Single-Issue Campaigns and Multidimensional Politics*

Georgy Egorov
Northwestern University

November 8, 2012

Abstract

In most elections, voters care about several issues, but candidates may have to choose only a few to build their campaign on. The information that voters will get about the politician depends on this choice, and it is therefore a strategic one. In this paper, I study a model of elections where voters care about the candidates’ competences (or positions) over two issues, e.g., economy and foreign policy, but each candidate may only credibly signal his competence or announce his position over at most one issue. Voters are assumed to get (weakly) better information if the candidates campaign on the same issue rather than on different ones. I show that the first mover (the challenger) will in equilibrium set the agenda for both himself and the opponent if campaigning on a different issue is uninformative, but otherwise the incumbent may actually be more likely to choose the other issue. The social (voters’) welfare is a non-monotone function of the informativeness of different-issue campaigns, but in any case the voters are better off if candidates are free to pick an issue rather than if an issue is set by exogenous events or by voters. If the decision to move first or second is endogenized, choosing to move first signals incompetence in one of the issues and thus politicians may wait until one has to make a move; however, the one moving first is more likely to be overall more competent and more likely to be elected. These results may help understand endogenous selection of issues in political campaigns and the dynamics of this decision.

Keywords: Elections, campaigns, issues, salience, probabilistic voting.
JEL Classification: D71, D74, C71.

Very Preliminary
Please Do Not Circulate
Comments Are Very Welcome.

*I thank David Austen-Smith, César Martinelli, Mattias Polborn and Konstantin Sonin for valuable comments.
1 Introduction

Political competition in one-dimensional policy space with two candidates is one of the most well-studied areas in political economy, and formal analysis goes at least as far as Downs (1957). It is not unrealistic: Duverger’s Law (Riker 1982, see also Lizzeri and Persico 2005) predicts emergence of two major candidates or parties in majoritarian (winner-take-all) elections, as is evident from the USA. But is that true that political preferences over multiple issues also endogenously lead to competition over a single dimension, perhaps the most important one, and if so, how is it chosen?

In this paper, I study endogenous selection of issues that political campaigns are about. I assume that voter preferences are two-dimensional (and separable), and politicians decide the issue they want to run on. The following friction is key: I assume that a politician can only credibly announce his position on (at most) one issue, but not on both.\(^1\) The motivation is that persuading voters about his/her position or about his/her talents in a particular sphere is hard, and there is only a limited amount of time. Many voters make their decision based on a single debate performance, or a single rally they attend, so losing focus may be costly for candidates. Even though occasionally candidates talk about several issues, the broad idea of the campaign is typically clear: e.g., Bill Clinton ran on economy in 1992 while George H.W. Bush on foreign policy; in 1972, George McGovern ran against the Vietnam war; in 2012, the Romney campaign attacked the economic record of Barack Obama, and the Obama campaign eventually switched from focusing on social issues to defending Obama’s economic record. Of course, in the course of long campaigns, the focuses of the campaigns, and even issue salience itself might change, but it is probably reasonably to assume that at each stage (e.g., in a given debate) the candidate has to adopt a particular line of attack or defense, which may or may not coincide with the one chosen by his opponent.

The second key assumption I make is that voters get better information about a candidate’s position or competence in a given issue if both of them choose to talk about it. In other words, there is a certain complementarity: the more likely a politician’s opponent to talk about, say, the economy, the more credibly the politician himself will sound if he chooses to talk about the economy as well. Indeed, the other politician may criticize his positions, check whether the facts are correct, and thus indirectly add credibility to the politician’s statements.\(^2\) On the other hand, if the other politician talks on another topic, then one’s own credibility is undermined.

\(^1\) Polborn and Yi (2006) consider politicians who can either run positive (revealing information about themselves) or negative (revealing information about their opponents) informative campaigns, but not both. Alternatively, one can assume that politicians have a limited resource (time) that they need to spend on different issues. Whenever this last problem has corner solutions, the two approaches yield identical results.

\(^2\) Bhattacharya (2011) models the process of information revelation through positive and negative ads.
do not model the process of making statements and having the other party check their veracity explicitly; but the assumption that voters learn more if politicians have a sensible debate on the same issue seems reasonable.\(^3\)

These two assumptions lead to a simple and tractable model of issue selection. We first study a basic case where one politician (e.g., the challenger) commits to build his campaign on one of the two dimensions. The other politician (e.g., the incumbent) observes this, and decides whether to reciprocate or to talk about the other dimension instead. Voters are Bayesian and they update on the politician’s decisions and the information that they observe, and vote, in a probabilistic way, for one of the two candidates. The model gives the following predictions. First, if campaigning on different issues gives politicians very little credibility, there is a strong unraveling effect, which forces all incumbents to respond the challengers and talk about the same issue. However, if a politician is quite likely to be able to credibly announce his position or establish his competence even when talking on a different issue, then divergence is possible, and in fact it is slightly more likely that the incumbent will choose a different issue. Second, politicians respond to voters: if voters consider an issue to be more important, it is more likely that politicians will choose it for their campaign. Third, the social welfare (as measured by the expected competence of the elected politician) does not necessarily increase as the ability of politicians to make credible announcements when the opponent campaigns on a different issue. The reason is that when this ability is low, politicians will endogenously choose the same issue, and this will help voters rather than hurt them. Finally, if politicians have some discretion whether to choose their issue first or second, those who choose to move first are likely to be more competent (and also disproportionately competent in one issue relative to the other). As such, there is a first-mover advantage, even though ex-ante, the expected competences of the two politicians are equal.

The idea that politicians may campaign on issues for strategic reasons is certainly not new. A sizeable part of the literature focuses on priming, i.e., that campaigning on an issue attracts voters’ attention, and in the end of the day the voters begin to put a higher weight on this issue, i.e., their preferences change. In this spirit, Aragonès, Castanheira, and Giani study a model where parties invest in generating alternative proposals to the status quo, and then advertise this proposal to increase the salience of this issue; they show that parties are most likely to invest in issues they hold ex-ante advantage in, although sometimes “issue stealing” is also possible. Amoros and Puy (2007) consider a model where campaigns allocate an advertisement budget to increase the salience of an issue (see also Colomer and Llavador, 2011, for a related model).

\(^3\)This may in fact be microfounded by introducing a third-party fact-checker which is active only some of the time; however, if both candidates campaign on the same issue, their opponents serve this role automatically.
Other papers emphasize signaling considerations; e.g., Morelli and van Weelden (2011) predict that a politician will spend a disproportionate amount of effort on a divisive issue for the purpose of credible signaling.

The paper most closely related to this one is Polborn and Yi (2006). There, politicians also campaign on their qualities, and there may be positive and negative information available about each of them (four dimensions totally). Politicians choose the campaign issue among two options: he can reveal positive information about himself or negative information about his opponent. The authors characterize a unique equilibrium, in which running a negative campaign reveals lack of positive information about oneself. This setting corresponds to the case where politicians have perfect ability to reveal their competence even if they talk about different issues. As such, this paper generalizes Polborn and Yi (2006) for the case of generic campaign issues where campaigning on different issues may undermine voters’ ability to learn the truth.\(^4\)

The model in this paper assumes that the relevant characteristic of candidates is their competence, and all voters have the same preferences (more competence is better), but similar forces would be in effect if candidates were competing on a more divisive issue; in this case, the counterpart of competence would be proximity of candidate’s ideal point (in a given dimension) to the median voter’s position. The model would predict that politicians would have an incentive to campaign on an issue where their position is close to the median voter’s one, and if politician turns out to be far from the median voter on the issue that he chose, voters would suspect that he is even more radical on the other issue. The latter interpretation, however, would require making additional assumptions: that the candidates cannot commit to any policy position other than their ideal one in the course of the campaign, and also that they cannot lie about their position (or, more precisely, that they cannot lie if the other party is campaigning on the same issue and can reveal this lie to voters).\(^5\) In the current model, voters’ preferences are aligned and competence is unambiguously good, so pandering must take the form of exaggerating one’s competence. The results are driven by the assumption that doing so is easier if the opponent talks about a different issue; the assumption that exaggeration is either infinitely costly (or at least competence is fully revealed) or totally uncontrolled to the point that the candidate has zero credibility makes the model tractable, but hardly drives the results.

