Let U_1 and U_2 be the utility functions for a_1 and a_2 respectively. Let $A_1 = \{s \in R \mid \exists\ x \in P.O,$ with $U_1(x^1) = s$, and $px^1 > p\omega^1$ for some $p \in \varphi(x)\}$. Note that A_1 is not empty since $U_1(\bar{x}^1) \in A_1$; hence $t_1 = \inf \{A_1\}$ exists. Let $A_2 = \{s \mid \exists\ x \in P.O.$ and $U_1(x^1) = s \leq t_1$ and

$p \cdot x^1 \leq p \cdot \omega^1$ for some $p \in \varphi(x)\}$. According to lemma 1.1 there exists an x Pareto optimal such that $U_1(x^1) = U_1(\omega^1)$. By theorem H1, there exists a $p \in \varphi(x)$ such that $px^1 \leq p\omega^1$ since $\omega^1 \underset{\sim a_1}{>} x^1$. Hence $U_1(x^1) \in A_2$ and $t_2 = \sup A_2$ exists.

Note that $t_2 \leq t_1$. If $t_2 < t_1$, there exists $t_3 \in (t_2, t_1)$. By the continuity of U_1, there exists a feasible allocation x such that $U_1(x^1) = t_3$. Hence lemma 2 tells us there exists $\hat{x} \in$ P.O. such that $U_1(\hat{x}^1) = t_3$. Then for some $p \in \varphi(\hat{x})$ either $p \cdot \hat{x}^1 > p\omega^1$ and $t_3 \in A_1$ or $p\hat{x}^1 \leq p\omega^1$ and $t_3 \in A_2$ (a contradiction in either case).

So suppose $t_2 = t_1$. If neither t_2 nor t_1 is a max or a min respectively, then again there is a t_3 such that $t_3 \notin A_1$ and $t_3 \notin A_2$ and we have the same argument as above.

If t_2 is a maximum (i.e., the supremum is actually achieved) there exists a sequence of Pareto optimal allocations $\{y_n\}$ such that $U_1(y_n^1) \in A_1$ and $U_1(y_n^1) \to t_1$. Since the Pareto optimal allocations are points in a compact set, a subsequence converges. Without loss of generality we may assume that $y_n \to y$.

Each of the y_n is a Pareto optimum, hence an efficiency allocation with efficiency prices p_n. If the y_n were initial allocations they would be competitive. In this way, we can consider a sequence of economies (with preferences the same as in this one and initial allocations y_n) such that the sequence converges. By theorem H2, there exists a subsequence y_{n_q} such that $p_{n_q} \to p$ and (p,y) is a competitive equilibrium for the limit economy. In other words, (p,y) is an efficiency equilibrium for the original economy, and y is Pareto optimal. $U_1(y^1) = t_1$ by continuity of U_1; $t_1 \notin A_1 \Rightarrow py^1 \leq p\omega^1$. But

$p_{n_q} y^1_{n_q} > p_{n_q} \omega^1 \ \forall \ n_q$ since $U_1(y^1_{n_q}) \varepsilon A_1$; hence $py^1 \geq p\omega^1$. It is easy to verify that for any utility level to trader 1, the set of possible values of efficiency allocations is convex. Consequently for some $p \varepsilon \varphi(y), py^1 = p\omega^1$ and y is a competitive allocation with $U_1(y^1) \leq U_1(\hat{x}^1_1)$.

The case where t_1 is a minimum is similarly handled. ||

That the hypothesis that the atomless portion is treated equally is necessary is shown by the following example.

Example: A disadvantageous syndicate of one type in an economy with two types of traders. Let μ be an atomless economy with two types of traders a_1 and a_2, $\mu(\{a_1\}) = \mu(\{a_2\}) = \frac{1}{2}$. There are two commodities. Figure 4 shows sample indifference curves for the two types. The indifference curves are again over net trades.

The allocation $\bar{x}$ characterized by net trades b to $\{a_1\}$ and d to $\{a_2\}$ is a competitive allocation, as $b = -d$ and at price p_2 both types of traders are maximizing over their budget sets. We will show that this is the unique competitive allocation and that there is an allocation $\tilde{x} \varepsilon \text{core}(\mu_{\{a_1\}})$ such that $\bar{x}(a) >_a \tilde{x}(a)$ for $a \varepsilon \{a_1\}$.

$k = -c$, so we see that the net trade k to traders of type a_1 is not part of a competitive allocation, since c is not maximum over type a_2's budget set with p_1 as the price, and at any other price if a_1 receives k, his net trade does not price out to 0. Any allocation which is less preferred than k by a_1 obviously cannot be a competitive equilibrium since his initial endowment is always in his budget set. Hence if there are any competitive equilibria less preferred than b by a_1, they must lie between the two indifference curves shown.

We will now construct families of indifference curves for a_1 and

a_2 which include those shown, such that $\{a_1\}$ will be a disadvantageous syndicate. All indifference curves for an agent will be the same shape as those shown, i.e., they will consist of two straight lines parallel to the segments given. The only matter we have not specified is the loci of the vertices of the indifference curves. Note that the construction of the indifference curves so far guarantees that the loci of the vertices between these indifference curves will constitute the offer curves for each type (along with segments of the price lines p_1 and p_2). Competitive equilibria are characterized by the condition that for x on the offer curve of a_1, $-x$ is on the offer curve of a_2.

Let $f = \frac{1}{2}(h + d)$, $g = -f$ (see figure 5). If g is on the offer curve of a_1, then the allocation $\tilde{x}$ given by net trades of:

d to $\frac{1}{2}$ of $\{a_2\}$

h to $\frac{1}{2}$ of $\{a_2\}$

g to $\{a_1\}$

will be shown to be a core point in $\mu_{\{a_1\}}$.

That it is an allocation follows from

$$\int_{\{a_1\}\cup\{a_2\}} \tilde{x} - \omega(a) = \int_{\{a_1\}} g + \int_{\frac{1}{2}\{a_2\}} h + \int_{\frac{1}{2}\{a_2\}} d = \int_{\{a_1\}} g + \int_{\{a_2\}} f = \tfrac{1}{2}(g + f) = 0.$$

At the price p_3 shown in figure 5, all traders are maximizing over their efficiency budget sets and all traders in the atomless part of the economy, $\{a_2\}$, have net trades which price out negatively at price p_3. Lastly, $\tilde{x}$ is individually rational for all traders; hence by theorem S', $\tilde{x}$ is a core allocation.

If we can construct the other indifference curves so that for no point x on the offer curve of a_1, $-x$ is on the offer curve for a_2, then $\tilde{x}$ is a disadvantageous allocation for $\{a_1\}$.

We can construct the indifference curves for a_1, so that the lines connecting g to b and k are the loci of the vertices as shown in figure 5. The dotted lines connecting f to d and c are the points which, if on the offer curve for a_2, would yield competitive equilibria. If, instead, we construct the indifference curves so that none of those points (except d) is on that offer curve, no new competitive equilibria will have been added. It is easy to see that many such offer curves can be constructed, for instance the heavy line shown connecting d and h.

In summary, we have shown that an economy $\mu_{\{a_1\}}$ with two types of traders can be constructed such that there is a core point, $\tilde{x}$, which is less desirable for a_1 than the unique competitive equilibrium.

Since proposition 1 tells us that this phenomenon cannot occur when all agents of one type are treated equally, we notice another interesting feature of this example. The allocation which treats traders of the same type (a_2) differentially, giving half of them h and half of them d, is a core point. However, if the aggregate allocation to $\{a_2\}$ is spread uniformly over $\{a_2\}$, the uniform allocation will be f which is <u>not</u> a core point. Thus if there is a rule such as "traders of the same preferences and initial holdings must be treated equally", the benefactors in the core sense would not necessarily be the small traders outside the syndicate as might be expected. Rather, as this example shows, the members of an organized syndicate might be the benefactors. An equal treatment rule would mean any core allocation would give all traders in $\{a_2\}$ the net trade d since this is the unique competitive equilibrium and proposition 1 says there will be no worse allocation for the atoms. In all this discussion "benefactor" is with respect to the core points. Whether or not this solution concept has merit in these cases will be discussed later.

SECTION 4.2

Having answered the question of how the number of types relates to the question of disadvantageousness, we turn to the relative number of economies in which there is a disadvantageous syndicate. Since the phenomenon of a disadvantageous syndicate is somewhat counterintuitive, it might be that the set of economies in which one is found is small in some sense; the phenomenon might merely be a minor pathology involving either a peculiar type of preference or arrangement of agents within an economy. Aumann's examples [1] are all constructed with innocent looking utility functions suggesting that the agents involved need not be pathological. The next result partially formalizes this by showing constructively that in a two-commodity world, any type of agent with strictly convex preferences can be embedded in an economy with three types such that if the agents of the given type form a syndicate, it will be disadvantageous. Note that the strict convexity of preferences implies that the demand correspondences are single-valued.

In section 4.4 below we will show that the set of economies in which there is a disadvantageous syndicate is not "small" in another sense; that given the set of all economies with two commodities and two types of agents, then the set of those with a disadvantageous syndicate is not a closed nowhere dense set. Thus there are economies which one cannot approximate by economies which do not contain a disadvantageous syndicates.

For simplicity in our construction we will consider agents with preferences over two commodities. It would appear that there is no structural reason that the result cannot be generalized to an arbitrary number of commodities, though we do not attempt it here.

Before beginning, we mention that in certain circumstances we could mimic the example in section 4.1 of a disadvantageous syndicate in an economy with two types of agents. In figure 6 we show an offer curve O_1 such that an agent a with such an offer curve could be made disadvantageous in a two-type economy. This can be done by constructing an offer curve O_2 which intersects $-O_1$ only at d and such that O_2 contains points a and c "outside" $-O_1$ but that the segment connecting a and c contains a point b between the origin and $-O_1$. This is the essence of the example in section 4.1.

However, if we look at the offer curve O_3 in figure 6, we see that it is not immediately possible to construct an offer curve with the properties O_2 has. This is due to the curvature of $-O_3$. Rather than attempt to work around this problem we construct an economy with two additional types of agents. We first construct what will turn out to be offer curves for the two types. The offer curves will be such that an allocation which will be disadvantageous for our given type of agent is guaranteed.

First we establish several lemmata.

For an agent a, define $\lambda(x) = \{p \in S \mid py \geq px \;\; \forall\, y \succsim_a x\}$. Note that since $\Omega \subset G(a,x)$ for any x, $p > 0$ for all $p \in \lambda(x)$.

<u>Lemma 2.1</u>: $\lambda(x)$ is convex-valued, compact-valued and upper semi-cortinuous.

