Falsiability, Identification and Rationality Discussion

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Falsiability, Identification and Rationality
Discussion

- Falsiability (exteding Chambers et.al)
- Identification (identifying the right problem)
- Logic of games (what to do about this?).

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Falsiability (exteding Chambers et.al)

Definition (Data Sets)

Let L' be a language with a finite number of constants and relation symbols such that $L' \subseteq L$. An L'-data set $\mathfrak{D}$ is a finite L'-structure.

Falsiability (exteding Chambers et.al)

Definition (Consistency of Data Sets)

A data set $\mathfrak{D}$ is consistent with an L-structure $\mathfrak{M} = (M, (R^M)_{R \in L}, (c^M)_{c \in L})$ if there is an inyective homomorphism of $\mathfrak{D}$ into $\mathfrak{M}$. We denote this by $\mathfrak{D} \rightarrow_{1-1} \mathfrak{M}$.

Falsiability (exteding Chambers et.al)

Definition (Falsiability)

Let $\mathfrak{T}$ be a class of strutures and $\mathfrak{M}$ any L-struture.

1. $\mathfrak{M}$ is falsified by the data set $\mathfrak{D}$ (i.e., $\mathfrak{D}$ falsifies $\mathfrak{M}$) if there is no inyective homomorphism of $\mathfrak{D}$ into $\mathfrak{M}$.
2. $\mathfrak{T}$ is falsified by the data set $\mathfrak{D}$ (i.e., $\mathfrak{D}$ falsifies $\mathfrak{T}$) if $\mathfrak{D}$ falsifies $\mathfrak{M}$ for all $\mathfrak{M} \in \mathfrak{T}$.
3. $\mathfrak{T}$ is falsifiable if there is some data set $\mathfrak{D}$ that falsifies $\mathfrak{T}$.

Falsiability (exteding Chambers et.al)

Definition (Empirical Content)

The empirical content $ec(\mathfrak{T})$ of theory $\mathfrak{T}$, is the class of all structures $\mathfrak{M}$ such that $\mathfrak{T}$ is not falsified by any data set $\mathfrak{D}$ consistent with $\mathfrak{M}$.

Falsiability (exteding Chambers et.al)

Theorem (Syntactic Characterization of Empirical Content)

For every class of L-strutures $\mathfrak{T}$, $ec(\mathfrak{T}) = \{\mathfrak{M} : \mathfrak{M} \models UNCAF(\mathfrak{T})\}$.

Falsiability (exteding Chambers et.al)

- We provide a structural characterization that motivates several generalizations and synthactic characterizations

Theorem

If $\mathfrak{T}$ is axiomatizable in a logic that satisfies the compacteness theorem then

$$ec(\mathfrak{T}) = \{\mathfrak{M} : \exists \mathfrak{A} \in \mathfrak{T}, \mathfrak{M} \rightarrow_{1-1} \mathfrak{A}\} \tag{1}$$

Contenido

Identification (identifying the right problem)

Definition (Data Sets)

Let L' be a language with a finite number of constants, functions and relation symbols such that $L' \subseteq L$. An L'-data set $\mathfrak{D}$ is a set $\mathfrak{D} = \{D, (R^{\mathfrak{D}})_{R\in L'}, (f^{\mathfrak{D}})_{f\in L'}, (c^{\mathfrak{D}})_{c\in L'}\}$ such that:

1. D is a finite non-empty set.
2. $R^{\mathfrak{D}}$ is an n-ary relation on D for every R n-ary relation symbol in L'.
3. $f^{\mathfrak{D}}$ is an n-ary *partial* function on D for every f n-ary function symbol in L'.
4. $c^{\mathfrak{D}}$ is an element of D for every constant symbol c in L.

Identification (identifying the right problem)

Definition (Consistency of Data Sets)

A data set $\mathfrak{D}$ is consistent with an L-structure $\mathfrak{M} = \{M, (R^{\mathfrak{M}})_{R \in L}, (f^{\mathfrak{M}})_{f \in L}, (c^{\mathfrak{M}})_{c \in L}\}$ if:

1. $D \subseteq M$
2. $R^{\mathfrak{D}} \subseteq R^{\mathfrak{M}}$
3. $f^{\mathfrak{D}} = f^{\mathfrak{M}} \mid dom(f^{\mathfrak{D}})$ where $dom(f^{\mathfrak{D}})$ is the domain of function $f^{\mathfrak{D}}$.
4. $c^{\mathfrak{D}} = c^{\mathfrak{M}}$ for every constant symbol c in L'.