\(^4\)Duggan and Martinelli (2011) study a model of media slant, where media are assumed to collapse a multi-dimensional policy position to a one-dimensional one.

\(^5\)There is a large literature on pandering to voters by partisan politicians as well as obscuring one’s positions, both on campaign trail and in office. Alesina and Cukierman (1990) suggest that incumbents have an incentive to be ambiguous (see also Heidhues and Lagerlof, 2003). Callander and Wilkie (2007) talk about lying on the campaign trail, as does Bhattacharya (2011). Kartik and McAfee (2007) consider signaling motive in policy choices; Acemoglu, Egorov, and Sonin (2012) suggest that signaling may make politicians choose policies further from the median voter rather than closer to his position.
The rest of the paper is organized as follows. Section 2 introduces the basic model. In Section 3, the equilibria in the basic case of equally important issues are studied, and comparative statics results are obtained. In Section 4, I study the implications of political competition on issues on social welfare. Section 5 allows one politician to choose whether he wants to be a leader or a follower in choosing his campaign issue. Section 6 discusses extensions of the basic model. Section 7 concludes. The Appendix contains the proofs.

2 Model

Consider a two-dimensional policy space, one dimension being economy ($E$) and the other being foreign policy ($F$). There are two politicians, which we will refer to as an incumbent and a challenger, and each politician has a two-dimensional type $a = (e, f)$, which corresponds to his ability in economic and foreign policy questions, respectively (in what follows, $a|_s$ will denote the projection of $a$ on $s \in \{E, F\}$). We consider an electorate with perfectly aligned preferences: there is a continuum of voters, the utilities of which if politician of type $(e, f)$ is elected is

$$U(e, f) = we + wff,$$

where $w_e$ and $w_f$ are weights that voters assign to the economy and foreign policy, respectively (we normalize it to $w_e + w_f = 2$). The politician’s type is his private information and is not known to the other politician and voters. At the time of voting, all voters have the same information on both the challenger and the incumbent: they know the history of the candidates’ moves as well as the move of the Nature, we capture this by $I$ for brevity. Voting is probabilistic: voter $j$ votes for the challenger if and only if

$$\mathbb{E}(U(e_c, f_c) - U(e_i, f_i) | I) > \theta + \theta_j,$$

where $\theta$ is a common shock and $\theta_j$ is voter $j$’s individual shock. As standard, we assume that $\theta$ is distributed uniformly on $[-\frac{1}{2A}, \frac{1}{2A}]$ and $\theta_j$ distributed uniformly on $[-\frac{1}{2B}, \frac{1}{2B}]$, where $A < \frac{1}{2}$ and $B < \frac{A}{2A+1}$.

During the campaign, each politician can only talk about one issue, economy or foreign policy. If both talk about the same issue, they have a reasonable conversation, during which the voters perfectly learn their competences on this dimension ($e_c$ and $e_i$ or $f_c$ and $f_i$). However, if they end up talking about different issues, it is much harder for them to do it credibly (e.g., the other side is not actively engaged in responding or fact-checking). In this case, the chance that the voters will actually find out his competence on this issue is $\mu \in [0, 1]$, where $\mu = 0$ corresponds to zero credibility, and $\mu = 1$ is the other extreme where a politician’s ability to
make credible announcements of his competence does not depend on the issue that his opponent chose. In some sense, lower $\mu$ corresponds to a higher noise in communication between politician and voters if there is nobody around to limit exaggerations or bluffing; not modeling noisy signals explicitly is a mere modeling shortcut aimed at simplifying the exposition.\footnote{One may assume that a politician may announce any competence, but is heavily penalized if he is found to have exaggerated. When politicians talk about the same issue, there is only chance $\mu$ that some third party is willing to check the politician’s announcements. However, when they talk about the same issue, there is always someone to do this, and in this case the politicians have to be credible, so voters learn the true competences on this issue. On the other hand, if nobody puts a check on politicians, then all announce that they are the most competent, and Bayesian voters learn nothing.}

More precisely, the type of challenger is given by $a_c = (e_c, f_c) \in \Omega_c = [0, 1]^2$ and $a_i = (e_i, f_i) \in \Omega_i = [0, 1]^2$. The challenger moves first and chooses the issue of his campaign, $d_c \in \{E, F\}$, and the incumbent observes this and chooses his issue $d_i \in \{E, F\}$. In other words, the set of strategies of the challenger is $S_c : \Omega_c \mapsto \{E, F\}$ and the set of incumbent’s strategies is $S_i : \Omega_i \times \{E, F\} \mapsto \{E, F\}$.\footnote{It should be emphasized that the two candidates do not have private information about the types of their opponent, and in particular the incumbent makes his decision knowing the issue that the challenger chose, but not his competence on that issue. This is a simplification of reality, but from a technical standpoint, it prevents politicians from strategically jamming the opponent’s signal if they know it is very high.} The Nature then decides whether each of the candidates is successful in announcing their competence: for the challenger, $c \in a c | d c$ if $d_c = d_i$, and if $d_c \neq d_i$, then $c = a c | d c$ with probability $\mu$ and $c = \emptyset$ otherwise (where $\emptyset$ stands for the absence of a credible announcement. Similarly, for the incumbent, $i = a i | d i$ if $d_c = d_i$, and if $d_c \neq d_i$, then $i = a i | d i$ with probability $\mu$ and $i = \emptyset$ otherwise. Each voter observes the entire history $I = \{d_c, d_i, c, i\}$ and uses this to create posterior

$$V (d_c, d_i, c, i) = \mathbb{E} (U (e_c, f_c) - U (e_i, f_i) | d_c, d_i, c, i).$$

After that, the common shock $\theta$ and idiosyncratic shocks $\theta_j$ is realized for each voter $j$, and each voter $j$ votes for the challenger if and only if (2) holds.

Both the incumbent and the challenger are expected utility maximizers. The incumbent solves the problem

$$\max_{d_i} \mathbb{E} (V (d_c, d_i, c, i) | d_c, e_i, f_i),$$

and the challenger maximizes

$$\max_{d_c} \mathbb{E} (V (d_c, d_i, c, i) | e_i, f_i)$$

where both in (3) and (4) the expectations is taken over the opponent’s type ($a_c$ and $a_i$, respectively), as well as the Nature’s moves.
The equilibrium concept is the following refinement of the standard Perfect Bayesian equilibrium (PBE) in pure strategies. First, consider monotone strategies. This means that if the challenger’s strategy satisfies $d_c(e_c, f_c) = E$, then $d_c(e'_c, f_c) = E$ for $e'_c > e_c$ and $d_c(e_c, f'_c) = E$ for $f'_c < f_c$, and similar requirements are true for $d_i(e_i, f_i; d_c = E)$ and $d_i(e_i, f_i; d_c = F)$. This refinement means that if some type chooses to discuss the economy, then increasing his competence in this dimension without changing the other does not make him switch to foreign policy, and vice versa. Second, consider “approachable” equilibria: I require that equilibrium strategies for a given $\mu$ are pointwise limits of equilibrium strategies for some sequence of $\mu_1, \mu_2, \ldots$ that converges to $\mu$ but never reaches it. This last requirement rules out unintuitive pooling equilibria for $\mu = 0$.