<u>Proof</u>: Convexity is obvious. To show upper semicontinuity, it is enough to show that the graph of the correspondence is closed (see Debreu [6]). Let $x_n \to x$, $p_n \to p$, $p_n \in \lambda(x_n)$ for each n; we must show $p \in \lambda(x)$. Let $y \succ_a x$. Then by continuity of preferences $\exists\, N$ such that $y \succ_a x_n$ for $n \geq N$. Hence $p_n(x_n - y) \leq 0$ for all $n \geq N$. But $p_n(x_n - y) \to p(x - y)$. Hence $p(x - y) \leq 0$, and $p \in \lambda(x)$. Hence $\lambda(x)$ is u.s.c.

If $p_n \in \lambda(x)$, $p_n \to p$, it is clear that $p \in \lambda(x)$, hence $\lambda(x)$ is closed. Since S is compact the result is established. ||

<u>Corollary</u>: $\bar{\lambda}(x) = \{px \mid p \in \lambda(x)\}$ is upper semi-continuous.

<u>Proof</u>: Immediate . ||

<u>Lemma 2.2</u>: Let $f(x)$ be a convex-valued, compact-valued upper semi-continuous correspondence defined from a connected set X into $\mathbb{R}$. If $f(a) \cap (m,\infty) \neq \emptyset, f(b) \cap (-\infty,m) \neq \emptyset$ then $m \in f(c)$ for some $c \in X$.

<u>Proof</u>: Suppose not; then f upper semi-continuous $\Rightarrow$

$$\mathcal{O}_1 = \{x \in X \mid f(x) \cap (m,\infty) \neq \emptyset\} \text{ and}$$

$$\mathcal{O}_2 = \{x \in X \mid f(x) \cap (-\infty,m) \neq \emptyset\}$$

are open (see Berge [3] p. 110). If $x \in \mathcal{O}_1 \cap \mathcal{O}_2$, $\exists\, y_1, y_2 \in f(x) \ni y_1 < m$, $y_2 > m$, and by convexity $m \in f(x)$; hence $\mathcal{O}_1 \cap \mathcal{O}_2 = \emptyset$. Thus we have found disjoint non-empty open sets $\mathcal{O}_1$ and $\mathcal{O}_2$ such that $\mathcal{O}_1 \cup \mathcal{O}_2 = X$, but this contradicts the connectedness of X and there must be $c \in X$ with $m \in f(c)$. $\|$

<u>Lemma 2.3</u>: Let a be a given agent with strictly convex preferences over two commodities. There is a price $\tilde{p} = (\tilde{p}_1, \tilde{p}_2)$ such that for any $p \in S^o$ with $p_1 > \tilde{p}_1$, $\xi(a,p) - \omega(a)$ is in the interior of the second quadrant.

<u>Proof</u>: Let $\tilde{p}$ be such that $\tilde{p}_1 = \max_{p \in \lambda(\omega(a))} p_1$. Suppose p is such that $p_1 > \tilde{p}_1$. Then $p \notin \lambda(\omega(a))$. Hence $\exists\, y \succ_a \omega(a)$ with $py < p\omega(a)$. Hence $\xi(a,p) - \omega(a) \neq 0$. Let $\bar{x} = (\bar{x}_1, \bar{x}_2) = \xi(a,p) - \omega(a)$. Since $p > 0$, $p\,\bar{x} = 0$ and $\bar{x} \neq 0$, $\bar{x}$ is in the interior of the second or the fourth quadrants. If $\bar{x}_1 > 0$ and $\bar{x}_2 < 0$, $\tilde{p}_1\bar{x}_1 + \tilde{p}_2\bar{x}_2 < p_1\bar{x}_1 + p_2\bar{x}_2 = 0 \Rightarrow$ $\bar{x} \in \gamma(a,\tilde{p}) - \omega(a) \Rightarrow \omega(a) \succ_a \bar{x} + \omega(a) \Rightarrow \bar{x} + \omega(a) \notin \xi(a,p)$, a contradiction. Hence $\bar{x}_1 < 0$, $\bar{x}_2 > 0$ and $\bar{x} \in$ interior of the second quadrant. $\|$

For a given agent a with strictly convex preferences, let N be the set of points of the offer curve translated by $\omega(a)$ which are in the interior of the second quadrant.

<u>Lemma 2.4</u>: Let $\bar{x} \in N$ and y be such that

i. $y \sim_a \bar{x}$

ii. $\bar{x}_1 < y_1 < 0$

Then $p \in \lambda(y) \Rightarrow py > 0$. (See figure 7)

Proof: We first show a) $py \neq 0$, then b) $py \nless 0$

a) Let $\bar{p} \in \lambda(\bar{x})$ such that $\bar{p}\,\bar{x} = \bar{p}_1\bar{x}_1 + \bar{p}_2\bar{x}_2 = 0$. Suppose there exists y with properties i and ii with $p \in \lambda(y)$ and $py = p_1y_1 + p_2y_2 = 0$. By the strict convexity of preferences, $\bar{p}y = \bar{p}_1y_1 + \bar{p}_2y_2 > 0$ and hence $\bar{p}_1 < p_1$. Similarly, $p\bar{x} = p_1\bar{x}_1 + p_2\bar{x}_2 > 0$ by strict convexity of preferences. Then $\bar{x}_1 < 0$, $\bar{x}_2 > 0$, $\bar{p}_1 < p_1$ and $\bar{p}_2 > p_2$ yield

$$0 = \bar{p}_1\bar{x}_1 + \bar{p}_2\bar{x}_2 > p_1\bar{x}_1 + p_2\bar{x}_2 > 0$$

which is a contradiction. Hence $py \neq 0$.

b) Let $\tilde{y} = \omega(a) + re_2$ [5/] be the intersection of the indifference curve through $\bar{x}$ and the translated axis. For all $p \in \lambda(\tilde{y})$, $p > 0$ and $\tilde{y} \geq 0 \Rightarrow p\tilde{y} > 0$. By corollary to lemma 2.1 and lemma 2.2, if $py < 0$ $\exists\, y' \sim y$ with $y_1 \leq y'_1 < \tilde{y}_1$ with $0 = p'y'$ for some $p' \in \lambda(y')$ which is impossible by a). ||

Let a_1 be any agent type with strictly convex preferences in a two commodity world. We will now construct two types of agents a_2 and a_3. Let $x \in N$ and $f \sim_{a_1} x$ with $x_1 < f_1 < 0$. By lemma 2.4 $\tilde{p} \in \lambda(f) \Rightarrow \tilde{p}\, f > 0$. Consider the ray $f + rz$ where z is the vector orthogonal to $\tilde{p}$ with norm 1 and $z_1 > 0$. Since f is in the interior of the second quadrant there exists $g = f + r_2 z$, $r_2 \in R^1_+$ with g in the interior of the second quadrant. Note that there are points of N on both sides of the line $f + rz$ $r \in R$ (the supporting hyperplane), hence $\exists$ points of N on the line since N is the continuous image of a connected set. Let $r_1 = \inf\{r \in R^1 \mid f + rz \in N\}$, $r_3 < r_1$ and $b = r_3 e + f$ (see figure 8).

The economy we construct, μ, contains $\{a_1\}$, $\{a_2\}$, $\{a_3\}$ with measures $\frac{1}{2}$, $\frac{-r_3}{2(r_2-r_3)}$, $\frac{r_2}{2(r_2-r_3)}$ respectively. We will consider an allocation $\hat{x}$ characterized by net trades

$$\hat{x}(a_1) = f$$
$$\hat{x}(a_2) = c = -g$$
$$\hat{x}(a_3) = d = -b$$

To see that this is an allocation,

$$\int \hat{x} = \tfrac{1}{2}(f) + \frac{r_3}{2(r_2-r_3)}(-f-r_2 z) - \frac{r_2}{2(r_2-r_3)}(-f-r_3 z)$$

$$= \left[\tfrac{1}{2} - \frac{r_3}{2(r_2-r_3)} + \frac{r_2}{2(r_2-r_3)}\right] f - \frac{r_3 r_2 z}{2(r_2-r_3)} + \frac{r_2 r_3 z}{2(r_2-r_3)} = 0.$$

We will next construct preferences for $\{a_2\}$ and $\{a_3\}$ such that at price $\tilde{p}$, $\hat{x}$ is an efficiency equilibrium. This is done by letting the indifference curves for $\{a_2\}$ and $\{a_3\}$ be "v's" and the loci of their vertices be as given below.

Let $\hat{p}$ be the price normal to the line through d and the origin. Note that $|d| > |\xi(a_1,\hat{p})|$ by the choice of b; hence the point $g = \frac{1}{2}d + \frac{1}{2}\xi(a_1,\hat{p})$ is such that $|g| > |\xi(a_1,\hat{p})|$. We let the line segment cg and the vertical segments shown in figure 9 emanating from c and g be the locus of vertices for a_2.

One vertex for a_3 will be d. Before we describe the other vertices we note that since $\xi(a_1,p)$ is continuous, $|\xi(a_1,p)|$ is bounded, by say M, over those prices p with $\bar{\bar{p}}^1 \leq p^1 \leq \bar{p}^1$ (where $\bar{p} \in \lambda(\omega(a_1))$ and $\bar{\bar{p}}$ is the price at which $\xi(a_1,\bar{\bar{p}}) \sim_{a_1} f)$ and which correspond to points of N (i.e., prices p such that $\xi(a_1,p) - \omega(a) \in N)$.

Since $|g|$ and $|d|$ are both greater than $|\xi(a_1,\hat{p}|$, $\left|\int_{\{a_2\}\cup\{a_3\}} \xi(a,\hat{p})\right| > \left|\int_{\{a_1\}} \xi(a,\hat{p})\right|$. By continuity of the locus of vertices for a_2 and the offer curve for a_1, $\left|\int_{\{a_2\}\cup\{a_3\}} \xi(a,p)\right| >$

$\left|\int_{\{a_1\}} \xi(a,p)\right|$ for some neighborhood of prices around $\hat{p}$, since at

$\hat{p}$ the relation holds. Then, in this neighborhood, we extend the locus of vertices for a_3 outward (i.e., so that $\left|\int_{\{a_2\}\cup\{a_3\}} \xi(a,p)\right|$ increases) until the distance from the origin is greater than $\frac{M}{\mu\{a_3\}}$ (see figure 9). These segments plus the extensions shown (where the extensions continue to stay more than $\frac{M}{\mu\{a_3\}}$ from the origin) will be the locus of vertices for a_3.

If we let the indifference curves for a_2 and a_3 be v's, then the offer curves for these traders will be exactly the vertices we described until a price line coincides with a side of the v's. Thus if we construct the sides of proper slope (i.e. so that all the indifference curves intersect the first quadrant) we can insure that $\bar{x}$ is individually rational and maximal over the budget sets at price $\tilde{p}$ for a_2 and a_3. Since $\bar{x}$ is maximal over the budget set for a_1, $\bar{x}$ is a core point for $\mu_{\{a_1\}}$ by theorem S'.