Identification (identifying the right problem)

Definition (Identification)

We say $\mathfrak{D}$ identifies $\mathfrak{T}$ over the universe $\widehat{D} \supseteq D$, if for any $\mathfrak{M} = \{M, (R^{\mathfrak{M}})_{R\in L}, (f^{\mathfrak{M}})_{f\in L}, (c^{\mathfrak{M}})_{c\in L}\}$ and $\mathfrak{N} = \{N, (R^{\mathfrak{N}})_{R\in L}, (f^{\mathfrak{N}})_{f\in L}, (c^{\mathfrak{N}})_{c\in L}\}$ in $\mathfrak{T}$, such $\widehat{D} \subseteq M \cap N$ we have:

1. $R^{\mathfrak{M}} \mid \widehat{D} = R^{\mathfrak{N}} \mid \widehat{D}$, for every R relation symbol in L'.
2. $f^{\mathfrak{M}} \mid \widehat{D} = f^{\mathfrak{N}} \mid \widehat{D}$, for every f funtion symbols in L'.
3. $c^{\mathfrak{M}} = c^{\mathfrak{N}}$ for every constant symbol c in L'.

Identification (identifying the right problem): Example

- Consider the following theory.

A formula that expresses Walras law$\forall x \forall x' (\bar{I}(x) = \bar{I}(x') \rightarrow \bar{Z}(x) = \bar{Z}$

$$(2)$$

This theory is satisfiable: consider aggregate demand Z of a Neoclassical economy, $M = R^l_{++} \times R^{ln}_{+}$, l is the number of commodities in the economy, n is the number of agents and:

1. $\bar{Z}^{\mathfrak{M}} : M \rightarrow M$, defined by $\bar{Z}(p, w) = (p, max\{Z, 0\}, ..., max\{Z, 0\})$ where Z is the excess demand function of a neoclassical exchange economy and $max\{Z, 0\} \equiv (max\{Z_1, 0\}, ..., max\{Z_l, 0\})$.
2. $\bar{I}^{\mathfrak{M}} : M \rightarrow M$, defined by $\bar{I}(p, w) = (p, p \odot w, ..., 0, p \odot w)$ where $p \odot w = (p \cdot w_1, ..., p \cdot w_n)$

Identification (identifying the right problem): Example

- Let $\mathfrak{T}$ be the class of all models of ϕ. Now consider the following data set.
  $\mathfrak{D} = \left\{ D, \overline{Z}^{\mathfrak{D}}, \overline{I}^{\mathfrak{D}}, \overline{P}^{\mathfrak{D}}, \cdot, 0 \right\}$ where $D \subseteq M$ and:
  1. $\overline{Z}^{\mathfrak{D}}(p, w) = (p, 0)$
  2. $\overline{I}^{\mathfrak{D}} = \overline{I}^{\mathfrak{M}} \mid D$

  Clearly $\mathfrak{D}$ is consistent with $\mathfrak{T}$. Observability of data set $\mathfrak{D}$ represents the partial observability of the equilibrium manifold.

Identification (identifying the right problem): Example

- Consider the following universe:

$$\widehat{D} = \{(\widehat{p}, \widehat{w}) \in M : \exists (p, w) \in D \text{ such that } \overline{I}^{\mathfrak{M}}(p, w) = \overline{I}^{\mathfrak{M}}(\widehat{p}, \widehat{w})\} \tag{3}$$

Then $\mathfrak{D}$ identifies $\mathfrak{T}$ over $\widehat{D}$.

Contenido

Logic of games (what to do about this?)

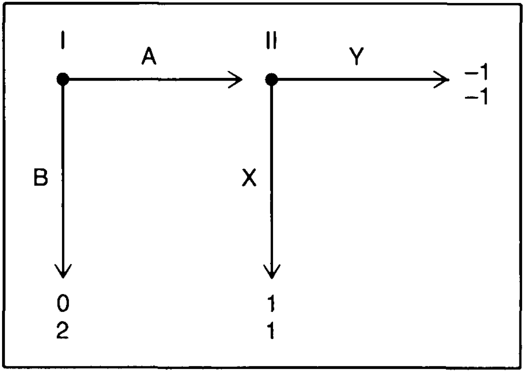
- Motivating idea: The Logic of Rational Play in Games of Perfect Information. Bonanno, G. 1991.
- Noncooperative games literature has studied estensively how to model the idea of rational behaviour in iunterative enviorments.
- An important result that motivtes a large literature is that not all Nash equilibrium ae acceptable: for example because of non credible threats.
- An axiomatization of what it means to be rational is most welcome.