### 3 Analysis

Let us first compute the probability of each politician to be elected for any possible $I = (d_c, d_i, \kappa_c, \kappa_i)$. For a given $\theta$, voter $j$ votes for the challenger with probability

$$\Pr(\theta_j < \mathbb{E}(U(e_c, f_c) - U(e_i, f_i) \mid I) - \theta) = \frac{1}{2} + B(\mathbb{E}(U(e_c, f_c) - U(e_i, f_i) \mid I) - \theta),$$

which is also the share of votes he gets. The challenger wins if and only if this exceeds $\frac{1}{2}$, which happens with probability

$$\Pr(\theta < \mathbb{E}(U(e_c, f_c) - U(e_i, f_i) \mid I)) = \frac{1}{2} + A\mathbb{E}(U(e_c, f_c) - U(e_i, f_i) \mid I).$$

Since $A$ is a constant and there are no transfers, the challenger maximizes

$$\mathbb{E}(\mathbb{E}(U(e_c, f_c) - U(e_i, f_i) \mid I))$$

and the incumbent minimizes it. Notice that at the time either politician makes a decision, he knows that he can affect the voters’ posterior about his opponent (e.g., the incumbent can make a very competent challenger appear worse if he chooses the opposite field, because the challenger could then fail to make a credible announcement). However, if he takes expectation conditional only on the information he knows at the time he makes the decision, the expected voters’ posterior is given and he cannot change it. This simplifies the problem a lot by effectively separating the problems of the challenger and the incumbent. From now on, we can just assume that the challenger and the incumbent maximize the expectations of $\mathbb{E}(U(e_c, f_c) \mid I)$ and $\mathbb{E}(U(e_i, f_i) \mid I)$, respectively; more precisely, the challenger maximizes

$$\mathbb{E}(\mathbb{E}(U(e_c, f_c) \mid d_c, \kappa_c) \mid e_c, f_c),$$

Polborn and Yi (2006) introduce a similar refinement in a game where politicians choose to run a positive campaign or a negative campaign.
and the incumbent maximizes

\[ E \left( E \left( U \left( e_i, f_i \right) \mid d_c, d_i, \kappa_i \right) \mid d_c, e_i, f_i \right). \]

The following proposition describes the equilibrium strategies in the case of equally important issues.

**Proposition 1** In every equilibrium, the challenger chooses the issue that he is more competent in: economy if \( e_c > f_c \) and foreign policy if \( f_c > e_c \) (and he is indifferent otherwise).

The incumbent’s response if the challenger chose \( E \) is the following:

(i) if \( \mu > \frac{1}{2} \), then the incumbent chooses \( E \) if \( e_i > \frac{5 - \mu + \sqrt{\mu(8 - 7\mu)}}{2} \) and chooses \( F \) if the opposite inequality holds;

(ii) if \( \mu < \frac{1}{2} \), then the incumbent always responds with \( E \);

(iii) if \( \mu = \frac{1}{2} \), then there are two equilibria: where the incumbent either chooses \( E \) if \( e_i > \frac{1}{2} \) and \( F \) if \( e_i < \frac{1}{2} \) (and if \( e_i = \frac{1}{2} \), then he chooses \( E \) iff \( f_i < \frac{1}{2} \)), and where he always chooses \( E \).

Similar characterization applies if the challenger chose \( F \).

Proposition 1 characterizes equilibria of the game. The challenger always chooses the issue that he is most competent in. The incumbent’s response critically depends on the chance that he will be heard if he chooses a different issue. If this chance is at least \( \frac{1}{2} \), then he will choose the same topic if he is sufficiently competent at it, but if he too incompetent in this dimension, he will choose a different one. However, if the chance is less than \( \frac{1}{2} \), then he will have to reciprocate and talk about the same issue regardless of his type. Intuitively, the voters are Bayesian updaters, and will punish the incumbent too harshly if he deviates to a different topic, and this leads to complete unraveling if \( \mu < \frac{1}{2} \). If \( \mu > \frac{1}{2} \), they punish him, too, but a relatively high \( \mu \) allows the incumbent to compensate with a credible announcement on the other dimension, where he may well excel. In other words, as \( \mu \) increases, the adverse selection on the dimension that challenger chose is counterveiled by additional credibility on the other issue, and \( \mu = \frac{1}{2} \) serves as the borderline case. Figure 1 illustrates the equilibrium strategies for different values of \( \mu \geq \frac{1}{2} \).

Where does the threshold \( \mu = \frac{1}{2} \) come from? Suppose that all challengers choose \( E \); when is it an equilibrium for the incumbent to always choose \( E \)? The type \((e_i, f_i) = (0, 1)\) is most prone to deviation. If he does not deviate, then the voters’ posterior about his ability is \( 0 + \frac{1}{2} = \frac{1}{2} \). But if he deviates, then with probability \( \mu \) he proves that \( f_i = 1 \), and thus his total ability is at least 1. Consequently, if \( \mu > \frac{1}{2} \), it is profitable for him to deviate; in contrast, if \( \mu \leq \frac{1}{2} \), then there exist out-of-equilibrium beliefs that make deviation unprofitable.\(^9\) Hence, unraveling

---

\(^9\)These beliefs are, for example: if the incumbent chooses \( F \) and is able to announce \( f_i \), then his \( e_i = 0 \) almost surely, and if he chooses \( F \) and is not able to make a credible announcement, then his type is almost surely \((e_i, f_i) = (0, 0)\).
equilibrium is possible only if $\mu \leq \frac{1}{2}$. This suggests that the threshold $\mu = \frac{1}{2}$ is a consequence of uniform distribution (rather than, e.g., equal weights on the two issues, as will be evident from Section 6).

The formal proof of Proposition 1 is in the Appendix, but the idea is relatively straightforward. The challenger’s problem is symmetric, and thus it is natural to expect a symmetric equilibrium, with politicians with $e_\text{c} > f_\text{c}$ and $e_\text{c} < f_\text{c}$ choosing the opposite strategies. At the same time, picking the issue where one has a disadvantage cannot happen in equilibrium: then deviating and choosing the other issue would send the voters a better signal on both issues. The problem of the incumbent is somewhat more complicated, and for simplicity let us focus on equilibria where both $E$ and $F$ are picked by positive shares of incumbents. To start, it is easy to see that the boundary separating the $E$-region and $F$-region must be linear. The second step is to show that all politicians of type $(0, z)$ with $z > 0$ prefer $F$ strictly. Indeed, suppose not, then without loss of generality $(0, z)$ has the highest $z$, and thus is indifferent between $E$ and $F$. For him, choosing $E$ yields $z$, and choosing $F$ yields exactly $z$ with probability $\mu$ and something higher than $z$ with probability $1 - \mu$ (because this type is the worst of those who can choose $E$). Hence, the payoff from $F$ is higher as long as $\mu < 1$ (and the extreme case $\mu = 1$ may be considered separately). We can then show that all politicians of type $(1, z)$ with $z < 1$ strictly prefer $E$; indeed, otherwise we can assume that $(1, z)$ is indifferent, and do similar calculations. For this type, choosing $E$ yields $1 + \frac{z}{2}$, and choosing $F$ yields $z + \frac{1}{2}$ (which is less than $1 + \frac{z}{2}$) with probability $\mu$; the payoff in the case where he chooses $F$ but voters do not learn his type is
harder to estimate, but one can show that it cannot exceed $z + \frac{1}{2}$ either. This means that type $(0,0)$ weakly prefers $F$ and type $(1,1)$ weakly prefers $E$; these statements are true for $\mu = 1$ as well.

The boundary must therefore intersect the upper and the lower sides of the unit square of incumbents’ types. Let us suppose that types $(a,0)$ and $(b,1)$ are indifferent, where $b > a$ ($b < a$ is ruled out, and $b = a$ is the borderline case). The average competence of types choosing $F$ is then $\frac{a^2 + ab + b^2 + a + 2b}{3(a+b)}$, and the indifference condition for type $(e_i, f_i)$ is

$$e_i + \frac{f_i}{2} = \mu \left( f_i + \frac{e_i}{2} \right) + (1 - \mu) \frac{a^2 + ab + a + b^2 + 2b}{3(a+b)}.$$ 

Substituting for $(a,0)$ and $(b,1)$ and solving, we find that a solution with $b > a$ exists if and only if $\mu < \frac{1}{2}$ and it is given by $a = \frac{4 - 5\mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)}$, $b = \frac{3\mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)}$. This yields part (i) of Proposition 1; part (ii) was established earlier, and part (iii) is the borderline case formally considered in the Appendix.