The loci of vertices have been constructed so that if at any price demands by a_2 and a_3 at that price are on those loci, $\left|\int_{\{a_2\}\cup\{a_3\}} \xi(a,p)\right| > \left|\int_{\{a_1\}} \xi(a,p)\right|$ and hence this price is not a competitive equilibrium price.

Since our conditions guarantee the existence of a competitive equilibrium, let us observe its location. As the price line rotates clockwise through the fourth quadrant, it eventually coincides with the left side of an indifference curves. If by construction both a_2 and a_3 have sides of the same slope, their aggregate demand will include $-\int_{\{a_1\}} \xi(a,p)$ and this price will be an equilibrium price. If we let the sides of the indifference curves be steep enough, we can insure that the equilibrium will be preferred by a_1 to $\bar{x}(a_1)$. Then $\{a_1\}$ is disadvantageous in such an economy.

It is perhaps interesting to note that the core allocation to our agent a_1 can be arbitrarily close (in both Euclidean distance and utility) to his initial endowment, while his competitive allocation arbitrarily far away. Thus, in some sense the "gap" between these can be made very large, increasing the degree of disadvantageousness. Note that the "gap" we are speaking of is the Euclidean distance between the core point and the competitive allocations, hence independent of the utility functions.

SECTION 4.3

Even though any type of agent (with strictly convex preferences over two commodities) can be embedded in an economy so that that type will be a disadvantageous syndicate, there is another, perhaps more interesting question. Given a type of agent already embedded in an economy, under what circumstances will that type be disadvantageous in the *given* economy?

We first note that under certain circumstances if in an economy with a syndicate we have the core equal to the set of competitive allocations, then the syndicate cannot be disadvantageous; (nor can it be "advantageous" either).

Shitovitz [13] has shown that if there is more than one syndicate and all syndicates are of the same type, the core and the set of competitive allocations coincide. If we let σ_i be the number of syndicates of the same kind i (two syndicates are of the same kind if they are of the same type and have the same measure), then the core and the set of competitive allocations coincide if the greatest common divisor of $(\sigma_1, \sigma_2, \ldots)$ is greater than 1. Gabszewicz and Mertens [10] have shown that if a syndicate contains a small enough proportion of the agents of a given type, then the equivalence theorem will again result. Each of these results then has an immediate corollary: under the conditions stated in the respective result, a syndicate cannot be disadvantageous.

All the above results have to do with the relation of the traders of one type which form a syndicate and those of that same type which do not belong to the syndicate. There is another set of circumstances in which *all* traders of a given type form a syndicate and the equivalence theorem still obtains. This is the situation where the traders forming the syndicate are not "extreme" in relation to the other traders of the economy. i.e., when for any price there are agents with demand similar to that of the agents in the syndicate. We will establish the result for two-commodity economies.

Lemma 3.1: Let a be an agent with preferences over two commodities. If

i. $\tilde{x}$ is individually rational

ii. $\tilde{x}$ is maximal over $B(\tilde{x},\tilde{p})$ (with respect to $P(a)$)

iii. $\tilde{p}[\tilde{x} - \omega(a)] < 0$

then $f = \tilde{x} - \omega(a)$ and $d \in \xi(a,\tilde{p}) - \omega(a)$ are in the same quadrant.

Proof: For $x \succ_a d + \omega(a)$, $\tilde{p}\, x > \tilde{p}(d + \omega(a))$. By continuity and monotonicity of preferences, $x \succsim_a d + \omega(a) \Rightarrow \tilde{p}x \geq \tilde{p}(d + \omega(a))$. Hence, $\tilde{p}f < 0 \Rightarrow d + \omega(a) \succ_a \tilde{x}$. Note that d must be in either the second or the fourth quadrants. We will show that the line segment $\alpha d + (1 - \alpha)f, \alpha \in (0,1)$ is contained in one quadrant.

Let $y = \alpha d + (1 - \alpha)f, \alpha \in (0,1)$; by convexity $y + \omega(a) \succ_a f + \omega(a)$. If $y \neq 0$ is in the third quadrant, we have $\omega(a) \succ_a y + \omega(a) \succ_a f + \omega(a)$ which violates individual rationality. $y = 0$ also yields $\omega(a) \succ_a f + \omega(a)$. By assumption $pd = 0$, and $pf < 0$. Hence $py < 0$. Hence y cannot be in the first quadrant. But if no point y between f and d is in the first or third quadrants or equal to zero, f and d must both be in the second or fourth quadrant. ||

Proposition 2: Let μ be an atomless economy with n types and two commodities. If for type $\bar{a}$ and every price p $\exists$ another type $\tilde{a}$ with

$\mu\{\tilde{a}\} > 0$ such that $\xi(\bar{a},p) - \omega(\bar{a}) = r\,[\xi(\tilde{a},p) - \omega(\tilde{a})]$ for some $r \in (0,\infty)$, then core $(\mu_{\{\bar{a}\}})$ = core $(\mu) = W(\mu)$.

Proof: It is clear that core $(\mu) \subset$ core $(\mu_{\{\bar{a}\}})$. We must show core $(\mu_{\{\bar{a}\}}) \subset$ core (μ). Let $\hat{x} \in$ core $(\mu_{\{\bar{a}\}})$. By Theorem S', $\exists\ \hat{p} \in \lambda(\hat{x})$ such that $\hat{p}\hat{x}(a) \leq \hat{p}\omega(a)$ a.e. $a \notin \{\bar{a}\}$. If $\hat{p}\hat{x}(\bar{a}) = \hat{p}\,\omega(\bar{a})$ then $\hat{p}\hat{x}(a) = \hat{p}\omega(a)$ a.e. $a \notin \{\bar{a}\}$ and $\hat{x} \in W(\mu) \subset$ core (μ) and the proposition is true. Suppose then that $\hat{p}(\hat{x}(\bar{a})) - \omega(\bar{a}) \neq 0$. Then $\hat{x}(\bar{a}) - \omega(\bar{a})$ is in the second or fourth quadrants. WLOG we will assume it is in the second.

If $\hat{p}\hat{x}(a) = \hat{p}\omega(a)$ a.e. $a \notin \{\bar{a}\}$, again $\hat{x} \in W(\mu) \subset$ core (μ). Suppose then that there is a non-null set of agents with $\hat{p}\hat{x}(a) < \hat{p}\omega(a)$. There must then be a non-null set of agents A_1 of the same type (say a_1), with $\hat{p}(\hat{x}(a) - \omega(a)) < 0$, and $\hat{x}(a) - \omega(a)$ is in the same quadrant as $\hat{x}(\bar{a}) - \omega(\bar{a})\ \forall\ a \in A_1$. By Lemma 3.1, $\xi(a,\hat{p}) - \omega(a)$ is in the same quadrant as $\hat{x}(a) - \omega(a)\ \forall\ a \in A_1$. By hypothesis all agents of type $\{\bar{a}\}$ have excess demand in the second quadrant. Since $\hat{x}$ is an allocation with $\hat{x}(a) - \omega(a)$ in the second quadrant for a non-null set of traders, there must be a non-null set of traders with $\hat{x}(a) - \omega(a)$ in the fourth quadrant, and hence a non-null set of the same type with this property. Thus we can find a set of traders A_2 of the same type $\{a_2\}$, $\mu(A_2) > 0$ such that $\forall\ a \in A_2$, $\xi(a,\hat{p}) - \omega(a)$ is in the quadrant opposite that in which $\xi(a,\hat{p}) - \omega(a)$ lies $\forall\ a \in A_1$.

Since $px > p\omega(a)\ \forall\ x \succ_a \xi(a,p)$, continuity and monotonicity of preferences give us $px \geq p\omega(a)\ \forall\ x \succsim_a \xi(a,p)$. Hence $\hat{p}\hat{x}(a) < \hat{p}\omega(a)\ \forall\ a \in A_1 \Rightarrow \xi(a,\hat{p}) \succ_a \hat{x}(a)\ \forall\ a \in A_1$. The continuity of preferences assures us that $\xi(a,\hat{p}) - \varepsilon(a)\, e \succ \hat{x}(a)$ is true for some $\varepsilon(a) > 0\ \ \forall\ a \in A_1$. If $B_n = \{a \in A_1 | \xi(a,\hat{p}) - \frac{1}{n} \succ_a \hat{x}(a)\}$; then $A_1 = \bigcup_{n=1}^{\infty} B_n$. Since $\mu(A_1) > 0$

then $\mu(B_n) > 0$ for some n. The non-atomicity of A_1 and A_2 allow us to find subsets $\bar{A}_1 \subset A_1$ and $\bar{A}_2 \subset A_2$ such that

i. $\exists\ \epsilon > 0$ with $\xi(a,\hat{p}) - \epsilon e >_a \hat{x}(a)\ \forall\ a \in \bar{A}_1$

ii. $-\mu(\bar{A}_1)\,[\xi(a_1,\hat{p}) - \omega(a_1)] = \mu(\bar{A}_2)[\xi(a_2,\hat{p}) - \omega(a_2)]\ a_1 \in A_1, a_2 \in A_2$ 6/

iii. $\mu(\bar{A}_1) > 0, \mu(\bar{A}_2) > 0$

Then consider

$$y(a) = \begin{cases} \xi(a,\hat{p}) - \epsilon e & \text{if } a \in \bar{A}_1 \\ \xi(a,\hat{p}) + r\,\epsilon\, e & \text{if } a \in \bar{A}_2 \end{cases}$$

where $r = \dfrac{\mu(\bar{A}_1)}{\mu(\bar{A}_2)}$. $y(a) >_a \hat{x}(a)\ \forall\ a \in \bar{A}_1$ by i above. Monotonicity gives us $y(a) >_a \hat{x}(a)\ \forall\ a \in \bar{A}_2$.

$$\int_{\bar{A}_1 \cup \bar{A}_2} y(a) - \omega(a) = \int_{\bar{A}_1} \xi(a,\hat{p}) - \epsilon e - \omega(a) + \int_{\bar{A}_2} \xi(a,\hat{p}) + r\epsilon e - \omega(a) =$$

$$\mu(A_1)[\xi(a_1,\hat{p}) - \omega(a_1)] + \mu(A_1)\epsilon e + \mu(A_2)[\xi(a_2,\hat{p}) - \omega(a_2)] +$$

$$\mu(A_2)\ r\ \epsilon\ e = 0.$$

Hence, the non-null coalition $\bar{A}_1 \cup \bar{A}_2$ blocks the allocation $\hat{x}$, a contradiction. Thus $\{a \notin \{\bar{a}\} \mid \hat{p}\hat{x}(a) < 0\}$ must be null, and $\hat{x} \in W(\mu) = \text{core}(\mu)$ ||

Q. E. D.

<u>Corollary</u>: Let μ and $\bar{a}$ be as in the proposition. Then $\{\bar{a}\}$ is not disadvantageous.

<u>Proof:</u> Immediate. ||

We conjecture that this result generalizes to any finite number of commodities. The needed hypothesis on the diversity of agents would be that at any price there be a non-null set of traders with demand in each orthant (except positive and negative).