Logic of games (what to do about this?)

- In my view there are three interesting ideas in this paper (most of them not the same as to what motivates the paper).
  1. I teaches how to represent n-person perfect information games as a set of propostions in propostional logic.
  2. An attempt is made to capture axiomatically, in this type of games with a few other restrictions, the notion of rationality.
  3. This is done so by avoiding modelling players beliefs. Therefore, in a sense it provides a different and probably very simplistic view on the question, what do we gain by modelling agents knowledge?

Logic of games (what to do about this?)

- Leading example (entry of a firm).



Logic of games (what to do about this?)

- Representing a game a perfect information game in propostional logic.
- Propostions: A (player I takes action A), X (player II takes action X), B (player I takes action B), Y (player II takes action Y), $\pi_i = t$ (players i payoff is t, $t \in mathcalR$).

Logic of games (what to do about this?)

- Representing the game in propositional logic.

$$(\Gamma^1_1) \quad A \vee B$$

$$(\Gamma^1_2) \quad \neg(A \wedge B)$$

$$(\Gamma^1_3) \quad (X \vee Y) \Leftrightarrow A$$

$$(\Gamma^1_4) \quad \neg(X \wedge Y)$$

$$(\Gamma^1_5) \quad B \Rightarrow ((\pi_{I} = 0) \wedge (\pi_{II} = 2))$$

$$(\Gamma^1_6) \quad X \Rightarrow ((\pi_{I} = 1) \wedge (\pi_{II} = 1))$$

$$(\Gamma^1_7) \quad Y \Rightarrow ((\pi_{I} = -1) \wedge (\pi_{II} = -1))$$

Logic of games (what to do about this?)

- Stratgies:
    1. Player I: $(A \vee B) \Rightarrow A, (A \vee B) \Rightarrow B$
    2. Player II: $(X \vee Y) \Rightarrow X, (X \vee Y) \Rightarrow Y$
- Strategy profiles are conjnctions f such formulas.

- Rational solution.
- Let R_i be the propostion i is rational.
- A strategy profile S is a rational solution of the game described by previous set of propositions Γ iff:

$$\Gamma \wedge R_1 \wedge R_2 \vdash S \tag{4}$$

Logic of games (what to do about this?)

- Charcaterizing rationality.
- Let $A_{i,h}$ player i takes action h.
- $\pi_i \geq t$ (players i payoff is at least t).
- $\pi_i \leq t$ (players i payoff is at most t).

Logic of games (what to do about this?)

DEFINITION OF PLAYER-*i*-ADMISSIBLE HYPOTHESIS. *A formula of the form*

$$\Gamma \wedge (A_{i1} \vee A_{i2} \vee \dots \vee A_{im}), \qquad \text{or} \qquad \textbf{(7a)}$$

$$\Gamma \wedge R_k \wedge (A_{i1} \vee A_{i2} \vee \dots \vee A_{im})$$
$$(\textit{for some or all } k \in \{1, \dots, n\} \setminus \{i\}) \qquad \textbf{(7b)}$$

where Γ *is the description of the game-tree,* R_k *is the proposition "player* k *is rational" (with* $k \neq i$*), and* A_{ih} *has the usual meaning ("player* i *takes action* A_h*,"* $h = 1, \dots, m$; $m \geq 1$).

Rule of inference of individual rationality (NERD):[10] If

$$H_i \Rightarrow [(A_{i1} \vee A_{i2} \vee \dots \vee A_{im}) \wedge (A_{ij} \Rightarrow \pi_i \leq \alpha) \wedge (A_{ik} \Rightarrow \pi_i \geq \beta) \wedge (\alpha < \beta)] \qquad \textbf{(8)}$$

is a theorem, then the following is a theorem

$$H_i \Rightarrow [A_{ij} \Rightarrow \neg R_i] \qquad \textbf{(9)}$$

Logic of games (what to do about this?)

- Each formula i the proof cannot contain R_i.
- Rationality characterizes choice in decision theory (one player, finite information games with a unique solution).
- Al rational solutions are equivalent (same play, same outcome).
- For nonrecursive games if there is a unique SPE it is rational.