Proposition 1 suggests that once the likelihood of voters learning politicians’ competences if they talk about different issues is sufficiently small ($\mu < \frac{1}{2}$), then it no longer matters: in equilibrium, both politicians will choose the same issue, and thus this possibility will not realize. This raises the question: are the politicians always more likely to talk about the same issue than about different issues? The answer is surprising: for $\mu \in \left( \frac{1}{2}, 1 \right)$, the exact opposite is true: the incumbent is more likely to choose foreign policy if the challenger chose economy. More precisely, we have the following result.

**Proposition 2** If $\mu < \frac{1}{2}$, both politicians will always choose to campaign on the same issue. However, if $\mu > \frac{1}{2}$, then the probability of having politicians talk about different issues is higher than $\frac{1}{2}$: it increases in $\mu$ on $\left( \frac{1}{2}, \frac{3}{4} \right)$ and decreases on $\left( \frac{3}{4}, 1 \right)$. Nevertheless, the probability that politicians will fail to make their announcement is strictly decreasing in $\mu$ on $\left( \frac{1}{2}, 1 \right)$.

It is true that politicians who choose the opposite issue have worse expected quality, and thus candidates have an incentive to pool with those who choose the same field. However, the result seems more intuitive if we recall that revealing one’s competence on one dimension but not the other punishes politicians who are equally competent on both issues disproportionately harshly, and these types are key to determining whether a majority of types chooses the same or different issues. In particular, assume, for example, that the type $\left( \frac{1}{2}, \frac{1}{2} \right)$ is indifferent, which would be true if the two shares were exactly equal. If reveals his competence in either issue, the voters’ posterior would be $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. However, it is easy to see that the average competence of agents choosing the opposite issue is (weakly) larger than this, and therefore the type $\left( \frac{1}{2}, \frac{1}{2} \right)$
would choose the opposite issue, too. The difference reaches its maximum at $\mu = \frac{4}{5}$, where campaigns on different issues are almost eight percentage points more likely than campaigns about the same issue.

Another natural question is whether the timing of the game gives an advantage to the incumbent or to the challenger. At first glance, the challenger gets to pick the issue that he prefers, and thus can guarantee that he will talk about the issue that he excels at. However, the offers are aware of this incentive and behavior, and discount the challenger’s competence on the other issue accordingly. The incumbent, on the other hand, does not have such flexibility, and if $\mu < \frac{1}{2}$ he is forced to choose the same issue as the challenger, which might not be his strong side. However, voters understand this, and do not infer the incumbent’s competence in the other dimension from his announcement, and thus incumbents who are incompetent in the issue that the challenger picked are not penalized further. It turns out that in the probabilistic voting model, these effects cancel each other out; the expected probability of winning only depends on the expected competence, which is equal in our case. In other words, before knowing his type, a politician is indifferent between being a first-mover and a second-mover in the game.

**Proposition 3** *The probability of incumbent winning and the challenger winning are equal.*

At the same time, a given type of politician need not be indifferent. For example, if $\mu < 1$, then all politicians who are equally competent on both dimensions, with $e = f < 1$, would prefer to be second-movers rather than first-movers. In Section 5, the incumbent will be given an option to choose an issue before the arrival of a challenger. It turns out that while not all incumbents will use this option, those who do are likely to be competent, and this in the end of the day will result in observable first-mover (incumbent’s) advantage.\(^{10}\)

4 **Social Welfare**

What are the consequences of strategic issue selection on social welfare? In this model, the relevant variable is the expected competence of the elected politician. The following lemma shows that in the probabilistic voting model as above, this could be computed with a simple formula that we employ throughout.

\(^{10}\)In Ashworth and Bueno de Mesquita (2008), incumbency advantage arises in a probabilistic voting model due to his ex-ante higher competence (which in turn is present because he had won elections before). Since this paper assumes, for simplicity, that the candidates are ex-ante symmetric, this effect is not present.
Lemma 1 The expected quality of elected politician equals

\[ 1 + A \left( \int_{(e_c, f_c, e_i, f_i) \in \Omega_c \times \Omega_i} \mathbb{E} \left( (U(e_c, f_c))^2 + (U(e_i, f_i))^2 \mid \mathcal{I} \right) d\lambda - 2 \right), \tag{7} \]

where \( \lambda \) is the uniform measure on \( \Omega_c \times \Omega_i \).

Proof. The expected competence of the elected politician equals

\[ \int_{\Omega} \left( \frac{1}{2} + A \mathbb{E} (U(e_c, f_c) - U(e_i, f_i) \mid \mathcal{I}) \right) d\lambda \]

\[ = 1 + A \int_{\Omega} \mathbb{E} \left( (U(e_c, f_c) - U(e_i, f_i))^2 \mid \mathcal{I} \right) d\lambda. \]

The result would follow if the expected utilities inside the integral were independent. They are not; a high realization of \( \mathbb{E} (U(e_c, f_c) \mid \mathcal{I}) \) means that most likely the politicians are talking about the same issue, and therefore the incumbent is likely to have high average competence. However, conditional on the choices of issues by both politicians, these variables are independent. In addition, the incumbent’s choice of issue does not depend on the challenger’s competence. This implies that social welfare may be computed using (7). Appendix fills in the details. ■

Lemma 1 simplifies the computations considerably. In particular, it shows that the expected competence of the elected politician only depends on the sum of variances of posterior beliefs that politicians’ equilibrium play generates. This allows to do the computations for the incumbent and the challenger separately, only taking into account the equilibrium strategies as described in Proposition 1. The following benchmark is useful: if both candidates completely revealed their competences to voters, then the expected quality of the winner would be

\[ 1 + A \left( 2 \int_0^1 \int_0^1 (x + y)^2 dx dy - 2 \right) = 1 + \frac{1}{3} A. \tag{8} \]

Here, \( \frac{1}{3} A \) is the extra benefit of having elections as opposed to picking the candidate randomly (which would yield expected competence 1). This is obviously the maximum one can achieve with probabilistic voting where a less competent candidate has a chance due to common shock. As the variance of common shock decreases (\( A \) becomes higher), the expected competence of the elected politician would increase.

Proposition 4 The expected competence of the elected politician \( C(A, \mu) \) is increasing in \( A \). It is nonmonotone in \( \mu \); more precisely, it equals \( 1 + \frac{5}{24} A \) if \( \mu \leq \frac{1}{2} \), and for \( \mu \geq \frac{1}{2} \), it monotonically increases from \( 1 + \frac{3}{16} A < 1 + \frac{5}{24} A \) to \( 1 + \frac{1}{4} A > 1 + \frac{5}{24} A \).
The nonmonotonicity is not surprising if one takes into account that for \( \mu < \frac{1}{2} \), the two politicians are guaranteed to discuss the same issue, whereas for \( \mu \) slightly exceeding \( \frac{1}{2} \) they will talk about different things half of the time, and thus there is a chance of one-fourth that they will fail to announce their respective competences. In fact, the expected competence exceeds \( 1 + \frac{5}{24} A \) only if \( \mu > 0.7 \). In all cases, this falls short of the maximal possible gain of \( \frac{1}{3} A \), although if \( \mu \) is close to 1, then 75\% of this gain is realized, and even in the worst-case scenario this chance exceeds 56\% \( (\frac{9}{16}) \).