SECTION 4.4

In section 4.2 we showed that any type agent could be embedded in an economy so that if that type forms a syndicate, it will be disadvantageous. Such economies still might be a very small set in relation to all economies. For instance, it might be conjectured that the set of economies having such a coalition is a closed nowhere dense set. We will show that such is not the case, at least not in a world of finite number of types. Rather, these economies are quite abundant in a precise sense.

Let the number of commodities, M, be fixed. Let $T_n = \{\mu \in \mathcal{E} \mid \text{supp}(\mu)$ contains n types of agents, each with equal measure$\}$. We will show that there is an open set of economies in T_n, $n \geq 2$ each containing a disadvantageous syndicate.

To prove the proposition we will take the disadvantageous economy with two types in the example of section 4.1 above and modify it as follows. We smooth the corners of the indifference curves for type 1.

In this way, we get a unique price supporting any allocation to type 1 (interior to his consumption set).

We will show the following.

Proposition 3: There is an open set of disadvantageous economies in T_2 containing this modified economy, τ.

Let $\tilde{x}$ denote the disadvantageous allocation and $\bar{x}$ the unique competitive equilibrium in this economy. To prove the proposition, we introduce several lemmas.

Lemma 4.1 will characterize an open ball of economies in T_n. Lemma 4.2 will show that for a given agent in a given economy μ in T_n, similar agents in similar economies are not substantially worse off (in utility) at the worst competitive allocations. Lemmas 4.3 and 4.4 show that our given type is not treated substantially worse at core points than similar agents in similar economies (given the characteristics of our economy τ). Together these say that in economies similar to τ, there is a type such that the competitive allocations do not get much worse nor the worst core allocations get much better than for our given type in τ. This will make the type disadvantageous.

Following the proof, we will indicate how it generalizes.

Lemma 4.1: Let $\mu \in T_n$. Then there exists $\epsilon > 0$ such that $\nu \in B_\epsilon(\mu) \cap T_n$ iff ν has n types and $d(a_i^\mu, a_i^\nu) < \epsilon \quad i = 1,\ldots,n$ for some suitable numbering of the respective agents.

Proof: Let $\epsilon < \frac{1}{n}$ and $\epsilon < \frac{1}{2} \min_{i,j} d(a_i^\mu, a_j^\mu)$.

Suppose $d(a_i^\mu, a_i^\nu) < \epsilon \quad i = 1,\ldots,n$, for some numbering. Then if E is any subset of $\operatorname{supp}(\mu)$,

$$\mu(E) = \frac{\text{number of } a_i^\mu \text{ in } \operatorname{supp}(\mu)}{n} \text{ and } B_\epsilon(E) = \bigcup_{a_i^\mu \in \operatorname{supp}(\mu)} B_\epsilon(a_i^\mu) \quad .$$

$B_\epsilon(a_i^\mu)$ contains exactly one $a_i^\nu \in \operatorname{supp}(\nu)$ by choice of ϵ. Hence

$\nu(B_\epsilon(E)) = \mu(E)$; similarly $\mu(B_\epsilon(E)) = \nu(E)$. Hence $\rho(\mu,\nu) \leq \epsilon$, i.e., $\nu \in B_\epsilon(\mu)$.

To show the implication in the other direction, let $\nu \in B_\epsilon(\mu)$. $\mu(a_i^\mu) \leq \nu(B_\epsilon(a_i^\mu)) + \epsilon \Rightarrow \exists\ a_i^\nu \in B_\epsilon(a_i^\mu)$ $i = 1,\ldots,n$ since $\epsilon < \frac{1}{n}$, i.e., $d(a_i^\mu, a_i^\nu) < \epsilon$ $i = 1,\ldots,n$. ||

Thus, if we restrict ourselves to a set of economies within ϵ of a given economy μ for a small enough ϵ, we are justified in speaking of the ith agent in each economy as being that agent within ϵ of the ith agent in μ.

If a sequence of economies ν_n converges to an economy ν and $f_n \in W(\nu_n)$, theorem H-2 tells us that $\int_E f_n d\nu_n \to \int_E f d\nu$ for any $E \in \mathcal{B}(\mathcal{A})$. The above lemma tells us that when there are a fixed finite number of types in the economies, we can find $E \in \mathcal{B}(\mathcal{A})$ and $N \geq 0$ such that for $n \geq N$, E contains only the ith agent of ν_n. Then $\int_E f_n d\nu_n = f_n(a_i^{\nu_n})\nu_n(a_i^{\nu_n}) \to f(a_i^\nu)\nu(a_i^\nu)$. If the measure of the type i is constant over n, we have that the allocations converge strongly. We use this fact later in the proof.

Lemma 4.2: Let $\mu \in T_n$. For $\delta > 0$ there exists an $\epsilon > 0$ such that $\nu \in B_\epsilon(\mu) \cap T_n \Rightarrow \inf_{x \in W(\nu)} U_i^\nu(x^i) \geq \inf_{x \in W(\mu)} U_i^\mu(x^i) - \delta$ for a given agent i.

Proof: For $\delta > 0$, let $\bar{\bar{x}} \in W(\mu)$ be such that $U_i^\mu(\bar{\bar{x}}^i) - \inf_{x \in W(\mu)} U_i^\mu(x^i) < \delta/2$ for a fixed i. $U(a,x)$ continuous implies that there exists $\epsilon_1 > 0$ such that $d_1(y,\bar{\bar{x}}^i) < \epsilon_1, d(a,a_i^\mu) < \epsilon_1 \Rightarrow |U(a,y) - U(a_i^\mu,\bar{\bar{x}}^i)| < \delta/2$. Theorem H2 tells us that $W(\mu)$ is upper semi-continuous, hence

$\exists\ \epsilon_2 > 0$ such that if $\nu \in B_{\epsilon_2}(\mu)$ and $\bar{x} \in W(\nu)$, then $\exists\ \hat{x} \in W(\mu)$ with $d_1(\bar{x}^i, \hat{x}^i) < \epsilon_1$. Then $\epsilon = \min(\epsilon_1, \epsilon_2)$ implies that if $\nu \in B_\epsilon(\mu)$ then $d(a_i^\nu, a_i^\mu) < \epsilon \leq \epsilon_1$ and $d_1(\bar{x}^i, \hat{x}^i) < \epsilon_1$ and hence $U_i^\nu(x^i) > U_i^\mu(\hat{x}^i) - \delta/2 \geq U_i^\mu(y^i) - \delta$ for all $x \in W(\nu)$, and $y \in W(\mu)$. ||

Lemma 4.3: Let $\delta > 0$. $\exists\ \epsilon > 0$ such that if $\mu \in B_\epsilon(\tau) \cap T_2$ then $\exists$ y an efficiency allocation in μ such that $|U_1^\tau(\tilde{x}(a_1^\tau)) - U_1^\mu(y(a_1^\mu))| < \delta$.

Proof: $\tilde{x} \in$ core $(\tau) \Rightarrow \exists$ p with $(p, \tilde{x})$ an efficiency equilibrium. Then $(p, \tilde{x})$ is a competitive equilibrium for the economy η with three types of agents $(\tilde{x}^1, P_1), (\sim 2, P_2)$ and $(\sim 3, P_2)$ with measures $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ respectively; i.e. η is the economy the same as τ except the initial endowment in η is now a core point in τ. Since any other $y \in W(\eta)$ must be individually rational, we must have $y(a) \gtrsim_a \tilde{x}(a)$ a.e. η; but $\tilde{x}(a) \gtrsim_a y(a)$ a.e. η by the Pareto optimality of $\tilde{x}$, hence $\tilde{x}(a) \sim_a y(a)$ a.e. η. By the upper semi-continuity of $W(\eta)$, $\exists\ \epsilon$ such that if $\eta' \in B_\epsilon(\eta)$ and η' contains three types, then $\exists\ \hat{x} \in W(\eta')$ such that $|U_i^{\eta'}(\hat{x}(a_i^{\eta'}) - U_i^\eta(\tilde{x}(a_i^\eta))| < \delta$ $i = 1,2,3$. This is due to the fact that the utility of all competitive allocations to a particular agent in η is the same. Now for any $\hat{\mu} \in B_\epsilon(\tau) \cap T_2$, there is associated with it an economy $\hat{\eta} \in B_\epsilon(\eta)$ where $\hat{\eta}$ contains agents with preferences identical to those of $\hat{\mu}$ and initial endowments $\tilde{x}^1$, d, and h. Hence $\hat{\mu} \in B_\epsilon(\tau) \cap T_2 \Rightarrow \exists$ an efficiency allocation y for $\hat{\mu}$ such that $|U_i^{\hat{\mu}}(y(a_i^{\hat{\mu}}) - U_i^\tau(\tilde{x}(a\ ))| < \delta$ $i = 1,2,3$. ||

Lemma 4.4: Let $\delta > 0$. $\exists\ \varepsilon > 0$ such that $\mu \in B_\varepsilon(\tau) \cap T_2 \Rightarrow \exists\ y\varepsilon \text{core}(\mu_{\{a_1\}})$ such that $|U_1^\tau(\tilde{x}(a_1^\tau)) - U_1^\mu(y(a_1^\mu))| < \delta$.

Proof: By lemma 4.3 $\exists\ \varepsilon > 0$ such that $\mu \in B_\varepsilon(\tau) \cap T_2 \Rightarrow$ there exists an efficiency allocation y for μ close in utility to $\tilde{x}$ for the first agent. To apply theorem S' we need to show we can find such an efficiency allocation which also is individually rational and satisfies $py(a) \leq p\omega(a)$ a.e. $a \in \{a_2^\mu\}$.

Since $U_2^\tau(\omega(a_2^\tau) + d) > U_2^\tau(\omega(a_2^\tau))$ and $U_2^\tau(\omega(a_2^\tau) + h) > U_2^\tau(\omega(a_2^\tau))$ we can find $\varepsilon_1 > 0$ such that $\mu \in B_{\varepsilon_1}(\tau) \cap T_2 \Rightarrow U_2^\mu(y(a)) > U_2^\mu(\omega(a_2^\mu))$ a.e. $a \in \{a_2^\mu\}$ by continuity of utility and ω, hence individual rationality will hold.

