

Mini-course on Stochastic Choice

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Northwestern, May 7–11, 2018

These lectures will discuss various models that generate stochastic choices. Randomness in choice is motivated both theoretically (since it naturally appears as a consequence of varying utility, information, or attention) as well as empirically (since individual choices typically exhibit variability, both in experimental and field data). I will mostly discuss the recently flourishing literature in decision theory, but I will also make connections to the discrete choice literature in econometrics. Both static and dynamic models will be discussed.

Lecture 1: Random Utility

- Random Utility/Discrete Choice
- Axioms
- Special Cases
- Random Expected Utility (REU)

Lecture 2: Other Static Models

- Learning
- Attention
 - Optimal Attention
 - Random Attention
- Deliberate Randomization

Lecture 3: Dynamic Models

- Dynamic Random Utility
- Dynamic Discrete Choice
- Decision Times

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