Consider, however, a different benchmark. What would happen if the candidates were forced to compete on an exogenously chosen dimension, so as to avoid the chance that voters will not get their announcements? In this case, the expected quality of the winner would equal

\[
1 + A \left( 2 \int_0^1 \int_0^1 \left( x + \frac{1}{2} \right)^2 \, dx \, dy - 2 \right) = 1 + \frac{1}{6} A. \tag{9}
\]

Notice that for all values of \( \mu \), the expected competence in the case where politicians are free to choose their issue is higher than this value. This seems surprising, because in this case, there is no loss due to possible failure to announce their competences on any dimension. However, there is a different force: with endogenous choice of issues, the announcement of a politician of his competence over one dimension carries quite a bit of information about his competence on the other issue, which is not the case if they were forced to talk on a given issue (this is not true for the incumbent if \( \mu < \frac{1}{2} \), but for challenger this is true for any \( \mu \)). It turns out that the latter effect dominates.

**Proposition 5** If politicians were forced to campaign on the same dimension, social utility would be lower regardless of the value of \( \mu \).

It should be emphasized that this result is due to the ability of politicians to coordinate in equilibrium; if it were not there, the results would be different. Indeed, suppose that politicians announce their positions simultaneously. One can check that in this case, if \( \mu > \frac{1}{2} \) then there is a unique equilibrium, where both politicians choose the issue they are more competent in. If \( \mu \leq \frac{1}{2} \), then there are three equilibrium: one in which both candidates choose economy \( E \), the other where both choose foreign policy \( F \), and a symmetric equilibrium where they choose the issue they excel in. We get the following comparison in this case.

**Proposition 6** For every \( \mu \), it is always an equilibrium for the challenger (incumbent) to choose \( E \) if and only if \( e_c > f_c \) \( (e_i > f_i) \). In this equilibrium, the expected quality of the elected politician equals \( 1 + \frac{1+\mu}{8} A \); this is lower than the expected quality with an exogenous issue whenever \( \mu < \frac{1}{3} \).
If $\mu \geq \frac{1}{2}$, there are also equilibria where both politicians choose the same issue; the expected competence of politicians is the same as in the case with exogenous issue.

Figure 2 illustrates the expected qualities of elected politicians under different scenarios. Proposition 6, in particular, suggests that campaigning on different issues may decrease social welfare, if this prevents voters from getting precise information about candidates’ policies.\(^\text{11}\) This will not happen if politicians choose issues in a well-established sequence, or at least if the one who moves second has a chance to observe the choice of the first politician and to respond. This leads to the following nontrivial implication. When candidates choose an agenda for an entire campaign, the decisions are unlikely to be made at once, and hence constraining the candidates with an exogenously given agenda is not a good idea. However, when it comes to some particular event, such as debates, where candidates are likely to prepare their strategy without observing the opponent’s plan, fixing a particular issue or set of related issues may make sense.

5 Dynamics of Campaign

So far, the challenger was assumed to be the first mover and the incumbent to be the second mover. Let us augment this game by assuming that before the challenger gets a chance to make his move, the incumbent may take an early move and commit to a particular issue for the future.

\(^\text{11}\)Caselli and Morelli (2004) and Mattozzi and Merlo (2007) consider very different models that lead to selection of incompetent politicians. In this paper, incompetent politicians may get elected because voters do not necessarily make a strong inference about a politician’s incompetence in the issue he is not campaigning on.
campaign. Suppose, furthermore, that the probability that the incumbent gets this chance is \( \lambda \in [0, 1] \), which is known to the voters, but they do not know whether the incumbent really got this chance, or had a chance but decided to pass. The question we address here is whether and which incumbents will decide to move first.

The equilibrium is easy to characterize in the extreme situations, with \( \lambda \) equal (or close) to 0 or 1.\(^{12}\) Suppose first that \( \lambda \) is close to 0; this means that when voters observe the incumbent making the second move, they do not update positively or negatively, as most likely he just did not have a chance to act. Then, if the incumbent gets a chance to move first, he will effectively compare the costs and benefits of being the first-mover and the second-mover. The following result shows that, incumbents who are very asymmetric will decide to act first, and those who are equally competent in both issues will pass.

**Proposition 7** Suppose that \( \lambda = 0 \), and also \( \mu < \frac{1}{2} \), so politicians are guaranteed to campaign on the same issue in equilibrium. In this case:

(i) the incumbent will move first if

\[
\left| \frac{1 - e_i}{1 - f_i} \right| > \frac{1}{2}, \tag{10}
\]

and will pass otherwise. If he moves first, he will choose the issue he is more competent in;

(ii) conditional on moving first, the chance that the incumbent wins is \( \frac{1}{2} + \frac{1}{6} A > \frac{1}{2} \);

(iii) the expected competence of the elected politician is higher if the incumbent chose to move first than if he chose to move second or did not have a chance to act.

Proposition 7 is intuitive, and the only role of the assumption \( \mu < \frac{1}{2} \) is to make the condition (10) so simple to analyze. The most competent incumbent, \((e_i, f_i) = (1, 1)\), will be indifferent between acting first and passing. However, if he is competent in one dimension but incompetent in the other, he will find waiting too risky, and thus candidates close to \((1, 0)\) and \((0, 1)\) will move first. On the other hand, candidates with \( e_i = f_i < 1 \) will strictly prefer to wait: they know that if they move first, their competence in the dimension they choose not to campaign on will be heavily discounted, and this is less of a problem if they are moving second. Indeed, in the latter case, the voters will think that they were forced to campaign on a dimension chosen by the challenger, and as such the penalty would not be so high. It is also intuitive that least competent

---

\(^{12}\)For Bayesian voters who observe that the incumbent did not make the first move, information on whether the incumbent did not have a chance to move first or had this chance but decided not to is payoff-relevant information, because this tells them something about his competence on the other issue. However, the probability that he had this chance will depend on the competence over the issue he campaigns on. This complicates the problem considerably; in particular, the boundaries of incumbent types choosing whether to move first or to pass, and later whether to campaign on economy or foreign policy need not be linear.
politicians (in particular, those with $e_i, f_i < \frac{1}{2}$) prefer to pass and become second-movers: for them, moving first and campaigning on either issue is going to release a (justified) negative signal about their competence in both dimensions; at the same time, if they move second, this would only be true for one dimension. These strategies are illustrated on Figure 3.

According to Proposition 7, if the incumbent gets a chance to move first, it is equally likely that he will use this chance and that he will not. However, there is another difference in the types of incumbents who choose either strategy. Not only the ones who act first are more likely to be competent in one dimension and challenged in the other, but they are more likely to be more competent overall. Indeed, suppose, for example, that such an incumbent has $e_i > f_i$; then simple calculation shows that his expected competence in economy $E$ is $\frac{5}{6}$, and his expected competence in foreign policy $F$ is $\frac{1}{3}$, which makes his overall expected competence $\frac{7}{6}$.

Consequently, incumbents who move first are also more competent than challengers, in expectation. This insight leads to the other two results in the proposition; first, there is first-mover advantage, in the sense that an incumbent who gets a chance to move first is more likely to win, and second, when campaigns choose their issues earlier, the expected competence of the elected politician is higher. The latter effect arises in the equilibrium because more competent politicians are more likely to move first.\footnote{Another effect, which is not modeled explicitly, is that deciding on an issue earlier gives politicians more time, and allows voters to get more precise signals about politicians’ competences, which again raises the expected competence of the winner.}

Suppose, however, that the incumbent is actually likely to be able to move first, and hence
voters who see an incumbent moving after the challenger decide that he decided to pass. As we saw, this tells them something negative about the overall competence of the incumbent. However, for incumbents close to the boundary (i.e., those for whom \( \frac{1 - \epsilon_i}{1 - f_i} \) is close to \( \frac{1}{2} \) or 2), the calculation looks differently. Moving first allows them to reveal their competence on the issue they are good at, but the cost is that they will be thought of as very incompetent on the other issue. On the other hand, if they wait, voters will think that their skills on the two issues are relatively close, and therefore announcing their competence on one issue will not hurt them on the other dimension, or will hurt very little. Hence incumbents close to the boundary have an incentive to pass, and this only increases the penalty for moving early. When \( \lambda = 1 \), this leads to full unraveling: no incumbent will move first (although for some values of \( \mu \), the extreme types \((1, 0)\) and \((0, 1)\) will be indifferent.