Consider again the economy η of lemma 4.3 with preferences the same as in τ but initial endowment $\tilde{x}$. Since the preferences are smooth, the price $\tilde{p}$ supporting $\tilde{x}$ as a competitive equilibrium is unique. Since $\tilde{x}(a)$ is the initial endowment for η, we have for any other competitive equilibrium $\tilde{y}, \tilde{p}\tilde{x}(a) = \tilde{p}\tilde{y}(a)$ a.e. η (since $\tilde{x}(a) \sim_a \tilde{y}(a)$ a.e. η). By the upper semi-continuity of $W(\cdot)$, we can find $\varepsilon_2 > 0$ such that if $\mu \in B_{\varepsilon_2}(\eta)$ then $(p,y) \in \overline{W}(\mu) \Rightarrow y$ and p are within an arbitrarily small preassigned distance of $\tilde{p}$ and some competitive allocation for η. In particular we can find ε_3 such that if $\mu \in B_{\varepsilon_3}(\eta)$ then $|py(a) - \tilde{p}\tilde{x}(a)|$ is arbitrarily small. Since $\tilde{p}\,\tilde{x}(a) < \tilde{p}\omega(a)$ all $a \in \{a_2^\tau\}$, we can find ε so that $py(a) < p\omega(a)$ for any $(p,y) \in \overline{W}(\mu)$ and a.e. $a \in \{a_2^\mu\} \cup \{a_3^\mu\}$.

Thus if we pick the smallest of the three ε_j's, $j = 1,2,3$, and consider $\nu \in B_\varepsilon(\tau) \cap T_2$ we are guaranteed an efficiency equilibrium y which is individually rational and such that $py(a) < p\,\omega(a)$ a.e. $a \in \{a_2^\nu\}$. We have shown, then, that

y is not blocked by a coalition containing $\{a_1^\mu\}$. We must only verify that $0 \notin \int_{\{a_2\}} G(a,y(a))$. All $a \in \{a_2^\nu\}$ have the same preferences, hence $\exists\ \bar{a} \in \{a_2^\nu\}$ such that $G(a,y(a)) \subset G(\bar{a},y(\bar{a}))\ \forall\ a \in \{a_2^\nu\}$. $0 \notin G(\bar{a},y(\bar{a}))$ by individual rationality. By convexity of preferences we can find a hyperplane h such that $hx > 0\ \forall\ x \in G(\bar{a},y(\bar{a}))$, hence $\forall\ x \in \bigcup_{a\in\{a_2^\nu\}} G(a,y(a))$. Thus $h\int_{\{a_2\}} g > 0\quad \forall\ g \in \mathcal{L}_G$ and $0 \notin \int_{\{a_2\}} G(a,y(a))$, and $y \in$ core $(\mu_{\{a_1\}})$ with utility to the syndicate as desired. ||

Proof of Proposition:

We can now find an ϵ such that if $\mu \in B_\epsilon(\tau) \cap T_2$ and we form a syndicate of the agents of the first type, there will be a core point with utility to the syndicate arbitrarily close to the utility of $\tilde{x}$ to the syndicate in τ, and secondly that the least desirable (to the syndicate) competitive equilibrium will be arbitrarily close in utility to the utility of the least desirable (to the syndicate) competitive equilibrium in τ. Hence for small ϵ the syndicate will be disadvantageous.

Q.E.D.

In the previous proof there is no use ever made of the fact that τ has only two types or two commodities. The only properties of τ and the disadvantageous allocation $\tilde{x}$ which were used were:

1. τ had a finite number of types
2. $\tilde{x}$ took on only a finite number of values
3. $\tilde{p}\,\tilde{x}(a) < \tilde{p}\omega(a)$ for all a not in the syndicate
4. $\tilde{x}(a) \succ_a \omega(a)$ for all a

5. $\tilde{p}$ is the unique price supporting $\tilde{x}$ as an efficiency equilibrium

For any economy with these properties the proof of the existence of an open set of economies each containing a disadvantageous synidicate is identical to that shown. For ease of exposition we include only the above.

The set of economies in which there is a disadvantageous syndicate is not open in $\mathcal{E}$. If we drop, for instance, the uniqueness of the price $\tilde{p}$ which supports $\tilde{x}$ as an efficiency equilibria (τ without the indifference curves appropriately smoothed), lemma 4.4 fails. There are economies arbitrarily close whose efficiency equilibria which are close to the disadvantageous allocation are all such that the net trades induced by them price out positive for the unsyndicated traders. Though this is actually only an indication of why the above proof fails, it is readily seen that a disadvantageous economy can be the limit of a sequence of non-disadvantageous economies.

5. CONCLUSIONS AND DISCUSSION

In [12], Postlewaite and Rosenthal present an example of a disadvantageous syndicates in a finite economy with money and a discussion of the plausibility of such a phenomenon actually occurring. From the example in section 4.1 and the construction in section 4.2, the reader will notice that a syndicate which is not disadvantageous can sometimes be made so by modifying slightly the preferences of either the syndicate or the unsyndicated traders. Thus any attempt to characterize or investigate the phenomenon must take account of the sensitivity of both the competitive correspondence and particularly of the core to such modifications. This sensitivity of the core to changes in the preferences and to syndicate formation leads us to agree with Aumann [1] that a reappraisal is needed of the use of the core in economic analysis, at least when syndicates are present.

If the core and Aumann's definition of disadvantageousness are used we have shown that disadvantageous syndicates are not "negligible" in two ways. First, in section 4.2, we showed that any type of agent with strictly convex preferences over two commodities can be embedded in an economy such that the syndicate of all traders of the given type will be disadvantageous. Since the pupose of this result is to show that pathologies of preferences are not the key to a syndicates' being disadvantageous, we do not feel that the restriction to the two-commodity case is a particular weakness. Also, as mentioned in section 4.2, we see no structural reason why the result cannot be generalized to an arbitrary, finite number of commodities. It also appears that the restriction to strictly convex preferences (rather than weakly convex preferences) can be weakened or dropped altogether, though the construction would be considerably more complex.

The second way in which we showed that disadvantageous syndicates are not a negligible phenomenon is the proof of existence of an open ball of economies (in the set of economies with a fixed number of types) with disadvantageous syndicates,around any economy with such a disadvantageous syndicate satisfying the five properties. This is slightly weaker than stating that the entire set of economies with disadvantageous syndicates is open, but nevertheless precludes the approximation of disadvantageous economies by "normal" economies in most cases.

Given this relative abundance of economies with disadvantageous syndicates, one might ask whether normal economies (i.e., those in which there is no disadvantageous syndicate) are in some sense negligible. For instance, economies with disadvantageous syndicates might be dense in the set of all economies. This does not seem to be the case, however. We conjecture that there are open sets of economies such that no economy in the set contains a disadvantageous syndicate, though we do not go into this question here.

In section 4.3 we proved an equivalence theorem for two commodity economies with a syndicate when there are other agents which are similar (in a precise sense) to the syndicate. We indicated in section 4.3 a possible generalization of this theorem to the case of an arbitrary, finite number of commodities. This generalization as well as several results in economies with more than one syndicate will be presented in a later paper.

We leave as an unexplored area the interesting problem of syndicates of heterogeneous traders. The main problems one encounteres here are, first, the question of distribution of the syndicates share among its members, and secondly, a characterization of core points in such economies.

We also leave as an open area the situation where more than one syndicate forms. Answers to some questions in this area will be forthcoming.

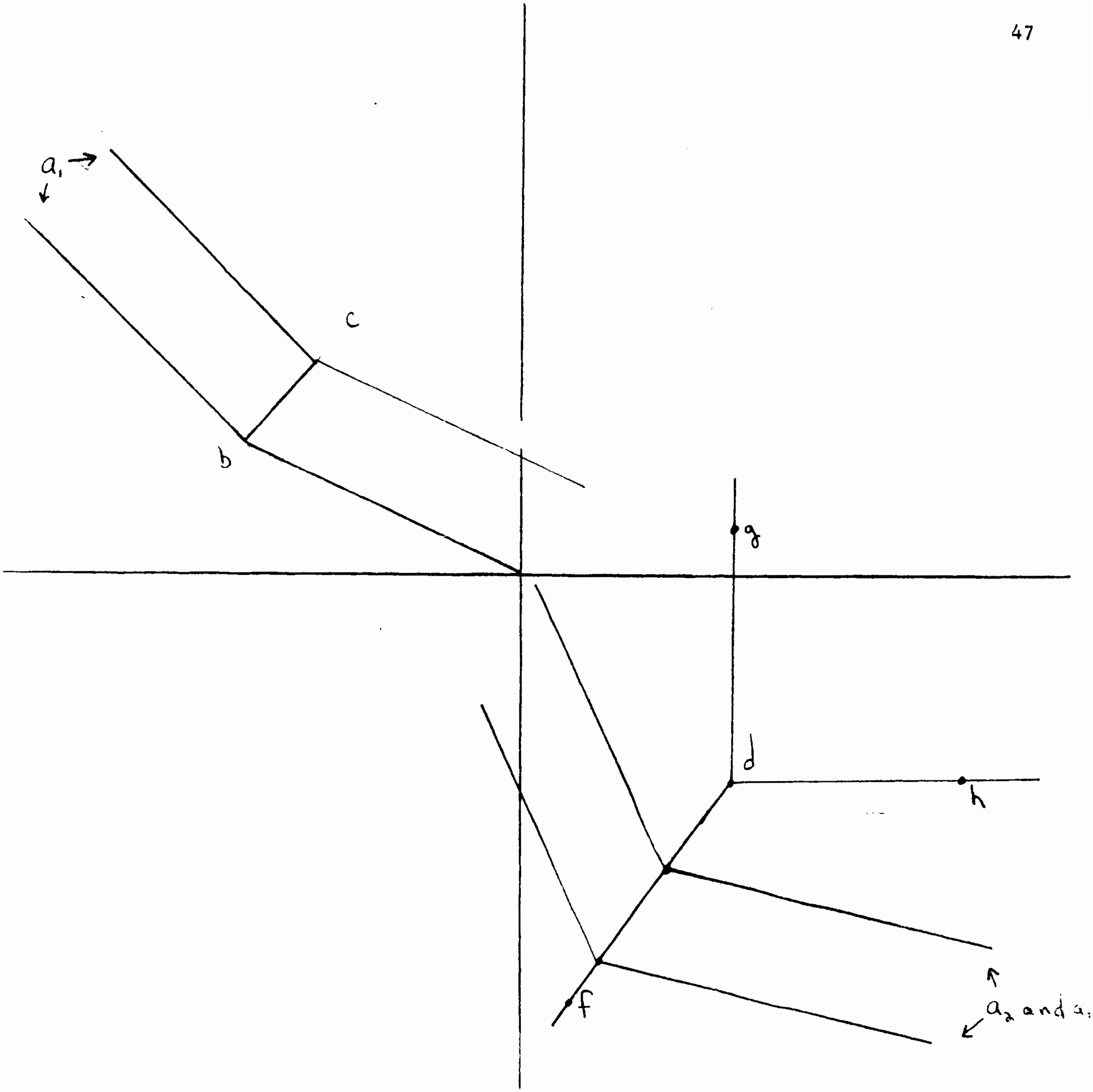

FIGURE 1.