**Proposition 8** If \( \lambda = 1 \), then the unique equilibrium requires the incumbent to postpone choosing the campaign issue until a challenger emerges and makes the choice. However, for \( \lambda < 1 \), a positive mass of incumbents will act first.

This result suggests that when voters know that a politician has a chance to move first, the best response for the politician is to wait. In this model, this means that strategies on choosing issues do not change, and neither does social welfare. More broadly, this result implies that while making a first announcement is an attractive option for at least some politicians, not using this option is even more attractive, as it serves as a positive signal of possessing balanced competence in the two issues, which is valuable in the environment where campaigns may be run on one issue only. As a result, the opportunity to make an early commitment to the issue of the campaign will not be used. This insight may be extended further, to a game where both politicians get, e.g., alternating opportunities to move first; backward induction will immediately suggest that both politicians will wait until the very last opportunity, and on the equilibrium path, waiting will be something expected rather than a positive signal. (Interestingly, off equilibrium path, once one politician makes the choice, the other one has no incentive to wait further.)

The insights obtained in the cases of very low or very high \( \lambda \) suggest the following implications. First, politicians are not likely to seize the very first opportunity to pick an issue, and the reason is not the aggregate uncertainty (i.e., they might want to learn, which issues voters find most important), but rather signaling considerations. Second, politicians will use opportunities which present a good reason not to wait. For example, if some event or story makes it impossible or very hard for a politician not to react, the politician might well make the first move (e.g., a stock market crash creates a good reason to start campaigning on economy). A politician may also want to use an opportunity which would otherwise go unnoticed (and therefore he would
not get enough credit for waiting). If none of these event types get realized, politicians are likely to wait until the last moment, when the remaining time becomes a binding constraint.

6 Extensions

In this section, we consider several extensions of the baseline model of Section 2.

6.1 Asymmetric Uncertainty

The baseline model assumed that the ex-ante distributions of the incumbent and challenger’s abilities are the same. This was obviously a simplification; first, the voters are likely to be better informed about the incumbent’s type, and also, the incumbent is more likely to be more competent on the grounds that he was selected into the office earlier. So suppose that the incumbent’s type is taken not from a uniform distribution on \([0, 1] \times [0, 1]\), but instead from a uniform distribution on \([e_1, e_2] \times [f_1, f_2]\); in particular, \(e_1 = e_2\) would imply that the voters are informed about the incumbent’s ability on economy, and \(f_1 = f_2\) would imply the same on foreign policy.

The equilibrium strategies are easy to characterize if \(\mu = 1\). In this case, denote \(\tilde{e}_i = e_i - e_1\) and \(\tilde{f}_i = f_i - f_1\); then the characterization of incumbent’s strategies in Proposition 1 applies, after replacing \(e_i\) and \(f_i\) with \(\tilde{e}_i\) and \(\tilde{f}_i\), respectively. In other words, strategy \(d_i\), only depend on the incumbent’s true type to the “floors” on both dimensions. This is intuitive: we saw that voters, in equilibrium, update negatively on the politician’s competence in the dimension he did not choose. If \(\mu \in \left(\frac{1}{2}, 1\right)\), then there is a chance that the incumbent will not be able to announce his type even on the dimension that he chooses. This is negative news for the incumbent, and any given incumbent is likely to switch to the same issue as the challenger, but not vice versa. The challenger’s strategies in this case are more complicated: he will have to take into account the effect that his choice will have on the likelihood of talking on the same issue (before, the situation was symmetric).

The results in the case \(\mu < \frac{1}{2}\), and the equilibrium has to be pooling, are somewhat different. In particular, it is no longer the case that the incumbent types must pool at the same issue as the challenger. Indeed, if, for example, \(e_2 - e_1\) is high, but \(f_2 - f_1\) is low (i.e., voters know well the incumbent’s competence on foreign policy, but less so on economy), then the incumbent will have an incentive to campaign on economy even if the challenger chose foreign policy. In such case, however, the challenger will make his initial decision effectively knowing the issue that the incumbent will campaign on. As a result, given that \(\mu < \frac{1}{2}\), he will also have to campaign on the issue where the incumbent’s qualities are less certain.
In short, when voters have more precise information about the incumbent’s ability on one dimension but not the other, the incumbent has incentives to campaign on the issue where there is more uncertainty. As a result, the challenger will also have incentives to campaign on that issue, especially if $\mu < \frac{1}{2}$. If so, the social welfare in this case will be the same as in the case where candidates had to campaign on a single issue, and the information that self-selection may carry (especially about the challenger) will be lost. This of course raises the question of whether or not learning about the incumbent during his tenure (but before a campaign) actually increases the informativeness of voters, and I leave this question for future research.

### 6.2 Asymmetric Issues

We have so far assumed that voters care about both issues equally. However, more generally, the weights voters put on economy and foreign policy may be different, i.e., $w_e \neq w_f$ is possible.

It is not hard to see, however, that the case of different weights is similar to the previous one in Subsection 6.1, with issue that has a higher weight corresponding to an issue where there is more uncertainty about both politicians types. (The difference is that both the incumbent and the challenger would have the same ex-ante distribution, with higher variance on one dimension.) The easiest way to see this is to renormalize the politicians’ types to the scale that voters use to evaluate them. Namely, one can define $\tilde{e}_c = w_e e_c$, $\tilde{f}_c = w_f f_c$, $\tilde{e}_i = w_e e_i$, and $\tilde{f}_i = w_f f_i$ (so that both $\tilde{a}_c$ and $\tilde{a}_i$ are distributed uniformly on $[0, w_e] \times [0, w_f]$), and from then on assume that the weights are equal: $\tilde{w}_e = \tilde{w}_f = 1$.

It will follow immediately that both the challenger and the incumbent are more likely to campaign on the issue that voters care about more. Indeed, the effective uncertainty about the politicians’ types would be greater over that dimension rather than over the issue that voters care little about. This is realistic: there is little reason for politicians to campaign on the issue that does not interest voters, and even more, doing so would be interpreted negatively as lack of competence (or occupying an extreme position) on the issue that concerns voters more. If the issues in the model were ones where voters had disagreement about, then campaigning on more divisive issues would sway fewer voters; and so the model predicts that campaigns would focus on candidates’ qualities on issues where preferences are the same, rather than on the issues where they are different.\footnote{The difference with Morelli and van Weelden (2011) is that there, taking positions on divisive issues serves as signaling, while the issues where voters’ preferences are similar does not have the competence component that voters care about a lot.}

One less trivial insight is the impact of different weight that voters put on issues on social welfare. In addition to the trade-off between losing information on the other important dimension
and the chance of learning nothing at all, a new problem is that politicians will end up talking about the less important issue. It turns out that this is not a concern: in equilibrium, politicians are more likely to choose the issue that voters care about more, and this still makes it optimal to give politicians the freedom to choose the issue for their campaign rather than force them to focus on a single (even ex-ante more important) issue. This result, of course, need not hold if politicians made choices simultaneously.

7 Conclusion

The paper studies the incentives of politicians to choose an issue to run their campaign on. Voters are rational, and the candidates cannot change their preferences, but can affect the information they possess. I assume that both candidates are more credible when they run on the same issue, and this creates a non-trivial interplay between their incentives. The first mover (the challenger) has a disproportionate influence on the course of the campaign, but this does not necessarily help him win. Whenever a politician gets a chance to postpone the announcement of his issue, he will, because this will signal his competence on the issues he will not focus his campaign on. The politicians who nevertheless choose to act first are more likely to be competent, and more likely to win. The model predicts that allowing politicians a free choice of campaign issues reveals more information in the course of campaign, and ultimately raises the chance of electing the most competent candidate; at the same time, if candidates make choices simultaneously, without a chance to effectively coordinate, the voters’ utility may fall. The model offers non-trivial insights on the nature of issue selection in campaigns, and studying dynamics of political campaigns as sequential issue selection seems to be an interesting avenue for future research.

References


Appendix: Proofs

Proof of Proposition 1. The statement about challenger’s strategy is relatively trivial. Indeed, if some type \((e_c, f_c)\) with, say, \(e_c > f_c\) is indifferent, then choosing \(d_c = E\) yields \(e_c + \frac{f_c}{2}\), and choosing \(d_c = F\) yields \(f_c + \frac{e_c}{2}\). The former is larger, which implies that \(d_c = E\) is a single best response, yielding a contradiction.

The case of incumbent is more complicated. First, consider the possibility of complete unraveling, i.e., situations where all incumbents choose \(E\) or all choose \(F\). As argued in the text, this may only constitute an equilibrium if \(\mu \leq \frac{1}{2}\). At the same time, for such values, choosing the same issue as the challenger is an equilibrium for properly chosen out-of-equilibrium beliefs. Moreover, if \(\mu = 0\), then choosing the opposite issue may also take place in a Perfect Bayesian equilibrium. However, since this only happens for one value of \(\mu\), the approachability requirement rules out such equilibrium.

Now suppose that both issues are chosen with a positive probability. It is straightforward to prove that the set of indifferent agents lie on the same line. The remainder of the proof consists of considering different possible positions of this line.

Consider the possibility that types \((a, 0)\) and \((1, b)\) are indifferent. Then the expected quality of types choosing \(F\) is \(\frac{b^2 + ba - 2b + 3}{3ab - 3b + 6} + \frac{ab^2 - b^2 + 3}{3ab - 3b + 6}\). The conditions that types \((a, 0)\) and \((1, b)\) are indifferent are then

\[
\begin{align*}
a &= \mu \frac{a}{2} + (1 - \mu) \frac{a^2 b + a b^2 + a b - b^2 - 2b + 6}{3ab - 3b + 6}, \\
1 + \frac{b}{2} &= \mu \left(b + \frac{1}{2}\right) + (1 - \mu) \frac{a^2 b + a b^2 + a b - b^2 - 2b + 6}{3ab - 3b + 6},
\end{align*}
\]

which does not have a solution except for \(a = 0, b = 1\) when \(\mu = 1\).

Now consider the possibility that types \((0, a)\) and \((b, 1)\) are indifferent. Now the expected quality of types choosing \(F\) is \(\frac{b}{3} + \left(a + \frac{2}{3}(1 - a)\right)\). The conditions that types \((0, a)\) and \((b, 1)\)
are indifferent are then

\[
\begin{align*}
\frac{a}{2} &= \mu a + (1 - \mu) \left( \frac{b}{3} + a + \frac{2}{3} (1 - a) \right), \\
b + \frac{1}{2} &= \mu \left( 1 + \frac{b}{2} \right) + (1 - \mu) \left( \frac{b}{3} + a + \frac{2}{3} (1 - a) \right),
\end{align*}
\]

which again does not have a solution except for \( a = 0, b = 1 \) when \( \mu = 1 \).

The third possibility is that types types \((0, a)\) and \((1, b)\) are indifferent, and \(a < b\). Then the expected quality of types choosing \(F\) is \(\frac{a + 2b - 3}{3(a + b - 2)} + \frac{a^2 + ab + b^2 - 3}{3(a + b - 2)}\). The conditions that types \((0, a)\) and \((1, b)\) are indifferent are then

\[
\begin{align*}
\frac{a}{2} &= \mu a + (1 - \mu) \left( \frac{a^2 + ab + a + b^2 + 2b - 6}{3(a + b - 2)} \right), \\
1 + \frac{b}{2} &= \mu \left( 1 + \frac{b}{2} \right) + (1 - \mu) \left( \frac{a^2 + ab + a + b^2 + 2b - 6}{3(a + b - 2)} \right),
\end{align*}
\]

and this again only has solution \( a = 0, b = 1 \) when \( \mu = 1 \).

The final non-trivial possibility is that types \((a, 0)\) and \((b, 1)\) are indifferent, and \(a < b\). Then the expected quality of types choosing \(F\) is \(\frac{a^2 + ab + b^2}{3(a + b)} + \frac{a + 2b}{3(a + b)}\). The conditions that types \((a, 0)\) and \((b, 1)\) are indifferent are indifferent are then

\[
\begin{align*}
a &= \mu a + (1 - \mu) \frac{a^2 + ab + a + b^2 + 2b}{3(a + b)}, \\
b + \frac{1}{2} &= \mu \left( 1 + \frac{b}{2} \right) + (1 - \mu) \frac{a^2 + ab + a + b^2 + 2b}{3(a + b)}.
\end{align*}
\]

For \( \mu > \frac{1}{2} \), this has solution \( a = \frac{4 - 5\mu + \sqrt{\mu(8 - 7\mu)}}{8 - 4\mu}, b = \frac{3\mu + \sqrt{\mu(8 - 7\mu)}}{8 - 4\mu} \), which delivers part (ii).

One last possibility to consider is the case of a vertical indiffERENCE line. It is straightforward to check that it may occur in an equilibrium only if \( \mu = \frac{1}{2} \), and the exact strategies are as required. This completes the proof.

**Proof of Proposition 2.** The probability that the incumbent choosing issue \(F\) when the challenger chose \(E\) (or vice versa) equals 0 if \( \mu < \frac{1}{2} \) and

\[
\frac{1}{2} \left( \frac{4 - 5\mu + \sqrt{\mu(8 - 7\mu)}}{8 - 4\mu} + \frac{3\mu + \sqrt{\mu(8 - 7\mu)}}{8 - 4\mu} \right) = \frac{2 - \mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)}
\]

if \( \mu > \frac{1}{2} \). Differentiating this with respect to \( \mu \) yields

\[
\frac{d}{d\mu} \left( \frac{2 - \mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)} \right) = \frac{4 - 5\mu}{2(2 - \mu)^2 \sqrt{\mu(8 - 7\mu)}}.
\]
This is increasing for \( \mu < \frac{4}{5} \) and decreasing for \( \mu > \frac{4}{5} \), and the maximal value equals \( \frac{1}{4} + \frac{\sqrt{3}}{6} \approx 0.539 \). This completes the proof. ■

**Proof of Proposition 3.** The expected probability of challenger winning is obtained by taking expectation of (6) over all possible realizations of \((e_c, f_c), (e_i, f_i)\), as well as \(\kappa_c\) and \(\kappa_i\). The law of iterated expectations implies that this equals \( \frac{1}{2} \), as thus the expected probability of incumbent winning also equals \( \frac{1}{2} \). This completes the proof. ■

**Proof of Lemma 1.** The expected competence of the elected politician: where we used 
\[
\mathbb{E}(U(e_c, f_c) | \mathcal{I}_c, \mathcal{I}_i) = \mathbb{E}(U(e_c, f_c)) = 1 \quad \text{(and similarly for } i). 
\]

Denote \( \Omega = \Omega_c \times \Omega_i \). Let us show that 
\[
\int_{\Omega} \mathbb{E}((U(e_c, f_c) U(e_i, f_i)) | \mathcal{I}) d\lambda = 1. 
\]