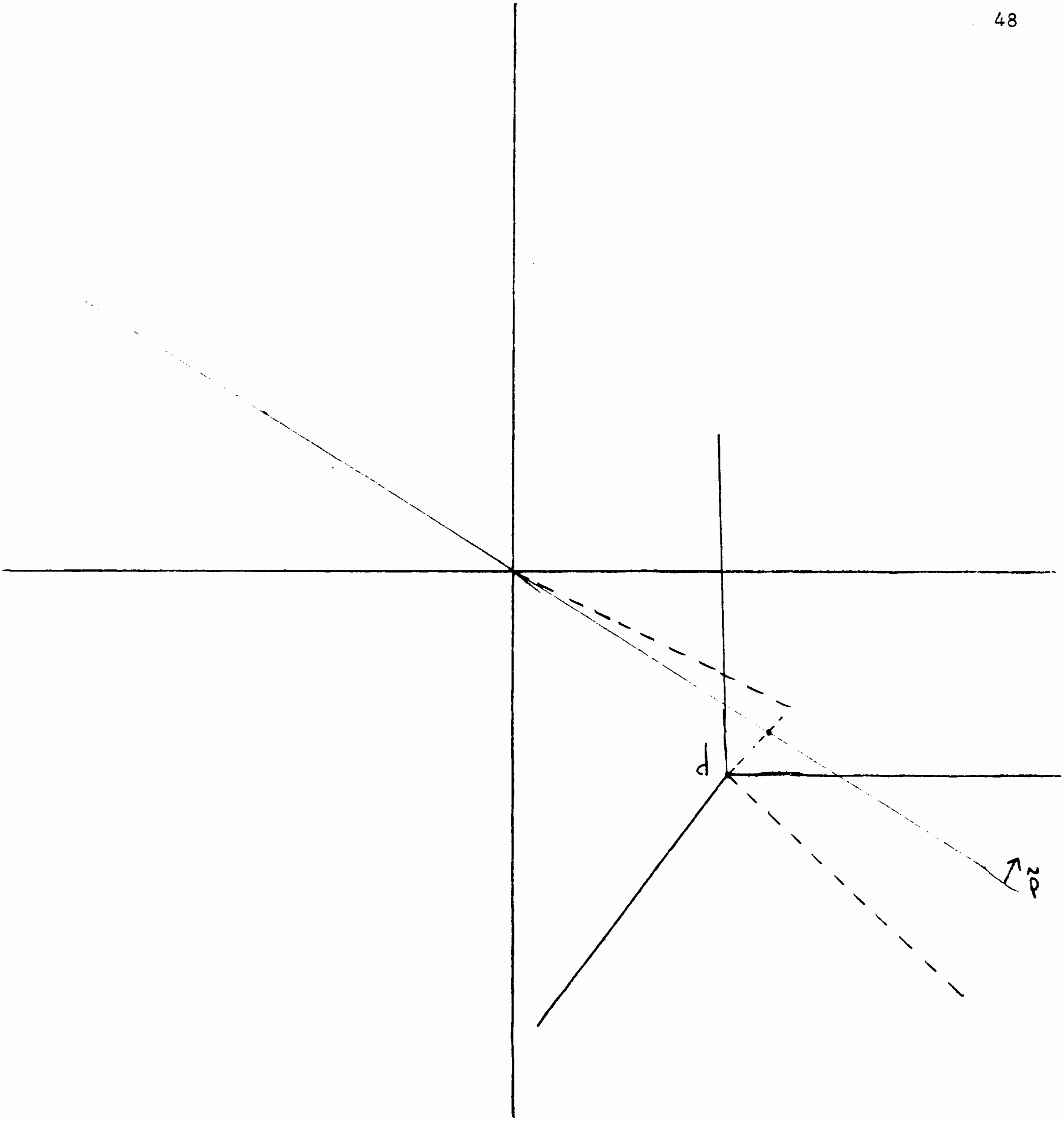

FIGURE 2

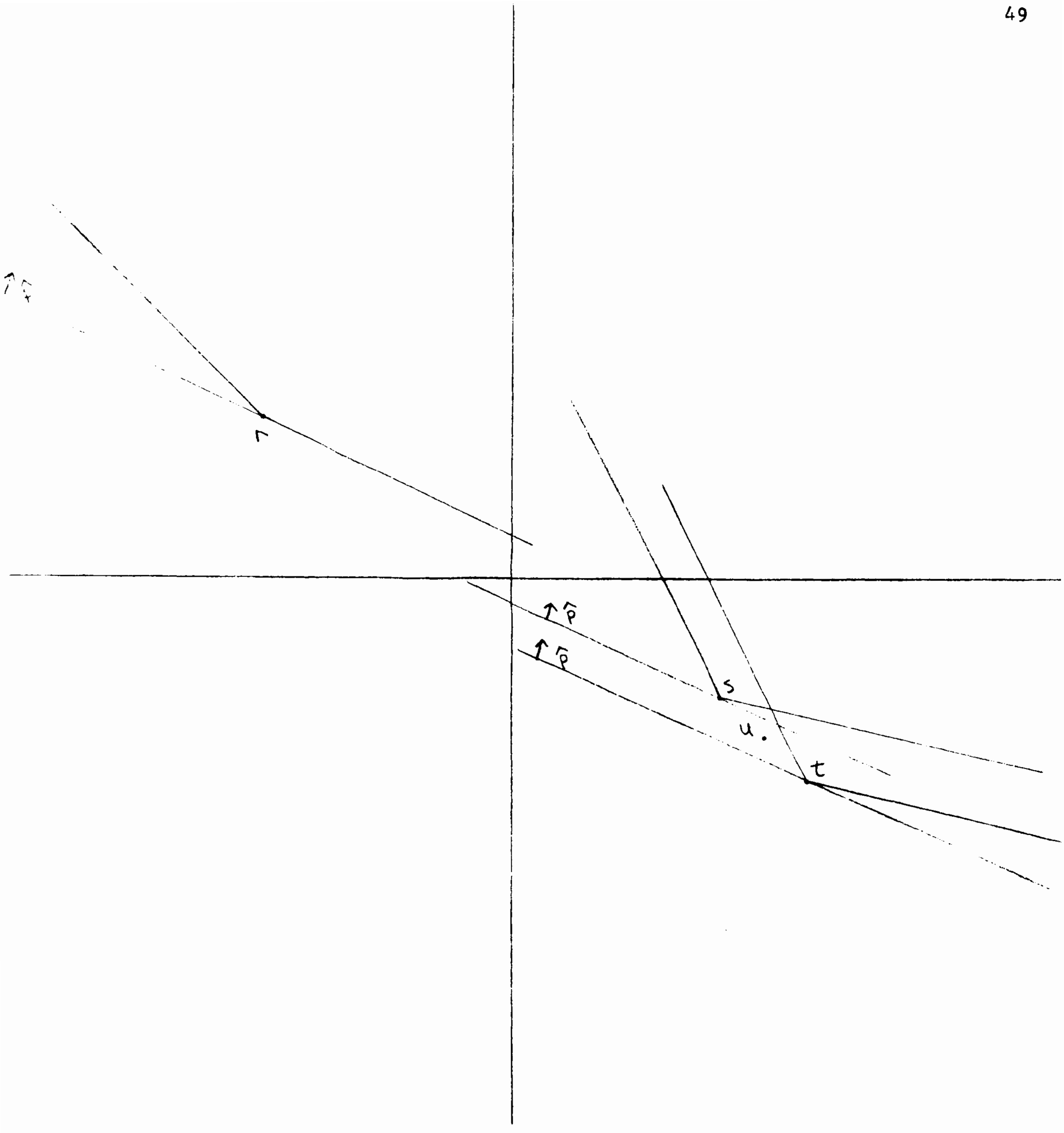

FIGURE 3

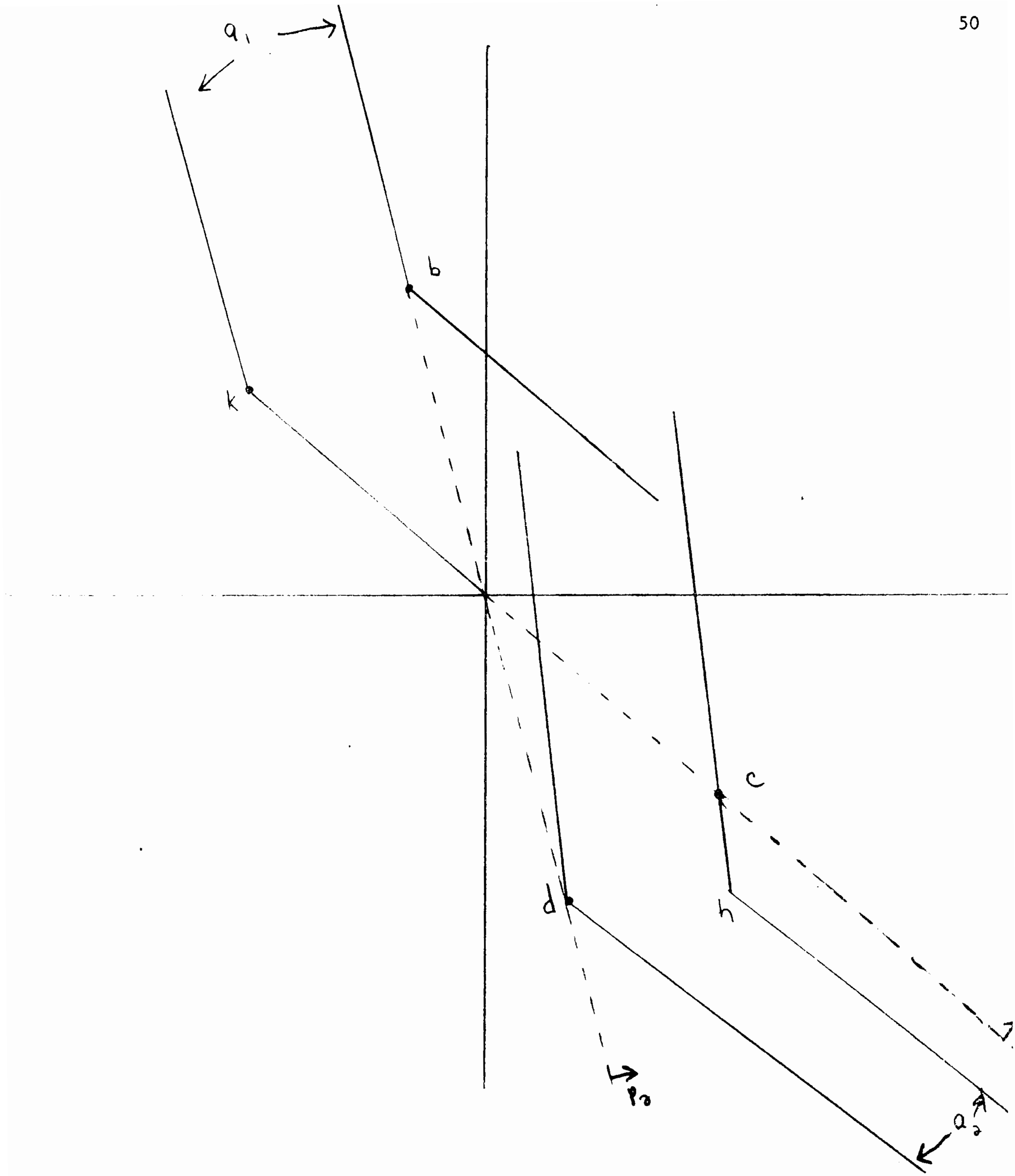

FIGURE 4

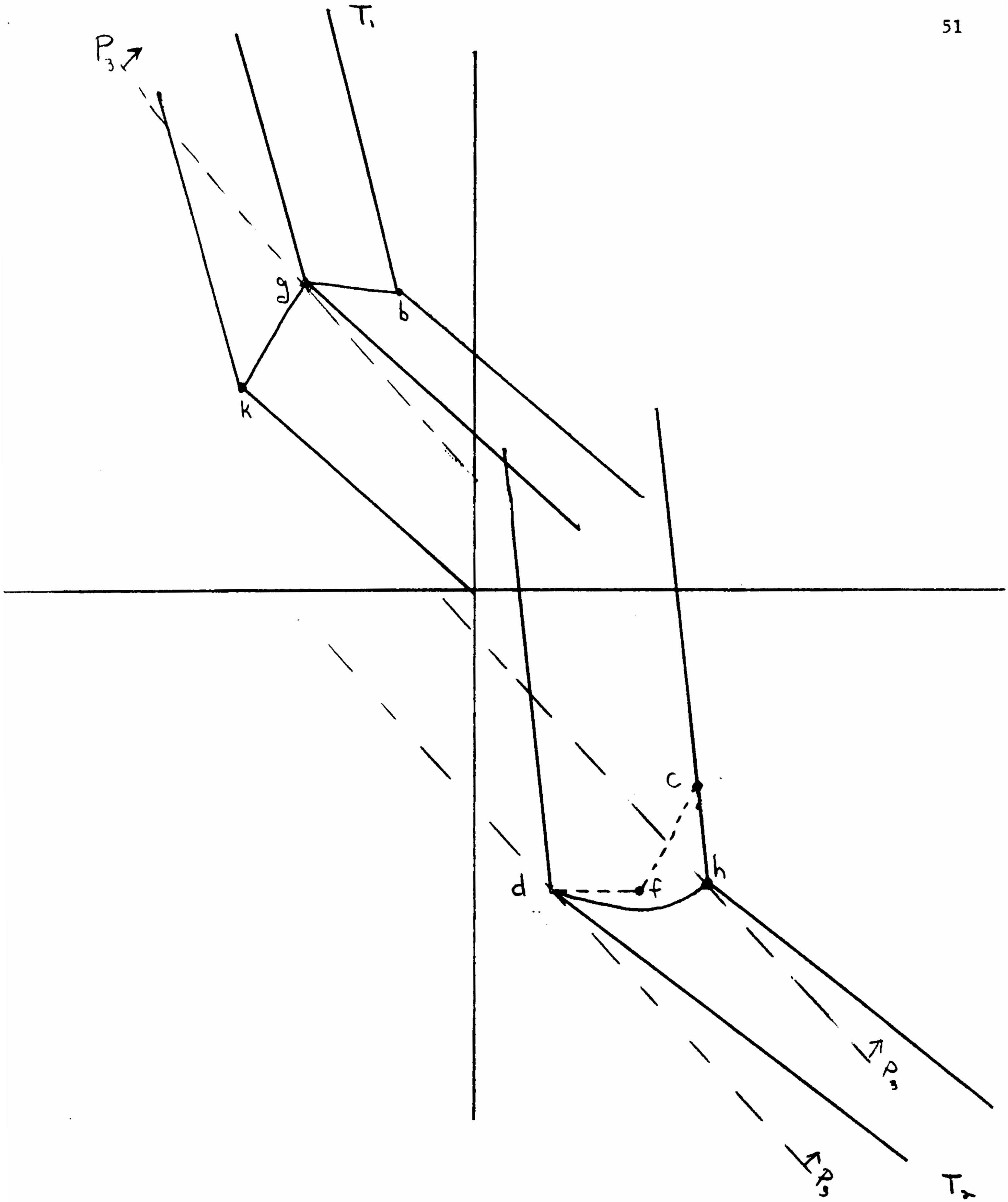

FIGURE 5

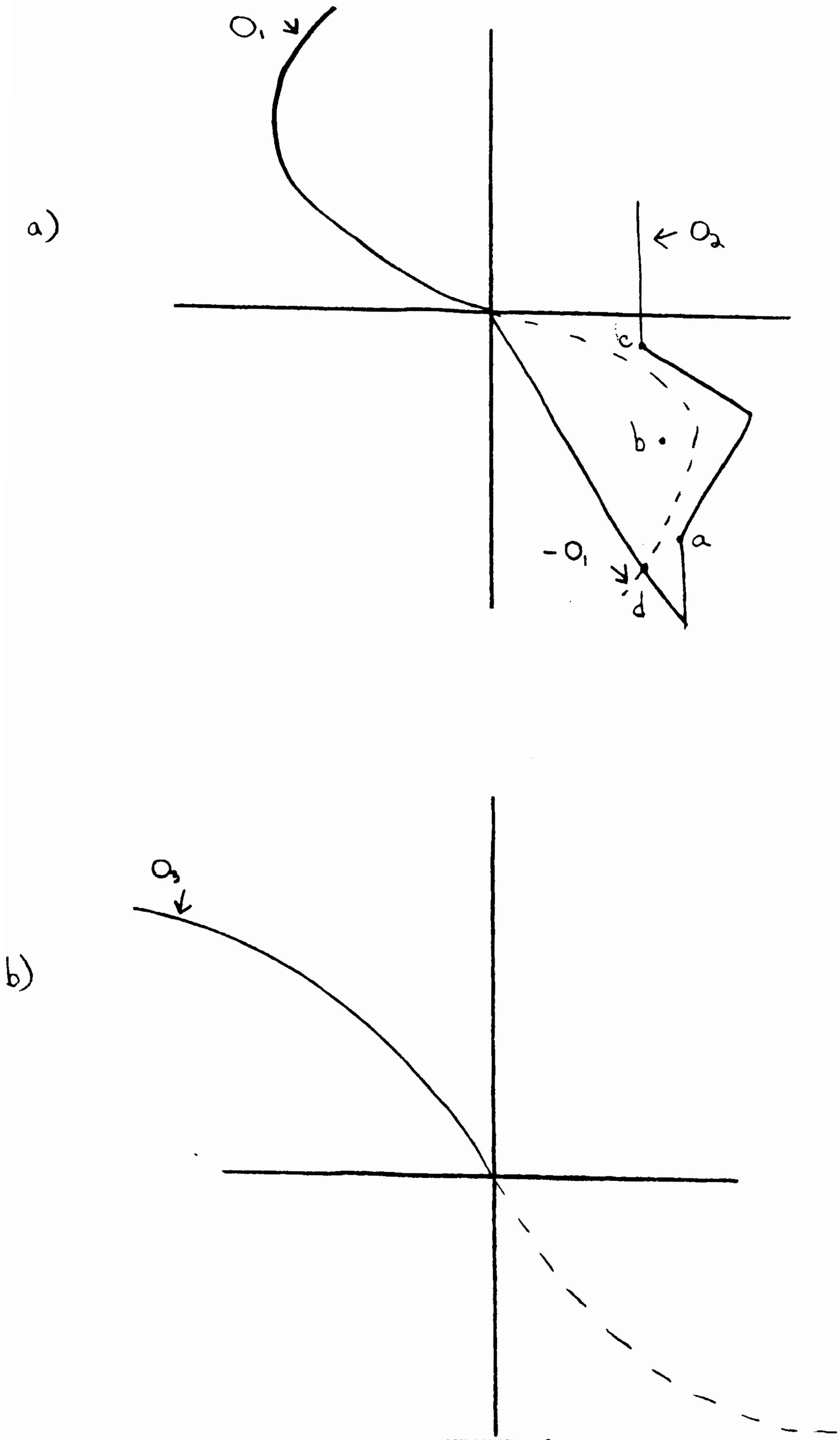

FIGURE 6

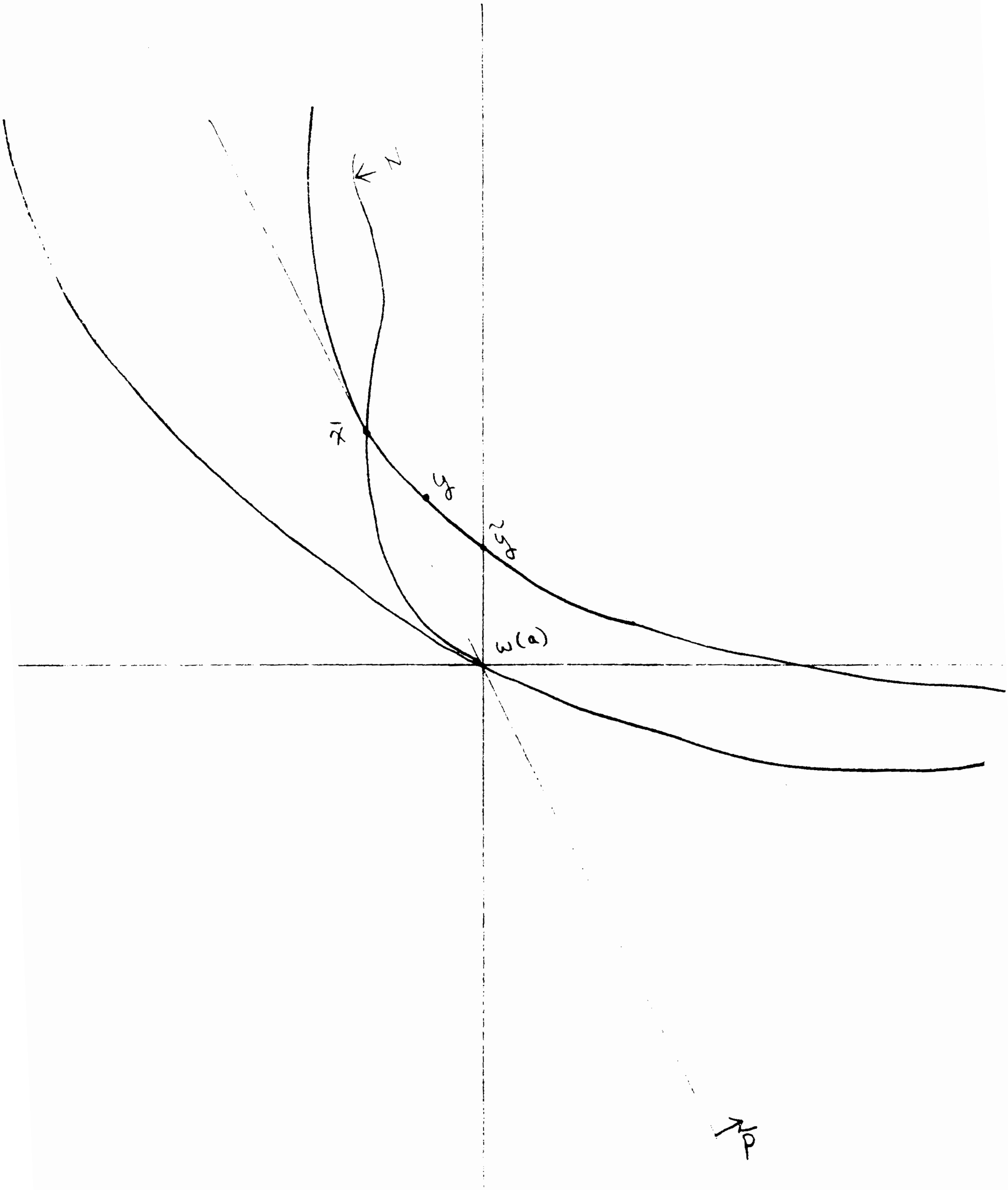

FIGURE 7

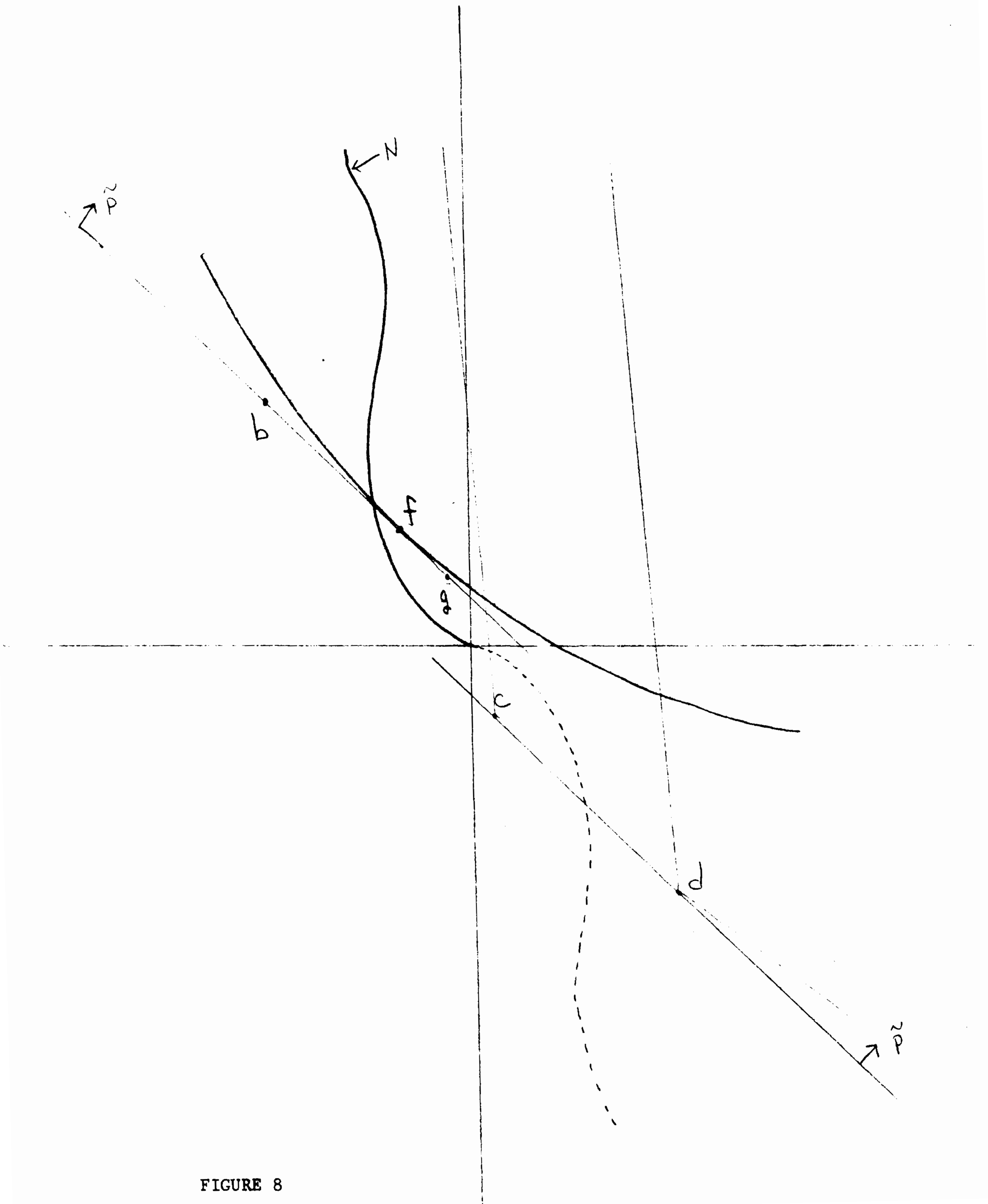

FIGURE 8

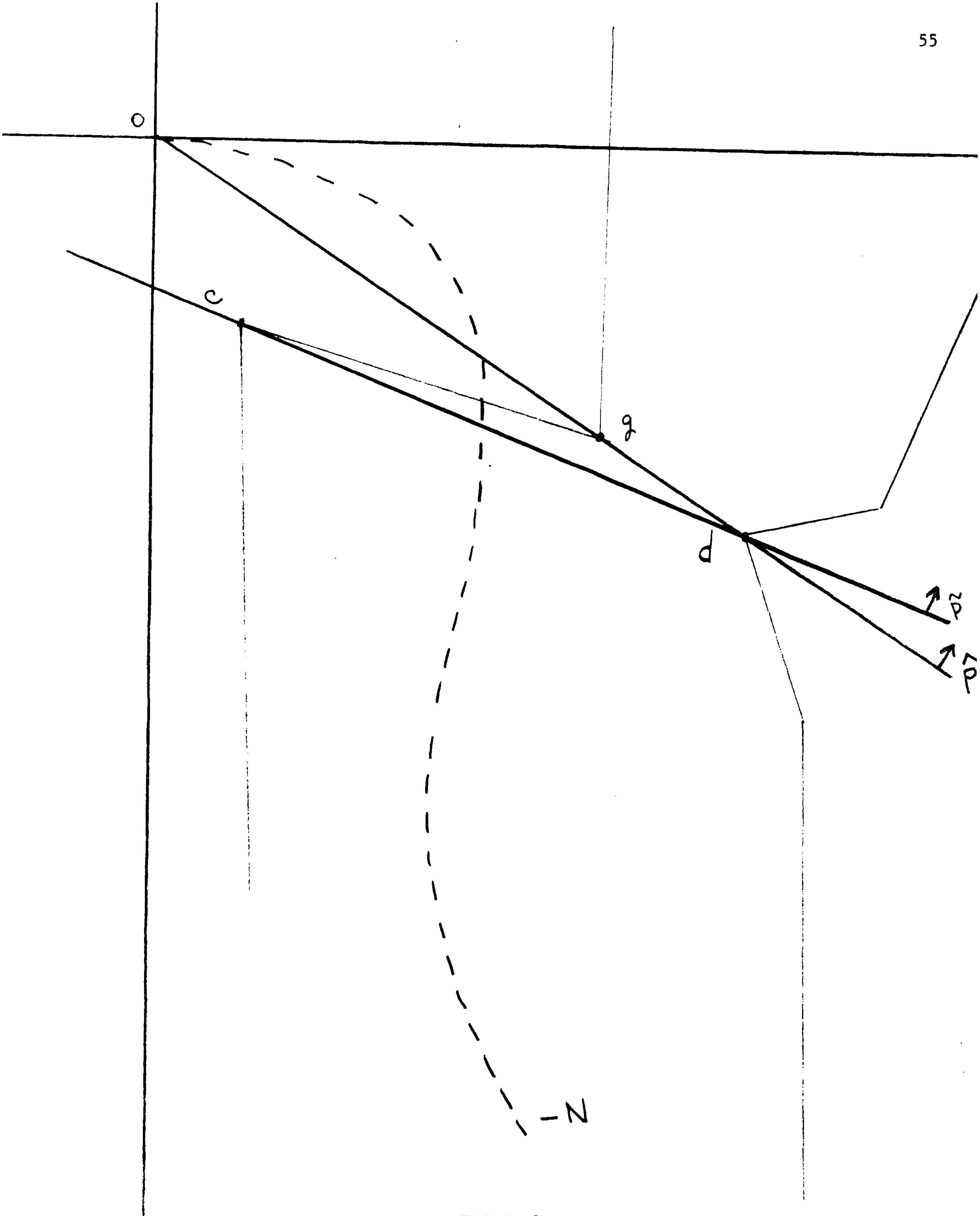

FIGURE 9

FOOTNOTES

1/ The interested reader is referred to Debreu [6] or Hildenbrand [7] for a discussion of these assumptions.

2/ This technique is originally due to T. Bewley.

3/ $\mathcal{E}$ actually depends on $\mathcal{A}$. However, since $\mathcal{A}$ is the only set of agents characteristics which we will be considering, we do not show the dependence.

4/ This result was shown for finite economies by Debreu.

5/ e_i is the vector of appropriate dimension with a 1 in the i^{th} position and o's elsewhere; e is the vector of all 1's.

6/ If for either a_1 or a_2 $\xi(a,\hat{p})$ is not single-valued, we choose a point in the correspondence arbitrarily.

BIBLIOGRAPHY

[1] Aumann, R. "Disadvantageous Monopolies" Journal of Economic Theory, 6 (1973) 1-11.

[2] Aumann, R."Markets with a Continuum of Traders" Econometrica, 32 (1964) 39-50.

[3] Berge, C. Topological Spaces New York; MacMillan, 1963.

[4] Billingsley, P. Convergence of Probability Measures New York; Wiley, 1968.

[5] Debreu, G., and Scarf, H. "A Limit Theorem on the Core of an Economy", International Economic Review, 4 (1963) 235-246.

[6] Debreu, G. Theory of Value New York, Wiley, 1959.

[7] Hildenbrand, Werner "On Economies with Many Agents" Journal of Economic Theory, 2 (1970) 167-188.

[8] Hildenbrand, Werner and Mertens, J. F. "Upper Hemi-Continuity of the Equilibrium Set Correspondence for Pure Exchange Economies," Econometrica, 40 (1972) 99-108.

[9] Jaskold-Gabszewicz, J. and Dreze, J. "Syndicates of Traders in an Exchange Economy", CORE D.P. 7020, 1970.

[10] Jaskold-Gabszewicz, J. and Mertens, J.F. "An Equivalence Theorem for the Core of an Economy Whose Atoms Are Not 'Too' Big", CORE D.P. 6916, 1969.

[11] Kannai, Y. "Continuity Properties of the Core of a Market", Econometrica, 38 (1970) 791-815.