We split the entire space of types into four regions, \((d_c, d_i) \in \{(E, E), (E, F), (F, E), (F, F)\}\) according to the dimension chosen in equilibrium (note that these need not be independent, as the second player’s choice depends on the first player’s one). We have 
\[
\int_{\Omega} \mathbb{E}((U(e_c, f_c) U(e_i, f_i)) | \mathcal{I}) d\lambda
\]

\[
= \sum_{(d_c, d_i) \in S} \mu_{(d_c, d_i)} \int_{\Omega_{(d_c, d_i)}} \mathbb{E}(U(e_c, f_c) U(e_i, f_i)) | \Omega_{(d_c, d_i)}, a_c | d_c, a_i | d_i) d\lambda
\]

\[
= \sum_{(d_c, d_i) \in S} \mu_{(d_c, d_i)} \int_{\Omega_{(d_c, d_i)}} \mathbb{E}(U(e_c, f_c) | \Omega_{(d_c, d_i)}, a_c | d_c) \mathbb{E}(U(e_i, f_i) | \Omega_{(d_c, d_i)}, a_i | d_i) d\lambda
\]

\[
= \sum_{d_c \in \{E, F\} \Omega_{d_c}} \sum_{d_i \in \{E, F\}} \mu_{(d_c, d_i)} \int_{\Omega_{d_i}(d_c)} \mathbb{E}(U(e_c, f_c) | \Omega_{d_c}, a_c | d_c) \mathbb{E}(U(e_i, f_i) | \Omega_{d_c}, a_i | d_i) d\lambda_{d_c} d\lambda_{d_i}
\]

\[
= \sum_{d_c \in \{E, F\} \Omega_{d_c}} \int_{\Omega_{d_c}} \mathbb{E}(U(e_c, f_c) | \Omega_{d_c}, a_c | d_c) \sum_{d_i \in \{E, F\}} \lambda_{d_i}(d_c) \mathbb{E}(U(e_i, f_i) | \Omega_{d_c}, a_i | d_i) d\lambda_{d_c}
\]

\[
= \sum_{d_c \in \{E, F\} \Omega_{d_c}} \mathbb{E}(U(e_c, f_c) | \Omega_{d_c}, a_c | d_c) \lambda_{d_c} \mathbb{E}(U(e_i, f_i) | \Omega_{d_c}) d\lambda_{d_c}
\]

\[
= 1 \sum_{d_c \in \{E, F\} \Omega_{d_c}} \mathbb{E}(U(e_c, f_c) | \Omega_{d_c}, a_c | d_c) d\lambda_{d_c} = 1 \cdot 1 = 1 - 1.
\]

(where \(a_c | d_c\) is the projection of type \(a_c\) on the dimension \(d_c\)). We used that \(\mathbb{E}(U(e_i, f_i) | \Omega_{d_c})\) does not in fact depend on \(d_c\) and equals 1. Consequently,

\[
1 + A \int_{\Omega} \mathbb{E}((U(e_c, f_c) - U(e_i, f_i))^2 | \mathcal{I}) d\mu = 1 + A \left( \int_{\Omega} \mathbb{E}((U(e_c, f_c))^2) + (U(e_i, f_i))^2 | \mathcal{I}) d\mu - 2 \right)
\]

23
Within each region, \( E(U(e_c, f_c) \mid I) \) and \( E(U(e_i, f_i) \mid I) \) are independent, and thus
\[
E((U(e_c, f_c)U(e_i, f_i)) \mid I) = E(U(e_c, f_c) \mid I)E(U(e_i, f_i) \mid I).
\]
This completes the proof. ■

**Proof of Proposition 4.** First, suppose \( \mu \leq \frac{1}{2} \). Without loss of generality, assume that the challenger has \( e_c > f_c \) and thus chooses \( E \), and the incumbent does the same. Computing the relevant integrals in the right-hand side of (7), we obtain
\[
\int_0^1 \left(\frac{3}{2} x\right)^2 2xdx + \int_0^1 \left( y + \frac{1}{2}\right)^2 dy = \frac{53}{24} = 2 + \frac{5}{24}.
\]

Now suppose that \( \mu \geq \frac{1}{2} \). We first calculate the integral for the challenger (again, assuming that \( e_c > f_c \) as the opposite case may be considered similarly). With probability \( \frac{1 - 2 - \mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)} \), the incumbent chooses \( E \), too. This gives \( \int_0^1 \left(\frac{3}{2} x\right)^2 2xdx = \frac{9}{8} \). With probability \( \frac{2 - \mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)} \), the incumbent chooses \( F \), and the integral equals \( \mu \cdot \frac{9}{8} + (1 - \mu) \cdot 1 \). Totally, the contribution of the challenger is
\[
\frac{9}{8} \left(1 - \frac{2 - \mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)}\right) + \frac{2 - \mu + \sqrt{\mu(8 - 7\mu)}}{4(2 - \mu)} \left(\frac{9}{8} + (1 - \mu) \cdot 1\right) = \left(\frac{9}{8} - \frac{1 - \mu}{32(2 - \mu)} \right) \left(2 - \mu + \sqrt{\mu(8 - 7\mu)}\right).
\]

Now, consider the contribution of the incumbent. The part of the integral coming from the set where he chooses \( E \) is (as before, \( a = \frac{4 - 5\mu + \sqrt{\mu(8 - 7\mu)}}{8 - 4\mu} \), \( b = \frac{3\mu + \sqrt{\mu(8 - 7\mu)}}{8 - 4\mu} \)):
\[
\int_a^b \left( x + \frac{1}{2} \frac{x - a}{b - a} \right)^2 \frac{x - a}{b - a} dx + \int_b^1 \left( x + \frac{1}{2} \right)^2 dx = \frac{13}{12} \frac{3a + 9b + 4ab^2 + 4a^2b + 8ab + 4a^2 + 4a^3 + 12b^2 + 4b^3}{48}.
\]
The part of the integral coming from the set where he chooses \( F \) may be found as follows: with probability \( \mu \), it is
\[
\int_0^1 \left( y + \frac{1}{2} (a + (b - a) y)\right)^2 (a + (b - a) y) dy = \frac{1}{48} \left(4a + 12b + 3ab^2 + 3a^2b + 8ab + 4a^2 + 3a^3 + 12b^2 + 3b^3\right),
\]
and with probability \( 1 - \mu \), it is
\[
\left(\frac{a^2 + ab + b^2}{3(a + b)} + \frac{a + 2b}{3(a + b)}\right)^2 \frac{a + b}{2} = \frac{1}{18(a + b)} (a^2 + ab + a + b^2 + 2b)^2.
\]
This means that the total integral for the challenger and the incumbent, after substituting for the values of \( a \) and \( b \), is
\[
2 + \frac{(19\mu + 6\mu^2 - 68)(2 - \mu)^2 + (29\mu^2 - 6\mu^3 - 52\mu + 24) \sqrt{\mu(8 - 7\mu)}}{192(2 - \mu)^3}.
\]
It may be shown directly that this is an increasing function of $\mu$, and that its value for $\mu = \frac{1}{2}$ is $2 + \frac{3}{16}$ and for $\mu = 1$ is $2 + \frac{1}{4}$. This completes the proof.

**Proof of Proposition 5.** Proved in the text.

**Proof of Proposition 6.** It was proved earlier that an equilibrium where both politicians always choose $E$ or always choose $F$ is only possible if $\mu > \frac{1}{2}$. On the other hand, it is obvious that picking one’s better dimension is an equilibrium for all values of $\mu$. For this strategy, each politician is able to announce his competence credibly with probability $\frac{1}{2} + \frac{1}{2} \mu$, and he fails to do so with probability $\frac{1}{2} (1 - \mu)$. Consequently, the integral in the right-hand side of (7) equals

$$2 \left( \left( \frac{1}{2} + \frac{1}{2} \mu \right) \int_0^1 \left( \frac{3}{2} x \right)^2 2x dx + \frac{1}{2} (1 - \mu) * 1 \right) = 2 + \frac{1 + \mu}{8}.$$

Therefore, the expected competence of the elected politician is monotonically increasing, and it exceeds the competence in the case of an exogenously given issue ($2 + \frac{1}{6}$) if and only if $\mu > \frac{1}{3}$. This completes the proof.

**Proof of Proposition 7.** The set of types that decide to wait should be symmetric around the line $e_i = f_i$. In addition, if the incumbent decides to act first, he should choose $E$ if and only if $e_i > f_i$, and choose $F$ otherwise. The remainder of the proof involves considering all possible cases as in the proof of Proposition 1 and applying (7).

**Proof of Proposition 8.** The proof is straightforward and is omitted in the current version.